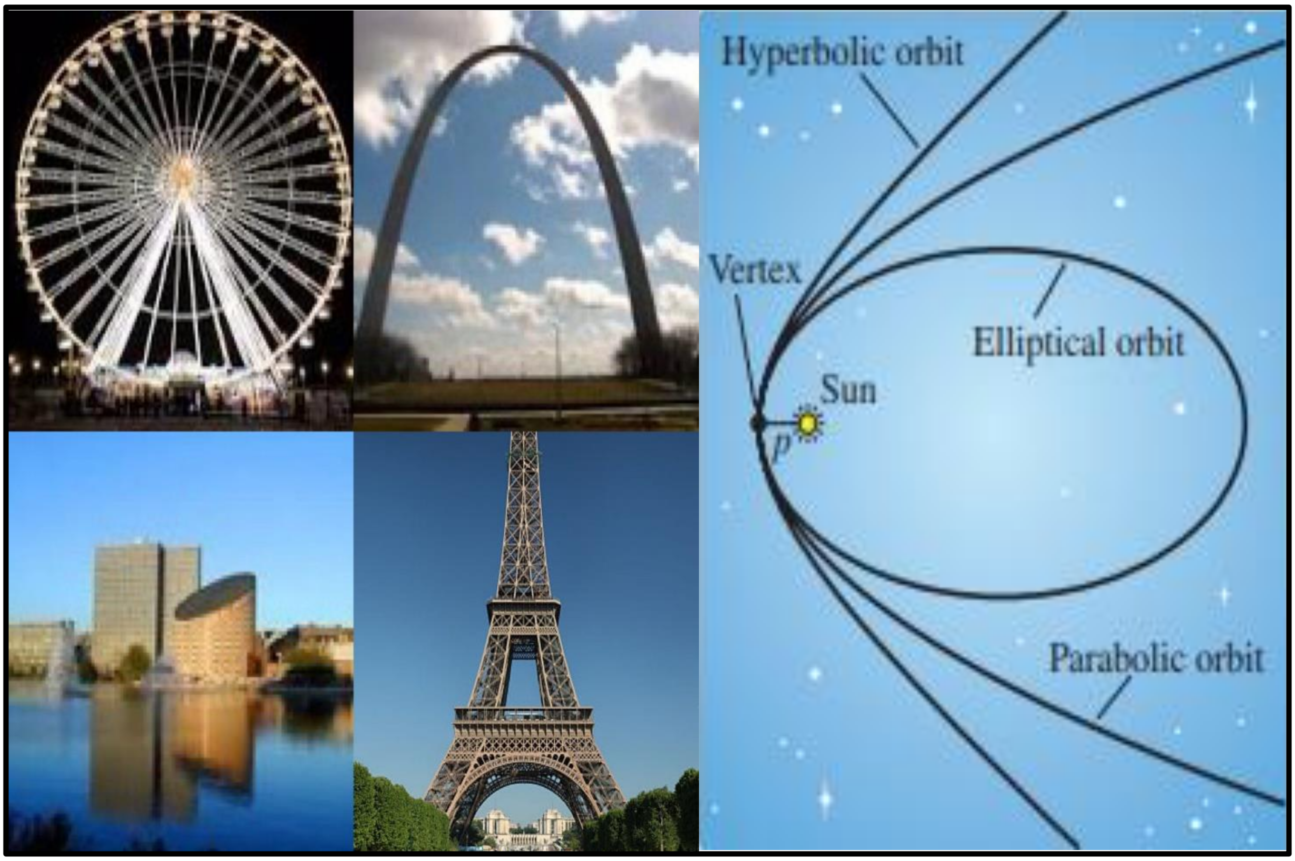
11

Precalculus

Quarter 1 – Module 1
ANALYTIC GEOMETRY



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Senior High School

Precalculus

MODULE 1:
Analytic Geometry

This instructional material was collaboratively developed and reviewed by educators from public and private schools, colleges, and/or universities. We encourage teachers and other education stakeholders to email their feedback, comments, and recommendations to the Department of Education at action@deped.gov.ph.

We value your feedback and recommendations.

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What This Module is About

The Precalculus course bridges basic mathematics and calculus. This course completes your foundational knowledge on algebra, geometry, and trigonometry. It provides you with conceptual understanding and computational skills that are prerequisites for Basic Calculus and future STEM courses.

Based on the Most Essential Curriculum Competencies (MELC) for Precalculus of the Department of Education, the primary aim of this Learning Manual is to give you an adequate stand-alone material that can be used for the Grade 11 Precalculus course.

The Module is divided into two units: analytic geometry and mathematical induction. Each unit is composed of lessons that bring together related learning competencies in the unit. Each lesson is further divided into sub-lessons that focus on one or two competencies for effective learning.

At the end of each lesson, more examples are given reinforce the ideas and skills being developed in the lesson. You have the opportunity to check your understanding of the lesson by solving the Supplementary Problems.

We hope that you will find this Learning Module helpful and convenient to use. We encourage you to carefully study this Module and solve the exercises yourselves with the guidance of your teacher. Although great effort has been put into this Module for technical correctness and precision, any mistake found and reported to the Team is a gain for other students. Thank you for your cooperation.

Module Content

The lessons that we will tackle are the following:

- ✓ Lesson 1 – Introduction of Conic Sections and the Circle
- ✓ Lesson 2 – The Parabola
- ✓ Lesson 3 – The Ellipse
- ✓ Lesson 4 – The Hyperbola
- ✓ Lesson 5 – Equation and Important Characteristics of Different Types of Conic Sections
- ✓ Lesson 6 – Solving Situational Problems Involving Conic Sections

What I Need to Know

Once you are done with this module, you should be able to:

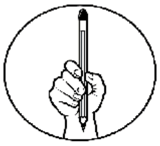
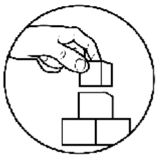
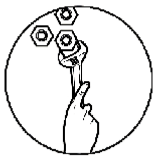
- ✓ (STEM_PC11AG-Ia-1) illustrate the different types of conic sections: parabola, ellipse, circle, hyperbola, and degenerate cases;
- ✓ (STEM_PC11AG-Ia-2) define circle;
- ✓ (STEM_PC11AG-Ia-3) determine the standard form of equation of a circle;
- ✓ (STEM_PC11AG-Ia-5) define a parabola;
- ✓ (STEM_PC11AG-Ib-1) determine the standard form of equation of a parabola;
- ✓ (STEM_PC11AG-Ic-1) define an ellipse;
- ✓ (STEM_PC11AG-Ic-2) determine the standard form of equation of an ellipse;
- ✓ (STEM_PC11AG-Id-1) define a hyperbola;
- ✓ (STEM_PC11AG-Id-2) determine the standard form of equation of a hyperbola;
- ✓ (STEM_PC11AG-Ie-1) recognize the equation and important characteristics of the different types of conic sections, and
- ✓ (STEM_PC11AG-Ie-2) solves situational problem involving conic sections.

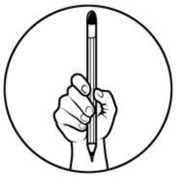
How to Learn From This Module

To complete the objectives of this module, you must DO THE FOLLOWING:

- Patiently read the text carefully and understand every sentence. Do not proceed to the next part of the module without fully understanding the previous text.
- Read the directions of each activity carefully. You will be guided as to the steps in answering the exercises and activities of this module.
- Do not proceed to the next part without completing the previous activities.

Icons of this Module

	What I Need to Know	This part contains learning objectives that are set for you to learn as you go along the module.
	What I know	This is an assessment as to your level of knowledge to the subject matter at hand, meant specifically to gauge prior related Knowledge
	What's In	This part connects previous lesson with that of the current one.
	What's New	An introduction of the new lesson through various activities, before it will be presented to you
	What is It	These are discussions of the activities as a way to deepen your discovery and under-standing of the concept.
	What's More	These are follow-up activities that are in-tended for you to practice further in order to master the competencies.
	What I Have Learned	Activities designed to process what you have learned from the lesson
	What I can do	These are tasks that are designed to show-case your skills and knowledge gained, and applied into real-life concerns and situations.



What I Know (Pre-Test)

Multiple Choice. Encircle the letter of the best answer.

1. It is the intersection of a plane and a double-napped cone.
 a. locus b. conic section c. circle d. radius

2. A collection of points satisfying a geometric property can also be referred to as a ______ of points.
 a. locus b. conic section c. circle d. radius

3. A ____________ is the set of all points (x, y) in a plane that are equidistant from a fixed point, called the center.
 a. locus b. conic section c. circle d. radius

4. A/n ____________ is the set of all points in the plane that are equidistant from a fixed point F(called the focus) and a fixed line *l* (called the directrix).
 a. circle b. ellipse c. parabola d. hyperbola

5. A/n ___________ is the set of all points in the plane the sum of whose distances from two fixed points F_1 and F_2 is a constant.
 a. circle b. ellipse c. parabola d. hyperbola

6. A/n ____________ is the set of all points in the plane, the difference of whose distances from two fixed points F_1 and F_2 is a constant.
 a. circle b. ellipse c. parabola d. hyperbola

7. $6x^2 + 12x + 6y^2 - 8y = 100$ is an example of a
 a. Circle b. parabola c. ellipse d. hyperbola

8. $4x^2 + 12x + y^2 - 8y = 64$ is an example of a
 a. Circle b. parabola c. ellipse d. hyperbola

9. $6y = 3x^2 - 15$ is an example of a
 a. Circle b. parabola c. ellipse d. hyperbola

10. $2x^2 + 87x - 25 = y^2 + 4y$ is an example of a
 a. Circle b. parabola c. ellipse d. hyperbola

11. $6x^2 - 15x = 3y^2 + 4y + 13$ is an example of a
 a. Circle b. parabola c. ellipse d. hyperbola

12. A hyperbola is the only conic that has
a. asymptotes b. focus c. vertex d. a minor axis

13. $\frac{x^2}{4} - \frac{y^2}{25} = 1$ has a major axis of length
a. 2 b. 3 c. 4 d. 5

Use $\frac{(x+3)^2}{9} - \frac{(y-2)^2}{4} = 1$ to answer the next 4 questions.

14. The center is at __________?
a. (-3, 2) b. (-3, -2) c. (3, 2) d. (3, -2)

15. The vertices are __________?
a. (0, 2), (-6, 2) b. (-3, 5), (-3, -1) c. (-1, 2), (-5, 2) d. (-3, 4), (-3, 0)

16. The foci are at __________?
a. $(-3 \pm \sqrt{7}, 2)$ $b.(-3 \pm \sqrt{13}, 2)$ c. $(-3, 2 \pm \sqrt{13})$ d. $(-3, 2 \pm \sqrt{7})$

17. The length of the major axis is ____________?
a. 3 b. 6 c. 9 d. 12

18. The standard form of $9x^2 - 4y^2 = 36$ is
a. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ b. $\frac{x^2}{36} - \frac{y^2}{9} = 1$ c. $\frac{x^2}{25} - \frac{y^2}{4} = 1$ d. $\frac{x^2}{5} - \frac{y^2}{20} = 1$

19. The earth's orbit is an ellipse with the sun at one of the foci. If the farthest distance of the sun from the earth is 105.5 million km and the nearest distance of the sun from the earth is 78.25 million km, find the eccentricity of the ellipse.
a. 0.15 b. 0.25 c. 0.35 d. 0.45

20. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ will have a minor and major axis with length _____ (In that order)
a. 3, 4 b. 6, 8 c. 9, 16 d. 18, 32

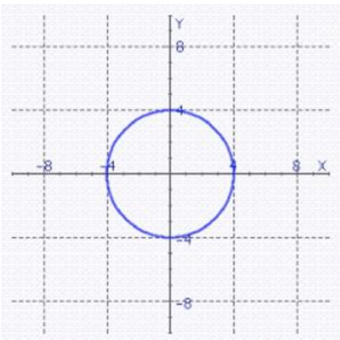
21. Which of the following shows the correct graph of the circle?

a. $x^2 + y^2 = 4$

b. $y^2 = x^2 + 16$

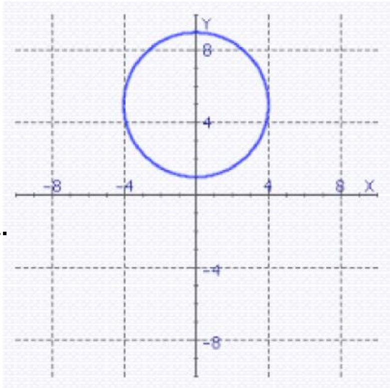
c. $x^2 + y^2 = 16$

d. $x^2 + y^2 = 1$

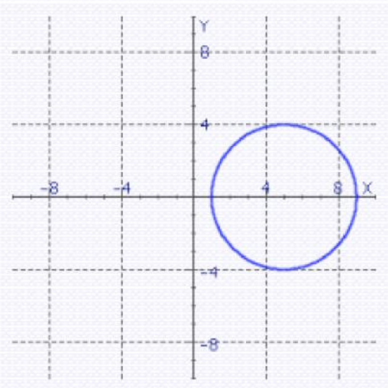


22. Which graph represents the equation $x^2 - 10x + y^2 = -9$?

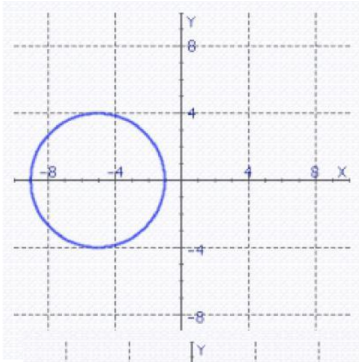
a.



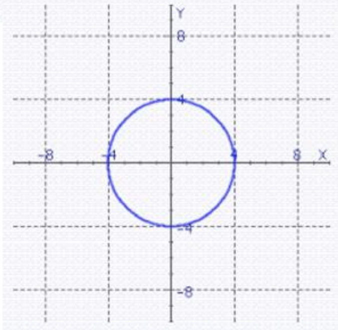
b



c.



d.



23. What is the focus and vertex of the parabola $(y-2)^2 + 16(x-3) = 0$?

a. Vertex: $V(-3,-2)$, Focus: $F(-3,14)$

b. Vertex: $V(-3,-2)$, Focus: $F(-3,-18)$

c. Vertex: $V(-3,-2)$, Focus: $F(-7,-2)$

d. Vertex: $V(3,2)$, Focus: $F(-1,2)$

24. Find the equation of the parabola with vertex at (5, 4) and focus at (-3, 4).

a $(y-4)^2 = -32(x-5)$

b $(y-4)^2 = 32(x-5)$

c. $(y+4)^2 = -32(x-5)$

d. $(y-4)^2 = 8(x-5)$

25. Find the center and foci of the ellipse $\frac{(x+5)^2}{5} + \frac{(y+9)^2}{9}$.

a. center: (5,9) , foci: (5,7),(5,11)

b. center (-5,-9) , foci: (-5,-11), (-5,-7)

c. center: (-5,-9) , foci: (-7,-9), (-3,-9)

d. center: (5,9) , foci: (3,-9), (7,-9)

26. Find the center and vertices of the ellipse $4x^2 + 9y^2 - 24x + 72y + 144 = 0$

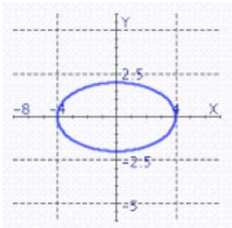
a. center: (-4,3) , vertices: (-7,3), (-1,3)

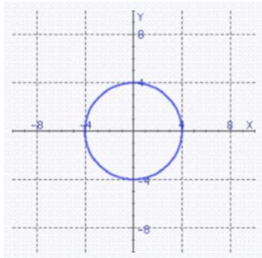
b. center: (-3,4) , vertices: (-5,4), (-1,4)

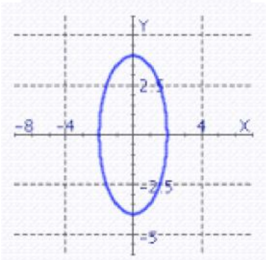
c. center: (3,-4) , vertices: (1,-4), (5,-4)

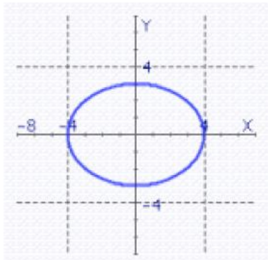
d. center: (3,-4) , vertices: (0,-4), (6,-4)

27. Which of the following shows the correct graphical representation of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$?

a.


c.


b.


d.


28. Find the center and vertices of the hyperbola $11x^2 - 25y^2 + 22x + 250y - 889 = 0$.

a. center: (1,-5) , vertices: (1,-10), (1,0)

b. center: (-1,5) ,vertices: (-1,0),(-1,10)

c. center: (-1,5) , vertices: (-6,5), (4,5)

d. center: (1,-5) , vertices: (-4,-5), (6,-5)

29. What are the vertices and asymptotes of the hyperbola $9y^2 - 16x^2 = 144$?

a. vertices: (0,-4),(0,4) , asymptote: $y = \pm\frac{4}{3}x$

b. vertices: (0,-4), (0,4) , asymptote: $y = \pm\frac{3}{4}x$

c. vertices: (-4,0), (4,0) , asymptote: $y = \pm\frac{4}{3}x$

d. vertices: (-4,0), (4,0) , asymptote: $y = \pm\frac{3}{4}x$

30. Find the standard form of the equation of the hyperbola with the given characteristics, vertices: (0,-6), (0,6) and foci (0,-7), (0,7).

a. $\frac{y^2}{36} - \frac{x^2}{49} = 1$

c. $\frac{x^2}{36} - \frac{y^2}{13} = 1$

b, $\frac{y^2}{36} - \frac{x^2}{13} = 1$

d. $\frac{x^2}{36} - \frac{y^2}{13} = 49$

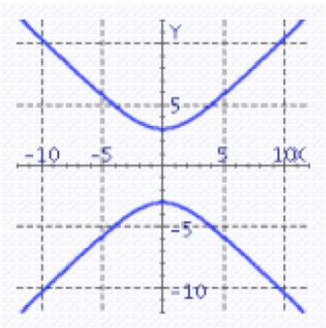
31. Write the equation of the ellipse that has its center at the origin with focus at (0, 4) and vertex at (0, 7).

a. $\frac{x^2}{49} + \frac{y^2}{33}$=1 c. $\frac{x^2}{33} + \frac{y^2}{49}$=-1

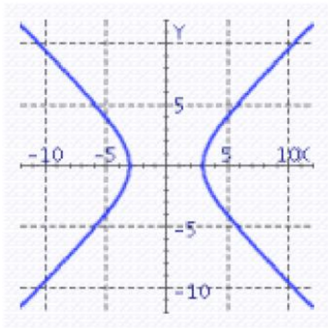
b. $\frac{x^2}{33} - \frac{y^2}{49}$=1 d. $\frac{x^2}{33} + \frac{y^2}{49}$=1

32. What is the graph of the hyperbola $9x^2 - 9y^2 = 81$?

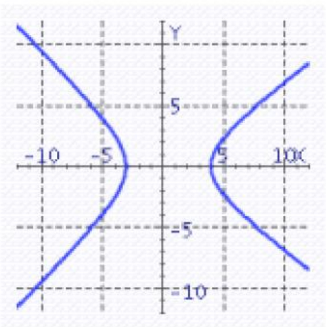
a.



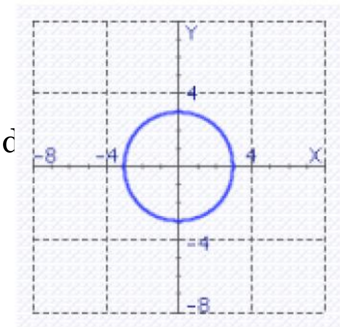
c,



b.



d



33. An arch 20 meters high has the form of parabola with vertical axis. The length of horizontal beam placed across the arch 9 meters from the top is 60 meters. Find the width of the arch at the bottom.

a. 44.72 meters b. 45.72 meters c. 89.44 meters d. 90.44 meters

34. A spotlight in a form of a paraboloid 9 inches deep has its focus 3 inches from the vertex. Determine the radius of the opening of the spotlight.

a. 9.39 inches b. 10.39 inches c. 11. 39 inches d. 12.39 inches

35. A bridge is supported on an elliptical arch of height of 7 meters and width at the base of 40 meters. A horizontal roadway is 2 meters above the center of the arch. How far is it above the arch at 8 meters from the center?

a. 0.58 meters b. 1.58 meters c. 2.58 meters d. 3.58 meters

LESSON 1 Introduction of Conic Sections and the Circle

What I Need to Know

Upon completion of this lesson, you should be able to:

- illustrate the different types of conic sections: parabola, ellipse, circle, hyperbola, and degenerate cases;
- determine the type of conic section defined by a given 2^{nd} degree equation in x and y;
- define a circle;
- determine the standard form of equation of circle;
- Graph a circle in a rectangular coordinate system;
- Derive and illustrate the equation of the circle;
- Find the center and the radius of the circle of an equation;
- Convert the general equation of circle to standard form and vice versa;

What's In

Activity 1: Recall

You had learned in your previous mathematics in junior high school about solving a quadratic equation by completing the squares. Let us recall your knowledge about the subject using these following examples.

In order to find the roots of a certain quadratic equation, the following steps will be used using completing the square method.

1. Rewrite the equation in the form $x^2 + bx = c$.
2. Add to both sides the term needed to complete the square.
3. Factor the perfect square trinomial.
4. Solve the resulting equation by using the square root property.

1. Solve the equation $x^2 + 8x + 5 = 0$ by completing the square.

Solution: $x^2 + 8x + 5 = 0$

$x^2 + 8x = -5$ *Rewrite the equation in the form* $x^2 + bx = c$.

$x^2 + 8x + 16 = -5 + 16$ *Add the appropriate constant to complete the square*.

$(x + 4)^2 = 11$ *Factoring the perfect square trinomial*

$x+4=\pm\sqrt{11}$ *Solve using the square root method.*

$x = -4 \pm \sqrt{11}$

$x_1=0.68$

$x_2= -7.38$

2. Find the roots of $x^2 + 10x - 4 = 0$ using completing the square method.

$x^2 + 10x = 4$ *Rewrite the equation in the form $x^2 + bx = c$.*

$x^2+10x+25=4+25$ *Add the appropriate constant to complete the square.*

$(x+5)^2=29$ *Factoring the perfect square trinomial*

$(x+5)=\pm\sqrt{29}$ *Solve using the square root method.*

$x = -5 \pm \sqrt{29}$

$X_1=0.39$

$X_2= -10.39$

Completing the square method is useful in discussing conic sections specially the equation of a circle.

What's New

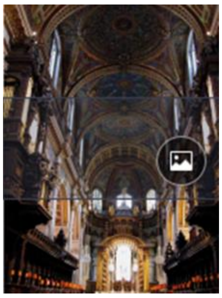
Geometric Figures or shapes are use in architectural designs. For this activity, identify the following shapes as circle, parabola, ellipse, or hyperbola as shown in the pictures being used in real-life. Write your answer on the space provided.

1) ___________

2)__________

3) ____________

4) ____________



5) ____________

6)___________

7) _____________

8) _____________

Photo source: file:///C:/Users/admin/Desktop/conic-sections.pdf and https://bit.ly/3iLqvXz

Does the activity ignite your interest to study more about geometric shapes particularly different conic sections like your answers in the activity? Can you name other architectural designs not in the pictures that used the idea of geometrical shapes? Does the shape matters on the durability, functionality and artistic designs?

Studying this module will help you appreciate nature and man's creation that would help daily life activities.

What is It

We present the conic sections, a particular class of curves which sometimes appear in nature and which have applications in other fields. In this lesson, we first illustrate how each of these curves is obtained from the intersection of a plane and a cone, and then discuss the first of their kind, circles. The other conic sections will be covered in the next lessons.

Conic sections (or conics), is a curved formed by a plane passing through a double-napped circular cone. One of the first shapes we learned, a circle, is a conic. When you throw a ball, the trajectory it takes is a parabola. The orbit taken by each planet around the sun is an ellipse. Properties of hyperbolas have been used in the design of certain telescopes and navigation systems. We will discuss circles in this lesson, leaving parabolas, ellipses, and hyperbolas for subsequent lessons.

- **Circle** (Figure 1.1) – is a special case of ellipse in which the plane is perpendicular to the axis of the cone.
- **Ellipse** (Figure 1.1) - when the (tilted) plane intersects only one cone to form a bounded curve
- **Parabola** (Figure 1.2) – the plane is parallel to a generator line of the cone
- **Hyperbola** (Figure 1.3) – the intersection is an unbounded curve and the plane is not parallel to a generator line of the cone and the plane intersects both halves of the cone.

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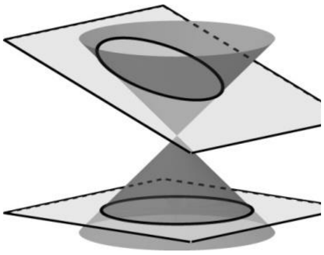


Figure 1.1

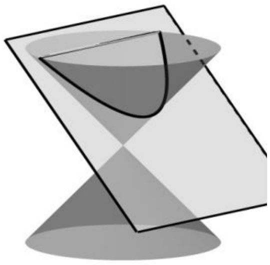


Figure 1.2

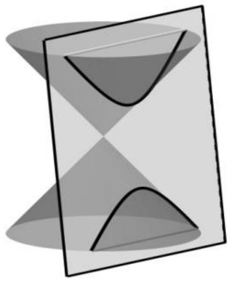


Figure 1.3

We can draw these conic sections on a rectangular coordinate plane and find their equations. To be able to do this, we will present equivalent definitions of these conic sections in subsequent sections, and use these to find the equations.

