Reliability Optimization for series systems under uncertain component reliability in the design phase

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Abstract

We consider an OEM who sells a series system for a customer under a Performance-Based Logistic (PBL) contract. During the design phase of the system, the OEM have to select an optimal design for each critical component in the system from all the possible alternatives with uncertain component reliability. The uncertainty in component reliabilities can lead to large deviations of the realized system availability from the expected system availability. Upon a failure of a critical component in the system, the failed part will be replaced by a as-good-as new component. According to the PBL contract, when the total system down time exceeds a predetermined level, the OEM should pay a penalty cost to the customer with respect to the actual total downtime and a penalty rate. In this case, we formulate the Life Cycle Costs (LCC) of this multi-stage system which are affected by the uncertain component reliability. The LCC consist of design costs, repair costs and downtime costs.

Keywords: Capital goods, Reliability optimization, Performance-based contracting, Life cycle costs

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1. Introduction

Capital goods are machines or products that are used by manufacturers to produce their end-products or that are used by service organizations to deliver their services. Advanced technical systems such as medical systems, manufacturing systems, defense systems are capital goods which are critical for the operational processes of their customers. System downtime of these capital goods can cause serious consequences (e.g. millions of euros of reduced production output, extra waiting time of passengers, failure of military missions). Therefore, customers of these complex systems such as hospitals, militaries and factories require high availability of these systems. On the other hand, the engineering systems involved in capital goods are becoming more and more complex due to the advancement of technologies. The maintenance and repair tasks are too challenging for customers to take care of by themselves. Thus, after-sale services such as maintenances and repairs are often needed by the customers.

As a result, integrating services as a major sustainable source of the profit of original equipment manufacturers (OEMs) has been widely recommended by a large amount of papers in recent decades. A study conducted by Accenture (**?**]) shows that after-sale services contribute only 25% of the total revenue across all manufacturing companies, but are responsible for $40\% - 50\%$ of the profit. Many OEMs thus has been transforming their business strategy from product-oriented to service-oriented. After selling a system, under the traditional material-based contract, the OEM is responsible for the repair of the system only within the warranty period, which is often short compared to the life cycle of the system. After the warranty period, the OEM will charge the customer for providing spare parts, maintenance, and other services to keep the availability of the systems above certain levels. This may lead to higher spare parts costs, repair costs, and labor costs, whereas system availabilities can be lower than a customer anticipated when buying the system. This undesired situation for the customer can be avoided by making better agreements with the OEM when a new system is being bought. A type of contract that may be attractive for both customers and OEMs is a Performance-Based Logistics (PBL) contract. Under a PBL contract, the OEM is responsible to meet a predetermined system availability target during a specified period, e.g., 5-10 years, and a penalty cost has to be paid to the customer when one fails to meet the system availability target. When designing the system, such system availability targets have to be taken into account.

In this paper, we attempt to solve a system design problem. During the design phase of a system, engineers have to select a certain design from all the possible alternatives for each critical component in the system. In real life, the outcome of any development process for a certain design is uncertain with respect to the reliability requirement. For example, since the failure mechanisms of some emerging technologies (e.g., Micro-Electro-Mechanical Systems) are complex, it is often difficult to predict the actual reliability behaviors of the critical components. Therefore newly-designed devices have been found to have uncertain component reliabilities. The uncertainty in component reliabilities can lead to large deviations of the realized system availability from the expected system availability (a point estimate for the system availability). In this case, the uncertainty in component reliabilities also needs to be considered in the decision making of system design.

The remainder of the paper is organized as follows:

2. Literature

3. Notations and assumptions

3.1. Notations

i	Index of critical components in the system, $i \in \{1, 2, ...n\}$
j	Index of rough design alternatives that can be used for component i, $j \in \{1, 2, ...m$
T	The time length of the service contract period
D_0	Predetermined accepted total downtime in the service contract
x_{ij}	Decision variable of whether choosing the jth design of component i or not
c_p	Penalty cost per time unit after the total system downtime exceeding D_0
c_a^{ij}	Design cost for the jth design of component i
c_r^{ij}	Repair cost per failure for the jth design of component i
r_{ij}	Repair time per failure for the jth design of component i
$r_i(\boldsymbol{x_i})$	Repair time per failure for component i
Λ_{ij}	Failure rate for the jth design of component i
$\Lambda_i(\boldsymbol{x_i})$	Failure rate of component i
μ_{ij}	The expected failure rate for the jth design of component i
$\mu_i(\boldsymbol{x_i})$	The expected failure rate of component i
σ_{ij}^2	The variance of the failure rate for the jth design of component i
$\sigma_i^2(\boldsymbol{x_i})$	The variance of the failure rate for component i
$S(x_{ij})$	Number of repairs for the jth design of component i during $[0, T]$
$S_i(\boldsymbol{x_i})$	Number of repairs for component i during $[0, T]$
$A_i(\boldsymbol{x_i})$	Design cost of component i
$A(\boldsymbol{x})$	Design cost of the system
$R_i(\boldsymbol{x_i})$	Expected repair cost of component i in $[0, T]$
$R(\boldsymbol{x})$	Expected repair cost of the system in $[0, T]$
$D_{ij}(x_{ij})$	Total downtime for the jth design of component i in $[0, T]$
$D_i(\boldsymbol{x_i})$	Total downtime of component i in $[0, T]$
$D(\boldsymbol{x})$	Total downtime of the system in $[0, T]$
$P(\boldsymbol{x})$	Expected penalty cost due to downtime exceeding D_0 in $[0, T]$

3.2. Assumptions

1. The components in discussion are all critical components. If a component fails, the entire system will stop functioning.
2. The system will be functioning for T years. The exploitation phase is denoted by $[0, T]$. We assume that the system are sold at time $t = 0$, and the design costs are also incurred at time $t = 0$.
3. During the exploitation phase of the system $[0, T]$, its total downtime should be less than or equal to D_0 years. If the total downtime exceeds D_0 years, the OEM will be charged for a penalty cost with respect to the extra downtime and the penalty rate c_p.
4. The failure rates of the components are uncertain in the design phase and distributed with the same general distribution function. Components from different rough designs have different failure rate distribution parameters and design costs. The jth design of component i has the failure rate Λ_{ij} with mean μ_{ij} and variance σ_{ij}^2 and design cost c_{ij}.
5. The life time of each component is independent and exponentially distributed. So the number of the failures for each component over $[0, T]$ has a Poisson distribution with a failure rate $\Lambda_i(\boldsymbol{x_i})$.
6. Once the rough design of a component has been chosen, the value of the component failure rate remains fixed through $[0, T]$.
7. Once a failure occurs, a repair will be performed. The failed part will be replaced by a ready-to-use part (repair by replacement) from the secondary supplier. The failure rate remains the same after each replacement. The repair cost c_r^{ij} is fixed for each rough design.

4. Model description

During the design phase of a system, engineers have to select a certain design from all the possible alternatives for each critical component in the system. Suppose the system is comprised of n critical components. If one of these critical components fails, the system as a whole stops working. Then the system can be seen to have a series system structure. For each critical component $i \in \{1, 2, ..., n\}$, one rough design needs to be selected from a set of all the possible alternatives, which is denoted by $\{1, 2, ..., m_i\}$. Each rough design candidate of component i in the set $\{1, 2, ..., m_i\}$ has different uncertain reliability parameters and cost parameters. We aim to find out the optimal combination of rough designs for the system to minimize the average total cost over the service period T of a PBL contract.

The lifetimes of the components are assumed to be independent and exponentially distributed. Then for a certain rough design j of component i ($i \in \{1, 2, ..., n\}$, $j \in \{1, 2, ..., m_i\}$), we denote its failure rate as Λ_{ij}, which can fully describe its failure process when its lifetime distribution is exponential. However, as we mentioned in the introduction section, the outcome of any development process for a certain design is uncertain. Therefore, the failure rate Λ_{ij} of a certain rough design j for component i is usually not known for sure before the development of the rough design. We use a certain distribution with a probability density function $f_{\Lambda_{ij}}(.)$ or a probability mass function $p_{\Lambda_{ij}}(.)$ to describe the random failure rate Λ_{ij} before the development of the rough design, which reflects the prior belief/information about the reliability uncertainty of the technologies used in the rough design. In the evaluation of the average total cost over the service period T, these design uncertainties will be taken into account for different combinations of rough designs.

The system will be sold together with a PBL contract over a service period T. The OEM is responsible for all the repairs within the service period, as a material-based contract. Moreover, the system availability should be above a predetermined level for the whole service period. Or in other words, the total downtime of the service period should be lower than a predetermined value D_0. A penalty cost will be paid by the OEM to compensate the customer if the total downtime exceeds the predetermined level. As a result, the average total cost of a system over $[0, T]$ consists of three parts: (a) design cost, (b) repair cost, and (c) penalty cost. A detailed description of the evaluation of these cost elements are given in the following subsections.

4.1. Design cost

Let c_a^{ij} denote the cost of designing component i according to a rough design j ($i \in \{1, 2, ..., n\}$, $j \in \{1, 2, ..., m_i\}$). It includes all the costs incurred to realize a certain rough design of component during the design phase, e.g., human resources, experimental equipment, testing or prototype units, etc. Define binary decision variable x_{ij} as

$$x_{ij} = \begin{cases} 0 & \text{not selecting rough design } j \text{ for component } i, \\ 1 & \text{selecting rough design } j \text{ for component } i. \end{cases}$$

Then the design cost for component i is given by

$$A_i(\boldsymbol{x_i}) = \sum_{j=1}^{m_i} c_a^{ij} x_{ij}, \tag{1}$$

where $\boldsymbol{x_i} = [x_{i1}, x_{i2}, ..., x_{i,m_i}]$ represents the selection of rough designs for component i. Since we assume the OEM can only select one rough design from all the possible candidates for each critical component, $\sum_{j=1}^{m_i} x_{ij} = 1$. And the total design cost for the system is given by

$$A(\boldsymbol{x}) = \sum_{i=1}^{n} A_i(\boldsymbol{x_i}) = \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_a^{ij} x_{ij}, \tag{2}$$

where $\boldsymbol{x}$ represents the selection plan of rough designs for all the critical components in the system.

