Generalizations on Inflection points/Mass matrix of scalar potential in Supergravity

April 12, 2018

Abstract

We explore the general form of the covariant derivatives of the scalar potential of supergravity. In section [1] we write down the general form which is usually used in metastability. In section [2] we calculate the mass matrix which we use to protect the models from tachyonic instabilities and achieve single field inflation as well. We demonstrate the stabilization in superconformal attractors too.

1 Inflection Point (Non-vanishing potential $V \neq 0$)

The scalar potential which arises from the minimal supergravity neglecting the D-terms is:

$$\boxed{V = e^G(G^{i\bar{j}}G_iG_{\bar{j}} - 3)} \tag{1}$$

where G is the real function $G = K + \log[WW^\dagger]$. K stands for the Kähler potential $K(\Phi\Phi^\dagger)$ and $W(\Phi)$ for the holomorphic superpotential. The second derivative $G_{i\bar{j}}$ denotes the Kähler metric which depends only on the Kähler potential.

In this section we explore the case where the potential has an inflection point which means the first and second derivatives of the potential vanish.

$$V \neq 0 \tag{2}$$
$$\nabla_j V = 0 \tag{3}$$
$$\nabla_i \nabla_j V = 0 \tag{4}$$

The first derivative is:

$$\begin{aligned}
\nabla_j V &= \nabla_j[e^G(G^{i\bar{j}}G_iG_{\bar{j}} - 3)] &(5)\\
&= \nabla_j(e^G)(G^{i\bar{j}}G_iG_{\bar{j}} - 3) + e^G[(\nabla_j(G^{i\bar{j}})G_iG_{\bar{j}} + G^{i\bar{j}}(\nabla_jG_i)G_{\bar{j}} + G^{i\bar{j}}G_i(\nabla_jG_{\bar{j}})] &(6)\\
&= G_jV + e^G[G^{i\bar{j}}(\nabla_jG_i)G_{\bar{j}} + G_j] &(7)\\
&= G_jV + e^G[G^i(\nabla_jG_i) + G_j] &(8)\\
& &(9)
\end{aligned}$$

thus,

$$\boxed{\nabla_j V = G_j V + e^G[G^i(\nabla_j G_i) + G_j]} \tag{10}$$

here we have used the compatibility of the metric $\nabla_j(G^{i\bar{j}}) = 0$ as well as the following calculation:

$$G^{i\bar{j}}G_i(\nabla_j G_{\bar{j}}) = G^{i\bar{j}}G_i[\partial_j G_{\bar{j}} - \Gamma^{\bar{k}}_{i\bar{j}}G_{\bar{k}}] = G^{i\bar{j}}G_i[\partial_j G_{\bar{j}}] = G^{i\bar{j}}G_i G_{j\bar{j}} = G_j \tag{11}$$

The specific Christoffel symbol vanishes according to Appendix C.4 from Wess-Bagger [1]. Now we write (10) in terms of G:

$$G_j e^G(G^i G_i - 3) + e^G(G^i\nabla_j G_i + G_j) = 0 \tag{12}$$

$$\Rightarrow G^i\nabla_j G_i + G_j G^i G_i - 2G_j = 0 \tag{13}$$

It doesn't look like we can get much information from this. If we set V=0 to the equation (10) we restore the known relation for symmetry breaking. I have chosen to keep the potential V in the relation (10) and not to expand it in terms of G because I followed the result of [2].

We want the potential to be positive thus from this equation we extract the information that either $(G^i\nabla_j G_i + G_j)$ or G_j has to be negative.
Afterward we calculate the second derivative:

$$\nabla_i\nabla_j V = [\nabla_i G_j]V + G_j[\nabla_i V] + e^G[G_i(\nabla_j G_k)G^k + G_i G_j] \tag{14}$$

$$+ e^G[G^k\nabla_i\nabla_j G_k + (\nabla_i G_k)(\nabla_j G^k) + \nabla_i G_j] \tag{15}$$

lets work out some of the terms in order to express it in more compact form:

$$(\nabla_i G_k)(\nabla_j G^k) = (\nabla_i G_k)(\nabla_j(G^{k\bar{j}}G_{\bar{j}})) = (\nabla_i G_k)G^{k\bar{j}}\partial_j G_{\bar{j}} = \nabla_i G_j \tag{16}$$

then we multiply $\nabla_j V$ by G_i and get:

$$G_i\nabla_j V = G_i G_j V + e^G(G_i G^k\nabla_j G_k + G_i G_j) \tag{17}$$

$$\Rightarrow e^G(G_i(\nabla_j G_k)G^k + G_i G_j) = G_i\nabla_j V - G_i G_j V \tag{18}$$

