

Python for Operational Research in Healthcare

There's a library for that...

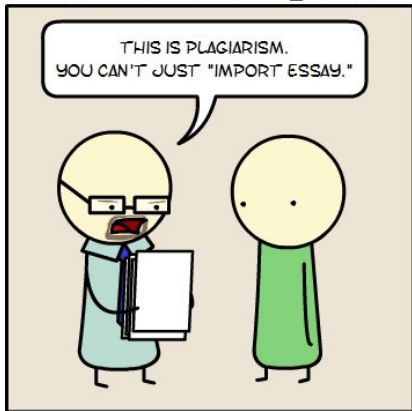
@GeraintPalmer

PyCon UK 2017





PYTHON



<http://i.imgur.com/ZyeC0.jpg>

The Basics

| | Sex | Height | Weight |
|---|-----|------------|------------|
| 0 | M | 187.306088 | 72.233276 |
| 1 | M | 170.595112 | 92.195728 |
| 2 | F | 157.637346 | 64.835601 |
| 3 | M | 162.010640 | 130.462244 |
| 4 | F | 154.017198 | 81.568846 |
| ⋮ | ⋮ | ⋮ | ⋮ |

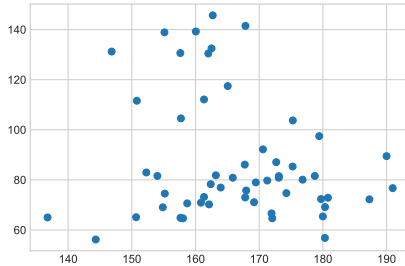
The Basics

| | Sex | Height | Weight |
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| : | : | : | : |

```
>>> import scipy.stats
```

```
>>> scipy.stats.ttest_ind(  
...     df[df['Sex']=='M']['Height'],  
...     df[df['Sex']=='F']['Height']  
... ).pvalue  
0.070033630470421021
```

Machine Learning



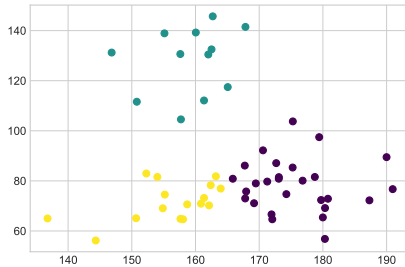
```
>>> from sklearn.cluster import KMeans
```

```
>>> kmeans = KMeans(n_clusters=3, random_state=0)
```

```
>>> kmeans.fit(df[['Height', 'Weight']])
```

```
>>> df['Cluster'] = kmeans.labels_
```

Machine Learning



```
>>> from sklearn.cluster import KMeans
```

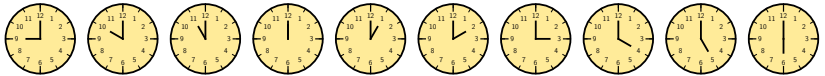
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```

```
>>> kmeans.fit(df[['Height', 'Weight']])
```

```
>>> df['Cluster'] = kmeans.labels_
```

Optimisation

7 8 9 9 7 5 4 8 4 3



Full Time

£7.50 per hour



4hrs, 1hr break, 3hrs



Part Time

£8 per hour



4hrs

$$\underline{x} = [F_9, F_{10}, F_{11}, P_9, P_{10}, P_{11}, P_{12}, P_1, P_2, P_3]$$

$$C = [\pounds60, \pounds60, \pounds60, \pounds32, \pounds32, \pounds32, \pounds32, \pounds32, \pounds32, \pounds32]$$

$$A = \begin{bmatrix} 11 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 17 \\ 8 \\ 9 \\ 9 \\ 7 \\ 5 \\ 4 \\ 8 \\ 4 \\ 3 \end{bmatrix}$$

min Cx
subject to:

$$A\underline{x} \geq B$$

$$\underline{x} \geq \underline{0}$$

```
>>> import pulp

>>> prob = pulp.LpProblem("Nurse Rostering", pulp.LpMinimize)
>>> x = pulp.LpVariable.dicts("x", range(10), cat=pulp.LpInteger)

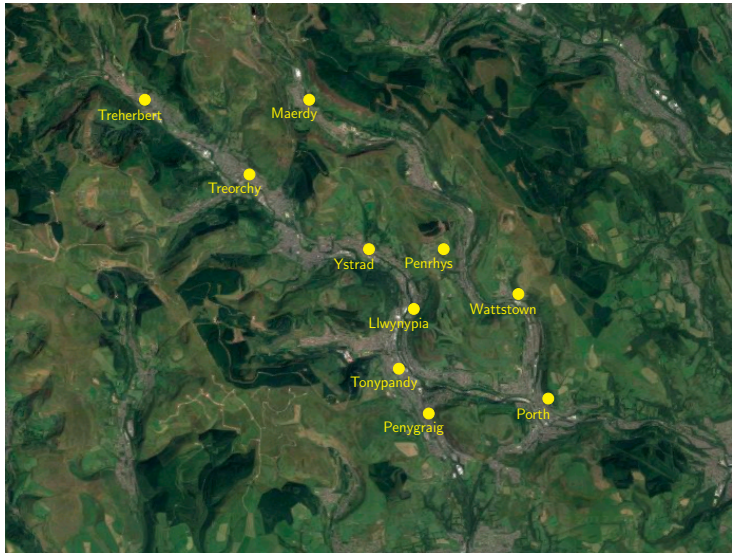
>>> objective_funtion = sum(C[i] * x[i] for i in range(10))
>>> prob += objective_funtion

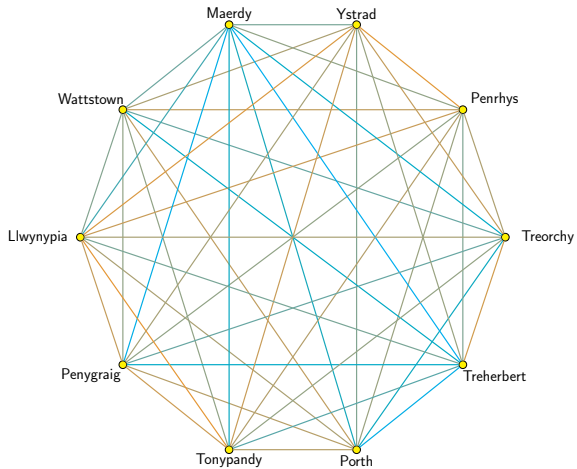
>>> for j in range(10):
...     prob += sum(A[j][i] * x[i] for i in range(10)) >= B[j]

>>> for j in range(10):
...     prob += x[j] >= 0

>>> prob.solve()
>>> [pulp.value(i) for i in x]
[-0.0, 1.0, 2.0, 7.0, -0.0, -0.0, -0.0, 4.0, -0.0, 1.0]
```

Graph Theory





```
>>> import networkx as nx

>>> towns = ['Porth', 'Wattstown', 'Penrhys', 'Maerdy',
...          'Penygraig', 'Tonypandy', 'Llwynypia', 'Ystrad',
...          'Treorchy', 'Treherbert']

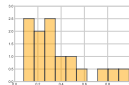
>>> D = nx.from_numpy_matrix(distances)
>>> D = nx.relabel_nodes(D,
...    {i: towns[i] for i in range(10)})

>>> ranks = nx.betweenness_centrality(D, weight='weight')
>>> max(ranks.keys(), key=lambda x: centrality[x])
'Ystrad'
```

Simulation

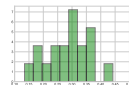
Low Priority

10 per hour

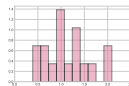


Medium Priority

5 per hour

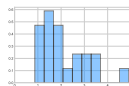


2 per hour



High Priority

$\frac{1}{2}$ per hour



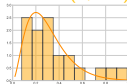
Simulation

Low Priority

10 per hour



Gamma(3, 0.1)



Medium Priority

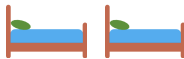
5 per hour



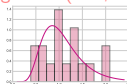
Lognormal(0, 0.4)



2 per hour



Lognormal(-1.2, 0.25)