There are other ways for a plane and the cones to intersect, to form what are referred to as degenerate conics: a point, one line, and two lines. See Figures 1.4, 1.5, and 1.6.

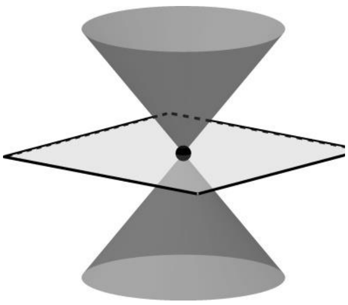


Figure 1.4

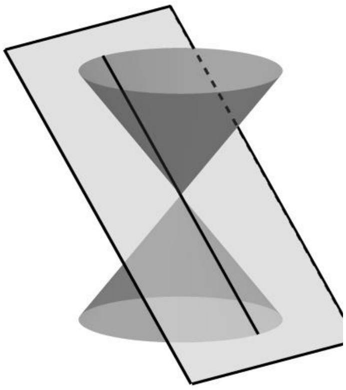


Figure 1.5

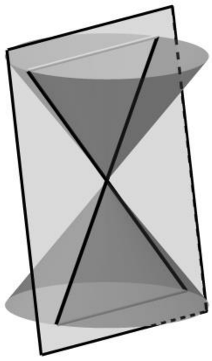


Figure 1.6

The graph of the second degree equation of the form $Ax^2 + Bx + Cy^2 + Dx + Ey + F = 0$ is determine by the values of $B^2 - 4ac$.

Table 1

Graphs of Quadratic Equations

Conic Section	**Value of $B^2 - 4ac$**	**Eccentricity**
Circle	$B^2 - 4ac$<0 or A=C	$e = 0$
Parabola	$B^2 - 4ac = 0$	$e = 1$
Ellipse	$B^2 - 4ac < 0, B \neq 0 \text{ or } A \neq C$	$0 < e < 1$
Hyperbola	$B^2 - 4ac > 0$	$e > 1$

Example1.1 Determine the type conic section that each general equation will produce.

1. $2x^2 + 4xy + 3y^2 + 12y - 1 = 0$
2. $3x^2 - 2y^2 + 6x + 10y - 16 = 0$
3. $3x^2 + 3y^2 + 18x - 16y + 31 = 0$
4. $x^2 + 4xy - 4x + y^2 - 12y = 0$

Solutions: We will collect all the values of A, B, C in each equation. Then solve for the value of $B^2 - 4ac$. Interpret the result based on table 1.

1. $2x^2 + 4xy + 3y^2 + 12y - 1 = 0$

   $A = 2, B = 4, C = 3$

   $B^2 - 4ac = 4^2 - 4(2)(3) = 16 - 24 = -8 < 0$

   Note that $B \neq 0$ and $A \neq C$. Thus, the conic section is an ellipse.

2. $3x^2 - 2y^2 + 6x + 10y - 16 = 0$

   $A = 3, B = 0, C = -2$

   $B^2 - 4ac = 0^2 - 4(3)(-2) = 0 + 24 = 24 > 0$

   Thus, the conic section is hyperbola.

3. $3x^2 + 3y^2 + 18x - 16y + 31 = 0$

   $A = 3, B = 0, C = 3$

   $B^2 - 4ac = 0^2 - 4(3)(3) = 0 - 36 = -36 < 0$

   Note that $B = 0$ and $A = C$. Thus, the conic section is a circle.

4. $4x^2 + 4xy - 4x + y^2 - 12y = 0$

   $A = 4, B = 4, C = 1$

   $B^2 - 4ac = 4^2 - 4(4)(1) = 16 - 16 = 0$

   The conic section is a parabola.

Definition and Equation of a Circle

A **circle** may also be considered a special kind of ellipse (for the special case when the tilted plane is horizontal). As we get to know more about a circle, we will also be able to distinguish more between these two conics.

See Figure 1.7, with the point $C(3,1)$shown. From the figure, the distance of $A(-2,1)$ from C is AC = 5. By the distance formula, the distance of $B(6,5)$ from C is BC = $\sqrt{(6-3)^2 + (5-1)^2} = 5$. There are other points P such that PC = 5. The collection of all such points which are 5 units away from C, forms a circle.

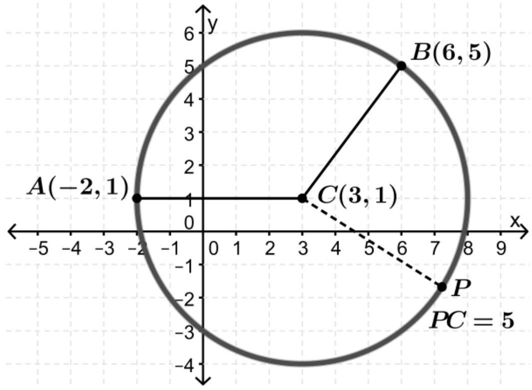


Figure 1.7

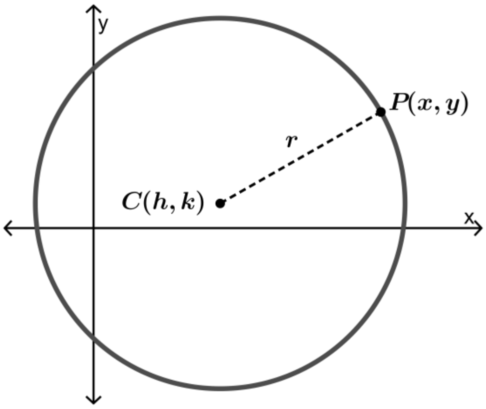


Figure 1.8

Let C be a given point. The set of all points P having the same distance from C is called a ***circle***. The point C is called the ***center*** of the circle, and the common distance its ***radius*.**

The term radius is both used to refer to a segment from the center C to a point P on the circle, and the length of this segment.

See Figure 1.8, where a circle is drawn. It has center C(h, k) and radius r > 0. A point P(x, y) is on the circle if and only if PC = r. For any such point then, its coordinates should satisfy the following.

$$PC = r$$

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2$$

This is the standard equation of the circle with center C(h, k) and radius r. If the center is the origin, then h = 0 and k = 0. The **standard equation** is then $x^2 + y^2 = r^2$.

Example 1.2. Graph the circle $x^2 + y^2 = 9$.

The given equation is in standard form with center at the origin C(0,0) and radius. We can rewrite the equation into this form $x^2 + y^2 = 3^2$.following the standard equation $x^2 + y^2 = r^2$. Thus, r=4. To be able to graph the circle, we take all the points that are 4 units from the center (0,0) to all directions along the plane. See Figure 1.9 below.

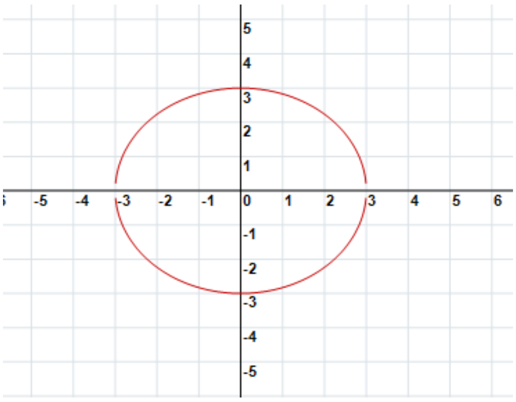


Figure 1.9

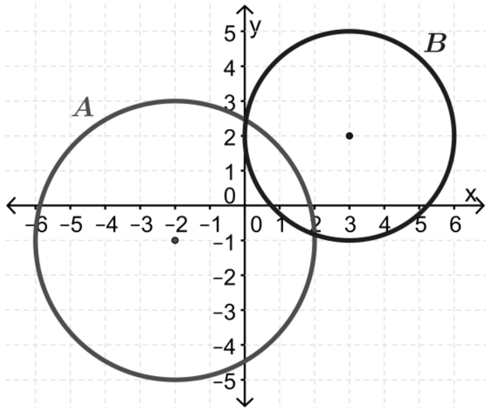


Figure 1.10

Example1.2. In each item, give the standard equation of the circle satisfying the given conditions.

(1) center at the origin, radius 4

(2) center (−4, 3), radius $\sqrt{7}$

(3) circle in Figure 1.7

(4) circle A in Figure 1.10

(5) circle B in Figure 1.10

(6) center (5,−6), tangent to the y-axis

(7) center (5, −6), tangent to the x-axis

(8) It has a diameter with endpoints A(−1, 4) and B(4, 2).

Solution:

(1) Since the center of the circle is the origin, then h=0 and k=0, the standard equation of the circle given radius (r=4) is $x^2 + y^2 = 16$.

(2) Since the center of the circle is not the origin, we will use the standard equation $(x-h)^2 + (y-k)^2 = r^2$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-(-4))^2 + (y-3) = (\sqrt{7})^2$$

$$(x+4)^2 + (y-3)^2 = 7$$

(3) The center is (3, 1) and the radius is 5

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-1) = (5)^2$$

$$(x-3)^2 + (y-1)^2 = 25$$

(4) By inspection, the center is (-2, -1) and the radius is 4. The equation is $(x + 2)^2 + (y + 1)^2 = 16$.

(5) Similarly by inspection, we have $(x - 3)^2 + (y - 2)^2 = 9$.

(6) The center is 5 units away from the y-axis, so the radius is r= 5 (you can make a sketch to see why). The equation is

$(x - 5)^2 + (y + 6)^2 = 25$.

(7) Similarly, since the center is 6 units away from the x-axis, the equation is $(x - 5)^2 + (y + 6)^2 = 36$.

(8) The center C is the midpoint of A and B: C= $\frac{-1+4}{2}, \frac{4+2}{2} = (\frac{3}{2}, 3)$. The radius is then r=AC=$\sqrt{(-1 - \frac{3}{2}) + (4 - 3)^2} = \sqrt{\frac{29}{4}}$. The circle has an equation $(x - \frac{3}{2})^2 + (y - 3)^2 = \frac{29}{4}$.

After expanding the standard equation, say for example the standard form in example 1.8, $(x - \frac{3}{2})^2 + (y - 3)^2 = \frac{29}{4}$, can be written as $x^2 + y^2 - 3x - 6y + 4 = 0$, an equation of the circle in **general form.** If the equation of a circle is given in the general form

$$Ax^2 + By^2 + Cx + Dy + E = 0, \qquad A \neq 0,$$

$$x^2 + y^2 + Cx + Dy + E = 0,$$

we can determine the standard form by completing the square in both variables. Steps below show the knowledge we had in our previous activity about completing the square.

$$(x - \frac{3}{2})^2 + (y - 3)^2 = \frac{29}{4}$$

$$-3x + \frac{9}{4} + Y^2 - 6y + 9 = \frac{29}{4}$$

$$x^2 + Y^2 - 3x - 6y = \frac{29}{4} - \frac{9}{4} - 9$$

$$x^2 + Y^2 - 3x - 6y = -4$$

$$x^2 + Y^2 - 3x - 6y + 4 = 0$$

In completing the square like the expression $(x^2 + 14x)$, means determining the term to be added that will produce a perfect polynomial square. Since the coefficient of x^2 is already 1, we take half the coefficient of x and square it, and we get 49. Indeed, $x^2 + 14x + 49 = (x + 7)^2$ is a perfect square. To complete the square in, say, $3x^2 + 18x$, we factor the coefficient of x^2 from the expression: $3(x^2 + 6x)$, then add 9 inside. When completing a square in an equation, any extra term introduced on one side should also be added to the other side.

Example 1.3. Identify the center and radius of the circle with the given equation in each item. Sketch its graph, and indicate the center.

(1) $x^2 - 6x + y^2$=7

(2) $x^2 + y^2 - 14x + 2y = -14$

(3) $16x^2 + 16y^2 + 96x - 40y = 315$

Solution. The first step is to rewrite each equation in standard form by completing the square in x and in y. From the standard equation, we can determine the center and radius.

(1) $x^2 - 6x + y^2 = 7$ (Given)

$x^2 - 6x + 9 + y^2 = 7 + 9$ (Adding 9 both sides)

$x^2 - 6x + 9 + y^2 = 16$ (Simplify right side of the equation)

$(x-3)^2 + y^2 = 16$ (Factoring into perfect square binomial)

$(x-3)^2 + y^2 = 4^2$

Center (3,0), r=4, see Figure 1.11

(2) $x^2 + y^2 - 14x + 2y = -14$ (Given)

$x^2 - 14x + y^2 + 2y = -14$ (Rearrange by terms)

$x^2 - 14x + 49 + y^2 + 2y + 1 = -14 + 49 + 1$ (Adding 49 & 1 both sides)

$x^2 - 14x + 49 + y^2 + 2y + 1 = 36$ (Simplify right side of the equation)

$(x-7)^2 + (y+1)^2 = 36$ (Factoring into perfect square binomial)

$(x-7)^2 + (y+1)^2 = 6^2$

Center (7,-1), r=6, see Figure 1.12

(3) $16x^2 + 16y^2 + 96x - 40y = 315$ (Given)

$16x^2 + 96x + 16y^2 - 40y = 315$ (Rearrange by terms)

$16(x^2 + 6x) + 16(y^2 - \frac{5}{2}y) = 315$ Applying common monomial factor

$$16(x^2+6x+9)+16\left(y^2-\frac{5}{2}y+\frac{25}{16}\right)=315+16(9)+16\left(\frac{25}{16}\right)$$ (Adding $16(9)+16(\frac{25}{16})$ both sides)

$$16(x+3)^2+16(y-\frac{5}{4})^2=484$$ (Simplify & factoring into perfect square binomial)

$$\frac{1}{16}\cdot\left\{16(x+3)^2+16(y-\frac{5}{4})^2\right\}=484\cdot\frac{1}{16}$$ (Multiplying $\frac{1}{16}$ both sides)

$$(x+3)^2+(y-\frac{5}{4})^2=\frac{121}{4}$$ (Simplify)

$$(x+3)^2+(y-\frac{5}{4})^2=\left(\frac{11}{2}\right)^2$$

Center (-3, $\frac{5}{4}$), r=$\frac{11}{2}$ or 5.5, see Fig. 1.13

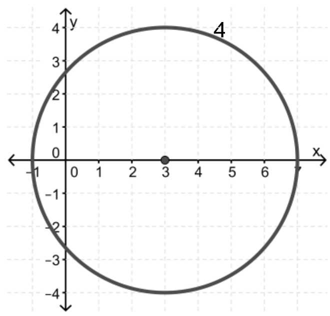


Figure 1.11

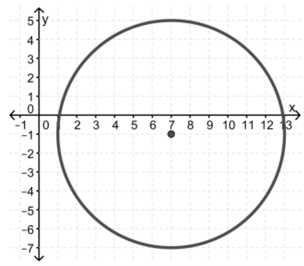


Figure 1.12

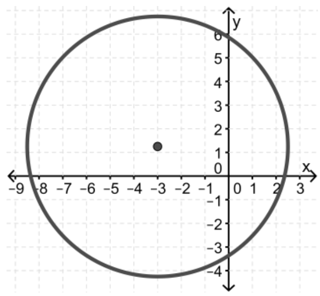
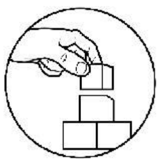


Figure 1.13

In the standard equation $(x-h)^2+(y-k)^2=r^2$ both the two squared terms on the left side have coefficient 1. This is the reason why the preceding example, we multiplied $\frac{1}{16}$ at the last equation.



What's More

Activity 1.1: Let Me try!

Let us find out if you really understood the discussed concept by answering these follow-up exercises.

1. Determine the type of conic section that each general equation will produce. Show your solution.

a. $x^2+y^2-2x-4y+1=0$	b. $x^2+y^2+8x-4y-2=0$
c. $5x^2+5y^2-9x-14y+26=0$	d. $9x^2+16y^2-54x+4xy-64y+1=0$

2. In each item, give the standard equation of the circle satisfying the given conditions.

a. center at the origin, contains (0, 3)	b. center (1, 5), diameter 8
c. circle A in Figure 1.10	d. circle B in Figure 1.10
e. circle C in Figure 1.10	f. center (-2,-3), tangent to the y- axis
g. center (-2,-3), tangent to the x-axis	h. contains the points (-2,0) and (8, 0), radius 5.

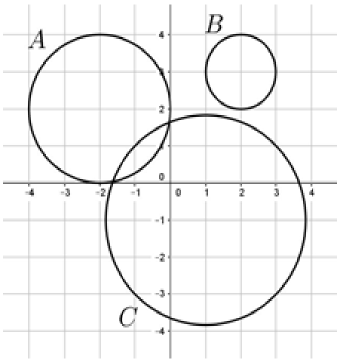


Figure1.10

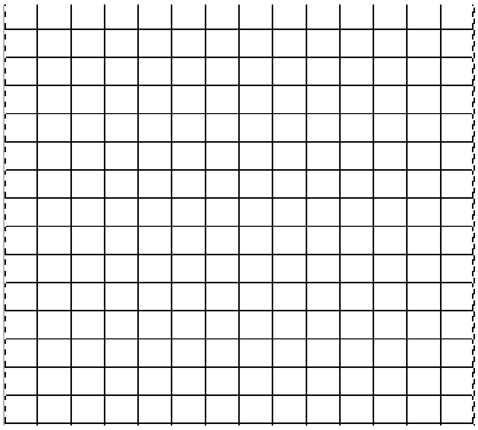
3. Identify the center and radius of the circle with the given equation in each item. Sketch its graph, and indicate the center.

(a) $x^2 + y^2 + 8y = 33$
b. $4x^2 + 4y^2 - 16x + 40y + 67 = 0$
c. $4x^2 + 12x + 4y^2 + 16y - 11 = 0$

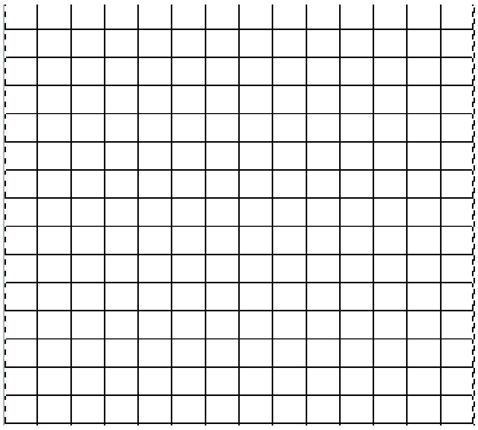
d. $x^2 + y^2 - 6x + 4y + 4 = 0$

e. $x^2 + y^2 - 4x - 8y + 20 = 0$

f. What is the equation of a circle having a diameter with endpoints at A(4,5) and B (-2,3). Sketch the graph.



g. What is the general equation of a circle whose center is at M(5,-3) and whose radius is 4 units. Sketch the graph.



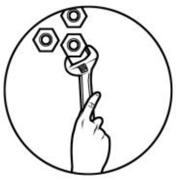
What I Have Learned

Let me check your knowledge by filling the blanks with a correct symbols/letter or terms in order to complete the statement/s.

There are four types of conic sections. When the plane is perpendicular to the axis of the cone and intersects each generator, a/an (1).__________is formed. However, when the plane is tilted slightly so that it intersects each generator, but only intersects one nappe of the cone, a/an (2)___________ is formed. When the plane is tilted further so that it is parallel to one and only one generator and intersects only one nappe of the cone, a /an(3) ____________ is formed. A hyperbola is generated when plane intersects both nappes.

Given a general equation of the conic sections, we can determine what type of conics by collecting the values (4)___, (5)___ and (6)___. Then solve the value of $B^2 - 4ac$ and interpret the result based on the table of the graphs of quadratic equations.

The first type of conic section is circle. It is defined as a set of all points in a plane equidistant from a fixed point called (7)_______ of the circle and the constant equal distance is called (8)________. The standard form of the equation of a circle is $((x-h)^2 + (y-k)^2 = r^2$ with the center: (9)_____, and radius: (10)____. However, when the circle has a center at origin: C(0,0), the standard equation would be (11)__________. This equation of the circle $Ax^2 + By^2 + Cx + Dy + E = 0$, is called (12)______________________. This equation can be converted into standard form using completing of (13) _______. To graph the equation of the circle into the coordinate plane, use the center represented by (14) _____. After locating the center, use the value of the (15)_______ to move in all directions and then connect the dots to form a circle.



What I Can Do

Performance Task: Let's do this!

Materials: Grid paper, Philippine new coins (1, 5 and 10 peso coins), ruler, and pen.

Procedures:

A. Center (0,0)

1. Use the grid paper below and draw 3 sets of Cartesian coordinate plane. Use 1 unit in labelling the x- and y- axes.
2. Locate the center (0,0) of each 3 sets of Cartesian plane by putting a visible dots.
3. Place each coin (designated set) at the center or on the dot in each Cartesian plane. Using pen, draw a circle by tracing the circumference (edge) of the each coin.
4. After you draw, remove the coins. From the center (dot), draw a line to any point of the circle.
5. Get your ruler, measure in centimeters the line (radius) you created in each Cartesian plane and record the values.
6. Solve the equation of each circle using the obtained value of the radius.
7. Compare the equation obtained and make an observation note of the activity.

Set A (1 peso coin)

Set B (5 peso coin)

Set C (10 peso coin)

Observation Note:

B. Center(h,k)

1. Use the grid paper below and draw 3 sets of Cartesian coordinate plane. Use 1 unit in labelling the x- and y- axes.
2. Locate the center (2,3) of each 3 sets of Cartesian plane by putting a visible dots.
3. Place each coin (designated set) at the center or on the dot in each Cartesian plane. Using pen, draw a circle by tracing the circumference (edge) of the each coin.
4. After you draw, remove the coins. From the center (dot), draw a line to any point of the circle.
5. Get your ruler, measure in centimeters the line (radius) you created in each Cartesian plane and record the values.
6. Solve the equation of each circle using the obtained value of the radius.
7. Compare the equation obtained and make an observation note of the activity.

Set A (1 peso coin)

Set B (5 peso coin)

Set C (10 peso coin)

Observation Note:

____

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Online connect! *For additional knowledge and information about the topics please visit the link indicated below.*

1. https://www.youtube.com/watch?v=HO2zAU3Eppo
2. https://www.youtube.com/watch?v=auD46ZWZxQo
3. https://www.youtube.com/watch?v=JUvo3GrgWHk
4. shorturl.at/bKU67

LESSON 2 The Parabola

What I Need to Know

Upon completion of this lesson, you should be able to:

- define a parabola;
- determine the standard form of equation of parabola
- graph a parabola in a Cartesian coordinate system;
- describe and discuss the parts of parabola;
- convert the general equation of parabola to standard form and vice versa.

What's In

Activity 2.1: Recall

Let us recall previous lessons in quadratic function. Write the correct answer of the following questions below.

Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is

$$\boldsymbol{d} =$$

Midpoint Formula

The midpoint between two points (x_1, y_1) and (x_2, y_2) is

$$\boldsymbol{M(x, y)} = \Big(\qquad\qquad\qquad \Big)$$

What is the standard form of a quadratic function? ______________________

What is Vertex form of a quadratic function? __________________________

What do you call the graph of quadratic function?________________________

Recalling this concepts are useful in studying the new lesson in this module as you go along the parts.

What's New

A parabola is one of the conic sections. You have learned from the previous lesson that it is formed when the plane intersects only one cone to form an unbounded curve. The same thing with circle, you will learn more about the opening of the graph, equation in standard form and general form.
Let us discover some important parts of the graph of a parabola.

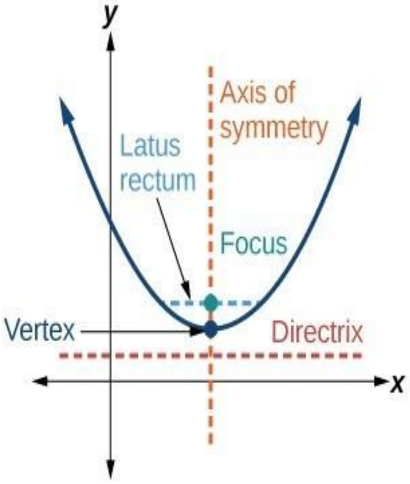


Figure 2.1

Follow-up Activity! Study figure 2.1 and fill in the blank to complete the statement.

Knowing that the graph of quadratic function is a parabola and you already had the idea on its part. But, there are new parts to be introduced in teaching parabola as one of the conics. A parabola is the set of all points in a plane equidistant from a fixed point and a fixed line. The fixed point is called ________ and the fixed line is called the ___________.The _______ of the parabola is the midpoint of the perpendicular segment from the focus to the directrix, while the line that passes through it and the focus is called the ________________. The line segment through the focus perpendicular to the axis of symmetry is called the ____________ whose length is 4ac.

What is It

Parabola is the set of all points in a plane equidistant from a fixed point and a fixed line.

