Deadlock in Queueing Networks

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SWORDS - 2015

Generic Queueing Networks

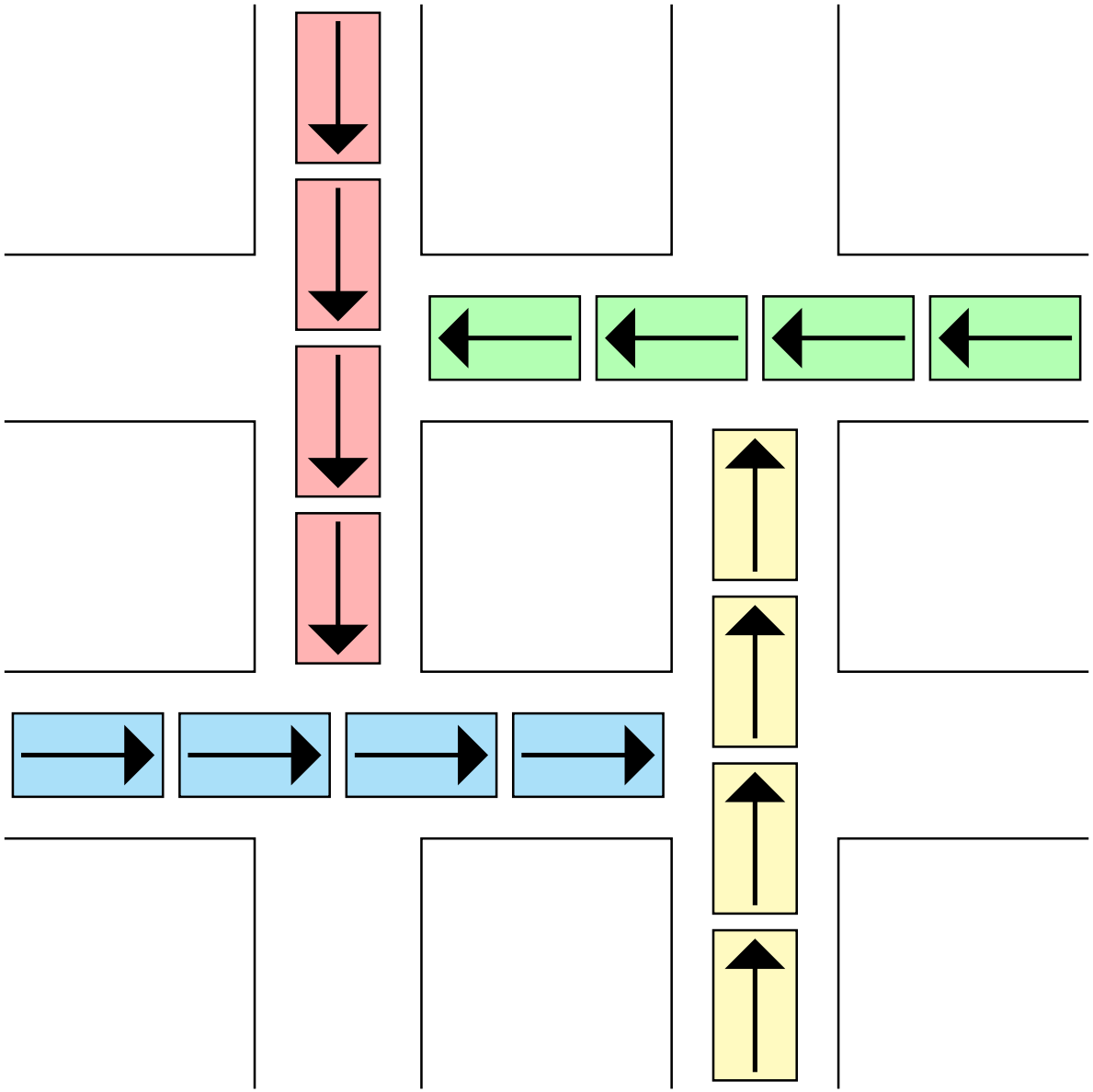