4.2. Repair cost

When a failure occurs in period $[0, T]$, a repair will be performed by the OEM. We assign c_r^{ij} as the repair cost for each failure of the jth rough design for component i ($i \in \{1, 2, ..., n\}$, $j \in \{1, 2, ..., m_i\}$) respectively. The repair cost c_r^{ij} corresponds to diagnosis cost, replacement cost, and other service costs for each repair. A failure-based policy for maintenance is assumed for this multi-component system in order to evaluate the maintenance cost over the service period T. Some other preventive maintenance policies, such as age/time-based policies or condition-based policies, can also be applied to the system, which may result in lower maintenance costs. However, at the design phase, it is usually hard to make a decision on the maintenance policies that are at the operational level. The evaluation of repair cost based on a failure-based policy is relatively accurate and conservative. Let $S_i(\boldsymbol{x_i})$ denote the total number of repairs for component i during $[0, T]$. Under such a failure-based policy and the assumption that the lifetimes of components are exponentially distributed, the expected number of repairs for component i during $[0, T]$, $\mathbb{E}[S_i(\boldsymbol{x_i})]$, equals the product of the failure rate and the service period $\Lambda_i(\boldsymbol{x_i})T$ (Barlow and Proschan 1967), which is still a random variable in our formulation due to the randomness of failure rate $\Lambda_i(\boldsymbol{x_i})$. The the expectation of $\mathbb{E}[S_i(\boldsymbol{x_i})]$ is given by

$$\mathbb{E}_{\Lambda_i(\boldsymbol{x_i})}\left\{\mathbb{E}\left[S_i(\boldsymbol{x_i})\right]\right\} = \mathbb{E}\left[\Lambda_i(\boldsymbol{x_i})T\right] = \mathbb{E}\left(\sum_{j=1}^{m_i} \Lambda_{ij} x_{ij} T\right) = \sum_{j=1}^{m_i} \mu_{ij} x_{ij} T, \tag{3}$$

where $\mathbb{E}_{\Lambda_i(\boldsymbol{x_i})}$ denotes the expectation over the distribution of $\Lambda_i(\boldsymbol{x_i})$, $\Lambda_i(\boldsymbol{x_i}) = \sum_{j=1}^{m_i} \Lambda_{ij} x_{ij}$ is the random failure rate of component i given a certain rough design $\boldsymbol{x_i}$, and μ_{ij} is the mean value of Λ_{ij}. Then the expected repair cost for component i, $R_i(\boldsymbol{x_i})$, is the product of the repair cost per failure and the expected number of repairs, which is also a random variable. The expectation of $R_i(\boldsymbol{x_i})$ over the distribution of $\Lambda_i(\boldsymbol{x_i})$ is given as

$$\mathbb{E}_{\Lambda_i(\boldsymbol{x_i})}\Big[R_i(\boldsymbol{x_i})\Big] = \mathbb{E}\Big(\sum_{j=1}^{m_i} \Lambda_{ij} c_r^{ij} x_{ij} T\Big) = \sum_{j=1}^{m_i} \mu_{ij} c_r^{ij} x_{ij} T. \tag{4}$$

Given that the failure processes of all the critical components are independent of each other, the expected system repair cost $R(\boldsymbol{x})$ in $[0, T]$ is the sum of $R_i(\boldsymbol{x_i}), \forall i \in \{1, 2, ..., n\}$. The expectation of $R(\boldsymbol{x})$ over all the distributions of $\Lambda_1(\boldsymbol{x_1}), \Lambda_2(\boldsymbol{x_2}), ..., \Lambda_n(\boldsymbol{x_n})$ is given as

$$\mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[R(\boldsymbol{x})\Big] = \mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[\sum_{i=1}^{n} R_i(\boldsymbol{x_i})\Big] = \sum_{i=1}^{n}\sum_{j=1}^{m_i} \mu_{ij} c_r^{ij} x_{ij} T. \tag{5}$$

where $\mathbb{E}_{\Lambda(\boldsymbol{x})}$ denotes the expectation over all the independent distributions of $\Lambda_1(\boldsymbol{x_1}), \Lambda_2(\boldsymbol{x_2}), ..., \Lambda_n(\boldsymbol{x_n})$. The variance of the expected system repair cost with respect to the random failure rates can be expressed as

$$\begin{aligned} Var_{\Lambda(\boldsymbol{x})}\Big[R(\boldsymbol{x})\Big] &= Var_{\Lambda(\boldsymbol{x})}\Big[\sum_{i=1}^{n} R_i(\boldsymbol{x_i})\Big] = \sum_{i=1}^{n} Var_{\Lambda_i(\boldsymbol{x_i})}\Big[R_i(\boldsymbol{x_i})\Big] \\ &= \sum_{i=1}^{n} Var_{\Lambda_i(\boldsymbol{x_i})}\Big(\sum_{j=1}^{m_i} \Lambda_{ij} c_r^{ij} x_{ij} T\Big) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m_i} (c_r^{ij} x_{ij} T)^2 Var(\Lambda_{ij}). \end{aligned} \tag{6}$$

4.3. Penalty cost

A period of system downtime $r_{ij} (r_{ij} << T)$ will be incurred due to a random failure of component i with rough design j in the system ($i \in \{1, 2, ..., n\}$, $j \in \{1, 2, ..., m_i\}$). Notice that while evaluating the repair costs in the previous section we ignore the downtime since the downtime is usually negligible compared with the service period T. However, under a PBL contract, when the total system downtime over the service period T exceeds a predetermined value D_0, a penalty cost should be paid by the OEM to customers with a rate c_p. The predetermined target value D_0 is at the same scale as r_{ij}. We assume the system downtime of each failure varies among different components with different rough designs. Hence the total system downtime $D(\boldsymbol{x})$

over the service period T is dependent on the number of failures $S_i(\boldsymbol{x_i})$ and the repair time per failure $r_i(\boldsymbol{x_i})$ for component i,$\forall i \in \{1, \ldots, n\}$ in $[0, T]$. The repair time per failure for component i, $r_i(\boldsymbol{x_i}) = \sum_{j=1}^{m_i} r_{ij}x_{ij}$, is a fixed value after the selection plan for component i has been made. Notice that the number of failures from component i, $S_i(\boldsymbol{x_i})$, is a Poisson distributed random variable, whose distribution is given as

$$\begin{aligned} Pr\Big[S_i(\boldsymbol{x_i}) = s_i\Big] &= \frac{e^{-\Lambda_i(\boldsymbol{x}_i)T}}{s_i!}\Big[\Lambda_i(\boldsymbol{x_i})T\Big]^{s_i} \\ &= \frac{e^{-\sum_{j=1}^{m_i}\Lambda_{ij}x_{ij}T}(\sum_{j=1}^{m_i}\Lambda_{ij}x_{ij}T)^{s_i}}{s_i!}. \end{aligned} \tag{7}$$

And the system downtime can be expressed as

$$D(\boldsymbol{x}) = r_1(\boldsymbol{x_1})S_1(\boldsymbol{x_1}) + r_2(\boldsymbol{x_2})S_2(\boldsymbol{x_2}) + \ldots + r_n(\boldsymbol{x_n})S_n(\boldsymbol{x_n}). \tag{8}$$

According to the service contract, the OEM should pay a penalty cost to the customer, when $D(\boldsymbol{x})$ exceeds the predetermined target downtime D_0. The expected penalty cost due to extra downtime exceeding D_0 is given as

$$\begin{aligned} P(\boldsymbol{x}) &= \mathbb{E}_S\Bigg\{\Big[D(\boldsymbol{x}) - D_0\Big]^+ c_p\Bigg\} \\ &= \mathbb{E}_S\Bigg\{\Big[\sum_{i=1}^{n} S_i(\boldsymbol{x_i})r_i(\boldsymbol{x_i}) - D_0\Big]^+ c_p\Bigg\} \\ &= \sum_{s_1=0}^{\infty} Pr\Big[S_1(\boldsymbol{x_1}) = s_1\Big] \cdots \sum_{s_n=0}^{\infty} Pr\Big[S_n(\boldsymbol{x_n}) = s_n\Big]\Big[\sum_{i=1}^{n} s_i r_i(\boldsymbol{x_i}) - D_0\Big]^+ c_p \\ &= \sum_{s_1=0}^{\infty}\frac{e^{-\Lambda_1(\boldsymbol{x_1})T}(\Lambda_1(\boldsymbol{x_1})T)^{s_1}}{s_1!} \cdots \sum_{s_n=0}^{\infty}\frac{e^{-\Lambda_n(\boldsymbol{x_n})T}(\Lambda_n(\boldsymbol{x_n})T)^{s_n}}{s_n!} \\ &\quad \Big[\sum_{i=1}^{n} s_i r_i(\boldsymbol{x_i}) - D_0\Big]^+ c_p. \end{aligned} \tag{9}$$

