Higher Derivatives with negative ξ coupling in quartic term

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0.1 Kähler and Superpotential

The purpose of this report is to calculate the scalar potential of F-field in N=1 higher derivative supergravity theories and investigate the behavior of the potential changing the sign of the coupling of the quartic term.
We start by doing explicitly the calculations with the same Kähler potential and Superpotential as the ones used in [1]. The results slightly differ to [2].

$$K \equiv -3\log(T + \bar{T} - C\bar{C}) \tag{1}$$
$$W = \frac{3}{\sqrt{\lambda_1}} C\Big(T - \frac{1}{2}\Big) \tag{2}$$

the derivatives of the Kähler potential are found to be:

$$K_{T\bar{T}} = \frac{1}{(T + \bar{T} - |C|^2)^2} \tag{3}$$
$$K_{T\bar{C}} = \frac{-3C}{(T + \bar{T} - |C|^2)^2} \tag{4}$$
$$K_{C\bar{T}} = \frac{-3\bar{C}}{(T + \bar{T} - |C|^2)^2} \tag{5}$$
$$K_{C\bar{C}} = \frac{3(T + \bar{T})}{(T + \bar{T} - |C|^2)^2} \tag{6}$$

0.2 F-term equation of motion

We will not write down the Lagranian and the procedure of minimization explicitly because it's very long and already written in [1], [2]. The solution of the equation of motion of the F-term can be compactly written into the single equation:

$$\alpha = X(1 + \beta X)^2 \tag{7}$$
$$\beta^2 X^3 + 2\beta X^2 + X - \alpha = 0 \tag{8}$$

the parameters of the polynomial are:

$$\alpha = \frac{A\bar{A}}{B^2} = \frac{|A|^2}{B^2} > 0 \tag{9}$$
$$\beta = \frac{2S}{B} \tag{10}$$
$$X = F_c\bar{F}_c \tag{11}$$

the sign of β depends on the sign of the coupling ξ. This coupling has crucial role in this analysis. The α, β parameters are given by complicated expresions of derivatives of the Kähler potential and superpotential:

$$A = e^{\frac{2K}{3}}\frac{K_{C\bar{T}}}{K_{T\bar{T}}}D_TW - e^{\frac{2K}{3}}D_CW \tag{12}$$
$$B = e^{\frac{K}{3}}K_{C\bar{C}} - e^{\frac{K}{3}}\frac{K_{T\bar{C}}K_{C\bar{T}}}{K_{T\bar{T}}} - \frac{32\xi}{\lambda_1^2}e^{\frac{K}{3}}\partial^\mu C\partial_\mu\bar{C} \tag{13}$$
$$S = \frac{16\xi}{\lambda_1^2}e^{\frac{2K}{3}} \tag{14}$$

Then we write (7) in the depressed dubic equation of the form:

$$t^3 + pt + q = 0 \tag{15}$$
$$p = \frac{3ac - b^2}{3a^2} \tag{16}$$
$$q = \frac{2b^3 - 9abc + 27a^2d}{27a^3} \tag{17}$$

In the case of equation (7) the parameters are $a = \beta^2$, $b = 2\beta$, $c = 1$, $d = -\alpha$ and the new parameters of the depressed cubic finally become:

$$p = -\frac{1}{3\beta^2} \tag{18}$$
$$q = -\frac{2 + 27\beta\alpha}{27\beta^3} \tag{19}$$

We notice that the parameter p is **always negative**. This rules out one of the hyperbolic solutions of the depressed cubic equation which especially worked out in [3].

At this point I would like to clarify an important issue which is crucial for the following analysis. Solving (7) I get a solution of the form $|F_c|^2 = X \propto \cosh(m)$ or $cos(m)$ depending on the signs of the couplings. However, both [2], [1] get a $|F_c|^2 = X \propto \cosh(m) - 1$ solution. The solution of (7) should be pretty straighforward but since the solution with the -1 gives the correct results I will continue working with it. If I don't consider the -1 only a sign changes in the final form of the potential but it is crucial in order to get th correct result.

