Dynamic Priority Classes (?)

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1 The Model

Here we consider an $M/M/c$ queue with K classes of customer. Order and label the customer classes $0, 1, 2, \dots, k$, with customer classes with lower labels having priority over customer classes of higher labels. The index k will be used to represent customer classes. Let:

- λ_k be the arrival rate of customers of class k,
- μ_k be the service rate of customers of class k,
- $\theta_{k_i k_j}$ be the rate at which customers of class k_i change to customers of class k_j.

Figure 1 shows an example with two classes of customer.

2 Markov Chain Formulation

Let $\underline{\mathbf{s}}_t = (s_0 t, s_1 t, \dots, s_K t) \in \mathbb{R}^K$ represent the state of the system at time step t, where $s_k t$ represents the number of customers of class k present at time step t.

Then the rates of change between $\underline{\mathbf{s}}_t$ and $\underline{\mathbf{s}}_{t+1}$ are given by Equation 1, where $\underline{\delta} = \underline{\mathbf{s}}_t - \underline{\mathbf{s}}_{t+1}$,

$$q_{\underline{\mathbf{s}}_t, \underline{\mathbf{s}}_{t+1}} = \begin{cases} \lambda_k & \text{if } \delta_k = 1 \text{ and } \delta_i = 0 \;\forall\; i \neq k \\ \min(s_k t, c)\mu_k & \text{if } \delta_k = 1 \text{ and } \delta_i = 0 \;\forall\; i \neq k \text{ and } s_{it} = 0 \;\forall\; i < k \\ W_{kt}\theta_{k_0 k_1} & \text{if } \delta_{k_0} = -1 \text{ and } \delta_{k_1} = 1 \text{ and } \delta_i = 0 \;\forall\; i \notin (k_0, k_1) \end{cases} \tag{1}$$

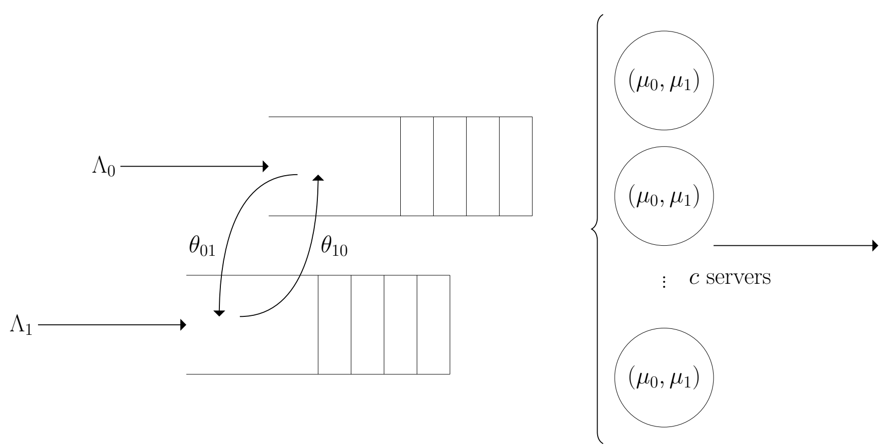

Figure 1: An example of a two-class priority queue.

and W_{kt}, representing the number of customers present but not in service at time step t, is given by Equation 2.

$$W_{kt} = s_{kt} - \min\left(c - \sum_{i<k} s_{it}, s_{kt}\right) \tag{2}$$