Queueing Networks for a Healthcare System

Deadlocking, Reinforcement Learning & Workforce Planning

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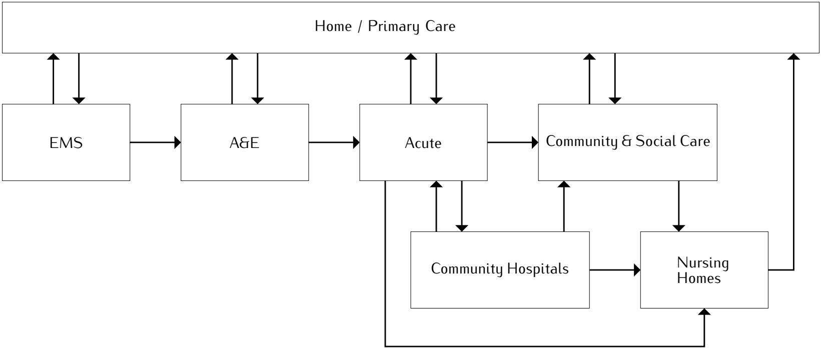
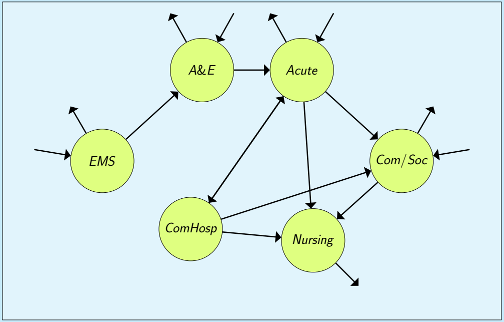
EURO 2015 - Glasgow

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

Aneurin Bevan University Health Board

Map of Healthcare System





Upload Parameters

Run Analysis

Run Simulation

View Results

Nodes:

EMS - *EDIT*

A&E - *EDIT*

Acute - *EDIT*

Community & Social Care - *EDIT*

Community Hospital - *EDIT*

Nursing Homes - *EDIT*

Community & Social Care

Winter Workforce Requirements:

Skill 1: ##### WTEs

Skill 2: ##### WTEs

Skill 3: ##### WTEs

Skill 4: ##### WTEs

Skill 5: ##### WTEs

Summer Workforce Requirements:

Skill 1: ##### WTEs

Skill 2: ##### WTEs

Skill 3: ##### WTEs

Skill 4: ##### WTEs

Skill 5: ##### WTEs

Spring Workforce Requirements:

Skill 1: ##### WTEs

Skill 2: ##### WTEs

Skill 3: ##### WTEs

Skill 4: ##### WTEs

Skill 5: ##### WTEs

Autumn Workforce Requirements:

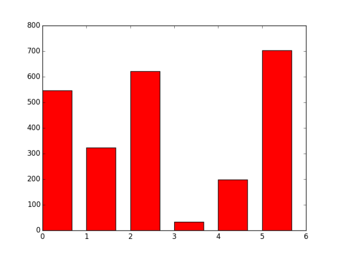
Skill 1: ##### WTEs

Skill 2: ##### WTEs

Skill 3: ##### WTEs

Skill 4: ##### WTEs

Skill 5: ##### WTEs



Performance Measures:

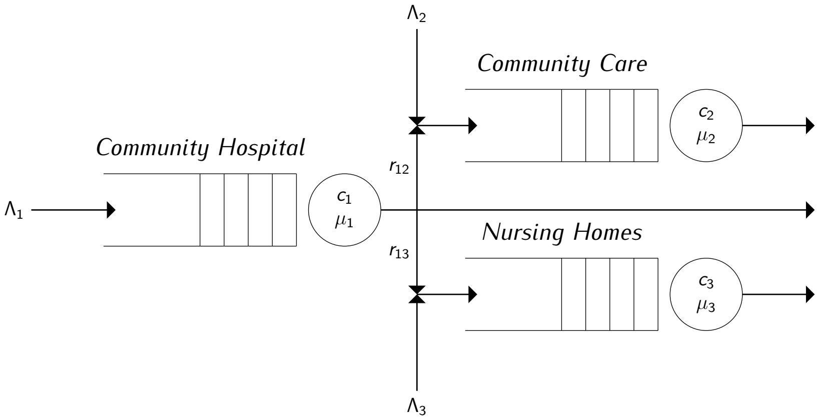
Winter Expected Occupancy: #####

Spring Exprected Occupancy: #####

Summer Expected Occupancy: #####

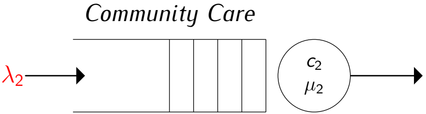
Autumn Expected Occupancy: #####

Jackson Networks



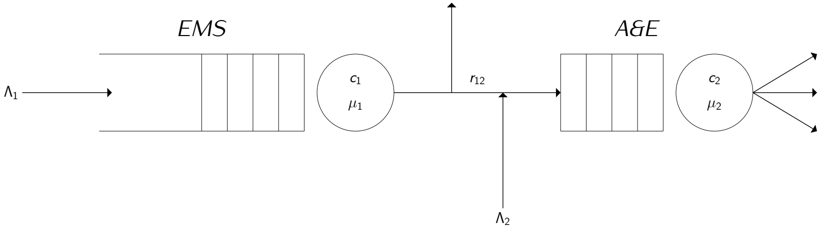
Jackson Networks

$$\lambda_i = \Lambda_i + \sum_j r_{ji}\lambda_j$$



$$P(k_1, k_2, \ldots, k_M) = \prod_{i=1}^{M} P_i(k_i)$$

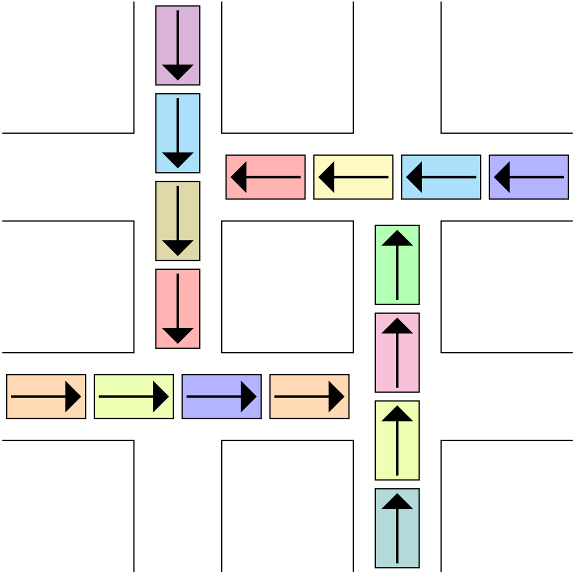
Restricted Networks

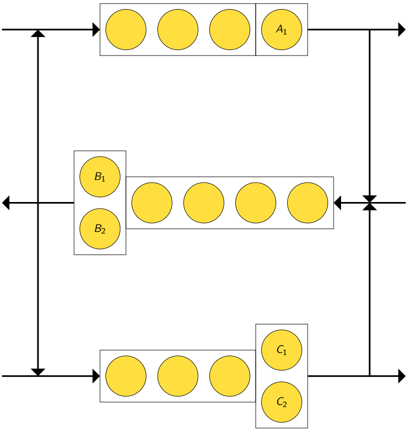
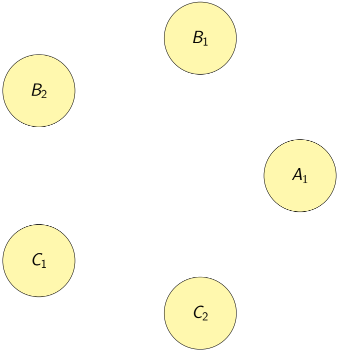


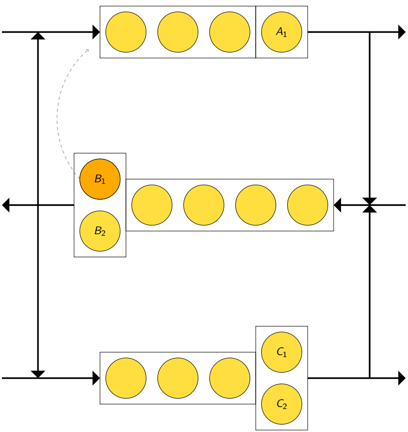
Restricted Networks

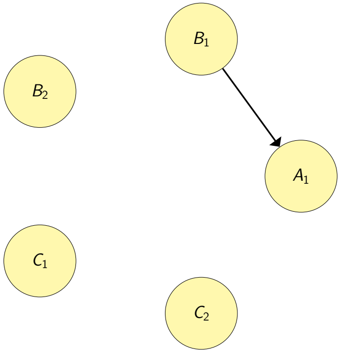
- Markov Chain Models
- Approximation Methods
- Simulation