0.3 The real solutions of equation (7),(15)

0.3.1 The one real solution

Then we analyze the rest of the solutions. Firstly, for **one real solution** $p < 0$, we have a hyperbolic expressed solution:

$$t = -2\frac{|q|}{q}\sqrt{-\frac{p}{3}}\left\{\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(\frac{-3|q|}{2p}\sqrt{\frac{-3}{p}}\right)\right] - 1\right\} \tag{20}$$

$$= \frac{2}{3\beta}\left\{\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(\frac{-3|q|}{2p}\sqrt{\frac{-3}{p}}\right)\right] - 1\right\} \tag{21}$$

$$= \frac{2}{3\beta}\left\{\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(-\frac{27}{2}\alpha\beta - 1\right)\right] - 1\right\} \tag{22}$$

$$\boxed{X = \frac{2}{3\beta}\left\{\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right)\right] - 1\right\}} \tag{23}$$

$\cosh(m) \geq 1$ for any value of $m = \frac{1}{3}\operatorname{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right)$. Thus the expression in the brackets is non-negative. Since we want $X = F_c\bar{F}_c > 0$, the parameter $\beta(\xi)$ should always be positive. Also $m > 0$ thus $\operatorname{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right) > 0$ and thus $\left(27\alpha\beta + 2\right) > 0$.

Hence the inequality:

$$4p^3 + 27q^2 > 0 \tag{24}$$

$$\Rightarrow -\frac{4}{27\beta^6} + 27\frac{(2 + 27\beta\alpha)^2}{27^2\beta^6} > 0 \tag{25}$$

$$\Rightarrow (2 + 27\beta\alpha)^2 > 4 \tag{26}$$

$$\Rightarrow \beta\alpha > 0 \quad \text{or} \quad \beta\alpha < -\frac{4}{27} \tag{27}$$

$$\Rightarrow \frac{|A|^2 S}{B^3} > 0 \quad \text{or} \quad \frac{|A|^2 S}{B^3} < -\frac{2}{27} \tag{28}$$

is always satisfied. The inequality $\beta\alpha > 0$ is always satisfied for $\xi > 0 \Rightarrow \beta > 0$ as we have assumed. The $\beta\alpha < -\frac{4}{27}$ is not acceptable solution of the inequality because both β and α have considered positive. In next paragraph we will see that when we express $\alpha\beta$ in terms of the *inflaton* field this inequality becomes:

$$\boxed{\left(e^{\sqrt{\frac{2}{3}}\phi} - 1\right)^2 \frac{\xi}{\lambda_1^3} > 0} \tag{29}$$

0.3.2 The three real solutions

Secondly, we check out the **three real** solutions of the depressed qubic equation (15) for $p < 0$:

$$t_k = 2\sqrt{-\frac{p}{3}}\left\{ \cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - \frac{2\pi k}{3}\right] - 1\right\} \quad \text{for} \quad \text{k=0,1,2} \tag{30}$$

plugging (18) in the equation above,

$$\begin{aligned} t_k &= \frac{2}{3\beta}\left\{ \cos\left[\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - \frac{2\pi k}{3}\right] - 1\right\} \\ &= \frac{2}{3\beta}\left\{ \cos\left[\frac{1}{3}\arccos\left(\frac{2+27\beta\alpha}{6\beta}3\beta\right) - \frac{2\pi k}{3}\right] - 1\right\} \end{aligned}$$

$$\boxed{X_k = \frac{2}{3\beta}\left\{ \cos\left[\frac{1}{3}\arccos\left(\frac{27}{2}\alpha\beta + 1\right) - \frac{2\pi k}{3}\right] - 1\right\}} \quad \text{for} \quad \text{k=0,1,2} \tag{31}$$

this solution should also satisfy the inequality:

$$4p^3 + 27q^2 \leq 0 \tag{32}$$

$$\Rightarrow -\frac{4}{27\beta^6} + 27\frac{(2+27\beta\alpha)^2}{27^2\beta^6} \leq 0 \tag{33}$$

$$\Rightarrow (2+27\beta\alpha)^2 \leq 4 \tag{34}$$

$$\bullet -1 \leq \frac{27}{2}\beta\alpha + 1 \leq 1 \Rightarrow -\frac{4}{27} \leq \beta\alpha \leq 0 \Rightarrow -\frac{4}{27} \leq \left(e^{\sqrt{\frac{2}{3}\phi}} - 1\right)^2 \frac{\xi}{\lambda_1^3} \leq 0 \tag{35}$$

$$\bullet -1 \leq 27\frac{|A|^2 S}{B^3} + 1 \leq 1 \Rightarrow -\frac{2}{27} \leq \frac{|A|^2 S}{B^3} \leq 0 \tag{36}$$

It is straightforward from the inequality $-1 \leq \frac{27}{2}\beta\alpha + 1 \leq 1$ that since $\alpha > 0$, β has to be negative in order to fullfil $\frac{27}{2}\beta\alpha + 1 \leq 1$. Thus these three real solutions correspond to $\beta(\xi) < 0$ and hence $\xi < 0$. It is now clear that the solutions (31) are defined only when the inequality above is satisfied.
Also, since we want $X > 0$ and $\cos(m)$ cannot be larger than unity β has to be negative or else for any value of m, $\cos(m) - 1 \leq 0$. We mention it once more, that for these solutions the inequality

$$\boxed{-1 \leq \frac{27}{2}\beta\alpha + 1 \leq 1} \tag{37}$$

has to be satisfied.

