

Using Queueing Network Modelling to Assess the Impact of the OPICP

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8th IMA International Conference on Quantitative Modelling
in the Management of Health and Social Care

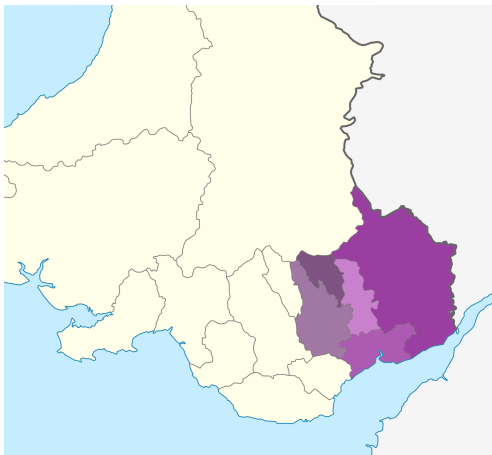


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Older People's Integrated Care Pathway

- Pathway focused around pro-active patient centred coordinated care
- Individuals identified through *risk stratification* as being at risk of admission to institutionalised care or becoming frequent users of high cost care
- Develop holistic personal *Stay Well Plans* for these individuals, utilising *low and no cost services*
- Aim to keep individuals and carers as well and as independent as possible

Elderly People's Flows Through Health System

Workforce Requirements

Simulation with Ciw

Open Source Python Library

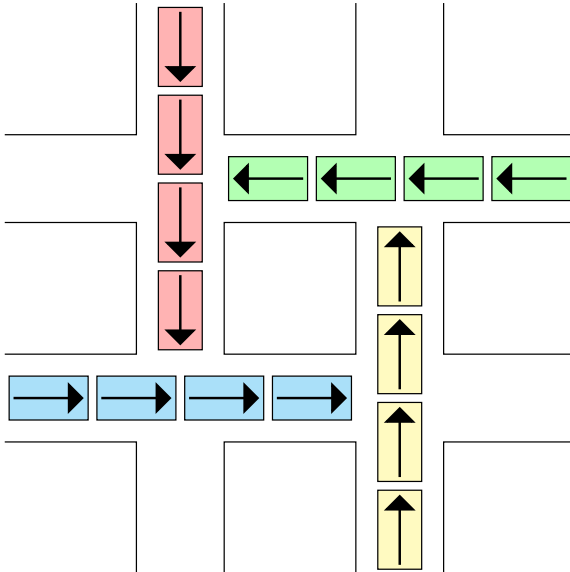


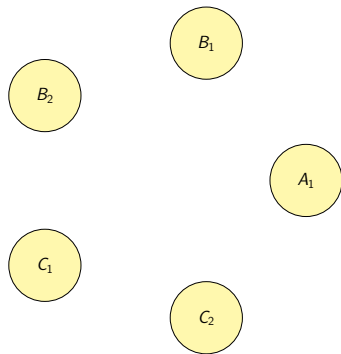
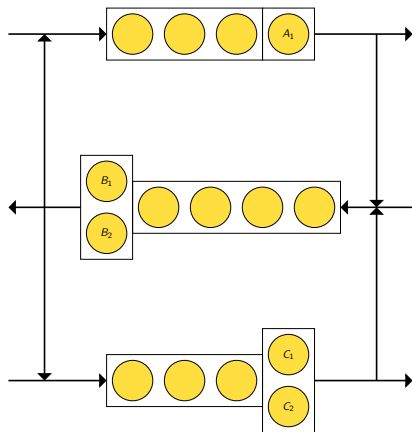
<https://github.com/geraintpalmer/Ciw>
<https://pypi.python.org/pypi/Ciw>
<http://ciw.readthedocs.org>

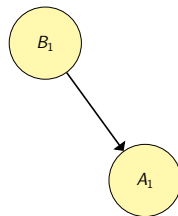
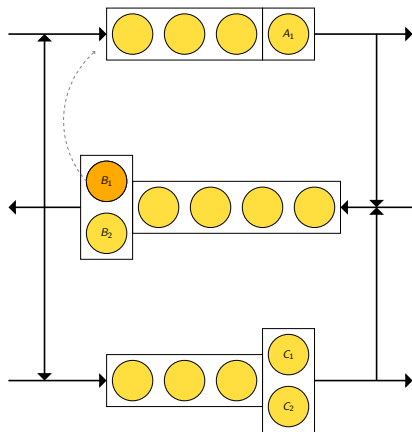
```
>>> import ciw
>>> params = {
...     'Number_of_nodes': 2,
...     'Arrival_distributions':
...         [['Exponential', 6.0],
...          ['Exponential', 6.0]],
...     'Service_distributions':
...         [['Exponential', 5.0],
...          ['Exponential', 6.0]],
...     'Transition_matrices':
...         [[0.5, 0.1],
...          [0.0, 0.4]],
...     'Number_of_servers': [3, 1],
...     'Simulation_time': 2000}
>>> Q = ciw.Simulation(params)
>>> Q.simulate_until_max_time()
>>> recs = Q.get_all_records()
```

