BSM-Decision support methods HT2020

Case-study Assignment

Group Y

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1. Proposed approach and motivation

The problem at hand is a multiple attribute decision making problem where the set of identified decision attributes are a mixture of quantitative and qualitative attributes. It is assumed that the preference of the decision-maker concerning the identified attributes, fulfills the independence conditions required for the use of the additive model.

Following the additive model, the weight corresponding to each attribute will be calculated, the value functions for each attribute will be defined and the weighted sum method will be employed to calculate each alternative's overall score.

Attributes A1, A2, A3, A4 are quantitative, these pose no additional difficulty as their value can be easily represented by real numbers in a consistent manner. On the contrary, attributes A5 and A6 are qualitative, presented in the form of linguistic terms. These linguistic terms carry an inherent degree of ambiguity and imprecision as the boundaries between them tend to be unclear and are often subjective to each individual's perception, affecting our ability to consistently compare them. Besides, attribute A6 is presented in the form of a linguistic interval adding uncertainty to the formerly mentioned ambiguity.

One way to cope with those issues, as suggested in (Venkata Rao, 2007) is to transform the linguistic variables into fuzzy numbers (Zadeh, 1965). The method of converting linguistic variables to fuzzy numbers suggested by (Chen & Hwang, 1992) will be employed here to transform attributes A5 and A6 into fuzzy numbers. This method was selected among others, as it also provides an easy to use procedure of converting these fuzzy numbers into crisp values if it is required to produce a numbers only decision matrix.

Both representations of attributes A5 and A6, fuzzy and crisp value, will be utilized. With A5 and A6 in their fuzzy format, the notion of the similarity degree between fuzzy numbers will be used to consistently compare them with the respective aspiration of the decision-maker and quantify the extent that these ratings fulfill this aspiration. This quantified aspiration fulfillment ratio will serve as their perceived value for the decision-maker. On the other hand, the crisp value representation of A5 and A6 will be employed during the calculations of the weighting method, where a numeric decision matrix is needed as input.

The selection of the weighting method is a critical decision as it can substantially affect the results. One of the constraints of the problem at hand is that no additional input from the decision-maker is available. Hence, subjective weighting methods, such as the trade-off method or the swing method (Eisenfuhr, Weber, & Langer, 2010), cannot be employed here, because the decision maker's indifference towards various alternatives is neither available nor can it be reliably deduced from the data at hand. One could assume the position of the decision-maker and argue some possible indifference pairs, nonetheless that would ultimately only add an extra layer of subjectivity to the problem as the actual decision maker's perspective may be quite different. Additionally, simply assuming equal importance between the different attributes would be difficult to decisively argue and largely considered a last resort method.

Instead, the current approach adopts an objective weighting method called the Shannon entropy weighting technique (S.E.W.T.) (Moarefi, Sweis, Hoseini-Amiri, & AlBalkhy, 2018).

The concept of Shannon entropy refers to a general measure of uncertainty embedded in a system (Shannon, 1948). In the context of a multiple attribute decision making problem, the greater the value of entropy corresponding to an attribute of the problem, the less the discriminative power of that attribute in the decision making process and hence the less its implied weight (Lotfi & Fallahnejad, 2010). Accordingly, the S.E.W.T. method defines a procedure to assign weights to attributes, based solely on the respective decision matrix, where the less the calculated entropy of an attribute, the higher the weight assigned to it. The ability to use a crisp value formulation of attributes A5 and A6 and hence produce a numbers only decision matrix for the problem will come in handy here and turn applying S.E.W.T. a pretty straight-forward procedure.

As far as the assignment of a value function to each attribute is concerned, special care is given so that all assigned value functions score in the [0,1] interval so that no additional normalization will be needed for the resulting value matrix.

The data for attributes A2 and A3 clearly define the intended value function according to the decision maker's aspiration and no additional assumptions are required. The decision-maker states that when the desired level for these attributes is breached, the loss of perceived value is linear up to a certain point and everything after that holds no perceived value. On the contrary, some assumptions will be made for the rest.

Regarding the value function of attribute A1 and A4, the decision-maker makes no mention of a linear fashion regarding the loss of value. Due to the nature of attributes A1 and A4, it can be assumed that even small deviations from the desired level set by the decision maker may have a substantial negative impact. Hence the loss of perceived value for these attributes was chosen to be non-linear and specifically of the parabola type.

Finally, as far as attributes A5 and A6 are concerned, the proposed approach will take advantage of the fuzzy format to create a well defined, objective, and easy to use value function. For any given rating X of these attributes, the degree of similarity between X and the optimal rating according to the decision-maker will be assigned as the value of X. In the case of attribute A5, the highest possible rating, H: High, will be assumed to be the optimal rating according to the decision-maker. The optimal rating for attribute A6 will be set to be the interval [DL, VL, L] following the stated decision maker aspirations.

2. Converting linguistic scale into fuzzy numbers

Following the paradigm of (Chen & Hwang, 1992), the linguistic scales of attributes A5 and A6 are turned into fuzzy numbers using triangular membership functions. The selected fuzzy numbers are presented in Table 1, while Figure 1 and Figure 2 present the new fuzzy scale of attributes A5 and A6 respectively. The corresponding membership functions are included in Appendix A.