Since the failure rates $\Lambda_1(\boldsymbol{x_1}), \Lambda_2(\boldsymbol{x_2}), ..., \Lambda_n(\boldsymbol{x_n})$ are random variables, the expected penalty cost is a random variable as well. Without loss of generality, we assume the failure rates are continuous random variables, with probability density functions $f_{\Lambda_{ij}}(.)$ over region $\mathcal{O}_{ij}$. Then for component i

with a certain design $\boldsymbol{x_i}$, the probability density function of the failure rate is $f_{\Lambda_i(\boldsymbol{x_i})}(.)$ over region $\mathcal{O}_i(\boldsymbol{x_i})$ and the expectation of $P(\boldsymbol{x})$ can be expressed as

$$\mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[P(\boldsymbol{x})\Big] = \int\cdots\int_{\lambda_i\in\mathcal{O}_i(\boldsymbol{x_i})} \sum_{s_1=0}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^{s_1}}{s_1!}\cdots\sum_{s_n=0}^{\infty}\frac{e^{-\lambda_n T}(\lambda_n T)^{s_n}}{s_n!} \Big[\sum_{i=1}^{n} s_i r_i(\boldsymbol{x_i}) - D_0\Big]^{+} c_p \prod_{i=1}^{n} f_{\Lambda_i(\boldsymbol{x_i})}(\lambda_i) d\lambda_1 \ldots d\lambda_n. \tag{10}$$

The variance of $P(\boldsymbol{x})$ can be expressed as

$$Var_{\Lambda(\boldsymbol{x})}\Big[P(\boldsymbol{x})\Big] = \int\cdots\int_{\lambda_i\in\mathcal{O}_i(\boldsymbol{x_i})} \sum_{s_1=0}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^{s_1}}{s_1!}\cdots\sum_{s_n=0}^{\infty}\frac{e^{-\lambda_n T}(\lambda_n T)^{s_n}}{s_n!} \left\{\Big[\sum_{i=1}^{n} s_i r_i(\boldsymbol{x_i}) - D_0\Big]^{+} c_p - \mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[P(\boldsymbol{x})\Big]\right\}^2 \prod_{i=1}^{n} f_{\Lambda_i(\boldsymbol{x_i})}(\lambda_i)\ d\lambda_1 \ldots d\lambda_n. \tag{11}$$

4.4. Optimization model

The OEM is interested in minimizing the expected total life cycle cost $\pi(\boldsymbol{x})$, which is the sum of the total design cost $A(\boldsymbol{x})$, the expected system repair cost $R(\boldsymbol{x})$ and the expected penalty cost $P(\boldsymbol{x})$. Due to the randomness of the failure rates $\Lambda_1(\boldsymbol{x_1}), \Lambda_2(\boldsymbol{x_2}), ..., \Lambda_n(\boldsymbol{x_n})$ in rough designs, the expected total life cycle cost $\pi(\boldsymbol{x})$ is random. If the decision maker is risk-neutral, the optimization model of this problem can be formulated as

$$\begin{aligned}
\text{(P)} \qquad & min_{\boldsymbol{x}} && \mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[\pi(\boldsymbol{x})\Big] \\
& \text{s.t.} && \sum_{j=1}^{m_i} x_{ij} = 1, \qquad \forall i \in \{1,2,...,n\} \\
& && x_{ij} \in \{0,1\}, \qquad \forall i \in \{1,2,...,n\}, j \in \{1,2,...,m_i\}
\end{aligned} \tag{12}$$

where $\pi(\boldsymbol{x}) = A(\boldsymbol{x}) + R(\boldsymbol{x}) + P(\boldsymbol{x})$. Notice that in the above optimization formulation, although we do not include a probability constraint on the distribution of $\pi(\boldsymbol{x})$, the variance of $\pi(\boldsymbol{x})$ still gets penalized by the third cost term $P(\boldsymbol{x})$ in the objective function. This optimization problem is difficult to solve because of the complicated form of the objective function. In the objective function, the expected penalty cost $P(\boldsymbol{x})$ is a multiple integration over the ranges of the random failure rates, which is difficult to calculate. Therefore, in the next section, an approximation method will be proposed to make the evaluation of the objective function easier.

5. Approximate evaluation

In this section, we will describe how to evaluate a given selection plan of rough designs for all the critical components in the system approximately. For a given policy $\boldsymbol{x} = [\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}]$, the design cost $A(\boldsymbol{x})$ and the expected repair cost $\mathbb{E}_{\Lambda(\boldsymbol{x})}\big[R(\boldsymbol{x})\big]$ can be determined from Equation (**??**) and (**??**). These two cost terms are linear functions of the decision variables $\boldsymbol{x} = [\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}]$.

For the exact evaluation of the expected penalty cost $\mathbb{E}_{\Lambda(\boldsymbol{x})}\big[P(\boldsymbol{x})\big]$ of a given policy $\boldsymbol{x}$, we will suffer from the "curse of dimensionality" when the number of critical components becomes large, since each critical component contributes a dimension in computing the convolution and integration in Equation (**??**). The computation time will become intractable as the number of cirtical components grows. (In a special case, if the repair time r for each rough design of each component is the same, the system downtime equals the product of the number of failures of the system and r. In this case, we only need to compute the integration part. But we will still face the "curse of dimensionality" in computing the integration part.) With the consideration of the real life situation, we introduce an approximation method to help estimate the expected penalty cost. Remember that the expected penalty cost is the expected exceeding downtime multiplied with the penalty cost rate, i.e., $\big[D(\boldsymbol{x}) - D_0\big]^+ c_p$. Then if we can find an approximation method to calculate the distribution of the system downtime $D(\boldsymbol{x})$ efficiently, it will become relatively easy to evaluate the expected penalty cost in the objective function.

5.1. Two-moment fits of the downtime distributions

As we mentioned before, the total downtime $D(\boldsymbol{x})$ is the summation of the downtimes for all critical components and the number of failures for each component is Poisson distributed. In the approximate evaluation, we approximate the downtime by fitting a mixed Erlang distribution to the first two moments of $D(\boldsymbol{x})$. From Equation (**??**), we can obtain the first moment, variance and second moment of $D(\boldsymbol{x})$ over the distributions of $S_i(\boldsymbol{x_i})$ ($i =$

$1, 2, \ldots, n)$ are given by

$$\mathbb{E}_S\Big[D(\boldsymbol{x})\Big] = \mathbb{E}_S\Big[\sum_{i=1}^{n} r_i(\boldsymbol{x_i})S_i(\boldsymbol{x_i})\Big] = \sum_{i=1}^{n} r_i(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T, \tag{13}$$

$$Var_S\Big[D(\boldsymbol{x})\Big] = \sum_{i=1}^{n} Var_S\Big[r_i(\boldsymbol{x_i})S_i(\boldsymbol{x_i})\Big] = \sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T, \tag{14}$$

$$\mathbb{E}_S\Big[D^2(\boldsymbol{x})\Big] = \Big[\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T\Big]^2 + \sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T. \tag{15}$$

The first moment μ_D, variance σ_D^2 and the coefficient of variation c_v of $D(\boldsymbol{x})$ over the distributions of $S_i(\boldsymbol{x_i})$ and $\Lambda_i(\boldsymbol{x_i})$ are given by

$$\mu_D(\boldsymbol{x}) = \mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T\Big] = T\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\mu_i(\boldsymbol{x_i}), \tag{16}$$

$$\begin{aligned}
\sigma_D^2(\boldsymbol{x}) &= \mathbb{E}_{\Lambda(\boldsymbol{x})}\mathbb{E}_S\Big[D^2(\boldsymbol{x})\Big] - \Big\{\mathbb{E}_{\Lambda(\boldsymbol{x})}\mathbb{E}_S\Big[D(\boldsymbol{x})\Big]\Big\}^2 \\
&= \mathbb{E}_{\Lambda(\boldsymbol{x})}\Big[\Big[\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T\Big]^2 + \sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\Lambda_i(\boldsymbol{x_i})T\Big] - \Big[T\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\mu_i(\boldsymbol{x_i})\Big]^2 \\
&= T^2\sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\sigma_i^2(\boldsymbol{x_i}) + T\sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\mu_i(\boldsymbol{x_i})
\end{aligned} \tag{17}$$

$$c_v(\boldsymbol{x}) = \frac{\sigma_D(\boldsymbol{x})}{\mu_D(\boldsymbol{x})} \tag{18}$$

$$\tag{19}$$

where $\mu_i(\boldsymbol{x_i})$ and $\sigma_i(\boldsymbol{x_i})$ is the mean and standard deviation of $\Lambda_i(\boldsymbol{x_i})$ respectively.

Given that $\mu_D > 0$, and $0 < c_v \leq 1$, according to Tijms 1994, we fit the downtime distribution to an Erlang$(k-1, k)$ distribution with parameters(k, θ, q_E) such that the first two moments of $D(\boldsymbol{x})$ equal the first two moments of the Erlang $(k-1, k)$ distribution. Thus the parameters of the Erlang $(k-1, k)$

distribution can be obtained as

$$k(\boldsymbol{x}) = \lceil \frac{1}{c_v^2(\boldsymbol{x})} \rceil, \tag{20}$$

$$q_E(\boldsymbol{x}) = \frac{1}{1+c_v^2(\boldsymbol{x})}\Big[k(\boldsymbol{x})c_v^2(\boldsymbol{x}) - \sqrt{k(\boldsymbol{x})\big[1+c_v^2(\boldsymbol{x})\big] - k^2(\boldsymbol{x})c_v^2(\boldsymbol{x})}\Big], \tag{21}$$

$$\theta(\boldsymbol{x}) = \frac{k(\boldsymbol{x}) - q_E(\boldsymbol{x})}{\mu_D(\boldsymbol{x})}. \tag{22}$$

Then for random failure rates $\Lambda_1(\boldsymbol{x_1})$, $\Lambda_2(\boldsymbol{x_2})$,..., $\Lambda_n(\boldsymbol{x_n})$ the expected exceeding total downtime can be approximated by

$$\begin{aligned} D_{E_1}^A(\boldsymbol{x}) &= \mathbb{E}_\Lambda \mathbb{E}_S \Big\{ \big[D(\boldsymbol{x}) - D_0\big]^+ \Big\} \\ &= \Big[\frac{k(\boldsymbol{x}) - q_E(\boldsymbol{x})}{\theta(\boldsymbol{x})} - D_0\Big] \sum_{j=0}^{k(\boldsymbol{x})-2} \frac{\big[\theta(\boldsymbol{x})D_0\big]^j}{j!} e^{-\theta(\boldsymbol{x})D_0} \\ &+ \Big[\frac{k(\boldsymbol{x}) - q_E(\boldsymbol{x})}{\theta(\boldsymbol{x})}\Big] \frac{\big[\theta(\boldsymbol{x})D_0\big]^{k(\boldsymbol{x})-1}}{\big[k(\boldsymbol{x})-1\big]!} e^{-\theta(\boldsymbol{x})D_0}. \end{aligned} \tag{23}$$

The derivation is given in Appendix.