0.4 The scalar potential

Finally we calculate the scalar potential,

$$\boxed{V_F = BX + 3SB^2 + V_T} \tag{38}$$

V_T is the scalar potential of the T chiral superfield:

$$\begin{aligned}
V_T &= e^K (K^{T\bar{T}})^{-1} D_T W D_{\bar{T}} \bar{W} - 3e^K W\bar{W} \qquad (39)\\
&= e^K \frac{(T+\bar{T}-C\bar{C})^2}{3(T+\bar{T})} \left\{ \Big[\frac{3}{\sqrt{\lambda_1}} C - \Big(\frac{3}{T+\bar{T}-C\bar{C}} \Big) \frac{3}{\sqrt{\lambda_1}} C \Big(T - \frac{1}{2} \Big) \Big] \right.\\
&\left. * \Big[\frac{3}{\sqrt{\lambda_1}} \bar{C} - \Big(\frac{3}{T+\bar{T}-C\bar{C}} \Big) \frac{3}{\sqrt{\lambda_1}} \bar{C} \Big(\bar{T} - \frac{1}{2} \Big) \Big] \right\} - 3e^K W\bar{W}
\end{aligned}$$

This part of the V_F potential will end up to be zero when we consider the fields $C = \bar{C} = 0$ to be stabilized. Subsequently we calculate the parameters B, S (12) of the equation (38),

$$\begin{aligned}
A &= e^{\frac{2K}{3}} \frac{K_{C\bar{T}}}{K_{T\bar{T}}} D_T W - e^{\frac{2K}{3}} D_C W = e^{\frac{2K}{3}} \left\{ \frac{K_{C\bar{T}}}{K_{T\bar{T}}} D_T W - D_C W \right\}\\
&= e^{\frac{2K}{3}} \left\{ -\bar{C} \Big[\frac{3}{\sqrt{\lambda_1}} C - \Big(\frac{3}{T+\bar{T}-C\bar{C}} \Big) \frac{3}{\sqrt{\lambda_1}} C \Big(T - \frac{1}{2} \Big) \Big] \right.\\
&\left. - \Big[\frac{3}{\sqrt{\lambda_1}} (T - \frac{1}{2}) + \Big(\frac{3\bar{C}}{T+\bar{T}-C\bar{C}} \Big) \frac{3}{\sqrt{\lambda_1}} C \Big(T - \frac{1}{2} \Big) \Big] \right\}\\
&= \frac{1}{(T+\bar{T}-C\bar{C})^2} \left\{ -\bar{C} \Big[\frac{3}{\sqrt{\lambda_1}} C - \Big(\frac{3}{T+\bar{T}-C\bar{C}} \Big) \frac{3}{\sqrt{\lambda_1}} C \Big(T - \frac{1}{2} \Big) \Big] \right.\\
&\left. - \Big[\frac{3}{\sqrt{\lambda_1}} (T - \frac{1}{2}) + \Big(\frac{3\bar{C}}{T+\bar{T}-C\bar{C}} \Big) \frac{3}{\sqrt{\lambda_1}} C \Big(T - \frac{1}{2} \Big) \Big] \right\}\\
&= -\frac{3}{\sqrt{\lambda_1}(T+\bar{T}-C\bar{C})^2} \Big(C\bar{C} + T - \frac{1}{2} \Big) \qquad (40)
\end{aligned}$$

$$\begin{aligned}
B &= e^{\frac{K}{3}} K_{C\bar{C}} - e^{\frac{K}{3}} \frac{K_{T\bar{C}} K_{C\bar{T}}}{K_{T\bar{T}}} - \frac{32\xi}{\lambda_1^2} e^{\frac{K}{3}} \partial^\mu C \partial_\mu \bar{C}\\
&= \frac{1}{T+\bar{T}-C\bar{C}} \left\{ \frac{3(T+\bar{T})}{(T+\bar{T}-C\bar{C})^2} - \frac{3C^2}{(T+\bar{T}-C\bar{C})^2} - \frac{32\xi}{\lambda_1^2} \partial^\mu C \partial_\mu \bar{C} \right\}\\
&= \frac{3}{(T+\bar{T}-C\bar{C})^2} - \frac{32\xi}{\lambda_1^2} \frac{\partial^\mu C \partial_\mu \bar{C}}{T+\bar{T}-C\bar{C}} \qquad (41)\\
S &= \frac{16\xi}{\lambda_1^2} e^{\frac{2K}{3}} = \frac{16\xi}{\lambda_1^2} \frac{1}{(T+\bar{T}-C\bar{C})^2} \qquad (42)
\end{aligned}$$

