Sample averages converge to population averages $\{$example $\bar{X}_n \xrightarrow[n\to\infty]{P} \mu\}$.
What about the distributions of sample averages? Do they converge to anything?
In general, the actual distribution of any specific sample variable (statistic) $\hat{\theta} = \hat{\theta}(X_1, \dots, X_n, n)$ will have an actual distribution $g(\hat{\theta}) \equiv P[\hat{\theta} \in [\hat{\theta}, \hat{\theta} + d\hat{\theta}]]$, so $\hat{\theta} \sim g$ in principle is a function of $f(X)$ and sample size n. For example we calculate

- If $X \sim N(\mu, \sigma^2) \equiv f(X) \longrightarrow \bar{X}_n \sim N(\mu, \sigma^2/n) \xrightarrow{(*)} \sqrt{n}(\bar{X}_n - \mu) \sim N(0, \sigma^2)$ and $\frac{n\hat{\sigma}^2}{\sigma^2} \equiv \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$
- If $X \sim \chi^2_1 \equiv f(X) \longrightarrow \sqrt{n}(\bar{X}_n - \mu) \sim \sqrt{n}(\chi^2_n/n - 1)$

In general the sample distribution is not calculable even for known $f(X)$. Thus, as $n \to \infty$...

- the next best thing is to examine whether it converges anywhere

A sequence of random $\{Z_1, \dots, Z_n\}$ with distributions $g_i(Z_i) \equiv P[Z_i \in [Z_i, Z_i + dZ_i]]$ converges in distribution to Z as $n \to \infty$ if $g_i \xrightarrow{n\to\infty} g$ where $g(Z) \equiv P[Z \in [Z, Z + dZ]]$ is the distribution of Z

$$Z_n \xrightarrow[d]{n\to\infty} Z$$

(*) In case $g(Z)$: degenerate $\{P[Z \equiv c] \equiv 1\}$ then $Z_n \xrightarrow[d]{n\to\infty} c$ which equivalent to $Z_n \xrightarrow[P]{n\to\infty} c$
Thus whenever $Z_n \xrightarrow[P]{n\to\infty} c$, it means automatically that $Z_n \xrightarrow[d]{n\to\infty} c$ $\{$a result of not great practical significance.$\}$