Table 1. Conversion of the linguistic scale of attributes A5 and A6 to fuzzy numbers.

A5 linguistic scale	A5 fuzzy number	A6 linguistic scale	A6 fuzzy number
H: *High*	(0.5, 1, 1)	DL: *Definitely Low*	(0, 0, 0.167)
M: *Medium*	(0.2, 0.5, 0.8)	VL: *Very Low*	(0, 0.167, 0.333)
L*: Low*	(0, 0, 0.5)	L: *Low*	(0.167, 0.333, 0.500)
		M: *Medium*	(0.333, 0.500, 0.667)
		H: *High*	(0.500, 0.667, 0.834)
		VH: *Very High*	(0.667, 0.834, 1)
		DH: *Definitely High*	(0.834, 1, 1)

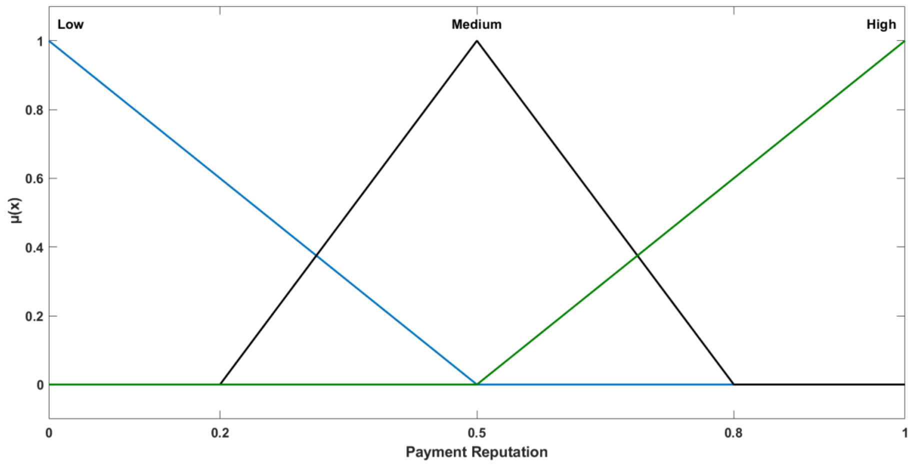

Figure 1. Attribute A5-Payment Reputation fuzzy scale.

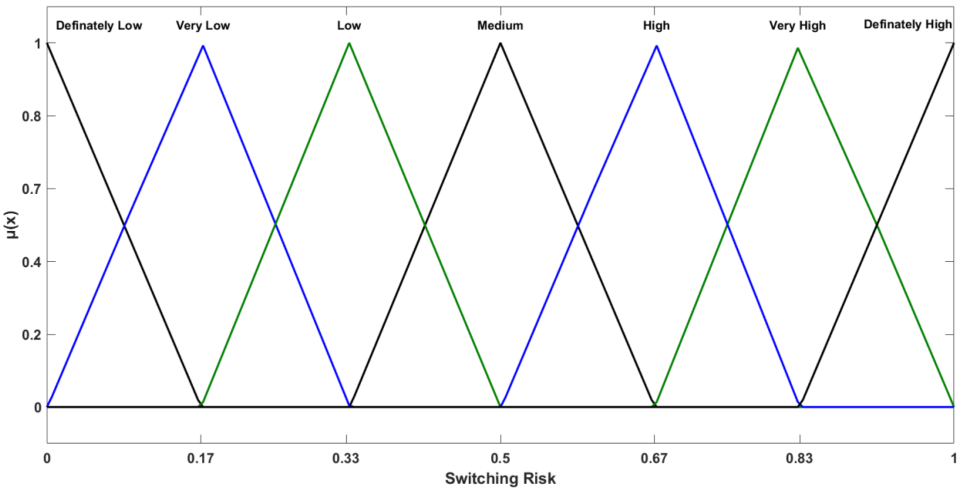

Figure 2. Attribute A6-Switching Risk fuzzy scale.

The decision matrix of the problem at hand contains all three possible ratings (H, M, L) of attribute A5 and the interval ratings [DL, VL, L], [VL, L, M] and [L, M, H] of attribute A6. Thus, these have to be converted into crisp value format to be used in the context of the S.E.W.T. (Moarefi et al., 2018) weighting method.

According to Chen and Hwang (1992), given a fuzzy number M_i with a membership function $\mu_{M_i}(x)$, its crisp value $M_T(M_i)$ can be calculated as:

$$M_T(M_i) = (M_R(M_i) + 1 - M_L(M_i))/2$$

Where:

$$M_R(M_i) = Sup_x[\,\mu_{max}(x) \cap \mu_{M_i}(x)\,]\,,\ M_L(M_i) = Sup_x[\,\mu_{min}(x) \cap \mu_{M_i}(x)\,]$$

and $\mu_{max}(x)$, $\mu_{min}(x)$ are the membership functions:

$$\mu_{max}(x) = \begin{cases} \mathrm{x}\,, 0 \le \mathrm{x} \le 1 \\ 0\,, \text{otherwise} \end{cases}\,,\ \mu_{min}(x) = \begin{cases} 1 - \mathrm{x}\,, 0 \le \mathrm{x} \le 1 \\ 0\,, \text{otherwise} \end{cases}$$

This method was used to calculate crisp values for the attribute A5, the results can be found in Table 2.

