WLLN: If $\begin{matrix} X_i \in \mathbb{R}^m \text{ iid} \\ E\|X\| < \infty \end{matrix}$ then $\boxed{\bar{X}_n \equiv \frac{1}{n}\sum_1^n X_i \xrightarrow[p]{n\to\infty} E[X]}$ $\{\bar{X}_n : \text{consistent}\}$

CLT: If $\begin{matrix} X_i \in \mathbb{R}^m \text{ iid} \\ E\|X\|^2 < \infty \end{matrix}$ then $\boxed{\sqrt{n}(\bar{X}_n - \mu) \xrightarrow[d]{n\to\infty} Z \sim N(0,\Sigma)}$

Delta Method: $\boxed{\text{If } \sqrt{n}(\hat{\theta}-\theta) \xrightarrow{d} Z \text{ then } \sqrt{n}(h(\hat{\theta})-h(\theta)) \xrightarrow[d]{} H^T_{(\theta)} \cdot Z}$, where

$$\theta \equiv \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_m \end{bmatrix} \in \mathbb{R}^m,\quad h(\theta) \equiv \underbrace{\begin{bmatrix} h_1(\theta_1,\dots,\theta_m) \\ \vdots \\ h_k(\theta_1,\dots,\theta_m) \end{bmatrix}}_{(k\times 1)} \in \mathbb{R}^k \Rightarrow \underbrace{h^T}_{(1\times k)} \in \mathbb{R}^k \Rightarrow H_{(\theta)} \equiv \frac{\partial h^T}{\partial \theta} \equiv \underbrace{\begin{bmatrix} \frac{\partial h_1}{\partial \theta_1} & \cdots & \frac{\partial h_k}{\partial \theta_1} \\ \vdots & & \vdots \\ \frac{\partial h_k}{\partial \theta_1} & \cdots & \frac{\partial h_k}{\partial \theta_m} \end{bmatrix}}_{m\times k} \Rightarrow H^T_{(\theta)} : k\times m \Rightarrow H^T_{(\theta)} \cdot Z : k\times 1$$

- If $Z \sim N(0,\Sigma)$ then $H^T_{(\theta)} \cdot Z \sim N(0, H^T \Sigma_\theta H)$. This is the case when
$\theta \equiv \mu \equiv E[X]$, thus $\hat{\theta} = \bar{X}_n \equiv \frac{1}{n}\sum_1^n X_i$ with $\Sigma_\theta \equiv \text{var}[X] \equiv E[(X-\mu)(X-\mu)^T]$
OR
$\theta \equiv E[g(X)]$, thus $\hat{\theta} \equiv \frac{1}{n}\sum_1^n g(X_i)$ with $\Sigma_\theta \equiv \text{var}[g(X)] = E[(g(X)-E[g(X)])(g(X)-E[g(X)])^T]$

- To practically use the latter we need an estimation for the value of $H^T \cdot \Sigma \cdot H$
We choose $\hat{\Sigma}_{h(\theta)} \equiv \hat{H}^T \hat{\Sigma} \hat{H} \equiv H^T(\hat{\theta}) \cdot \frac{1}{n-1}\left(\sum_1^n (g(X_i)-\hat{\theta})(g(X_i)-\hat{\theta})^T\right) \cdot H(\hat{\theta})$ which is consistent: $\hat{\Sigma}_{h(\theta)} \xrightarrow{p} \Sigma_\theta$

- For $h(\theta) : \mathbb{R}$, thus $\mathbb{R}^m \xrightarrow{h()} \mathbb{R}$, we get the T-statistic: $\sqrt{n}\frac{(h(\hat{\theta})-h(\theta))}{\sqrt{\hat{\Sigma}_{h(\theta)}}} \xrightarrow{d} \xi \sim \frac{N(0, H^T\Sigma_\theta H)}{\sqrt{\Sigma_{h(\theta)}}} \equiv N(0,1)$