Collecting the A,B,C parameters we write them in a box,

$$\boxed{\begin{aligned}
A &= -\frac{3}{\sqrt{\lambda_1}(T+\bar{T}-C\bar{C})^2} \Big(C\bar{C} + T - \frac{1}{2} \Big) \qquad (43)\\
B &= \frac{3}{(T+\bar{T}-C\bar{C})^2} - \frac{32\xi}{\lambda_1^2} \frac{\partial^\mu C \partial_\mu \bar{C}}{T+\bar{T}-C\bar{C}} \qquad (44)\\
S &= \frac{16\xi}{\lambda_1^2} \frac{1}{(T+\bar{T}-C\bar{C})^2} \qquad (45)
\end{aligned}}$$

Then we redifine the T field in order to restore the Strobinsky-like potential:

$$T = \frac{1}{2}e^{\sqrt{\frac{2}{3}}\phi} + ib \tag{46}$$

and setting for $C = \bar{C} = 0$ as well as b=0. The explanation of this consideration is discussed in [2].

$$\boxed{\begin{aligned} A &= -\frac{3}{2\sqrt{\lambda_1}}\frac{1}{e^{2\sqrt{\frac{2}{3}}\phi}}(e^{\sqrt{\frac{2}{3}}\phi} - 1) && (47)\\ B &= \frac{3}{e^{2\sqrt{\frac{2}{3}}\phi}} && (48)\\ S &= \frac{16\xi}{\lambda_1^2}\frac{1}{e^{2\sqrt{\frac{2}{3}}\phi}} && (49) \end{aligned}}$$

the parameters of the cubic equation become

$$\alpha = \frac{1}{\lambda_1}(e^{\sqrt{\frac{2}{3}}\phi} - 1)^2 \tag{50}$$

$$\beta = \frac{32\xi}{3\lambda_1^2} \tag{51}$$

We write down explicitly the most general solution of the cubic equation for k=0 :

$$X_0 = \frac{2}{3\beta}\left\{ \cos\left[\frac{1}{3}\arccos\left(\frac{27}{2}\alpha\beta + 1\right)\right] - 1 \right\}$$

$$\boxed{X_0 = \frac{\lambda_1^2}{16\xi}\left\{ \cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi} - 1)^2 + 1\right)\right] - 1 \right\}} \tag{52}$$

Checking the solution (52) for $\xi < 0$ the solution is negative while for $\xi > 0$ is positive. Now we explicitly write down the form of the scalar potential (38)

$$BX_0 = \frac{3}{e^{2\sqrt{\frac{2}{3}}\phi}} * \frac{\lambda_1^2}{16\xi}\left\{ \cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi} - 1)^2 + 1\right)\right] - 1 \right\} \tag{53}$$

$$3SX_0^2 = 3 * \frac{16\xi}{\lambda_1^2}\frac{1}{e^{2\sqrt{\frac{2}{3}}\phi}} * \frac{\lambda_1^2}{16\xi}\left\{ \cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi} - 1)^2 + 1\right)\right] - 1 \right\}^2 \tag{54}$$

The potential V_T for $C = \bar{C} = 0$ is zero and doesn't contribute to the V_F potential.

$$
\begin{aligned}
V_F =& BX + 3SX^2 \qquad (55)\\
=& \frac{3\lambda_1^2}{16\xi} e^{-2\sqrt{\frac{2}{3}}\phi} \Bigg\{ \cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi}-1)^2+1\right)\right]-1 \\
& + \left(\cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi}-1)^2+1\right)\right]-1\right)^2 \Bigg\}\\
=& \frac{3\lambda_1^2}{16\xi} e^{-2\sqrt{\frac{2}{3}}\phi} \cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi}-1)^2+1\right)\right]\left\{\cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi}-1)^2+1\right)\right]-1\right\}
\end{aligned}
$$

$$\boxed{V_F = \frac{3\lambda_1^2}{16\xi} e^{-2\sqrt{\frac{2}{3}}\phi}\chi(\chi-1)} \qquad (56)$$

for

$$\chi = \cos\left[\frac{1}{3}\arccos\left(144\frac{\xi}{\lambda_1^3}(e^{\sqrt{\frac{2}{3}}\phi}-1)^2+1\right)\right] \qquad (57)$$

Then we plot the scalar potential V_F including higher order corrections with different sign in the quartic term.