The same method was used to convert the intervals [DL, VL, L], [VL, L, M] and [L, M, H,] of attribute A6 into crisp values. Instead of individually converting each interval component, such as DL, H etc., which would lead to crisp values in interval form after the conversion, each interval was treated as a single entity. This means that for example, [DL, VL, L] was treated as being a rating of its own instead of the interval between ratings DL and L. Thus, each interval was represented by the corresponding fuzzy set of its components and handled in the same manner. The resulting crisp values can be found in Table 3.

Table 2. Crisp values of attribute A5.

A5 linguistic scale	M_R	M_L	M_T
L: *LOW*	0.3333	1	0.1666
M: *MEDIUM*	0.6154	0.6154	0.5000
H: *HIGH*	1	0.3333	0.8334

Table 3. Crisp values of attribute A6

A6 linguistic scale	M_R	M_L	M_T
[DL, VL, L]	0.4284	1	0.2142
[VL, L, M]	0.5714	0.8571	0.3571
[L, M, H]	0.7140	0.7144	0.4998

3. Calculating the attribute weights

Given a multiple attribute decision making problem that involves m alternatives and n attributes, a decision matrix $X = [\,x_{i,j}\,]_{(m\,x\,n)}$ can be constructed where the element $x_{i,j}$ corresponds to the rating of alternative i with respect to attribute j.

The Shannon entropy weighting technique S.E.W.T. (Moarefi et al., 2018), suggests that the weight w_j corresponding to attribute j can be calculated as follows:

First, the initial decision matrix X needs to be normalised. The normalised matrix $P = [\,p_{i,j}\,]_{(m\,x\,n)}$ can be calculated as:

$$p_{i,j} = \frac{x_{i,j}}{\sum_{i=1}^{m} x_{i,j}}\,, \qquad (i = 1, \dots, m)\ (j = 1, \dots, n)$$

Then, the entropy E_j corresponding to attribute j is calculated as:

$$E_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} p_{i,j}\, ln\,(p_{i,j}), \qquad (j = 1, \dots, n)$$

Setting the degree of diversification d_j corresponding to attribute j as:

$$d_j = 1 - E_j, \qquad (j = 1, \ldots, n)$$

The weight w_j of attribute j can be calculated as:

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}, \qquad (j = 1, \ldots, n)$$

The decision matrix for the current problem is presented in Table 4. The crisp value format is used for attributes A5 and A6. Since the available precision of attributes A1 to A4 is two decimals, attributes A5 and A6 were rounded to two decimals as well. The normalized decision matrix and the resulting entropy and weights for each attribute are presented in Table 5.

Table 4. Decision matrix of the problem.

Alternative	A1	A2	A3	A4	A5	A6
P1	2.35	0.20	16.78	16.56	0.83	0.36
P2	0.55	0.38	11.34	24.55	0.17	0.50
P3	2.48	0.12	24.32	16.64	0.50	0.21
P4	2.62	0.27	20.55	22.56	0.83	0.36
P5	3.36	0.66	12.67	13.65	0.50	0.21
$\sum_{i=1}^{m} x_{i.j}$	11.36	1.63	85.66	93.96	2.83	1.64

Table 5. Normalized decision matrix and resulting attribute weights.

Alternative	A1	A2	A3	A4	A5	A6
P1	0.206866	0.122699	0.195891	0.176245	0.293286	0.219512
P2	0.048415	0.233129	0.132384	0.261281	0.060071	0.304878
P3	0.218310	0.073620	0.283913	0.177097	0.176678	0.128049
P4	0.230634	0.165644	0.239902	0.240102	0.293286	0.219512
P5	0.295775	0.404908	0.147910	0.145275	0.176678	0.128049
$\sum_{i=1}^{m} p_{i.j} \ln(p_{i.j})$	-1.50341	-1.45285	-1.56964	-1.58599	-1.50094	-1.55423
E_j	0.934121	0.902707	0.975274	0.985432	0.932589	0.965697
d_j	0.065879	0.097293	0.024726	0.014568	0.067411	0.034303
w_j	0.216579	0.319853	0.081286	0.047894	0.221616	0.112772

Hence attribute A1 will have a weight of 0,216579, A2 will have 0,319853, A3 will have 0,081286, A4 will have 0,047894, A5 will have 0,221616 and A6 will have 0,112772. As mentioned in section 1., the higher the value of entropy calculated for an attribute, the lesser is the weight assigned to this particular attribute via the S.W.E.T. method.

4. Defining the value functions.

4.1 Attribute A1 –Supply quantity.

Supply quantities within the decision maker's ideal range of [2.0, 3.0] tonnes, will be awarded 1 point, the maximum possible score. As mentioned in section 1., the loss of value for supply quantities near this ideal range, specifically within (1.0, 2.0) and (3.0, 4.0), is assumed nonlinear and of the parabola type. All other possible supply quantities will receive zero points. The supply function for A1 is defined here as:

$$V_{A1}(x) = \begin{cases} (x-1)^2\,, & 1 < x < 2 \\ 1\,, & 2 \leq x \leq 3 \\ (4-x)^2\,, & 3 < x < 4 \\ 0\,, & otherwise \end{cases}$$

A graphical representation of this value function is given in Figure 3 below.