Open Unrestricted Queueing Networks

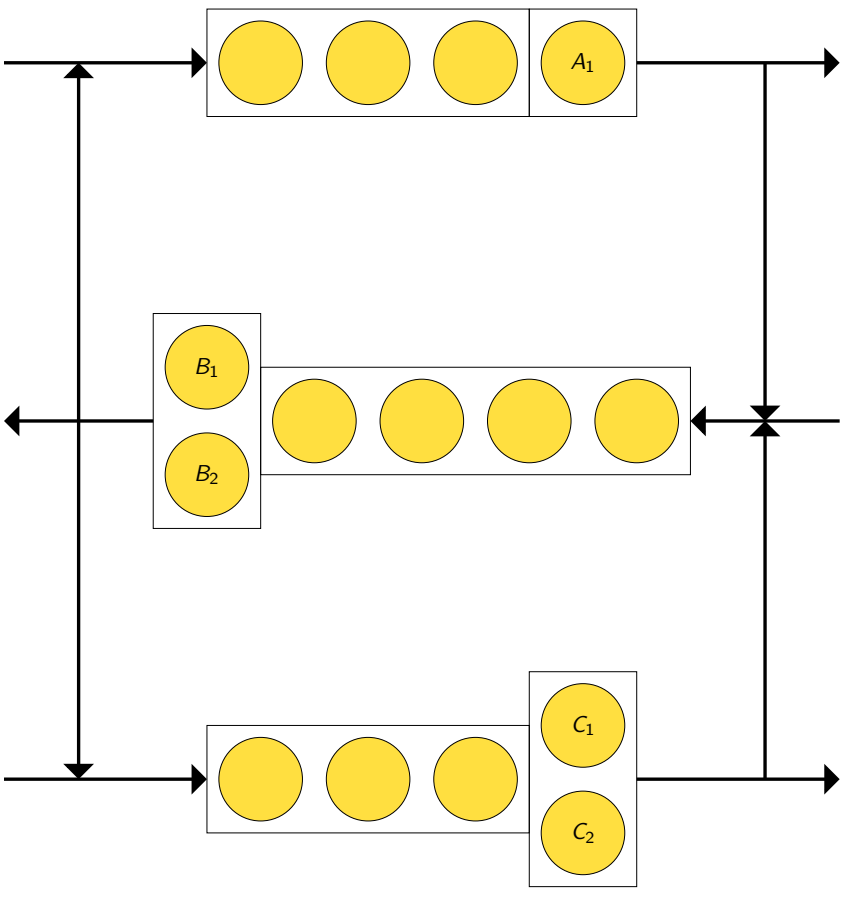
- Simple analytically
- Jackson networks
- Product-form solution

