

# Responses to Reviewers

We would like to express our gratitude to the editor and reviewers for their useful suggestions and remarks on the manuscript. These recommendation resulted in a significant restructure of parts of the paper, along with many other stylistic and content changes throughout. These changes include:

- Restructure of the introduction.
- Inclusion of a motivating real world example.
- Rewritten definition of deadlock.
- Inclusion of further references.
- Improvements to diagrams and plots.
- Removal of superfluous Markov chain models.

The rest of this document will address specific concerns raised by the editor and reviewers.

## Responses to the Editor

- The editor wrote:

“Please ensure that you have cited recent and relevant publications in EJOR and other OR journals, to avoid the impression that your article would be misplaced in EJOR and to link your findings better to current OR trends (cf. Report #5).”

Some recent publications, including some from EJOR, have now been cited:

- “An analytic finite capacity queueing network model capturing the propagation of congestion and blocking”, Osorio, C. and Bierlaire, M., 2009.
- “Application of a simulation-based dynamic traffic assignment model”, Florian, M., Mahut, M. and Tremblay, N., 2008.
- “A sufficient condition for the liveness of weighted event graphs”, Marchetti, O. and Munier-Kordon, A., 2009.
- “Modeling recirculating conveyors with blocking”, Schmidt, L. C and Jackman, J., 2000.
- “Survey of research in the design and control of automated guided vehicle systems”, Vis, I.F.A., 2006.
- “From bed-blocking to delayed discharges: precursors and interpretations of a contested concept”, Manzano-Santaella, A., 2010.

- The editor wrote:

“I also would suggest to include a line of acknowledgement to the referees in your article.”

This has been included as the final section.

## Responses to Reviewer 1

- Reviewer 1 wrote:

“The Markov model presentation is quite repetitive and it could be simplified since the models are simple and the results can be easily related. I suggest the author to reduce Section 3 to avoid useless repetition of the models details and the observations on the model behaviour. The model definition and analysis is quite standard technique and there is no particular original contribution in this part of the work.”

Of the original five networks modelled, only three are now considered. The single server one node system was a special case of the multi server one node system. The single server two node system was a special case of the multi server two node system. The single server  $\Omega$  system was kept, along with multi server  $\Omega_1$  and  $\Omega_2$  systems, as discussion on these models are needed for the section on the bound on the expected time to deadlock.

- Reviewer 1 wrote:

“The bound definition could be interesting, however this section of the work is quite confused. It is not clear how to apply the bounds.”

This section has been expanded and rewritten. A table (Table 2) of values showing how the bound performs is now included. This should also help clarify the meaning of the bound. Further work could build on the ideas presented in this section, to derive bounds for larger networks that have not yet been (and maybe unable to be) modelled analytically. This note on further work is added to the conclusions section.

- Reviewer 1 wrote:

“It seems that the bound is defined just for one of the considered models, but it is not clear its applicability and accuracy”

The bound is only defined for model  $\Omega$ , in terms of  $\Omega_1$  and  $\Omega_2$ . However, a discussion of further work, to apply the ideas presented here to networks that are not modelled explicitly, is included in the conclusions. For the considered models, accuracy and tightness of the bound is analysed and discussed in this section.

- Reviewer 1 wrote:

“the figures are not readable for the state transition diagrams (Fig. 9, 12, 15, 18)”

and:

“the graphs of the expected time to deadlock cannot be read (Fig. 7, 10, 13, 16, 17, 23)”

The clarity of these diagrams and plots has been improved. We appreciate the reviewer pointing this out.

- Reviewer 1 wrote:

“the presentation of the bound definition of the models (p. 24) can be simplified.”

This section has been simplified.

## Responses to Reviewer 2

- Reviewer 2 wrote:

“There is no doubt that deadlock is a bad phenomenon in queuing networks. However, it is difficult to understand the importance of deadlock detection by applied researcher. Therefore, the author(s) should explain the importance of deadlock detection by using a concrete example. The author(s) should identify problems that occur because of deadlock in practice.”

and:

“To ensure the paper is reader friendly, the author(s) should indicate which operational problems can be solved by using this study's result.”

and:

“The case is not really made for the importance of blocking/deadlock in practice for the systems discussed. I have seen real-world instances and am personally convinced of the importance of these phenomena, but the paper has to make the case. In my view, the paper is crying out for a motivating real-world example;”

A motivational example is given in an introductory section. This example shows a potential relationship between community care and secondary care, and reflects the situation modelled as an  $\Omega_2$  system. Furthermore, an article is referenced in which potential deadlock is described as occurring in a real healthcare scenario, a Swiss hospital. We now emphasise that deadlock is a phenomenon and problem of the model, not of reality. In practice customers are swapped, but current modelling of circular Type I blocking does not account for this, and will deadlock. This emphasises the potential application of the work in developing simulation and analytical models that may exhibit deadlock despite the system itself not.

