

# Dynamic Priority Classes (?)

authors (?)

## 1 The Model

Here we consider an  $M/M/c$  queue with  $K$  classes of customer. Order and label the customer classes  $0, 1, 2, \dots, k$ , with customer classes with lower labels having priority over customer classes of higher labels. The index  $k$  will be used to represent customer classes. Let:

- $\lambda_k$  be the arrival rate of customers of class  $k$ ,
- $\mu_k$  be the service rate of customers of class  $k$ ,
- $\theta_{k_i k_j}$  be the rate at which customers of class  $k_i$  change to customers of class  $k_j$ .

Figure 1 shows an example with two classes of customer.

## 2 Markov Chain Formulation

Let  $\underline{s}_t = (s_0 t, s_1 t, \dots, s_K t) \in \mathbb{R}^K$  represent the state of the system at time step  $t$ , where  $s_k t$  represents the number of customers of class  $k$  present at time step  $t$ .

Then the rates of change between  $\underline{s}_t$  and  $\underline{s}_{t+1}$  are given by Equation 1, where  $\underline{\delta} = \underline{s}_t - \underline{s}_{t+1}$ ,

$$q_{\underline{s}_t, \underline{s}_{t+1}} = \begin{cases} \lambda_k & \text{if } \delta_k = 1 \text{ and } \delta_i = 0 \forall i \neq k \\ B_{kt} \mu_k & \text{if } \delta_k = 1 \text{ and } \delta_i = 0 \forall i \neq k \text{ and } \sum_{i < k} s_{it} < c \\ (s_k - B_{kt}) \theta_{k_0 k_1} & \text{if } \delta_{k_0} = -1 \text{ and } \delta_{k_1} = 1 \text{ and } \delta_i = 0 \forall i \notin (k_0, k_1) \end{cases} \quad (1)$$

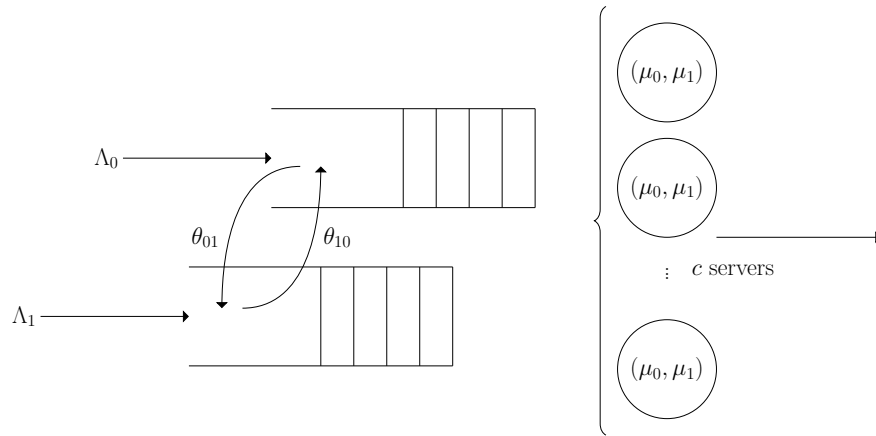


Figure 1: An example of a two-class priority queue.

and  $B_{kt}$ , representing the number of customers of class  $k$  currently in service at time step  $t$ , is given by Equation 2.

$$B_{kt} = \min \left( c - \min \left( \sum_{i < k} s_{it}, c \right), s_{kt} \right) \quad (2)$$