

# Queueing Networks for a Healthcare System

## Deadlocking & Reinforcement Learning

Geraint Palmer  
Paul Harper, Vincent Knight

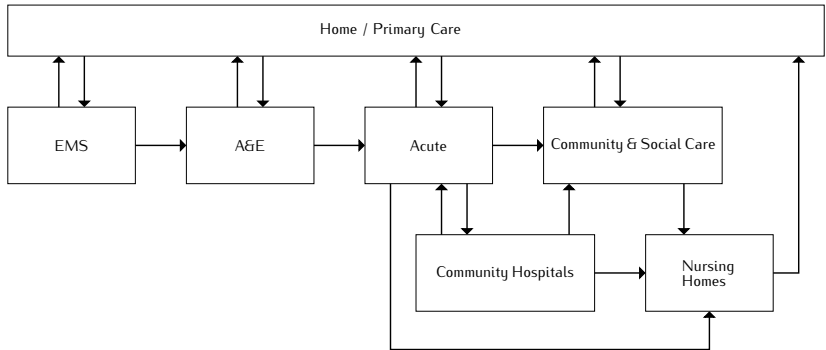
Young OR 19 – Aston University 2015



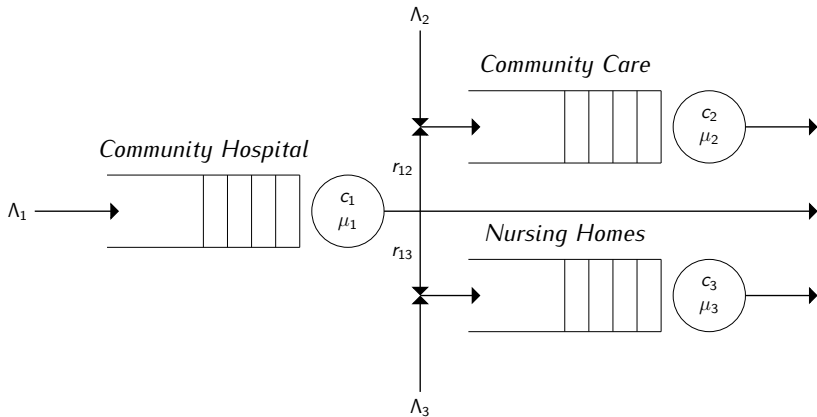
# Aneurin Bevan University Health Board



# Map of Healthcare System

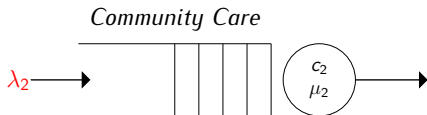


# Jackson Networks



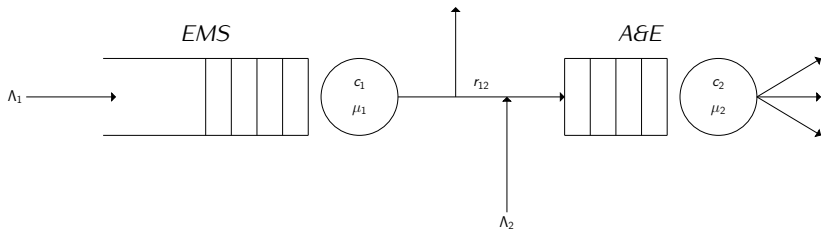
# Jackson Networks

$$\lambda_i = \Lambda_i + \sum_j r_{ji} \lambda_j$$



