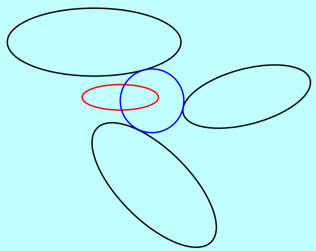
Apollonius Circle Conflict

George M. Tzoumas   Ioannis Z. Emiris

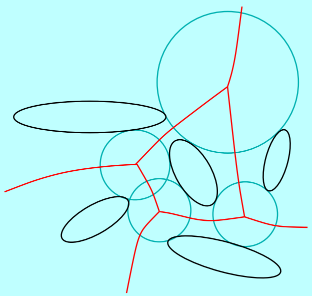
National University of Athens

Problem definition

What does a *query ellipse* do (outside, intersects or tangent) with respect to the *Apollonius circle* of the three black ellipses?



Motivation



This is the most critical test when one tries to compute the Voronoi diagram of ellipses on the plane.

Predicates

More predicates are actually needed for the computation of the Voronoi diagram. We have efficiently solved the following ones:

- compute a point inside an ellipse — *the center is computed as a rational expression of the parameters of the ellipse*
- compare distances between a point and two ellipses — *we proved that the distance is an algebraic number of degree 4 (optimally) and simply compare two such algebraic numbers*
- relative position of an ellipse with respect to an external bitangent line of two other ones — *we compute all bitangents by efficiently solving a 2×2 system of degree 2, then decide the predicate by evaluating polynomials of at most degree 2 on algebraic numbers of the same degree*

Algebraic approach for main predicate

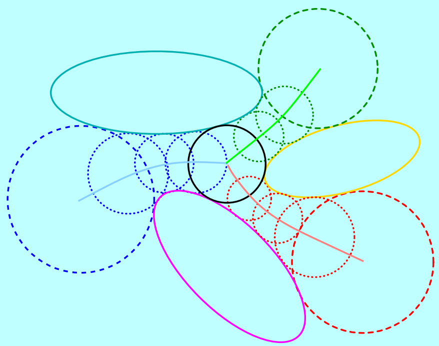
- Solving equations *numerically* could lead to *incorrect* results, due to numerical *errors*.
- Solving *symbolically* is too difficult due to the *high degree* of equations involved.
- *Optimal* systems for this problem are a 6×6 one of degree $\{3, 3, 3, 2, 2, 2\}$ and a 3×3 one of degree $\{6, 6, 6\}$.
- There are 184 complex circles tangent to three different conics in the plane. This bound is *tight*. The above systems have mixed volumes of 184.
- Can all such circles be *real*? We have configurations where 23 circles exist. Note that $184 = 8 \times 23$.

Subdivision-based approach

- *Iterative* — intermediate computations using full precision. Problem precision progressively increases.

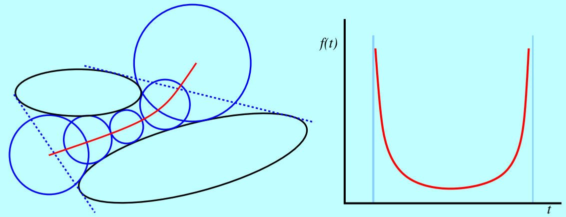
Basic Idea

Slide a circle along the boundary of an ellipse while keeping it tangent to two ellipses. Its center moves on the bisector.

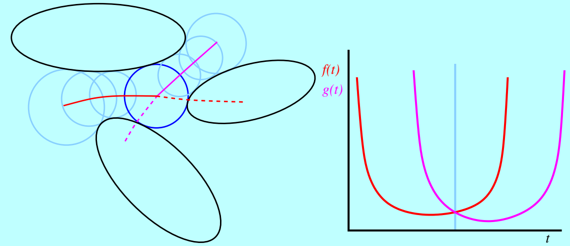


We use the parametric form of the ellipse $x(t),\ y(t)$, where $t \in (-\infty, \infty)$.

Let function f be the radius of the bitangent circle as it slides along the boundary.

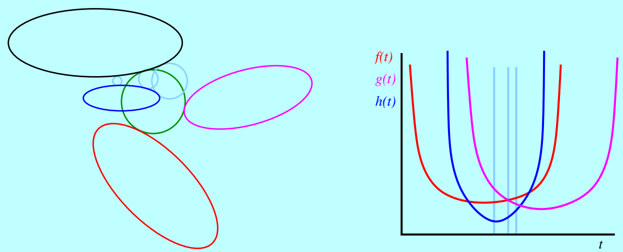


The Apollonius circle is located at the intersection of two such functions.



Deciding the predicate

The *bisection* method, properly adapted to exploit the *geometric* behaviour of the functions, allows us to quickly isolate the intersection point and decide the predicate.



An upper bound on the number of iterations needed in the worst case is provided by the gap theorem [Canny87]. This is 1 million iterations for 32-bit input quantities. We are currently working on improving the termination condition.

Sample times from our *Maple* implementation:

5s	no confilct
20s	conflict
64s	conflict where two ellipses almost coincide