We replace the equations (16) and (17) to equation (14) and we arrive at the following equation for the second derivative of the scalar potential:

$$\boxed{\nabla_i\nabla_j V = e^G(G^k\nabla_i\nabla_j G_k + 2\nabla_i G_j) + G_i\nabla_j V + G_j\nabla_i V + (\nabla_i G_j - G_i G_j)V} \tag{19}$$

The result above is in agreement with [1]. Now we use the second (2) inflection point condition $\nabla_i V = 0$ and get:

$$\nabla_i \nabla_j V = e^G (G^k \nabla_i \nabla_j G_k + 2\nabla_i G_j) + (\nabla_i G_j - G_i G_j) V \tag{20}$$

This term should be also equal to zero due to the third of the conditions (2).

$$\boxed{e^G (G^k \nabla_i \nabla_j G_k + 2\nabla_i G_j) + (\nabla_i G_j - G_i G_j) V = 0} \tag{21}$$

this result contains the potential term and can be expressed only in G terms and its derivatives.

$$G^k \nabla_i \nabla_j G_k + (\nabla_i G_j) G^k G_k - \nabla_i G_j + 3 G_i G_j - G_i G_j G^k G_k = 0 \tag{22}$$

The second conjugate derivative of the first derivative of the potenial is:

$$\nabla_{\bar{j}} \nabla_i V = \nabla_{\bar{j}} [G_i V + e^G (G^k (\nabla_i G_k) + G_i)] \tag{23}$$

$$= [\nabla_{\bar{j}} G_i] V + G_i \nabla_{\bar{j}} V + G_{\bar{j}} e^G (G^k \nabla_i G_k + G_i) + e^G [\nabla_{\bar{j}} G_i + \nabla_{\bar{j}} G^k \nabla_i G_k + G^k \nabla_{\bar{j}} \nabla_i G_k] \tag{24}$$

$$= g_{i\bar{j}} V + G_i \nabla_{\bar{j}} V + G_{\bar{j}} \nabla_i V - G_{\bar{j}} G_i V + e^G [g_{i\bar{j}} + \nabla_{\bar{j}} G^k \nabla_i G_k + G^k \nabla_{\bar{j}} \nabla_i G_k] \tag{25}$$

the last term in the brackets becomes:

$$G^k [\nabla_{\bar{j}}, \nabla_i] G_k = G^k R^m_{\bar{j}ik} G_m = G^k G^{\bar{q}} R^m_{\bar{j}ik} g_{m\bar{q}} = G^k G^{\bar{q}} R_{\bar{j}ik\bar{q}} = -R_{i\bar{j}k\bar{q}} G^k G^{\bar{q}} \tag{26}$$

but also:

$$G^k [\nabla_{\bar{j}}, \nabla_i] G_k = G^k \nabla_{\bar{j}} \nabla_i G_k - G^k \nabla_i \nabla_{\bar{j}} G_k = G^k \nabla_{\bar{j}} \nabla_i G_k - G^k \nabla_i g_{k\bar{j}} = G^k \nabla_{\bar{j}} \nabla_i G_k \tag{27}$$

thus using (27) and (26),

$$\boxed{G^k \nabla_{\bar{j}} \nabla_i G_k = -R_{i\bar{j}k\bar{q}} G^k G^{\bar{q}}} \tag{28}$$

pluging this into (23) we arrive at the following equation:

$$\boxed{\nabla_{\bar{j}} \nabla_i V = (g_{i\bar{j}} - G_{\bar{j}} G_i) V + G_i \nabla_{\bar{j}} V + G_{\bar{j}} \nabla_i V + e^G [g_{i\bar{j}} + \nabla_{\bar{j}} G^k \nabla_i G_k - R_{i\bar{j}k\bar{q}} G^k G^{\bar{q}}]} \tag{29}$$

The result above is in agreement with [1] as well for V=0.

