- Strong Law of Large Numbers (SLLN) {proven using Kolmogorov's inequality}

To overcome the dependence of the covergence to the σ^2 of the distribution of X we define a different kind of convergence and use the Kolmogorov inequality instead of Chebyshev's.

A sequence of random $\{Z_1, \dots, Z_n\}$ converges almost surely to c as $n \to \infty$ if $\lim_{n\to\infty} P[Z_n = c] = 1$

$$Z_n \xrightarrow[n\to\infty]{a.s} c$$

$$\boxed{\text{If } \begin{array}{l} X_i : \text{iid} \\ E[X] \equiv \mu < \infty \end{array} \Rightarrow \lim_{n\to\infty} P[\bar{X}_n = \mu] = 1 \text{ thus } \bar{X}_n \xrightarrow[n\to\infty]{a.s} \mu}$$

(*) The convergence of SLLN is uniform, as it does not depend on σ^2, and thus does not lead to any "anomalous" behaviour as $n \gg 1$ but yet $n \neq \infty$ {unlike the WLLN...}

There is the analogous

Continuous Maping Theorem (CMT) : $\boxed{\text{If } \begin{array}{l} Z_n \xrightarrow[n\to\infty]{a.s} c \\ h(\,): \text{continuous at } c \end{array} \Rightarrow h(Z_n) \xrightarrow[n\to\infty]{a.s} h(c)}$

$\{\text{If } Z_n \xrightarrow[n\to\infty]{a.s} c \Rightarrow h(Z_n) \xrightarrow[n\to\infty]{a.s} h(c)\}$

! The value of LLN : Given that $E[\bar{X}_n] = \mu$ why do we care about the LLN result?
- In order to destimate μ through $\mu = E[\bar{X}_n]$ we need to take many times (if not ∞) n-sample, calculate the $\bar{X}_n$ each time and finally calculate $E[\bar{X}_n]$ to estimate μ. {This inpractical!}
- With LLN we can instead estimate $\mu \simeq \bar{X}_n$ by taking only once a big enough sample ($n \gg 1$)