Open Restricted Queueing Networks

- Markov chain models
- Approximation methods
- Simulation

Deadlock





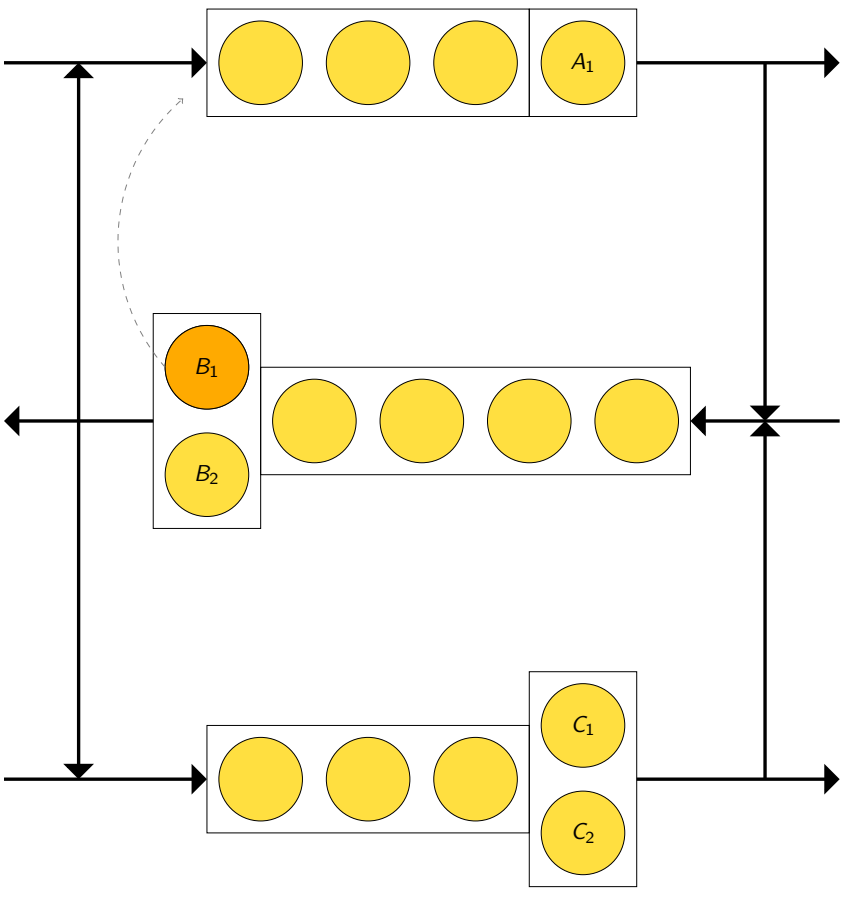
B_1

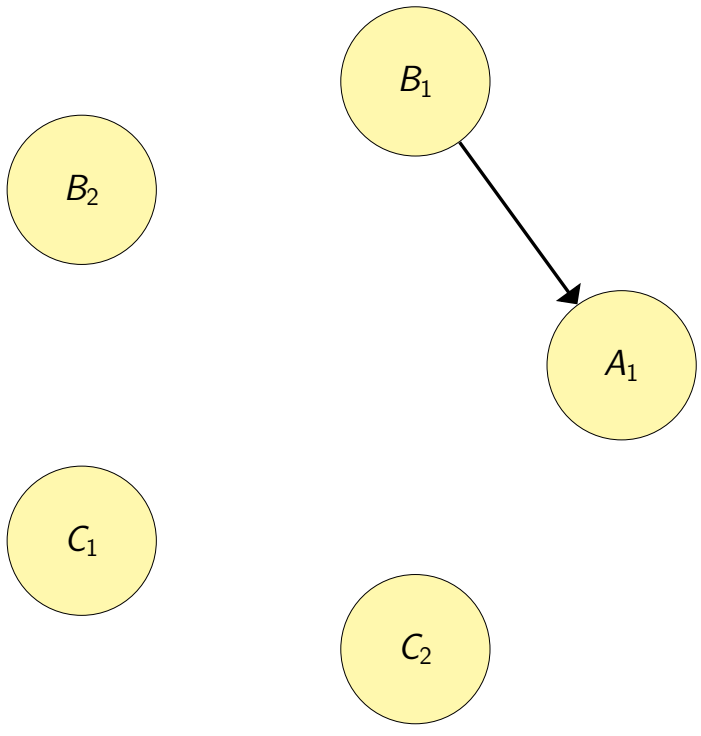
B_2

A_1

C_1

C_2


A_1
B_1
B_2
C_1