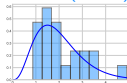


High Priority

$\frac{1}{2}$ per hour



Gamma(4, 0.5)



```

>>> import ciw

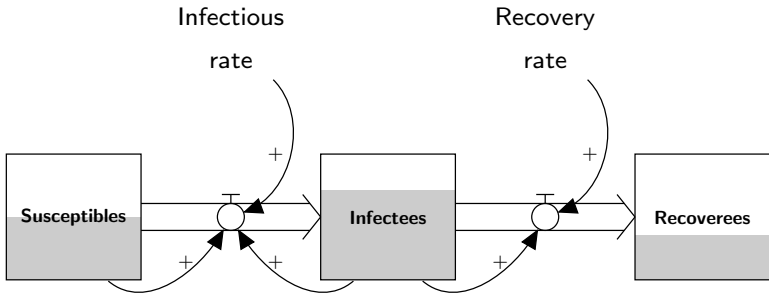
>>> N = ciw.create_network(
...     Arrival_distributions={
...         'Class 0': [['Exponential', 10.0]],
...         'Class 1': [['Exponential', 5.0]],
...         'Class 2': [['Exponential', 2.0]],
...         'Class 3': [['Exponential', 0.5]]},
...     Service_distributions={
...         'Class 0': [['Gamma', 3, 0.1]],
...         'Class 1': [['Lognormal', -1.2, 0.25]],
...         'Class 2': [['Lognormal', 0, 0.4]],
...         'Class 3': [['Gamma', 4, 0.5]]},
...     Number_of_servers=[10],
...     Priority_classes={
...         'Class 0': 2, 'Class 1': 1, 'Class 2': 1, 'Class 3': 0
...     }
... )

>>> average_waits = []
>>> for i in range(10):
...     ciw.seed(1)
...     Q = ciw.Simulation(N)
...     Q.simulate_until_max_time(30)
...     recs = Q.get_all_records()
...     waits = [r.waiting_time for r in recs if r.arrival_date >= 6]
...     average_waits.append(np.mean(waits))

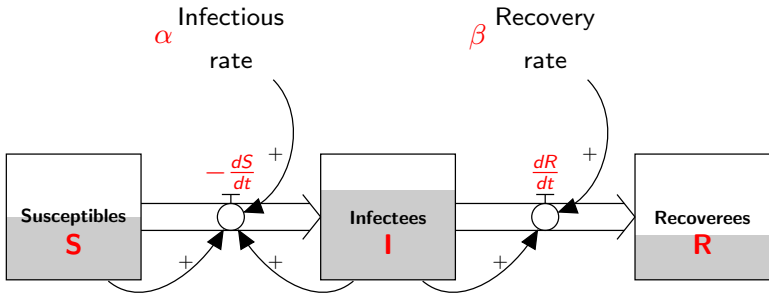
>>> np.mean(average_waits)
0.11887656711933872

```


System Dynamics



System Dynamics



$$\frac{dS}{dt} = -\alpha SI$$

$$\frac{dR}{dt} = \beta I$$

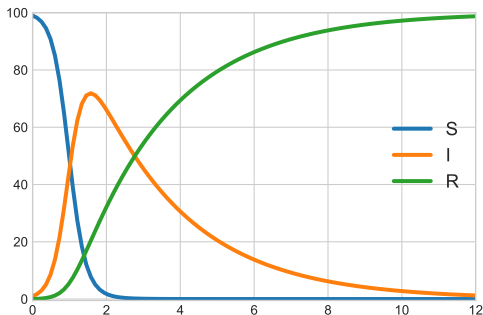
$$\frac{dI}{dt} = -\frac{dS}{dt} - \frac{dR}{dt}$$

```
>>> from scipy.integrate import odeint

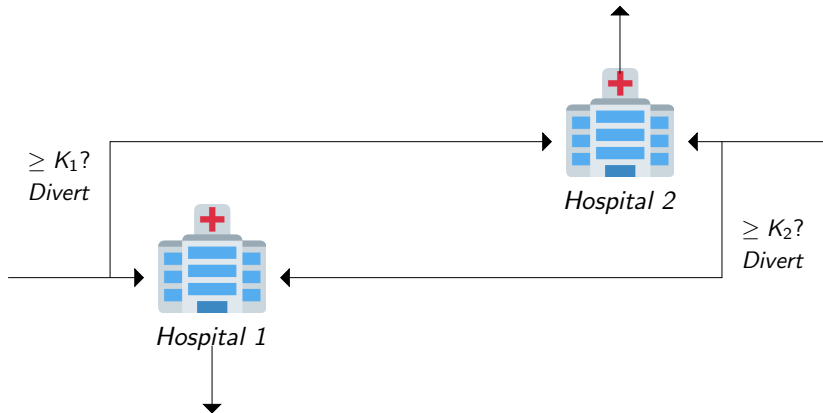
>>> def dsir(sir, t, a, b):
...     s, i, r = sir
...     return -a*s*i, a*s*i - b*i, b*i

>>> t = np.linspace(0, 12, 100)
>>> a, b = 0.05, 0.4

>>> sirs = odeint(func=dsir, y0=[99, 1, 0], t=t, args=(a, b))
>>> plt.plot(t, sirs)
```