If $c_v \geq 1$, we fit the downtime distribution to an Hyperexponential distribution with parameters$(\theta_1, \theta_2, q_H)$ such that the first two moments of $D(\boldsymbol{x})$ equal the first two moments of the Hyperexponential distribution. Thus the parameters of the Hyperexponential distribution can be obtained as

$$\theta_1(\boldsymbol{x}) = \frac{2}{\mu_D(\boldsymbol{x})}\Bigg(1 + \sqrt{\frac{c_v^2(\boldsymbol{x}) - \frac{1}{2}}{c_v^2(\boldsymbol{x}) + 1}}\Bigg), \tag{24}$$

$$\theta_2(\boldsymbol{x}) = \frac{4}{\mu_D(\boldsymbol{x})} - \theta_1(\boldsymbol{x}), \tag{25}$$

$$q_H(\boldsymbol{x}) = \frac{\theta_1(\boldsymbol{x})(\theta_2(\boldsymbol{x})\mu_D(\boldsymbol{x}) - 1)}{\theta_2(\boldsymbol{x}) - \theta_1(\boldsymbol{x})}. \tag{26}$$

Then the expected exceeding total downtime can be approximated by

$$D_{E_2}^A(\boldsymbol{x}) = \frac{q_H(\boldsymbol{x})}{\theta_1(\boldsymbol{x})} e^{-\theta_1(\boldsymbol{x})D_0} + \frac{1 - q_H(\boldsymbol{x})}{\theta_2(\boldsymbol{x})} e^{-\theta_2(\boldsymbol{x})D_0} \tag{27}$$

The derivation can be found in Appendix **??**.

The procedure of the approximation evaluation method is illustrated in the following algorithm.

Algorithm 1

Step 1 Compute the $\mu_D(\boldsymbol{x})$, $\sigma_D^2(\boldsymbol{x})$ and $c_v(\boldsymbol{x})$ of the downtime distribution, check the value of $c_v(\boldsymbol{x})$: if $0 < c_v(\boldsymbol{x}) \leq 1$, go to step 2; if $c_v(\boldsymbol{x}) > 1$, go to step 3.

Step 2 Fit the first two moments of an Erlang$(k-1, k)$ distribution to be equal to the first two moments of the downtime distribution.

Step 2-a Let $k(\boldsymbol{x})$, $\theta(\boldsymbol{x})$ and $q_E(\boldsymbol{x})$ be the parameters of the fitted Erlang$(k-1, k)$ distribution, and compute the values of $k(\boldsymbol{x})$, $\theta(\boldsymbol{x})$ and $q_E(\boldsymbol{x})$ according to Equation (**??**),(**??**) and (**??**).

Step 2-b Calculate the expected exceeding total downtime $D_{E1}^A(\boldsymbol{x})$ according to Equation (**??**).

Step 3 Fit the first two moments of a Hyperexponential distribution to be equal to the first two moments of the downtime distribution.

Step 3-a Let $\theta_1(\boldsymbol{x})$, $\theta_2(\boldsymbol{x})$ and $q_H(\boldsymbol{x})$ be the parameters of the fitted Hyperexponential distribution, compute the values of these parameters according to Equation (**??**), (**??**) and (**??**).

Step 3-b Compute the expected exceeding total downtime $D_{E2}^A(\boldsymbol{x})$ according to Equation (**??**) .

5.2. Accuracy of the approximation

To assess the accuracy of the approximation of the expected exceeding downtime $D_E^A(\boldsymbol{x})$, we use Monte Carlo simulation method to generate the simulation results $D_E^S(\boldsymbol{x})$ according to **??**, so that the comparison between $D_E^A(\boldsymbol{x})$ and $D_E^S(\boldsymbol{x})$ can be made. A full factorial test bed is set up to show the accuracy of the approximation under different parameter settings. We investigate the effect of three factors: the number of the critical components n, standard deviation $\sigma_i(\boldsymbol{x_i})$ of the failure rate for each component and the predetermined targeted downtime D_0. Notice that although the standard deviation σ_i is a function of the design $\boldsymbol{x_i}$, we will treat it as a parameter of the approximation model or simulation model to assess the approximation accuracy without loss of generality. Similarly, for $\mu_i(\boldsymbol{x_i})$ and $r_i(\boldsymbol{x_i})$ in the calculation of $D_E^A(\boldsymbol{x})$ and $D_E^S(\boldsymbol{x})$, we will also treat them as parameters (μ_i, r_i) of the approximation model or simulation model.

The instance space Ω is thus defined by all combinations of the choices for the three factors, i.e., $(n_\alpha, \sigma_\beta, D_\gamma) \in \Omega, \forall \alpha \in \{5, 50, 100\}, \beta \in \{0.2, 0.35, 0.5\}, \gamma \in \{01, 02, 03, 04\}$. The values of choices for each factor are shown in Table **??**. In the test bed, the factor D_γ is a coefficient to generate different values of D_0 by the following expression,

$$D_0(\boldsymbol{x}) = D_\gamma \sum_{i=1}^{n} \mu_i r_i T, \tag{28}$$

where $\sum_{i=1}^{n} \mu_i r_i T$ is the expected total downtime. Thus, if D_γ is one, the targeted downtime D_0 is set to be equal to the expected total downtime. There are 36 instances in the test bed Ω. The setting of the fixed parameters in the test bed is given in Table **??**. r_i varies for different i by taking a value from 1, 3, 5. μ_i varies for different i as a sequence. Both lognormal distribution and uniform distribution are assumed as the distributions for the failure rates in this study.

Table 1: The parameter setting of the test bed

n_α	$\sigma_\beta(\forall i \in \{1, ..., n_\alpha\})$	D_γ
5, 50, 100	$0.2\mu_i$, $0.35\mu_i$, $0.5\mu_i$	$1, 1.1, 1.2, 1.3$

Table 2: Parameter values for μ_i and r_i, $i = \{1, 2, ..., n_\alpha\}$

n_α	T	μ_i	r_i
5	10	$\{0.500, 0.250, 0.167, 0.125, 0.100\}$	$\{1, 3, 5, 1, 3\}$
50	10	$\{1, ..., 1/(\frac{1}{\mu_{i-1}} + 0.182), ..., 0.101\}$	$\{1, 3, 5, 1, 3, 5, ..., 3\}$
100	10	$\{1, ..., 1/(\frac{1}{\mu_{i-1}} + 0.091), ..., 0.100\}$	$\{1, 3, 5, 1, 3, 5, ..., 1\}$

We present the results of the test bed in Table **??**,**??**, **??**, and **??**. To see how the approximation results deviate from the simulation results for each instance, we first compute the proportion of the expected exceeding downtime in the targeted total downtime, i.e., D_E^A/D_0 for the approximation model and D_E^S/D_0 for the simulation model, as well as the confidence intervals of the simulation results. The confidence intervals of all the simulation results are relatively small, which shows the accuracy of our simulation results. Then the accuracy of our approximation is assessed by the gaps between D_E^A/D_0 and D_E^S/D_0. For lognormally-distributed failure rates, the gaps are denoted as G_l in Table **??** and **??**; the average gap is 0.0004 and the maximum gap is 0.0019. For uniformly-distributed failure rates, the gaps are denoted as G_u in Table **??** and **??**; the average gap is 0.0005 and the maximum gap is 0.0023. From these results, we can get the conclusion that our approximation is relatively accurate under different parameter settings.

Furthermore, our approximation method is more accurate when n is large (e.g., 50 or 100). This is due to the fact that the total downtime is the sum of n independent random variables, and when n is large the total downtime converges in distribution to a normal random variable regardless of the individual underlying distributions. Note that the computation times of the approximation model for all the instances are negligible compared with the simulation model. In order to better demonstrate the accuracy of our approximation method, we also compare the probability density functions of the downtime distribution estimated from the simulation model with the ones estimated from the approximation model, as shown in Figure **??** and Figure **??**. It is obvious that the downtime distributions estimated from the simulation model are approximately the same as the ones estimated from the approximation model for various settings of n and σ.

6. Optimization

In the previous section, we have discussed how a given policy, which is the selection of rough designs for all the critical components in the system can be evaluated in an approximate way. In this section, we will describe a method of finding a feasible policy for problem (P) with as low life cycle costs as possible. The problem is formulated as a binary integer programming with a nonlinear objective function.