C_2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A_1
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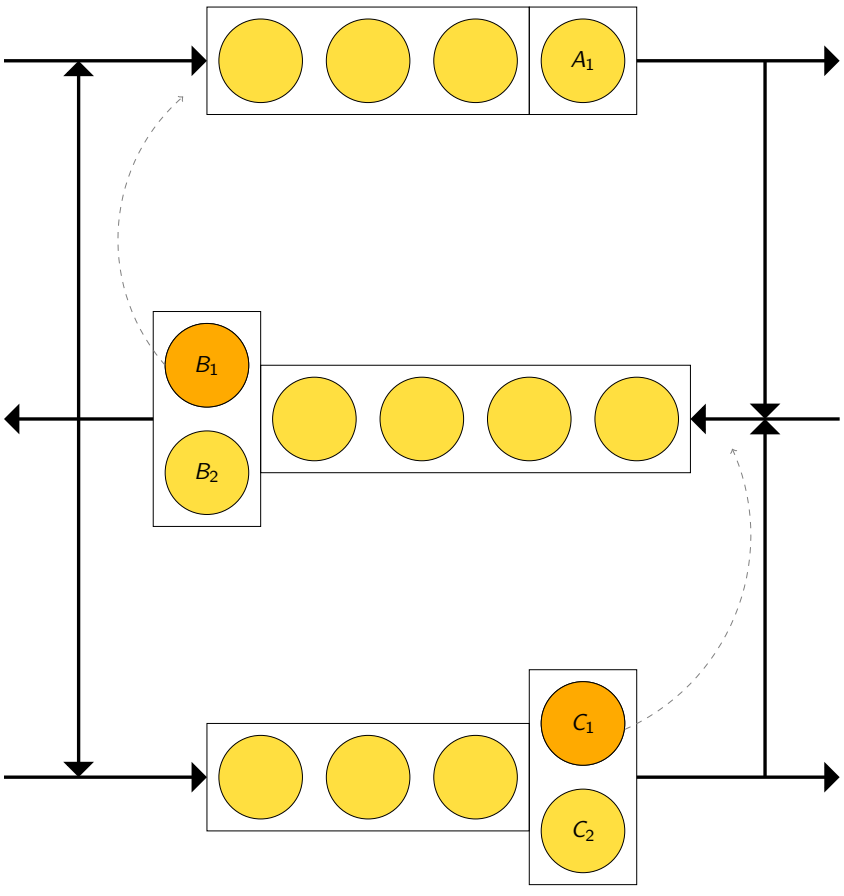
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B_1
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C_1
