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PHY 307

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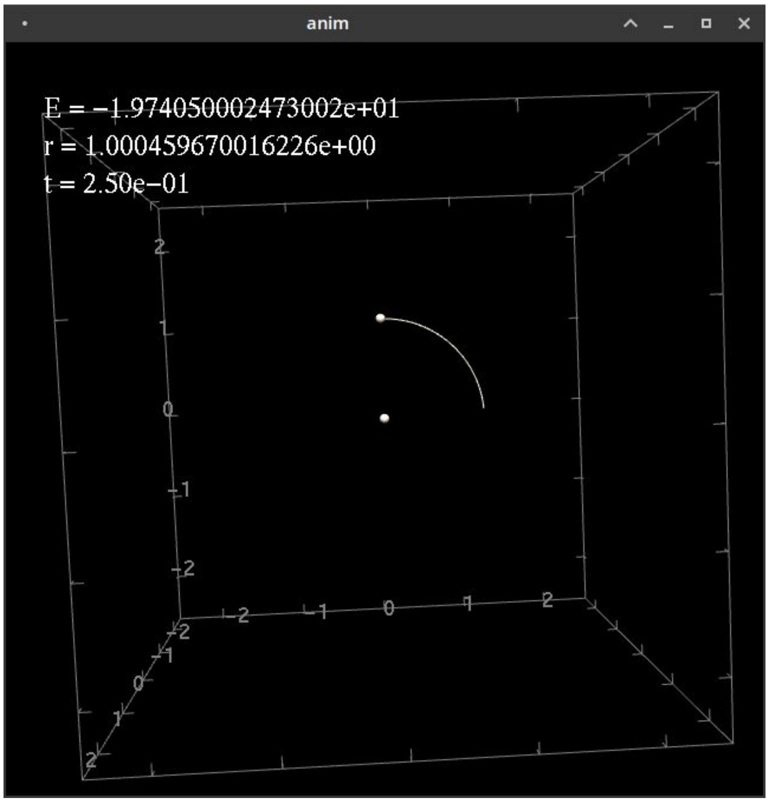
Project 5: Newtonian Gravity

The power in computational physics comes from its ability to simulate objects too small even for microscopes up to celestial bodies and even clusters of galaxies. In this project we explored the higher end of the scale and simulated the Earth in its orbit around the Sun. There are some simplifications that allow for better understanding of the simulation. For example, in this lab we measured distance in astronomical units (the average distance from the earth to the sun), time in years, and mass in solar masses. If the Earth travels in a perfectly circular orbit, it has a velocity of 2π AU per year. Letting the force of gravity on earth equal the centripetal force it experiences in this circular, and substituting these numbers in we find that G, Newton's gravitational constant, is equal to $4\pi^2$. These numbers are easier to grasp and understand than when using traditional units like meters and seconds.

To ensure our simulation is behaving as intended, we can run several checks to make sure everything is accurate. For example, if Earth is travelling in a circular orbit then its distance from the Sun should stay relatively fixed (the Sun moves slightly due to the gravitational force from Earth but this is negligible). The Earth should also complete one orbit in a year, as well as conserve energy. Printing out the distance between the Sun and Earth as well as the energy shows constant values when Earth starts at a position (1,0,0) with an initial velocity of $(0,2\pi,0)$, the initial conditions for a circular orbit. There is some variation due to the computational nature of the simulation, but this may be neglected. Letting the simulation run until t = 0.25 years shows the Earth stopping a quarter of the way into its orbit, so our simulation is behaving accurately.

¼ of the orbit is completed when t = 0.25s



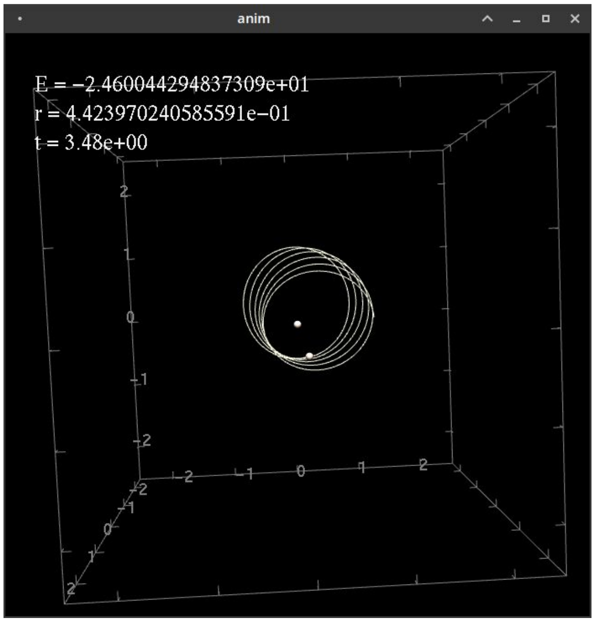
Next we began simulating different orbits a body could take. If we increase the initial velocity of Earth slightly, it begins to travel in an elliptical path, and if we increase the velocity even more Earth leaves the Sun's pull. The Earth was shown to move faster at the lowest point in its orbit, and slowest at the highest point. This makes sense when considering the conservation of energy.