2 Mass Matrix of Scalar Potential ($V = 0$)

We start this analysis by writing down the mass matrix which consists of the second covariant holomorphic anti-holomorphic derivatives.

$$M = \begin{bmatrix} M_{\bar{a}b} & M_{\bar{a}\bar{b}} \\ M_{ab} & M_{a\bar{b}} \end{bmatrix} = \begin{bmatrix} \bar{D}_{\bar{a}} D_b V & \bar{D}_{\bar{a}} \bar{D}_{\bar{b}} V \\ D_a D_b V & D_a \bar{D}_{\bar{b}} V \end{bmatrix} \tag{30}$$

Here I have changed the notation of the covariant derivatives $\nabla_i = D_i$ to follow the bibliography [3], [4]. Since we want to calculate the mass matrix, the potential as well as the first derivative of the potential vanish (in contrast to the previous analysis).
We use the scalar potential expressed in superpotential terms:

$$V = e^K (D_i W D^i \bar{W} - 3W\bar{W}) \tag{31}$$

continously we take the first derivative of this potential ignoring the derivative of the exponential since it will give the potential which vanish at minimum.

$$\begin{aligned} D_b V &= e^K D_b (D_a W D^a \bar{W} - 3W\bar{W}) && (32) \\ &= e^K (D_b D_a W \bar{D}^a \bar{W} - 3\bar{W} D_b W + D_a W D_b \bar{D}^a \bar{W}) && (33) \\ &= e^K (D_b D_a W \bar{D}^a \bar{W} - 2\bar{W} D_b W) \qquad \texttt{we used } D_b \bar{D}^a \bar{W} = \delta^a_b \bar{W} && (34) \end{aligned}$$

Next we take the second covariant derivative of the scalar potential:

$$\begin{aligned} D_c D_b V &= e^K (D_c D_b D_a W \bar{D}^a \bar{W} + D_b D_a W D_c \bar{D}^a \bar{W} - 2D_c \bar{W} D_b W - 2\bar{W} D_c D_b W) && (35) \\ &= e^K (D_c D_b D_a W \bar{D}^a \bar{W} + (D_b D_a W) \delta^a_c \bar{W} - 2\bar{W} D_c D_b W) && (36) \\ &= e^K (D_c D_b D_a W \bar{D}^a \bar{W} - \bar{W} D_c D_b W) && (37) \end{aligned}$$

Afterwards we want to calculate the conjugate covaritant derivative of the first derivative of the potential. I will start from the general relation (29) instead of calculating the conjugate derivative of (35).(SKIP THESE CALCULATIONS AGO TO THE STABILIZATION PARAGRAPH). We have,

$$\boxed{D_{\bar{b}} D_a V = e^G [g_{a\bar{b}} + D_{\bar{b}} G^k D_a G_k - R_{a\bar{b}k\bar{q}} G^k G^{\bar{q}}]} \tag{38}$$

Now we wish to re-write this relation in terms of the superpotential W. But lets calculate again the first derivative expressed in G_i:

$$
\begin{aligned}
D_jV &= e^K W\bar{W}[g^{i\bar{j}}G_{\bar{j}}(D_jG_i) + G_j] && (39)\\
&= e^K[g^{i\bar{j}}(\bar{W}G_{\bar{j}})(WD_jG_i) + (WG_j)\bar{W}] && (40)\\
&= e^K[g^{i\bar{j}}(\bar{D}_{\bar{j}}\bar{W})(WD_jG_i) + (D_jW)\bar{W}] && (41)\\
&= e^K[g^{i\bar{j}}(\bar{D}_{\bar{j}}\bar{W})(D_j(WG_i) - (D_jW)G_i) + (D_jW)\bar{W}] && (42)\\
&= e^K[g^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W}D_j(D_iW) - g^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W}(D_jW)G_i + (D_jW)\bar{W}] && (43)\\
&= e^K[g^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W}D_j(D_iW) - g^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W}(D_jW)G_i + (D_jW)\bar{W}] \qquad \texttt{use}\ G^iG_i = 3 && (44)\\
&= e^K[g^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W}D_j(D_iW) - 2(D_jW)\bar{W}] && (45)\\
&= e^K[D_j(D_iW)\bar{D}^i\bar{W} - 2(D_jW)\bar{W}] && (46)
\end{aligned}
$$

the result is in agreement with [3] and (32). In the third row we used the relations:

$$
\begin{aligned}
WG_k &= W*(K_k + \frac{W_k}{W}) = WK_k + W_k = D_kW && (47)\\
\bar{W}G_{\bar{k}} &= \bar{W}*(K_{\bar{k}} + \frac{\bar{W}_{\bar{k}}}{\bar{W}}) = \bar{W}K_{\bar{k}} + \bar{W}_{\bar{k}} = \bar{D}_{\bar{k}}\bar{W} && (48)
\end{aligned}
$$