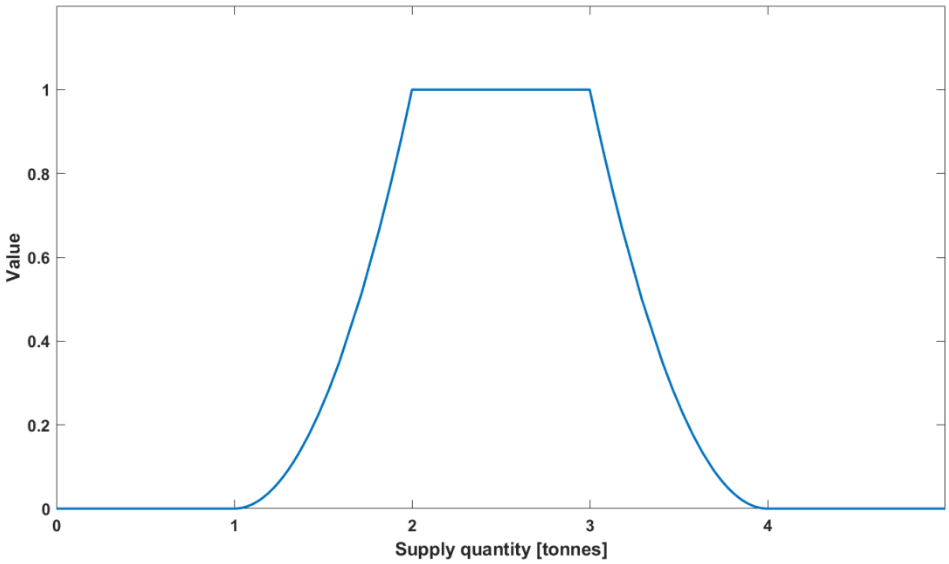

Figure 3. Value function for A1-Supply quantity.

4.2 Attribute A2 –Supply stability.

Similarly to A1, supply stabilities within the decision maker's ideal range of [0, 0.3] tonnes, will be awarded 1 point, the maximum possible score. It is known that the management cost linearly increases for supply stabilities in the interval (0.3, 0.6], hence the value function will linearly decrease in this interval. The management cost is unfavorable for supply stabilities above 0.6 tonnes, thus the value function will award anything above 0.6 tonnes with zero points. A graphical representation of this function is given in Figure 4. The value function for A2 is defined here as:

$$V_{A2}(x) = \begin{cases} 1\,, & 0 \leq x \leq 0.3 \\ \dfrac{0.6 - x}{0.3}\,, & 0.3 < x \leq 0.6 \\ 0\,, & otherwise \end{cases}$$

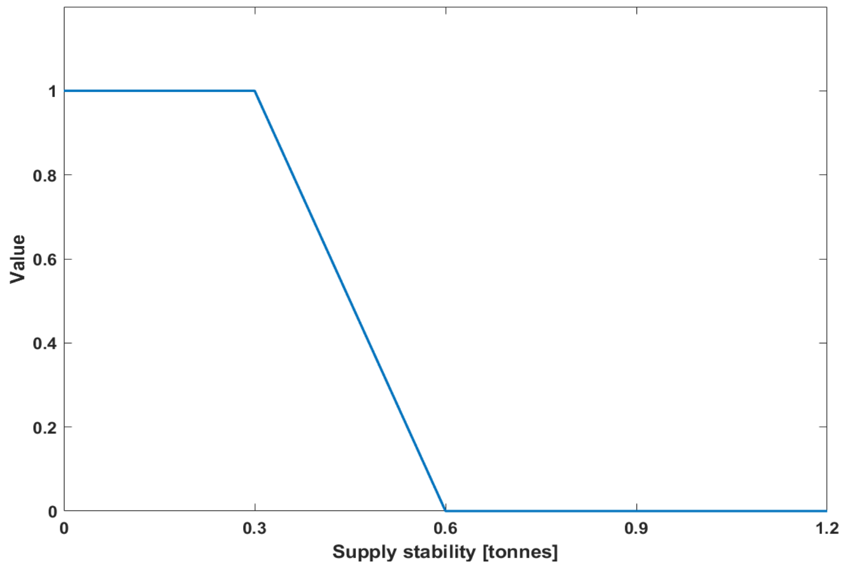

Figure 4. Value function for A2-Supply stability.