This part presents how to convert general form of a parabola to its standard form and vice versa. Table 2.1 presents the general and standard equations of the parabola with vertex at origin and at (h,k).

Table 2.1

General and Standard Equations of the Parabola

Vertex	**General Form**	**Standard Form**
(0,0)	y^2+ Dx+ F=0	$y^2 = 4cx$
		$y^2 = -4cx$
	y^2+ Dx+ F=0	$x^2 = 4cy$
		$x^2 = -4cy$
(h,k)	y^2+ Dx+ Ey + F=0	$(y-k)^2 = 4c(x-h)$
		$(y-k)^2 = -4c(x-h)$
	x^2+ Dx+ Ey + F=0	$(x-h)^2 = 4c(y-k)$
		$(x-h)^2 = -4c(y-k)$

Example 2.1Convert the general equations to standard form:

a. $y^2 + 12x + 2y + 25 = 0$　　　　b. $2x^2 - 12x - y + 16 = 0$

Solution: a. $y^2 + 12x + 2y + 25 = 0$

$$y^2 + 2y = -12x - 25$$

$$y^2 + 2y + 1 = -12x - 25 + 1$$

$$(y+1)^2 = -12x - 24$$

$$(y+1)^2 = -12(x+2)$$

b. $2x^2 - 12x - y + 16 = 0$

$$2x^2 - 12x = y - 16$$

$$2(x^2 - 6x) = y - 16$$

$$2(x^2 - 6x + 9) = y - 16 + 2(9)$$

$$2(x-3)^2 = y - 16 + 18$$

$$2(x-3)^2 = y + 2$$

$$(x-3)^2 = \frac{y+2}{2}$$

Example 2.2 Convert the general equations to standard form:

a. $(y-3)^2 = 7(x-8)$ b. $(x+2)^2 = -8(y+5)$

Solution: a. $(y-3)^2 = 7(x-8)$

$$y^2 - 6y + 9 = 7x - 56$$

$$y^2 - 6y + 9 - 7x + 56 = 0$$

$$y^2 - 7x - 6y + 65 = 0$$

b. $(x+2)^2 = -8(y+5)$

$$x^2 + 4x + 4 = -8y - 40$$

$$x^2 + 4x + 4 + 8y + 40 = 0$$

$$x^2 + 4x + 8y + 44 = 0$$

Consider the point F(0, 2) and the line ℓ having equation y = 2, as shown in Figure 1.14. What are the distances of A(4, 2) from F and from ℓ? (The latter is taken as the distance of A from Aℓ, the point on ℓ closest to A). How about the distances of B(−8, 8) from F and from ℓ (from Bℓ)?

AF = 4 and $AA_\ell = 4$

BF=$\sqrt{(-8-0)^2 + (8-2)^2} = 10$ and $BB_\ell = 10$

There are other points P such that PF = PPℓ (where Pℓ is the closest point on line ℓ). The collection of all such points forms a shape called a parabola.

> Let F be a given point, and ℓ a given line not containing F. The set of all points P such that its distances from F and from ℓ are the same, is called a *parabola*. The point F is its *focus* and the line ℓ its *directrix*.

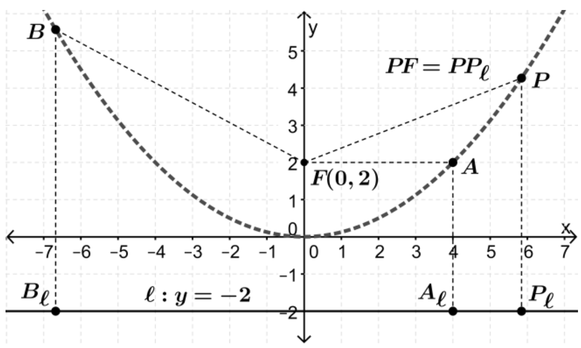


Figure 1.14

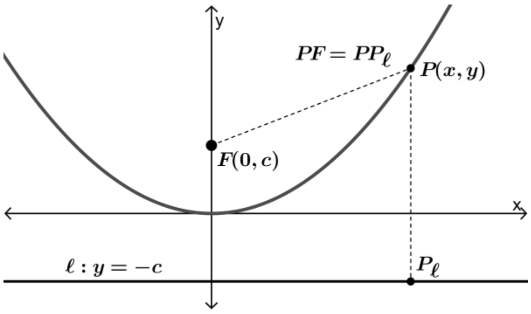


Figure 1.15

Consider a parabola with focus F(0, c) and directrix ℓ having equation y = c. See Figure 1.26. The focus and directrix are c units above and below, respectively, the origin. Let P(x, y) be a point on the parabola so PF = PPℓ, where Pℓ is the point on ℓ closest to P. The point P has to be on the same side of the directrix as the focus (if P was below, it would be closer to ℓ than it is from F).

$$PF = PP_\ell$$

$$\sqrt{x^2 + (y - c)^2} = y - (-c) = y + c$$

$$x^2 + y^2 - 2cy + c^2 = y^2 + 2cy + c^2$$

$$x^2 = 4cy$$

The vertex V is the point midway between the focus and the directrix. This equation, x2 = 4cy, is then the standard equation of a parabola opening upward with vertex V (0, 0).

Suppose the focus is F(0,-c) and the directrix is y = c. In this case, a point P on the resulting parabola would be below the directrix (just like the focus). Instead of opening upward, it will open downward. Consequently, PF = $\sqrt{x2 + (y + c)2}$ and PPℓ = $c - y$(you may draw a version of Figure 1.15 for this case). Computations similar to the one done above will lead to the equation $x^2 = -4cy$.

We collect here the features of the graph of a parabola with standard equation $x^2 = 4cy$ or $x^2 = -4cy$, where c > 0.

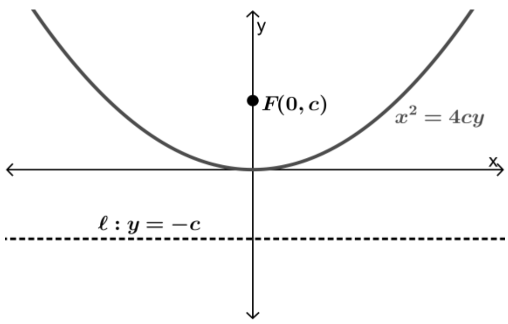


Figure 1.16

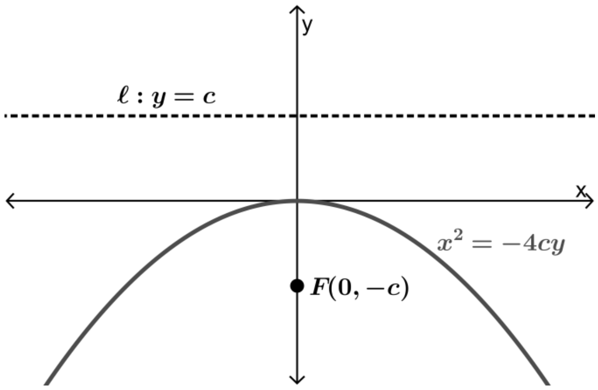


Figure 1.17

(1) *vertex* : origin V (0, 0)

- If the parabola opens upward, the vertex is the lowest point. If the parabola opens downward, the vertex is the highest point.

(2) *directrix* : the line y = −c or y = c

- The directrix is c units below or above the vertex.

(3) *focus*: F(0, c) or F(0, −c)

- The focus is c units above or below the vertex.
- Any point on the parabola has the same distance from the focus as it has from the directrix.

(4) *axis of symmetry* : x = 0 (the y-axis)

- This line divides the parabola into two parts which are mirror images of each other.

Example 2.3. Determine the focus and directrix of the parabola with the given equation. Sketch the graph, and indicate the focus, directrix, vertex, and axis of symmetry.

(1) $x^2 = 12y$ (2) $x^2 = -6y$

Solution.

(1) The vertex is V (0, 0) and the parabola opens upward. From $4c = 12$, $c = 3$. The focus, c = 3 units above the vertex, is F(0, 3). The directrix, 3 units below the vertex, is $y = -3$. The axis of symmetry is $x = 0$.

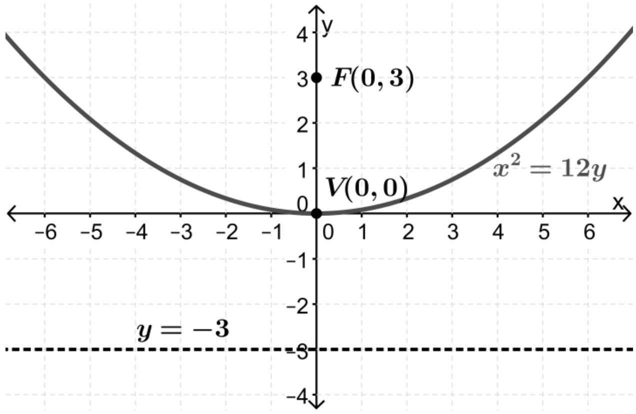
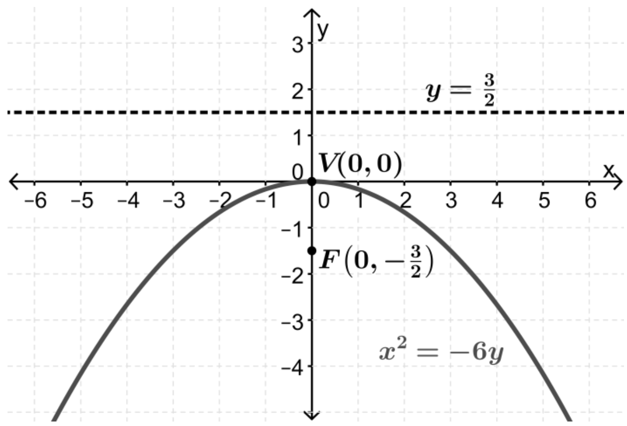


Figure 1.18

(2) The vertex is V (0,0) and the parabola opens downward. From 4c=6, c=$\frac{3}{2}$. The focus, c=$\frac{3}{2}$ units below the vertex, is F (0,-$\frac{3}{2}$). The directrix, $\frac{3}{2}$ units above the vertex, is y=$\frac{3}{2}$. The axis of symmetry is x=0.

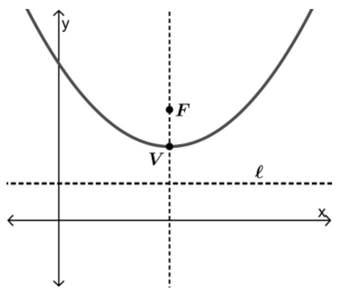
Figure 1.19

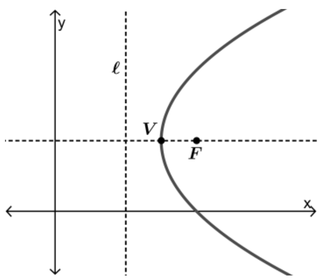


Example 2.4. What is the standard equation of the parabola in Figure 1.14?

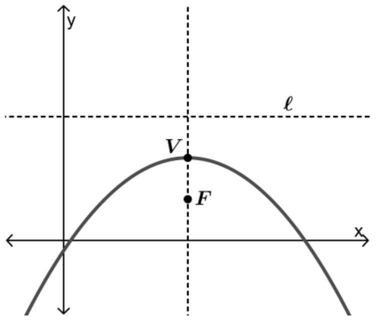
Solution: From the figure, we deduce that c=2. The equation is thus $x^2 = 8y$

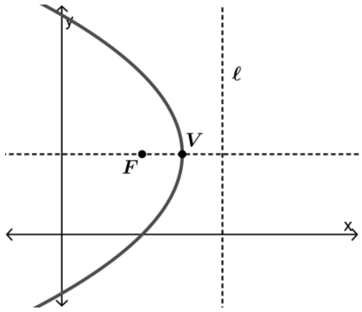
In all four cases below, we assume that c > 0. The vertex is V (h, k), and it lies between the focus F and the directrix ℓ. The focus F is c units away from the vertex V, and the directrix is c units away from the vertex. Recall that, for any point on the parabola, its distance from the focus is the same as its distance from the directrix.





$(x - h) = 4c(y - k)$ $\qquad\qquad$ $(y - k) = 4c(x - h)$





$(x - h) = -4c(y - k)$ $\qquad\qquad$ $(y - k) = -4c(x - h)$

directrix ℓ: horizontal	directrix ℓ: vertical
axis of symmetry: x=h, vertical	axis of symmetry: y=k, horizontal

Note the following observations:

- The equations are in terms of x h and y k: the vertex coordinates are subtracted from the corresponding variable. Thus, replacing both h and k with 0 would yield the case where the vertex is the origin. For instance, this replacement applied to (x h)2 = 4c(y k) (parabola opening upward) would yield x2 = 4cy, the first standard equation we encountered (parabola opening upward, vertex at the origin).
- If the x-part is squared, the parabola is “vertical”; if the y-part is squared, the parabola is “horizontal.” In a horizontal parabola, the focus is on the left or right of the vertex, and the directrix is vertical.
- If the coefficient of the linear (non-squared) part is positive, the parabola opens upward or to the right; if negative, downward or to the left.

Example 2.4. Figure 1.20 shows the graph of parabola, with only its focus and vertex indicated. Find its standard equation. What are its directrix and its axis of symmetry?

Solution. The vertex is V (5, −4) and the focus is F(3, −4). From these, we deduce the following: h = 5, k = −4, c = 2 (the distance of the focus from the vertex). Since the parabola opens to the left, we use the template (y − k)2 =−4c(x − h). Our equation is

$$(y + 4)^2 = -8(x - 5).$$

Its directrix is c = 2 units to the right of V , which is x = 7. Its axis is the horizontal line through V : y = −4.

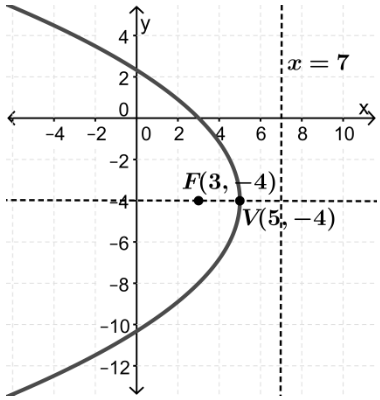


Figure 1.20

The standard equation $(y + 4)^2 = -8(x - 5)$from the preceding example can be rewritten as y^2 + 8x + 8y 24 = 0, an equation of the parabola in general form.

If the equation is given in the general form Ax2 + Cx + Dy + E = 0 (A and C are nonzero) or By^2 + Cx+ Dy + E = 0 (B and C are nonzero), we can determine the standard form by completing the square in both variables.

Example 2.5. Determine the vertex, focus, directrix, and axis of symmetry of the parabola with the given equation. Sketch the parabola, and include these points and lines.

(1) $y^2 - 5x + 12y = -16$ (2) $5x^2 + 30x + 24y = 51$

Solution.

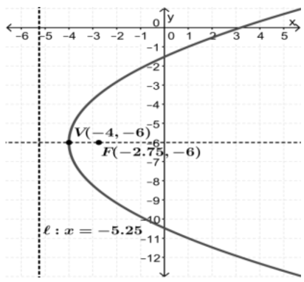
(1) We complete the square on y, and move x to the other side.

$$y^2 + 12y = 5x - 16$$

$$y^2 + 12y + 36 = 5x - 16 + 36 = 5x + 20$$

$$(y + 6)^2 = 5(x + 4)$$

The parabola opens to the right. It has vertex V (−4, −6). From 4c = 5, we get c = 5 = 1.25. The focus is c = 1.25 units to the right of V: F(−2.75, −6). The (vertical) directrix

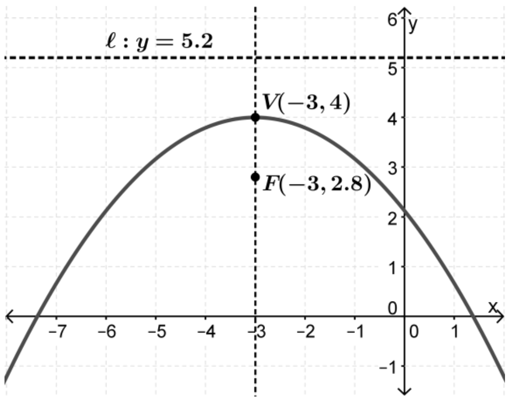


is c = 1.25 units to the left of V: x = −5.25. The (horizontal) axis is through V: y = −6.

(2) We complete the square on x, and move y to the other side.

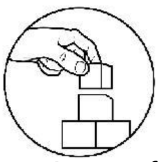
$$5x^2 + 30x = -24y + 51$$
$$5(x^2 + 6x + 9) = -24y + 51 + 5(9)$$
$$5(x + 3)^2 = -24y + 96$$
$$5(x + 3)^2 = -24(y - 4)$$
$$(x + 3)^2 = \frac{-24}{5}(y - 4)$$

In the last line, we divided by 5 for the squared part not to have any coefficient. The parabola opens downward. It has vertex V(-3,4). From 4c = $\frac{24}{5}$, we get c = 6 = 1.2. The focus is c = 1.2 units below V:F(−3, 2.8). The (horizontal) mdirectrix is c = 1.2 units above V: y = 5.2. The (vertical) axis is through V : x = −3.



Example 2.6 A parabola has focus F(7, 9) and directrix y = 3. Find its standard equation.

Solution. The directrix is horizontal, and the focus is above it. The parabola then opens upward and its standard equation has the form $(x - h)^2 = 4c(y-k)$. Since the distance from the focus to the directrix is 2c = 9-3 = 6, then c = 3. Thus, the vertex is V (7, 6), the point 3 units below F. The standard equation is then (x − 7)   = 12(y − 6).



What's More

Activity 2.1: Let Me try!

Let us find out if you really understood the discussed concept by answering these follow-up exercises.

1. Convert the following general form to standard form of a parabola.

a. $4x^2 - 24x - 40y - 4 = 0$	c. $y^2 - 6x + 2y - 17 = 0$
b. $x^2 - 2x - 6y + 25 = 0$	d. $2y^2 - x - 12y + 18 = 0$

2. Convert the following standard form to general form of a parabola.

a. $(y + 1)^2 = -2(x + 2)$	c. $(x - 3)^2 = 12(y - 4)$
b. $(x - 1)^2 = 8(y - 3)$	d. $(y - 3)^2 = -\frac{3(x-3)}{2}$

3. Determine the vertex, focus, directrix, and axis of symmetry of the parabola with the given equation. Sketch the parabola, and include these points and lines

a. $y^2 = 20x$
b. $3x^2 = -12y$
c. . $x^2 - 6x - 2y + 9 = 0$

d. $3y^2 + 8x + 24y + 40 = 0$

4. A parabola has focus F(11, 8) and directrix x = -17. Find its standard equation

5. Find the equation of a parabola with vertex at the origin whose properties are given below.

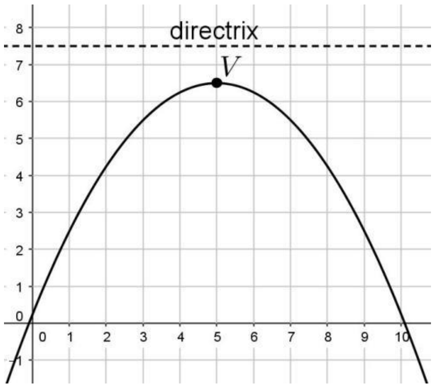
a. length of latus rectum is 10 and parabola opens downward

b. equation of directrix is y=8

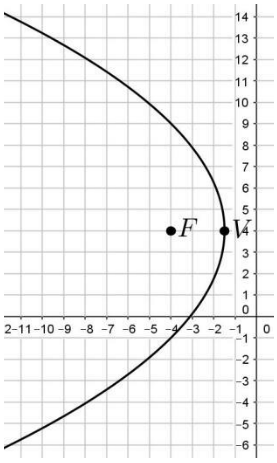
c. focus at (0,4)

d. diretcrix is x=7

6. Determine the standard equation of the parabola in Figure below given only its vertex and directrix. Then determine its focus and axis of symmetry



7. Determine the standard equation of the parabola in the figure given only its focus and vertex. Determine its directrix and axis of symmetry.



8. Find an equation of the parabola with vertex at (-1,4) and y=5 as the line of directrix. Draw the general appearance of this graph.

What I Have Learned

Let me check your knowledge by filling the blanks with a correct symbols/letter or terms in order to complete the statement/s.

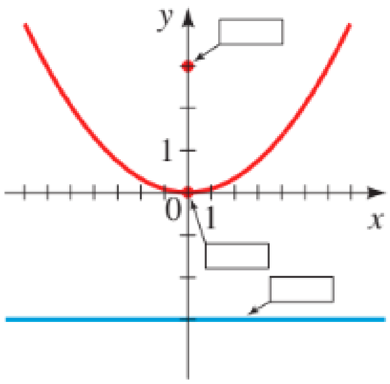
1. A parabola is the set of all points in the plane that are equidistant from a fixed point called the _________ and fixed line called the __________ of the parabola.

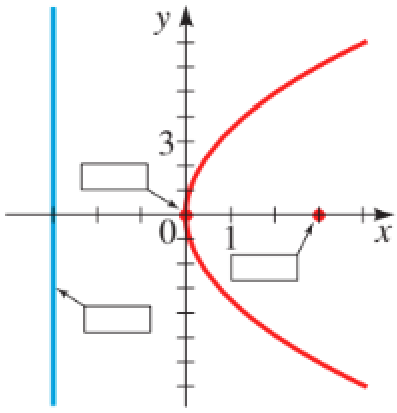
2. The graph of the equation $x^2 = 4cy$ is a parabola with focus F(__,__) and directrix y=____. So the graph of $x^2 = 12y$ is a parabola with focus F(__,__) and directrix y=_____.

3. The graph of the equation $y^2 = 4cx$ is a parabola with focus F(__,__) and directrix x=_____. So the graph of $y^2 = 12x$ is a parabola with focus F(__,__) and directrix x= ___.

4. Label the focus, directrix and vertices on the graphs given for the parabolas below.

a. $x^2 = 12y$ of $y^2 = 12x$





What I Can Do

Performance Task: Let's do this!

Materials: Flashlight, ball and ring

1. Get a flashlight and perform the task in the picture. A flashlight is held to form a lighted area on the ground, as shown in the figure. Is it possible to angle

the flashlight in such a way that the boundary of the lighted area is a parabola? Explain your answer.

Answer:

2. In a basketball game, it is crucial to be able to execute a throw which creates a parabola that can deliver the ball through a hoop with ease. Shooting at a 90 degree is optimal yet impractical as in order to shoot to this degree, one must be directly under the rim. This is where a parabola is used to complete the objective of shooting the ball to acquire points.

Go to the basketball court (if available or accessible, if not innovate) and perform the following.

a. Standing and facing on one of the posts, move your feet backward 3 times and then perform shooting the ball. Repeat the process of moving your feet 7 times, 10 times, 12, times 15 times and 20 times and then shoot the ball.

2. While doing the activity of shooting the ball in different distances, what can you say on the following?

a. Does shooting the ball create parabolic arc? Explain

b. Do you think the parabolic arcs formed are of the same measurement? Explain

c. Are the parabolic arcs formed in shooting the ball are dependent on the distance of a person throwing the ball? Or are they related? Explain

d. Do you think that a player or famous player studied the shooting style in order to get the perfect shoot?

Online connect! *For additional knowledge and information about the topics please visit the links/url indicated below.*

1. shorturl.at/eikV1
2. shorturl.at/cfRT5
3. https://www.youtube.com/watch?v=ZJf9shWlMz0
4. shorturl.at/HKSU6

LESSON 3 The Ellipse

What I Need to Know

Upon completion of this lesson, you should be able to:

- define an ellipse;
- determine the standard form of equation of an ellipse;
- graph an ellipse in a Cartesian coordinate system;
- discuss the parts of an ellipse;
- convert the general equation of an ellipse to standard form and vice versa.

What's In

Activity 2.1: Recall

Let us recall previous lessons in the graphs of quadratic equations. We will prove that a given equation without graphing will result to a certain conic section. Now, let us try circle.

Table 1

Graphs of Quadratic Equations

Conic Section	**Value of $B^2 - 4ac$**	**Eccentricity**
Circle	$B^2 - 4ac$<0 or A=C	$e = 0$
Parabola	$B^2 - 4ac = 0$	$e = 1$
Ellipse	$B^2 - 4ac < 0, B \neq 0 \; or \; A \neq C$	$0 < e < 1$
Hyperbola	$B^2 - 4ac > 0$	$e > 1$

1. $x^2 + y^2 + 6x - 16 = 0$.

We will collect the values of A, B, and C.

A= 1, B=0, and C=1. Solving for $B^2 - 4ac$.

$B^2 - 4ac = 0^2 - 4(1)(1) = 0 - 4 = -4 < 0$

Note that B=0, A=C. Thus, the conic section is a circle.

2. $4x^2 + 4y^2 - 4x - 12y + 1 = 0$

We will collect the values of A, B, and C.

A= 4, B=0, and C=4. Solving for $B^2 - 4ac$.

$B^2 - 4ac$=$0^2 - 4(4)(4) = 0 - 64 = -64 < 0$

Note that B=0, A=C. Thus, the conic section is a circle.

3. $4x^2 + 9y^2 - 48x + 72y + 144 = 0$

We will collect the values of A, B, and C.