The derivative of the potential should vanish at the global minima, we write the relation above which we will use soon:

$$
D_jV = 0 \Rightarrow D_jD_iW\bar{D}^i\bar{W} = 2(D_jW)\bar{W} \tag{49}
$$

Next we calculate the second covariant derivative of the potential, we start from equation (38):

$$
\begin{aligned}
\bar{D}_{\bar{j}}D_iV &= e^G[g_{i\bar{j}} + \bar{D}_{\bar{j}}G^kD_iG_k - R_{i\bar{j}k\bar{q}}G^kG^{\bar{q}}] && (50)\\
&= e^KW\bar{W}[g_{i\bar{j}} + g^{k\bar{m}}\bar{D}_{\bar{j}}G_{\bar{m}}D_iG_k - R_{i\bar{j}k\bar{q}}G^kG^{\bar{q}}] && (51)\\
&= e^K[g_{i\bar{j}}W\bar{W} + g^{k\bar{m}}(\bar{W}\bar{D}_{\bar{j}}G_{\bar{m}})(WD_iG_k) - R_{i\bar{j}k\bar{q}}(WG^k)(\bar{W}G^{\bar{q}})] && (52)\\
&= e^K[g_{i\bar{j}}W\bar{W} + g^{k\bar{m}}(\bar{W}\bar{D}_{\bar{j}}G_{\bar{m}})(WD_iG_k) - R_{i\bar{j}k\bar{q}}(D^kW)(\bar{D}^{\bar{q}}\bar{W})] && (53)
\end{aligned}
$$

In the second row we have used the metric compatibility in order to lower the index of the second term. Since we want to express the above relation only in terms of the superpotential we write the second term as:

$$
\begin{aligned}
g^{k\bar{m}}(\bar{W}\bar{D}_{\bar{j}}G_{\bar{m}})(WD_iG_k) &= g^{k\bar{m}}\Big(\bar{D}_{\bar{j}}(G_{\bar{m}}\bar{W}) - G_{\bar{m}}\bar{D}_{\bar{j}}\bar{W}\Big)*\Big(D_i(G_kW) - G_kD_iW\Big) && (54)\\
&= g^{k\bar{m}}\Big(\bar{D}_{\bar{j}}\bar{D}_{\bar{m}}\bar{W} - G_{\bar{m}}\bar{D}_{\bar{j}}\bar{W}\Big)*\Big(D_iD_kW - G_kD_iW\Big) && (55)
\end{aligned}
$$

and we again make use of the relation which comes from the first derivative of the potential:

$$
\begin{aligned}
(D_aD_bW)\bar{D}^b\bar{W} &= 2(D_aW)\bar{W} && (56)\\
(\bar{D}_{\bar{a}}\bar{D}_{\bar{b}}\bar{W})D^{\bar{b}}\bar{W} &= 2(\bar{D}_{\bar{a}}\bar{W})W && (57)
\end{aligned}
$$

then

$$
\begin{aligned}
&g^{k\bar{m}}\Big(\bar{D}_{\bar{j}}\bar{D}_{\bar{m}}\bar{W}-G_{\bar{m}}\bar{D}_{\bar{j}}\bar{W}\Big)*\Big(D_iD_kW-G_kD_iW\Big) && (58)\\
&=g^{k\bar{m}}\Big(\bar{D}_{\bar{j}}\bar{D}_{\bar{m}}\bar{W}D_iD_kW+G_{\bar{m}}G_k\bar{D}_{\bar{j}}\bar{W}D_iW-G_kD_iW\bar{D}_{\bar{j}}\bar{D}_{\bar{m}}\bar{W}-G_{\bar{m}}\bar{D}_{\bar{j}}\bar{W}D_iD_kW\Big) && (59)\\
&=g^{k\bar{m}}\Big(\bar{D}_{\bar{j}}\bar{D}_{\bar{m}}\bar{W}D_iD_kW+G_{\bar{m}}G_k\bar{D}_{\bar{j}}\bar{W}D_iW-2g_{i\bar{m}}\bar{D}_{\bar{j}}\bar{W}D_kW-2g_{k\bar{j}}\bar{D}_{\bar{m}}\bar{W}D_iW\Big) && (60)\\
&=\Big(\bar{D}_{\bar{j}}\bar{D}^k\bar{W}D_iD_kW+3\bar{D}_{\bar{j}}\bar{W}D_iW-2\bar{D}_{\bar{j}}\bar{W}D_iW-2\bar{D}_{\bar{j}}\bar{W}D_iW\Big) && (61)\\
&=\Big(\bar{D}_{\bar{j}}\bar{D}^k\bar{W}D_iD_kW-\bar{D}_{\bar{j}}\bar{W}D_iW\Big) && (62)
\end{aligned}
$$