4.3 Attribute A3 –Delay time.

Delay times within the decision maker's ideal range of [0, 15) minutes will be awarded 1 point, the maximum possible score. It is known that delay times in the interval [15, 25] result in a linearly increasing loss, hence the value function will linearly decrease for delay times in this interval. Delay times above 25 minutes are unacceptable. Consequently, delay times above this threshold will be awarded zero points by the value function. The supply function for A3 is defined here as:

$$V_{A3}(x) = \begin{cases} 1\,, & 0 \leq x < 15 \\ \dfrac{25 - x}{10}\,, & 15 \leq x \leq 25 \\ 0\,, & otherwise \end{cases}$$

A graphical representation of this value function is given in Figure 5 below

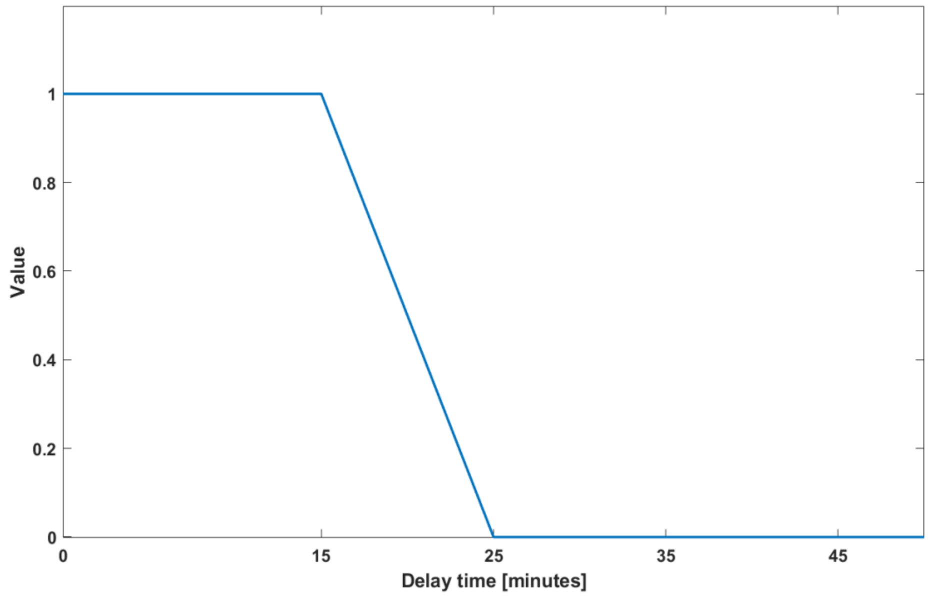

Figure 5. Value function for A3-Delay time.

4.4 Attribute A4 –Delay time.

Ratios that meet or exceed the decision-makers desired level of 20% will be awarded 1 point, the maximum possible score. It is known that ratios below 20% cause a capacity waste until 0%. As mentioned in section 1., the loss of value associated with this waste of capacity is here assumed nonlinear and of the parabola type. A graphical representation of this value function is given in Figure 6. The supply function for A4 is defined here as:

$$V_{A4}(x) = \begin{cases} \left(\frac{x}{20}\right)^2, & 0 \leq x < 20 \\ 1, & x > 20 \end{cases}$$

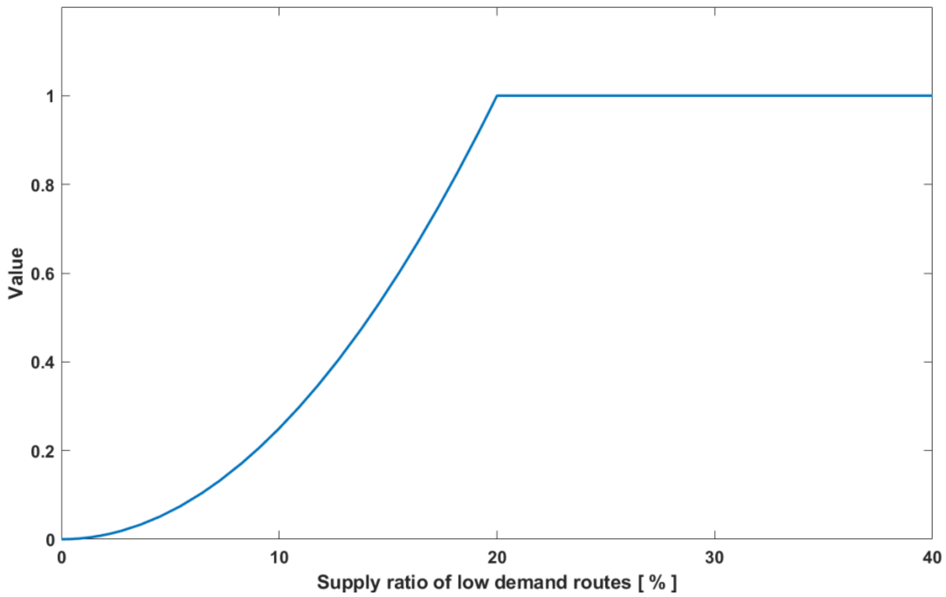

Figure 6. Value function for A4- Supply ratio of low demand routes.

4.5 Attribute A5 –Payment reputation.

As mentioned in section 1., it is assumed that the desired level for attribute A5 according to the decision-maker is H: High. Let $\widehat{H}$ be the fuzzy number representing the rating H: High and $\hat{R}$ a fuzzy number representing any one of the possible ratings of attribute A5. Then the value of $\hat{R}$ can be defined consistently and objectively as the degree of similarity $D(\widehat{H}, \hat{R})$ between $\widehat{H}$ and $\hat{R}$, where:

$$D(\widehat{H}, \hat{R}) = \frac{\int_{-\infty}^{+\infty}(\widehat{H}(x) \cap \hat{R}(x))\mathrm{dx}}{\int_{-\infty}^{+\infty}(\widehat{H}(x) \cup \hat{R}(x))\mathrm{dx}}$$

The ratings H: High, M: Medium and L: Low are present in the decision matrix, thus their value will now be calculated using D as the degree of similarity. The union of $\widehat{H}(x)$ and itself is still $\widehat{H}(x)$, hence $(\widehat{H}(x) \cup \widehat{H}(x)) = \widehat{H}(x)$. Same goes for the intersection of $\widehat{H}(x)$ and itself, meaning also $(\widehat{H}(x) \cap \widehat{H}(x)) = \widehat{H}(x)$. As a result, $D(\widehat{H}, \widehat{H})$=1 and H: high is awarded 1 point, the maximum score possible.