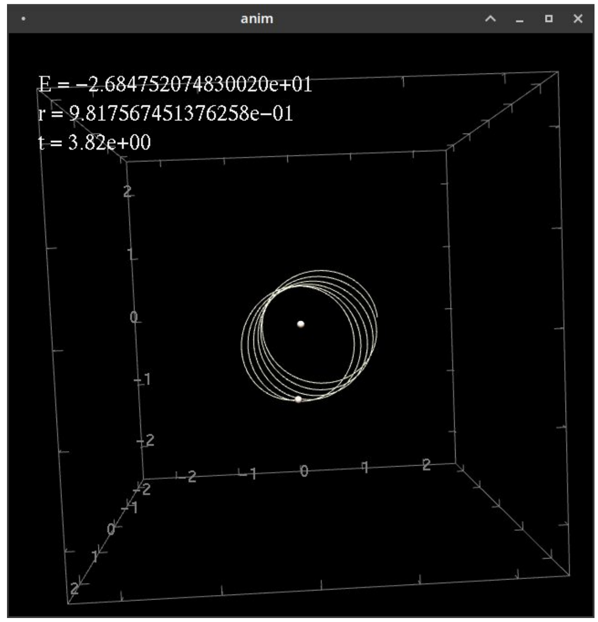
Because this is a simulation we are free to change any parameters we like and draw conclusions from what we see. As long as the specific orbital energy is negative, we will stay in orbit around the sun. The closer our orbit is to the sun, the more negative the specific energy gets, which is due to the fact that the gravitational potential energy is inversely proportional to the radius of the orbit. As our orbit gets bigger, and we increase our initial velocity making an almost hyperbolic orbit, the specific energy remains negative but is very small. If we continue to increase the initial velocity, the specific energy becomes positive and Earth ends up on an escape trajectory. These conclusions were drawn by changing the initial velocity and position of Earth.

We can change more than just the initial conditions however. What happens if Newton's Law of Gravity was just slightly different? Say, $GMm/r^{\wedge}(2+\delta)$ where δ is some small number whose value is near zero ($|\delta| > 0$)? Adding a δ of 0.1 to the gravitational force in the simulation causes Earth's orbit to 'rotate' around the Sun while Earth is in an elliptical orbit. Instead of travelling on the same path every orbit as before, the orbit rotates a little in the direction of the planet's motion. Subtracting δ from the exponent of the radius in the force equation causes a similar reaction but in the opposite direction. This orbital precession is shown below; both cases have the same initial conditions. Earth's orbit is counterclockwise to the camera.

Orbital Precession with $\delta > 0$

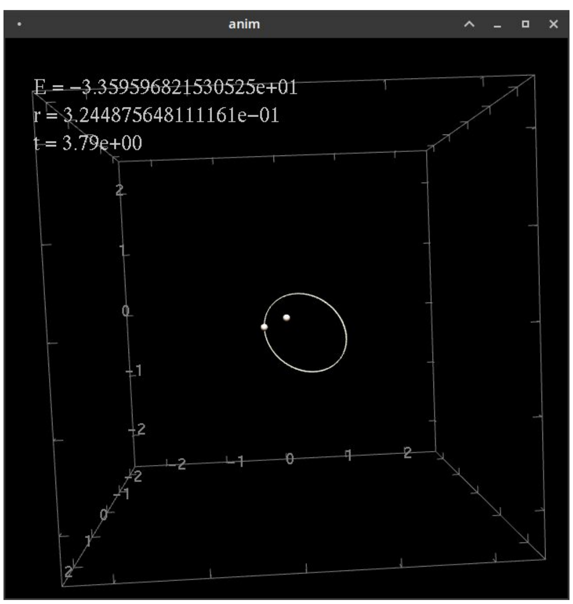
Orbital Precession with $\delta < 0$

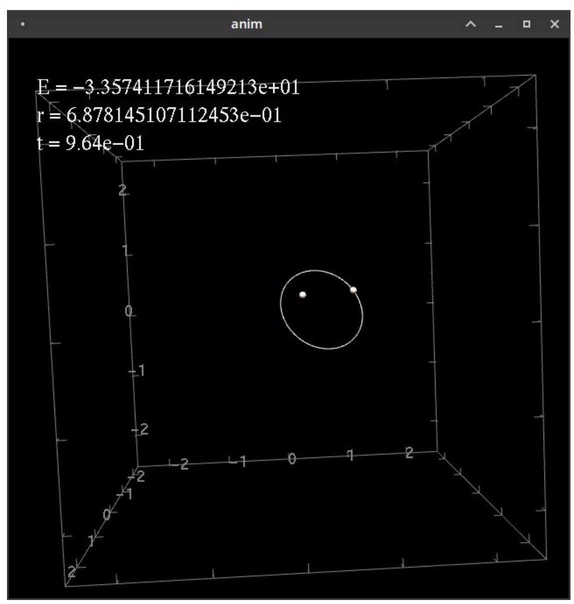




The final change I performed was changing the location of the Sun. After some testing, this is exactly analogous to moving the initial position of the planet in the opposite direction. They produce the same results.

Changing the initial position of the planet Changing position of the sun





The orbits are exactly the same. It may be notated the semi major-axis is no longer horizontal.

In summary this lab was a basic introduction to the simulation of celestial bodies, as well as a demonstration of the power of computational physics in allowing us to play with reality to see what would happen if things were different.