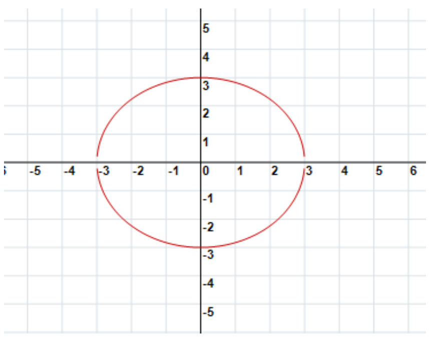
A= 4, B=0, and C=9. Solving for $B^2 - 4ac$.

$B^2 - 4ac$=$0^2 - 4(4)(9) = 0 - 144 = -144 < 0$

Note that B=0, A$\pm$C. Thus, the conic section is not a circle but an ellipse.

Comparing the three equations, 1 and 2 are both circles, while 3 is an ellipse. Although the values of B=0 for the three equations, only 1 and 2 have same values of A and C, while equation 3 has different values of A and C. For a circle, the values of A and C should be the same, while in ellipse should be different. Circles and ellipse are quite related in terms of their graphs, but do not be confused in determining the two by evaluating their A, B, and C values and solving $B^2 - 4ac$ in their standard form in order to be précised in sketching the graph.

Let us show the graph of a circle to prove that A and C are of the same values.



From the graph, the center is C (h,k) which is (0,0), and the radius r is 3. Using the standard form

$(x - h)^2 + (y - k)^2 = r^2$,

$(x - 0)^2 + (y - 0)^2 = 3^2$

$x^2 + y^2 = 9$ is the equation of the circle.

A=1 and C=1, then A=C.

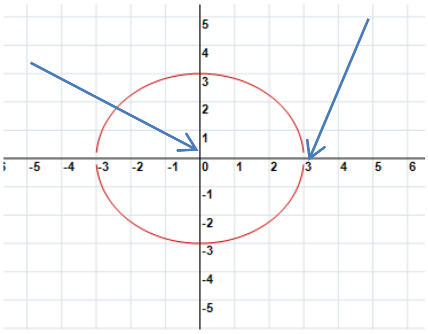
Recalling these concepts are useful in teaching ellipse.

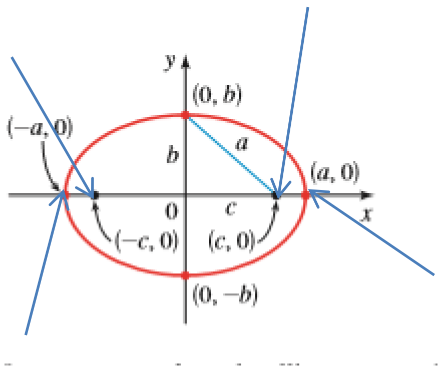
What's New

Unlike circle and parabola, an ellipse is one of the conic sections that most students have not encountered formally before. Its shape is a bounded curve which looks like a flattened circle. The orbits of the planets in our solar system around the sun happen to be elliptical in shape. Also, just like parabolas, ellipses have reflective properties that have been used in the construction of certain structures. These applications and more will be encountered in this lesson.

Name the parts of the two figures below using the terms found in the box.

first vertex	first focus	center	radius
second focus	second vertex		





Can you tell the difference between the graphs? Let us leave the question unanswered and do some discussions and activities in order for you to understand better the topic.

What is It

An **Ellipse** is a set of all points in a plane, the sum whose distances from two fixed points is constant. The fixed points are called **foci.**

This section presents how to convert general form of ellipse to its standard form and vice versa.

Example 3.1. Convert the following general equation to standard form.

a. $9x^2 + 8y^2 = 288$ b. $3x^2 + 4y^2 + 24x - 16y + 52 = 0$

Solution:

a. $9x^2 + 8y^2 = 288$

$$\frac{9x^2}{288} + \frac{8y^2}{288} = \frac{288}{288}$$

$$\frac{x^2}{32} + \frac{y^2}{36} = 1$$

The standard form is $\frac{x^2}{32} + \frac{y^2}{36} = 1$

b. $3x^2 + 4y^2 + 24x - 16y + 52 = 0$

$(3x^2 + 24x) + (4y^2 - 16y) + 52 = 0$ Regroup the terms

$3(x^2 + 8x) + 4(y^2 - 4y) = -52$ Apply common Factor

$3(x^2 + 8x + 16) + 4(y^2 - 4y + 4) = -52 + 3(16) + 4(4)$ Apply completing the square

$3(x^2 + 8x + 16) + 4(y^2 - 4y + 4) = -52 + 48 + 16$ Simplify right side

$3(x^2 + 8x + 16) + 4(y^2 - 4y + 4) = 12$ Simplify right side

$3(x + 4)^2 + 4(y - 2)^2 = 12$ Factoring completely

$\frac{3(x+4)^2}{12} + \frac{4(y-2)^2}{12} = \frac{12}{12}$ Divide both sides by 12

$\frac{(x+4)^2}{4} + \frac{(y-2)^2}{3} = 1$ Simplify

The standard form is $\frac{(x+4)^2}{4} + \frac{(y-2)^2}{3} = 1$.

Example 3.2. Convert the following standard form to general form:

a. $\frac{(y-2)^2}{25} + \frac{(x-3)^2}{9} = 1$ b. $\frac{(x-1)^2}{100} + \frac{(y+1)^2}{36} = 1$

Solution:

a. $\frac{(y-2)^2}{25} + \frac{(x-3)^2}{9} = 1$

$$225\left[\frac{(y-2)^2}{25} + \frac{(x-3)^2}{9} = 1\right]$$

$$9(y-2)^2 + 25(x-3)^2 = 225$$

$$9(y^2 - 4y + 4) + 25(x^2 - 6x + 9 = 225$$

$$9y^2 - 36y + 36 + 25x^2 - 150x + 225 = 225$$

$$9y^2 - 36y + 36 + 25x^2 - 150x + 225 - 225 = 0$$

$$25x^2 + 9y^2 - 150x - 36y + 36 = 0$$

The general form is $25x^2 + 9y^2 - 150x - 36y + 36 = 0$

b. . $\frac{(x-1)^2}{100}+\frac{(y+1)^2}{36}=1$

$$3600\left[\frac{(x-1)^2}{100}+\frac{(y+1)^2}{36}=1\right]$$

$$36(x-1)^2+100(y+1)^2=3600$$

$$36(x^2-2x+1)+100(y^2+2y+1)=3600$$

$$36x^2-72x+36+100y^2+200y+100=3600$$

$$36x^2-72x+36+100y^2+200y+100-3600=0$$

$$36x^2+100y^2-72x+200y-3464=0$$

The general form is $36x^2+100y^2-72x+200y-3464=0$.

Consider the points F1(−3, 0) and F2(3, 0), as shown in Figure 1.22. What is the sum of the distances of A(4, 2.4) from F1 and from F2? How about the sum of the distances of B(and C(0, −4)) from F1 and from F2?

AF1 + AF2 = 7.4 + 2.6 = 10

BF1 + BF2 = 3.8 + 6.2 = 10

CF1 + CF2 = 5 + 5 = 10

There are other points P such that PF1 + PF2 = 10. The collection of all such points forms a shape called an ellipse.

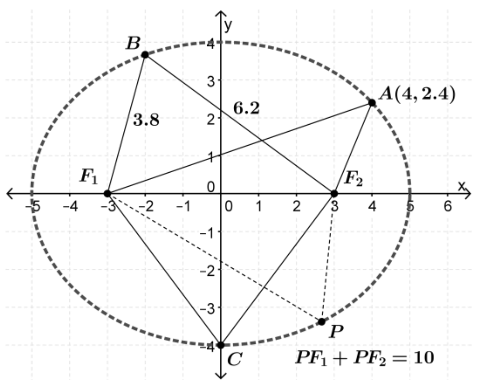


Figure 1.22

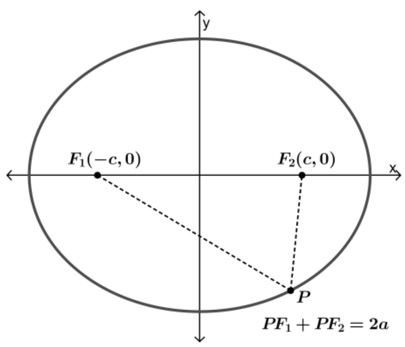


Figure 1.23

Let F_1 and F_2 be two distinct points. The set of all points P, whose distances from F_1 and from F_2 add up to a certain constant, is called an *ellipse*. The points F_1 and F_2 are called the *foci* of the ellipse.

Given are two points on the x-axis, F1(-c, 0) and F2(c, 0), the foci, both c units away from their center (0, 0). See Figure 1.23. Let P(x, y) be a point on the ellipse. Let the common sum of the distances be 2a (the coefficient 2 will make computations simpler). Thus, we have PF1 + PF2 = 2a

$$PF_1 = 2a - PF_2$$

$$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

$$x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2cx + c^2 + y^2$$

$$a\sqrt{(x-c)^2 + y^2} = a^2 - cx$$

$$a^2[x^2 - 2cx + c^2 + y^2] = a^4 - 2a^2cx + c^2x^2$$

$$(a^2 - c^2)x^2 + a^2y^2 = a^4 - a^2c^2 = a^2(a^2 - c^2)$$

$b^2x^2 + a^2y^2 = a^2b^2$ by letting $b = \sqrt{(a^2 - c^2)}$, so a>b

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

When we let b=$a^2 - c^2$, we assumed a > c. To see why this is true, look at ΔPF1F2 in Figure 1.23. By the Triangle Inequality, PF1 + PF2 > F1F2, which implies 2a > 2c, so a > c.

We collect here the features of the graph of an ellipse with standard equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,where a>b. Let $c = \sqrt{(a^2 - b^2)}$.

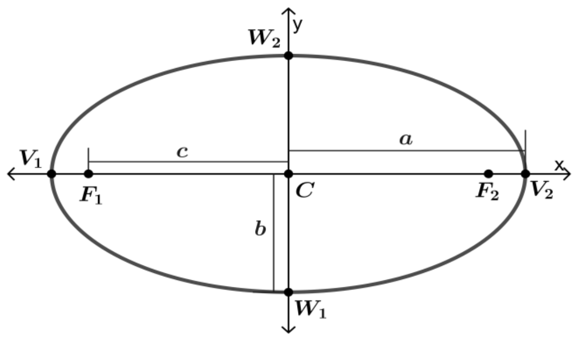


Figure 1.24

(1) *center* : origin (0, 0)
(2) *foci* : $F_1(-c, 0)$ and $F_2(c, 0)$
   - Each focus is c units away from the center.

- For any point on the ellipse, the sum of its distances from the foci is 2a.

(3) *vertices:* $V_1(-1,0)$ and V_2 (1,0)

- The vertices are points on the ellipse, collinear with the center and foci.
- If y = 0, then x = ±a. Each vertex is a unit away from the center. The segment V1V2 is called the major axis. Its length is 2a. It divides the ellipse into two congruent parts.

(4) *covertices:* $W_1(0, -b)$ and $W_2(0, b)$

- The segment through the center, perpendicular to the major axis, is the minor axis. It meets the ellipse at the covertices. It divides the ellipse into two congruent parts.
- If x = 0, then y = ±b. Each covertex is b units away from the center.
- The minor axis W_1W_2 is 2b units long. Since a > b, the major axis is longer than the minor axis.

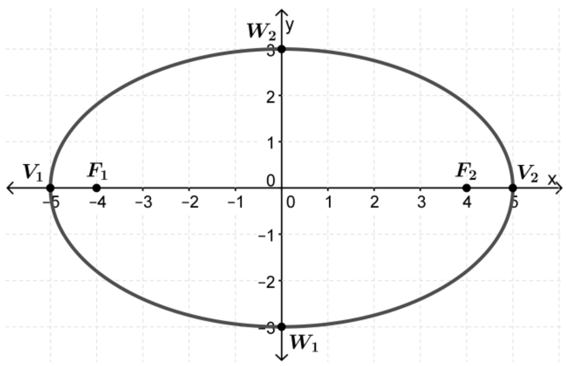
Example 3.3. Give the coordinates of the foci, vertices, and covertices of the ellipse with equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Sketch the graph, and include these points.

Solution. With $a^2 = 25$ and $b^2 = 9$, we have a = 5, b = 3, and

$c = \sqrt{(a^2 - b^2)} = 4.$

foci: $F_1(-4, 0), F_2(4, 0)$ vertices: $V_1(-5, 0), V_2(5, 0)$

covertices: $W_1(0, -3), W_2(0, 3)$

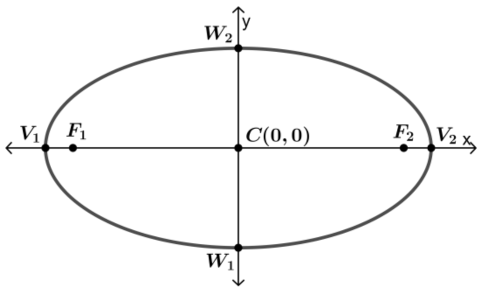


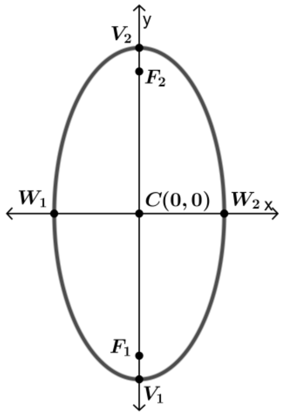
Example 3.4. Find the (standard) equation of the ellipse whose foci are F1(3, 0) and F2(3, 0), such that for any point on it, the sum of its distances from the foci is 10. See Figure 1.22.

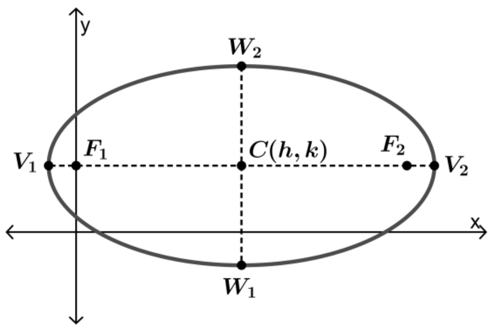
Solution. We have 2a = 10 and c = 3, so a = 5 and $b = \sqrt{(a^2 - c^2) = 4}$. The equation is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

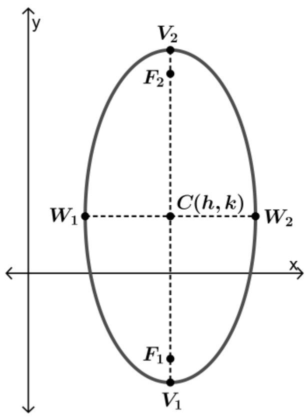
The ellipses we have considered so far are “horizontal” and have the origin as their centers. Some ellipses have their foci aligned vertically, and some have centers not at the origin. Their standard equations and properties are given in the box. The derivations are more involved, but are similar to the one above, and so are not shown anymore.

In all four cases below, a > b and The foci F_1 and F_2 are $c = \sqrt{(a^2 - b^2)}$ units away from the center. The vertices V_1 and V_2 are a units away from the center, the major axis has length 2a, the covertices W_1 and W_2 are b units away from the center, and the minor axis has length 2b. Recall that, for any point on the ellipse, the sum of its distances from the foci is 2a.

<table>
<tr><th>Center</th><th colspan="2">Corresponding Graphs</th></tr>
<tr><td>(0, 0)</td><td>

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$</td><td>

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, b > a$</td></tr>
<tr><td>(h, k)</td><td>

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$,a>b</td><td>

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$,b>a</td></tr>
<tr><td></td><td>major axis: horizontal
minor axis: vertical</td><td>major axis: vertical
minor axis: horizontal</td></tr>
</table>

In the standard equation, if the x-part has the bigger denominator, the ellipse is horizontal. If the y-part has the bigger denominator, the ellipse is vertical

Example 3.5. Give the coordinates of the center, foci, vertices, and covertices of the ellipse with the given equation. Sketch the graph, and include these points.

1. $\frac{(x+3)^2}{24} + \frac{(y-5)^2}{49} = 1$
2. $9x^2 + 16y^2 - 126x + 64y = 71$

Solution: (1) From $a^2 = 49$ and $b^2 = 24$, we have a=7, b=$2\sqrt{6} \approx 4.9$, and

Solution: (1) From $a^2 = 49$ and $b^2 = 24$, we have a=7, b=$2\sqrt{6} \approx 4.9$, and

$c = \sqrt{(a^2 - b^2)} = 5$. The ellipse is vertical.

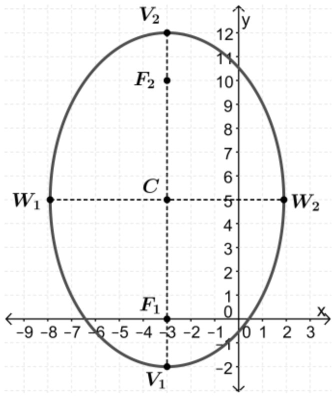
Center: (-3,5)

Foci: F_1(-3,0), F_2(-3,10)

Vertices: V_1 (-3,-2), V_2(-3,12)

Covertices: W_1(-3,-$2\sqrt{6}$, $5) \approx (-7.9,5)$

W_2(-3+$2\sqrt{6}$, $5) \approx (1.9,5)$



(2) We first change the given equation to standard form.

$$9(x^2 - 14x) + 16(y^2 + 4y) = 71$$
$$9(x^2 - 14x + 49) + 16(y^2 + 4y + 4) = 71 + 9(49) + 16(4)$$
$$9(x - 7)^2 + 16(y + 2)^2 = 576$$
$$\frac{(x - 7)^2}{64} + \frac{(y + 2)^2}{36} = 1$$

We have $a = 8$ and $b = 6$. Thus, $c = \sqrt{(a^2 - b^2)} = 2\sqrt{7}$=5.3. The ellipse is horizontal.

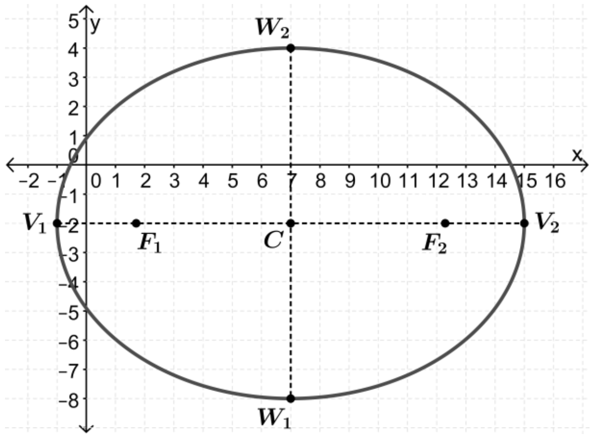
Center: (7,-2)

Foci: F_1(7-$2\sqrt{7}$,-2) $) \approx 1.7, -2)$

F_2 (7+$2\sqrt{7}$,-2) $) \approx 12.3, -2)$

Vertices: V_1 (-1,-2), V_2(15,-2)

Covertices: W_1(7,-8), W_2(7,4)



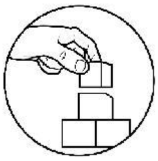
Example 3.6. The foci of an ellipse are (-3,-6) and (-3, 2). For any point on the ellipse, the sum of its distances from the foci is 14. Find the standard equation of the ellipse.

Solution. The midpoint (−3, −2) of the foci is the center of the ellipse. The ellipse is vertical (because the foci are vertically aligned) and c=4. From the given sum, 2a=14 so a=7. Also, $b = \sqrt{(a^2 - c^2)} = \sqrt{33}$. The equation is

$$\frac{(x+3)^2}{33} + \frac{(y+2)^2}{49} = 1$$

Example 3.7. An ellipse has vertices (2-$\sqrt{61}$,– 5) and (2+$\sqrt{61}$,– 5) and its minor axis is 12 units long. Find the standard equation and its foci.

Solution: The midpoint (2, −5) of the vertices is the center of the ellipse, which is horizontal. Each vertex is $a = \sqrt{61}$ units away from the center. From the length of the minor axis, 2b = 12 so b = 6. The standard equation is $\frac{(x-2)^2}{61} + \frac{(y+5)^2}{36} = 1$. Each focus is $c = \sqrt{(a^2 - b^2) = 5}$ units away from (2, -5), so their coordinates are (-3,-5) and (7,-5).



What's More

Activity 3.1: Let Me try!

Let us find out if you really understood the discussed concept by answering these follow-up exercises.

1. Convert the following general form to standard form of an ellipse.

a. $16x^2 + 4y^2 - 32x + 16y - 32 = 0$	c. $9x^2 + 4y^2 - 72x - 24y + 144 = 0$
b. $4x^2 + 9y^2 + 48x + 72y + 144 = 0$	d. $49x^2 + 9y^2$=441

2. Convert the following standard form to general form of an ellipse .

a. $\frac{(y-3)^2}{9} + \frac{(x-4)^2}{4} = 1$	c. $\frac{(x-1)^2}{4} + \frac{(y-1)^2}{2} = 1$
b$\frac{(x-2)^2}{20} + \frac{(y-3)^2}{36} = 1$	d. . $\frac{(y+3)^2}{4} + \frac{(x-2)^2}{1} = 1$

For numbers 3-6, give the coordinates of the center, foci, vertices, and covertices of the ellipse with the given equation. Sketch the graph, and include these points.

3. . $\frac{(x-7)^2}{64} + \frac{(y+2)^2}{25} = 1$

4. $16x^2 + 96x + 7y^2 + 14y + 39 = 0$

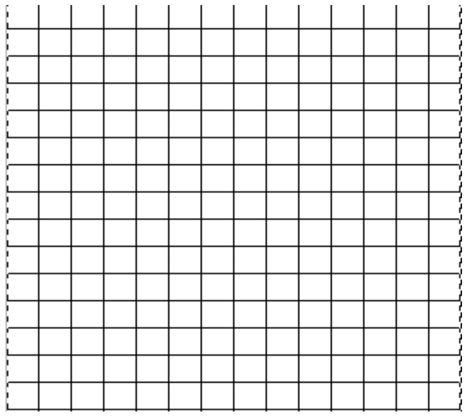
5. . Give the coordinates of the foci, vertices, and covertices of the ellipse with equation $\frac{x^2}{169} + \frac{y^2}{144} = 1.$ Then sketch the graph and include these points.

6. $25x^2 + 9y^2 = 225$

For numbers 7-10, answer as directed.

7. Find the standard equation of the ellipse whose foci are F1(0,-8) and F2(0, 8), such that for any point on it, the sum of its distances from the foci is 34.

8. 8. Find an equation of an ellipse with center at (0,0), one focus at (3,0), and a vertex (-4,0). Sketch the graph.



9. The covertices of an ellipse are (5,6) and (5,8). For any point on the ellipse, the sum of its distances from the foci is 12. Find the standard equation of the ellipse.

10. An ellipse has foci $(-4-\sqrt{15},3)$ and $(-4+\sqrt{15},3)$, and its major axis is 10 units long. Find its standard equation and its vertices.

What I Have Learned

Let me check your knowledge by filling the blanks with a correct symbols/letter or terms in order to complete the statement/s.

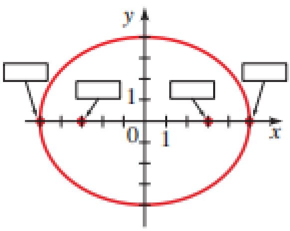
1. An ellipse is the set of all points in the plane for which the __________ of the distances from two fixed points F_1 and F_2 is constant. The points F_1 and F_2 are called the _______ of the ellipse.

2. The graph of equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with a>b is an ellipse with vertices (__,__) and (__,__) and foci $(\pm c, 0)$,where c = ________. So the graph of $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$ is an ellipse with vertices (_,_) and (_,__) and foci (_,__) and (__,__).

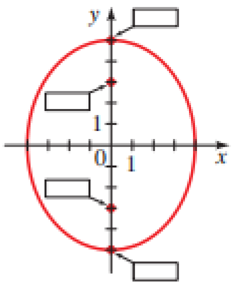
3. The graph of the equation $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ with a>b>0 is an ellipse with vertices (__,___ and (__,__) and foci (0,$\pm c$)where c=___. So the graph of $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$ is an ellipse with vertices (__,__) and (__,__) and foci (__,__) and (__,__).

4. Label the vertices and foci on the graphs given for the ellipses:

a. $\frac{x^2}{5^2} + \frac{y^2}{4^2}$



b. $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$



What I Can Do

DISCOVERY■ DISCUSSION■ WRITING

Materials: bond paper, cylindrical bottle, compass

1. A flashlight shines on a wall, as shown in the figure. What is the shape of the boundary of the lighted area? Explain your answer.

2. Get a piece of bond paper and wrapped around a cylindrical bottle, and then use a compass to draw a circle on the paper, as shown in the figure. When the paper is laid flat, is the shape drawn on the paper an ellipse? Explain your findings.