we put this result into (63)

$$
\begin{aligned}
&\bar{D}_{\bar{j}}D_iV=e^K[g_{i\bar{j}}W\bar{W}+\bar{D}_{\bar{j}}\bar{D}^k\bar{W}D_iD_kW-\bar{D}_{\bar{j}}\bar{W}D_iW-R_{i\bar{j}k\bar{q}}(D^kW)(\bar{D}^{\bar{q}}\bar{W})] && (63)\\
&D_jD_iV=e^K(D_jD_iD_kW\bar{D}^k\bar{W}-D_jD_iW\bar{W}) && (64)
\end{aligned}
$$

However in [3] some extra term appear which are already contained in out result but in different form:

$$
\begin{aligned}
V=0\Rightarrow & e^K(D^iW\bar{D}_i\bar{W}-3W\bar{W})=0 && (65)\\
\Rightarrow & g_{a\bar{b}}D^iW\bar{D}_i\bar{W}-2g_{a\bar{b}}W\bar{W}-g_{a\bar{b}}W\bar{W}=0 && (66)\\
\Rightarrow & g_{a\bar{b}}W\bar{W}=g_{a\bar{b}}D^iW\bar{D}_i\bar{W}-2g_{a\bar{b}}W\bar{W} && (67)
\end{aligned}
$$

and the missing terms appear!

2.0.1 Stabilization

The rest of the analysis follows the idea of [4] but is more analytically written and explained and then applied to the attractor model. Until now we have not assumed a specific form of superpotential either of Kähler potential. We work for superpotentials of the from $W=Sf(\Phi)$. The inflaton will be the real part of Φ while S is the chiral superfield which accomodates the goldstino. We want the supersymmetry to be spontaneously broken during inflation thus $F_\Phi=e^{\frac{K}{2}}D_\Phi W=0$ which implies $S=0$ and $F_S=e^{\frac{K}{2}}D_SW\neq 0$ which implies $f(\Phi)\neq 0$. Thus the superpotential vanishes as well since it's a product of the S superfield. Since $W=0$,

$$
V=e^K(D^iW\bar{D}_i\bar{W})=e^K(D^SW\bar{D}_S\bar{W}+D^\Phi W\bar{D}_\Phi\bar{W})=F^S\bar{F}_S=3H^2 \tag{68}
$$

where in the last equality we used the background equation and assumed the velocity to be negligible and $M_P=1$. So the matrix becomes:

$$
M_{i\bar{j}}=(g_{i\bar{j}}g_{k\bar{m}}-g_{i\bar{m}}g_{j\bar{k}}-R_{i\bar{j}k\bar{m}})\bar{F}^kF^{\bar{m}}+D_iF_k\bar{D}_{\bar{j}}\bar{F}^k \tag{69}
$$

BE CAREFULL! The last term in the mass matrix is related to the slow-roll parameters. In [4] (paragraph before relation (30), sentence before relation (19), also (40),(41)) is everywhere assumed that ϵ,$\eta \ll$1. For large η and ϵ these terms should not be neglected.

Lets make some assumptions of the Kähler potential form. If it respects a discrete symmetry $S \to -S$ odd derivatives of the Kähler potential cancel out. The Levi-Cevita connections are odd derivative quantities of the Kähler thus they vanish. This turns to the simplification:

$$R_{i\bar{j}m\bar{k}} = K_{i\bar{j}m\bar{k}} - \Gamma^{n}_{im} g_{n\bar{n}} \Gamma^{\bar{n}}_{\bar{j}\bar{k}} = K_{i\bar{j}m\bar{k}} \tag{70}$$

which is the most crucial term of this analysis.

