

Deadlock in Queueing Networks

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SWORDS - 2015



Generic Queueing Networks

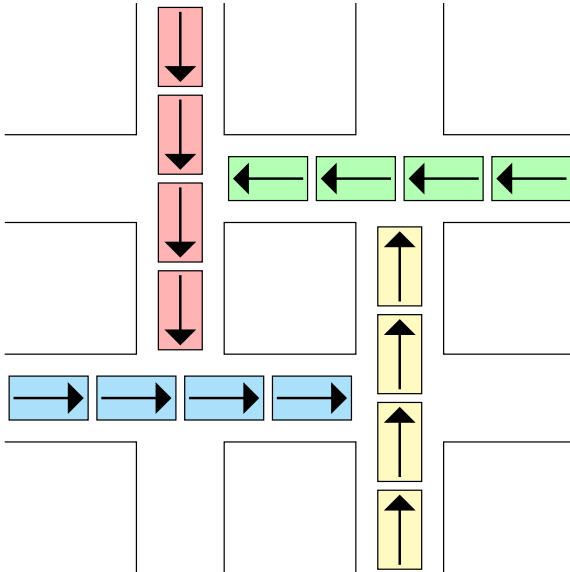
Open Unrestricted Queueing Networks

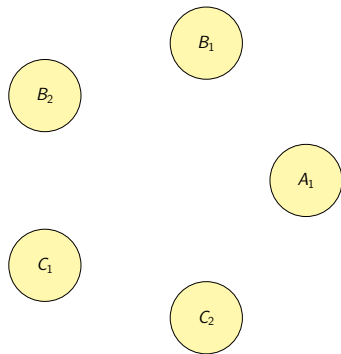
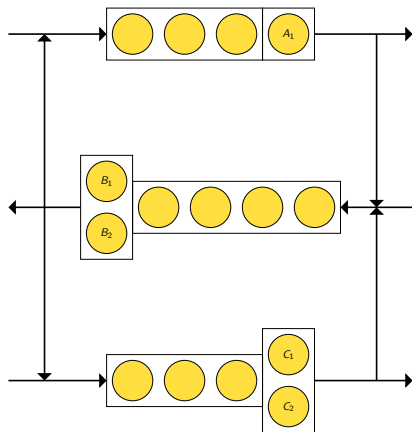
- Simple analytically
- Jackson networks
- Product-form solution

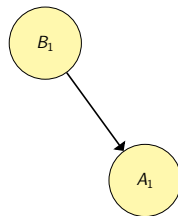
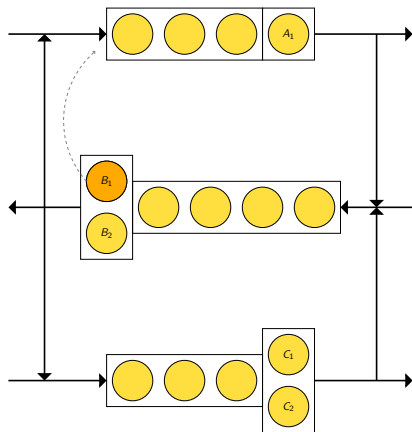
Open Restricted Queueing Networks

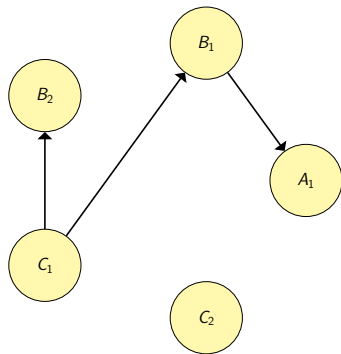
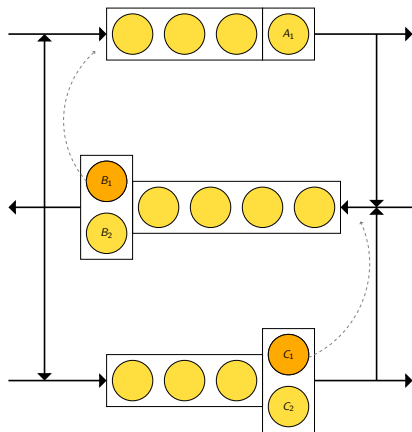
- Markov chain models
- Approximation methods
- Simulation