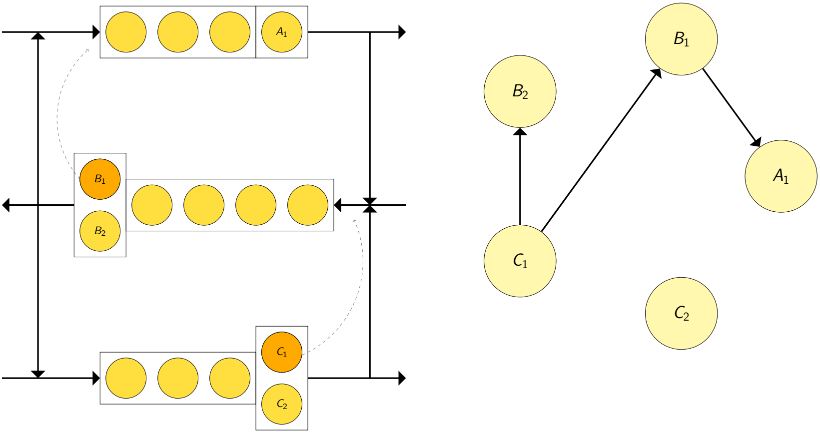
Deadlock



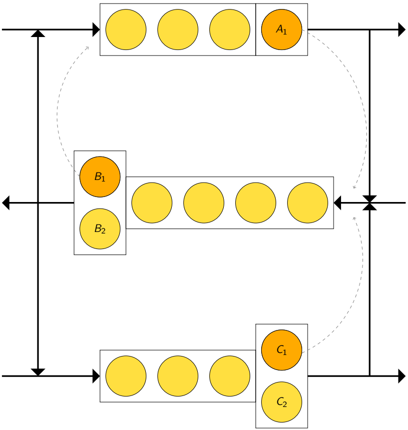

A1
B1
B2
C1
C2

B1
B2
A1
C1
C2

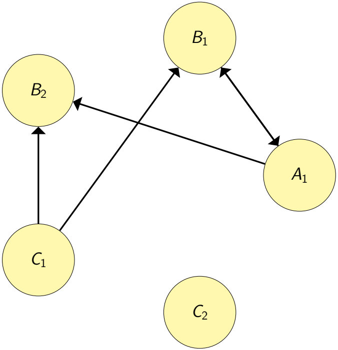

A_1
B_1
B_2
C_1
C_2

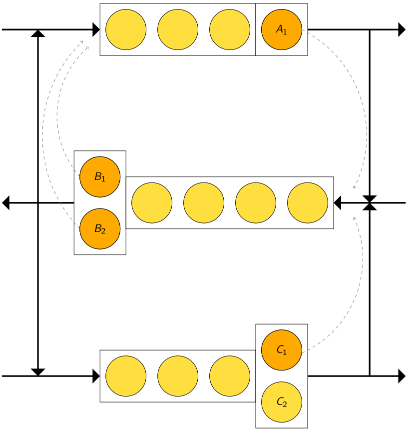

B_1
B_2
A_1
C_1
C_2

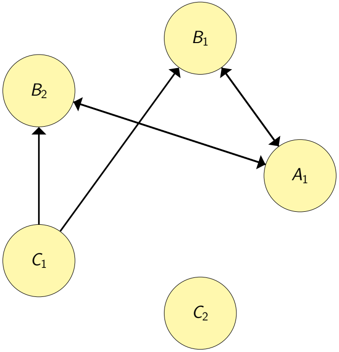


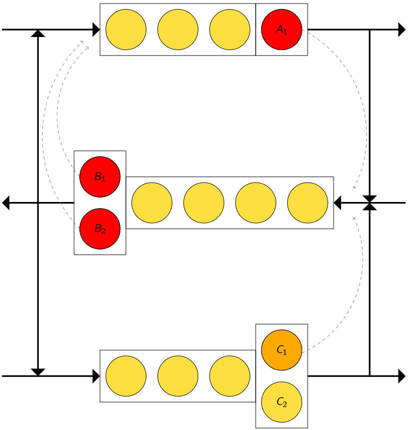
A_1
B_1
B_2
C_1
C_2
B_1
B_2
A_1
C_1
C_2

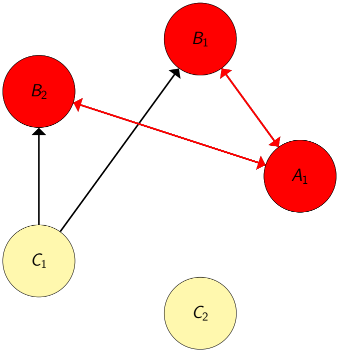