If $\hat{L}(x)$ is the fuzzy number representing the rating L: Low, we can see that, as presented in Figure 1, the membership functions of $\widehat{H}(x)$ and $\hat{L}(x)$ do not intersect at any point. Consequently, $\int_{-\infty}^{+\infty}\widehat{(H(x) \cap \hat{L}(x))}\mathrm{dx} = 0$. Meaning that $D(\widehat{H}, \hat{L})$=0 and alternatives with rating L: Low are awarded zero points, the lowest score possible.

Let $\widehat{M}(x)$ be the fuzzy number representing the rating M: Medium. The similarity degree $D(\widehat{H}, \widehat{M})$ can be either calculated directly using the membership functions found in Appendix A and performing the integration as usual, or by taking advantage of the simple triangular shape of the membership functions and calculating the integrals by the respective surface they cover.

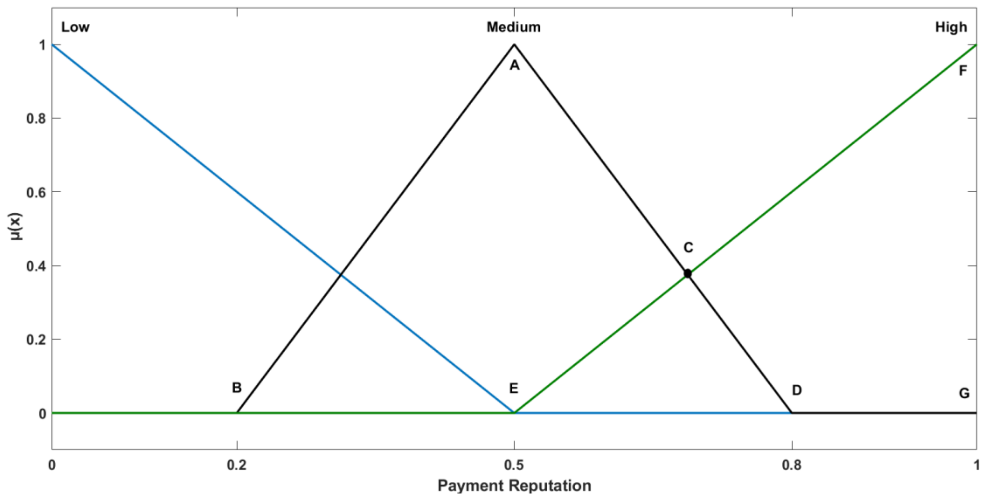

Figure 7. Using the surface area method to calculate the similarity degree between High and Low

As we can see in Figure 7, using the surface area way we have:

$$D(\widehat{H}, \widehat{M}) = \frac{\int_{-\infty}^{+\infty} (\widehat{H}(x) \cap \widehat{M}(x))\mathrm{dx}}{\int_{-\infty}^{+\infty} (\widehat{H}(x) \cup \widehat{M}(x))\mathrm{dx}} = \frac{S_{CED}}{S_{ABD} + S_{FEG} - S_{CED}} = \frac{0.0562}{0.4938} = 0.1138$$

Hence, alternatives with rating M: Medium will be awarded 0.1138 points. Table 6 sums up the calculated value for each rating.

Table 6. Values for the ratings of attribute A5 -Payment Reputation.

A5 linguistic scale	$\boldsymbol{Value}$
L: *LOW*	0
M: *MEDIUM*	0.1138
H: *HIGH*	1

4.6 Attribute A6 –Switching risk.

Essentially, the same method as the one used for the value function of attribute A5 will be employed. The only difference being that ratings of attribute A6 are in the form of intervals hence fuzzy sets instead of fuzzy numbers will be used to represent them.

The desired level for attribute A6 according to the decision maker is [DL, VL, L], hence the degree of similarity will be measured against this rating. Let $\hat{A} = (\widehat{DL}, \widehat{VL}, \hat{L})$ be the fuzzy set representing the desired level of attribute A6 and $\hat{B} = (\hat{B}_i, \hat{B}_{i+1}, \dots, \hat{B}_n)$ a fuzzy set representing a possible interval rating of attribute A6, then value of $\hat{B}$ will be defined as the degree of similarity $D(\hat{A}, \hat{B})$ between fuzzy sets $\hat{A}$ and $\hat{B}$, where:

$$D(\hat{A}, \hat{B}) = \frac{\int_{-\infty}^{+\infty} [(\widehat{DL}(x) \cup \widehat{VL}(x) \cup \hat{L}(x)) \cap (\hat{B}_i(x) \cup \hat{B}_{i+1}(x) \cup \dots \cup \hat{B}_n(x))]\mathrm{dx}}{\int_{-\infty}^{+\infty} (\widehat{DL}(x) \cup \widehat{VL}(x) \cup \hat{L}(x) \cup \hat{B}_i(x) \cup \hat{B}_{i+1}(x) \cup \dots \cup \hat{B}_n(x))\mathrm{dx}}$$

The intervals (DL, VL, L), (VL, L, M) and (L, M, H) are present in the decision matrix, thus their value will now be calculated using D as the degree of similarity. The union of a fuzzy set by itself as well as the intersection of a fuzzy set by itself gives the same fuzzy set, hence $D(\hat{A}, \hat{A}) = 1$. Consequently, as expected, the interval (DL, VL, L) is awarded 1 point, the maximum possible score.