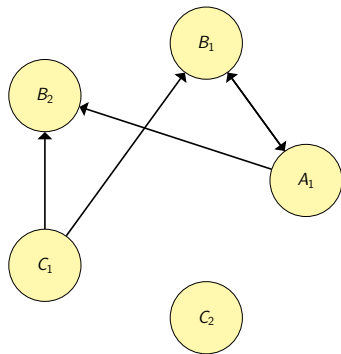
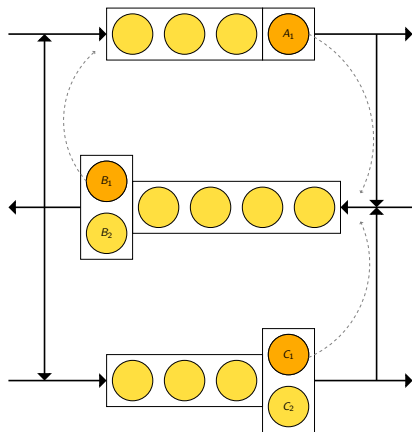
Deadlock

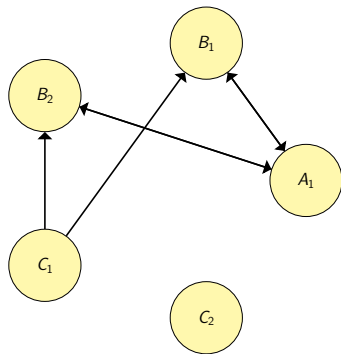
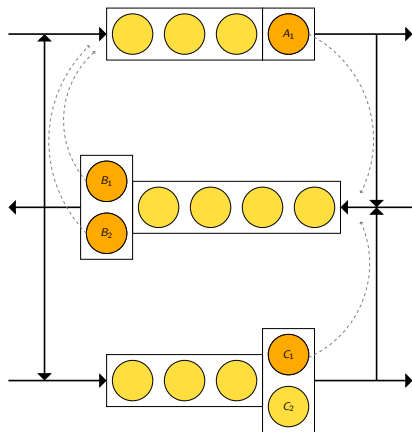


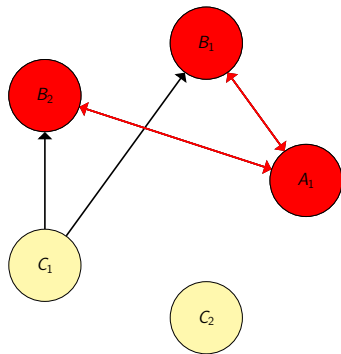
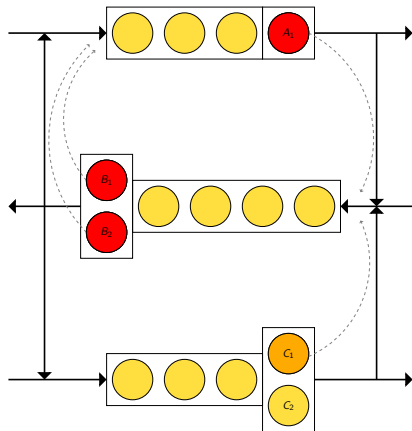




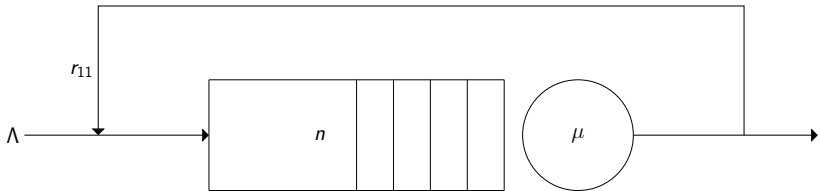








Markovian Model of Deadlock



(i)