A_1
B_1
B_2
C_1
C_2


B_1
B_2
A_1
C_1
C_2

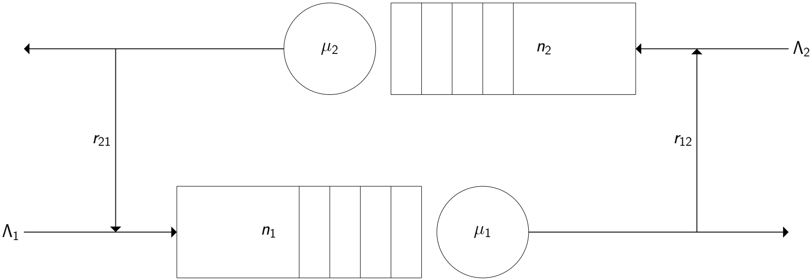

A_1
B_1
B_2
C_1
C_2


B_1
B_2
A_1
C_1
C_2


A_1
B_1
B_2
C_1
C_2


B_1
B_2
A_1
C_1
C_2

Markovian Model of Deadlock



$$(i, j)$$

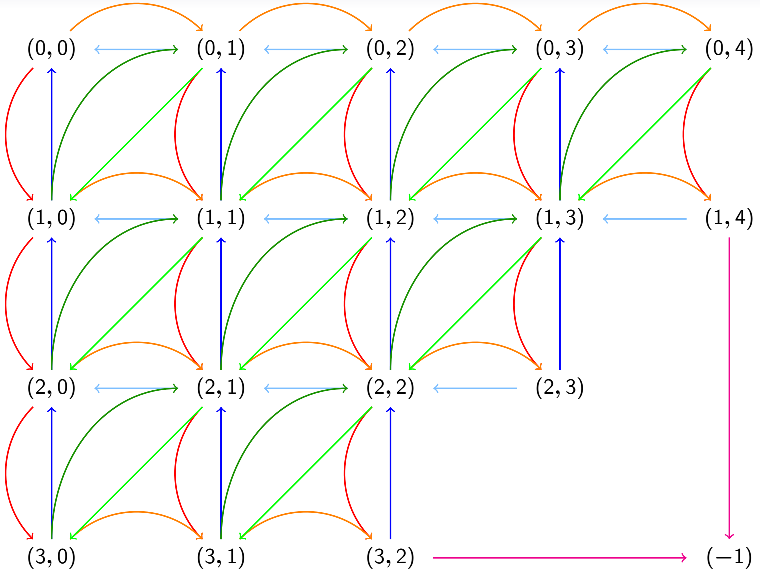
$$S = \{(i,j) \in \mathbb{N}^{(n_1+2\times n_2+2)} | 0 \leq i+j \leq n_1+n_2+2\} \cup \{(-1)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