- Reviewer 2 wrote:

“Why does this study consider only Type 1 deadlock? The author(s) must indicate why this study does not consider deadlocks of Types 2 and 3.”

Although this paper does only consider blocking of Type I, consideration of Types II and III blocking are now noted as future research directions. It is noted that blocking of type III with random destinations (RS-RD) cannot reach deadlock, although could be considered a method of deadlock prevention.

- Reviewer 2 wrote:

“The author(s) should justify some assumptions. For example, why does this study pay attention to no pre-emption?”

In the literature review we state that there are four conditions that a system must satisfy for deadlock to occur. We then show that open restricted queueing network satisfy all four conditions. One of the conditions is no-preemption, and in queueing networks this is satisfied if there are either no priorities, or priorities with no-preemption. This work focuses on systems without priorities. This paragraph is clarified in the text.

- Reviewer 2 wrote:

“This paper has some references missing. For example, in [14], the journal name is mentioned as Operations Research. However, in [17] and [30], the journal name is mentioned as Operations research. The author(s) should check the format of the references.”

This has been addressed, we appreciate the reviewer taking the time to point these out.

## Responses to Reviewer 5

- Reviewer 5 wrote:

“Many of the terms used in the paper are not defined/explained with the clarity I would wish;”

- The introduction has been restructured, and now defines terms before using them.
- The definition of deadlock has been rewritten for clarity, and now does not use any undefined terms.
- A list of graph theoretical terms has been provided prior to discussion on graph theory.

- Reviewer 5 wrote:

“It is not made clear just how much of an advance the central result of Section 2 (Theorem 1) is on previous work. Is the construction in Definition 2 new, for example?”

and:

“The contribution of Section 2 needs to be more clearly set in the context of the foregoing literature. The suggestion on p.4 is that rather similar approaches (using wait-for graphs) have been used previously to analyse other systems. Please identify more clearly what is the novel contribution here;”

Yes the construction in Definition 2 is new. This method is a generic method for all FIFO queueing networks under Type I blocking, whereas previous work seems bespoke to the system concerned. New introductory sentences to this Section have been included to explain and clarify this and to put the work in context.

- Reviewer 5 wrote:

“I struggle to see the importance of much of the material in Sections 3 and 4. I would have thought that after the general discussion preceding Section 3.1 a single illustrative example would suffice. Further, while the bounding result in Section 4 seems correct, what is its significance and where does it leave us? Is it meant to point to a general approach to bounding expected times to deadlock in more complex systems? If this is the intention, the case does not seem to have been made. The bound proposed does not appear particularly tight. But more than that, no case is made regarding why and when such a bound would be of value.”

The bound isn’t very tight, and there is a discussion on its tightness and accuracy in this section. We have also included a table (Table 2) of example values to illustrate this. The ideas presented in the proof of the bound, however, could be used to develop bounds for systems that have not been modelled analytically, by considering the networks embedded within. Notes on this further work have been included in the conclusions.

The number of examples in the modelling section has been reduced from 5 to 3. Three remain as discussion on these three systems is required in order to define the bound.

- Reviewer 5 wrote:

“Abstract. It is not good to have a number of unfamiliar yet undefined terms (eg, restricted queueing network, recursive upstream blocking, deadlock etc) in an abstract.”

Some unfamiliar terms have been replaced, although some have remained. I feel that “open restricted queueing network”, will be familiar to readers interested in queueing networks. As this is the central

idea of the paper, I do not think that the abstract could be written without the word “deadlock”. The abstract would not be an appropriate place to define this term either.

- Reviewer 5 wrote:

“Further, the upper bound is on the expected time to deadlock. The time to deadlock is a random variable;”

The description of the bound has been updated to clarify that it is for the expected time to deadlock.

- Reviewer 5 wrote:

“Introduction: I found this very long and quite unclear in parts. While it collects together some interesting material, I feel it lacks clarity and direction. I strongly suggest that you begin with a clear and precise description of the phenomena of blocking/deadlock in the systems of interest to you. Explain why they matter. Then describe succinctly the contribution of the paper. Proceed to a review of the literature (or perhaps this could be a separate section?) and end with a summary of the contents of the paper;”

The introduction has been restructured, following this suggestion. Literature review is now a section of its own, a motivating example is provided. The structure of the introduction is now:

- Define “open restricted queueing networks”, and explain Type I blocking
- Define and explain deadlock.
- Explain notation used throughout paper
- Motivating example (separate section)
- Literature review (separate section)

- Reviewer 5 wrote:

“We really need to understand what blocking is and why you are taking the view of it you are. For example, in the one node single-server model of Section 3.1, why does the proposed -1 state mean deadlock for the system? In your deadlock state, why cannot the served customer to be rerouted to the queue when the waiting space is full take up the very space she vacates when she leaves service? Presumably your notion of blocking forbids this, but please explain clearly what your notions of blocking are and why they are natural/important for applications;”

