et the case where the true distribution of X is described by a parametric model $f(X/\theta)$

$\theta \in \Theta$ ← parameter space : unknown parameter
$F(x/\theta)$: known ∀ given θ
} How to specify the correct $\theta = \theta_0$???

ake n-sample of i.i.d $X_1, \ldots, X_n$ and compute how possible is to get these data ∀ given θ:

$$\boxed{L_n(\theta) \equiv f(X_1, \ldots, X_n / \theta) \overset{iid}{\equiv} \prod_1^n f(X_i / \theta) : \text{likelihood function}}$$

he value of θ that maximizes $-L_n(\theta)$ is the $\boxed{\text{maximum likelihood estimator of } \theta \text{ (MLE)} : \hat{\theta} \equiv \arg\max_{\theta \in \Theta} L_n(\theta)}$

equivalently we can define the log-likelihood function $l_n(\theta) = \log L_n(\theta) \equiv \sum_1^n \log f(X_i / \theta)$ and maximize it!

(1) MLE-$\hat{\theta}$ is a sample analog of the true parameter-θ_0 :

this means!!!

$\{ E[\log f(X/\theta)] \equiv \int \log f(X/\theta) \cdot f(X/\theta_0) \, dx \}$

Define $\boxed{\text{expected log density} : l(\theta) \equiv E[\log f(X/\theta)]} \xrightarrow[\text{prove}]{\text{can}} \theta_0 \equiv \arg\max_{\theta \in \Theta} l(\theta)$

Define sample analog of $l(\theta)$ average log likelihood: $\bar{l}_n(\theta) \equiv \frac{1}{n} \sum \log f(X_i / \theta) \Longrightarrow \hat{\theta} \equiv \arg\max \bar{l}_n(\theta)$

(2) If $\hat{\theta}$: MLE of θ then ∀ transformation $\beta \equiv h(\theta)$, $\hat{\beta} \equiv h(\hat{\theta})$: MLE of $\beta = h(\theta)$