2 Uncorrelated VS Independendent

$\forall$ variables X, Y:

- $E[X+Y] = E[X] + E[Y]$
- $var[X+Y] = var[X] + var[Y] + 2\,cov[X,Y]$ where covariance $cov[X,Y] \equiv E[(X-E[X])(Y-E[Y])] = E[X \cdot Y] - E[X] \cdot E[Y]$
- $E[X \cdot Y] \leq E[X] \cdot E[Y]$

correlation $cor[X,Y] \equiv \frac{|cov[X,Y]|}{\sqrt{var[X] \cdot var[Y]}} \leq 1$ Cauchy-Schwarz

If $cov[X,Y], cor[X,Y] \neq 0$ then X, Y: correlated (if $cor[X,Y] = \pm 1$ then linear dependence $Y = a + b \cdot X$)

If $cov[X,Y] = cor[X,Y] \equiv 0$ then X, Y: uncorrelated (there can still be non-linear dependence i.e: $Y = X^2$)

X, Y are statistically independent iff $f(x,y) \equiv f_X(x) \cdot f_Y(y)$. Then:

- $f_{X|Y}(x/y) = f_X(x)$ and $f_{Y|X}(y/x) = f_Y(y)$
- X, Y: uncorrelated $\{cor[X,Y] = 0\}$
- $E[X+Y] = E[X] + E[Y]$, $var[X+Y] = var[X] + var[Y]$, $E[X \cdot Y] = E[X] \cdot E[Y]$

and even more generally

- If X, Y: statistically independent then any $g(X), h(Y)$: statistically independent.