# Deadlock in Open Restricted Queueing Networks

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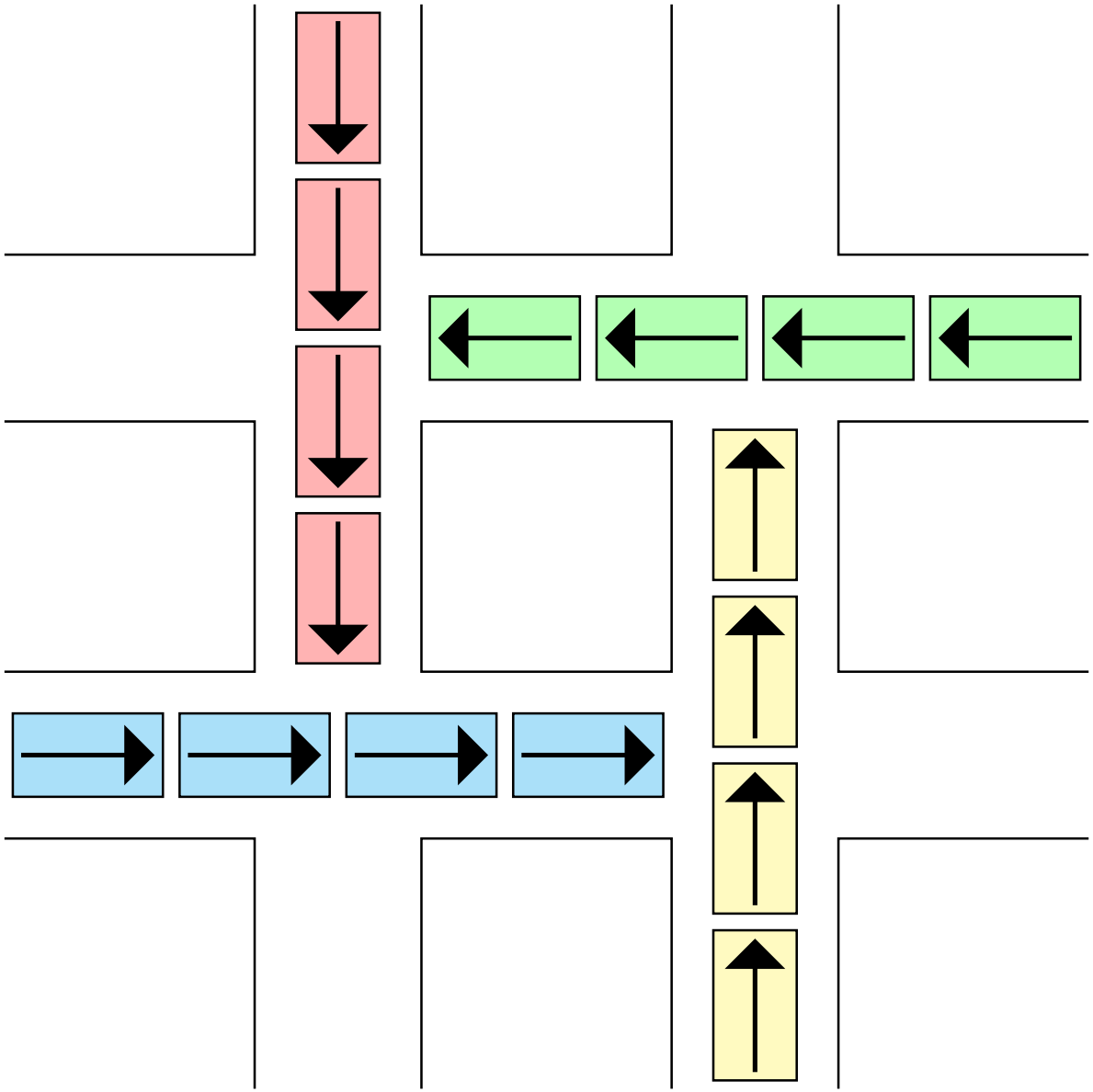
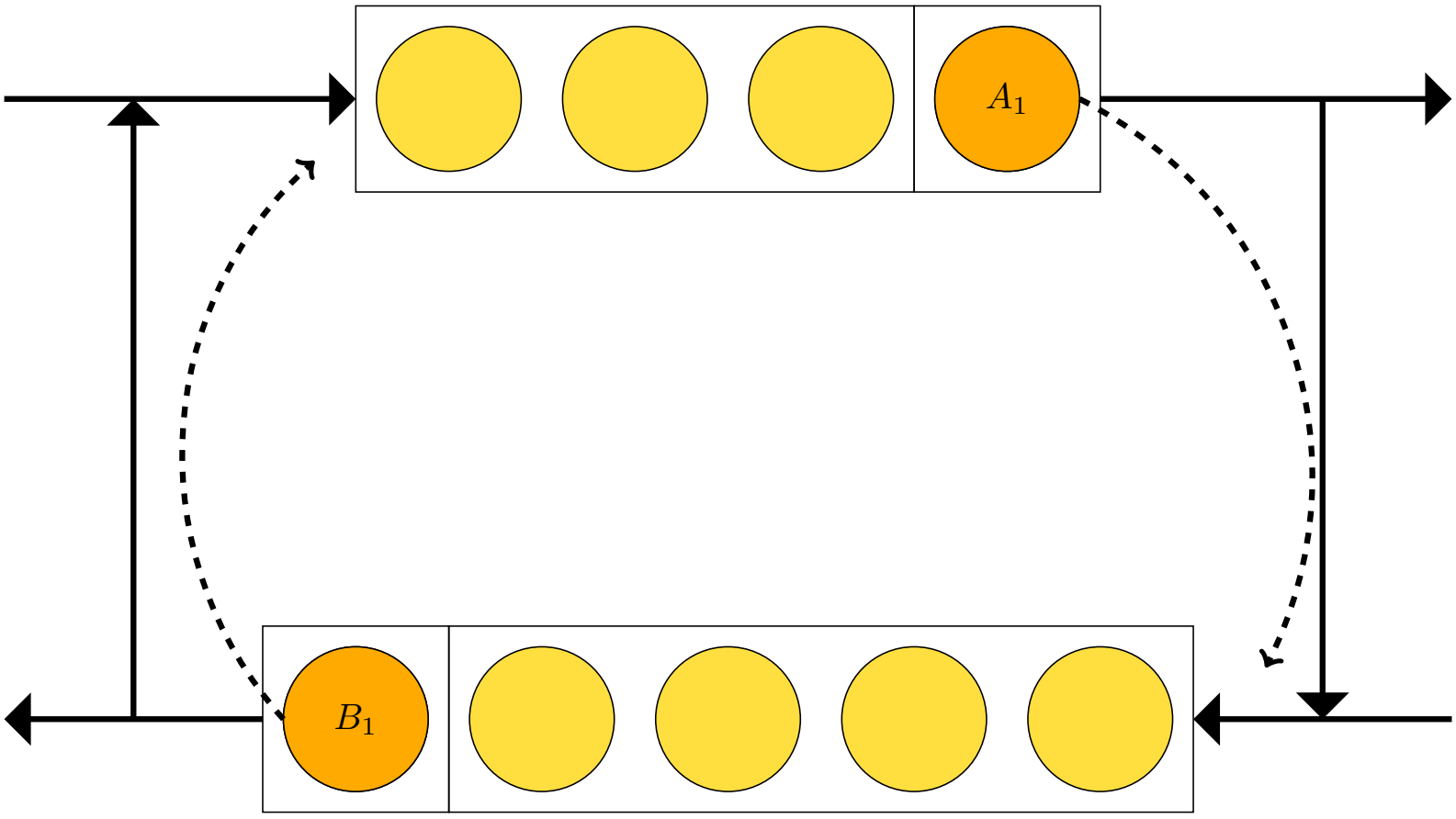
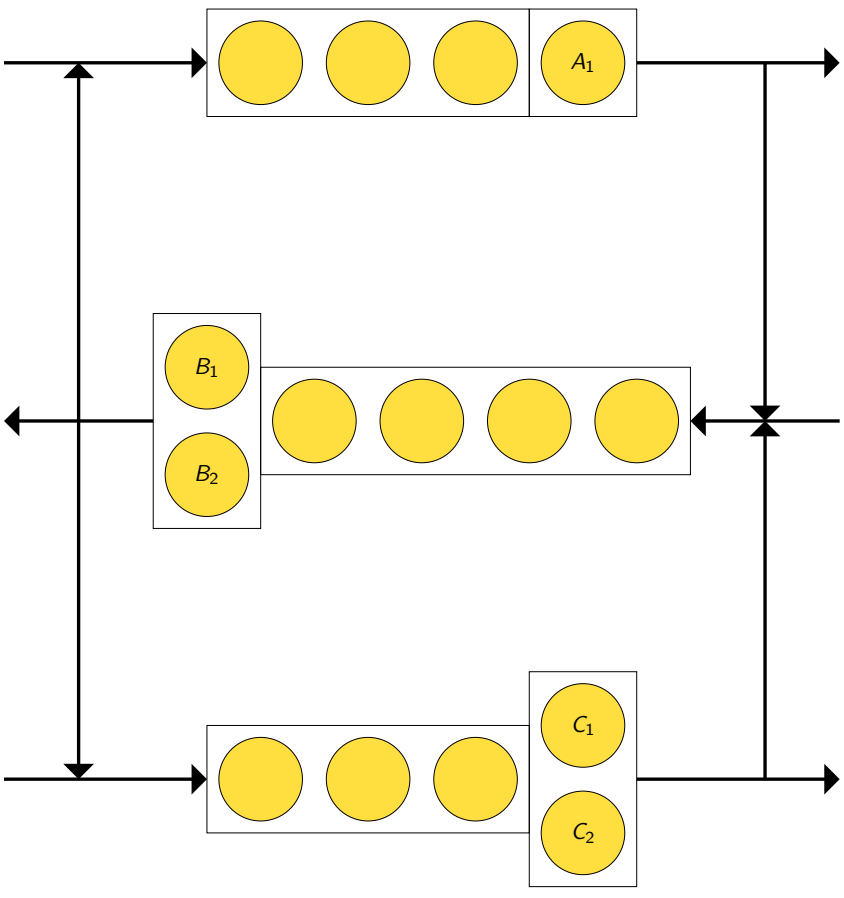
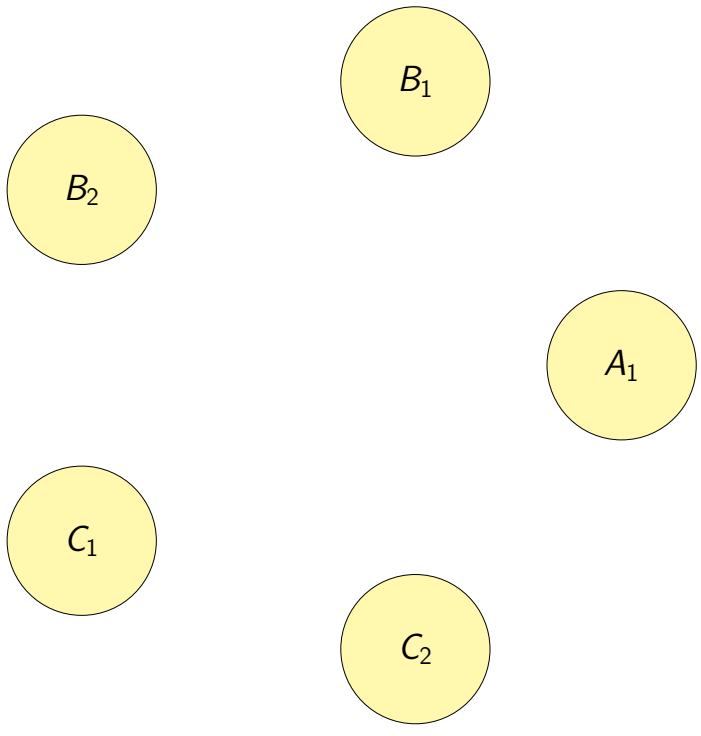
13th March 2017