Online connect! *For additional knowledge and information about the topics please visit the links/url indicated below.*

1. shorturl.at/cknx6
2. shorturl.at/pGTZ1
3. shorturl.at/agAO6

LESSON 4 The Hyperbola

What I Need to Know

Upon completion of this lesson, you should be able to:

- Determine hyperbola and its properties.
- determine the standard form of equation of hyperbola;
- sketch hyperbola given various equations;
- discuss the parts of hyperbola;

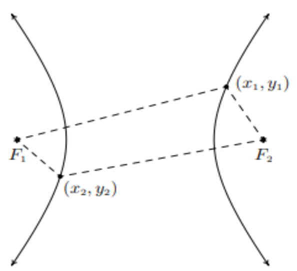
What's In

Just like ellipse, a hyperbola is one of the conic sections that most students have not encountered formally before. Its graph consists of two unbounded branches which extend in opposite directions. It is a misconception that each branch is a parabola. This is not true, as parabolas and hyperbolas have very different features.

Hyperbolas can be used in so-called "trilateration", or positioning problems. It is possible to locate the place from which a sound, such as gunfire, emanates. Long Range Aid to Navigation (**LORAN** for short) system, of ship or aircraft utilizes hyperbolas.

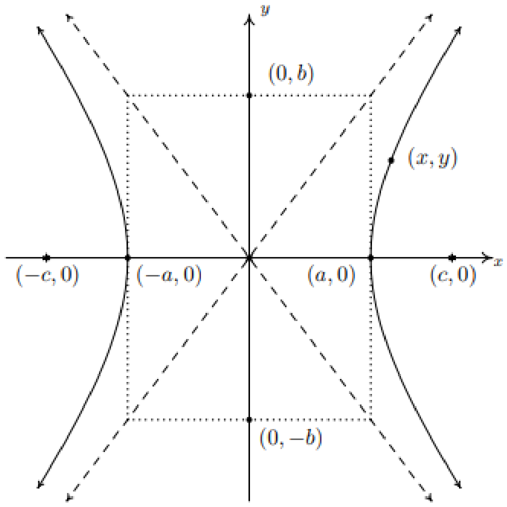
<u>Definition and Equation of a Hyperbola</u>

Given two distinct points F_1 and F_2 in the plane and a fixed distance ***d***, a **hyperbola** is the set of all points (x,y) in the plane such that the absolute value of the difference of each of the distances from F_1 and F_2 to (x,y) is ***d.*** The points F_1 and F_2 are called the foci of the hyperbola.



In the figure above: the distance of F_1 to (x_1,y_1) - distance of F_2 to (x_1,y_1) = d
And the distance of F_2 to (x_2,y_2) - distance of F_2 to (x_2,y_2) = d.

Suppose we wish to derive the equation of a hyperbola. For simplicity, we shall assume that the center is (0, 0), the vertices are (a, 0) and (−a, 0) and the foci are (c, 0) and (−c, 0). We label the endpoints of the conjugate axis (0, b) and (0, −b). (Although b does not enter into our derivation, we will have to justify this choice as you shall see later.) As before, we assume a, b, and c are all positive numbers. Schematically we have



Since (a, 0) is on the hyperbola, it must satisfy the conditions of hyperbola. That is, the distance from (−c, 0) to (a, 0) minus the distance from (c, 0) to (a, 0) must equal the fixed distance d. Since all these points lie on the x-axis, we get

$$\text{distance from } (-c, 0) \text{ to } (a, 0) - \text{distance from } (c, 0) \text{ to } (a, 0) = d$$

$$(a + c) - (c - a) = d$$

$$2a = d$$

In other words, the fixed distance d from the definition of the hyperbola is actually the length of the transverse axis! (Where have we seen that type of coincidence before?) Now consider a point (x, y) on the hyperbola. Applying the definition, we get

$$\text{distance from } (-c, 0) \text{ to } (x, y) - \text{distance from } (c, 0) \text{ to } (x, y) = 2a$$

$$\sqrt{(x-(-c))^2+(y-0)^2}-\sqrt{(x-c)^2+(y-0)^2}=2a$$

$$\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=2a$$

$\sqrt{(x+c)^2+y^2} = 2a + \sqrt{(x-c)^2+y^2}$, Transpose the second radical to the right

$\left(\sqrt{(x+c)^2+y^2}\right)^2 = \left(2a+\sqrt{(x-c)^2+y^2}\right)^2$, Square both sides of the equation

$x^2+2cx+c^2+y^2 = 4a^2+4a\sqrt{x^2-2cx+c^2+y^2}+x^2-2cx+c^2+y^2$, Cancel like terms

$4cx = 4a^2+4a\sqrt{x^2-2cx+c^2+y^2}$, Combine like terms and divide the Eq. by 4

$cx-a^2 = a\sqrt{(x-c)^2+y^2}$, Transpose a^2 to the left side of the Eq.

$\left(cx-a^2\right)^2 = \left(a\sqrt{(x-c)^2+y^2}\right)^2$, Square again both sides of the equation

$c^2x^2-2a^2cx+a^4 = a^2\left(x^2-2cx+c^2+y^2\right)$, Distribute a^2 to the quantity

$c^2x^2-a^2x^2-a^2y^2 = a^2c^2-a^4$, Factor out the common term

$\left(c^2-a^2\right)x^2-a^2y^2 = a^2\left(c^2-a^2\right)$, Regroup the terms

$b^2x^2-a^2y^2 = a^2b^2$, Replace $\left(c^2-a^2\right)$ with b^2 since $b^2=c^2-a^2$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, Divide both sides of the Eq. by a^2b^2

What's New

Activity 1

You'll need: Patty paper Ruler Sharpie Compass Colored paper Tape or glue stick As you do each of the following, be careful not to smudge your work. For any step that includes the use of a Sharpie, wait about 30 seconds after marking before you do any folding.

1. Using a Sharpie, draw a point near the center of your wax paper.

2. Label this point O.

3. Using a pencil and a compass, draw a circle of radius 4 cm†, using the point you drew in step 1 as the center.

4. Using a Sharpie, draw another point somewhere outside your circle. Where you put the point will affect the final result. Don't put your point too close to the edge of the paper or too close to the circle, or it will be difficult to do the rest of the activity. Try to arrange it so that everyone in your group has their point at different distances and in different positions from their circle.

5. Label this point F.

6. Fold the paper so that the point F lands on the circle (or the circle lands on F).

7. Crease the paper.

8. “Slide” the point along the circle just a little bit so that a different place on the circle is over the point F.

9. Crease again.

10. Keep sliding, folding and creasing the paper so that different places on the circle land on the point. The closer together your creases are, the more refined your shape will be.

11. Keep doing this until you have gone all the way around the circle.

12. Unfold your paper. Do you see a definitive shape?

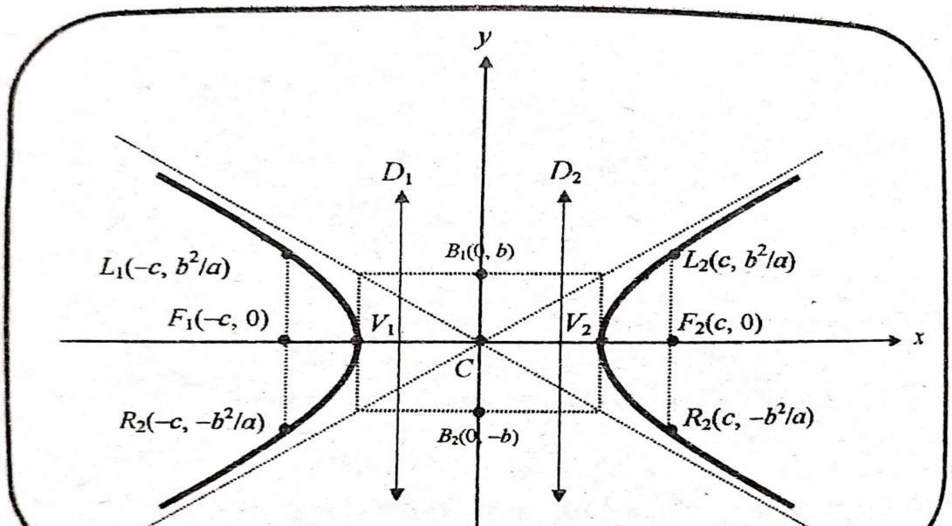
13. Carefully darken the outline of your shape with a Sharpie. You’ve just drawn a hyperbola!

14. Tape or glue the edges of your patty paper to a piece of colored paper.

15. Write “The Hyperbola” and your name at the top of the colored paper.

What is It

We collect here the features of the graph of a hyperbola with standard equation.



Graph of a Hyperbola

(1) ***center*** : origin (0, 0)

(2) ***foci*** : $F_1(-c, 0)$ and $F_2(c, 0)$

- Each focus is c units away from the center.
- For any point on the hyperbola, the absolute value of

the difference of its distances from the foci is 2a.

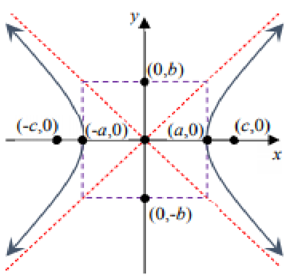
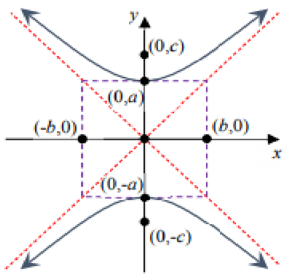
(3) ***vertices***: $V_1(-a, 0)$ and $V_2(a, 0)$

- The vertices are points on the hyperbola, collinear with the center and foci.
- If $y = 0$, then $x = \pm a$. Each vertex is a units away from the center.
- The segment V_1V_2 is called the ***transverse axis***. Its length is 2a.

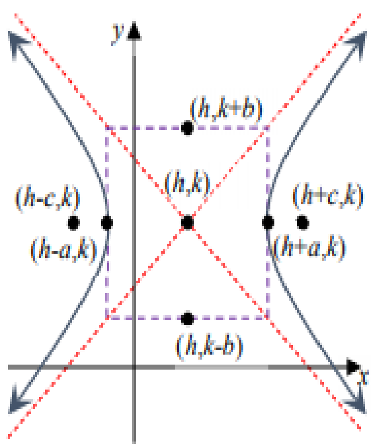
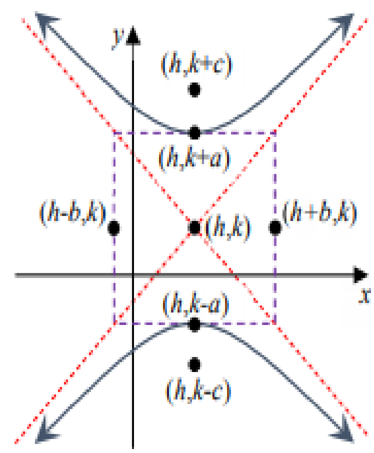
(4) ***asymptotes***: $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$, the lines ℓ_1 and ℓ_2 in the figure

- The asymptotes of the hyperbola are two lines passing through the center which serve as a guide in graphing the hyperbola: each branch of the hyperbola gets closer and closer to the asymptotes, in the direction towards which the branch extends. (We need the concept of limits from calculus to explain this.)
- An aid in determining the equations of the asymptotes: in the standard equation, replace 1 by 0 and in the resulting equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$, solve for y.
- To help us sketch the asymptotes, we point out that the asymptotes ℓ_1 and ℓ_2 are the extended diagonals of the ***auxiliary rectangle*** drawn in Figure 1.45. This rectangle has sides 2a and 2b with its diagonals intersecting at the center C. Two sides are congruent and parallel to the transverse axis V_1V_2.The other two sides are congruent and parallel to the ***conjugate axis***, the segment shown which is perpendicular to the transverse axis at the center, and has length 2b.

Equation of a Hyperbola Centered at the Origin in Standard Form

Opens	Horizontally	Vertically
Standard Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices	$(-a, 0)$ and $(a, 0)$	$(0, -a)$ and $(0, a)$
Foci	$(-c, 0)$ and $(c, 0)$ where $b^2 = c^2 - a^2$	$(0, -c)$ and $(0, c)$ Where $b^2 = c^2 - a^2$
Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$
Construction Rectangle Vertices	(a, b), $(-a, b)$, $(a, -b)$, $(-a, -b)$	(b, a), $(-b, a)$, $(b, -a)$, $(-b, -a)$
Graph		

Equation of a Hyperbola Centered at (h, k) in Standard Form

Opens	Horizontally	Vertically
Standard Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Foci	$(h \pm c, k)$ where $b^2 = c^2 - a^2$	$(h, k \pm c)$ where $b^2 = c^2 - a^2$
Asymptotes	$y-k=\pm\frac{b}{a}(x-h)$	$y-k=\pm\frac{a}{b}(x-h)$
Construction Rectangle Vertices	$(h \pm a, k \pm b)$	$(h \pm b, k \pm a)$
Graph		

Activity 2

HOW TO SKETCH A HYPERBOLA

1. **Sketch the Central Box.** This is the rectangle centered at the origin, with sides parallel to the axes, that crosses one axis at $\pm a$, the other at $\pm b$.
2. **Sketch the Asymptotes.** These are the lines obtained by extending the diagonals of the central box.
3. **Plot the Vertices.** These are the two x-intercepts or the two y-intercepts.
4. **Sketch the Hyperbola.** Start at a vertex, and sketch a branch of the hyperbola, approaching the asymptotes. Sketch the other branch in the same way.

Example 1. **Sketch the graph of the hyperbola with the equation $4x^2 - y^2 = 16$. Include and label the foci, vertices, endpoints, asymptotes, transverse and conjugate axis, and rectangle constructions.**

Step 1. **Rewrite the equation in standard form.**

$$4x^2 - y^2 = 16$$

$$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

$$\frac{x^2}{(2)^2} - \frac{y^2}{(4)^2} = 1$$

Step 2. **Analysis.**

a.) **The transverse axis is horizontal because x^2 is positive.**

b.) $c^2 = a^2 + b^2$

c.) $a = 2$, $b = 4$

Example 1 (Continued):

Step 2. **Analysis.**

d.) $c^2 = a^2 + b^2$

$c^2 = (2)^2 + (4)^2$

$c^2 = 4 + 16 = 20$

$c = \pm\sqrt{20} = \pm 2\sqrt{5}$

e.) **Foci:** $\left(-2\sqrt{5}, 0\right), \left(2\sqrt{5}, 0\right)$

f.) **Vertices:** $(-2, 0), (2, 0)$

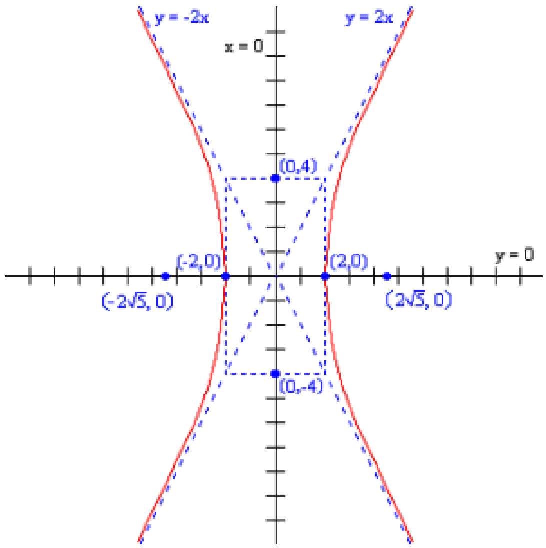
g.) **Endpoints:** $(0, -4), (0, 4)$

h.) **Asymptotes:** $y = \pm\frac{4}{2}x = \pm 2x$

i.) **Transverse axis:** **y = 0**

j.) **Conjugate axis:** **x = 0**

Step 3. **Graph**



Example 2. **Sketch the graph of the hyperbola with the equation $9y^2 - x^2 = 9$. Include and label the foci, vertices, endpoints, asymptotes, transverse and conjugate axis, and rectangle constructions.**

Step 1. **Rewrite the equation in standard form.**

$$9y^2 - x^2 = 9$$
$$\frac{9y^2}{9} - \frac{x^2}{9} = \frac{9}{9}$$
$$\frac{y^2}{1} - \frac{x^2}{9} = 1$$
$$\frac{y^2}{(1)^2} - \frac{x^2}{(3)^2} = 1$$

Step 2. **Analysis.**

a.) **The transverse axis is vertical because y^2 is positive.**

b.) $c^2 = a^2 + b^2$

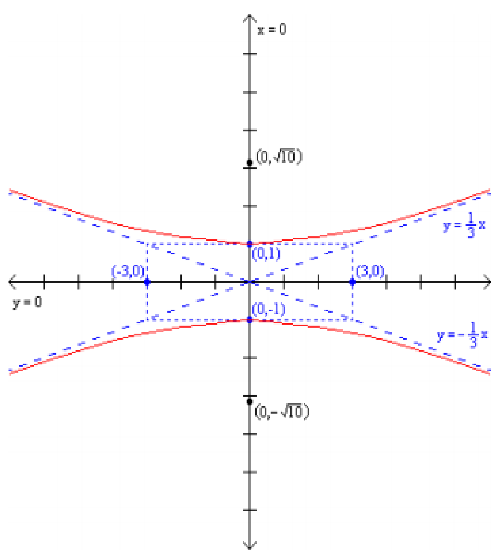
c.) $a = 3$, $b = 1$

d.) $c^2 = a^2 + b^2$

$$c^2 = (3)^2 + (1)^2$$
$$c^2 = 9 + 1 = 10$$
$$c = \pm\sqrt{10}$$

Example 2 (Continued):

Step 3. **Graph**



Example 3

Give the coordinates of the center, foci, vertices, and asymptotes of the hyperbola with the given equation. Sketch the graph, and include these points and lines, the transverse and conjugate axes, and the auxiliary rectangle.

1.) $\frac{(y+2)^2}{25} - \frac{(x-7)^2}{9} = 1$ 2.) $4x^2 - 5y^2 + 32x + 30y = 1$

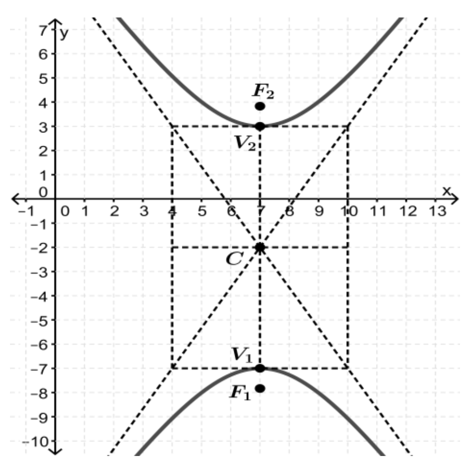
Solution: 1.) From $a^2 = 25$, and $b^2 = 9$, we have a = 5, b = 3, and

$c = a^2 + b^2 = \sqrt{34}$, $c \approx 5.8$. The hyperbola is vertical. To determine the asymptotes, we write $\frac{(y+2)^2}{25} - \frac{(x-7)^2}{9} = 0$, which is equivalent to $y + 2 = \pm\frac{5}{3}(x-7)$. We can then solve for y.

center: C(7, -2) foci: $F_1(7,-2-\sqrt{34}) \approx (7,-7.8)$ and $F_2(7,-2+\sqrt{34}) \approx (7,3.8)$

asymptotes: $y = \frac{5}{3}x - \frac{41}{3}$ and $y = -\frac{5}{3}x + \frac{29}{3}$

The conjugate axis drawn has its endpoints b = 3 units to the left and right of the center.



Solution 2.) We first change the given equation to standard form.

$$4(x^2+8x)-5(y^2-6y)=1$$
$$4(x^2+8x+16)-5(y^2-6y+9)=1+4(16)-5(9)$$
$$4(x+4)^2-5(y-3)^2=20$$
$$\frac{(x+4)^2}{5}-\frac{(y-3)^2}{4}=1$$

We have $a=\sqrt{5}\approx 2.2$ and b = 2. Thus, $c=\sqrt{a^2+b^2}=3$. The hyperbola is horizontal. To determine the asymptotes, we write $\frac{(x+4)^2}{5}-\frac{(y-3)^2}{4}=0$ which is equivalent to $y-3=\pm\frac{2}{\sqrt{5}}(x+4)$, and solve for y.

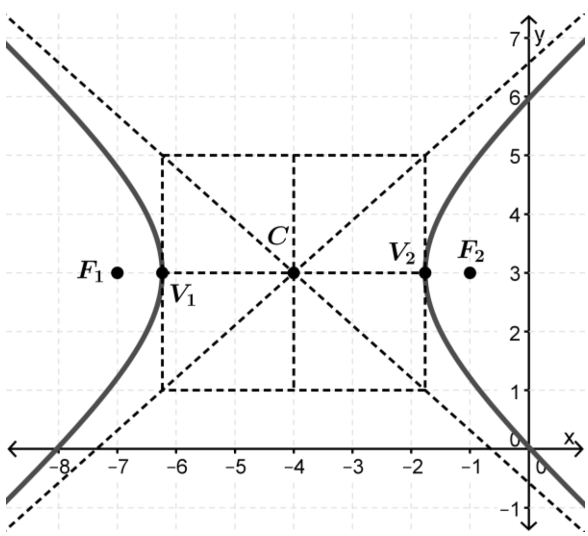
center: c(-4,3)

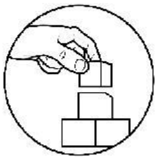
foci: F_1(-7,3) and F_2(-1,3)

vertices: $V_1(-4-\sqrt{5},3)\approx(-6.2,3)$ and $V_2(-4+\sqrt{5},3)\approx(-1.8,3)$

asymptotes: $y=\frac{2}{\sqrt{5}}x+\frac{8}{\sqrt{5}}+3$ and $y=-\frac{2}{\sqrt{5}}x-\frac{8}{\sqrt{5}}+3$

The conjugate axis drawn has its endpoints b = 2 units above and below the center.





What's More

Let us find out if you really understood the concept about hyperbola by answering these exercises.

A. Reduce each of the following equations of hyperbolas in standard form.

1.) $4x^2 - y^2 + 2y - 5 = 0$

2.) $36x^2 - 81y^2 + 24x + 328 = 0$

3.) $49x^2 - 25y^2 + 98x + 200y + 874 = 0$

4.) $28x^2 - 64y^2 - 28x - 128y - 505 = 0$

B. Determine the foci, vertices, and asymptotes of the hyperbola with equation $\frac{x^2}{16} - \frac{y^2}{33} = 1$. Sketch the graph, and include these points and lines, the transverse and conjugate axes, and the auxillary rectangle.

For each equation of the hyperbola, find the center, foci, vertices, endpoints of conjugate axis. Determine the equation of the asymptotes and sketch the graph.

1.) $\frac{(y+6)^2}{25} - \frac{(x-4)^2}{39} = 1$ 2.) $9x^2 + 126x - 16y^2 - 96y + 153 = 0$

What I Have Learned

Let me check your knowledge by filling the blanks with a correct symbols/letter or terms in order to complete the statement/s.

1. A hyperbola is the set of all points in the plane for which the ________ of the distances from two fixed points F1 and F2 is constant. The points F1 and F2 are called the _______ of the hyperbola.

2. The graph of the equation $\frac{x^2}{a^2} - \frac{x^2}{a^2} = 1$ with a>0,b>0 is a hyperbola with vertices (___,___) and (__,__) and foci$(\pm c, 0)$ where c= ________. So the graph of

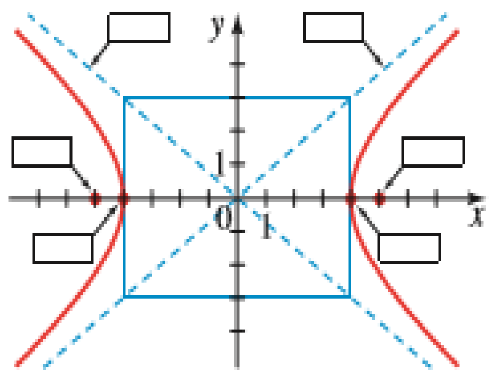
$\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$ is a hyperbola with vertices (___, ___) and (___,___) and foci (___,___) and (___,___).

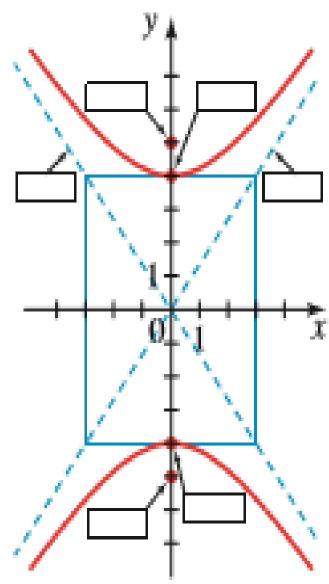
3. The graph of the equation $\frac{y^2}{a^2} - \frac{x^2}{a^2} = 1$ with a>0,b>0 is a hyperbola with vertices (___,___) and (___,___) and foci $(0, \pm c)$, where c= _________. So the graph of $\frac{y^2}{4^2} - \frac{x^2}{3^2} = 1$ is a hyperbola with vertices (__,__) and (___,__) and foci (___,___) and (___,___).

4 Label the vertices, foci, and asymptotes on the graphs given for the hyperbola.

a. $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

b. $\frac{y^2}{4^2} - \frac{x^2}{3^2} = 1$





What I can Do

DISCOVERY■ DISCUSSION■ WRITING

1. Light from a Lamp. The light from a lamp forms a lighted area on a wall, as shown in the figure. Why is the boundary of this lighted area a hyperbola? How can one hold a flashlight so that its beam forms a hyperbola on the ground?