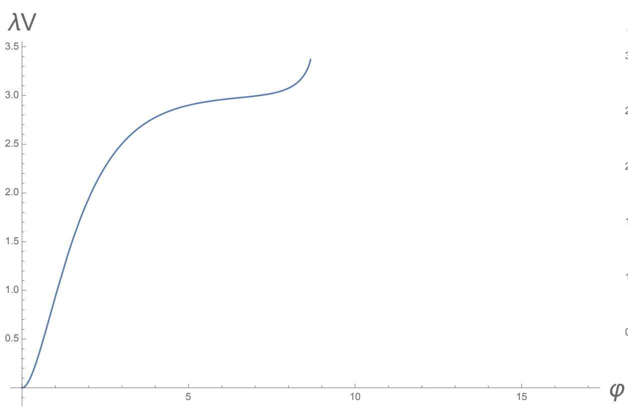

Figure 1: Potential with negative coupling ξ and $s = -10^{-8}$

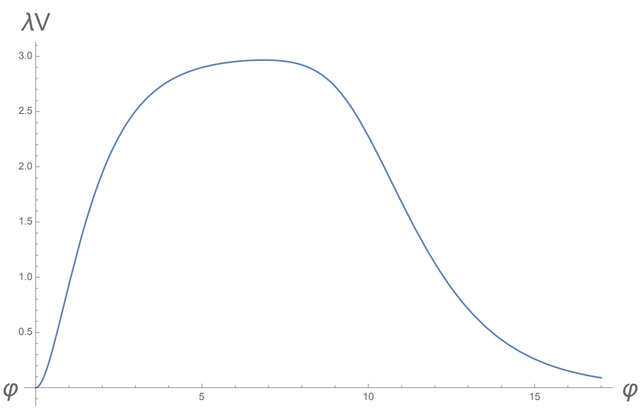

Figure 2: Potential with positive coupling ξ and $s = 10^{-8}$

In Figure[1] we have used the solution (31) of the qubic equation with $\xi < 0$ while for Figure[2] we have used the hyperbolic solution (20) to construct the potential (55) with $\xi > 0$.

The behavior of the potential in Figure[1] stops for a certain value of inflaton, name it ϕ_c. This happens because the inequality (89)

$$4p^3 + 27q^2 \leq 0 \qquad (58)$$

$$\bullet -1 \leq 1 + \frac{27}{2}\beta\alpha \leq 1 \Rightarrow \qquad \boxed{-\frac{4}{27} \leq (e^{\sqrt{\frac{2}{3}}\phi}-1)^2\frac{\xi}{\lambda_1^3} \leq 0} \qquad (59)$$

$$\bullet -1 \leq 1 + 27\frac{|A|^2 S}{B^3} \leq 1 \Rightarrow -\frac{2}{27} \leq \frac{|A|^2 S}{B^3} \leq 0 \qquad (60)$$

is *not satisfied* anymore. The function becomes undetermined for specific critical value of the inflaton, ϕ_c. Furthermore, for the specific Kähler and superpotential (1) this inequality is violated and the only parameter we can change is s which contains ξ and λ. The last one is costrained by Planck measurments [2]. So changing the value of s we *don't* fix the behavior of the potential just make the plateau flatter and for $s \Rightarrow 0$ we recover the Starobinsky potential.
From another perspective we can approach this problem by checking the general term where the problem appears,

$$\frac{|A|^2 S}{B^3} = \frac{16\xi}{\lambda_1^2} * \frac{\left\{\frac{K_{C\bar{T}}}{K_{T\bar{T}}} D_T W - D_C W\right\}^2\Big|_{C=\bar{C}=0}}{\left\{K_{C\bar{C}} - \frac{K_{C\bar{T}} K_{T\bar{C}}}{K_{T\bar{T}}}\right\}^3\Big|_{C=\bar{C}=0}} \tag{61}$$

we express the inequality quantity in this form to make clear that changing the Kähler potential to a more general one, worked out in [4] :

$$K = -3 \log \left\{T + \bar{T} + f(C, \bar{C})\right\} \tag{62}$$

may help to move ϕ_c to higher scales.