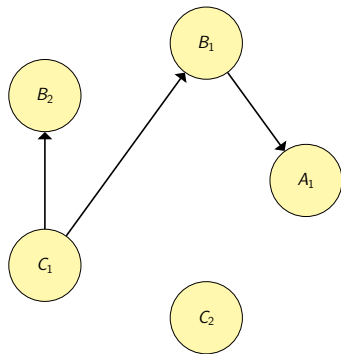
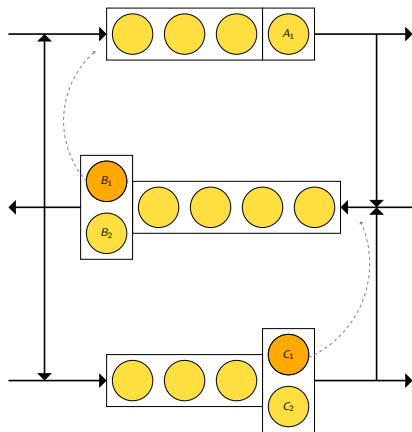
Generality

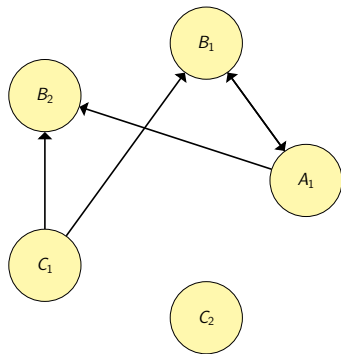
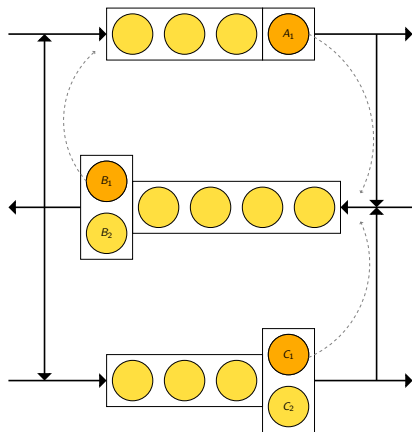
Deadlock

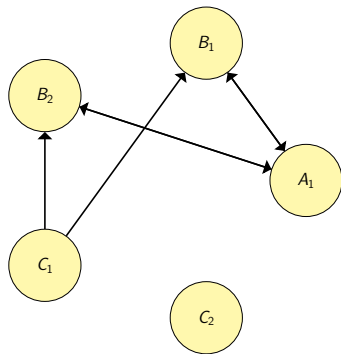
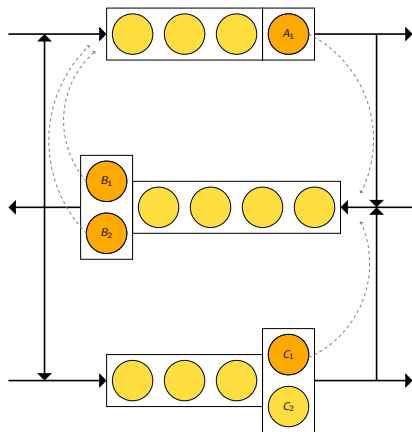


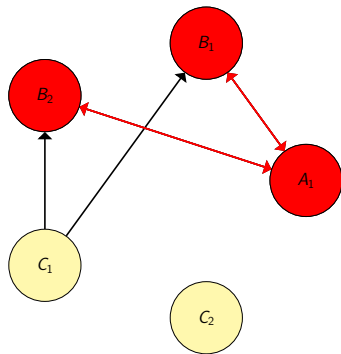
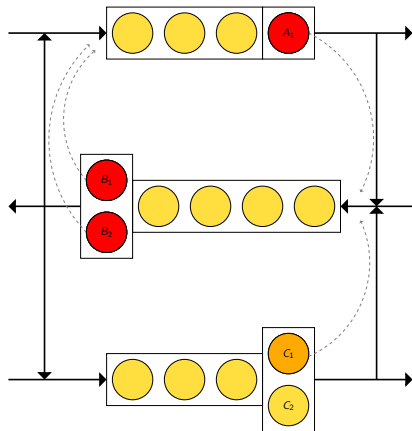