The reviewer has touched on an excellent and important point. The fact that in reality a customer could take up the very space they vacate when they leave service in reality is of course important, however in a standard analytical model as well as most commercial simulation packages this would not happen: the model would become deadlocked. This is why an understanding of this phenomenon from a mathematical point of view is vital as it helps design modelling frameworks able to easily deal with deadlock. For example in the simulation library Ciw, all the theoretic deadlock detection methods have been implemented (based on the work presented here) and can be used by the user if they so choose. We appreciate the reviewer pointing out that this needed clarification and feel that the numerous amendments made to the paper ensure that this is now clear.

The  $-1$  state is simply used as a point of notation, any other symbol could be used.

- Reviewer 5 wrote:

“It would be great to include in the Introduction a motivating real-world example related to an open restricted queueing network;”

This has now been given as its own section, showing the relationship between community care and secondary care, and is in fact an  $\Omega_2$  system described here. Further, a article is referenced in which potential deadlock is described as occurring in a real healthcare scenario, a Swiss hospital. We now emphasise that deadlock is a phenomenon and problem of the model, not of reality. In practice customers are swapped, but current modelling of circular Type I blocking does not account for this, and will deadlock. This emphasises the potential application of the work in developing simulation and analytical models that may be constrained by deadlock despite the system itself often being able to recover from deadlock.

- Reviewer 5 wrote:

“There are several instances in the paper of using a term and only defining it later. For example, the terms open and restricted at the foot of p.1;”

This has been addressed when restructuring the introduction. Definitions for these terms are given at the beginning of the paper.

- Reviewer 5 wrote:

“Definition 1 is far from clear to me. What is recursive upstream blocking? Is the cessation of service referred to in the definition understood to be permanent? The term deadlock would rather imply so. Please clarify. Your reference on p.4 to different types of deadlock suggests to me that further clarification around this definition would be very helpful.”

The definition of deadlock has been rewritten for clarity, and to avoid using undefined terms. It now reads: “When there is a subset of blocked customers who are blocked directly or indirectly by customers in that subset only, then the system is said to be in deadlock.” The text also now explains that deadlock implies permanent cessation of services.

- Reviewer 5 wrote:

“On p.3 l.4 up, what is  $B_j$ ?”

$B_j$  is the total capacity of each node in the network. It corresponds to  $n_j + c_j$  when using the notation of this paper. This has been clarified in the text.

- Reviewer 5 wrote:

“The discussion around Figure 2 is far from clear;”

The discussion on this figure has been rewritten and hopefully this clarifies the differences between the definitions of deadlocks defined in this paper, and the deadlocks defined by those authors.

- Reviewer 5 wrote:

“Much of the graph-theoretic terminology used in Section 2 (including weakly and strongly connected components) will be unfamiliar to many readers. What is a wait-for graph? These and some other key terms need to be defined/explained;”

More clarification of wait-for graphs, and the contribution of this paper, is given, with new introductory sentences before defining the state digraph. In addition, a list of graph theoretical terms has been provided before discussion on the digraph begins.

- Reviewer 5 wrote:

“On p.8 you say that the simulation model described is there to verify the results of Section 3. In fact, the latter concern the mean time to deadlock while the simulation results you report give a much richer description of the distributions of the time to deadlock (based on order statistics, it appears-thus possibly not explicitly including the mean);”

This is correct, much more information of the distribution of the time to deadlock is available from the simulation results. This is clarified in the text. The simulation results and analytical results are used to verify one another.

- Reviewer 5 wrote:

“In your description on p.9 of the Markov chain modelling, you begin by discussing transition rates but later down the page appear to convert to transition probabilities (the canonical form of an absorbing Markov Chain). Since transition rates are the key model descriptors you use throughout, I suggest you stick to those;”

Previously there was a confusing duplication of terminology: ‘Transition probabilities’ were used to describe both the probability of a customer joining another node after service, and also the transition probabilities of a Markov chain. We have now used ‘Routing probabilities’ to denote the first case, and retained ‘Transition probabilities’ to denote the second case.

- Reviewer 5 wrote:

“P.12 I believe you need to apply the function  $\omega_1$  to both sides of the inequalities in 1.3 of Remark 2;”

Yes, this has been addressed.

- Reviewer 5 wrote:

“P.12, 1.3 up Here (and elsewhere) the superscript of  $\mathbb{N}$  (the natural numbers) is used incorrectly;”

In the description of the state space of the two server system, the following was changed:  $\mathbb{N}^{(n_1+2 \times n_2+2)}$  is replaced with  $\mathbb{N}^{(n_1+2) \times (n_2+2)}$ .