$$P(k_1, k_2, \dots, k_M) = \prod_{i=1}^M P_i(k_i)$$

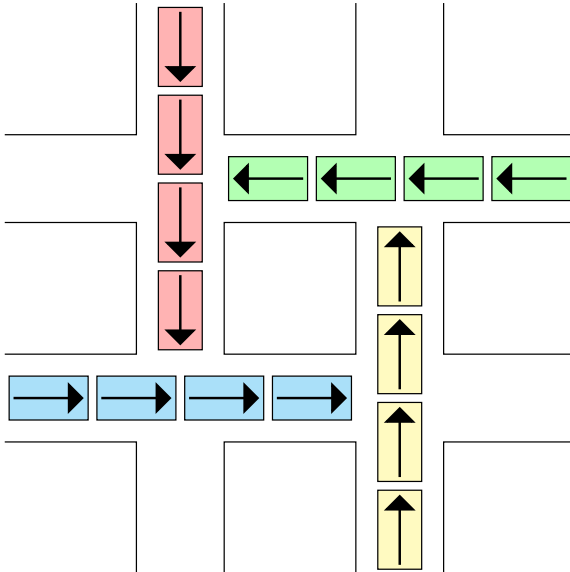
# Restricted Networks



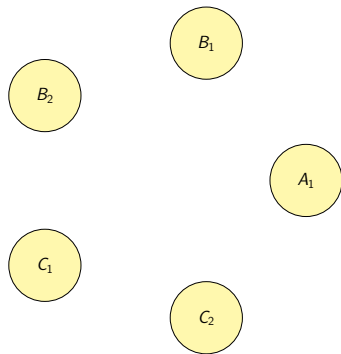
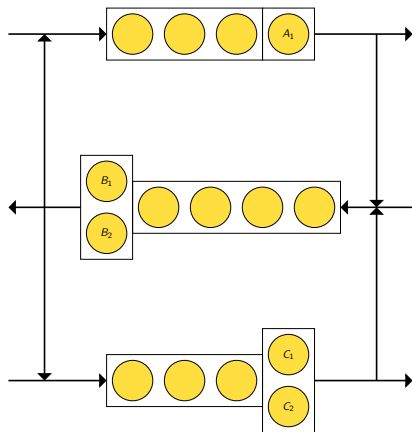
# Restricted Networks

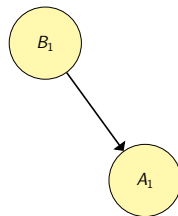
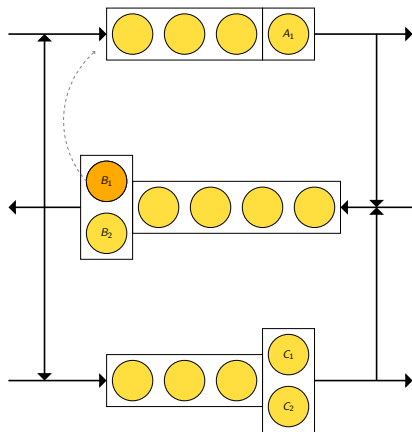
- Markov Chain Models
- Approximation Methods
- Simulation

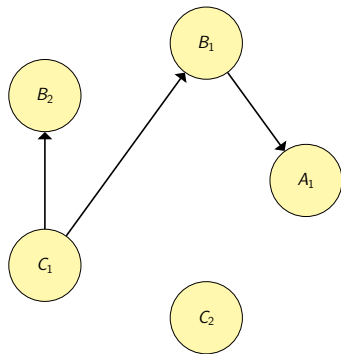
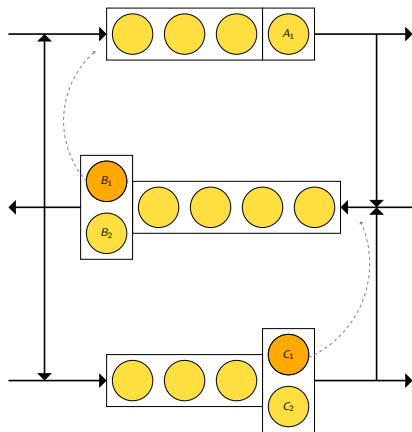
# Deadlock

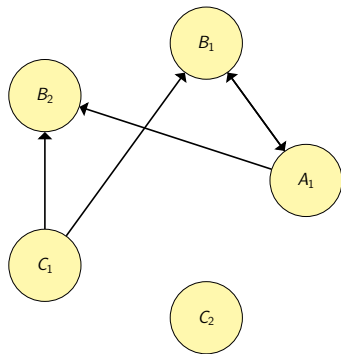
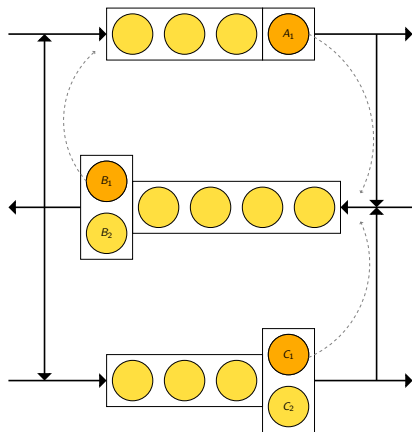