Tachyonic Instability We start by calculating the matrix elements of the mass matrix:

$$\begin{aligned}
M_{\Phi\bar{\Phi}} =&(g_{\Phi\bar{\Phi}} g_{k\bar{m}} - g_{\Phi\bar{m}} g_{\Phi\bar{k}} - R_{\Phi\bar{\Phi}k\bar{m}}) \bar{F}^k F^{\bar{m}} + D_\Phi F_k \bar{D}_{\bar{\Phi}} \bar{F}^k && (71)\\
=&(g_{\Phi\bar{\Phi}} g_{S\bar{S}} - g_{\Phi\bar{S}} g_{\Phi\bar{S}} - R_{\Phi\bar{\Phi}S\bar{S}}) \bar{F}^S F^{\bar{S}} + D_\Phi F_S \bar{D}_{\bar{\Phi}} \bar{F}^S + D_\Phi F_\Phi \bar{D}_{\bar{\Phi}} \bar{F}^\Phi && (72)\\
=&(g_{\Phi\bar{\Phi}} - R_{\Phi\bar{\Phi}S\bar{S}}) V + D_\Phi F_S \bar{D}_{\bar{\Phi}} \bar{F}^S + D_\Phi F_\Phi \bar{D}_{\bar{\Phi}} \bar{F}^\Phi && (73)\\
M_{S\bar{S}} =&(g_{S\bar{S}} g_{k\bar{m}} - g_{S\bar{m}} g_{S\bar{k}} - R_{S\bar{S}k\bar{m}}) \bar{F}^k F^{\bar{m}} + D_S F_k \bar{D}_{\bar{S}} \bar{F}^k && (74)\\
=&(g_{S\bar{S}} g_{S\bar{S}} - g_{S\bar{S}} g_{S\bar{S}} - R_{S\bar{S}S\bar{S}}) \bar{F}^S F^{\bar{S}} + D_S F_S \bar{D}_{\bar{S}} \bar{F}^S + D_S F_\Phi \bar{D}_{\bar{S}} \bar{F}^\Phi && (75)\\
=&- R_{S\bar{S}S\bar{S}} \bar{F}^S F^{\bar{S}} + D_S F_S \bar{D}_{\bar{S}} \bar{F}^S + D_S F_\Phi \bar{D}_{\bar{S}} \bar{F}^\Phi && (76)
\end{aligned}$$

we notify again that these masses contain the covariant derivative terms which have been neglected in relation (34) in [4].

$$\begin{aligned}
D_S F_S =&D_S(e^{\frac{K}{2}} D_S W) = D_S(e^{\frac{K}{2}}(\partial_S W + K_S W)) = 0 && (77)\\
D_\Phi F_S =&D_\Phi(e^{\frac{K}{2}} D_S W) = D_\Phi(e^{\frac{K}{2}}(\partial_S W + K_S W)) && (78)\\
=&D_\Phi(e^{\frac{K}{2}} \partial_S W) = D_\Phi(e^{\frac{K}{2}} f(\Phi)) = \partial_\Phi f(\Phi) && (79)\\
D_S F_\Phi =&D_S(e^{\frac{K}{2}} D_\Phi W) = D_S(\partial_\Phi W + K_\Phi W) = \partial_\Phi f(\Phi) && (80)\\
D_\Phi F_\Phi =&D_\Phi(e^{\frac{K}{2}} D_\Phi W) = D_\Phi(\partial_\Phi W + K_\Phi W) = 0 && (81)
\end{aligned}$$

in the last row we've assumed that out Kähler potential respect a Φ=$\bar{\Phi}$ and a shift symmetry that make is vanish solging this way the possible η-problem as well. After all that the masses are:

$$\begin{aligned}
M_{\Phi\bar{\Phi}} =&(g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}}) V + (\partial_\Phi f(\Phi))^2 && (82)\\
=&(g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}} + \epsilon) V && (83)\\
M_{S\bar{S}} =&- K_{S\bar{S}S\bar{S}} V + (\partial_\Phi f(\Phi))^2 = (-K_{S\bar{S}S\bar{S}} + \epsilon) V && (84)
\end{aligned}$$

an extra $|\partial_\Phi f(\Phi)|^2$ term appears if we down assume slow roll aproximation. We used the first and second slow roll parameters:

$$\epsilon = \frac{1}{2}\Big(\frac{\partial_\Phi V}{V}\Big)^2 = \frac{(\partial_\Phi f)^2}{f^2} \tag{85}$$

$$\eta = \frac{(\partial_\Phi V)^2}{V} = \frac{(\partial^2_\Phi f)}{f} + \frac{(\partial_\Phi f)^2}{f^2} = \frac{(\partial^2_\Phi f)}{f} + \epsilon \tag{86}$$