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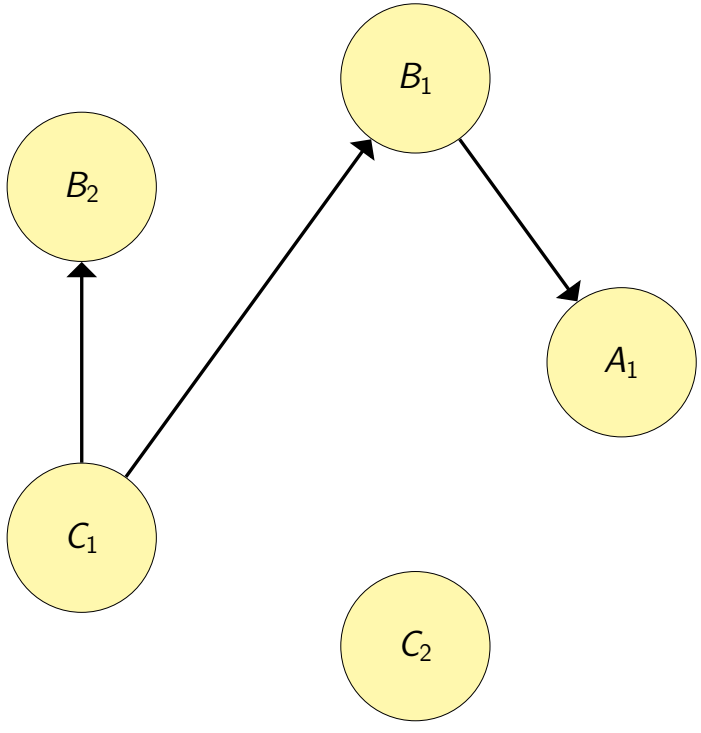
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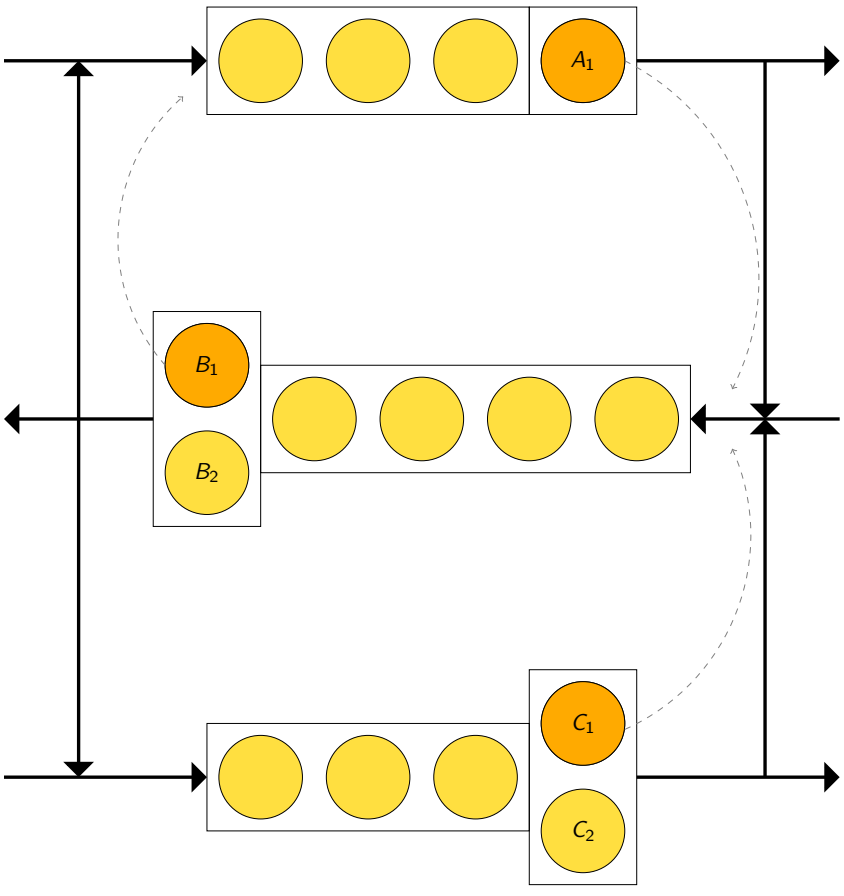
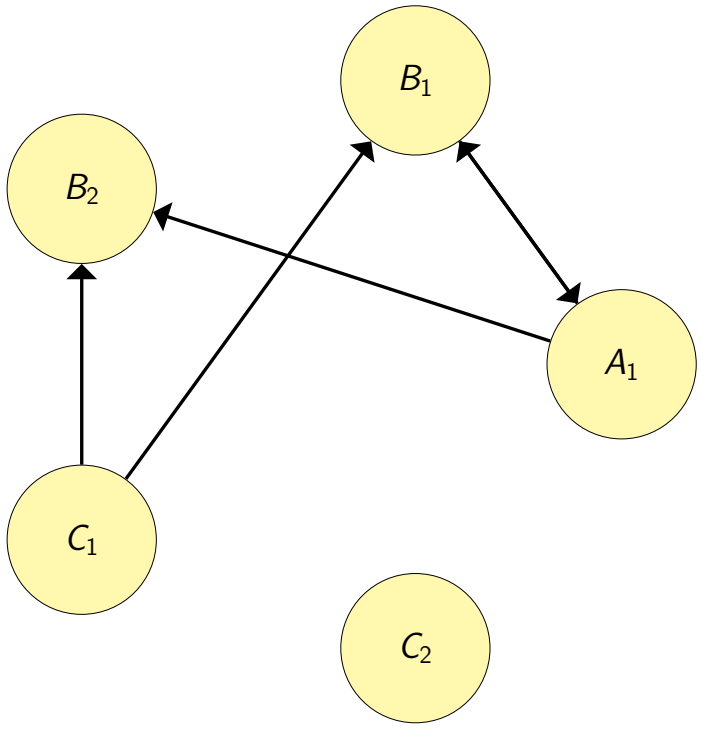

