Extending Models of Deadlock in Queueing Networks
Baulking / Scheduled Vacations

1 Background

Consider the queueing network shown in Figure 1. This system has one server. Customers arrive randomly at a rate Λ per time unit. Customers are served in a First-In-First-Out discipline, and service times are random with mean $\frac{1}{\mu}$. Once a service is completed, there is a probability r_{11} of rejoining the queue, and so a probability $1 - r_{11}$ of leaving the system. (When we say randomly, we mean according to an Exponential distribution).

There is only enough room for n customers to wait in the queue. If the queue is full and a customer arrives from the outside, then that customer is turned away and lost. If the queue is full and a customer finishing service wishes to rejoin the queue then that customer is blocked, remains with the server until room becomes available. For this particular system, room won't become available until the blocked customer himself moves, and so that customer is in fact waiting for himself to move before he can move. This causes *deadlock*, when no more movement occurs.

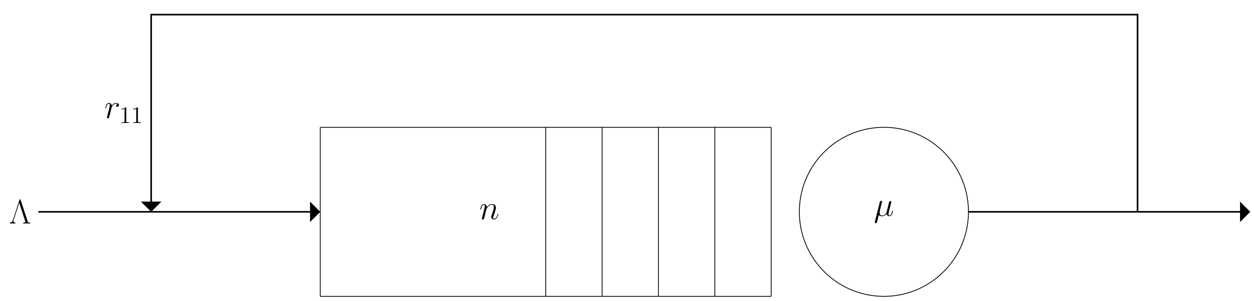

Figure 1: A one node queueing network that deadlocks.

Deadlocking systems like this one has been studied by us (please see reading material). This involved detecting when deadlock occurs in computer simulations of queueing network; building Markov chain models of such networks; and investigating the parameters' effects on the time until deadlock ω.

2 Queues with Baulking

In some cases, customers may choose not to join a queue if the queue is too long at the time of arrival, called baulking. This are usually modelled as a baulking function $b(n)$ which returns a probability of baulking when the customer finds n customers in the queue. These functions may take on a variety of forms, for example:

$$b(n) = \frac{n}{N}$$

$$b(n) = 1 - \frac{\beta}{n}$$

where N denotes the maximum queue size, and β is some parameter representing willingness to join the queue.

Now consider the deadlocking system in Figure 1, where arriving customers baulk. Let the time to deadlock of this system be denoted by $\hat{\omega}$. No work has been done on this system so far, and we would like you to investigate!

3 Queues with Scheduled Vacations

Imagine an online shop where orders can arrive to a queue at any time of the day or night. The shop hires one server who processes orders from 9am to 5pm, seven days a week. From 5pm to 9am next day, that server is off duty and no orders can be processed, however orders continue to arrive. That server is on a scheduled vacation. (Note: if the server is mid way through processing an order at 5pm, then he will complete that service before going off duty.) Let's call u the time on duty (9am - 5pm), and v the time on vacation (5pm - 9am).

Now consider the deadlocking system in Figure 1, where the server has scheduled vacations. Let the time to deadlock of this system be denoted by $\bar{\omega}$. No work has been done on this system so far, and we would like you to investigate!

4 Rough Plan

Week 1 (Baulking)

- Familiarise yourself with Markov Chains, deadlock work, and Python.
- Get lots of data using Ciw (various parameter sets, baulking functions parameters).
- Find analytical results for each parameter set (edit current Markov chain).

Week 2

- Using Ciw data already obtained, plot $\hat{\omega}$ and investigate the parameters' effect on $\hat{\omega}$ (similar to the plots in reading material, violin plots with analytical mean running through).
- Any other analysis that might arise: Is the behaviour intuitive / explainable? Do any behaviours change? Is there still a threshold? etc.

Week 3-4

- Do the same for vacations.
- Find relationship between ω and $\bar{\omega}$ / $\hat{\omega}$? (I have some ideas. Maybe use regression analysis. DIFFICULT)