$\boxed{5}$ RANDOM VECTORS

Let a multivariate random variable $X \equiv \begin{pmatrix} X_1 \\ \vdots \\ X_m \end{pmatrix} \equiv (X_1 \cdots X_n)^T \in R^m$ (also called random vector)

with

joint cumulative distribution function (CDF): $F(x) \equiv P[X \leq x] \equiv P[X_1 < x_1, \ldots, X_m < x_m]$ where $x \equiv \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$

joint probability distribution function (PDF): $f(x) \equiv P[X \in [x, x+dx]] \equiv \frac{\partial^m F(x)}{\partial x_1 \cdots \partial x_m} = f(x_1, \ldots, x_m)$

Then

Expectation of $X \in R^m$: μ $E[X] \equiv \begin{bmatrix} E[X_1] \\ \vdots \\ E[X_m] \end{bmatrix} \equiv \begin{bmatrix} \int x_1 \cdot f(x) dx \\ \vdots \\ \int x_m \cdot f(x) dx \end{bmatrix}$ where $f(x)dx \equiv f(x_1, \ldots, x_m)\, dx_1 \cdot \ldots \cdot dx_m$

Variation or covariance matrix of $X \in R^m$: $\Sigma \equiv var[X] \equiv E[(X - E[X]) \cdot (X - E[X])^T] \equiv \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 & \ldots & \sigma_{1m}^2 \\ \sigma_{21}^2 & \sigma_2^2 & \ldots & \sigma_{2m}^2 \\ \vdots & \vdots & \ddots & \\ \sigma_{m1}^2 & \ldots & & \sigma_m^2 \end{bmatrix}$

where $\sigma_i^2 \equiv E[(X_i - E[X_i])(X_i - E[X_i])] \equiv Var[X_i]$

$\sigma_{ij}^2 \equiv E[(X_i - E[X_i])(X_j - E[X_j])] \equiv cov(X_i, X_j)$

(*) $\forall X \in R^m$: $E\|X\| \equiv E[\sqrt{X^T \cdot X}] < \infty$ iff $E|X_i| \equiv E[|X_i|] < \infty \; \forall i = 1, \ldots, m$

(*) $\forall X \in R^m$ with $\underset{\in R^m}{E[X] = \mu}$, $\underset{\in R^m \times R^m}{var[X] = \Sigma}$: the $A \cdot X \in R^k$ (where $A: k \times m$) with $E[AX] = A \cdot E[X] = A \cdot \mu \in R^k$, $Var[AX] = A \cdot var[X] \cdot A^T = A \cdot \Sigma \cdot A^T \in R^k \times R^k$