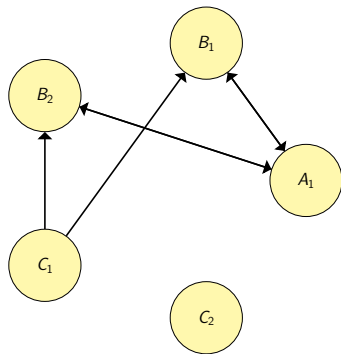
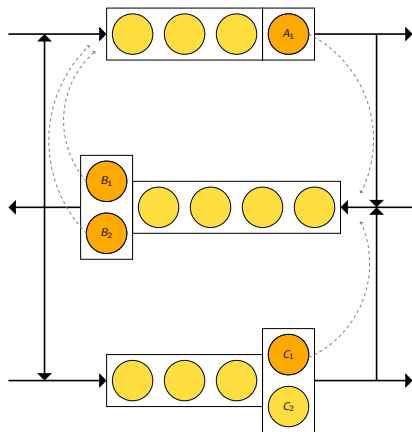


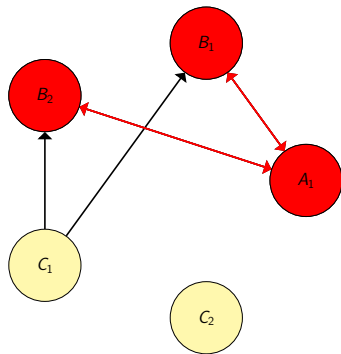
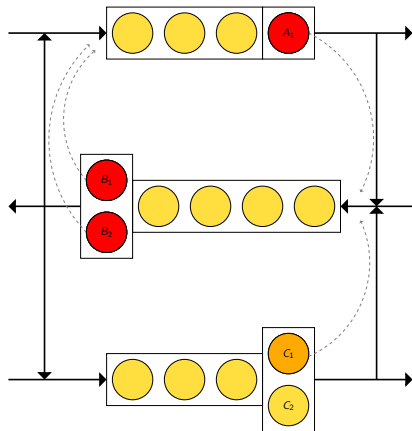






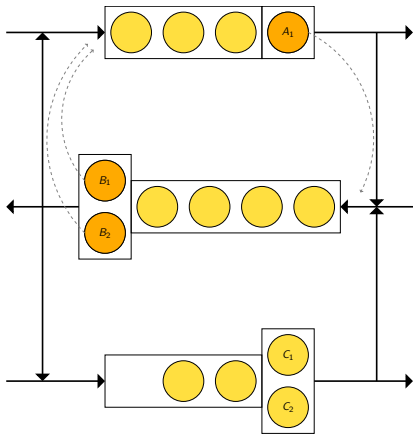




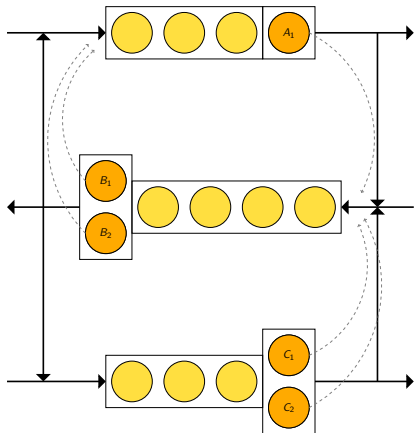


# Types of Deadlock

Transient Deadlock

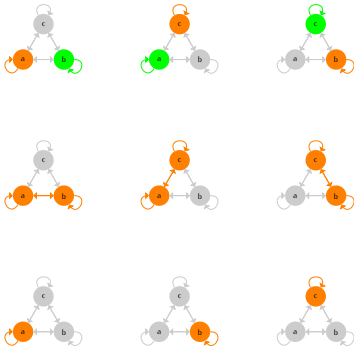


Absorbing Deadlock

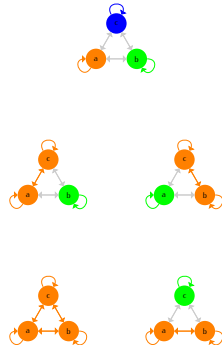


# Deadlock Configurations

## Transient Deadlock

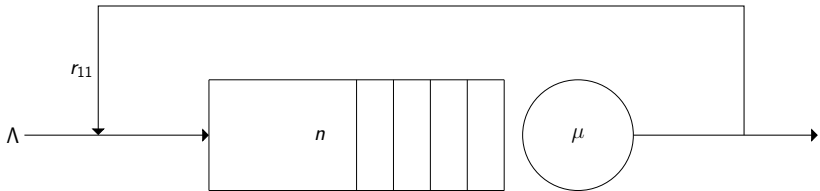


## Absorbing Deadlock





# Markovian Model of Deadlock



(i)