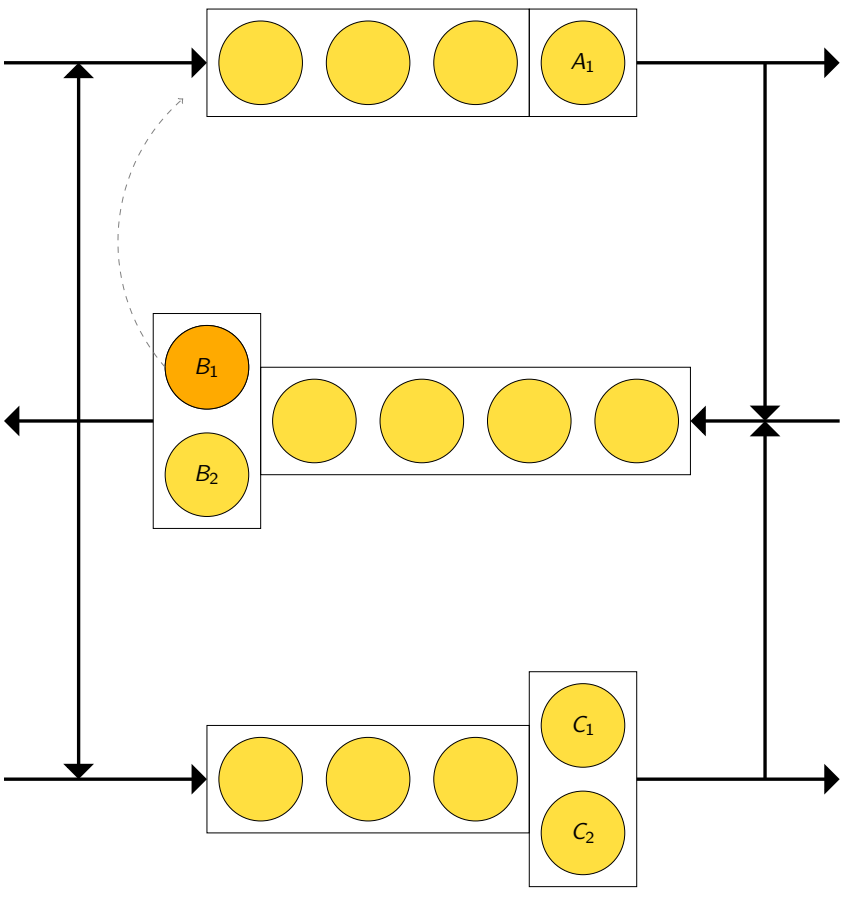
# Deadlock

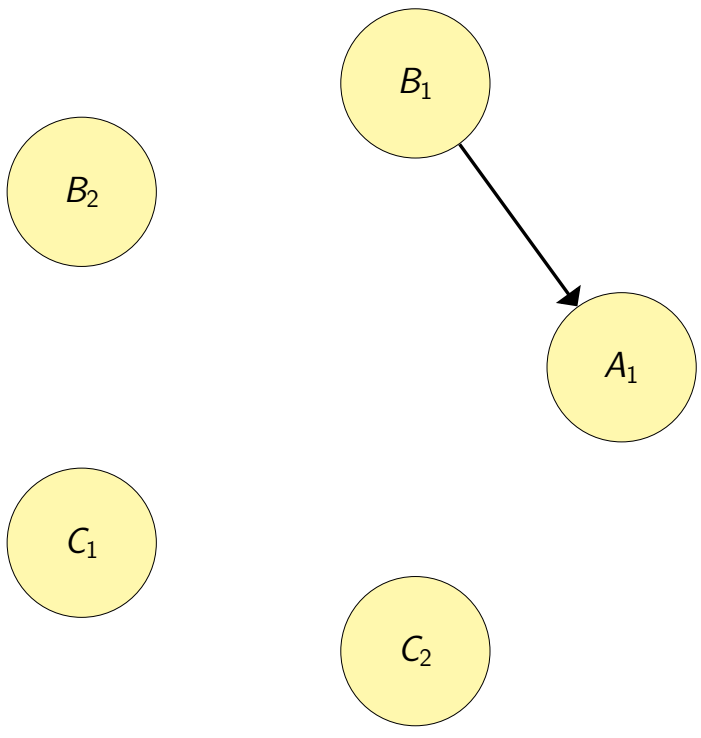


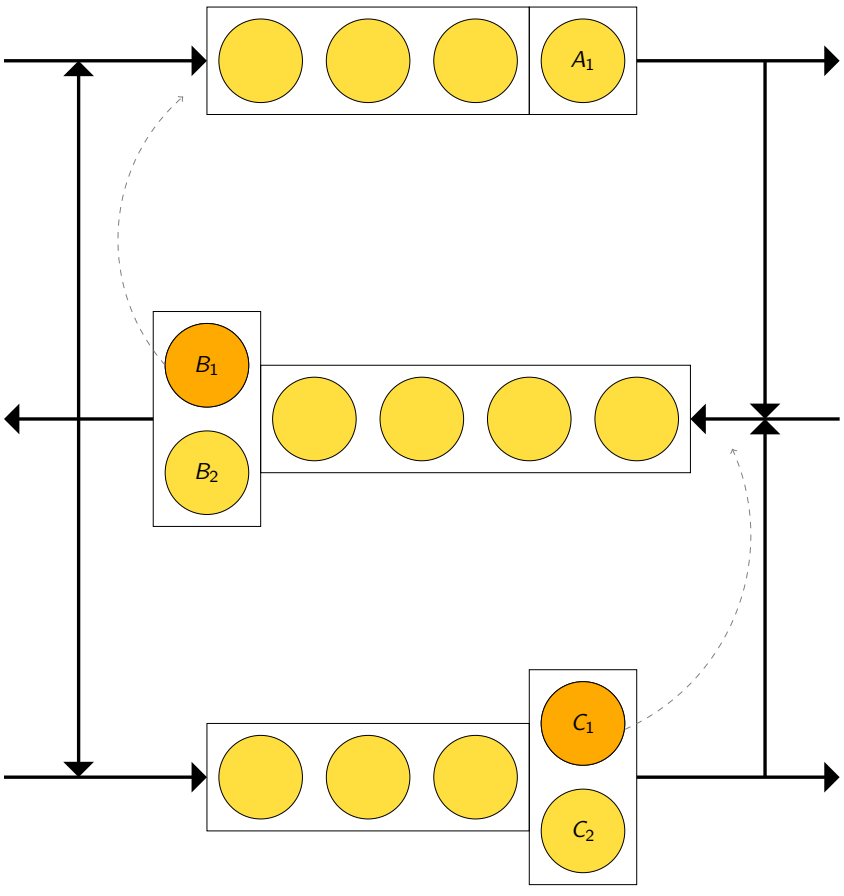

$A_1$
$B_1$

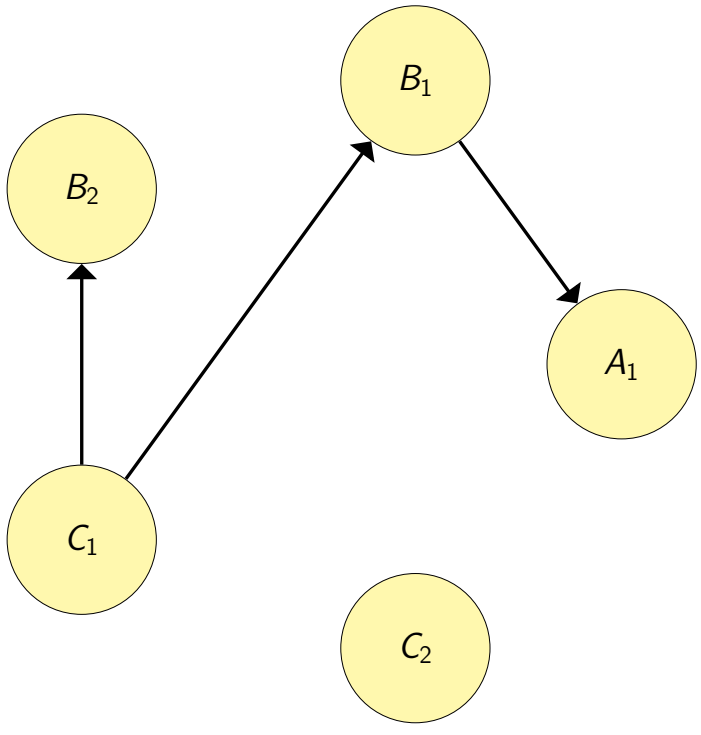

$A_1$
$B_1$
$B_2$
$C_1$
$C_2$

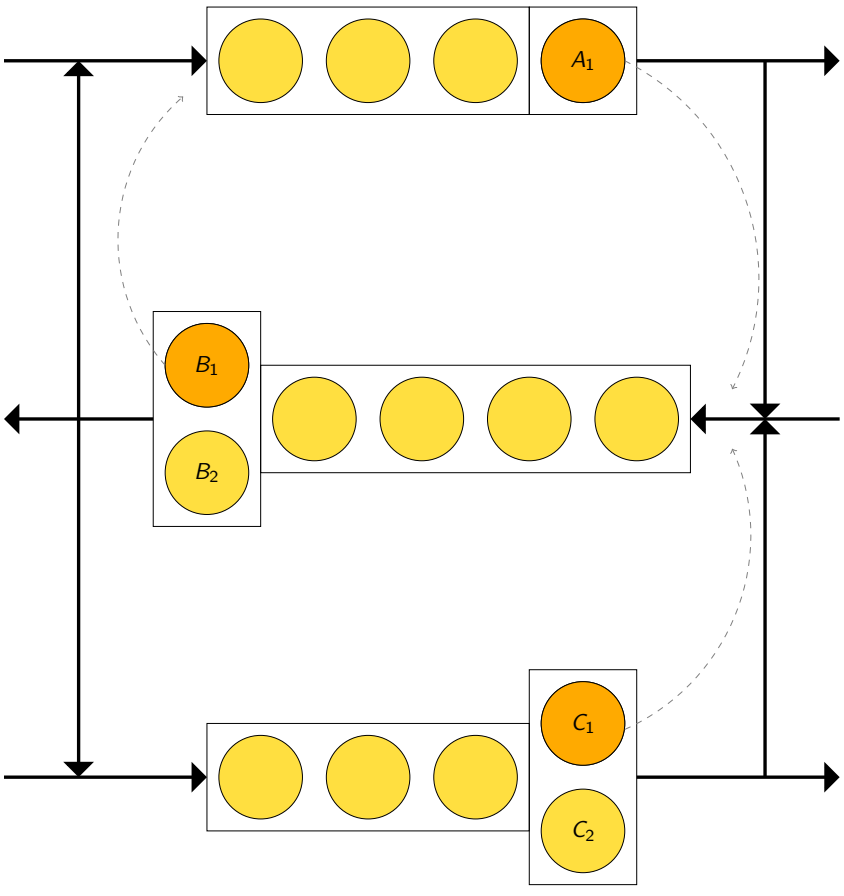

$B_1$
$B_2$
$A_1$
$C_1$
$C_2$

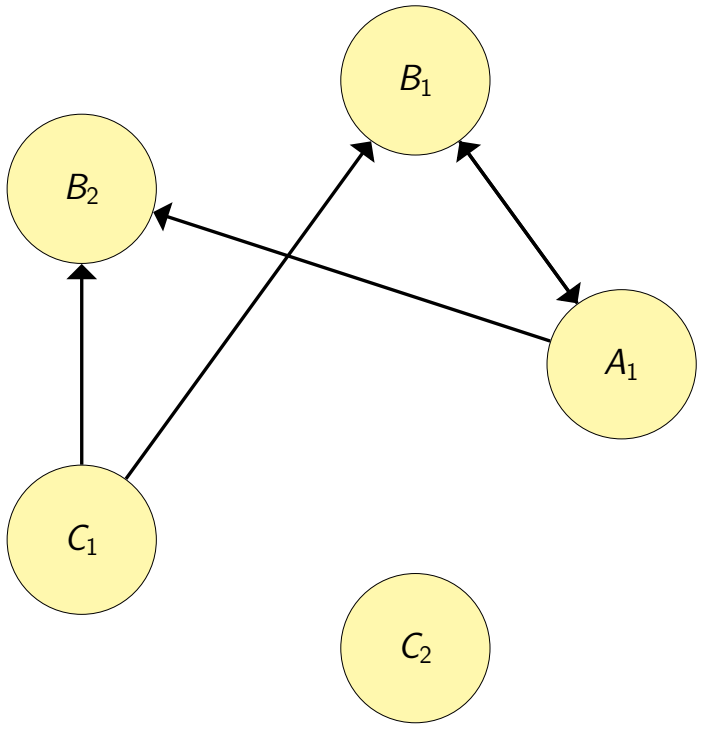

$A_1$
$B_1$
$B_2$
$C_1$
$C_2$

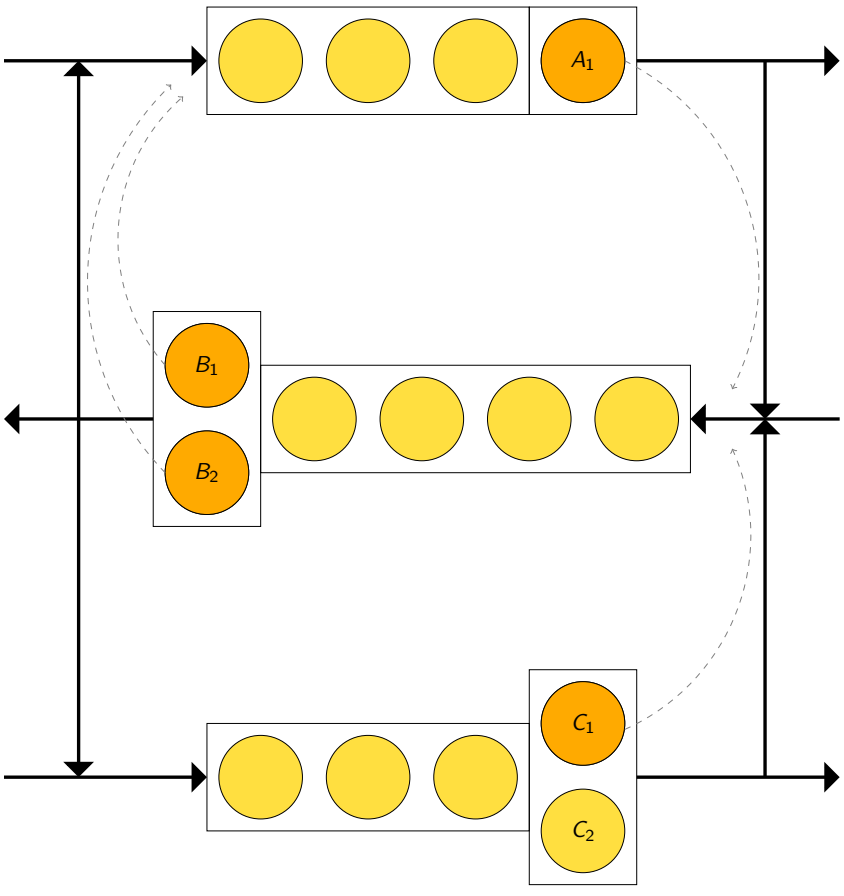

$B_1$
$B_2$
$A_1$
$C_1$
$C_2$

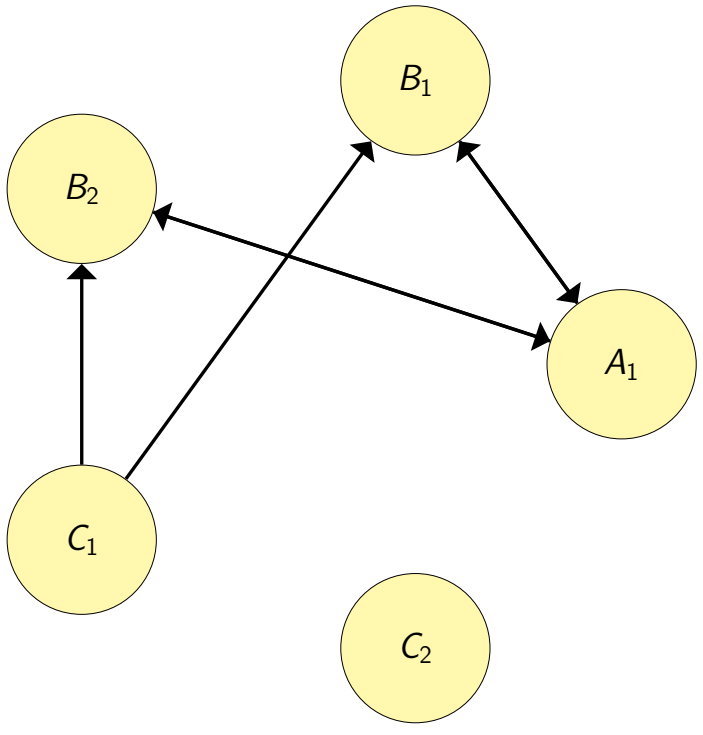

$A_1$
$B_1$
$B_2$
$C_1$
$C_2$

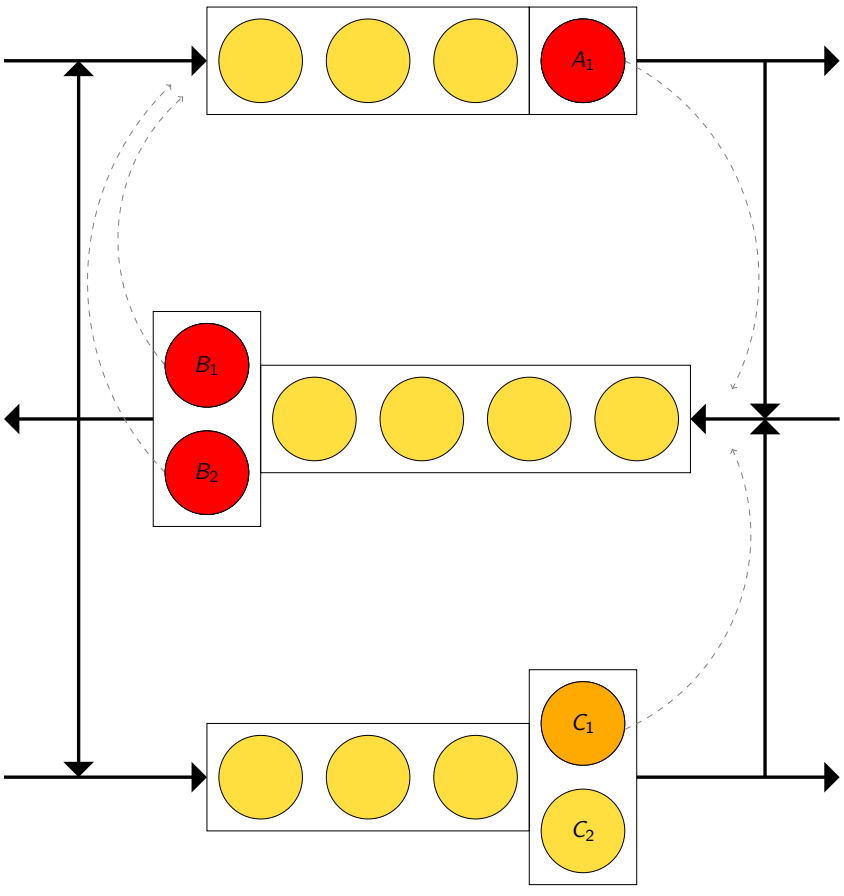

$B_1$
$B_2$