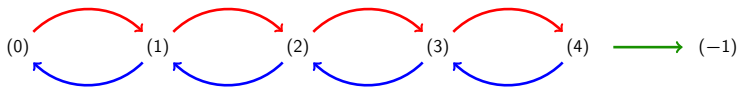
$$S = \{i \in \mathbb{N} \mid 0 \leq i \leq n+1\} \cup \{-1\}$$

$$\text{Define } \delta = i_2 - i_1$$

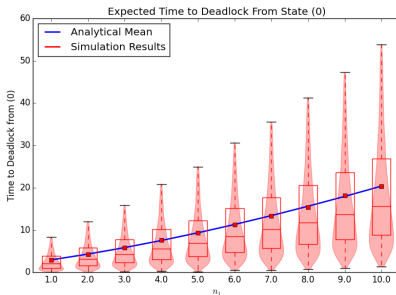
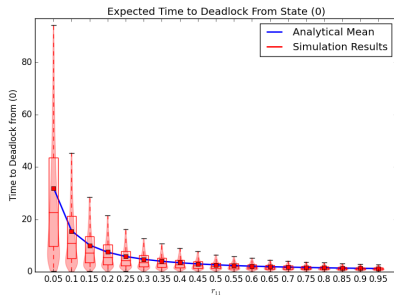
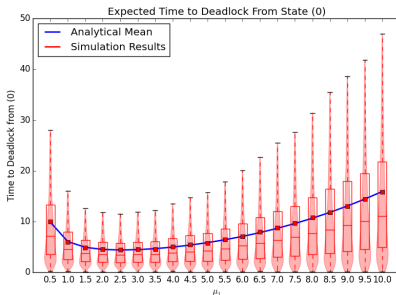
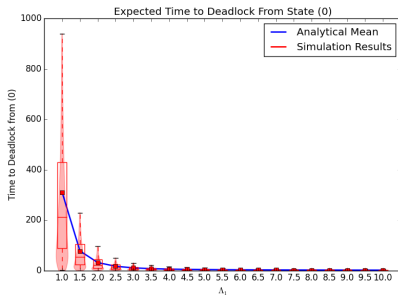
$$q_{i_1, i_2} = \begin{cases} \left\{ \begin{array}{ll} \Lambda & \text{if } i < n+1 \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = 1 \\ \left\{ \begin{array}{ll} (1 - r_{11})\mu & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{array} \right\} & \end{cases}$$

$$q_{i, -1} = \begin{cases} r_{11}\mu & \text{if } i = n+1 \\ 0 & \text{otherwise} \end{cases}$$

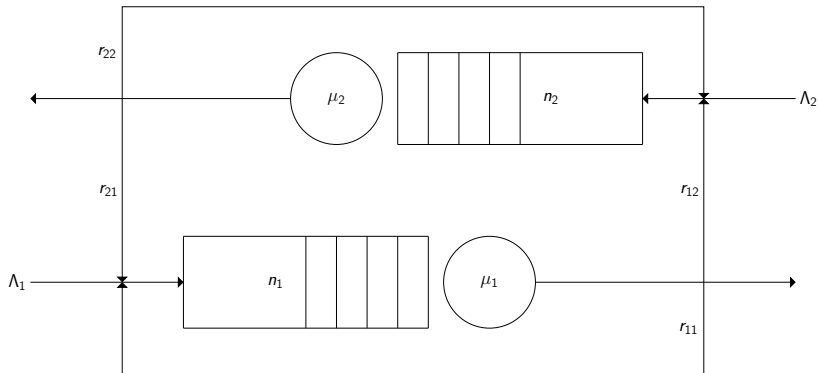
$$q_{-1, s} = 0$$



Times to Deadlock



Markovian Model of Deadlock



(i, j)

$$S = \{(i, j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} \mid 0 \leq i + j \leq n_1 + n_2 + 2\} \cup \{(-1)\}$$