We want to calculate the rest of the matric elements.

$$
\begin{aligned}
D_c D_b V &= e^K (D_c D_b D_a W \bar{D}^a \bar{W} - \bar{W} D_c D_b W) && (87)\\
&= D_c D_b F_a \bar{F}^a - D_c F_b \bar{W} && (88)
\end{aligned}
$$

and the rest of matrix elements are:

$$
\begin{aligned}
M_{SS} &= D_S D_S F_S \bar{F}^S + D_S D_S F_\Phi \bar{F}^\Phi - D_S F_S \bar{W} && (89)\\
&= D_S D_S F_S \bar{F}^S + D_S D_S F_\Phi \bar{F}^\Phi && (90)\\
&= 0 + D_S(\partial_\Phi f(\Phi))\overline{f(\Phi)} && (91)\\
&= 0 && (92)\\
M_{\Phi\Phi} &= D_\Phi D_\Phi F_S \bar{F}^S + D_\Phi D_\Phi F_\Phi \bar{F}^\Phi - D_\Phi F_\Phi \bar{W} && (93)\\
&= D_\Phi D_\Phi F_S \bar{F}^S + 0 && (94)\\
&= \overline{f(\Phi)} \partial_{\Phi\Phi} f(\Phi) && (95)
\end{aligned}
$$

Finally we can write down the masses of the S and Φ superfields.

$$
m^2_{S\pm} = \begin{bmatrix} M_{\bar{S}S} & M_{\bar{S}\bar{S}} \\ M_{SS} & M_{S\bar{S}} \end{bmatrix} = \begin{bmatrix} M_{\bar{S}S} & 0 \\ 0 & M_{S\bar{S}} \end{bmatrix} \qquad (96)
$$

in the followng calculation we use that $M_{\bar{\Phi}\Phi} = M_{\Phi\bar{\Phi}}$ and find the eigenvalues as well:

$$
m^2_{\Phi\pm} = \begin{bmatrix} M_{\bar{\Phi}\Phi} & M_{\bar{\Phi}\bar{\Phi}} \\ M_{\Phi\Phi} & M_{\Phi\bar{\Phi}} \end{bmatrix} = \begin{bmatrix} M_{\bar{\Phi}\Phi} + \sqrt{M_{\bar{\Phi}\bar{\Phi}} M_{\Phi\Phi}} & 0 \\ 0 & M_{\bar{\Phi}\Phi} - \sqrt{M_{\bar{\Phi}\bar{\Phi}} M_{\Phi\Phi}} \end{bmatrix} \qquad (97)
$$

The eigenvalues of the matrices give the masses:

$$
\begin{aligned}
m^2_{Im\Phi} &= M_{\bar{\Phi}\Phi} - \sqrt{M_{\bar{\Phi}\bar{\Phi}} M_{\Phi\Phi}} = (g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}} + \epsilon)V - f\partial_{\Phi\Phi} f && (98)\\
&= (g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}} + \epsilon)V - f^2 \frac{\partial_{\Phi\Phi} f}{f} = (g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}} + \epsilon - \frac{\partial_{\Phi\Phi} f}{f})V && (99)\\
&= (g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}} + 2\epsilon - \eta)V && (100)\\
m^2_{ReS} &= m^2_{ImS} = M_{\bar{S}S} = (-K_{S\bar{S}S\bar{S}} + \epsilon)V && (101)
\end{aligned}
$$

I'm actually missing a 2 factor in the first mass term $m^2_{Im\Phi} = V(2(g_{\Phi\bar{\Phi}} - K_{\Phi\bar{\Phi}S\bar{S}}) + 2\epsilon - \eta)$ but I cannot understand where it comes from. In order to ensure non-tachyonic instabilities the mass should be positive and in order to ensure single field inflation the fields, except the inflaton, should be heavier than H^2.

$$m^2_{Im\Phi} \geqslant H^2 \Rightarrow 3H^2(2(g_{\Phi\bar\Phi} - K_{\Phi\bar\Phi S\bar S}) + 2\epsilon - \eta) \geqslant H^2 \tag{102}$$

$$\Rightarrow (2(g_{\Phi\bar\Phi} - K_{\Phi\bar\Phi S\bar S}) + 2\epsilon - \eta) \geqslant \frac{1}{3} \tag{103}$$

$$\boxed{\Rightarrow K_{\Phi\bar\Phi S\bar S} \leqslant -\frac{1}{6} + \epsilon - \frac{\eta}{2} + g_{\Phi\bar\Phi}} \tag{104}$$

$$m^2_{ReS,ImS} \geqslant H^2 \Rightarrow 3H^2(-K_{S\bar S S\bar S} + \epsilon) \geqslant H^2 \tag{105}$$

$$\boxed{\Rightarrow K_{S\bar S S\bar S} \leqslant -\frac{1}{3} + \epsilon} \tag{106}$$