1 General Kähler potential

We try a more general form of $f(C, \bar{C})$ instead of the term $C\bar{C}$:

$$\boxed{f(C, \bar{C}) = 1 + \gamma_n \frac{C^n + \bar{C}^n}{m^n} - 2\frac{C\bar{C}}{m^2} + \frac{1}{9}\zeta \frac{C^2 \bar{C}^2}{m^4}} \tag{63}$$

Next we calculate the Kähler metric elements for the new general Kähler potential:

$$K_{T\bar{T}} = \frac{3}{\left(T + \bar{T} + f(C, \bar{C})\right)^2} \tag{64}$$

$$K_{T\bar{C}} = \frac{3 f(C, \bar{C})_{\bar{C}}}{\left(T + \bar{T} + f(C, \bar{C})\right)^2} \tag{65}$$

$$K_{C\bar{T}} = \frac{3 f(C, \bar{C})_C}{\left(T + \bar{T} + f(C, \bar{C})\right)^2} \tag{66}$$

$$K_{C\bar{C}} = 3\frac{-f(C, \bar{C})_{C\bar{C}}\left(T + \bar{T} + f(C, \bar{C})\right) + f(C, \bar{C})_{\bar{C}} f(C, \bar{C})_C}{\left(T + \bar{T} + f(C, \bar{C})\right)^2} \tag{67}$$

where the indexes "$C\bar{C}$" denote the partial derivative to the respective fields.Then we calculate again the parameters A,B,S (12) of the equation (38) for the general Kähler potential,

$$
\begin{aligned}
A &= e^{\frac{2K}{3}}\frac{K_{C\bar{T}}}{K_{T\bar{T}}}D_T W - e^{\frac{2K}{3}}D_C W = e^{\frac{2K}{3}}\left\{\frac{K_{C\bar{T}}}{K_{T\bar{T}}}D_T W - D_C W\right\} \\
&= e^{\frac{2K}{3}}\left\{f(C,\bar{C})_C D_T W - D_C W\right\} \\
&= e^{\frac{2K}{3}}\left\{(\gamma_n n\frac{C^{n-1}}{m^n} - 2\frac{\bar{C}}{m^2} + \frac{2}{9}\zeta\frac{C\bar{C}^2}{m^4})D_T W - D_C W\right\} \\
&= \frac{3}{\sqrt{\lambda_1}}e^{\frac{2K}{3}}\left\{\left(\gamma_n n\frac{C^n}{m^n} - 2\frac{\bar{C}C}{m^2} + \frac{2}{9}\zeta\frac{C^2\bar{C}^2}{m^4}\right) - \left(T - \frac{1}{2}\right)\right\} && (68) \\
&= \frac{3}{\sqrt{\lambda_1}}\frac{1}{\left(T + \bar{T} + f(C,\bar{C})\right)^2}\left\{\left(\gamma_n n\frac{C^n}{m^n} - 2\frac{\bar{C}C}{m^2} + \frac{2}{9}\zeta\frac{C^2\bar{C}^2}{m^4}\right) - \left(T - \frac{1}{2}\right)\right\} && (69)
\end{aligned}
$$

$$
\begin{aligned}
B &= e^{\frac{K}{3}}K_{C\bar{C}} - e^{\frac{K}{3}}\frac{K_{T\bar{C}}K_{C\bar{T}}}{K_{T\bar{T}}} - \frac{32\xi}{\lambda_1^2}e^{\frac{K}{3}}\partial^\mu C\partial_\mu\bar{C} \\
&= -\frac{1}{T + \bar{T} + f(C,\bar{C})}\left\{\frac{3\left(\frac{2}{m^2} + \frac{4}{9}\zeta\frac{C\bar{C}}{m^4}\right)}{T + \bar{T} + f(C,\bar{C})} + \frac{32\xi}{\lambda_1^2}\partial^\mu C\partial_\mu\bar{C}\right\} && (70) \\
S &= \frac{16\xi}{\lambda_1^2}e^{\frac{2K}{3}} = \frac{16\xi}{\lambda_1^2}\frac{1}{(T + \bar{T} + f(C,\bar{C}))^2} && (71)
\end{aligned}
$$

We assume that fields $C = \bar{C} = 0$ are stabilized during inflation and the parameters become:

$$
\begin{aligned}
A &= -\frac{3}{\sqrt{\lambda_1}}\frac{(T - \frac{1}{2})}{(T + \bar{T} + 1)^2} && (72) \\
B &= -\frac{6}{m^2}\frac{1}{(T + \bar{T} + 1)^2} && (73) \\
S &= \frac{16\xi}{\lambda_1^2}\frac{1}{(T + \bar{T} + 1)^2} && (74)
\end{aligned}
$$