Intervals whose fuzzy set does not intersect the optimal interval represented by $\hat{A}$ at any point, will be awarded zero points, the lowest score possible. Hence, any interval having High or worse as its lower end, such as (H, VH, DH) or (VH,DH), receives zero points.

The fuzzy sets for both intervals (VL, L, M) and (L, M, H) intersect $\hat{A}$, thus their value must be calculated. Let the fuzzy set $\hat{C} = (\widehat{\mathrm{VL}}, \hat{\mathrm{L}}, \widehat{\mathrm{M}})$ represent the interval (VL, L, M) and the fuzzy set $\widehat{K} = (\hat{\mathrm{L}}, \widehat{\mathrm{M}}, \widehat{\mathrm{H}})$ represent the interval (L, M, H) , as shown in Figure 8, it holds:

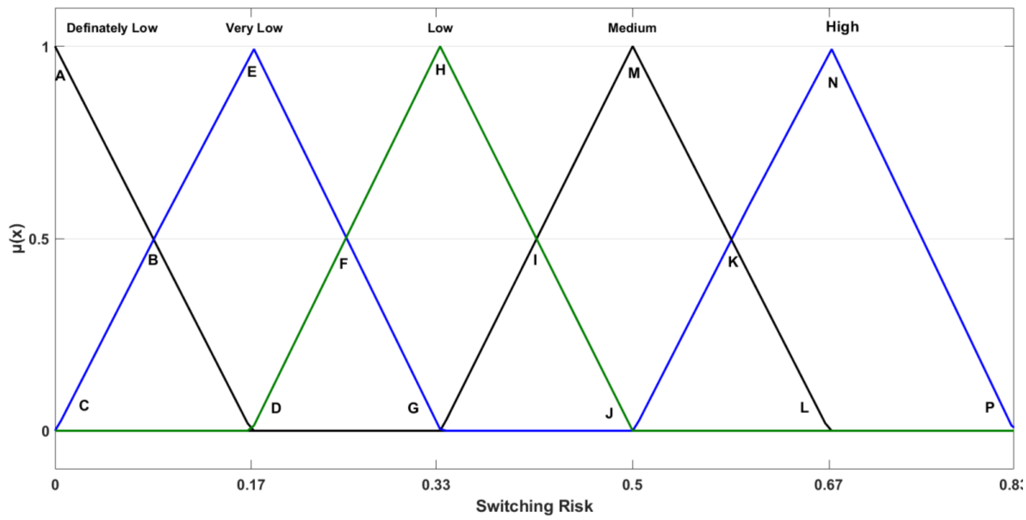

Figure 8. Using the surface area method to calculate the similarity degree.

$$D(\hat{A},\hat{C}) = \frac{S_{ECG} + S_{HDJ} - S_{FDG}}{S_{ACD} + S_{ECG} + S_{HDJ} + S_{MGL} - S_{BCD} - S_{FDG} - S_{IGJ}} = 0.6364$$

$$D(\hat{A},\hat{K}) = \frac{S_{HDJ}}{S_{ACD} + S_{ECG} + S_{HDJ} + S_{MGL} + S_{NJP} - S_{BCD} - S_{FDG} - S_{IGJ} - S_{KJL}} = 0.2857$$

As a result interval (VL, L, M) will be awarded 0.6364 points while interval (L, M, H) will be awarded 0.2857 points. Table 7 sums up the calculated value for each interval.

Table 7. Values for the ratings of attribute A6 –Switching risk.

A6 linguistic interval	*Value*
(DL, VL, L)	1
(VL, L, M)	0.6364
(L, M, H)	0.2857

5. Results and discussion.

Using the value functions described for each attribute in 4., the unweighted value matrix presented in Table 8 was created for the problem at hand.

Table 8. Unweighted value matrix for the problem

Alternative	A1	A2	A3	A4	A5	A6
P1	1	1	0.8220	0.6856	1	0.6364
P2	0	0.7333	1	1	0	0.2857
P3	1	1	0.0680	0.6922	0.1138	1
P4	1	1	0.4450	1	1	0.6364
P5	0.4096	0	1	0.4658	0.1138	1

Using the weights found in Table 5 for each attribute, the value matrix was weighted and the overall score of each alternative was summed. The weighted value matrix of the problem, along with the overall score of each alternative, are found in Table 9. It should be noted that, as mentioned in section 1., special care was given so that all value functions award points in the same [0,1] interval, so no additional normalization was required.