According to Eq. (**??**), the expected penalty costs $\mathbb{E}_{\Lambda((X))}[P((X))]$ of problem (P) can be written as:

$$\begin{aligned}\mathbb{E}_{\Lambda(\boldsymbol{x})}[P_A(\boldsymbol{x})] \quad = \quad & c_p(\frac{k(\boldsymbol{x})-q(\boldsymbol{x})}{\theta(\boldsymbol{x})}-D_0)\sum_{j=0}^{k(\boldsymbol{x})-2}\frac{(\theta(\boldsymbol{x})D_0)^j}{j!}e^{-\theta(\boldsymbol{x})D_0} \\ & +c_p(\frac{k(\boldsymbol{x})-q(\boldsymbol{x})}{\theta(\boldsymbol{x})})\frac{(\theta(\boldsymbol{x})D_0)^{k(\boldsymbol{x})-1}}{(k(\boldsymbol{x})-1)!}e^{-\theta(\boldsymbol{x})D_0} \qquad (29)\end{aligned}$$

Table 3: Simulation and approximation results in the case of lognormal distributed failure rates

	D_E^A/D_0	D_E^S/D_0	Confidence interval of D_E^S/D_0	G_l
$(n_5, \sigma_{0.2}, D_{01})$	14,58	14,70	(14,68, 14,72)	0,0012
$(n_5, \sigma_{0.2}, D_{02})$	9,62	9,59	(9,57, 9,60)	0,0003
$(n_5, \sigma_{0.2}, D_{03})$	6,26	6,11	(6,10, 6,13)	0,0015
$(n_5, \sigma_{0.2}, D_{04})$	4,02	3,84	(3,83, 3,85)	0,0019
$(n_5, \sigma_{0.35}, D_{01})$	15,65	15,73	(15,70, 15,75)	0,0008
$(n_5, \sigma_{0.35}, D_{02})$	10,58	10,55	(10,53, 10,56)	0,0004
$(n_5, \sigma_{0.35}, D_{03})$	7,09	6,97	(6,95, 6,98)	0,0013
$(n_5, \sigma_{0.35}, D_{04})$	4,71	4,55	(4,53, 4,56)	0,0016
$(n_5, \sigma_{0.5}, D_{01})$	17,15	17,15	(17,13, 17,17)	0,0000
$(n_5, \sigma_{0.5}, D_{02})$	11,96	11,89	(11,87, 11,91)	0,0006
$(n_5, \sigma_{0.5}, D_{03})$	8,29	8,18	(8,16, 8,20)	0,0011
$(n_5, \sigma_{0.5}, D_{04})$	5,72	5,60	(5,58, 5,61)	0,0012
$(n_{50}, \sigma_{0.2}, D_{01})$	4,28	4,29	(4,28, 4,29)	0,0001
$(n_{50}, \sigma_{0.2}, D_{02})$	1,00	0,98	(0,98, 0,98)	0,0003
$(n_{50}, \sigma_{0.2}, D_{03})$	0,15	0,14	(0,14, 0,14)	0,0001
$(n_{50}, \sigma_{0.2}, D_{04})$	0,02	0,01	(0,01, 0,01)	0,0000
$(n_{50}, \sigma_{0.35}, D_{01})$	4,82	4,82	(4,81, 4,83)	0,0000
$(n_{50}, \sigma_{0.35}, D_{02})$	1,36	1,34	(1,34, 1,35)	0,0002
$(n_{50}, \sigma_{0.35}, D_{03})$	0,28	0,27	(0,27, 0,27)	0,0001
$(n_{50}, \sigma_{0.35}, D_{04})$	0,04	0,04	(0,04, 0,04)	0,0000
$(n_{50}, \sigma_{0.5}, D_{01})$	5,56	5,54	(5,53, 5,55)	0,0002
$(n_{50}, \sigma_{0.5}, D_{02})$	1,88	1,89	(1,89, 1,90)	0,0001
$(n_{50}, \sigma_{0.5}, D_{03})$	0,51	0,52	(0,52, 0,52)	0,0002
$(n_{50}, \sigma_{0.5}, D_{04})$	0,11	0,12	(0,12, 0,12)	0,0001
$(n_{100}, \sigma_{0.2}, D_{01})$	3,05	3,05	(3,04, 3,05)	0,0000
$(n_{100}, \sigma_{0.2}, D_{02})$	0,35	0,34	(0,33, 0,34)	0,0001
$(n_{100}, \sigma_{0.2}, D_{03})$	0,02	0,01	(0,01, 0,01)	0,0000
$(n_{100}, \sigma_{0.2}, D_{04})$	0,00	0,00	(0,00, 0,00)	0,0000
$(n_{100}, \sigma_{0.35}, D_{01})$	3,44	3,44	(3,43, 3,44)	0,0000
$(n_{100}, \sigma_{0.35}, D_{02})$	0,53	0,52	(0,52, 0,53)	0,0000
$(n_{100}, \sigma_{0.35}, D_{03})$	0,04	0,04	(0,04, 0,04)	0,0000
$(n_{100}, \sigma_{0.35}, D_{01})$	0,00	0,00	(0,00, 0,00)	0,0000

Table 4: (Continued) Simulation and approximation results in the case of lognormal distributed failure rates

	D_E^A/D_0	D_E^S/D_0	Confidence interval of D_E^S/D_0	G_l
$(n_{100}, \sigma_{0.5}, D_{01})$	3,97	3,96	(3,96, 3,97)	0,0001
$(n_{100}, \sigma_{0.5}, D_{02})$	0,82	0,83	(0,82, 0,83)	0,0001
$(n_{100}, \sigma_{0.5}, D_{03})$	0,10	0,11	(0,11, 0,11)	0,0001
$(n_{100}, \sigma_{0.5}, D_{04})$	0,01	0,01	(0,01, 0,01)	0,0000

The we have the optimization problem formulated as:

$$
\begin{aligned}
(P_A) \qquad\qquad & min_{\boldsymbol{x}} \qquad && \mathbb{E}_\Lambda\Big[\pi_A(\boldsymbol{x})\Big] \\
& \text{s.t.} && \sum_{j=1}^{m_i} x_{ij} = 1, \qquad \forall i \in \{1, 2, ..., n\}
\end{aligned}
$$

where $\pi_A(\boldsymbol{x}) = A(\boldsymbol{x}) + R(\boldsymbol{x}) + P_A(\boldsymbol{x})$.

6.1. Exact analysis

6.2. Numerical example

7. Conclusion

8. Appendices:

AppendixA. Procedures of the Monte Carlo simulation

Step 1 First, we generated the sequences of r_i, μ_i and σ_i, $i = \{1, 2, ..., n\}$. Then we get D_0 immediately from Eq. (**??**). Furthermore, we take one sample $\hat{\Lambda}_i$ from $\Lambda_i \sim G(\mu_i, \sigma_i)$ for each component to simulate its failure rate λ_i, where $G(\mu_i, \sigma_i)$ is a general distribution with parameter mean μ_i and standard deviation σ_i. Given that the number of failures of each component s_i is Poisson distributed with parameter $\lambda_i T$, we take one sample $\hat{S}_i$ from $S_i \sim$Pois$(\hat{\Lambda}_i T)$. Together with r_i, we get one simulation result $\hat{D}^S_{E_p}$ of the proportion of the exceeded total downtime D_{E_p} computed as following:

$$
\hat{D}^S_{E_p} = \frac{\sum_{i=1}^n \hat{S}_i r_i - D_0}{\sum_{i=1}^n \hat{S}_i r_i} \tag{A.1}
$$

Table 5: Simulation and approximation results in the case of uniform distributed failure rates

	D_E^A/D_0	D_E^S/D_0	Confidence interval of D_E^S/D_0	G_u
$(n_5, \sigma_{0.2}, D_{01})$	14,58	14,70	(14,68, 14,7)	0,0012
$(n_5, \sigma_{0.2}, D_{02})$	9,62	9,60	(9,58, 9,6)	0,0002
$(n_5, \sigma_{0.2}, D_{03})$	6,26	6,12	(6,1, 6,12)	0,0014
$(n_5, \sigma_{0.2}, D_{04})$	4,02	3,83	(3,82, 3,83)	0,0019
$(n_5, \sigma_{0.35}, D_{01})$	15,65	15,82	(15,8, 15,82)	0,0017
$(n_5, \sigma_{0.35}, D_{02})$	10,58	10,57	(10,55, 10,57)	0,0001
$(n_5, \sigma_{0.35}, D_{03})$	7,09	6,97	(6,96, 6,97)	0,0012
$(n_5, \sigma_{0.35}, D_{04})$	4,71	4,53	(4,52, 4,53)	0,0018
$(n_5, \sigma_{0.5}, D_{01})$	17,15	17,38	(17,35, 17,38)	0,0023
$(n_5, \sigma_{0.5}, D_{02})$	11,96	12,01	(11,99, 12,01)	0,0005
$(n_5, \sigma_{0.5}, D_{03})$	8,29	8,18	(8,17, 8,18)	0,0011
$(n_5, \sigma_{0.5}, D_{04})$	5,72	5,54	(5,53, 5,54)	0,0018
$(n_{50}, \sigma_{0.2}, D_{01})$	4,28	4,28	(4,28, 4,28)	0,0001
$(n_{50}, \sigma_{0.2}, D_{02})$	1,00	0,98	(0,97, 0,98)	0,0003
$(n_{50}, \sigma_{0.2}, D_{03})$	0,15	0,13	(0,13, 0,13)	0,0002
$(n_{50}, \sigma_{0.2}, D_{04})$	0,02	0,01	(0,01, 0,01)	0,0000
$(n_{50}, \sigma_{0.35}, D_{01})$	4,82	4,83	(4,83, 4,83)	0,0001
$(n_{50}, \sigma_{0.35}, D_{02})$	1,36	1,33	(1,33, 1,33)	0,0003
$(n_{50}, \sigma_{0.35}, D_{03})$	0,28	0,25	(0,25, 0,25)	0,0003
$(n_{50}, \sigma_{0.35}, D_{04})$	0,04	0,03	(0,03, 0,03)	0,0001
$(n_{50}, \sigma_{0.5}, D_{01})$	5,56	5,58	(5,57, 5,58)	0,0002
$(n_{50}, \sigma_{0.5}, D_{02})$	1,88	1,84	(1,83, 1,84)	0,0005
$(n_{50}, \sigma_{0.5}, D_{03})$	0,51	0,46	(0,45, 0,46)	0,0005
$(n_{50}, \sigma_{0.5}, D_{04})$	0,11	0,08	(0,08, 0,08)	0,0002
$(n_{100}, \sigma_{0.2}, D_{01})$	3,05	3,04	(3,04, 3,04)	0,0000
$(n_{100}, \sigma_{0.2}, D_{02})$	0,35	0,33	(0,33, 0,33)	0,0002
$(n_{100}, \sigma_{0.2}, D_{03})$	0,02	0,01	(0,01, 0,01)	0,0000
$(n_{100}, \sigma_{0.2}, D_{04})$	0,00	0,00	(0,00, 0,00)	0,0000
$(n_{100}, \sigma_{0.35}, D_{01})$	3,44	3,44	(3,44, 3,44)	0,0001
$(n_{100}, \sigma_{0.35}, D_{02})$	0,53	0,51	(0,51, 0,51)	0,0002
$(n_{100}, \sigma_{0.35}, D_{03})$	0,04	0,03	(0,03, 0,03)	0,0001
$(n_{100}, \sigma_{0.35}, D_{04})$	0,00	0,00	(0,00, 0,00)	0,0000