$A_1$
$C_1$
$C_2$

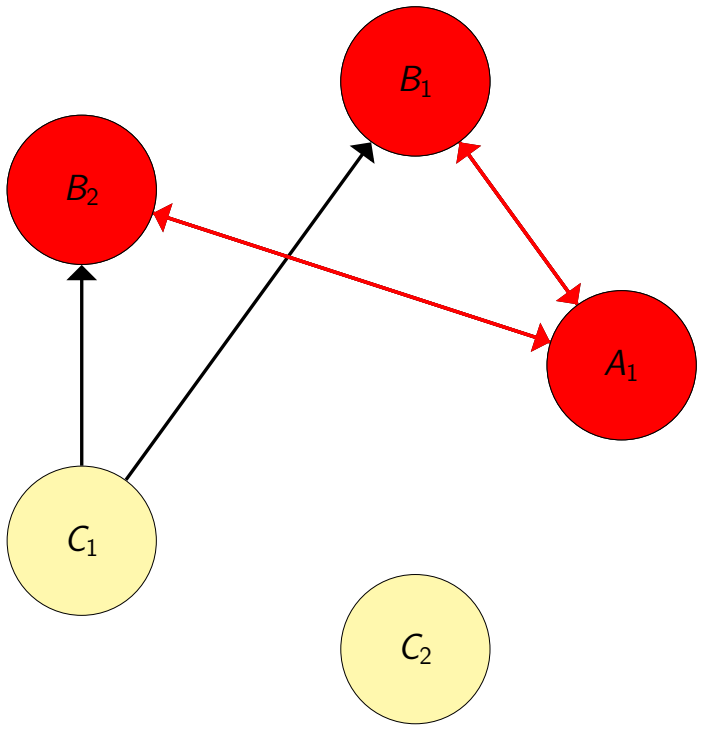

$A_1$
$B_1$
$B_2$
$C_1$
$C_2$


$B_1$
$B_2$
$A_1$
$C_1$
$C_2$

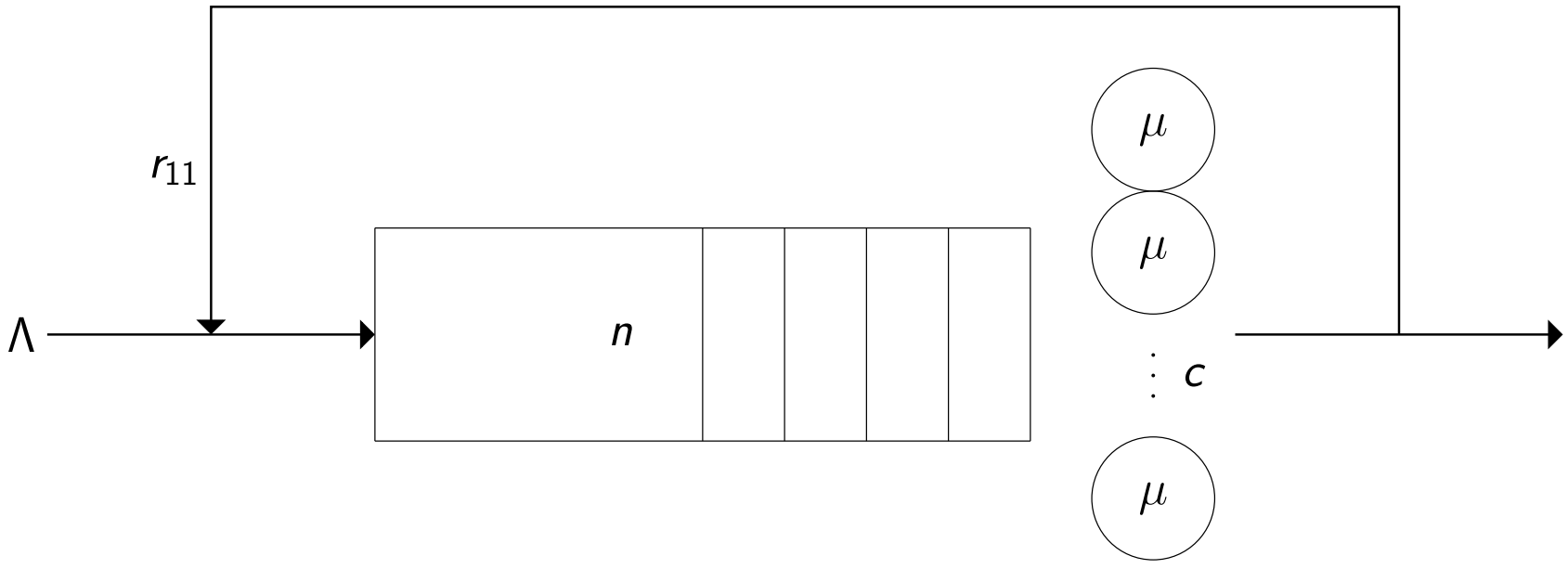

$A_1$
$B_1$
$B_2$
$C_1$
$C_2$


$B_1$
$B_2$
$A_1$
$C_1$
$C_2$


$A_1$
$B_1$
$B_2$
$C_1$
$C_2$


$B_1$
$B_2$
$A_1$
$C_1$
$C_2$

# Markovian Model of Deadlock



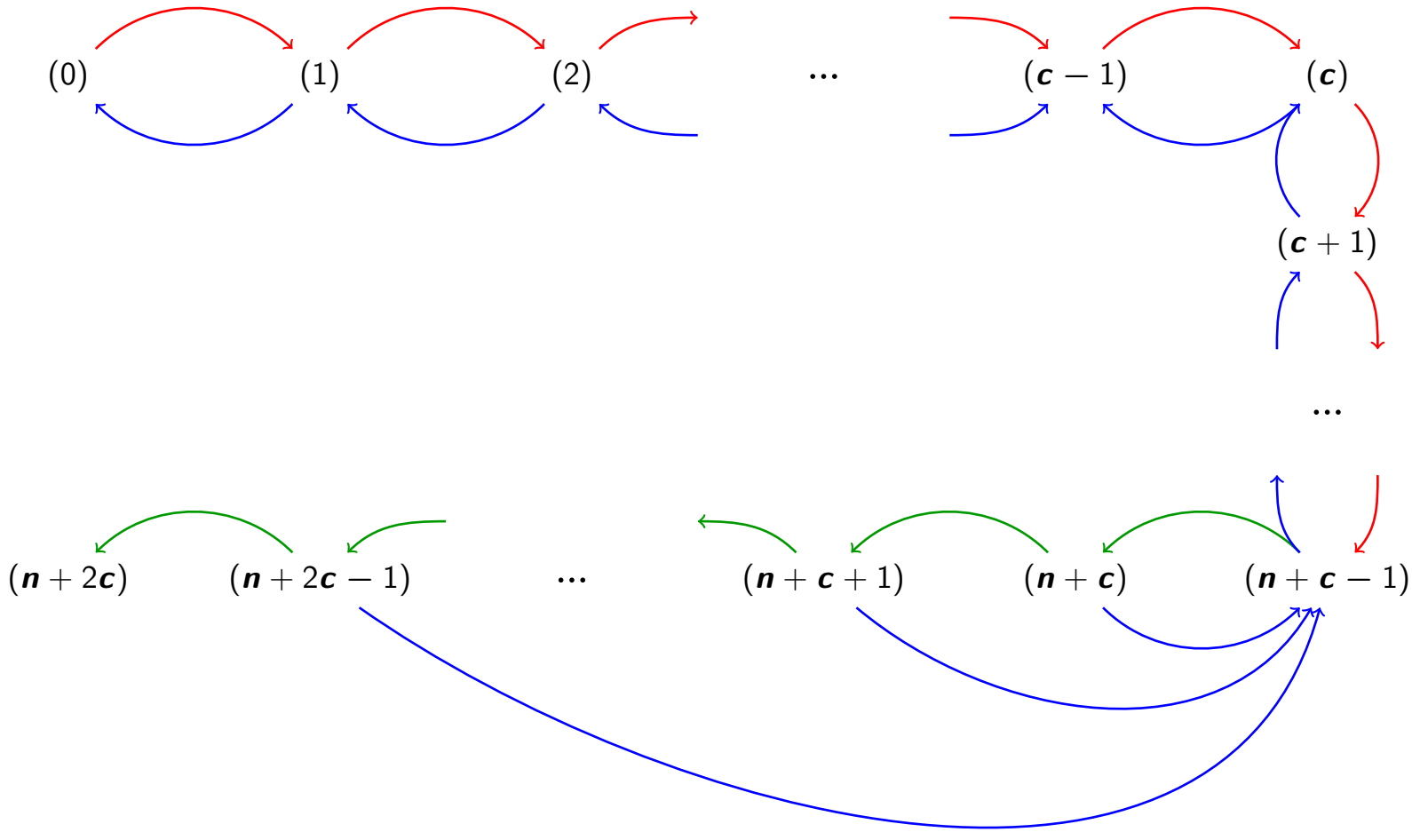

$(i)$

$$S = \{i \in \mathbb{N} \mid 0 \leq i \leq n + 2c\}$$

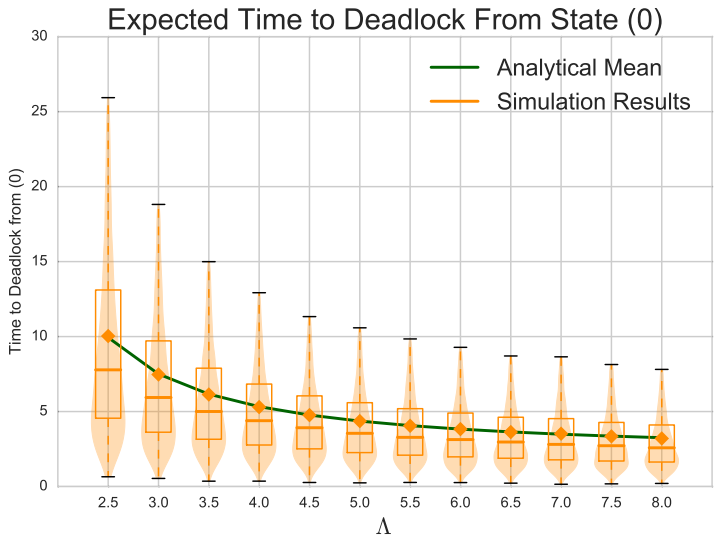
Define $\delta = i_2 - i_1$

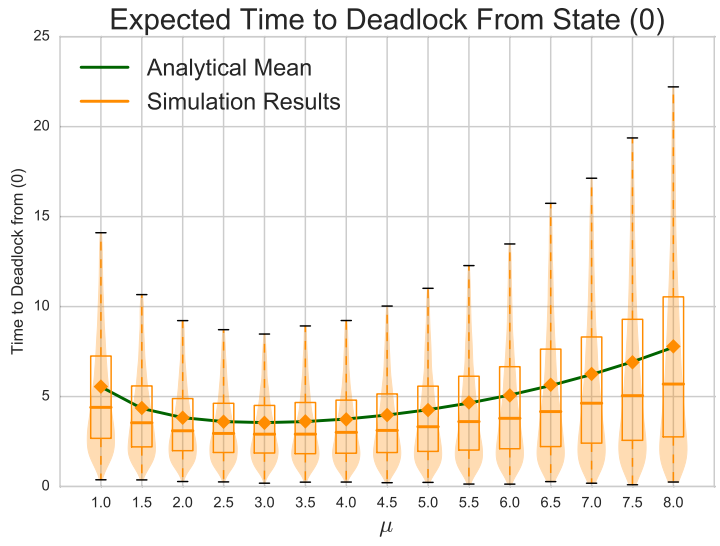