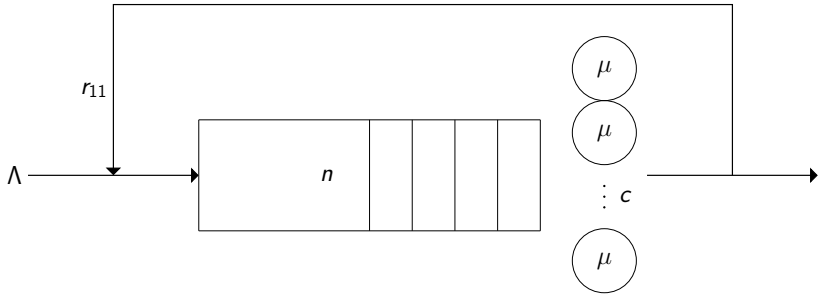


Deadlock Detection in Ciw

```
>>> import ciw
>>> params = {'Number_of_nodes': 1,
...           'Arrival_distributions': [['Exponential', 6.0]],
...           'Service_distributions': [['Exponential', 5.0]],
...           'Transition_matrices': [[0.5]],
...           'Number_of_servers': [1],
...           'Queue_capacities': [3],
...           'Detect_deadlock': True}
>>> Q = ciw.Simulation(params)
>>> times_to_deadlock = Q.simulate_until_deadlock()
>>> times_to_deadlock[((0, 0),)]
1.1707879982560288
```

Three Deadlocking Queueing Networks

Markovian Model of Deadlock



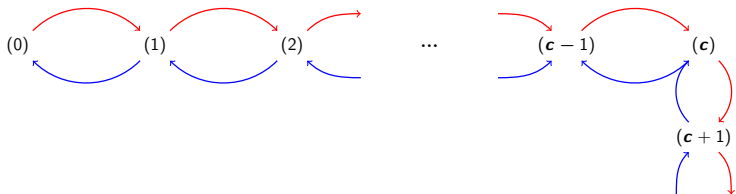
(i)

$$S = \{i \in \mathbb{N} \mid 0 \leq i \leq n + 2c\}$$

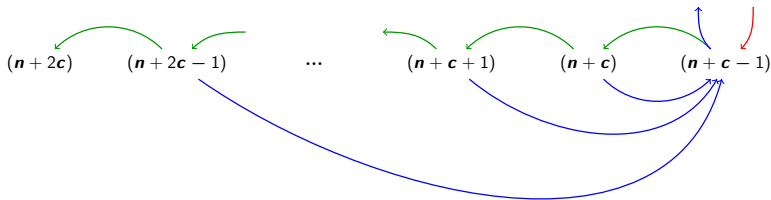
$$\text{Define } \delta = i_2 - i_1$$

$$q_{i_1, i_2} = \left\{ \begin{array}{ll} (1 - r_{11})\mu \min(i, c) & \text{if } \delta = 1 \\ 0 & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 < n + c$$

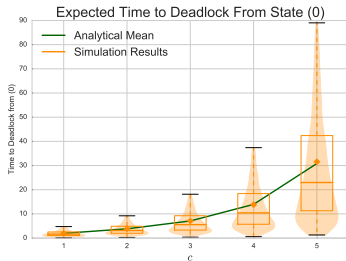
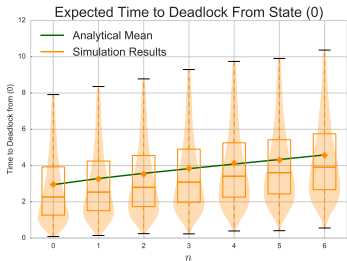
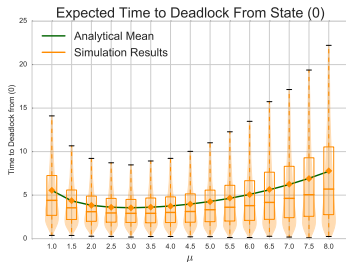
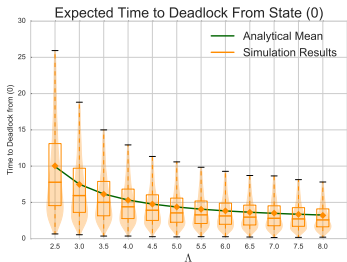
$$q_{i_1, i_2} = \left\{ \begin{array}{ll} (c - b)r_{11}\mu & \text{if } \delta = 1 \\ (1 - r_{11})(c - b)\mu & \text{if } \delta = -b - 1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 = n + c + b \quad \forall \quad 0 \leq b \leq c$$