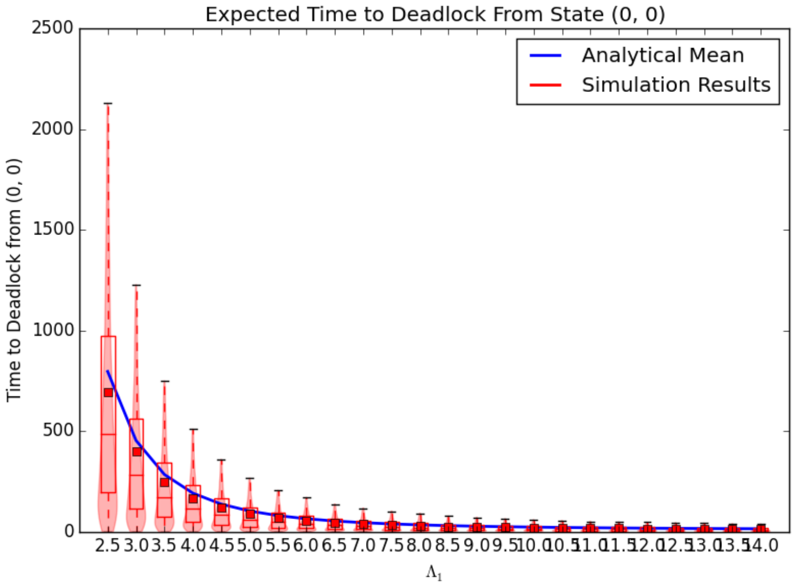
$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \left.\begin{matrix} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,0) \\ \left.\begin{matrix} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,1) \\ \left.\begin{matrix} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,0) \\ \left.\begin{matrix} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (0,-1) \\ \left.\begin{matrix} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (-1,1) \\ \left.\begin{matrix} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{matrix}\right\} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

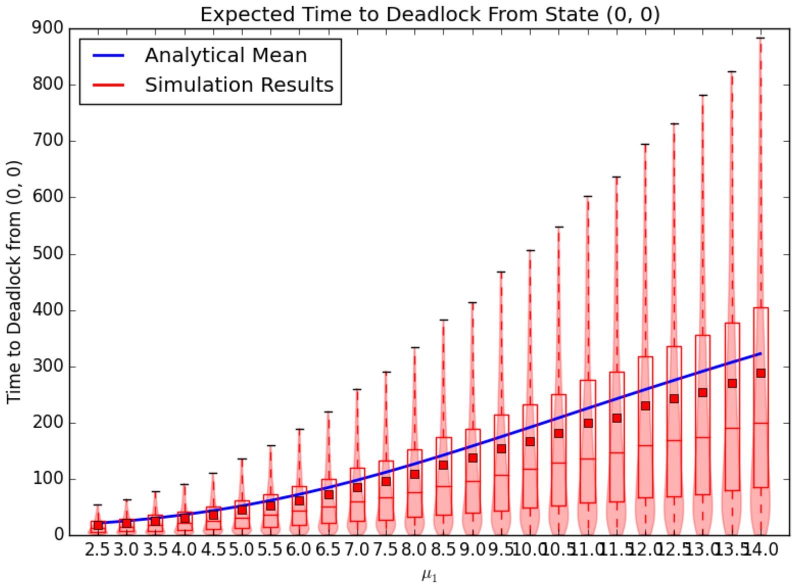
$$q_{-1,s} = 0$$


(0, 0)
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(3, 0)
(3, 1)
(3, 2)
(−1)