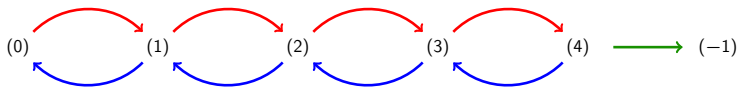
$$S = \{i \in \mathbb{N} | 0 \leq i \leq n+1\} \cup \{-1\}$$

$$\text{Define } \delta = i_2 - i_1$$

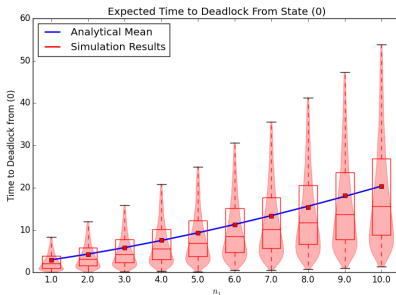
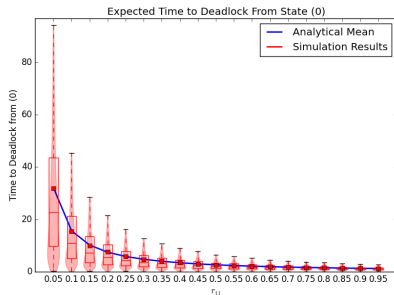
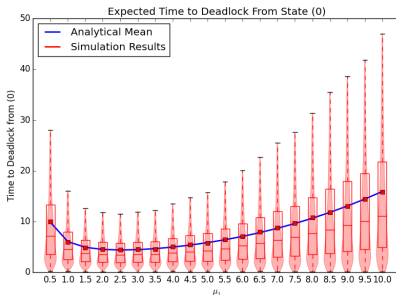
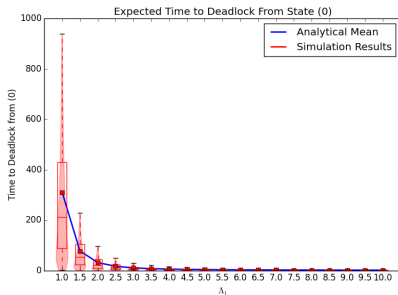
$$q_{i_1, i_2} = \begin{cases} \left\{ \begin{array}{ll} \Lambda & \text{if } i < n+1 \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = 1 \\ \left\{ \begin{array}{ll} (1 - r_{11})\mu & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{array} \right\} & \end{cases}$$

$$q_{i, -1} = \begin{cases} r_{11}\mu & \text{if } i = n+1 \\ 0 & \text{otherwise} \end{cases}$$