Then we express the T field in real and imaginary part [46] and assume that the imaginary part b is also stabilized as well. Finally the parameters become:

$$
\boxed{
\begin{aligned}
A &= -\frac{3}{2\sqrt{\lambda_1}}\frac{e^{\sqrt{\frac{2}{3}}\phi} - 1}{(e^{\sqrt{\frac{2}{3}}\phi} + 1)^2} && (75) \\
B &= -\frac{6}{m^2}\frac{1}{(e^{\sqrt{\frac{2}{3}}\phi} + 1)^2} && (76) \\
S &= \frac{16\xi}{\lambda_1^2}\frac{1}{(e^{\sqrt{\frac{2}{3}}\phi} + 1)^2} && (77)
\end{aligned}
}
$$

We point out that B has the same sign compared to the first case. α is always positive cause it's fraction of squared values and the sign of β depends on the sign of ξ. Now $\alpha\beta$ takes the form,

$$\alpha = \frac{|A|^2}{B^2} \tag{78}$$

$$\beta = \frac{2S}{B} = -\frac{16}{3}\frac{\xi m^2}{\lambda_1^2} \tag{79}$$

$$\alpha\beta = \frac{2S|A|^2}{B^3} = -\frac{72}{216}\frac{m^6\xi}{\lambda_1^3}\left(e^{\sqrt{\frac{2}{3}}\phi} - 1\right)^2 \tag{80}$$

1.1 The real solutions of the cubic equation

In this paragraph I will follow the same analysis was done in section [0.3] but for the general Kähler potential.

1.1.1 The one real solution for general Kähler

So we have written again explicitly the one real solution:

$$\boxed{X = \frac{2}{3\beta}\left\{\cosh\left[\frac{1}{3}\,\text{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right)\right] - 1\right\}} \tag{81}$$

and $\cosh(m) \geq 1$ for any value of $m = \frac{1}{3}\,\text{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right)$. Thus the expression in the brackets is non-negative. Since we want $X = F_c\bar{F}_c > 0$, the parameter $\beta(\xi)$ should always be positive. That means for $\beta(\xi) > 0$ we have $\xi < 0$. Also $m > 0$ so, $\text{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right) > 0$ and thus $\left(27\alpha\beta + 2\right) > 0$.

Hence the inequality:

$$4p^3 + 27q^2 > 0 \tag{82}$$

$$\Rightarrow -\frac{4}{27\beta^6} + 27\frac{(2 + 27\beta\alpha)^2}{27^2\beta^6} > 0 \tag{83}$$

$$\Rightarrow (2 + 27\beta\alpha)^2 > 4 \tag{84}$$

$$\Rightarrow \beta\alpha > 0 \quad \text{or} \quad \beta\alpha < -\frac{4}{27} \tag{85}$$

$$\Rightarrow \frac{|A|^2 S}{B^3} > 0 \quad \text{or} \quad \frac{|A|^2 S}{B^3} < -\frac{2}{27} \tag{86}$$

is always satisfied. The inequality $\beta\alpha > 0$ is always satisfied for $\xi < 0 \Rightarrow \beta > 0$ as we have assumed. The $\beta\alpha < -\frac{4}{27}$ is *not acceptable* solution of the inequality because both β and α have considered positive. In next paragraph we will see that when we express $\alpha\beta$ in terms of the *inflaton* field the inequality becomes:

$$\boxed{\alpha\beta > 0 \Rightarrow -\frac{72}{216}\frac{m^6\xi}{\lambda_1^3}\left(e^{\sqrt{\frac{2}{3}}\phi}-1\right)^2 > 0} \tag{87}$$

which is always satisfied for $\xi < 0$.

1.1.2 The three real solutions for general Kähler

Secondly, we check out the **three real** solutions of the depressed qubic equation (15) for $p < 0$ given by:

$$\boxed{X_k = \frac{2}{3\beta}\left\{\cos\left[\frac{1}{3}\arccos\left(\frac{27}{2}\alpha\beta+1\right)-\frac{2\pi k}{3}\right]-1\right\}} \quad \text{for} \quad \text{k=0,1,2} \tag{88}$$

this solution should also satisfy the inequality:

$$4p^3 + 27q^2 \leq 0 \tag{89}$$

$$\Rightarrow -\frac{4}{27\beta^6} + 27\frac{(2+27\beta\alpha)^2}{27^2\beta^6} \leq 0 \tag{90}$$