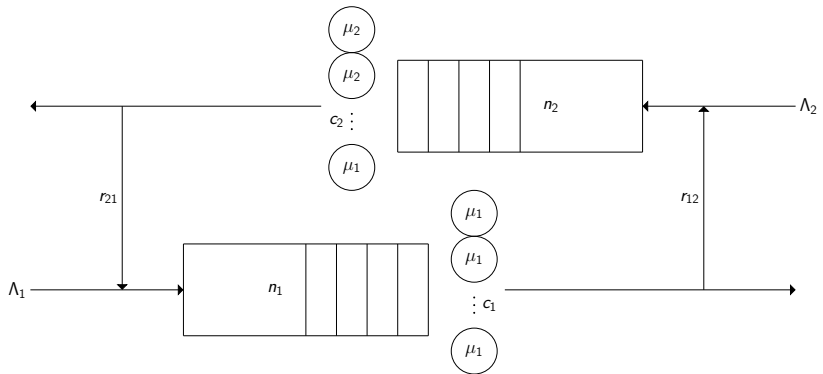
...



Times to Deadlock



Markovian Model of Deadlock



(i, j)

$$S = \{(i, j) \in \mathbb{N}^{(n_1+c_1+c_2) \times (n_2+c_2+c_1)} \mid i \leq n_1 + c_1 + j, j \leq n_2 + c_2 + i\}$$

$$\delta = (i_2, j_2) - (i_1, j_1)$$

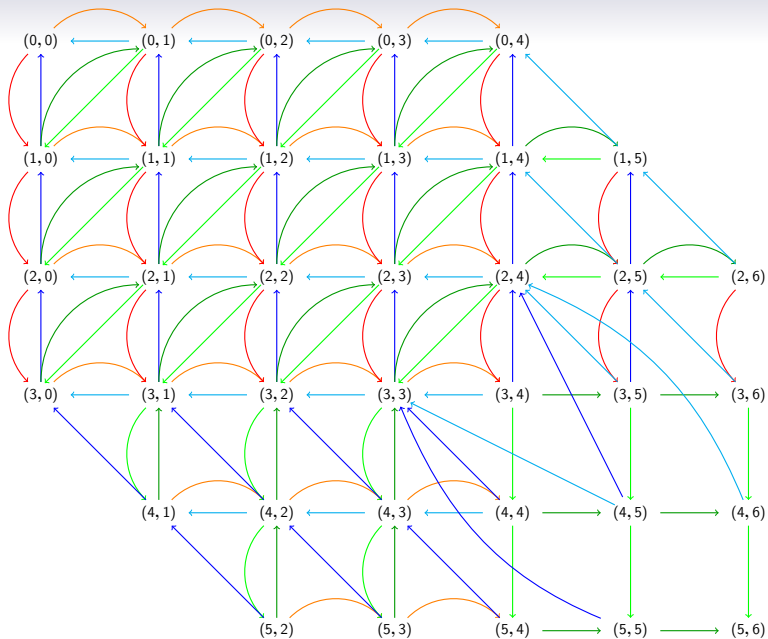
$$b_1 = \max(0, i_1 - n_1 - c_1)$$

$$b_2 = \max(0, i_2 - n_2 - c_2)$$

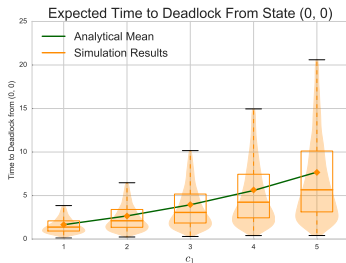
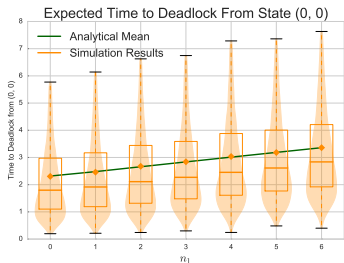
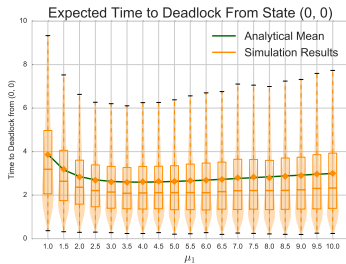
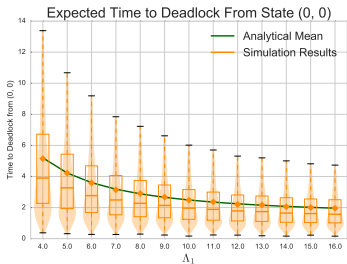
$$s_1 = \min(i_1, c_1) - b_2$$