Online connect! *For additional knowledge and information about the topics please visit the links/url indicated below.*

Hyperbola (Part 1) - Conic Sections Class 11 CBSE
https://www.youtube.com/watch?v=WEyYaIWIUp0

Hyperbola (Part 2) - Conic Sections Class 11 CBSE
https://www.youtube.com/watch?v=Ni0gjU8-Pn4

LESSON 5 Equation and Important Characteristics of the Different Types of Conic Sections

What I Need to Know

Upon completion of this lesson, you should be able to:

- recognize the equation and important characteristics of the different types of conic sections discuss the parts of hyperbola;

What's In

s lesson, we will identify the conic section from a given equation. We will properties of the identified conic section. We will also look at problems that use the properties of the different conic sections. This will allow us to synthesize what has been covered so far.

Identifying the Conic Section by Inspection

The equation of a circle may be written in standard form

$$Ax^2 + Ay^2 + Cx + Dy + E = 0,$$

that is, the coefficients of x^2 and y^2 are the same. However, it does not follow that if the coefficients of x^2 and y^2 are the same, the graph is a circle.

General Equation	Standard Equation	Graph
(A) $2x^2 + 2y^2 - 2x + 6y + 5 = 0$	$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 0$	point
(B) $x^2 + y^2 - 6x - 8y + 50 = 0$	$(x-3)^2 + (y-4)^2 = -25$	empty set

For a circle with equation $(x-h)^2 + (y-k)^2 = r^2$, we have $r^2 > 0$. This is not the case for the standard equations of (A) and (B).

In (A), becase the sum of two squares can only be 0 if and only if each square is 0, it follows that $x - \frac{1}{2} = 0$ and $y + \frac{3}{2} = 0$. The graph is thus the single point $\left(\frac{1}{2}, -\frac{3}{2}\right)$.

In (B), no real values of x and y can make the nonnegative left side equal to the negative right side. The graph is then the empty set.

Let us recall the general form of the equations of the other conic sections.

We may write the equations of conic sections we discussed in the general form $Ax^2 + By^2 + Cx + Dy + E = 0$.

Some terms may vanish, depending on the kind of conic section.

(1) *Circle:* both x^2 and y^2 appear, and their coefficients are the same

$$Ax^2 + Ay^2 + Cx + Dy + E = 0$$

Example: $18x^2 + 18y^2 - 24x + 48y - 5 = 0$
Degenerate cases: a point, and the empty set

(2) *Parabola:* exactly one of x^2 or y^2 appears

$Ax^2 + Cx + Dy + E = 0$ (D ƒ= 0, opens upward or downward)
$By^2 + Cx + Dy + E = 0$ (C ƒ= 0, opens to the right or left)

Examples: $3x^2 - 12x + 2y + 26 = 0$ (opens downward)
$-2y^2 + 3x + 12y - 15 = 0$ (opens to the right)

(3) *Ellipse:* both x^2 and y^2 appear, and their coefficients A and B have the same sign and are unequal

Examples: $2x^2 + 5y^2 + 8x - 10y - 7 = 0$ (horizontal major axis) $4x^2 + y^2 - 16x - 6y + 21 = 0$ (vertical major axis)

If A = B, we will classify the conic as a circle, instead of an ellipse.

Degenerate cases: a point, and the empty set

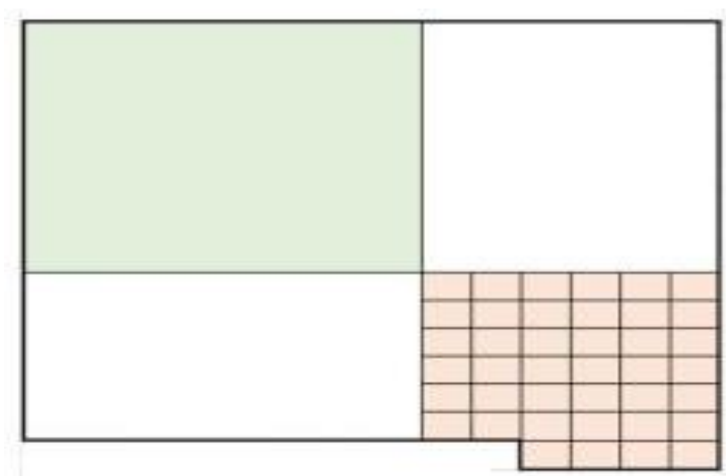
(4) *Hyperbola:* both x^2 and y^2 appear, and their coefficients A and B have different signs

Examples: $5x^2 - 3y^2 - 20x - 18y - 22 = 0$ (horizontal transverse axis)
$-4x^2 + y^2 + 24x + 4y - 36 = 0$ (vertical transverse axis)

Degenerate case: two intersecting lines.

What's New

Activity 1



Verify a Model

Alex says he can represent the enclosed region as

$(x+6)^2+4$

Julie says she can represent the enclosed region as

$x^2+12x+40$.

Which student do you agree with? Justify your choice.

You can draw on the picture to justify your thinking.

What is It

The following examples will show the possible degenerate conic (a point, two intersecting lines, or the empty set) as the graph of an equation following a similar pattern as the non-degenerate cases.

$$(1)\ 4x^2+9y^2-16x+18y+25=0 \Rightarrow \frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{2^2}=0$$

$$\Rightarrow \text{one point } (2,-1)$$

$$(2)\ 4x^2+9y^2-16x+18y+61=0 \Rightarrow \frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{2^2}=-1$$

$$\Rightarrow \text{empty set}$$

$$(3)\ 4x^2-9y^2-16x-18y+7=0 \Rightarrow \frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{2^2}=0$$

$$\Rightarrow \text{two lines}: y+1=\pm\frac{2}{3}(x-2)$$

A Note on Identifying a Conic Section by Its General Equation

It is only after transforming a given general equation to standard form that we can identify its graph either as one of the degenerate conic sections (a point, two intersecting lines, or the empty set) or as one of the non-degenerate conic sections (circle, parabola, ellipse, or hyperbola).

General Equations of Conics		
Classifying a Conic from Its General Equation		
The Graph of $Ax^2+Cy^2+Dx+Ey+F=0$ is one of the following.		
1. Circle	$A=C$	$A\neq 0$
2. Parabola	$AC=0$	$A=0$ or $C=0$, but not both.
3. Ellipse	$AC>0$	A and C have like signs.
4. Hyperbola	$AC<0$	A and C have unlike signs.
NOTE: The test above is valid when the graph is a non-degenerate conic(circle, parabola, ellipse, or hyperbola)		

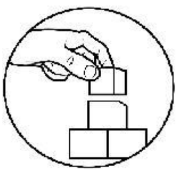
Example:	
Classifying Conics from General Equations	
Equation	Solution
$4x^2 - 9x + y - 5 = 0$	AC = 4(0) = 0. Parabola
$4x^2 - y^2 + 8x - 6y + 4 = 0$	AC = 4(-1) = -4 < 0. Hyperbola
$2x^2 + 4y^2 - 4x + 12y = 0$	AC = 2(4) = 8 > 0. Ellipse
$2x^2 + 2y^2 - 8x + 12y + 2 = 0$	A = C = 2. Circle

Activity 2

Classify the graph of the equation as circle, a parabola, an ellipse, or a hyperbola.

Show your proof.

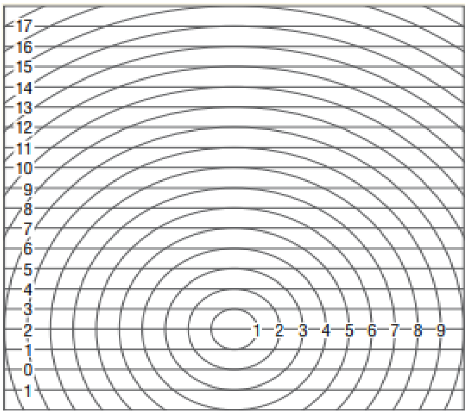
1.) $9x^2 + 4y^2 - 18x + 16y - 119 = 0$
2.) $x^2 + y^2 - 4x - 6y - 23 = 0$
3.) $16x^2 - 9y^2 + 32x + 54y - 209 = 0$
4.) $x^2 + 4x - 8y + 20 = 0$
5.) $y^2 + 12x + 4y + 28 = 0$
6.) $4x^2 + 25y^2 + 16x + 250y + 541 = 0$
7.) $x^2 + y^2 + 2x - 6y = 0$
8.) $y^2 - x^2 + 2x - 6y - 8 = 0$
9.) $3x^2 + 3y^2 - 6x + 6y + 5 = 0$
10.) $2x^2 - 4y^2 + 4x + 8y - 3 = 0$



What's More

Activity 3

Mark the point at the intersection of circle 1 and line 1. Mark both points that are on line 2 and circle 2. Continue this process, marking both points on line 3 and circle 3, and so on. Then connect the points with a smooth curve.

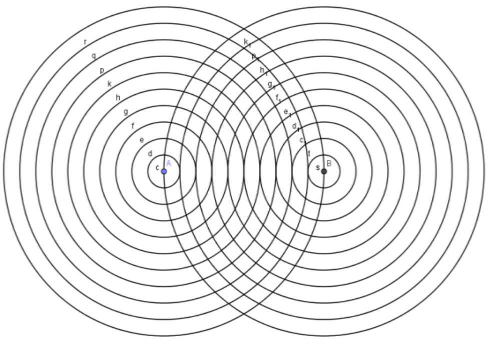


Activity 4

An ellipse is the set of points such that the sum of distance from two fixed points is constant. The two fixed points are called foci.

- Use graph paper like that shown. It contains two small circles and a series of concentric circles from each. The concentric circles are tangent to each other as shown.
- Choose the constant 13. Mark the points at the intersections of circle 9 and circle 4, because 9 + 4 = 13. Continue this process until you have marked the intersection of all circles whose sum is 13.
- Connect the points to form a smooth curve. The curve is an ellipse whose foci are the centers of the two small circles on the graph paper.

Circles in the figure are named by letters, circle 1 is circle c, circle 2 is circle d, and so on and so forth

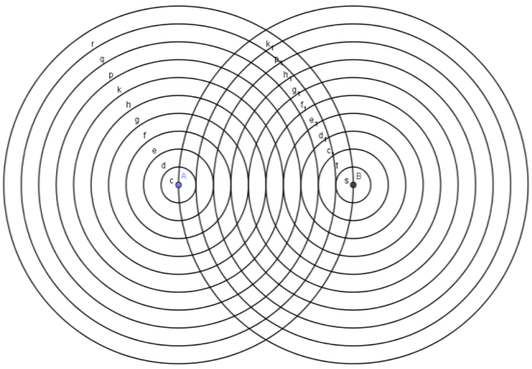


Activity 5

A hyperbola is the set of points such that the difference of the distances from two fixed points is constant. The two fixed points are called the foci.

- Use the same type of graph paper that you used for the ellipse in Activity 2. Choose the constant 7. Mark the points at the intersections of circle 9 and circle 2, because $9 - 2 = 7$. Continue this process until you have marked the intersections of all circles whose difference in radius is 7.
- Connect the points to form a hyperbola.

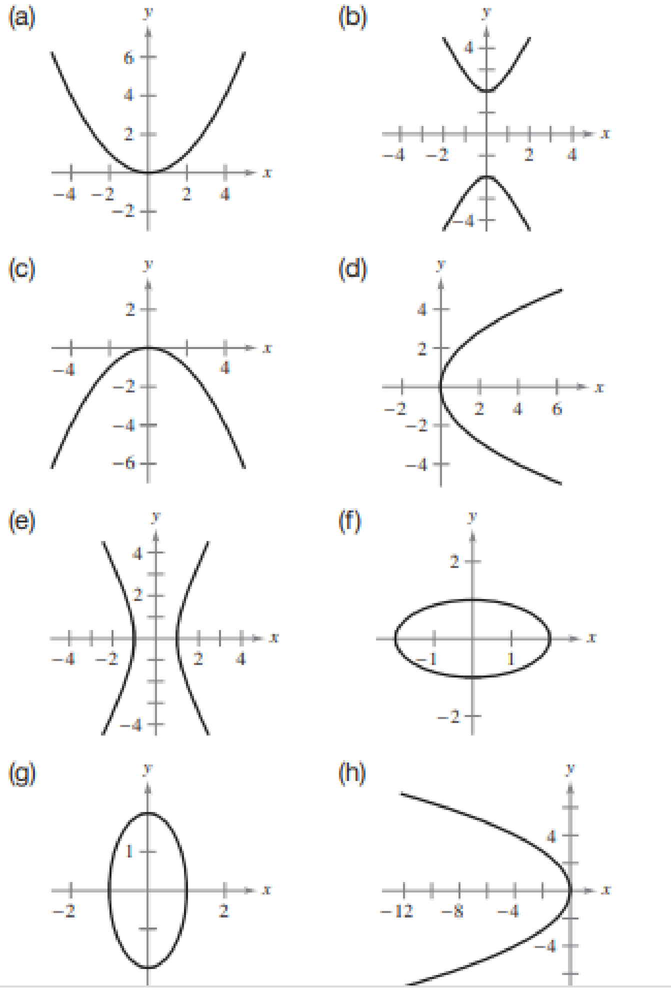
Circles in the figure are named by letters, circle 1 is circle c, circle 2 is circle d, and so on and so forth.



What I have Learned

Activity 6

Matching In Exercises 1–8, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]



1. $x^2 = 4y$

2. $x^2 = -4y$

3. $y^2 = 4x$

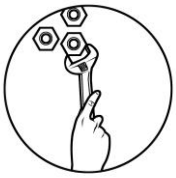
4. $y^2 = -4x$

5. $\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$

6. $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

7. $\dfrac{x^2}{1} - \dfrac{y^2}{4} = 1$

8. $\dfrac{y^2}{4} - \dfrac{x^2}{1} = 1$



What I can Do

Activity 7

Model and Analyze

1. Use the type of graph paper you used in Activity 1. Mark the intersection of line 0 and circle 2. Then mark the two points on line 2 and circle 3, the two points on line 2 and circle 4, and so on. Draw the new parabola. Continue this process and make as many parabolas as you can on one sheet of the graph paper. The focus is always the center of the small circle. Why are the resulting graphs parabolas?

2. In Activity 4 you drew an ellipse such that the sum of the distances from two fixed points was 13. Choose 10, 11, 12, 14, and so on for that sum, and draw as many ellipses as you can on one piece of the graph paper.

a. Why can you not start with 9 as the sum?

b. What happens as the sum increases? decreases?

3. In Activity 5 you drew a hyperbola such that the difference of the distances from two fixed points was 7. Choose other numbers and draw as many hyperbolas as you can on one piece of graph paper. What happens as the difference increases? decreases?

Online connect! *For additional knowledge and information about the topics please visit the links/url indicated below.*

- Determining What Type of Conic Section from General Form

https://www.youtube.com/watch?v=auD46ZWZxQo

- Conic Sections - Circles, Ellipses, Parabolas, Hyperbola - HowTo Graph & Write In Standard Form

https://www.youtube.com/watch?v=PLrgwD9TleU

LESSON 6 Solving Situational Problems Involving Conic Sections

What I Need to Know

Upon completion of this lesson, you should be able to:

- solve situational problems involving conic sections

What's In

There are four conics in the conics sections- Parabolas, Circles, Ellipses and Hyperbolas. We see them everyday, but we just don't notice them. They appear everywhere in the world and can be man-made or natural. The applications of conics can be seen everyday all around us. Conics are found in architecture, physics, astronomy and navigation. If you get lost, you can use a GPS and it will tell you where you are (a point) and it will lead you to your destination (another point). Bridges, buildings and statues use conics as support systems. Conics are also used to describe the orbits of planets, moons and satellites in our universe.

What's New

Activity 1

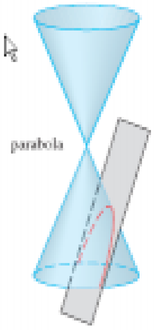
Mathematics is fun, isn't it? To develop your artistic minds and also to apply your brilliant ideas about the different conic sections you will be task to create or draw a figure resembling the different types of conic sections. It's like a tessellation but not a tessellation because it can have gaps or overlapping which is not true for tessellation. Use A-4 size bond paper for your drawing. Add colors on your drawing.

Use the sample below as your guide.

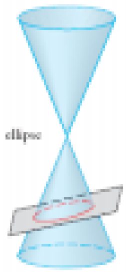
What Is It

Since conics have so many real-life applications, it is essential that we know how to solve problems involving conic sections. Below are the different conic sections that you have learned in the previous lessons with their corresponding properties and equations that will help you in solving situational problems involving conics.

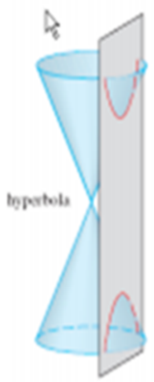
Parabola		
	Vertical Axis	Horizontal axis
equation	$(x-h)^2=4p(y-k)$	$(y-k)^2=4p(x-h)$
Axis of symmetry	$x=h$	$y=k$
Vertex	(h,k)	(h,k)
Focus	$(h,k+p)$	$(h+p,k)$
Directrix	$y=k-p$	$x=h-p$
Direction of opening	$p>0$ then up; $p<0$ then down	$p>0$ then rignt; $p<0$ then left



Ellipse		
	Vertical Major Axis	Horizontal Major axis
equation	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
center	(h,k)	(h,k)
Vertices	$(h,k\pm a)$	$(h\pm a,k)$
Foci	$(h,k\pm c)$	$(h\pm c,k)$
Major axis equation	$2a$=length of major axis	
Minor axis equation	$2b$=length of minor axis	
Equation that relates a, b, and c	$a^2=b^2+c^2$	
Eccentricity of an ellipse	$e=(c/a)$	

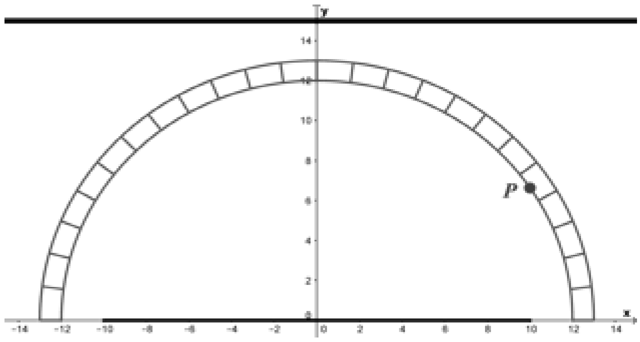


Hyperbola		
	Vertical Transverse Axis	Horizontal Transverse axis
equation	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$
center	(h,k)	(h,k)
Vertices	$(h, k\pm a)$	$(h\pm a, k)$
Foci	$(h, k\pm c)$	$(h\pm c, k)$
Assymptote equation	$y=k\pm\frac{a}{b}(x-h)$	$y=k\pm\frac{b}{a}(x-h)$
Equation relating a, b, and c	$c^2=a^2+b^2$	



Classifying conic sections	Circles	Parabola	Ellipse	Hyperbola
$Ax^2+Cy^2+Dx+Ey+F=0$	A=C	AC=0, Both are not 0	AC>0	AC<0

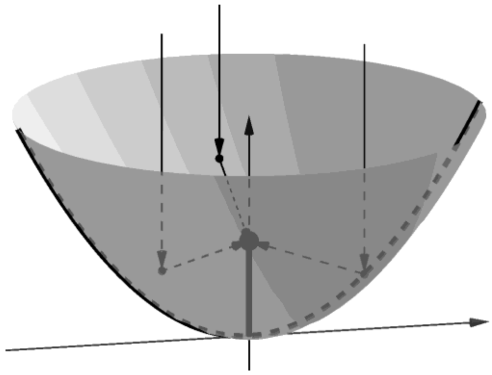
1. A street with two lanes, each 10 ft wide, goes through a semicircular tunnel with radius 12 ft. How high is the tunnel at the edge of each lane? Round off to 2 decimal places.

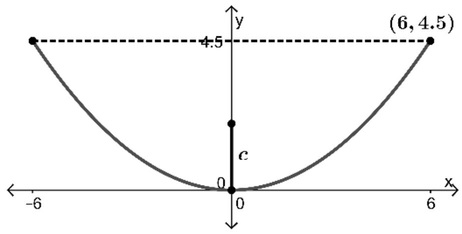


Solution: We draw a coordinate system with origin at the middle of the highway, as shown. Because of the given radius, the tunnel's boundary is on the circle $x^2 + y^2 = 12^2$. Point P is the point on the arc just above the edge of a lane, so its x-coordinate is 10. We need its y-coordinate. We then solve $10^2 + y^2 = 12^2$ for y > 0, giving us $y = 2\sqrt{11} \approx 6.63$ft.

2. A satellite dish has a shape called a paraboloid, where each cross-section

is a parabola. Since radio signals (parallel to the axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. How far should the receiver be from the vertex, if the dish is 12 ft across, and 4.5 ft deep at the vertex?





Solution: The second figure above shows a cross-section of the satellite disc drawn on a rectangular coordinate system, with the vertex at the origin. From the problem, we deduce that (6, 4.5) is a point on the parabola. We need the distance of the focus from the vertex, i.e., the value of c in $x^2 = 4cy$.

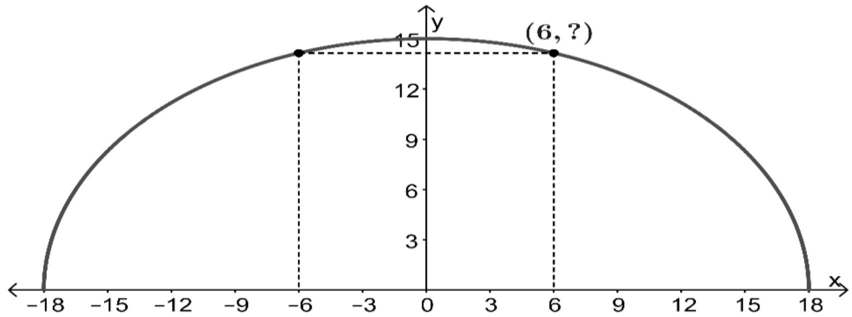
$$x^2 = 4cy$$

$$6^2 = 4c(4.5)$$

$$c = \frac{6^2}{4 \bullet 4.5} = 2$$

Thus, the receiver should be **2 ft** away from the vertex.

3. A tunnel has the shape of a semi-ellipse that is 15 ft high at the center, and 36 ft across at the base. At most how high should a passing truck be, if it is 12 ft wide, for it to be able to fit through the tunnel? Round off your answer to two decimal places.



Solution: Refer to the figure above. If we draw the semi-ellipse on a rectangular coordinate system, with its center at the origin, an equation of the ellipse which contains it, is $\frac{x^2}{18^2} + \frac{y^2}{15^2} = 1$

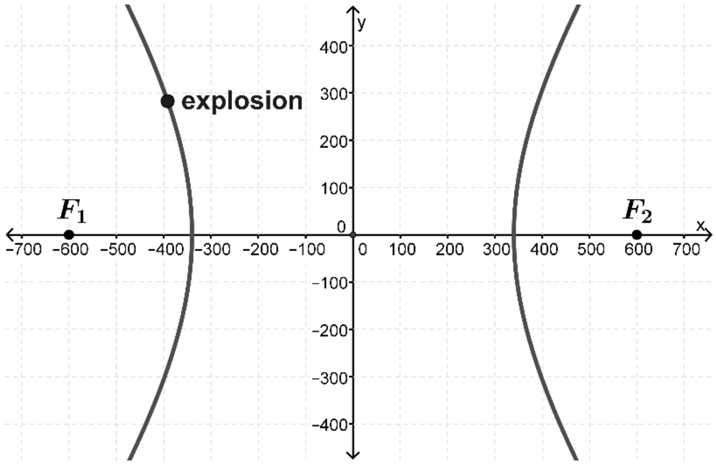
To maximize its height, the corners of the truck, as shown in the figure, would have to just touch the ellipse. Since the truck is 12 ft wide, let the point (6, n) be the corner of the truck in the first quadrant, where n > 0, is the (maximum) height of the truck. Since this point is on the ellipse, it should fit the equation. Thus, we have

$$\frac{6^2}{18^2}+\frac{n^2}{15^2}=1$$

$$n=10\sqrt{2}\approx 14.14\text{ft}$$

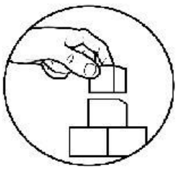
4. An explosion was heard by two stations 1200 m apart, located at $F_1(-600, 0)$ and $F_2(600, 0)$. If the explosion was heard in F_1 two seconds before it was heard in F_2, identify the possible locations of the explosion. Use 340 m/s as the speed of sound.

Solution. Using the given speed of sound, we can deduce that the sound traveled 340(2) = 680 m farther in reaching F_2 than in reaching F_1. This is then the difference of the distances of the explosion from the two stations. Thus, the explosion is on a hyperbola with foci are F_1 and F_2, on the branch closer to F_1.



We have c = 600 and 2a = 680, so a = 340 and $b^2 = c^2 - a^2 = 244400$. The explosion could therefore be anywhere on the left branch of the hyperbola

$$\frac{x^2}{115600}+\frac{y^2}{244400}=1.$$



What's More

Activity 2 Solve the following problems, show all your solutions.