Table 9. Weighted value matrix for the problem and overall score

Alternative	A1	A2	A3	A4	A5	A6	Overall score
P1	0.216579	0.319853	0.066817	0.032836	0.221616	0.071768	0.929469
P2	0	0.234548	0.081286	0.047894	0	0.032219	0.395947
P3	0.216579	0.319853	0.005527	0.033152	0.025220	0.112772	0.713103
P4	0.216579	0.319853	0.036172	0.047894	0.221616	0.071768	0.913882
P5	0.088711	0	0.081286	0.022309	0.025220	0.112772	0.330297

As a result, the alternatives are ranked $P1 \succ P4 \succ \mathrm{P3} \succ \mathrm{P2} \succ \mathrm{P5}$, meaning that the two best alternatives are $P1$ and $P4$. We can see that the scores of alternatives $P1$ and $P4$ have a very narrow overall score difference, while all other alternatives score significantly lower.

Regarding the confidence in these results, it should be noted that an objective method was used to weight the attributes as no input from the decision-maker was available. The decision-maker's perception regarding the attribute weights might turn out to be quite different than the one objectively produced here. Additional input from the decision-maker in that regard would certainly improve the current approach and increase the confidence in the produced results.

Furthermore, attribute A6 is a potential cause of concern as the decision-maker used a wide interval when judging the alternatives with regards to the attribute A6-switching risk. There is a chance the decision-maker could be proven to be overly optimistic or overly pessimistic in his judgment, meaning that the actual ratings of the alternatives would fall closer to either end of the interval rather than the middle of it. Apart from affecting the alternatives perceived value, this would also affect the weights calculated by the S.W.E.T. method. The crisp values used to represent the fuzzy sets for attribute A6, are more geared to represent the middle of the interval rather than its extreme ends.

Nonetheless, given the resulting overall score as well as the status of the unweighted value matrix, the conclusion regarding the best two alternatives should prove to be pretty resilient against both small and medium scale changes in attribute weights and shifts in scores regarding attribute A6. No other alternative seems to be able to catch up with $P1$ and $P4$. Even if all weights were assumed to be even, the last resort assumption that was dismissed, alternatives $P1$ and $P4$ would still have the two highest scores by a significant margin.

Finally, the assumptions made while drafting the selected value functions were minor and arguably not significant enough that their dismissal would cause a change regarding the resulting top two scoring alternatives. Consequently, there is considerable confidence that $P1$ and $P4$ truly are the best two alternatives.

6. References

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Appendix A

A5 Linguistic scale	Membership function
H: *High*	$\mu_{HIGH}(x) = \begin{cases} \frac{x-0.5}{0.5}, & 0.5 \leq x \leq 1 \\ 1, & x = 1 \\ 0, & otherwise \end{cases}$
M: *Medium*	$\mu_{MEDIUM}(x) = \begin{cases} \frac{x-0.2}{0.3}, & 0.2 \leq x \leq 0.5 \\ \frac{0.8-x}{0.3}, & 0.5 \leq x \leq 0.8 \\ 0, & otherwise \end{cases}$
L: *Low*	$\mu_{LOW}(x) = \begin{cases} 1, & x = 0 \\ \frac{0.5-x}{0.5}, & 0 \leq x \leq 0.5 \\ 0, & otherwise \end{cases}$

A6 Linguistic scale	Membership function
DL: *Definitely Low*	$\mu_{DL}(x) = \begin{cases} 1, & x = 0 \\ \frac{0.167-x}{0.167}, & 0 \leq x \leq 0.167 \\ 0, & otherwise \end{cases}$
VL : *Very Low*	$\mu_{VL}(x) = \begin{cases} \frac{x}{0.167}, & 0 \leq x \leq 0.167 \\ \frac{0.333-x}{0.167}, & 0.167 \leq x \leq 0.333 \\ 0, & otherwise \end{cases}$
L: *Low*	$\mu_{L}(x) = \begin{cases} \frac{x-0.167}{0.167}, & 0.167 \leq x \leq 0.333 \\ \frac{0.5-x}{0.167}, & 0.333 \leq x \leq 0.5 \\ 0, & otherwise \end{cases}$
M: *Medium*	$\mu_{M}(x) = \begin{cases} \frac{x-0.333}{0.167}, & 0.333 \leq x \leq 0.5 \\ \frac{0.667-x}{0.167}, & 0.5 \leq x \leq 0.667 \\ 0, & otherwise \end{cases}$
H: *High*	$\mu_{H}(x) = \begin{cases} \frac{x-0.5}{0.167}, & 0.5 \leq x \leq 0.667 \\ \frac{0.834-x}{0.167}, & 0.667 \leq x \leq 0.834 \\ 0, & otherwise \end{cases}$

VH: *Very High*	$\mu_{VH}(x) = \begin{cases} \dfrac{x - 0.667}{0.167}, & 0.667 \leq x \leq 0.834 \\ \dfrac{1 - x}{0.167}, & 0.834 \leq x \leq 1 \\ 0, & otherwise \end{cases}$
DH: *Definitely High*	$\mu_{DH}(x) = \begin{cases} \dfrac{x - 0.834}{0.167}, & 0.834 \leq x \leq 1 \\ 1, & x = 1 \\ 0, & otherwise \end{cases}$