$$q_{i_1,i_2} = \left\{ \begin{array}{rr} \Lambda & \text{if } \delta = 1 \\ (1 - r_{11})\mu \min(i, c) & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 < n + c$$

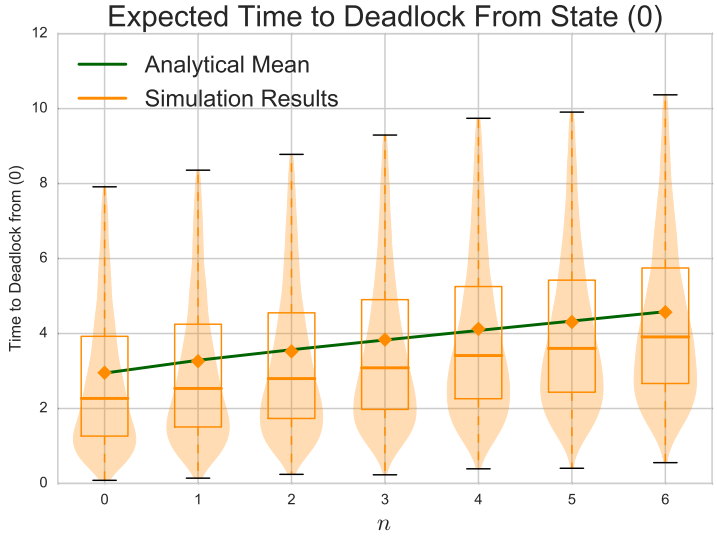
$$q_{i_1,i_2} = \left\{ \begin{array}{rr} (c - b)r_{11}\mu & \text{if } \delta = 1 \\ (1 - r_{11})(c - b)\mu & \text{if } \delta = -b - 1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 = n + c + b \quad \forall \quad 0 \leq b \leq c$$

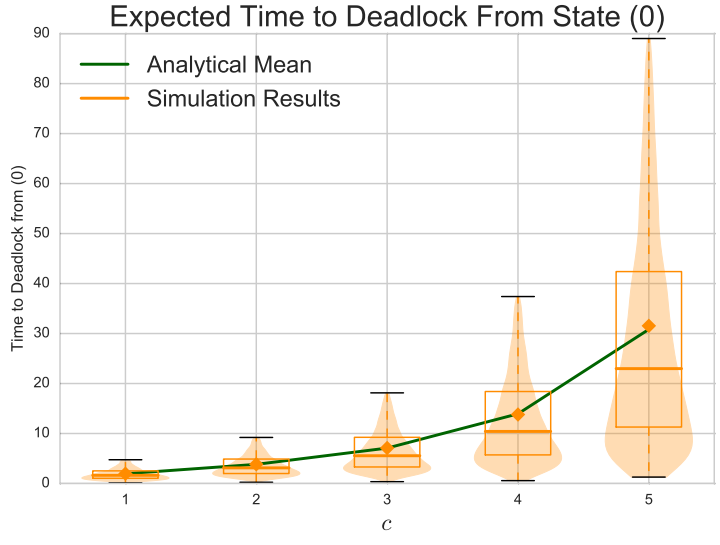

(0)
(1)
(2)
...
(c − 1)
(c)
(c + 1)
...
(n + 2c)
(n + 2c − 1)
...
(n + c + 1)
(n + c)
(n + c − 1)