$$s_2 = \min(i_2, c_2) - b_1$$

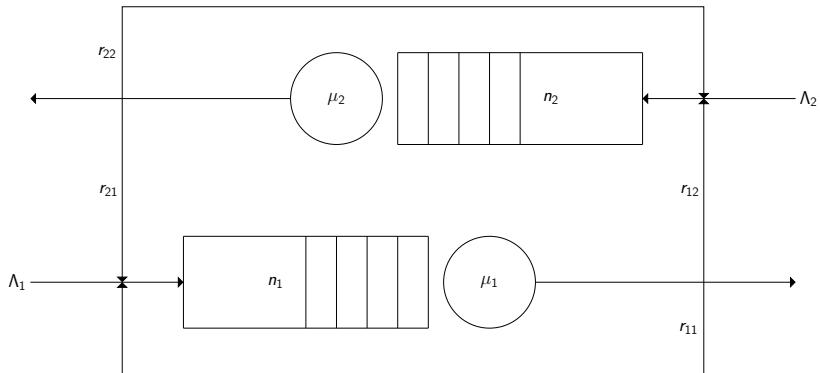
	$j_1 < n_2 + c_2$	$j_1 = n_2 + c_2$	$j_1 > n_2 + c_2$
$i_1 < n_1 + c_1$	Λ_1 if $\delta = (1, 0)$ Λ_2 if $\delta = (0, 1)$ $r_{12}s_1\mu_1$ if $\delta = (-1, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, -1)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, 0)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (0, -1)$	Λ_1 if $\delta = (1, 0)$ $r_{12}s_1\mu_1$ if $\delta = (0, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, -1)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, 0)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (0, -1)$	Λ_1 if $\delta = (1, 0)$ $r_{12}s_1\mu_1$ if $\delta = (0, 1)$ $r_{21}s_2\mu_2$ if $\delta = (0, -1)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, 0)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (-1, -1)$
$i_1 = n_1 + c_1$	Λ_2 if $\delta = (0, 1)$ $r_{12}s_1\mu_1$ if $\delta = (-1, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, 0)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, 0)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (0, -1)$	$r_{12}s_1\mu_1$ if $\delta = (0, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, 0)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, 0)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (0, -1)$	$r_{12}s_1\mu_1$ if $\delta = (0, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, 0)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, 0)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (-1, -1)$
$i_1 > n_1 + c_1$	Λ_2 if $\delta = (0, 1)$ $r_{12}s_1\mu_1$ if $\delta = (-1, 0)$ $r_{21}s_2\mu_2$ if $\delta = (1, 0)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, -1)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (0, -1)$	$r_{12}s_1\mu_1$ if $\delta = (0, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, 0)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-1, -1)$ $(1 - r_{21})s_2\mu_2$ if $\delta = (0, -1)$	$r_{12}s_1\mu_1$ if $\delta = (0, 1)$ $r_{21}s_2\mu_2$ if $\delta = (1, 0)$ $(1 - r_{12})s_1\mu_1$ if $\delta = (-\min(b_1 + 1, b_2 + 1), -\min(b_1, b_2 + 1))$ $(1 - r_{21})s_2\mu_2$ if $\delta = (-\min(b_1 + 1, b_2), -\min(b_1 + 1, b_2 + 1))$



Times to Deadlock



Markovian Model of Deadlock



(i, j)

$$S = \{(i, j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} \mid 0 \leq i + j \leq n_1 + n_2 + 2\} \cup \{(-1)\}$$