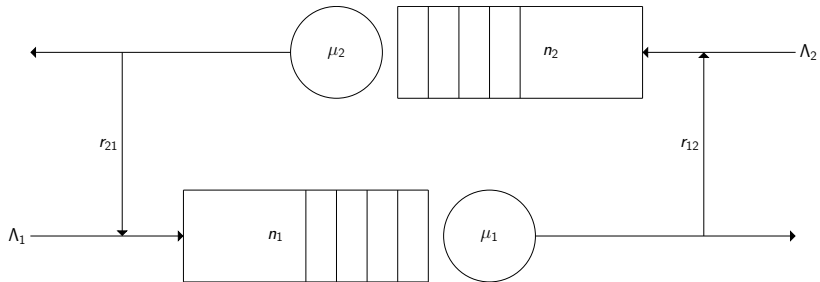
$$q_{-1, s} = 0$$



# Times to Deadlock



# Markovian Model of Deadlock



$(i, j)$

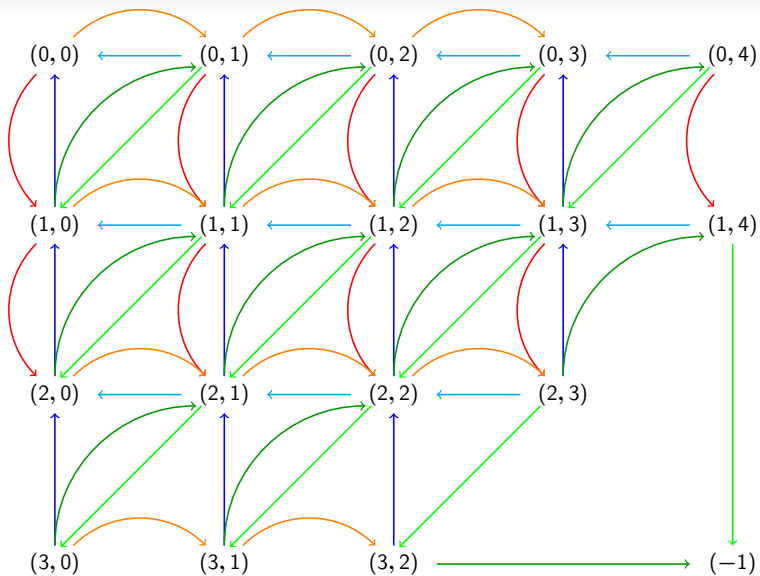
$$S = \{(i, j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} | 0 \leq i + j \leq n_1 + n_2 + 2\} \cup \{(-1)\}$$

$$\text{Define } \delta = (i_2, j_2) - (i_1, j_1)$$

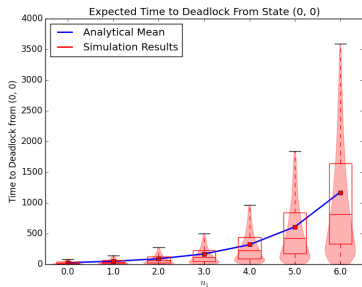
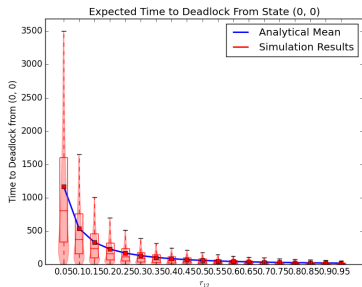
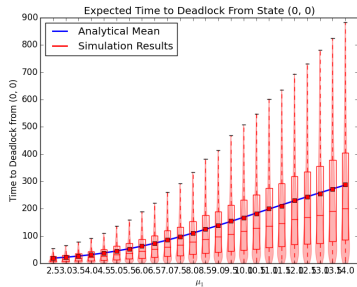
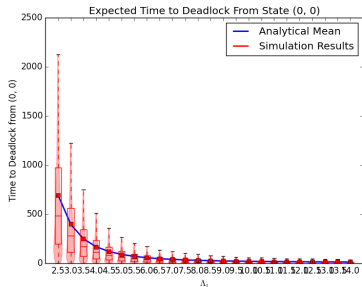
$$q_{(i_1, j_1), (i_2, j_2)} = \left\{ \begin{array}{ll} \left. \begin{array}{l} \Lambda_1 \quad \text{if } i_1 \leq n_1 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (1, 0) \\ \left. \begin{array}{l} \Lambda_2 \quad \text{if } j_1 \leq n_2 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (0, 1) \\ \left. \begin{array}{l} (1 - r_{12})\mu_1 \quad \text{if } j_1 < n_2 + 2 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (-1, 0) \\ \left. \begin{array}{l} (1 - r_{21})\mu_2 \quad \text{if } i_1 < n_1 + 2 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (0, -1) \\ \left. \begin{array}{l} r_{12}\mu_1 \quad \text{if } j_1 < n_2 + 2 \text{ and } (i_1, j_1) \neq (n_1 + 2, n_2) \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (-1, 1) \\ \left. \begin{array}{l} r_{21}\mu_2 \quad \text{if } i_1 < n_1 + 2 \text{ and } (i_1, j_1) \neq (n_1, n_2 + 2) \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (1, -1) \\ 0 & \text{otherwise} \end{array} \right.$$

$$q_{(i_1, j_1), (-1)} = \left\{ \begin{array}{ll} r_{21}\mu_2 & \text{if } (i, j) = (n_1, n_2 + 2) \\ r_{12}\mu_1 & \text{if } (i, j) = (n_1 + 2, n_2) \\ 0 & \text{otherwise} \end{array} \right.$$