1. A piece of a broken plate was dug up in an archaeological site. It was put on top of a grid with the arc of the plate passing through A(7, 0), B(1, 4) and C(7, 2). Find its center, and the standard equation of the circle describing the boundary of the plate.

2. The cable of a suspension bridge hangs in the shape of a parabola. The towers supporting the cable are 400 ft apart and 150 ft high. If the cable, at its lowest, is 30 ft above the bridge at its midpoint, how high is the cable 50 ft away (horizontally) from either tower?

3. The orbit of a planet has the shape of an ellipse, and on one of the foci is the star around which it revolves. The planet is closest to the star when it is at one vertex. It is farthest from the star when it is at the other vertex. Suppose the closest and farthest distances of the planet from this star, are 420 million kilometers and 580 million kilometers, respectively. Find the equation of the ellipse, in standard form, with center at the origin and the star at the x-axis. Assume all units are in millions of kilometers.

4. LORAN navigational transmitters A and B are located at (-130 , 0) and (130, 0) respectively. A receiver P on a fishing boat somewhere in the first quadrant listens to the pair (A, B) of the transmissions and computes the difference of the distance from boat A and B as 240 miles. Find the equation of the hyperbola on which P is located.
5. A tunnel through a mountain for a four lane highway is to have an elliptical opening. The total width of the highway (not the opening) is to be 16*m*, and the height at the edge of the road must be sufficient for a truck 4*m* high to clear if the highest point of the opening is to be 5*m* approximately. How wide must the opening be?

6. At a water fountain, water attains a maximum height of 4*m* at horizontal distance of 0.5*m* from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75*m* from the point of origin

What I have Learned

Solving Situational Problems Involving Conic Sections is possible by applying the strategies below and by using the equations and characteristics of the different conic sections previously learned

Problem Solving Strategies:

A. Collect information
   1. write down what is/are given
   2. write down what is asked for

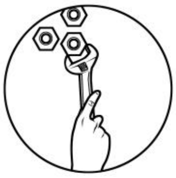
B. Visualize
   1. If possible draw a sketch
   2. Label the sketch

C. Analysis
   1. Write down any related formulas or principles
   2. Write out the sequence of what you must do

D. Solve numerically(Solution)
   1. Substitute numbers from the problem into your formula
   2. Evaluate the resulting equation to get an answer usually numerical.
   3. Don’t forget units (minutes, meter, miles, etc.)

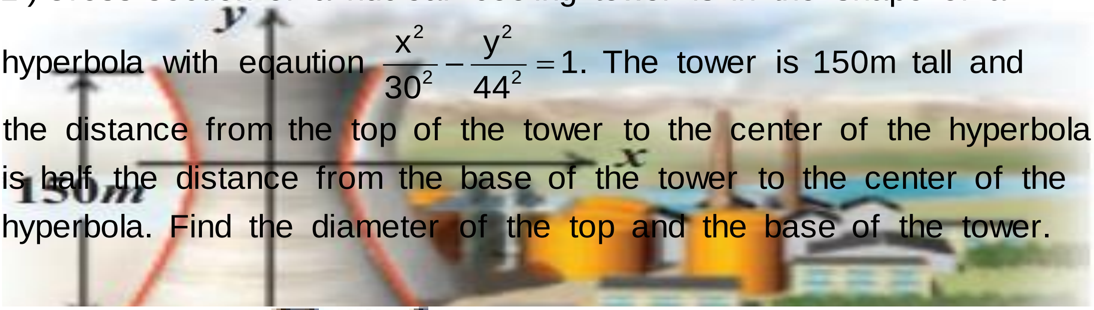
E. Verification/Checking

Check your answer for reasonableness.

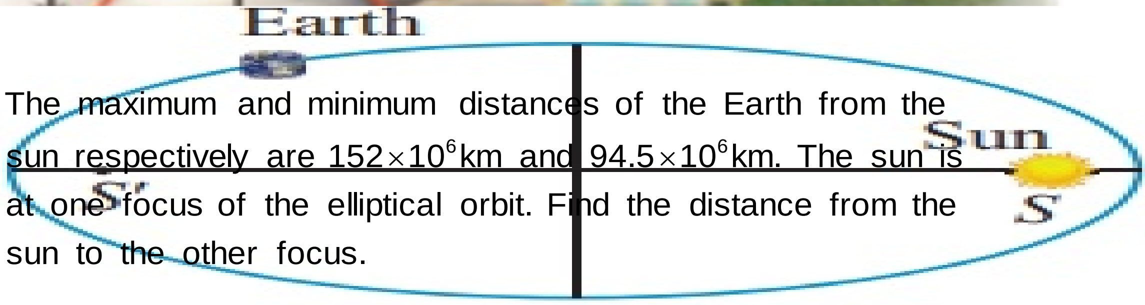


What I Can

Activity 3 Answer the following multi-step problems involving conics. Show your complete solutions.



1.) Cross section of a nuclear cooling tower is in the shape of a hyperbola with eqaution $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the center of the hyperbola is half the distance from the base of the tower to the center of the hyperbola. Find the diameter of the top and the base of the tower.



2.) The maximum and minimum distances of the Earth from the sun respectively are 152×10^6km and 94.5×10^6km. The sun is at one focus of the elliptical orbit. Find the distance from the sun to the other focus.

Online connect! *For additional knowledge and information about the topics please visit the links/url indicated below.*

8.6 Conic Sections - Word Problems
https://www.youtube.com/watch?v=1eVzYUEi93o

Solve a word problem involving parabolas
https://www.youtube.com/watch?v=oXKkgIRnfEU

How to solve a word problem involving ellipsis
https://www.youtube.com/watch?v=KTiuwijOch4

Solving Applied Problems Involving Hyperbolas
https://www.youtube.com/watch?v=upsEcpmuPKA

Summary

Circle is the set of all points in a plane that are equidistant from a given point in plane, called **center**. Any segment whose endpoints are the center and a point on the circle is **radius** of the circle.

Standard form of the equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$, where (h,k) is the center and r is the radius.

General Form of the equation of the circle

$x^2 + y^2 + Dx + Ey + F = 0$

Parabola can be defined as the set of all points in a plane that are the same distance from a given point called focus and a given line called directrix.

Equations of Parabola:

General Form	Standard Form
y^2+ Dx+ F=0	$y^2 = 4cx$
	$y^2 = -4cx$
y^2+ Dx+ F=0	$x^2 = 4cy$
	$x^2 = -4cy$
y^2+ Dx+ Ey + F=0	$(y-k)^2 = 4c(x-h)$
	$(y-k)^2 = -4c(x-h)$
x^2+ Dx+ Ey + F=0	$(x-h)^2 = 4c(y-k)$
	$(x-h)^2 = -4c(y-k)$

The **vertex of the parabola** lies halfway between the focus and the directrix, and the **axis of symmetry** is the line that runs through the focus perpendicular to the directrix.

An **ellipse** is the set of all points in the plane the sum of whose distances from two fixed points *F*1 and *F*2 is a constant. These two fixed points are the **foci** (plural of **focus**) of the ellipse.

Equation of Ellipse

General form	Standard Form
$Ax^2 + Cy^2 + F = 0, A < C$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$
$Ax^2 + Cy^2 + F = 0, A > C$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1, a > b$
$Ax^2 + Cy^2 + Dx + Ey + F = 0, A < C$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)}{b^2} = 1, a > b$
$Ax^2 + Cy^2 + Dx + Ey + F = 0, A > C$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1, a > b$

A **hyperbola** is the set of all points in the plane, the difference of whose distances from two fixed points *F*1 and *F*2 is a constant. These two fixed points are the **foci** of the hyperbola.

General form	Standard Form
$Ax^2 - Cy^2 + F = 0$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$Cy^2 - Ax^2 + F = 0$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
$Ax^2 - Cy^2 + Dx + Ey + F = 0$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$Cy^2 - Ax^2 + Dx + Ey + F = 0$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

center : origin (0, 0)

foci : $F_1(-c, 0)$ and $F_2(c, 0)$

- Each focus is c units away from the center.

Vertices: $V_1(-a, 0)$ and $V_2(a, 0)$

- The vertices are points on the hyperbola, collinear with the center and foci.

Asymptotes: $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$, the lines ℓ_1 and ℓ_2 in the figure

- The asymptotes of the hyperbola are two lines passing through the center which serve as a guide in graphing the hyperbola:

Assessment (Post-test)

Multiple Choice: Encircle the letter of the best answer.

1. It is the intersection of a plane and a double-napped cone.
a. locus b. conic section c. circle d. radius

2. A collection of points satisfying a geometric property can also be referred to as a ______ of points.
a. locus b. conic section c. circle d. radius

3. A ____________ is the set of all points (x, y) in a plane that are equidistant from a fixed point, called the center.
a. locus b. conic section c. circle d. radius

4. A/n ____________ is the set of all points in the plane that are equidistant from a fixed point F(called the focus) and a fixed line *l* (called the directrix).
a. circle b. ellipse c. parabola d. hyperbola

5. A/n ___________ is the set of all points in the plane the sum of whose distances from two fixed points F_1 and F_2 is a constant.
a. circle b. ellipse c. parabola d. hyperbola

6. A/n ____________ is the set of all points in the plane, the difference of whose distances from two fixed points F_1 and F_2 is a constant.
a. circle b. ellipse c. parabola d. hyperbola

7. $6x^2 + 12x + 6y^2 - 8y = 100$ is an example of a
a. Circle b. parabola c. ellipse d. hyperbola

8. $4x^2 + 12x + y^2 - 8y = 64$ is an example of a
a. Circle b. parabola c. ellipse d. hyperbola

9. $6y = 3x^2 - 15$ is an example of a
a. Circle b. parabola c. ellipse d. hyperbola

10. $2x^2 + 87x - 25 = y^2 + 4y$ is an example of a

a. Circle b. parabola c. ellipse d. hyperbola

11. $6x^2 - 15x = 3y^2 + 4y + 13$ is an example of a

a. Circle b. parabola c. ellipse d. hyperbola

12. A hyperbola is the only conic that has

a. asymptotes b. focus c. vertex d. a minor axis

13. $\frac{x^2}{4} - \frac{y^2}{25} = 1$ has a major axis of length

a. 2 b. 3 c. 4 d. 5

Use $\frac{(x+3)^2}{9} - \frac{(y-2)^2}{4} = 1$ to answer the next 4 questions.

14. The center is at _________?

a. (-3, 2) b. (-3, -2) c. (3, 2) d. (3, -2)

15. The vertices are _________?

a. (0, 2), (-6, 2) b. (-3, 5), (-3, -1) c. (-1, 2), (-5, 2) d. (-3, 4), (-3, 0)

16. The foci are at _________?

$a. \left(-3 \pm \sqrt{7}, 2\right)$ $b. \left(-3 \pm \sqrt{13}, 2\right)$ c. $\left(-3, 2 \pm \sqrt{13}\right)$ d. $\left(-3, 2 \pm \sqrt{7}\right)$

17. The length of the major axis is ___________?

a. 3 b. 6 c. 9 d. 12

18. The standard form of $9x^2 - 4y^2 = 36$ is

$a. \frac{x^2}{4} - \frac{y^2}{9} = 1$ b. $\frac{x^2}{36} - \frac{y^2}{9} = 1$ c. $\frac{x^2}{25} - \frac{y^2}{4} = 1$ d. $\frac{x^2}{5} - \frac{y^2}{20} = 1$

19. The earth's orbit is an ellipse with the sun at one of the foci. If the farthest distance of the sun from the earth is 105.5 million km and the nearest distance of the sun from the earth is 78.25 million km, find the eccentricity of the ellipse.

a. 0.15 b. 0.25 c. 0.35 d. 0.45

20. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ will have a minor and major axis with length _____ (In that order)

a. 3, 4 b. 6, 8 c. 9, 16 d. 18, 32

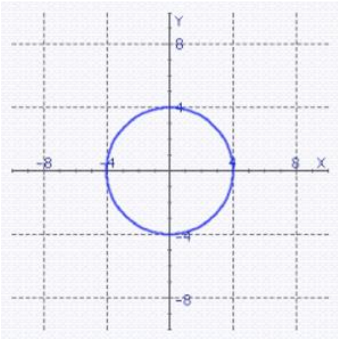
21. Which of the following shows the correct graph of the circle?

a. $x^2 + y^2 = 4$

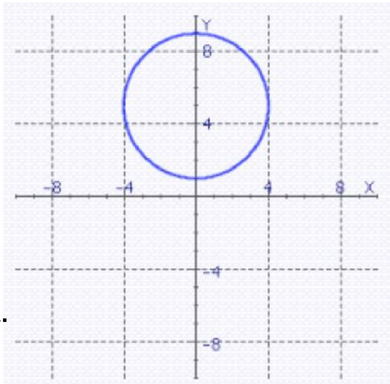
b. $y^2 = x^2 + 16$

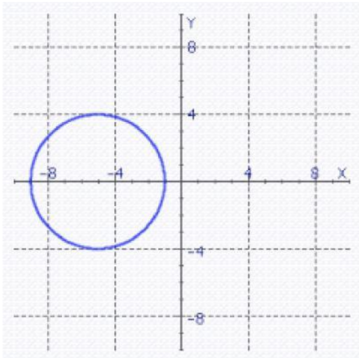
c. $x^2 + y^2 = 16$

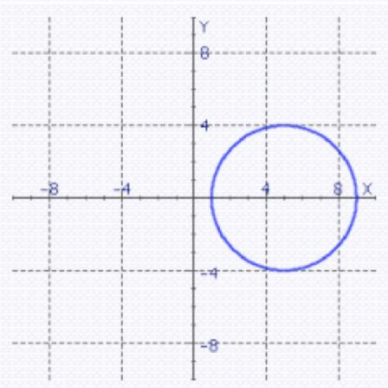
d. $x^2 + y^2 = 1$

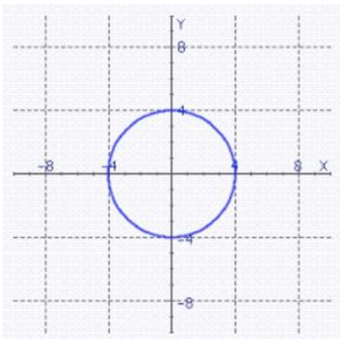


22. Which graph represents the equation $x^2 - 10x + y^2 = -9$?

a. 

c. 

b 

d. 

23. What is the focus and vertex of the parabola $(y-2)^2 + 16(x-3) = 0$?

a. Vertex: $V(-3,-2)$, Focus: $F(-3,14)$

b. Vertex: $V(-3,-2)$, Focus: $F(-3,-18)$

c. Vertex: $V(-3,-2)$, Focus: $F(-7,-2)$

d. Vertex: $V(3,2)$, Focus: $F(-1,2)$

24. Find the equation of the parabola with vertex at (5, 4) and focus at (-3, 4).

a $(y-4)^2 = -32(x-5)$

b $(y-4)^2 = 32(x-5)$

c. $(y+4)^2 = -32(x-5)$

d. $(y-4)^2 = 8(x-5)$

25. Find the center and foci of the ellipse $\frac{(x+5)^2}{5} + \frac{(y+9)^2}{9}$.

a. center: (5,9) , foci: (5,7),(5,11)

b. center (-5,-9) , foci: (-5,-11), (-5,-7)

c. center: (-5,-9) , foci: (-7,-9), (-3,-9)

d. center: (5,9) , foci: (3,-9), (7,-9)

26. Find the center and vertices of the ellipse $4x^2 + 9y^2 - 24x + 72y + 144 = 0$

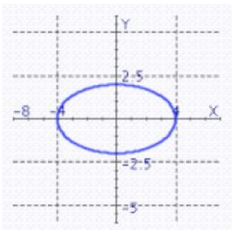
a. center: (-4,3) , vertices: (-7,3), (-1,3)

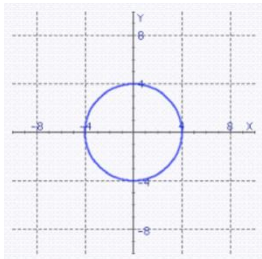
b. center: (-3,4) , vertices: (-5,4), (-1,4)

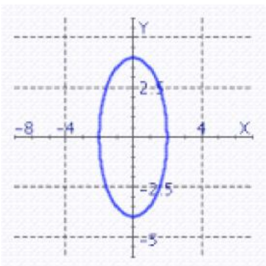
c. center: (3,-4) , vertices: (1,-4), (5,-4)

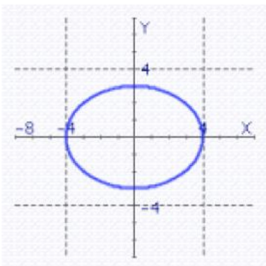
d. center: (3,-4) , vertices: (0,-4), (6,-4)

27. Which of the following shows the correct graphical representation of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$?

a. 

c. 

b. 

d. 

28. Find the center and vertices of the hyperbola $11x^2 - 25y^2 + 22x + 250y - 889 = 0$.

a. center: (1,-5) , vertices: (1,-10), (1,0)

b. center: (-1,5) ,vertices: (-1,0),(-1,10)

c. center: (-1,5) , vertices: (-6,5), (4,5)

d. center: (1,-5) , vertices: (-4,-5), (6,-5)

29. What are the vertices and asymptotes of the hyperbola $9y^2 - 16x^2 = 144$?

a. vertices: (0,-4),(0,4) , asymptote: $y = \pm\frac{4}{3}x$

b. vertices: (0,-4), (0,4) , asymptote: $y = \pm\frac{3}{4}x$

c. vertices: (-4,0), (4,0) , asymptote: $y = \pm\frac{4}{3}x$

d. vertices: (-4,0), (4,0) , asymptote: $y = \pm\frac{3}{4}x$

30. Find the standard form of the equation of the hyperbola with the given characteristics, vertices: (0,-6), (0,6) and foci (0,-7), (0,7).

a. $\frac{y^2}{36} - \frac{x^2}{49}$=1

c. $\frac{x^2}{36} - \frac{y^2}{13}$=1

b, $\frac{y^2}{36} - \frac{x^2}{13}$=1

d. $\frac{x^2}{36} - \frac{y^2}{13}$=49

31. Write the equation of the ellipse that has its center at the origin with focus at (0, 4) and vertex at (0, 7).

f. $\frac{x^2}{49} + \frac{y^2}{33}$=1

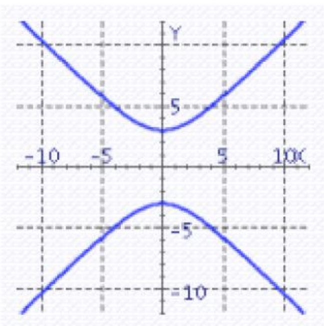
c. $\frac{x^2}{33} + \frac{y^2}{49}$=-1

g. $\frac{x^2}{33} - \frac{y^2}{49}$=1

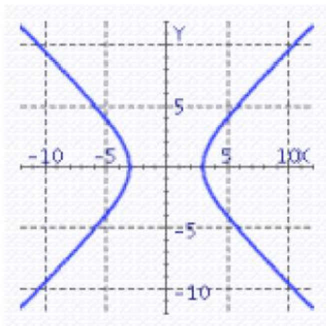
d. $\frac{x^2}{33} + \frac{y^2}{49}$=1

32. What is the graph of the hyperbola $9x^2 - 9y^2 = 81$?

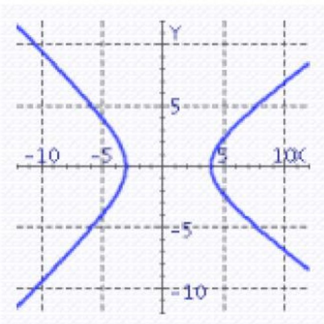
a.



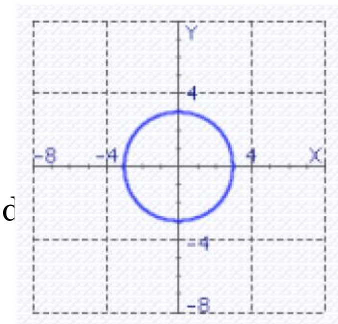
c,



b.



d



33. An arch 20 meters high has the form of parabola with vertical axis. The length of horizontal beam placed across the arch 9 meters from the top is 60 meters. Find the width of the arch at the bottom.

a. 44.72 meters b. 45.72 meters c. 89.44 meters d. 90.44 meters

34. A spotlight in a form of a paraboloid 9 inches deep has its focus 3 inches from the vertex. Determine the radius of the opening of the spotlight.

a. 9.39 inches b. 10.39 inches c. 11. 39 inches d. 12.39 inches

35. A bridge is supported on an elliptical arch of height of 7 meters and width at the base of 40 meters. A horizontal roadway is 2 meters above the center of the arch. How far is it above the arch at 8 meters from the center?

a. 0.58 meters b. 1.58 meters c. 2.58 meters d. 3.58 meters

Key Answers

Pretest

1. B	11. D	21. A	31.D
2. A	12. A	22. B	32.C
3. C	13.D	23.D	33.C
4. C	14.A	24.A	34.B
5. B	15.A	25.B	35.C
6. D	16.B	26.D	
7. A	17.B	27.A	
8. C	18.A	28.C	
9. B	19.A	29.A	
10. D	20. B	30.B	

What's New

1.Hyperbola 2. Hyperbola 3. Parabola 4. Parabola
5. Ellipse 6. Ellipse 7. Circle 8. Circle

What's More

1. a. Circle b. circle c. circle d. ellipse

2. (a) The radius is 3, so the equation is $x^2 + y^2 = 9$.

(b) The radius is 8/2 = 4, so the equation is $(x - 1)^2 +(y5)=16$

(c) The center is $(-2,2)$ and the radius is 2, so the equation is

$$(x+2)^2+(y-2)^2 = 4.$$

(d) The center is $(2, 3)$ and the radius is 1, so the equation is

$$(x-2)^2+(y-3)^2 = 1.$$

(e) The center is $(1, -1)$ and by the Pythagorean Theorem, the radius (see Figure A) is $\sqrt{2^2 + 2^2} = 8$, so the equation $(x-1)^2 + (x+1)^2 = 8$

(f) The radius is 3, so the equation is $(x + 2)^2 + (y + 3)^2 = 9$.

(g) The radius is 2, so the equation is $(x + 2)^2 + (y + 3)^2 = 4$.

(h) The distance between (2, 0) and (8, 0) is 10; since the radius is 5, these two points are endpoints of a diameter. Then the circle has center at (3, 0) and radius 5, so its equation is

$$(x - 3)^2 + y^2 = 25.$$

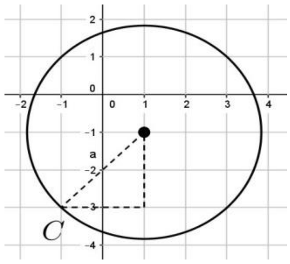


Figure A

3. (a) Thus, the circle's standard form is $x^2 + (y + 4)^2 = 7^2$, center (0,-4) and radius 7, see Figure B.

(b) Thus, the circle's standard form is $(x-2)^2+(y+5)^2=\left(\frac{7}{2}\right)^2$,center (2,-5) and radius 3.5, see Figure C.

(c) Thus, the circle's standard form is $(x+\frac{3}{2})^2+(y+2)^2=3^2$, center $(-\frac{3}{2},-2)$ and radius 3,see figure D.

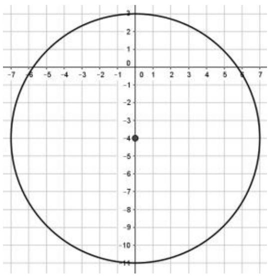


Figure B

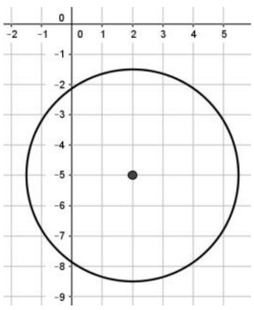


Figure C

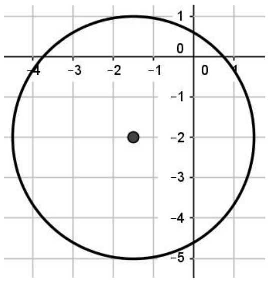


Figure D

d. Thus, the circle's standard form is $(x-3)^2+(y+2)^2=9$,center (3,2) and radius 3, see Figure E.

(e) Thus, the circle's standard form is$(x-2)^2+(y-4)^2=0$, center (2,4) and radius 0, see Figure F

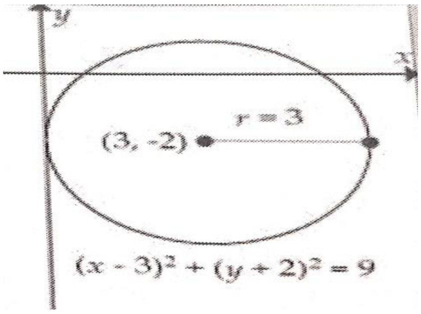


Figure E

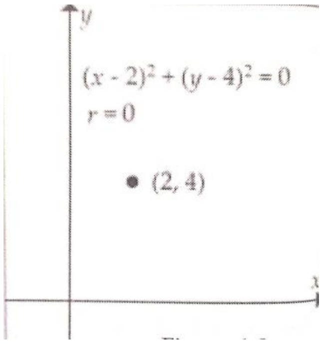


Figure F

f. Equation of the circle in standard form is $(x-1)^2+(y-4)^2=10$, see Figure G.

g. Equation of the circle in general form is $x^2+y^2-10x+6y+18=0$, see Figure H.