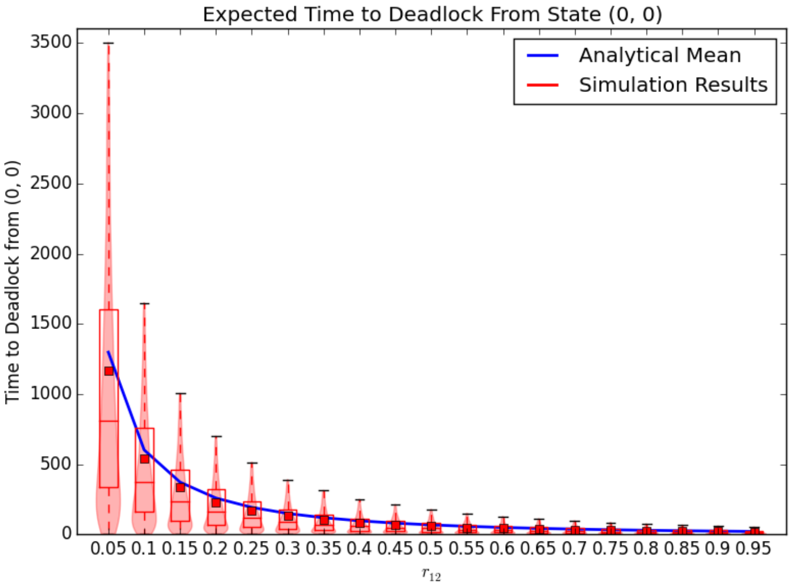
Times to Deadlock - Λ_1



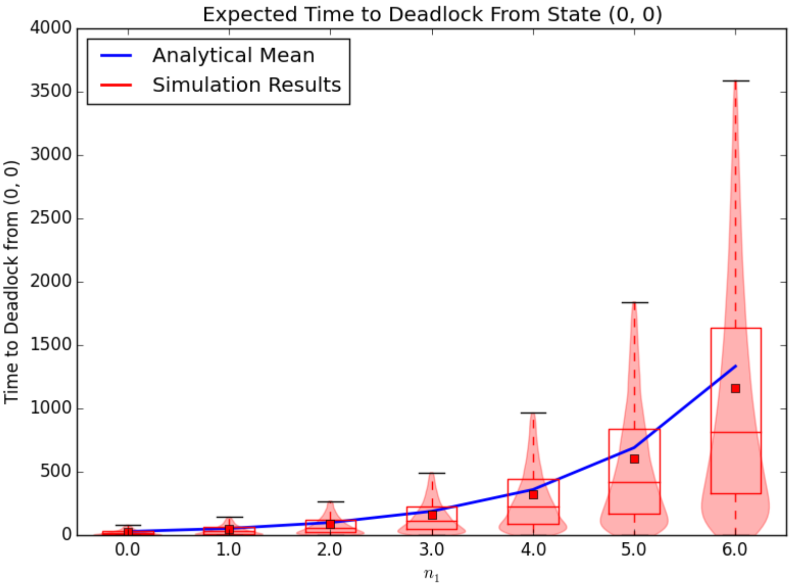
Times to Deadlock - μ_1



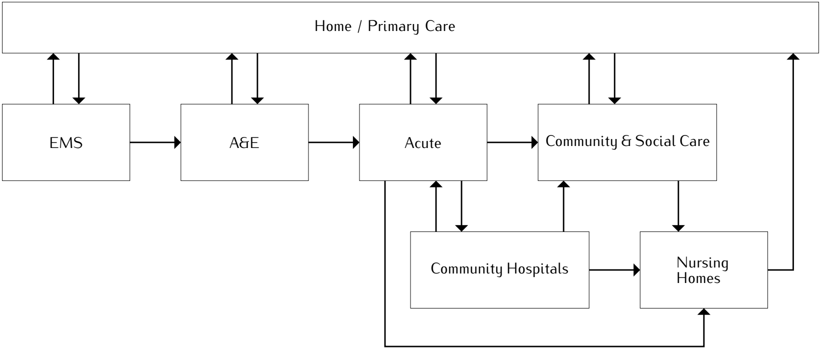
Times to Deadlock - r_{12}



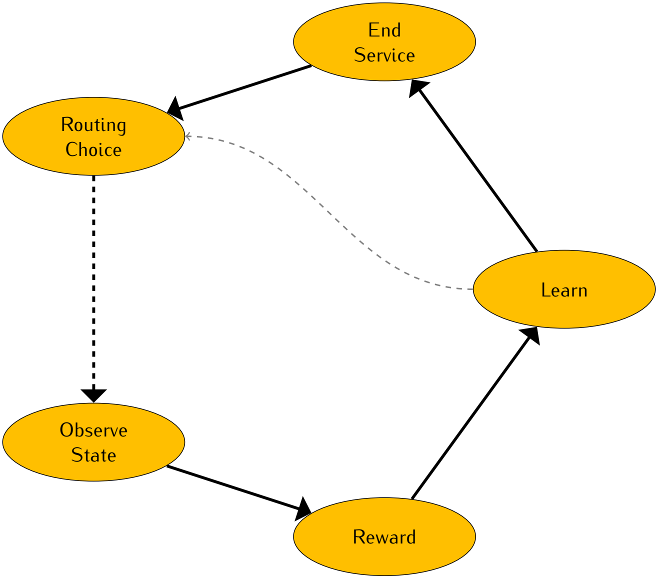
Times to Deadlock – n_1



Reinforcement Learning



Reinforcement Learning



Reinforcement Learning

Q-Learning

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

Diolch - Thank You

https://github.com/geraintpalmer/Presentations
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