Game Theory



| | $K_2 = 0$ | $K_2 = 40$ | $K_2 = 65$ | $K_2 = \infty$ |
|----------------|--------------|--------------|--------------|----------------|
| $K_1 = 0$ | (59.6, 59.6) | (60.2, 59.6) | (51.8, 68.6) | (0.0, 119.0) |
| $K_1 = 40$ | (59.6, 60.2) | (59.6, 59.6) | (50.6, 67.9) | (38.4, 80.9) |
| $K_1 = 65$ | (68.6, 51.8) | (67.9, 50.6) | (59.6, 59.6) | (56.7, 62.5) |
| $K_1 = \infty$ | (119.0, 0.0) | (80.9, 38.4) | (62.5, 56.7) | (59.6, 59.6) |

| | $K_2 = 0$ | $K_2 = 40$ | $K_2 = 65$ | $K_2 = \infty$ |
|----------------|--------------|--------------|--------------|----------------|
| $K_1 = 0$ | (59.6, 59.6) | (60.2, 59.6) | (51.8, 68.6) | (0.0, 119.0) |
| $K_1 = 40$ | (59.6, 60.2) | (59.6, 59.6) | (50.6, 67.9) | (38.4, 80.9) |
| $K_1 = 65$ | (68.6, 51.8) | (67.9, 50.6) | (59.6, 59.6) | (56.7, 62.5) |
| $K_1 = \infty$ | (119.0, 0.0) | (80.9, 38.4) | (62.5, 56.7) | (59.6, 59.6) |

```
>>> import nash
```

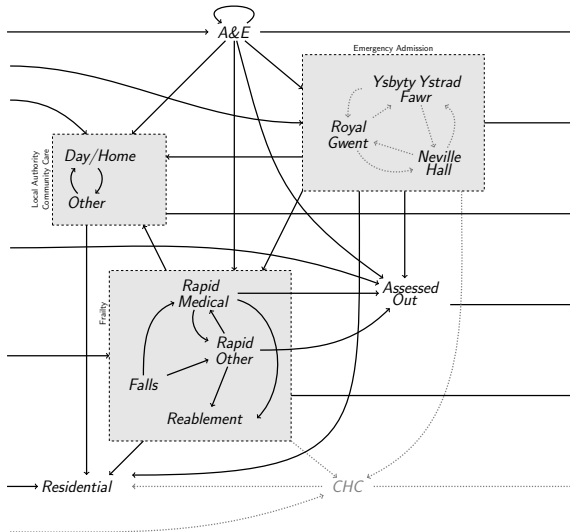
```
>>> g = nash.Game(-01, -02)
```

```
>>> eqs = g.support_enumeration()
```

```
>>> list(eqs)
```

```
[(array([ 0.,  1.,  0.,  0.]), array([ 0.,  1.,  0.,  0.])))]
```

My PhD Work...



```
pip install pandas
pip install numpy
pip install matplotlib
pip install scipy==0.19.1
pip install sklearn==0.17.1
pip install pulp==1.6.8
pip install networkx==1.11
pip install ciw==1.1.3
pip install nashpy==0.0.11
```

Emojis thanks to Twemoji.