

Let a random X with unknown $f(x)$. We are interested in the value of a parameter θ determined by $f(x)$. In particular we want to test the null hypothesis $H_0: \theta = \theta_0$ against the alternative hypothesis $H_1: \theta \neq \theta_0$.

From a random sample $\{X_1, \dots, X_n\}$ construct an appropriate statistic $T(X_1, \dots, X_n)$.

Calculate the null sampling distribution $G_0(x) \equiv P[T \leq x | \frac{F}{\theta} = \frac{F_0}{\theta_0}]$.

If the (measured) $T(x_1, \dots, x_n)$ $\begin{cases} \rightarrow x \in C: \text{accept (or fail to reject)} H_0 \\ \rightarrow x \notin C: \text{reject } H_0 \end{cases}$ where $C \subseteq \mathbb{R}$: critical region.

But how should we define C ?

Define the C such that $\underbrace{P[\text{reject } H_0 | H_0: \text{true}]}_{\text{Type-I error}} \leq \underbrace{\alpha}_{\text{significance level}} \in (0, 1)$.
 (If possible) $\Rightarrow \begin{cases} \text{1-sided: } 1 - G_0(T) \leq \alpha \Rightarrow C = G_0^{-1}(1 - \alpha) \\ \text{2-sided: } 1 - G_0(c) + G_0(-c) = \alpha \Rightarrow G_0(c) = 1 - \alpha/2 \Rightarrow C = G_0^{-1}(1 - \alpha/2) \end{cases}$
 power

Among various appropriate statistics $\{(T^1, G_0^1), (T^2, G_0^2), \dots\}$ choose the one with $\max 1 - P[\text{Accept } H_0 | H_1: \text{true}]$
 type-II error

In some cases: Reject H_0 when $T > C$ \Rightarrow Then we can define $p \equiv 1 - G_0(T)$ $\xRightarrow{G_0(c) = 1 - \alpha}$ Reject H_0 when $p < \alpha$
 p-value