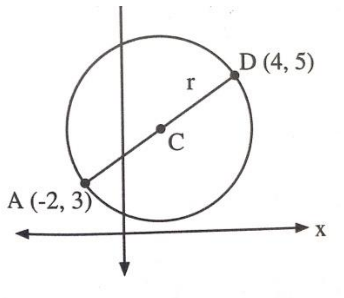


Figure G

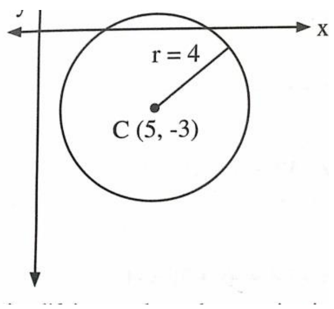


Figure H

What I Have Learned

1. circle	**2. Ellipse**	**3. Parabola**	**4. A**	**5, B**
6. C	**7. Focus**	**8. Radius**	**9. (h,k)**	**10.r**
1`1. $x^2+y^2=r^2$	**12. General form**	**13. Square**	**14. (h,k)`**	**15. Radius**

What I Can Do

A. center (0,0)
1 peso coin=2.3 cm 5 peso coin=2.5 cm 10 peso coin=2.7 cm
1. $x^2+y^2=1.32$ 2. $x^2+y^2=1.56$ 3. $x^2+y^2=1.82$

Discussion: The bigger the measurement of the coin the bigger the radius.
B. center (2,3)
1. $(x-2)^2+(y-3)^2=1.32$ 2. $(x-2)^2+(y-3)^2=1.56$ 3. $(x-2)^2+(y-3)^2=1.82$
Discussion: The center changes from origin to the given point (2,3) and so with the equation .

Lesson 2: The Parabola

What's In

d=$\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

midpoint= $\left(\frac{x1+x2}{2}, \frac{y1+y2}{2}\right)$

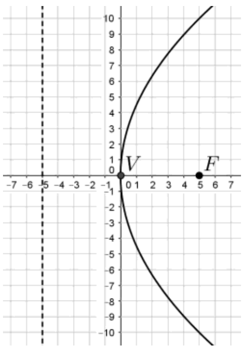
Standard Form: ax^2+bx+c
Vertex Form: $F(x)= a(x-h)^2+k$
Parabola

What's New
Focus
Directrix
Vertex
Axis of symmetry
Latus rectum

What's More

1. a $(x-3)^2 - 10(y+1)$
   b. $(x-1^2 = 6(y-4)$
   c. $(y+1)^2 = 6(x+3)$
   d. $(y-3)^2 = \frac{x}{2}$

2. a. $y^2 + 2y + 2x + 6 = 0$
   b. $x^2 - 2x - 8y + 25 = 0$
   c. $.x^2 - 6x - 12y + 57 = 0$
   d. $2y^2 - 12y + 3x + 9 = 0$
3. a Vertex: V (0, 0), opens to the right 4c = 20 c = 5
   Focus: F(5, 0), Directrix: x =-5. See Figure L.
   b. Vertex: V (0, 0), opens downward, c = $\overset{\Rightarrow}{4}$ c = 1
   Focus: F(0, 1), Directrix: y = 1. See Figure M. –



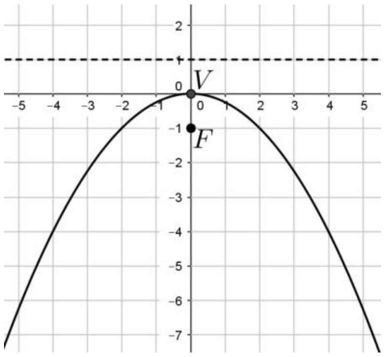


Figure L

Figure M

c. Vertex: V(3,0), parabola opens upward

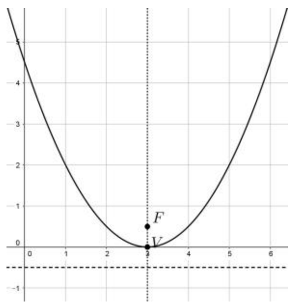
$4c = 2 \quad \rightarrow c = \frac{1}{2}$, Focus: F(3, $\frac{1}{2}$)

Directrix: $y = -\frac{1}{2}$, Axis of symmetry: x=3, See Figure N.

d. Vertex: V(1,-4), parabola opens to the left

$4c = \frac{8}{3} \rightarrow c = \frac{2}{3}$, Focus: F($\frac{1}{3}, -4$)

Directrix: $x = \frac{5}{3}$, Axis of symmetry: $y = -4$. See Figure O.



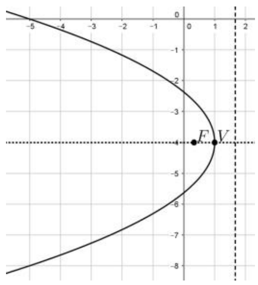


Figure N

Figure O

4. Since the focus is 6 units to the right of the directrix, the parabola opens to the right with 2c = 6. Then c = 3 and V (−14, 8). Hence, the equation is

$(y - 8)^2 = 12(x + 14)$.

5. a. 4p=10. Since the parabola opens downward, use the equation $x^2 = -4py$. Hence, the equation is $x^2 = -10y$.

b. Since the given directrix is $y = 8$, it indicates that the parabola opens downward. Use the equation $x^2 = -4py$. From the equation of the directrix, we can conclude that p=8 so 4p=32. The equation is $x^2 = -32y$.

c. If focus is at (0,4), then p=4 and 4p=16. Since the location of the focus is located at the positive y-axis, then the parabola opens upward so use the equation $x^2 = 4py$. Hence, the equation of the parabola is $x^2 = 16y$.

d. Since the directrix is x=7, then p=7 and 4p=28. The parabola opens to the left, use the equation $y^2 = -4px$, we get $y^2 = -28x$.

6. V$\left(5, \frac{13}{2}\right)$, directrix: $y = \frac{15}{2}$, $c = 1 \rightarrow 4c = 4$, parabola opens downward

Equation: $\left(y - \frac{13}{2}\right)^2 = -4(x - 5)$

Focus: $(5, \frac{11}{2})$, Axis of symmetry: x=5

7. V$\left(-\frac{3}{2}, 4\right)$,F(-4,4)

C=$\frac{5}{2} \rightarrow 4c = 10$

Parabola opens to the left

Equation; $(y - 4)^2 = -10\left(x \frac{3}{2}\right)$

Directrix: x=1,Axis:y=4

8. $(x + 1)^2 = -4(y - 4)$ or $x^2 + 2x + 4y - 15 = 0$,see Figure Q

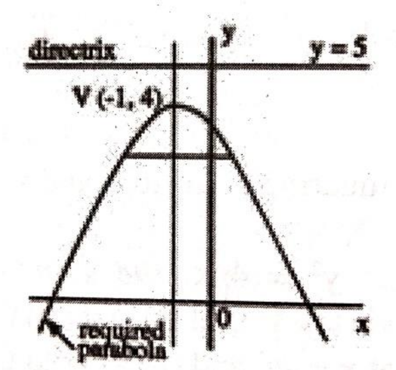
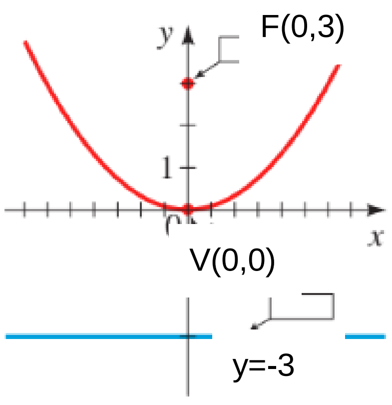
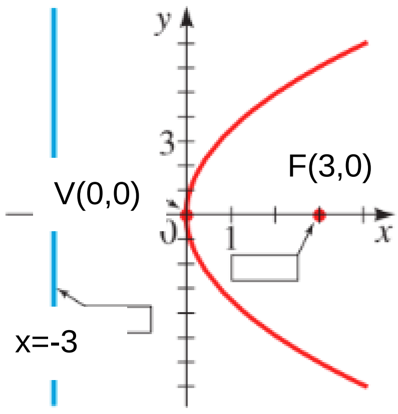


Figure Q

What I Have Learned

1. focus, directrix
2. F(0,c), y=-c, F(0,3), y=-3
3. F(c,0),x=-c,F93,0),x-3
4.



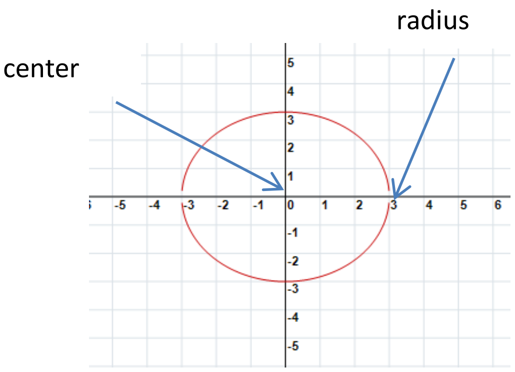


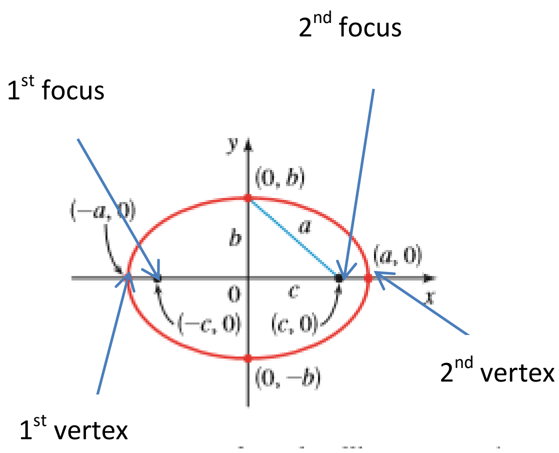
What I Can Do

Answers vary.

Lesson 3: The Ellipse

What's New





What's More

1. $a.\frac{(x-1)^2}{4}+\frac{(y+2)^2}{16}=1$

   b. $\frac{(x+6)^2}{38}+\frac{(y+4)^2}{16}=1$

c. $\frac{(x-4)^2}{4} + \frac{(y-3)^2}{9} = 1$

d. $\frac{(x)^2}{9} + \frac{(y)^2}{49} = 1$

2. a. $4y^2 - 24y + 9x^2 - 72x + 144 = 0$

b. $36x^2 - 144x + 20y^2 - 120y - 396 = 0$

c. $x^2 - 2x + 2y^2 - 4y \pm 1 = 0$

d. $y^2 + 6y + 4x^2 - 16x + 21 = 0$

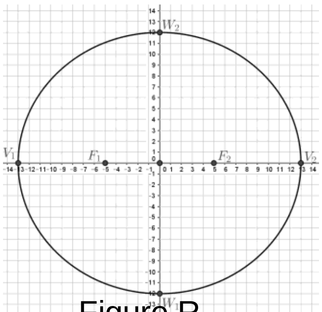


Figure R

3. Center: C(7,-2)
Foci: $F_1(7-\sqrt{39},-2)$, $F_2\ (7-\sqrt{39},-2)$,
Vertices; V_1(-1,-2), V_2(15,-2)
Covertices: W_1(7,-7), W_2(7,3)
See Figure R

4. Center: C(-3,-1)
Foci: F1(-3,-4),F2(-3,2)
Vertices: V1(-3,-5), V2(-3,3)

Covertices: W1$(-3-\sqrt{7},-1)$, W2$(-3+\sqrt{7},-1)$

See Figure S

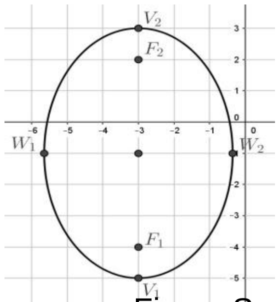
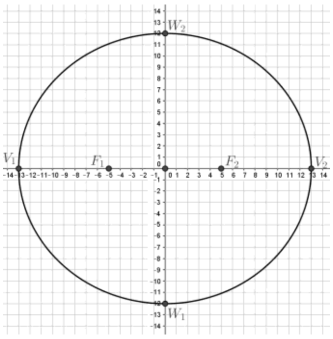


Figure S

5. Foci: F_1(-5,0), F_2(5,0)

Vertices: V_1(-13,0), V_2(13,0)

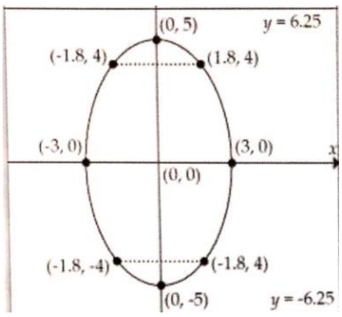
Covertices: W_1(0,-12), W_2(0,12)



6. Foci: F_1(0,4), F_2(0,-4)

Vertices: V_1(0,5),V_2(0,-5)

Covertices: W_1(3,0), W_2(-3,0)



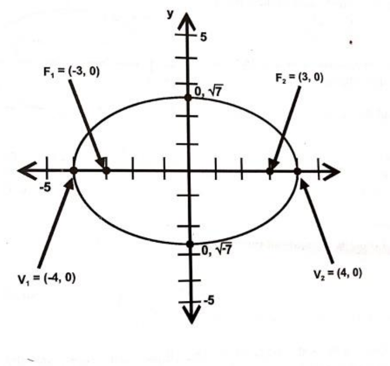
7. The ellipse is vertical and has center at (0,0).

$$2a = 34 \rightarrow a = 17$$

$$c = 8 \rightarrow b = \sqrt{17^2 - 8^2} = 15$$

The equation is $\frac{x^2}{225} + \frac{y^2}{289} = 1$

8. Since its center is at (0,0), the major axis coincide with the x-axis, one focus at (3,0) and a vertex at (-4,0), then using the obtained equation, $b^2 = a^2 - c^2 = 16 - 9 = 7$. So an equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{7} = 1.$



9. The ellipse is horizontal with center at the midpoint (5,7) of the convertices. Also, 2a=12 so a=6 while b=1. The equation is $\frac{(x-5)^2}{36} + \frac{(y-7)^2}{1} = 1.$

10. The ellipse is horizontal with center at the midpoint (-4,3) of the foci, also $c = \sqrt{15}$. Since the length of the major axis is 10, 2a=10 and a=5. Thus $b = \sqrt{5^2 - 15}$=$\sqrt{10}$. Therefore, the equation of the ellipse $\frac{(x+4)^2}{25} + \frac{(y-3)^2}{10} = 1$ and its vertices are (-9,3) and (1,3).

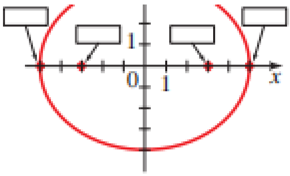
What I Have Learned

1. sum, foci

2. (a,0), (-a,0), c=$\sqrt{a^2 - b^2}$, Vertices(-5,0), (5,0), Foci(-3,0),(3,0)

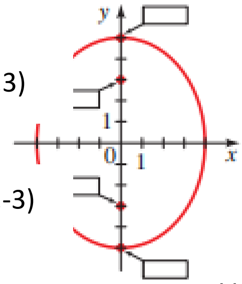
3. Vertices (0,-a), (0,a), c=$\sqrt{a^2 - b^2}$, Vertices(0,-5), (0,5), Foci (0,-3), (0,3)

V1(-5,0) F1(-3,0) F2(3,0) V2(5,0)



V2(0,5)

F2(0,3)



F1(0,-3)

V1(0,-5)

What Can I Do

Answer may Vary.

Lesson4-The Hyperbola

What's New

This is a folding activity. For reference you can watch the video using

this link https://www.youtube.com/watch?v=nEISCCjObPg

What's More

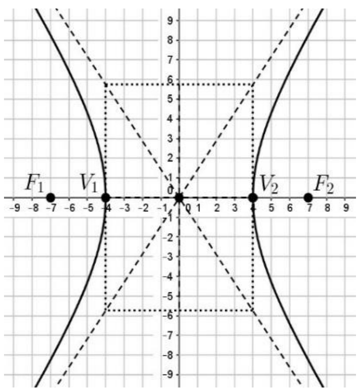
A.

1.) $x^2 - \frac{(y-1)^2}{4} = 1$

2.) $\frac{y^2}{4} - \frac{\left(x+\frac{1}{3}\right)^2}{9} = 1$

3.) $\frac{(y-4)^2}{49} - \frac{(x-1)^2}{25} = 1$

4.) $\frac{\left(x-\frac{1}{2}\right)^2}{16} - \frac{(y+1)^2}{7} = 1$



B. Solution: The hyperbola is horizontal.

$a^2 = 16 \Rightarrow a = 4,$

$b^2 = 33 \Rightarrow b = \sqrt{33},$

$c = \sqrt{16+33} = 7$

center : (0,0)

center : (0,0)

foci: $F_1(-7,0), F_2(7,0)$

asymptotes : $y = \pm\frac{\sqrt{33}}{4}x$

The conjugate axis has endpoints

$(0,-\sqrt{33})$ and $(0,\sqrt{33})$

What I Have Learned

Activity # 2

1. difference; foci

2. $(-a,0)$ and $(a,0)$; $\sqrt{a^2+b^2}$;
   $(-4,0)$ and $(4,0)$ $(-5,0)$ and $(5,0)$

3. $(0,a)$ and $(0,-a)$; $\sqrt{a^2+b^2}$;
   $(0,4)$ and $(0,-4)$; $(0,5)$ and $(0,-5)$

4. a) $V(-4,0)$ and $(4,0)$ b) $V(0,4)$ and $(0,-4)$
   $F(-5,0)$ and $(5,0)$ $F(0,5)$ and $(0,-5)$
   Asymp. $y = \pm \frac{3}{4}x$ Asymp. $y = \pm 3x$

Activity # 4

Solution : The hyperbola is vertical.

$a^2 = 25 \Rightarrow a = 5,$

$b^2 = 39 \Rightarrow b = \sqrt{39}$

$c = \sqrt{25+39} = 8$

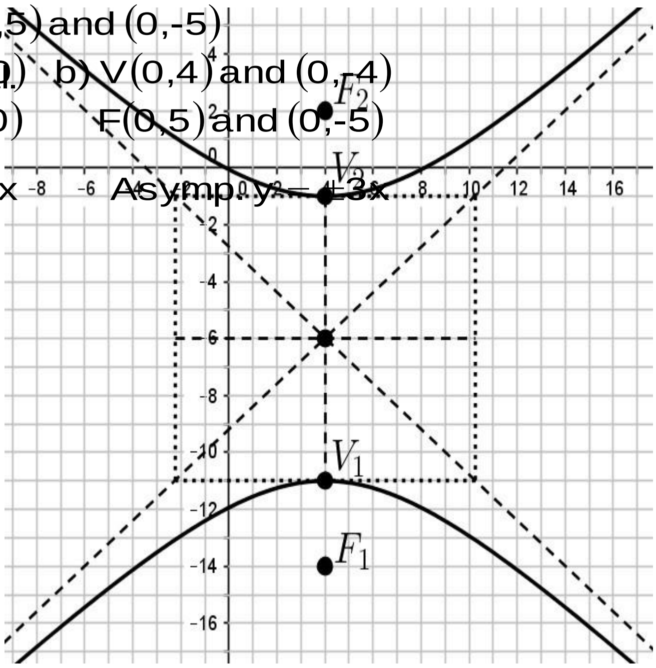
center : (-4,6)

center : (0,0)

foci: $F_1(4,-14), F_2(4,-1)$

asymptotes : $y + 6 = \pm \frac{5}{\sqrt{39}}(x-4)$

The conjugate axis has endpoints $b = \sqrt{39}$ units to the left and to the right of the center.



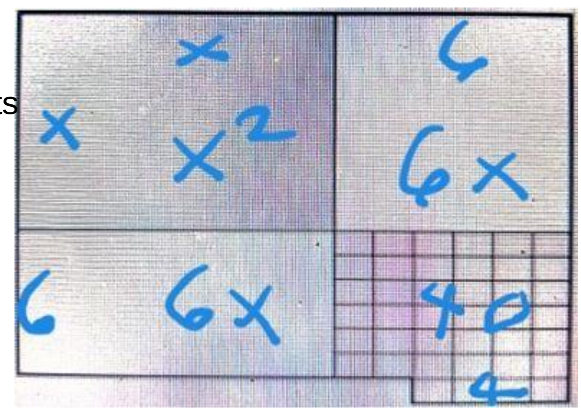
What I Can Do

Answer may vary.

Lesson 5: Equation and Important Characteristics of the Different Types of Conic Sections

What's New :Activity # 1

I agree to both Alex and Julie since both their equations represents the enclosed region. One is general form, the other is standard form.



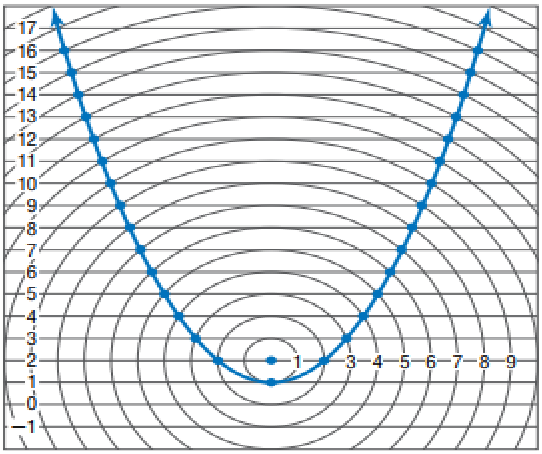
What Is It: Activity # 2

1. Ellipse 2. Circle 3. Hyperbola 4. Parabola 5. Parabola

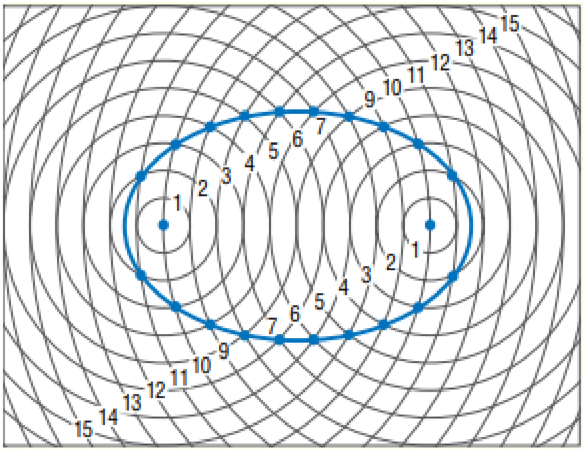
6. Ellipse 7. Circle 8. Hyperbola 9. Circle 10. Hyperbola

What's More

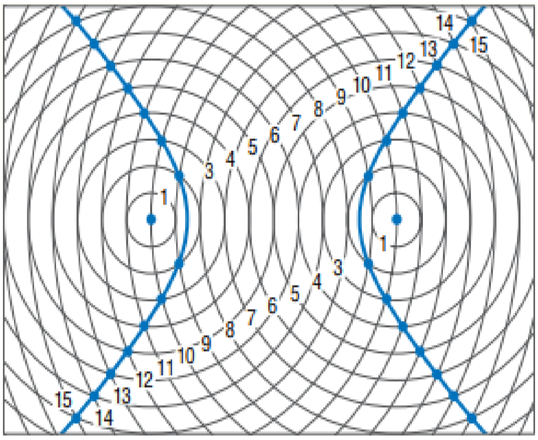
Activity # 3



Activity # 4



Activity # 5



What I Have Learned

Activity # 6

1. a 2. c 3. d 4. h 5. g 6. f 7. e 8. B

What I Can Do Activity # 7

1. Because these are array of points that are equidistant from the focus.
2. a. It's intersection points wont form an ellipse or not on the ellipse.
   b. When sum increases, the intersection points are outside the ellipse, whereas when sum decreases, the intersection points are inside the ellipse.

Lesson 6: Solving Problems involving Conic Sections

What's New : Activity # 1

This is an output based activity, an artwork.

What's More: Activity # 2

1.) $(x-1)^2+(y+6)^2=100$

2.) 97.5 ft

3.) $\frac{x^2}{250000}+\frac{y^2}{243600}=1$

4.) $\frac{x^2}{14400}+\frac{y^2}{2500}=1.$

5.) The opening is 9.6m

6) The height of the water fountain at 0.75m from the origin is 3m.

What I Can Do

Activity # 3

1.) The diameter of the bottom tower is 74.49m
The diameter of the top of the tower is 45.41m

2.) 575×10^5km

Posttest

1. B	11. D	21. A	31.D
2. A	12. A	22. B	32.C
3. C	13.D	23.D	33.C
4. C	14.A	24.A	34.B
5. B	15.A	25.B	35.C
6. D	16.B	26.D	
7. A	17.B	27.A	
8. C	18.A	28.C	
9. B	19.A	29.A	
10. D	20. B	30.B	

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