$$\text{Define } \delta = (i_2, j_2) - (i_1, j_1)$$

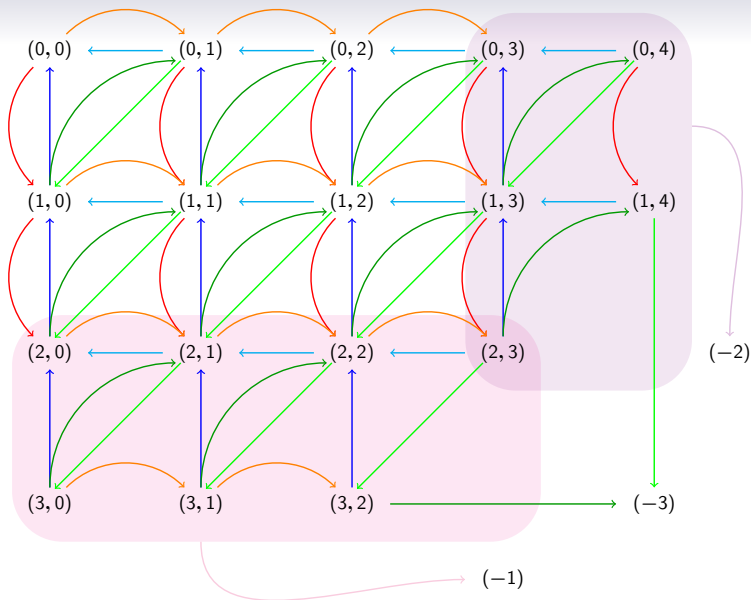
$$q_{(i_1, j_1), (i_2, j_2)} = \begin{cases} \left. \begin{array}{ll} \left. \begin{array}{ll} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = (1, 0) \\ \left. \begin{array}{ll} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = (0, 1) \\ \left. \begin{array}{ll} (1 - r_{12})\mu_1 & \text{if } j_1 < n_2 + 2 \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = (-1, 0) \\ \left. \begin{array}{ll} (1 - r_{21})\mu_2 & \text{if } i_1 < n_1 + 2 \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = (0, -1) \\ \left. \begin{array}{ll} r_{12}\mu_1 & \text{if } j_1 < n_2 + 2 \text{ and } (i_1, j_1) \neq (n_1 + 2, n_2) \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = (-1, 1) \\ \left. \begin{array}{ll} r_{21}\mu_2 & \text{if } i_1 < n_1 + 2 \text{ and } (i_1, j_1) \neq (n_1, n_2 + 2) \\ 0 & \text{otherwise} \end{array} \right\} & \text{if } \delta = (1, -1) \\ 0 & \text{otherwise} \end{array} \right\}$$

$$q_{(i_1, j_1), (-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2 + 2 \\ 0 & \text{otherwise} \end{cases}$$

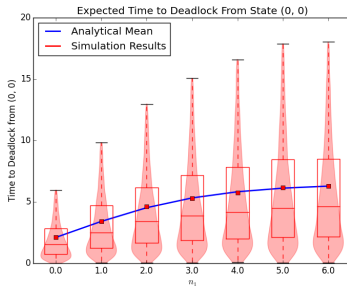
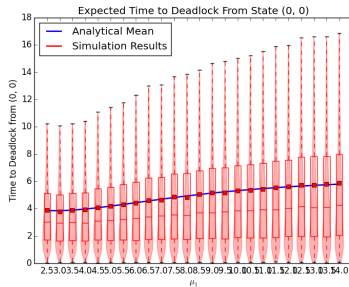
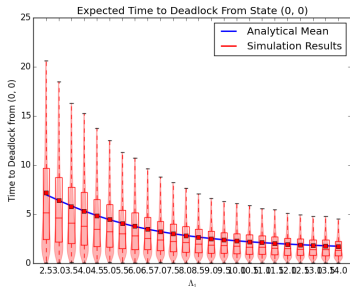
$$q_{(i_1, j_1), (-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1 + 2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1, j_1), (-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i, j) = (n_1, n_2 + 2) \\ r_{12}\mu_1 & \text{if } (i, j) = (n_1 + 2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1, s} = q_{-2, s} = q_{-3, s} = 0$$



Times to Deadlock



Diolch - Thank You

<https://github.com/geraintpalmer/Presentations>
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