# Times to Deadlock

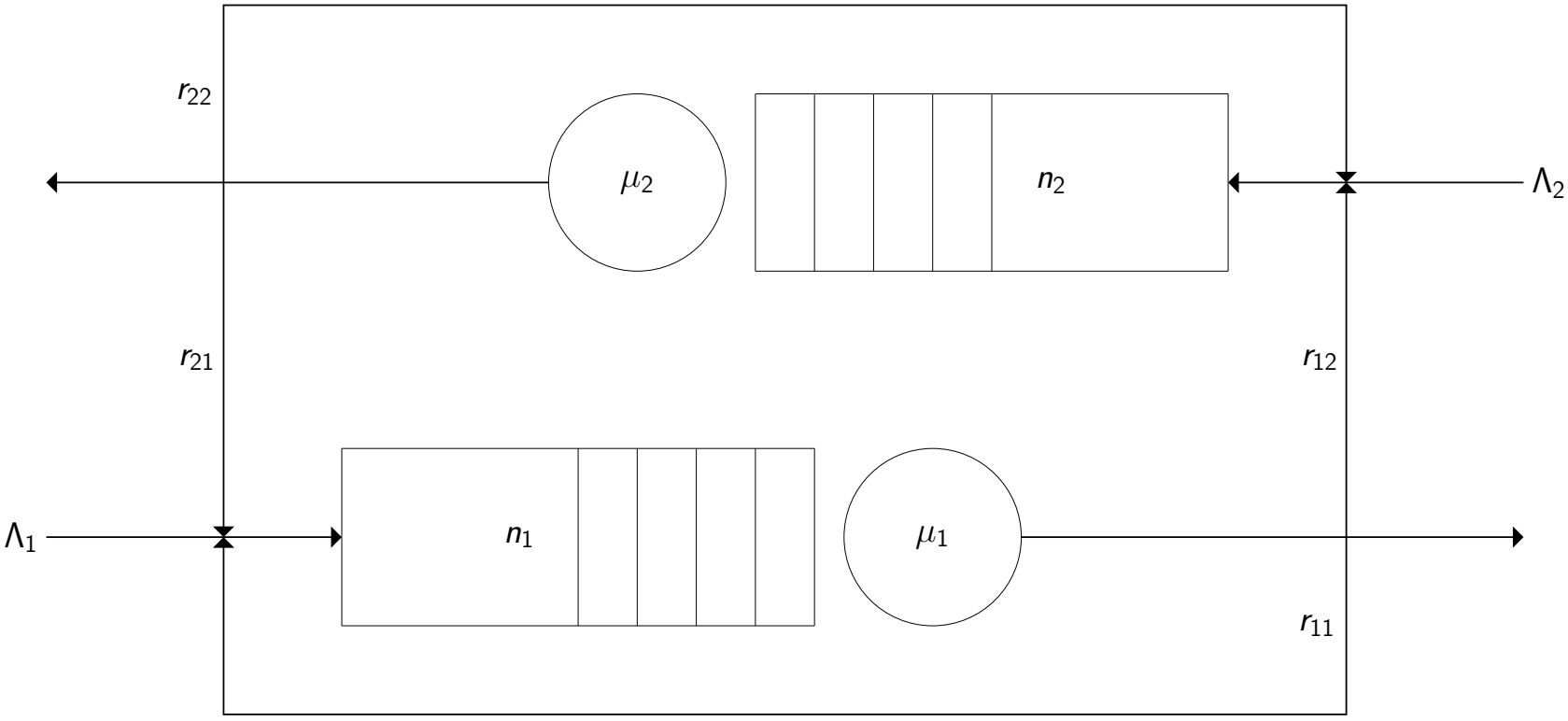


# Markovian Model of Deadlock




$(i, j)$

$$S = \{(i,j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} \mid 0 \leq i+j \leq n_1+n_2+2\} \cup \{(-1),(-2),(-3)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

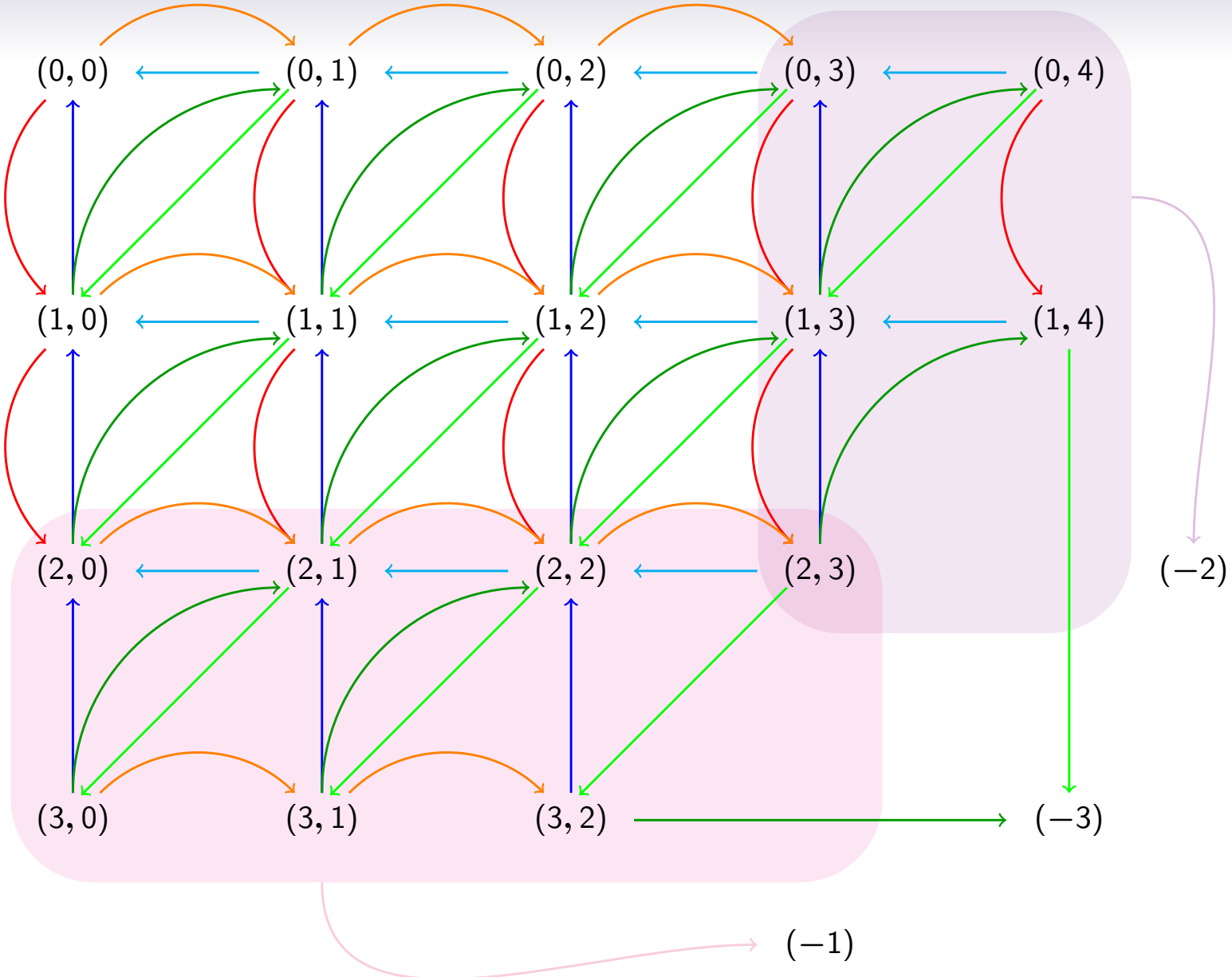
$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \begin{cases} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (1,0) \\ \begin{cases} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (0,1) \\ \begin{cases} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (-1,0) \\ \begin{cases} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (0,-1) \\ \begin{cases} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \text{ and } (i_1,j_1) \neq (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (-1,1) \\ \begin{cases} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \text{ and } (i_1,j_1) \neq (n_1, n_2+2) \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2+2 \\ 0 & \text{otherwise} \end{cases}$$

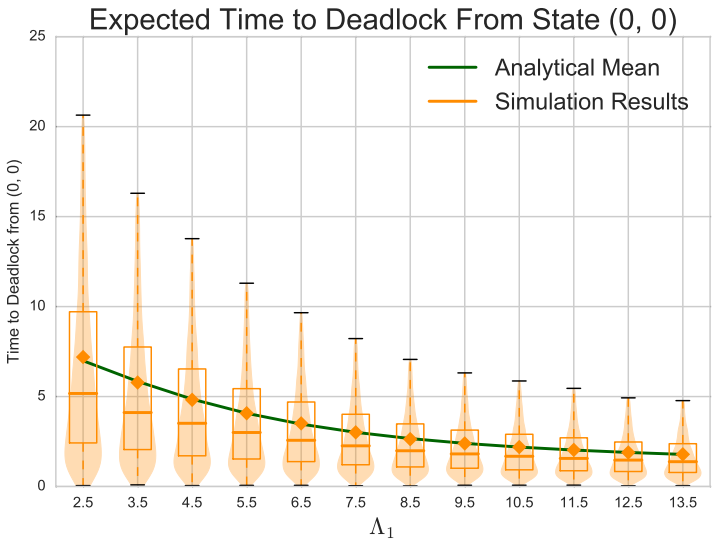
$$q_{(i_1,j_1),(-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1+2 \\ 0 & \text{otherwise} \end{cases}$$

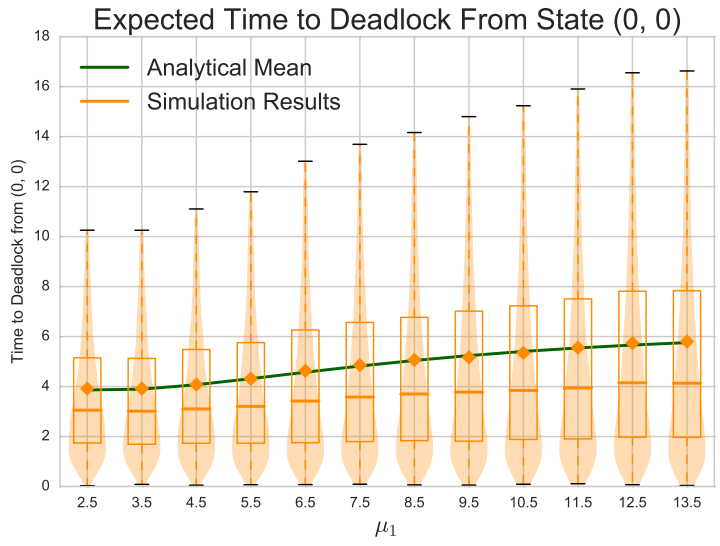
$$q_{(i_1,j_1),(-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