Superconformal Attractors We now explore the instabilities in superconformal attractor models. The Kähler potential doesn't have a γ stabilizer which controls the possible instabilities in ImΦ. We choose the R-symmetry gauge $X^0 = \bar X^0 = \sqrt{3}$.

$$K(X, \bar X) = -3\,\mathrm{Log}\Big(-|X^0|^2 + |X^1|^2 + |S|^2 - 3\zeta \frac{(S\bar S)^2}{|X^0|^2 - |X^1|^2}\Big) \tag{107}$$

in this gauge the Kähler becomes:

$$K = -3\,\mathrm{Log}\Big(1 - \frac{|\Phi|^2 + |S|^2}{3} + \zeta\frac{(S\bar S)^2}{3 - |\Phi|^2}\Big) \tag{108}$$

I have calculated the $K_{S\bar S S\bar S}$ and $K_{\Phi\bar\Phi S\bar S}$ using Mathematica (the file is in the latex folder) and the curvatures are $K_{\Phi\bar\Phi S\bar S} = \frac{1}{3}$ and $K_{S\bar S S\bar S} = \frac{2}{3} - 4\zeta$ in agreement with [7]. We put those into the general relations (97) to achieve ReΦ single field inflation:

$$m^2_{Im\Phi} = 3H^2(2(g_{\Phi\bar\Phi} - K_{\Phi\bar\Phi S\bar S}) + 2\epsilon - \eta) \geqslant H^2 \tag{109}$$

$$\Rightarrow \boxed{(2(g_{\Phi\bar\Phi} - \frac{1}{3}) + 2\epsilon - \eta) \geqslant \frac{1}{3}} \tag{110}$$

$$m^2_{ReS,ImS} = 3H^2(-K_{S\bar S S\bar S} + \epsilon) \geqslant H^2 \tag{111}$$

$$\Rightarrow \boxed{\zeta \geqslant \frac{1-\epsilon}{4}} \tag{112}$$

those are the relations should be satisfied in order to make these fields heavy enough to achieve single field inflation only by ReΦ. The constaint for tachyonic stabilizations is $m^2 \geqslant 0$.

$$m^2_{ReS,ImS} \geqslant 0 \tag{113}$$

$$\Rightarrow (-K_{S\bar S S\bar S} + \epsilon) \geqslant 0 \tag{114}$$

$$\Rightarrow -\frac{2}{3} + 4\zeta + \epsilon \geqslant 0 \tag{115}$$

$$\Rightarrow \zeta \geqslant \frac{1}{6} - \frac{\epsilon}{4} \tag{116}$$

for $\epsilon \ll 1$, $\zeta \geqslant \frac{1}{6}$ in agreement with [7]. However near the inflection point the slow roll parameters are large.

What are the values of ϵ and η when the SRA is violated? He have discussed in a different document that slow roll approximation has to be violated in order PBHs to be produced [5]. This happens when the potentials are extremely flat and the deceleration term dominates over the Hubble friction term [6]. In USR (ultra slow roll) the Klein Gordon becomes:

$$\ddot{\phi} + 3H\dot{\phi} = 0 \tag{117}$$

the second slow roll parameter becomes (I've done the calculations explicitly):

$$\epsilon_2 = \frac{\dot{\epsilon}}{\epsilon H} = \frac{2\ddot{\phi}}{H\dot{\phi}} + 2\epsilon = -6 - \frac{2V'}{H\dot{\phi}} + 2\epsilon \tag{118}$$

where $\epsilon = -\dot{H}/H^2$ is the exact first slow roll parameter. In USR $V' = 0$, so the slow roll condition becomes:

$$\epsilon_2 = -6 + 2\epsilon \tag{119}$$

thus for $\epsilon \ll 1$ the second slow roll parameter $\epsilon_2 \approx -6$ and near the inflection point $\epsilon_2 = -4$. Thus we cannot neglect the slow roll parameters which appear in the masses (97). This will change the stabilizer value.

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