$$q_{-1, s} = 0$$

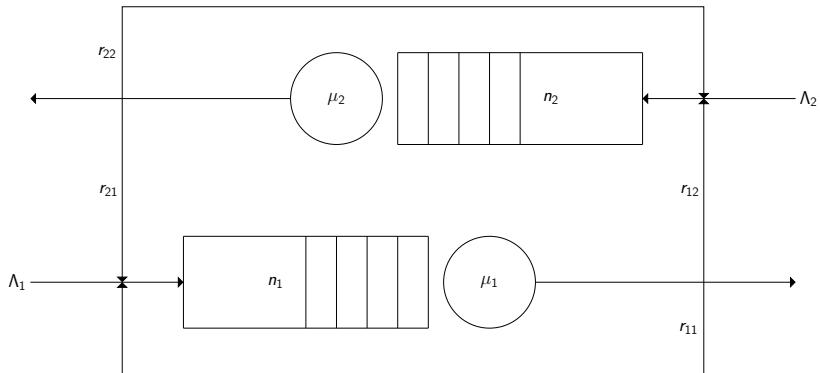


# Times to Deadlock





# Markovian Model of Deadlock



$(i, j)$

$$S = \{(i, j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} | 0 \leq i + j \leq n_1 + n_2 + 2\} \cup \{(-1)\}$$

$$\text{Define } \delta = (i_2, j_2) - (i_1, j_1)$$

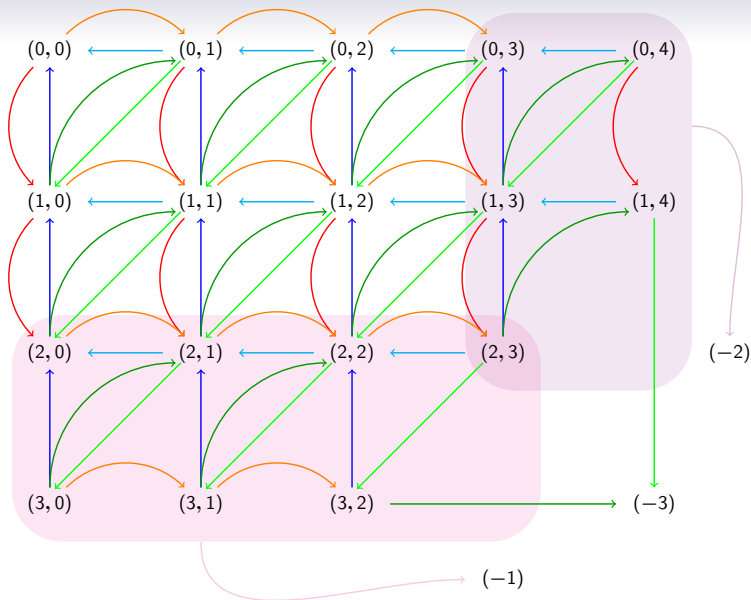
$$q_{(i_1, j_1), (i_2, j_2)} = \left\{ \begin{array}{ll} \left. \begin{array}{l} \Lambda_1 \quad \text{if } i_1 \leq n_1 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (1, 0) \\ \left. \begin{array}{l} \Lambda_2 \quad \text{if } j_1 \leq n_2 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (0, 1) \\ \left. \begin{array}{l} (1 - r_{12})\mu_1 \quad \text{if } j_1 < n_2 + 2 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (-1, 0) \\ \left. \begin{array}{l} (1 - r_{21})\mu_2 \quad \text{if } i_1 < n_1 + 2 \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (0, -1) \\ \left. \begin{array}{l} r_{12}\mu_1 \quad \text{if } j_1 < n_2 + 2 \text{ and } (i_1, j_1) \neq (n_1 + 2, n_2) \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (-1, 1) \\ \left. \begin{array}{l} r_{21}\mu_2 \quad \text{if } i_1 < n_1 + 2 \text{ and } (i_1, j_1) \neq (n_1, n_2 + 2) \\ 0 \quad \text{otherwise} \end{array} \right\} & \text{if } \delta = (1, -1) \\ 0 & \text{otherwise} \end{array} \right.$$

$$q_{(i_1, j_1), (-1)} = \left\{ \begin{array}{ll} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2 + 2 \\ 0 & \text{otherwise} \end{array} \right.$$