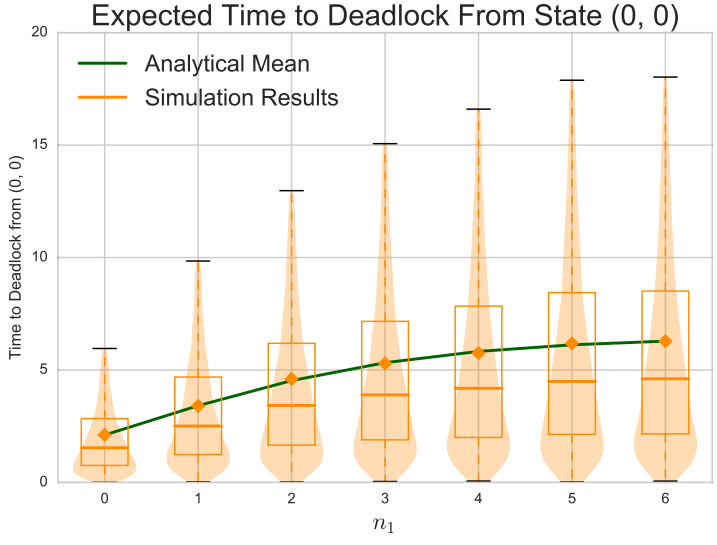
$$q_{-1,s} = q_{-2,s} = q_{-3,s} = 0$$

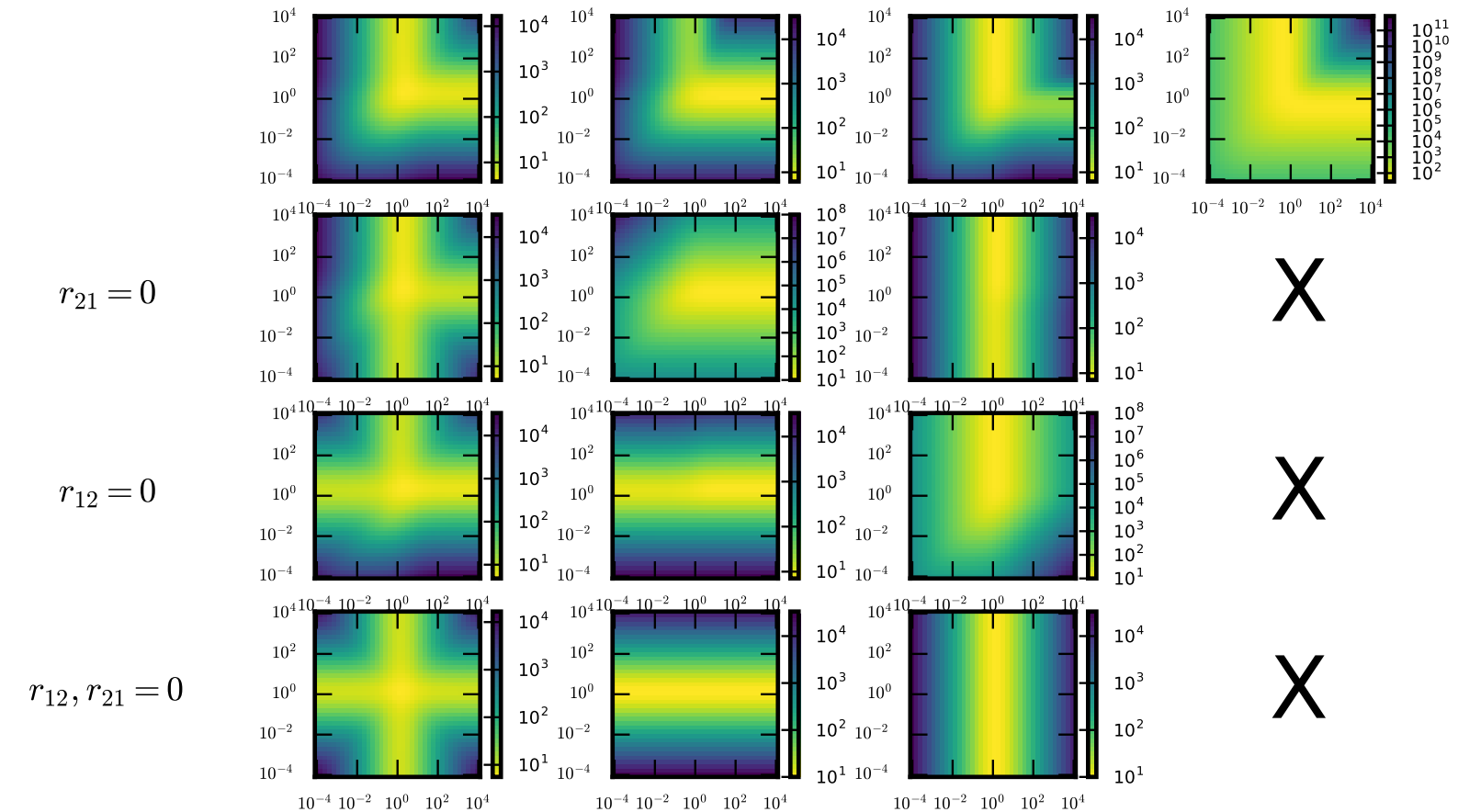

(0, 0)
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(3, 0)
(3, 1)
(3, 2)
(−2)
(−3)
(−1)

# Times to Deadlock

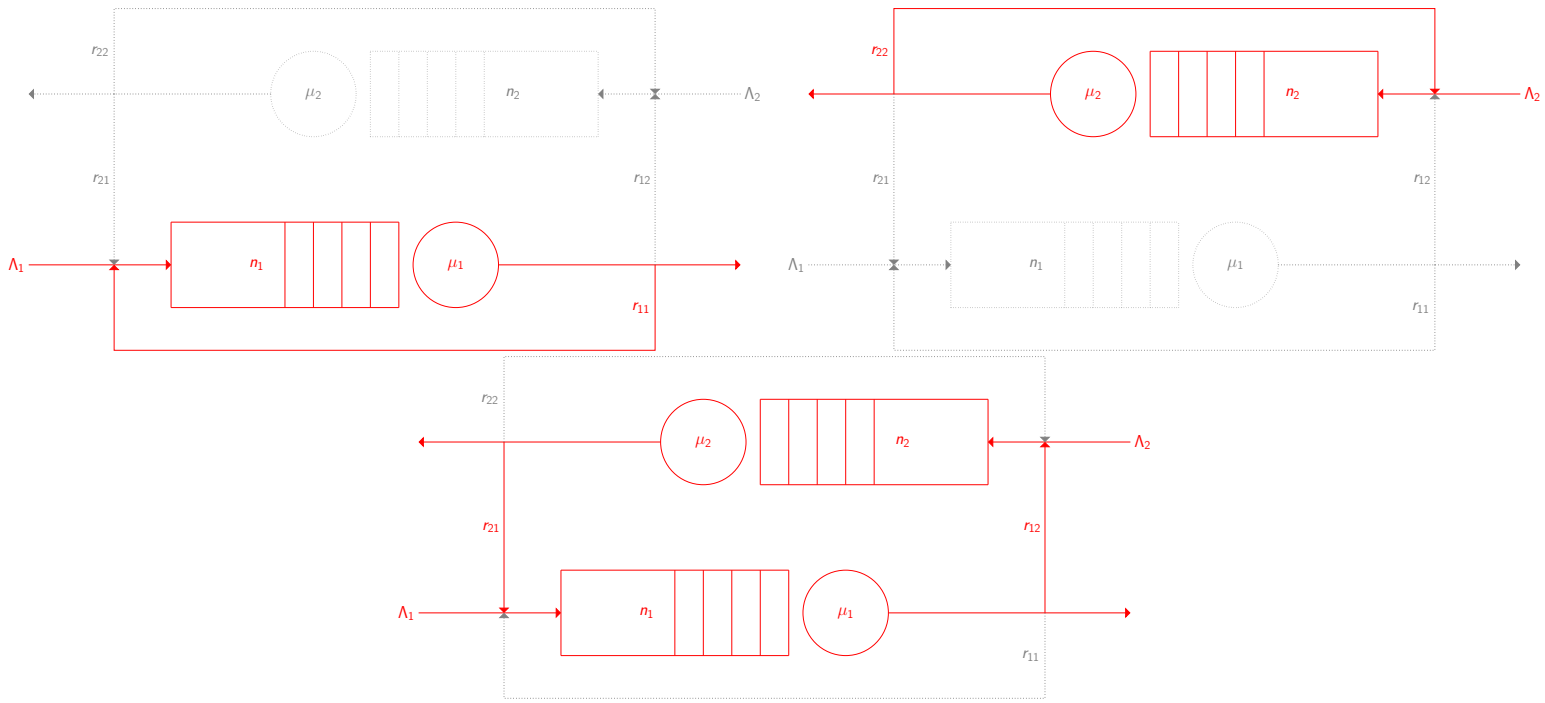


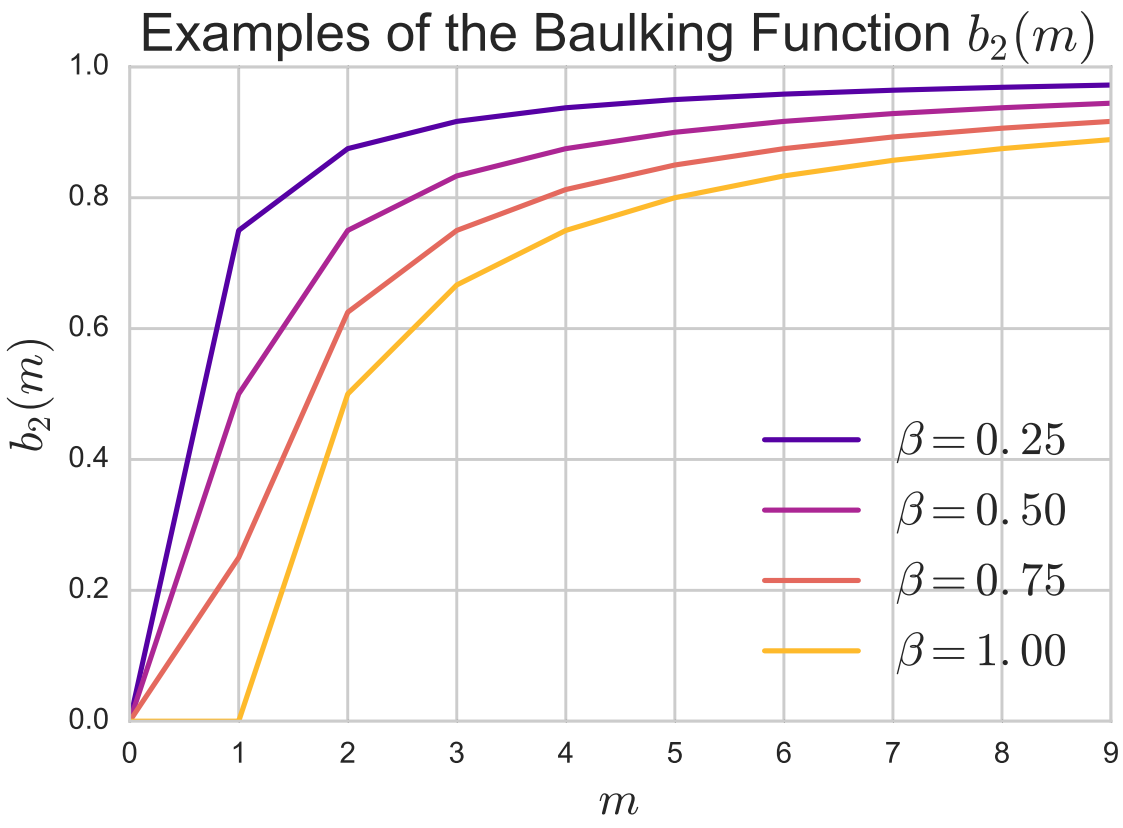

$r_{22} = 0$
$r_{11} = 0$
$r_{11}, r_{22} = 0$
$r_{21} = 0$
$r_{12} = 0$
$r_{12}, r_{21} = 0$
X
X
X

