Chebyshev/Kolmogorov Inequalities

- <u>Chebyshev's Inequality</u>

$$\boxed{\forall \underset{\text{variable}}{\text{random}}\ X \text{ and } \delta>0 : P[|X-E[X]|\geq\delta] \leq \frac{var[X]}{\delta}}$$

(*) Important application: For $\bar{X}_n$ we get $P[|\bar{X}_n - E[\bar{X}_n]|\geq\delta] \leq \frac{var[\bar{X}_n]}{\delta} \Rightarrow P[|\bar{X}_n-\mu|\geq\delta]\leq \sigma^2/n\cdot\delta$

Now, we can answer to the question:
"How big must be the n-sample to ensure that the probability-P of the estimate $\bar{X}_n$ being further than $\pm\delta$ from the true μ, is less than or equal to P_0?"

$P\leq P_0 \Rightarrow \sigma^2/n\cdot\delta \leq P_0 \Rightarrow n \geq \sigma^2/\delta\cdot P_0$

(+) result does not depend on distribution of X
(−) bound can be quite imprecise
(−) result varies significantly across distributions with very different σ^2

- <u>Kolmogorov's Inequality</u>

strengthens Chebyshev's inequality

$$\boxed{\text{If } \begin{array}{l} X_i\text{'s: independent} \\ \text{and} \\ E[X]=0 \\ E[X^2]<\infty \end{array} \Longrightarrow \forall\varepsilon>0\ P\left[\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} X_i\right|>\varepsilon\right] \leq \sum_{i=1}^{n} E[X_i^2]/\varepsilon^2}$$