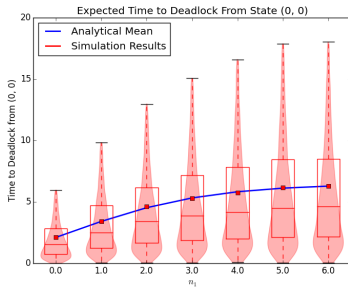
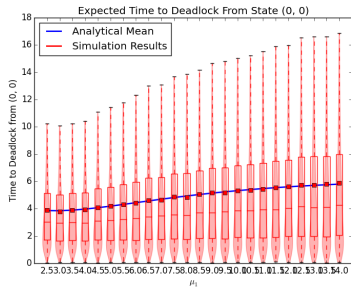
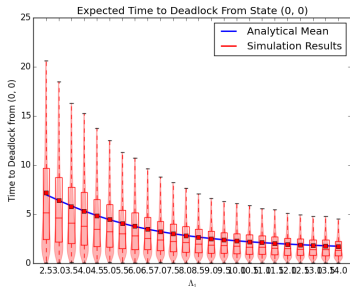
$$q_{(i_1, j_1), (-2)} = \left\{ \begin{array}{ll} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1 + 2 \\ 0 & \text{otherwise} \end{array} \right.$$

$$q_{(i_1, j_1), (-3)} = \left\{ \begin{array}{ll} r_{21}\mu_2 & \text{if } (i, j) = (n_1, n_2 + 2) \\ r_{12}\mu_1 & \text{if } (i, j) = (n_1 + 2, n_2) \\ 0 & \text{otherwise} \end{array} \right.$$

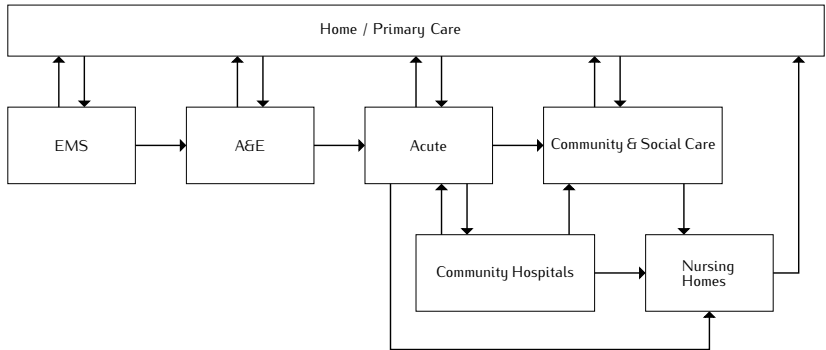
$$q_{-1, s} = q_{-2, s} = q_{-3, s} = 0$$



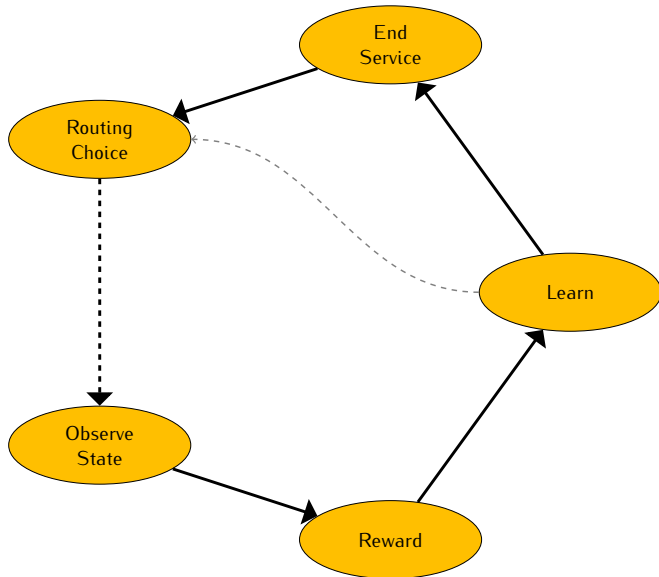
# Times to Deadlock



# Reinforcement Learning



# Reinforcement Learning



# Reinforcement Learning

## Q-Learning

$$Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$$

# Reinforcement Learning

$s_n$	$a_1$	$a_2$	$a_3$
$n = 0$	10.2	3.3	-2.1
$n = 1$	9.4	2.1	-0.7
$n = 2$	9.5	3.8	1.6
$n = 3$	8.3	5.4	5.3
$n = 4$	3.1	9.2	6.7
$n = 5$	4.0	6.1	6.7
$n = 6$	0.2	6.3	7.5

Table: Example table of  $Q(s, a)$



# Diolch - Thank You

<https://github.com/geraintpalmer/Presentations>  
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