$$\text{Define } \delta = (i_2, j_2) - (i_1, j_1)$$

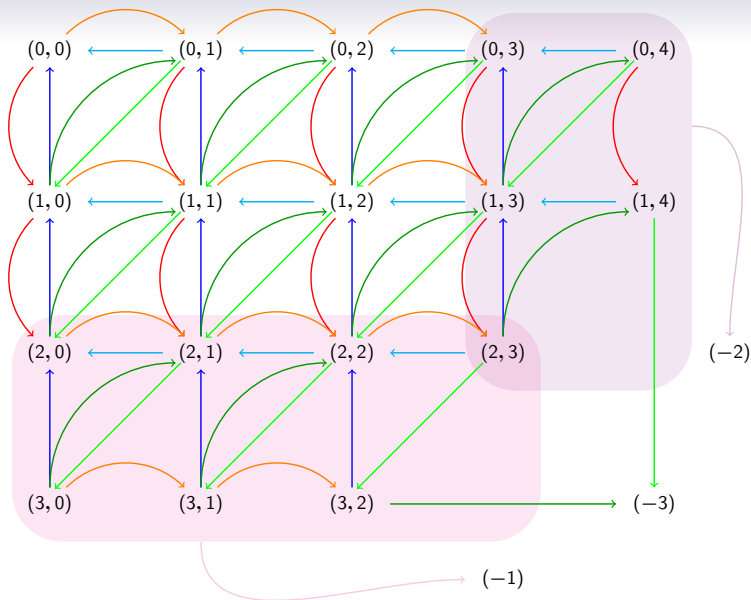
$$q_{(i_1, j_1), (i_2, j_2)} = \begin{cases} \left. \begin{array}{ll} \begin{array}{l} \Lambda_1 \\ 0 \end{array} & \begin{array}{l} \text{if } i_1 \leq n_1 \\ \text{otherwise} \end{array} \end{array} \right\} & \text{if } \delta = (1, 0) \\ \left. \begin{array}{ll} \begin{array}{l} \Lambda_2 \\ 0 \end{array} & \begin{array}{l} \text{if } j_1 \leq n_2 \\ \text{otherwise} \end{array} \end{array} \right\} & \text{if } \delta = (0, 1) \\ \left. \begin{array}{ll} \begin{array}{l} (1 - r_{12})\mu_1 \\ 0 \end{array} & \begin{array}{l} \text{if } j_1 < n_2 + 2 \\ \text{otherwise} \end{array} \end{array} \right\} & \text{if } \delta = (-1, 0) \\ \left. \begin{array}{ll} \begin{array}{l} (1 - r_{21})\mu_2 \\ 0 \end{array} & \begin{array}{l} \text{if } i_1 < n_1 + 2 \\ \text{otherwise} \end{array} \end{array} \right\} & \text{if } \delta = (0, -1) \\ \left. \begin{array}{ll} \begin{array}{l} r_{12}\mu_1 \\ 0 \end{array} & \begin{array}{l} \text{if } j_1 < n_2 + 2 \text{ and } (i_1, j_1) \neq (n_1 + 2, n_2) \\ \text{otherwise} \end{array} \end{array} \right\} & \text{if } \delta = (-1, 1) \\ \left. \begin{array}{ll} \begin{array}{l} r_{21}\mu_2 \\ 0 \end{array} & \begin{array}{l} \text{if } i_1 < n_1 + 2 \text{ and } (i_1, j_1) \neq (n_1, n_2 + 2) \\ \text{otherwise} \end{array} \end{array} \right\} & \text{if } \delta = (1, -1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1, j_1), (-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2 + 2 \\ 0 & \text{otherwise} \end{cases}$$

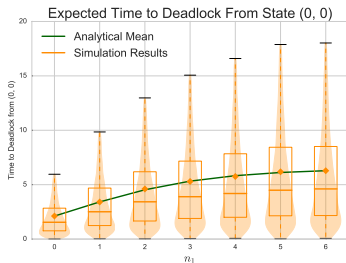
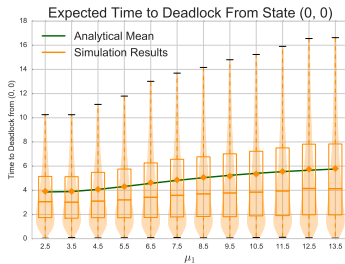
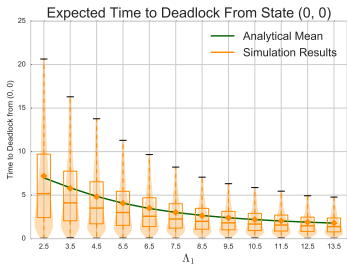
$$q_{(i_1, j_1), (-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1 + 2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1, j_1), (-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i, j) = (n_1, n_2 + 2) \\ r_{12}\mu_1 & \text{if } (i, j) = (n_1 + 2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1, s} = q_{-2, s} = q_{-3, s} = 0$$



Times to Deadlock



Recap about Health Stuff

Thank You

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