$$\Rightarrow (2+27\beta\alpha)^2 \leq 4 \tag{91}$$

$$\boxed{-1 \leq \frac{27}{2}\beta\alpha + 1 \leq 1} \tag{92}$$

When this inequality is violated the function becomes undetermined like in Figure[1].It is straightforward from the inequality $-1 \leq \frac{27}{2}\beta\alpha + 1 \leq 1$ that since $\alpha > 0$, β **has to be negative** in order to fullfil $\frac{27}{2}\beta\alpha \leq 0$. Thus these three real solutions correspond to $\beta(\xi) < 0$ and hence $\xi > 0$.
Also, since we want $X > 0$ and $\cos(m)$ cannot be larger than unity, β has to be negative or else for any value of m, $\cos(m) - 1 \leq 0$. We mention it once more, that for these solutions the inequality

$$\boxed{-1 \leq 1 - \frac{9}{2}\frac{m^6\xi}{\lambda_1^3}\left(e^{\sqrt{\frac{2}{3}}\phi}-1\right)^2 \leq 1} \tag{93}$$

has to be satisfied.

1.2 The scalar potential for general Kähler

We calculate again the scalar potential this case for th general Kähler. For instance we choose [88] the one of the three solutions of the qubic eqaution, for $k = 0$. This corresponds to $\xi > 0$ while the hyperbolic one [81] corresponds to $\xi < 0$.

$$
\begin{aligned}
V_F &= BX + 3SX^2 \\
&= \frac{3}{4}\frac{\lambda_1^2}{\xi m^4}\frac{1}{(e^{\sqrt{\frac{2}{3}}\phi}+1)^2}\left\{\cos\left[\frac{1}{3}\arccos\left(\frac{27}{2}\alpha\beta+1\right)\right]\right\} * \left\{\cos\left[\frac{1}{3}\arccos\left(\frac{27}{2}\alpha\beta+1\right)\right]-1\right\} \\
&= \frac{3}{4}\frac{\lambda_1^2}{\xi m^4}\frac{1}{(e^{\sqrt{\frac{2}{3}}\phi}+1)^2}\chi * (\chi - 1)
\end{aligned} \tag{94}
$$

where

$$
\chi = \cos\left[\frac{1}{3}\arccos\left(\frac{27}{2}\alpha\beta+1\right)\right] \tag{95}
$$

$$
\chi = \cos\left[\frac{1}{3}\arccos\left(1-\frac{9}{2}\frac{m^6\xi}{\lambda_1^3}\left(e^{\sqrt{\frac{2}{3}}\phi}-1\right)^2\right)\right] \tag{96}
$$

Lets plot again the scalar potential of F-term for the two solutions, the hyperbolic one which corresponds to $\xi < 0$ and [94] which corresponds to $\xi > 0$.

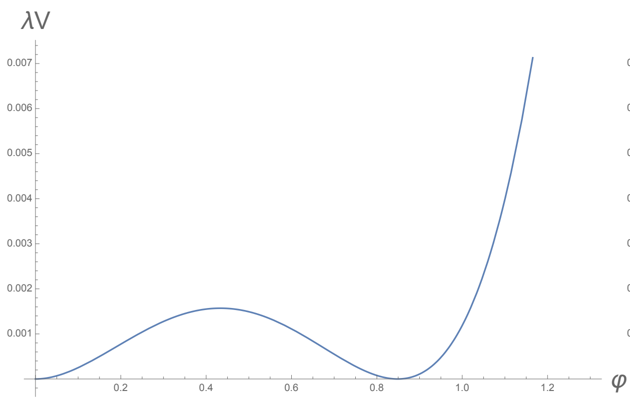

Figure 3: Potential with negative coupling ξ, $s = -10^{-6}$ and m=1

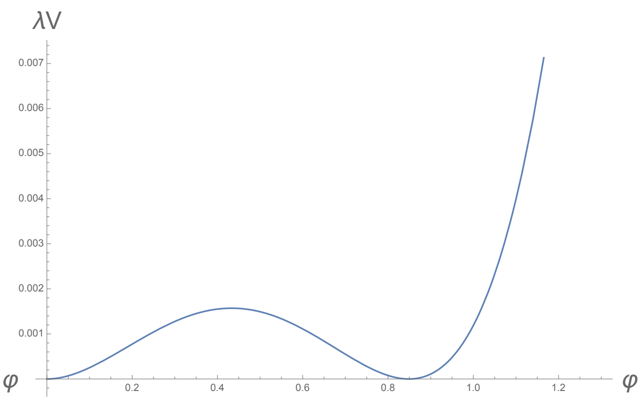

Figure 4: Potential with positive coupling ξ, $s = 10^{-6}$ m=1

Suprisingly we get the same graph for both solutions.

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