# $\omega \leq \min(\omega_{1_1}, \omega_{1_2}, \omega_2)$

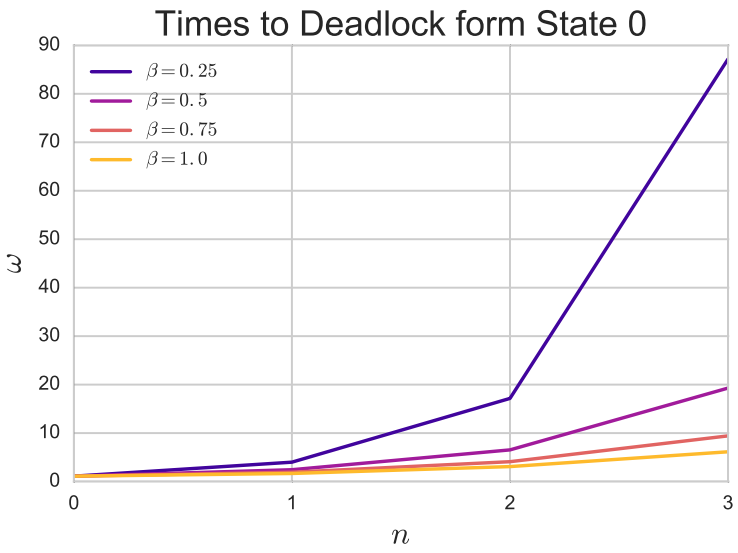


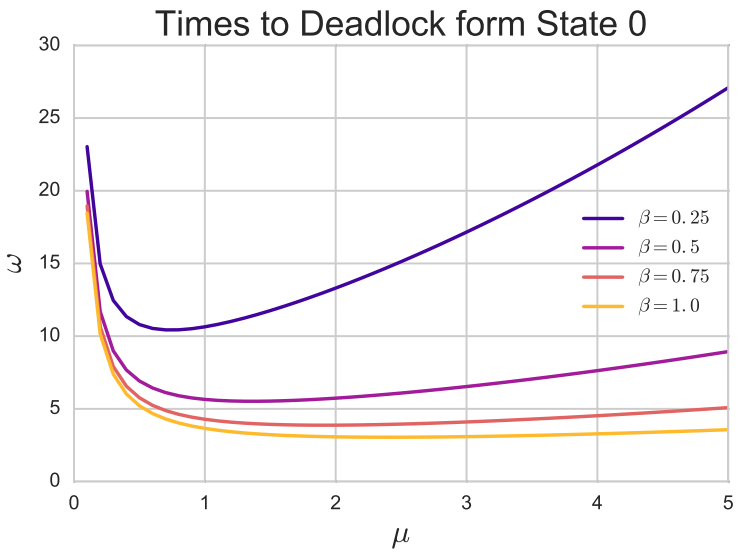
# Baulking

$$b_2(m) = \begin{cases} 0 & \text{if } m = 0 \\ 1 - \frac{\beta}{m} & \text{otherwise} \end{cases}$$

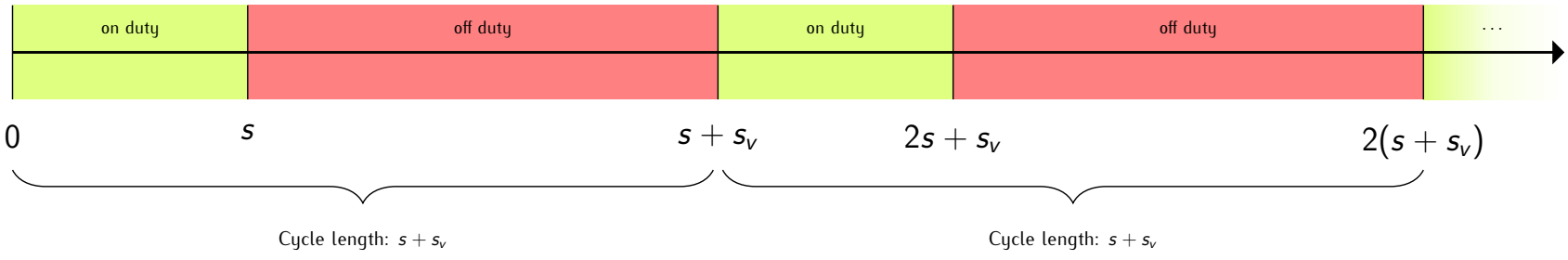


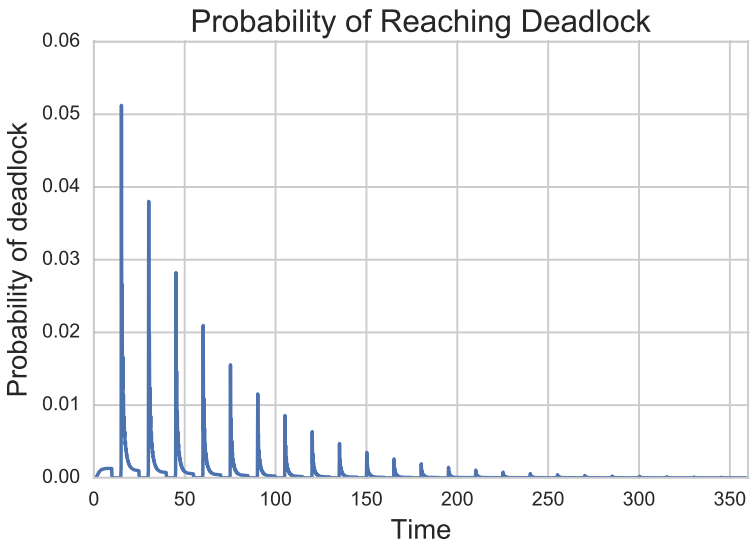
# Baulking

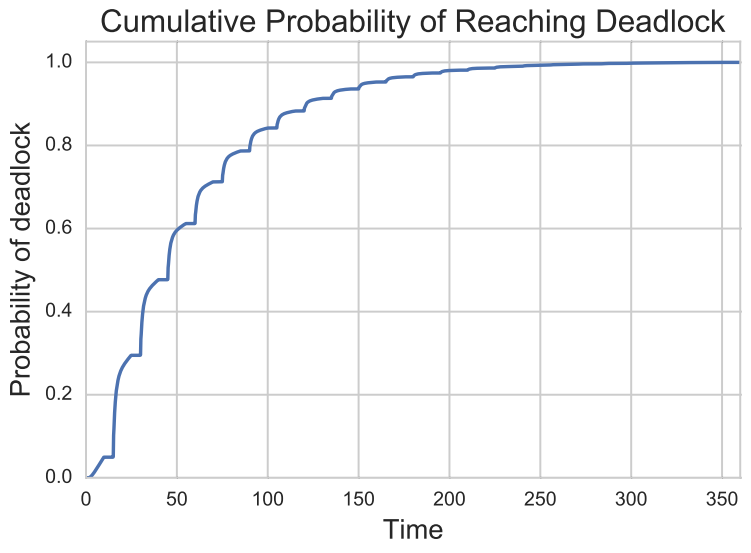

Times to Deadlock form State 0
$\beta = 0.25$
$\beta = 0.5$
$\beta = 0.75$
$\beta = 1.0$
$\omega$
$n$
0
10
20
30
40
50
60
70
80
90
0
1
2
3

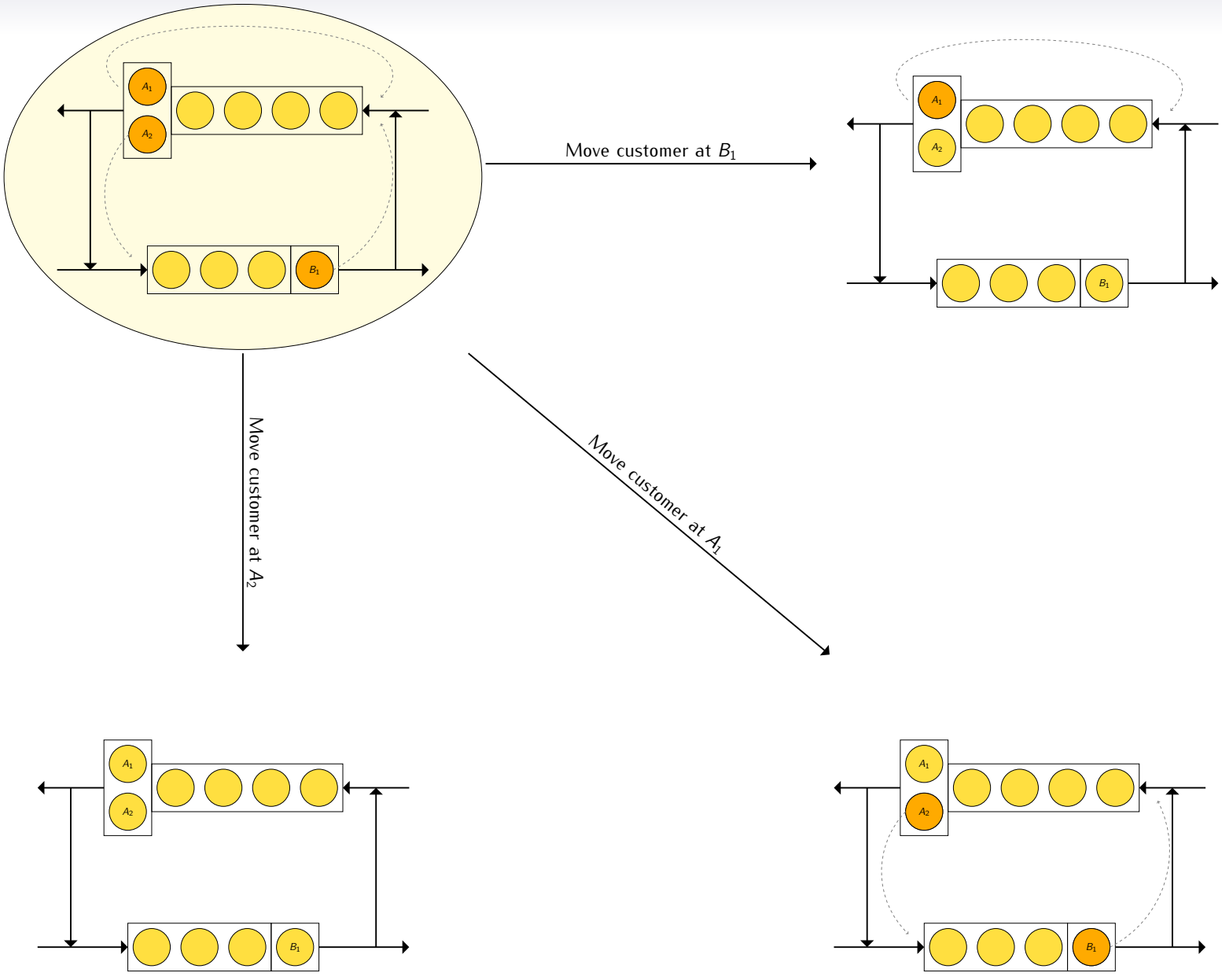

Times to Deadlock form State 0
$\beta = 0.25$
$\beta = 0.5$
$\beta = 0.75$
$\beta = 1.0$
$\omega$
$\mu$
0
5
10
15
20
25
30
0
1
2
3
4
5