Table 6: (Continued) Simulation and approximation results in the case of uniform distributed failure rates

	D_E^A/D_0	D_E^S/D_0	Confidence interval of D_E^S/D_0	G_u
$(n_{100}, \sigma_{0.5}, D_{01})$	3,97	3,98	(3,97, 3,98)	0,0001
$(n_{100}, \sigma_{0.5}, D_{02})$	0,82	0,79	(0,79, 0,79)	0,0003
$(n_{100}, \sigma_{0.5}, D_{03})$	0,10	0,09	(0,08, 0,09)	0,0001
$(n_{100}, \sigma_{0.5}, D_{04})$	0,01	0,00	(0,00, 0,00)	0,0000

Table 7: Parameter explanation of the numerical example

Parameter	Explanation
c_a^{ij}	acquistion costs of each desgin
$c_r^{ij} = c_a^{ij} * \{5\%, 10\%, 20\%\}$	repair costs of each design
r_{ij}	repair time, randomly generated by a $U(2, 48)$ distribution
μ_{ij}	mean of the failure rate, μ_{i1} are randomly generated from $U(0.1, 0.3)$, μ_{2i} are randomly generated from $U(0.4, 0.6)$
σ_{ij}	$\sigma_{i1} = \mu_{i1} * \{20\%, 35\%, 50\%\}$, $\sigma_{i2} = \mu_{i2} * \{35\%, 50\%, 20\%\}$

Step 2 Repeat step 1 for 10000 times to get 10000 $\hat{D}_{E_p}^S$, we take the expected value $\bar{D}_{E_p}^S$ of all the $\hat{D}_{E_p}^S$ and compare it with the approximation result $D_{E_p}^A$ according to Eqs. (**??**) or (**??**) to get a value of the absolute gap between $\bar{D}_{E_p}^S$ and $D_{E_p}^A$.

Step 3 Repeat step 3 for 50 times to generate final simulation results of D_{E_p}, $D_{E_p}^S$. And the mean value *avg* G, maximum value *max* G and the confidence interval for $D_{E_p}^S$ of the gap between $D_{E_p}^S$ and $D_{E_p}^A$. The 95% percent confidence interval is given as:

$$(D_{E_p}^S - t_{(49,2.5\%)}\sqrt{\frac{S^2(50)}{50}}, \quad D_{E_p}^S + t_{(49,2.5\%)}\sqrt{\frac{S^2(50)}{50}})$$

Table 8: Parameter design of the numberical example

Comp.	Alternative design 1					Alternative design 2				
Number	c_a^{i1}	c_r^{i1}	r_{i1}	μ_{i1}	σ_{i1}	c_a^{i2}	c_r^{i2}	r_{i2}	μ_{i2}	σ_{i2}
1	10000	500	12	0,23	0,05	5000	250	32	0,47	0,17
2	9675	967,5	32	0,17	0,06	4837,5	483,75	25	0,41	0,20
3	9350	1870	28	0,15	0,07	4675	935	16	0,45	0,09
4	9025	451,25	10	0,25	0,05	4512,5	225,625	35	0,60	0,21
5	8700	870	41	0,14	0,05	4350	435	14	0,46	0,23
6	8375	1675	5	0,25	0,12	4187,5	837,5	25	0,51	0,10
7	8050	402,5	11	0,21	0,04	4025	201,25	27	0,53	0,18
8	7725	772,5	12	0,24	0,08	3862,5	386,25	38	0,55	0,27
9	7400	1480	12	0,14	0,07	3700	740	2	0,54	0,11
10	7075	353,75	8	0,29	0,06	3537,5	176,875	23	0,52	0,18
11	6750	675	20	0,14	0,05	3375	337,5	15	0,55	0,27
12	6425	1285	34	0,21	0,11	3212,5	642,5	4	0,46	0,09
13	6100	305	31	0,20	0,04	3050	152,5	9	0,53	0,18
14	5775	577,5	8	0,26	0,09	2887,5	288,75	21	0,57	0,29
15	5450	1090	16	0,29	0,14	2725	545	10	0,58	0,12
16	5125	256,25	41	0,16	0,03	2562,5	128,125	15	0,42	0,15
17	4800	480	47	0,18	0,06	2400	240	17	0,44	0,22
18	4475	895	33	0,27	0,13	2237,5	447,5	27	0,51	0,10
19	4150	207,5	9	0,16	0,03	2075	103,75	27	0,50	0,18
20	3825	382,5	19	0,20	0,07	1912,5	191,25	33	0,45	0,23
21	3500	700	33	0,25	0,12	1750	350	9	0,54	0,11
22	3175	158,75	46	0,12	0,02	1587,5	79,375	30	0,43	0,15
23	2850	285	5	0,11	0,04	1425	142,5	19	0,48	0,24
24	2525	505	37	0,13	0,07	1262,5	252,5	30	0,44	0,09
25	2200	110	21	0,28	0,06	1100	55	9	0,53	0,19
26	1875	187,5	6	0,19	0,07	937,5	93,75	37	0,57	0,28
27	1550	310	34	0,14	0,07	775	155	26	0,42	0,08

Table 9: Optimization results of the numerical example for problem P_A by complete enumeration

n	D_0	$\mathbb{E}_\Lambda\left[\pi(\boldsymbol{x})\right]$	$\boldsymbol{x}$	CPU time(s)
2	167,45	24882	11	0
3	224,45	37013	112	0
4	341,95	42103	1121	0
5	402,85	49398	11212	0
6	472,85	58924	122121	0
7	555,95	65847	1221211	0
8	674,85	73646	12212211	0
9	688,65	81178	122122112	0
10	760,05	87580	1221221121	0
11	815,3	94690	12212111222	0
12	860,2	99836	122122112122	0
13	915,05	103603	1221221121222	1
14	985,3	109704	12212211212221	2
15	1037,5	115784	122122112122212	4
16	1101,8	119121	1221221121222122	8
17	1181,5	123020	12212211212221222	16
18	1294,9	129466	122121112222212222	32
19	1369,6	131993	1221221121222122221	64
20	1462,85	135476	22212111212221222211	129
21	1528,4	138689	122122112222212222112	261
22	1620,5	141239	1221221122222122221121	526
23	1668,85	143169	12212211222221222211211	1062
24	1758,9	145565	222122112122212222112111	2119
25	1812,15	146897	2221221121222122221121112	4330
26	1923,3	146089	22212211222221222211211121	8732
27	2001,7	147558	222122112222212222112111211	17485

Table 10: Optimization results for deterministic model setting

n	D_0	$\pi(\boldsymbol{x}_D)$	$\boldsymbol{x}_D$	$\mathbb{E}_\Lambda\left[\pi(\boldsymbol{x}_D)\right]$	Δ	$\%\Delta$
2	167,45	17971	12	34048	9166	26,92%
3	224,45	26853	122	50456	13443	26,64%
4	341,95	35304	2211	73908	31804	43,03%
5	402,85	41655	22112	84661	35263	41,65%
6	472,85	50945	222121	83758	24834	29,65%
7	555,95	59009	2211221	106884	41037	38,39%
8	674,85	65132	22222111	128448	54803	42,67%
9	688,65	72828	222221112	134873	53695	39,81%
10	760,05	77309	2221222121	137112	49532	36,12%
11	815,30	82700	22212211222	145550	50860	34,94%
12	860,20	88708	222122212122	152331	52495	34,46%
13	915,05	92566	2221222121222	153838	50235	32,65%
14	985,30	98663	22212221221221	159213	49509	31,10%
15	1037,50	104549	222122212212212	168340	52556	31,22%
16	1101,80	107650	2221222122122122	172421	53300	30,91%
17	1181,50	111106	22212221221221222	176535	53515	30,31%
18	1294,90	115625	222122212212212222	196383	66917	34,07%
19	1369,60	117644	2221222122222122221	197933	65940	33,31%
20	1462,85	121862	22212221222222222111	206425	70949	34,37%
21	1528,40	125390	222222112222212222112	209560	70871	33,82%
22	1620,50	126496	2221222122222222221121	211114	69875	33,10%
23	1668,85	129659	22212221222222222211211	196555	53386	27,16%
24	1758,90	131297	222222212222212222112111	223816	78251	34,96%
25	1812,15	132689	2222222122222122221121112	223276	76378	34,21%
26	1923,30	132177	22222221222222222211211121	227036	80947	35,65%
27	2001,70	134161	222222212222222222112111211	217550	69992	32,17%

