

1 discrete random variable

$$X \in \mathcal{H} \equiv \{\tau_0, \tau_1, \dots\}$$

→ probability mass function:

$$\pi(\tau_i) \equiv P[X = \tau_i]$$

↑ random variable
↑ support
↑ discrete values

$$P[X \in \mathcal{H}] \equiv \sum_{i=0}^{\infty} \pi(\tau_i) \equiv 1$$

• expectation: $\mu \equiv E[X] \equiv \sum_{i=0}^{\infty} \tau_i \cdot \pi(\tau_i) \leadsto E[g(X)] \equiv \sum_i g(\tau_i) \cdot \pi(\tau_i)$

• variance: $\sigma^2 \equiv \text{var}[X] \equiv E[(X - \mu)^2] \equiv \sum_{i=0}^{\infty} (\tau_i - \mu)^2 \cdot \pi(\tau_i) \leadsto E[g(X)] \equiv \sum_i (g(\tau_i) - E[g(X)])^2 \cdot \pi(\tau_i)$

• standard deviation: $\sigma \equiv \sqrt{\sigma^2}$

1 continuous random variable

$$X \in \mathcal{H} \equiv \{-\infty, +\infty\}$$

→ probability density function:

$$f(x) \cdot dx \equiv P[X \in [x, x+dx]]$$

$$P[X \in \mathcal{H}] \equiv \int_{-\infty}^{\infty} f(x) \cdot dx \equiv 1$$

• expectation: $\mu \equiv E[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx \leadsto E[g(X)] \equiv \int_{-\infty}^{\infty} g(x) \cdot f(x) \cdot dx$

• variance: $\sigma^2 \equiv \text{var}[X] = E[(X - \mu)^2] \equiv \int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) \cdot dx \leadsto \text{var}[g(X)] \equiv \int_{-\infty}^{\infty} (g(x) - E[g(X)])^2 \cdot f(x) \cdot dx$

• standard deviation: $\sigma \equiv \sqrt{\sigma^2}$

! important results

$$\begin{aligned} &\rightarrow \text{var}[X] = E[X^2] - (E[X])^2 \\ &\rightarrow E[a + b \cdot X] = a + b \cdot E[X] \\ &\rightarrow \text{var}[a + b \cdot X] = b^2 \cdot \text{var}[X] \end{aligned}$$

$a, b \in \mathbb{R}$