

# Responses to Reviewers

We were very pleased to see that 2 of the 3 reviewers are in agreement and believe that our paper is now worthy of publication in EJOR. We would like to express our thanks to Reviewer 5 for further useful suggestions and remarks to improve the manuscript. The rest of this document will address specific issues raised by this remaining reviewer.

## Responses to Reviewer 5

- Reviewer 5 wrote:

“At the end of the Introduction, I would like to see a stronger and clearer statement of (what the authors see as) the main contributions of the paper. This should in essence answer the questions Why is this work important? Why does it merit publication in EJOR?”

A paragraph has been added that clearly summarises the importance and placement of this work in the literature.

- Reviewer 5 wrote:

“Now that (as it seems to me) Section 4 is the centrepiece of the work, the material here needs improvement. The proofs of Theorem 1 and Proposition 1 need some serious work to make them mathematically compelling.”

and

“Reading the proof of Theorem 1, it is not at all clear to me that you have proved the statement in the result. For a start, the statement is if and only if and it is not at all clear to me that you have the implication going both ways. There is no mention of strongly connected components in the proof and yet that is a key property of a knot. I want to be able to see absolutely clearly that you can start from Suppose a deadlock state arises at  $t$  and from there infer that  $D(t)$  must contain a knot and then repeat with the implications reversed.”

and

“So far as Proposition 1 is concerned, I assume that (starting from Theorem 1) the claim is that in the three examples of queueing networks listed the statements  $D(t)$  contains a knot and  $D(t)$  contains a weakly connected component without a sink node are equivalent (ie, are if and only if). Is that what you have proved? Please make it much clearer. As with Theorem 1, in the proofs the implications appear to go only one way, which is a concern. One further point, please explain the value of Proposition 1. Is it easier to check that  $D(t)$  has a weakly component without a sink node than that it contains a knot? Do you use Proposition 1 in any of your own analyses?”

The reviewer is completely correct. The Proofs of these have now been re written to follow the structure proposed.

Upon reflection we felt that Proposition 1 should be named Theorem 2 as it is not used in the rest of the analyses however could potentially be of use in applications of deadlock detection.

- Reviewer 5 wrote:

“You do not seem to have addressed my concerns about Section 6. I do not see any explanation given as to why what you have done here makes any useful contribution to the work. In Figure 14 you have already demonstrated an ability to compute the mean time to deadlock analytically for these systems. Presumably you can also estimate these quantities by simulation, using the ideas in Section 4. I imagine that you can also do the latter for rather larger and more complex networks than the ones discussed here. What value then is there in producing (what are often) poor bounds for these quantities? If the bounds were rather better, I could imagine that they might have a role in system design to (for example) ameliorate deadlock by identifying embedded sub-systems which are vulnerable to early deadlock. However, anything of that kind seems a long way off.”

and

“My own personal preference would be for a shorter paper which omits Section 6”

After a lot of reflection and discussion, we are in agreement with the reviewer. Section 6 has been removed. We agree that although interesting, the content in its current form does not have as much usefulness as other sections. We express our thanks to the reviewer as we feel that this particular suggestion greatly improves the overall presentation of the work and gives it better focus.

- Reviewer 5 wrote:

“p.1 Abstract: These models are compared to results obtained ..... I do not think this is quite what you mean;”

This has been replaced with: “The expected time to deadlock of these models are compared to results obtained...”.

- Reviewer 5 wrote:

“p.1 Last four lines of the first para of Section 1. Please reword from These deadlocks can be real..... to achieve greater clarity; ”

These sentences have been expanded.

- Reviewer 5 wrote:

“p.2 Definition 1 and material following. Please aim for greater clarity here. If you relate Figure 1 to Definition 1, what exactly is the subset of customers in the latter which is relevant here? Is it just  $A_{sub1}$  and  $B_{sub1}$ ?”

The following sentence has been included for clarity: “This system is in deadlock as there is a subset of blocked customers (the customer with server  $A_1$  and the customer with server  $B_1$ ) who are blocking eachother only.”

- Reviewer 5 wrote:

“p.3 bottom para to the end of Section 2. Please try to reword for clarity. This discussion could be clearer on the distinction between real-world deadlock and the model-based variety. It should also explain more clearly why the latter (which is the focus of the paper) matters;”

These last couple of paragraphs of this section have been reworded.

- Reviewer 5 wrote:

“pp4-6 The literature review is very long. If it is possible to lose some of the material toward the end of Section 3, that might help readability;”

The literature review has been made more concise.

- Reviewer 5 wrote:

“p.7 l.3 up In the definition of weakly connected does the either/or include both and? In other words, is a strongly connected vertex pair necessarily weakly connected?”

Yes a strongly connected component is also weakly connected. This has been made explicit in the definitions.

- Reviewer 5 wrote:

“p.10, l.2 up Should it not be ....to check whether any strongly connected component....”

Yes, as there will be many situations where there does not exist a strongly connected component in  $D(t)$ . This has been changed.

- Reviewer 5 wrote:

“p.11, l.11 You claim In this section.....expressions for their expected time to deadlock are found. What are you referring to here? I do not see any expressions unless you are claiming (5.1) meets this description.”

Expressions are not found, but a numerical method is presented that gives the times to deadlock. This has been clarified in the text.

- Reviewer 5 wrote:

“p.11, l.3up In (5.1) you give a formula for the expected number of time steps until absorption for a (discrete) Markov chain. Will you not need to uniformise your continuous chains to translate this easily into the expected time to deadlock? If so, should this be mentioned? How exactly did you compute the analytical mean times to deadlock in Figure 8 etc?”

The continuous time until deadlock is found by multiplying the expected number of steps until deadlock with the time step used in the discretisation process. This has now been clarified in the text.

- Reviewer 5 wrote:

“p.15, l.9 ....the rate at which  $i$  are (is?) reduced for most states.... Please explain. What does this mean?”

This sentence has been reworded.

- Reviewer 5 wrote:

“p.15 Are Remarks 1 and 2 mathematical certainties or empirical observations? If the former, can they be proved rigorously?”

These remarks are empirical observations, and have now been rewritten so that this fact is clearer. Future work would be to prove these remarks more rigorously.

- Reviewer 5 wrote:

“p.16, l.3 I think possibilities should be probabilities here;”

This sentence has been reworded.

- Reviewer 5 wrote:

“p.16, bottom In your description of state space  $S$ , the index applied to  $N$  should be 2 rather than the one given. The space  $S$  consists of pairs of integers. A similar comment applies on p.19. Also please rewrite the material following the characterisation of  $S$ , some of which seems confusing and contradictory;”

These have been fixed.

- Reviewer 5 wrote:

“p.18 (and later). Having commented extensively on the analytical/simulation results given in Figure 8, you say very little about the Figures 11 and 14.”

There is not much to say about Figure 11 as the behaviour observed here is the same as that observed in Figure 8. A sentence explaining this has been added. We feel that this fact is in itself interesting. A paragraph of discussion about Figure 14 has been added, explaining the phenomenon that the time to deadlock functions stabilise as parameters increase or decrease.