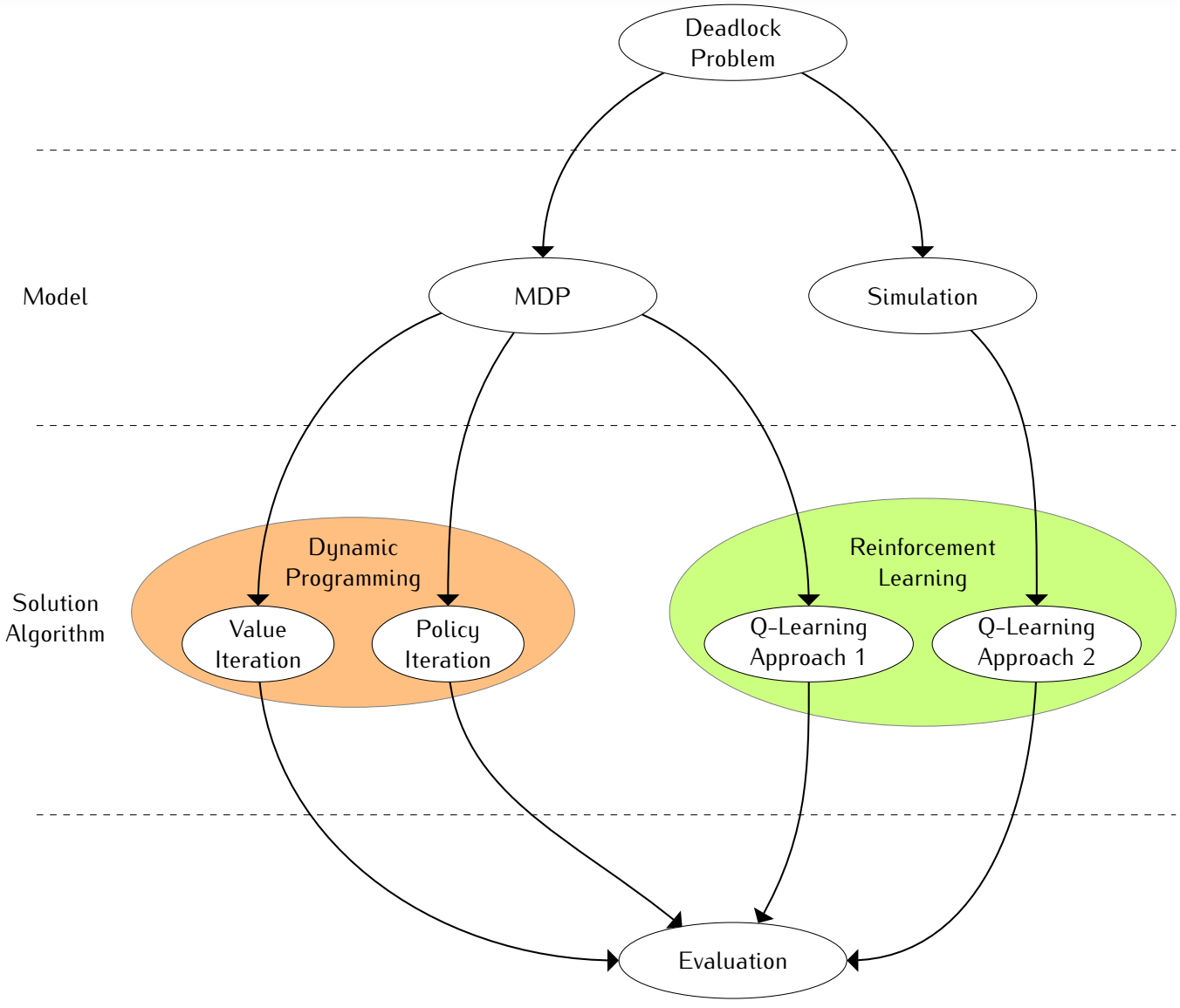
# Scheduled Vacations

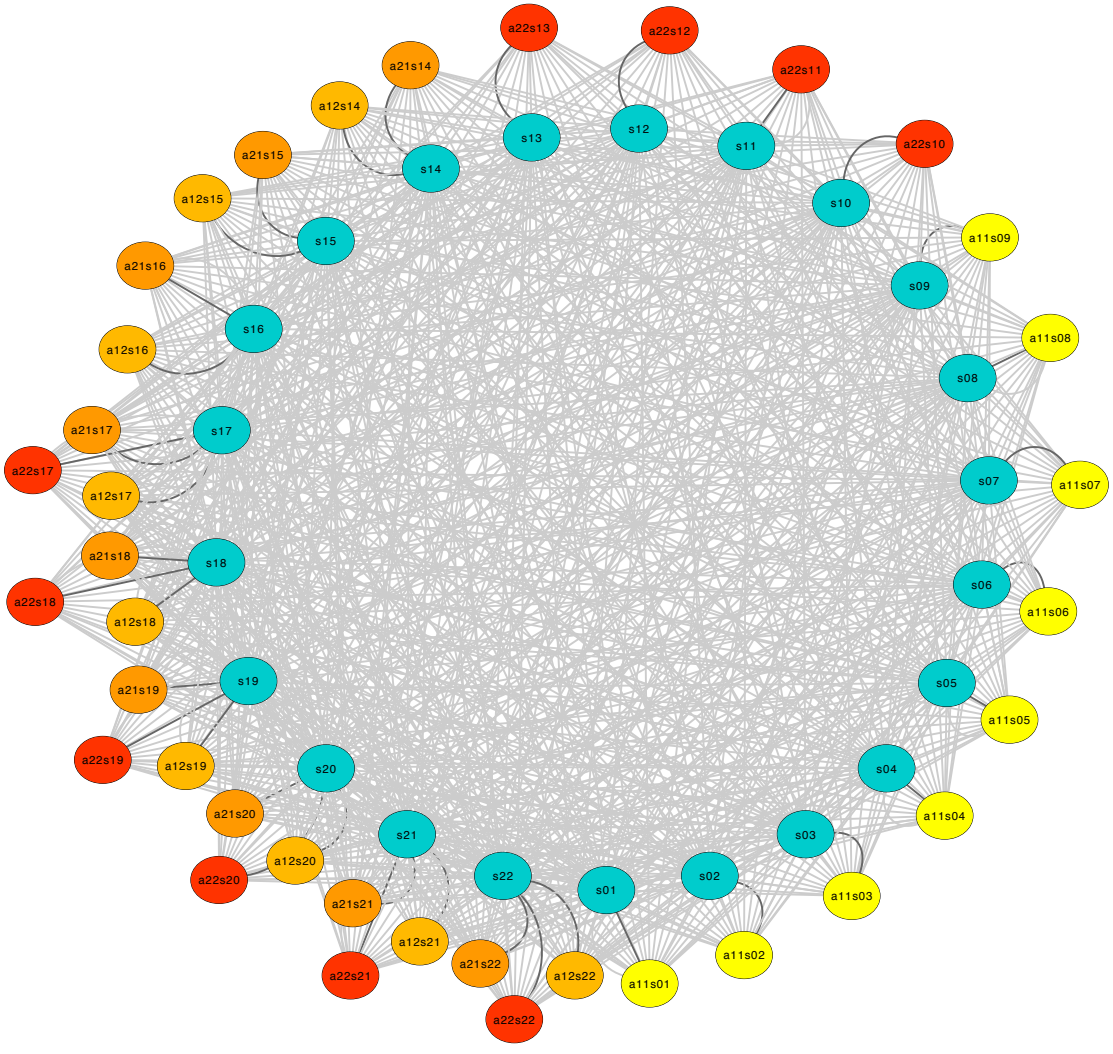


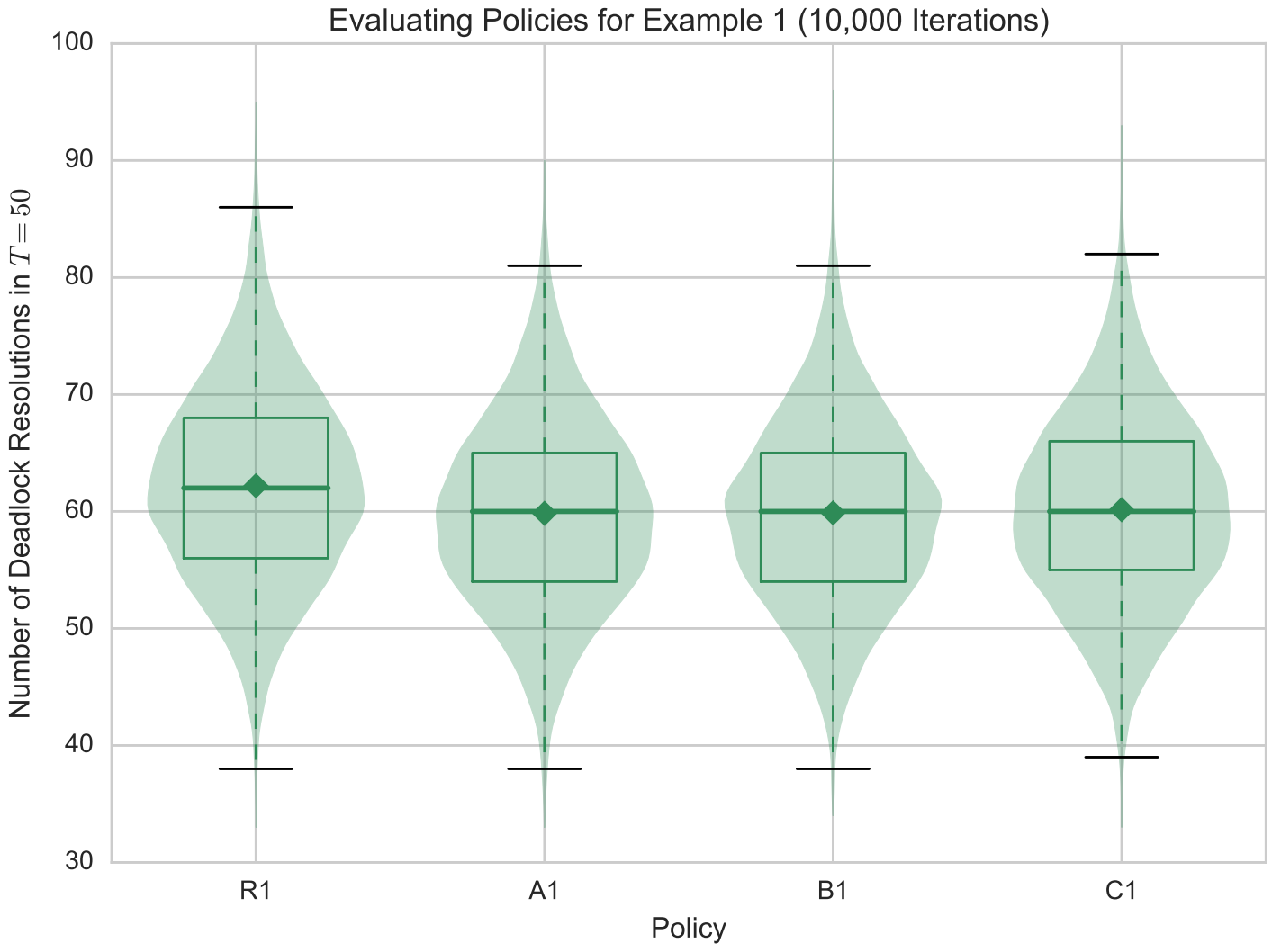

Probability of Reaching Deadlock
Probability of deadlock
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0
50
100
150
200
250
300
350
Time


Cumulative Probability of Reaching Deadlock
Probability of deadlock
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150
200
250
300
350
Time


$A_1$
$A_2$
$B_1$
Move customer at $B_1$
Move customer at $A_2$
Move customer at $A_1$


Deadlock Problem
Model
MDP
Simulation
Dynamic Programming
Reinforcement Learning
Solution Algorithm
Value Iteration
Policy Iteration
Q-Learning Approach 1
Q-Learning Approach 2
Evaluation


a22s13
a22s12
a21s14
a22s11
a12s14
s13
s12
s11
a22s10
a21s15
s14
s10
a12s15
a11s09
s15
a21s16
s09
s16
a11s08
a12s16
s08
a21s17
s17
a22s17
s07
a11s07
a12s17
a21s18
s18
s06
a22s18
a11s06
a12s18
a21s19
s19
s05
a11s05
a22s19
a12s19
s20
s04
a21s20
a11s04
s21
s03
a12s20
a22s20
s22
s01
s02
a11s03
a21s21
a12s21
a11s02
a22s21
a21s22
a12s22
a11s01
a22s22


Evaluating Policies for Example 1 (10,000 Iterations)
Number of Deadlock Resolutions in $T = 50$
100
90
80
70
60
50
40
30
R1
A1
B1
C1
Policy