AppendixB. Derivations for the two-moment fits method

AppendixB.1. The first two moments of the downtime distribution

The mean $\mu_D(\boldsymbol{x})$ of $\mathbb{E}_S\Big[D(\boldsymbol{x})\Big]$ are given by:

$$\mu_D(\boldsymbol{x}) = \mathbb{E}_{\Lambda(\boldsymbol{x})}\mathbb{E}_S\Big[D(\boldsymbol{x})\Big] = T\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\mu_i(\boldsymbol{x_i}) = \frac{k(\boldsymbol{x}) - q(\boldsymbol{x})}{\theta(\boldsymbol{x})} \tag{B.1}$$

The variance $\sigma_D(\boldsymbol{x})$ of $D(\boldsymbol{x})$ is given by:

$$\begin{aligned}
\sigma_D^2(\boldsymbol{x}) &= \mathbb{E}_{\Lambda(\boldsymbol{x})}\mathbb{E}_S\Big[D^2(\boldsymbol{x})\Big] - \Big\{\mathbb{E}_{\Lambda(\boldsymbol{x})}\mathbb{E}_S\Big[D(\boldsymbol{x})\Big]\Big\}^2 \\
&= \mathbb{E}_{\Lambda(\boldsymbol{x})}\Bigg[\Bigg[\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\lambda_i(\boldsymbol{x_i})T\Bigg]^2 + \sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\lambda_i(\boldsymbol{x_i})T\Bigg] - \Bigg[T\sum_{i=1}^{n} r_i(\boldsymbol{x_i})\mu_i(\boldsymbol{x_i})\Bigg]^2 \\
&= \sum_{i=1}^{n} r_i^2(\boldsymbol{x_i})\mu_i(\boldsymbol{x_i})T \\
&= \frac{\sqrt{k(\boldsymbol{x}) - q^2(\boldsymbol{x})}}{\lambda(\boldsymbol{x})}
\end{aligned} \tag{B.2}$$

AppendixB.2. Identities for the Erlang$(k-1,k)$ distribution

Consider an Erlang$(k-1,k)$ distribution X with parameters (k,θ,q), the probability density function of is

$$e_{k-1,k}(x) = q\theta^{k-1}\frac{x^{k-2}}{(k-2)!}e^{-\theta x} + (1-q)\theta^k\frac{x^{k-1}}{(k-1)!}e^{-\theta x}$$

the cumulative distribution function of X is

$$E_{k-1,k}(x) = q\Bigg(1 - \sum_{j=0}^{k-2}\frac{(\theta x)^j}{j!}e^{-\theta x}\Bigg) + (1-q)\Bigg(1 - \sum_{j=0}^{k-1}\frac{(\theta x)^j}{j!}e^{-\theta x}\Bigg)$$

Then the first partial moment of the Erlang$(k-1,k)$ can be described as:

$$\begin{aligned}
\mathbb{E}[(X - X_0)^+] &= \int_0^\infty (x - X_0)^+ e_{k-1,k}(x)dx \\
&= \frac{q(k-1)}{\theta}\Big[1 - E_k^\theta(X_0)\Big] + \frac{k(1-q)}{\theta}\Big[1 - E_{k+1}^\theta(X_0)\Big] - X_0\Big[1 - E_{k-1,k}^\theta(X_0)\Big] \\
&= (\frac{k-q}{\theta} - X_0)\sum_{j=0}^{k-2}\frac{(\theta X_0)^j}{j!}e^{-\theta X_0} + (\frac{k-q}{\theta})\frac{(\theta X_0)^{k-1}}{(k-1)!}e^{-\theta X_0}
\end{aligned}$$

AppendixB.3. Identities for the Hyperexponential distribution

For a Hyperexponential distribution X with parameters (θ_1, θ_2, q), the probability density function is given as:

$$h_2(x) = q\theta_1 e^{-\theta_1 x} + (1-q)\theta_2 e^{-\theta_2 x}$$

the cumulative distribution function is given as:

$$H_2(x) = q(1 - e^{-\theta_1 x}) + (1-q)(1 - e^{-\theta_2 x})$$

And the first partial moment is given as:

$$\begin{aligned}
\mathbb{E}[(X - X_0)^+] &= \int_0^\infty (x - X_0)^+ h_2(x) dx \\
&= \int_{X_0}^\infty x q\theta_1 e^{-\theta_1 x} dx + \int_{X_0}^\infty x(1-q)\theta_2 e^{-\theta_2 x} dx - X_0 \Big[1 - H_2(X_0)\Big] \\
&= qX_0 e^{-\theta_1 X_0} + (1-q)X_0 e^{-\theta_2 X_0} + \frac{q}{\theta_1}\Big[1 - F_{\theta_1}(X_0)\Big] + \frac{1-q}{\theta_2}\Big[1 - F_{\theta_2}(X_0)\Big] \\
&\quad - X_0 \Big[1 - H_2(X_0)\Big]
\end{aligned}$$

AppendixC. Detail results of the test bed

AppendixD. Figures of the total downtime of the test bed

Table C.11: Simulation and approximation results in the case of lognormal distributed failure rates and mixed Erlang distribution

	$D^{A}_{E_p}$	$D^{S_l}_{E_p}$	Confidence interval of $D^{S_l}_{E_p}$	$avg\ G_l$	$max\ G_l$
$(n_5, \sigma_{0.2}, D_{01})$	14,58	14,67	(14.60, 14.73)	0,0009	0,0056
$(n_5, \sigma_{0.2}, D_{02})$	9,62	9,59	(9.54, 9.63)	0,0003	0,0040
$(n_5, \sigma_{0.2}, D_{03})$	6,26	6,11	(6.07, 6.15)	0,0015	0,0047
$(n_5, \sigma_{0.2}, D_{04})$	4,02	3,82	(3.79, 3,85)	0,0020	0,0041
$(n_5, \sigma_{0.35}, D_{01})$	16,09	16,15	(16.06, 16.23)	0,0006	0,0089
$(n_5, \sigma_{0.35}, D_{02})$	11,00	10,98	(10.91, 11.05)	0,0002	0,0062
$(n_5, \sigma_{0.35}, D_{03})$	7,46	7,33	(7.27, 7.38)	0,0013	0,0059
$(n_5, \sigma_{0.35}, D_{04})$	5,02	4,85	(4.81, 4.89)	0,0017	0,0041
$(n_5, \sigma_{0.5}, D_{01})$	18,34	18,20	(18.11, 18.28)	0,0014	0,0073
$(n_5, \sigma_{0.5}, D_{02})$	13,04	12,87	(12.79, 12.96)	0,0017	0,0099
$(n_5, \sigma_{0.5}, D_{03})$	9,25	9,11	(9.05, 9.16)	0,0014	0,0052
$(n_5, \sigma_{0.5}, D_{04})$	6,54	6,41	(6.36, 6.47)	0,0013	0,0059
$(n_{50}, \sigma_{0.2}, D_{01})$	4,28	4,28	(4.26, 4.29)	0,0000	0,0012
$(n_{50}, \sigma_{0.2}, D_{02})$	1,00	0,98	(0.97, 0.99)	0,0003	0,0010
$(n_{50}, \sigma_{0.2}, D_{03})$	0,15	0,14	(0.14, 0.14)	0,0001	0,0004
$(n_{50}, \sigma_{0.2}, D_{04})$	0,02	0,01	(0.01, 0.01)	0,0000	0,0001
$(n_{50}, \sigma_{0.35}, D_{01})$	5,05	5,04	(5.02, 5.07)	0,0001	0,0017
$(n_{50}, \sigma_{0.35}, D_{02})$	1,52	1,51	(1.50, 1.52)	0,0001	0,0010
$(n_{50}, \sigma_{0.35}, D_{03})$	0,34	0,34	(0.33, 0.34)	0,0000	0,0003
$(n_{50}, \sigma_{0.35}, D_{04})$	0,06	0,06	(0.06, 0.06)	0,0000	0,0002
$(n_{50}, \sigma_{0.5}, D_{01})$	6,12	6,08	(6.05, 6.11)	0,0004	0,0028
$(n_{50}, \sigma_{0.5}, D_{02})$	2,31	2,32	(2.30, 2.34)	0,0001	0,0013
$(n_{50}, \sigma_{0.5}, D_{03})$	0,72	0,76	(0.75, 0.77)	0,0003	0,0011
$(n_{50}, \sigma_{0.5}, D_{04})$	0,19	0,23	(0.22, 0.23)	0,0004	0,0007
$(n_{100}, \sigma_{0.2}, D_{01})$	3,05	3,04	(3.02, 3.05)	0,0001	0,0016
$(n_{100}, \sigma_{0.2}, D_{02})$	0,35	0,33	(0.33, 0.34)	0,0002	0,0006
$(n_{100}, \sigma_{0.2}, D_{03})$	0,02	0,01	(0.01, 0.01)	0,0000	0,0001
$(n_{100}, \sigma_{0.2}, D_{04})$	0,00	0,00	(0.00, 0.00)	0,0000	0,0000
$(n_{100}, \sigma_{0.35}, D_{01})$	3,60	3,60	(3.59, 3.62)	0,0000	0,0011
$(n_{100}, \sigma_{0.35}, D_{02})$	0,61	0,61	(0.61, 0.62)	0,0000	0,0003
$(n_{100}, \sigma_{0.35}, D_{03})$	0,05	0,05	(0.05, 0.06)	0,0000	0,0001
$(n_{100}, \sigma_{0.35}, D_{04})$	0,00	0,00	(0.00, 0.00)	0,0000	0,0000

Table C.12: (Continued) Simulation and approximation results in the case of lognormal distributed failure rates and mixed Erlang distribution

	$D^{A}_{E_p}$	$D^{S_l}_{E_p}$	Confidence interval of $D^{S_l}_{E_p}$	$avg\ G_l$	$max\ G_l$
$(n_{100}, \sigma_{0.5}, D_{01})$	4,38	4,36	(4.34, 4.38)	0,0002	0,0016
$(n_{100}, \sigma_{0.5}, D_{02})$	1,07	1,10	(1.09, 1.11)	0,0003	0,0010
$(n_{100}, \sigma_{0.5}, D_{03})$	0,17	0,19	(0.19, 0.20)	0,0002	0,0005
$(n_{100}, \sigma_{0.5}, D_{04})$	0,02	0,03	(0.03, 0.03)	0,0001	0,0002

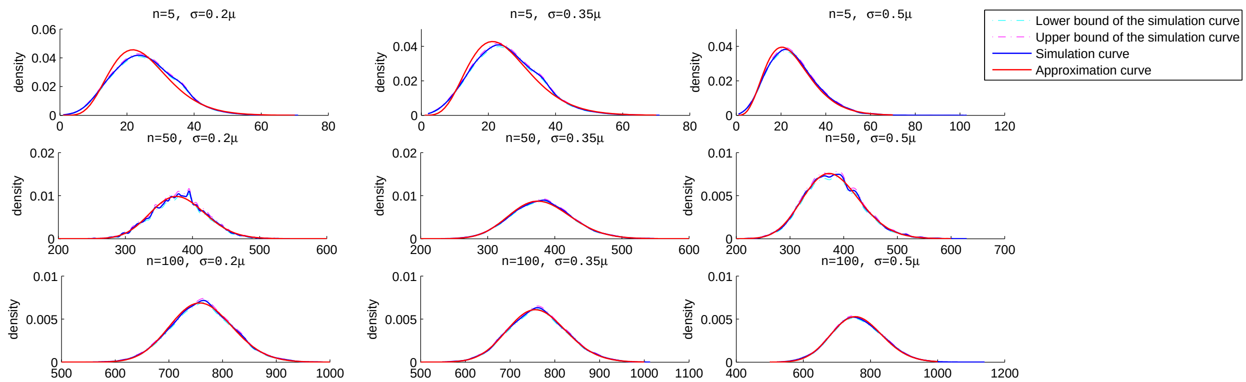

Figure D.1: The simulation curve and approximation curve of the downtime density distribution under lognormal distributed failure rate

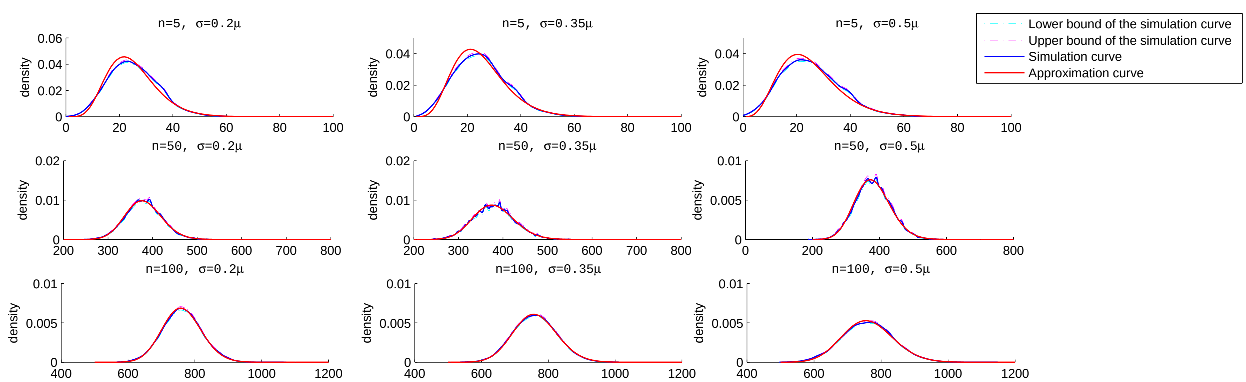

Figure D.2: The simulation curve and approximation curve of the downtime density distribution under uniform distributed failure rate

Table C.13: Simulation and approximation results in the case of uniform distributed failure rates and mixed Erlang distribution

	$D_{E_p}^{A}$	$D_{E_p}^{S_u}$	Confidence interval of $D_{E_p}^{S_u}$	*avg* G_u	*max* G_u
$(n_5, \sigma_{0.2}, D_{01})$	14,58	14,68	(14,68, 14,74)	0,0010	0,0082
$(n_5, \sigma_{0.2}, D_{02})$	9,62	9,59	(9,59, 9,64)	0,0003	0,0051
$(n_5, \sigma_{0.2}, D_{03})$	6,26	6,13	(6,13, 6,17)	0,0013	0,0037
$(n_5, \sigma_{0.2}, D_{04})$	4,02	3,84	(3,84, 3,87)	0,0018	0,0043
$(n_5, \sigma_{0.35}, D_{01})$	15,65	15,79	(15,79, 15,86)	0,0014	0,0075
$(n_5, \sigma_{0.35}, D_{02})$	10,58	10,58	(10,58, 10,64)	0,0000	0,0058
$(n_5, \sigma_{0.35}, D_{03})$	7,09	6,94	(6,94, 6,99)	0,0015	0,0043
$(n_5, \sigma_{0.35}, D_{04})$	4,71	4,55	(4,55, 4,59)	0,0016	0,0047
$(n_5, \sigma_{0.5}, D_{01})$	17,15	17,32	(17,32, 17,41)	0,0017	0,0076
$(n_5, \sigma_{0.5}, D_{02})$	11,96	11,97	(11,97, 12,04)	0,0002	0,0058
$(n_5, \sigma_{0.5}, D_{03})$	8,29	8,18	(8,18, 8,22)	0,0012	0,0039
$(n_5, \sigma_{0.5}, D_{04})$	5,72	5,50	(5,50, 5,54)	0,0022	0,0051
$(n_{50}, \sigma_{0.2}, D_{01})$	4,28	4,27	(4,27, 4,29)	0,0001	0,0018
$(n_{50}, \sigma_{0.2}, D_{02})$	1,00	0,97	(0,97, 0,98)	0,0003	0,0011
$(n_{50}, \sigma_{0.2}, D_{03})$	0,15	0,14	(0,14, 0,14)	0,0002	0,0004
$(n_{50}, \sigma_{0.2}, D_{04})$	0,02	0,01	(0,01, 0,01)	0,0000	0,0001
$(n_{50}, \sigma_{0.35}, D_{01})$	4,82	4,83	(4,83, 4,85)	0,0001	0,0015
$(n_{50}, \sigma_{0.35}, D_{02})$	1,36	1,33	(1,33, 1,34)	0,0003	0,0010
$(n_{50}, \sigma_{0.35}, D_{03})$	0,28	0,25	(0,25, 0,25)	0,0003	0,0006
$(n_{50}, \sigma_{0.35}, D_{04})$	0,04	0,03	(0,03, 0,03)	0,0001	0,0002
$(n_{50}, \sigma_{0.5}, D_{01})$	5,56	5,58	(5,58, 5,60)	0,0002	0,0021
$(n_{50}, \sigma_{0.5}, D_{02})$	1,88	1,83	(1,83, 1,85)	0,0005	0,0015
$(n_{50}, \sigma_{0.5}, D_{03})$	0,51	0,45	(0,45, 0,46)	0,0005	0,0009
$(n_{50}, \sigma_{0.5}, D_{04})$	0,11	0,08	(0,08, 0,09)	0,0002	0,0005
$(n_{100}, \sigma_{0.2}, D_{01})$	3,05	3,04	(3,04, 3,05)	0,0000	0,0010
$(n_{100}, \sigma_{0.2}, D_{02})$	0,35	0,33	(0,33, 0,34)	0,0001	0,0004
$(n_{100}, \sigma_{0.2}, D_{03})$	0,02	0,01	(0,01, 0,01)	0,0000	0,0001
$(n_{100}, \sigma_{0.2}, D_{04})$	0,00	0,00	(0,00, 0,00)	0,0000	0,0000
$(n_{100}, \sigma_{0.35}, D_{01})$	3,44	3,43	(3,43, 3,44)	0,0001	0,0012
$(n_{100}, \sigma_{0.35}, D_{02})$	0,53	0,51	(0,51, 0,51)	0,0002	0,0006
$(n_{100}, \sigma_{0.35}, D_{03})$	0,04	0,03	(0,03, 0,04)	0,0001	0,0002
$(n_{100}, \sigma_{0.35}, D_{04})$	0,00	0,00	(0,00, 0,00)	0,0000	0,0000

Table C.14: (Continued) Simulation and approximation results in the case of uniform distributed failure rates and mixed Erlang distribution

	$D_{E_p}^{A}$	$D_{E_p}^{S_u}$	Confidence interval of $D_{E_p}^{S_u}$	$avg\ G_u$	$max\ G_u$
$(n_{100}, \sigma_{0.5}, D_{01})$	3,97	3,97	(3,97, 3,99)	0,0000	0,0016
$(n_{100}, \sigma_{0.5}, D_{02})$	0,82	0,79	(0,79, 0,80)	0,0003	0,0008
$(n_{100}, \sigma_{0.5}, D_{03})$	0,10	0,08	(0,08, 0,09)	0,0002	0,0003
$(n_{100}, \sigma_{0.5}, D_{04})$	0,01	0,00	(0,00, 0,01)	0,0000	0,0001