A_1
B_1
B_2
C_1
C_2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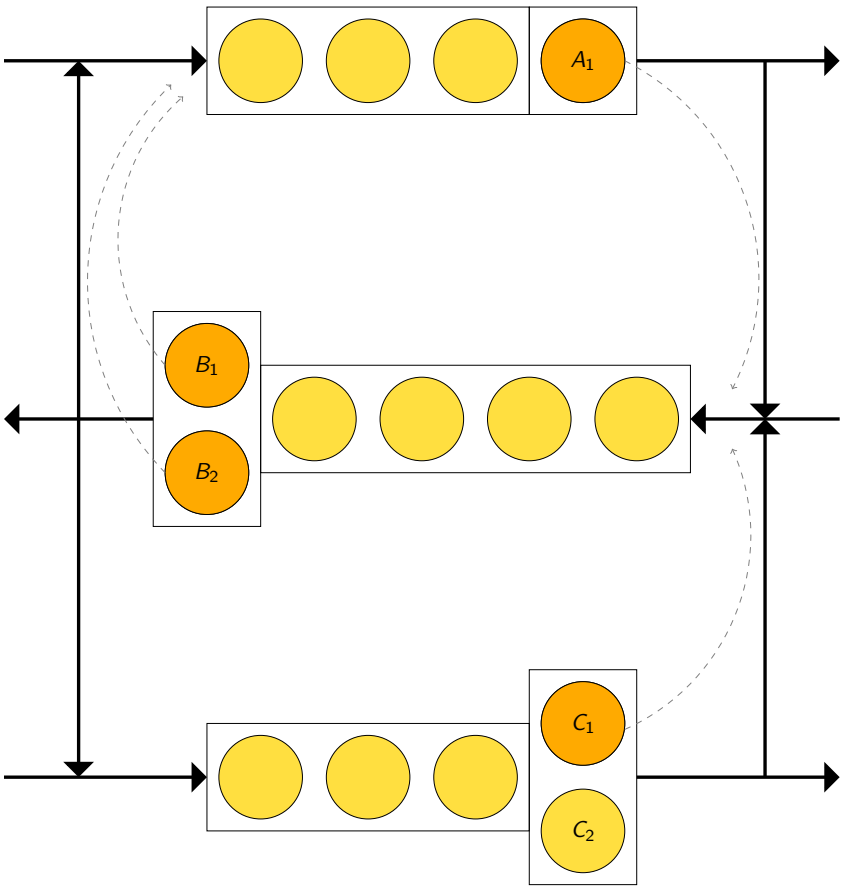
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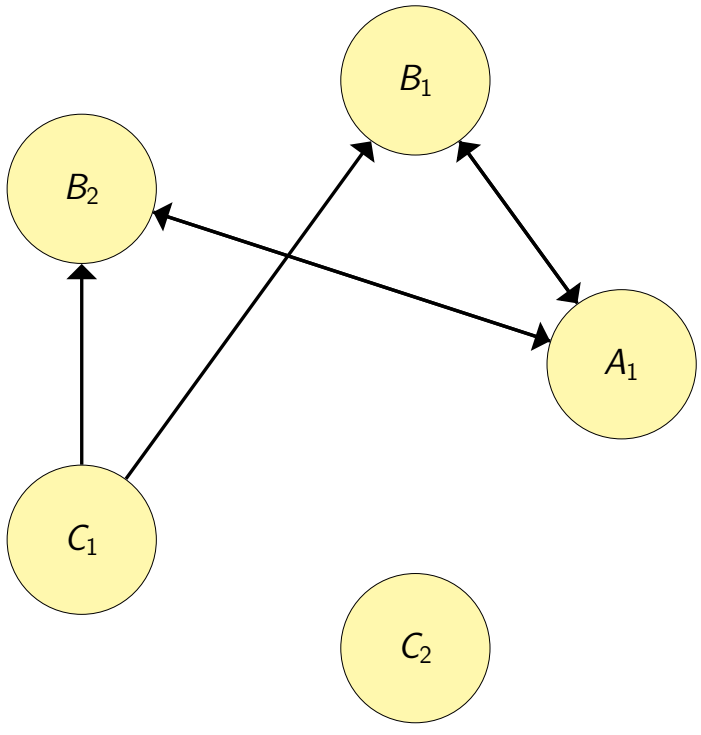
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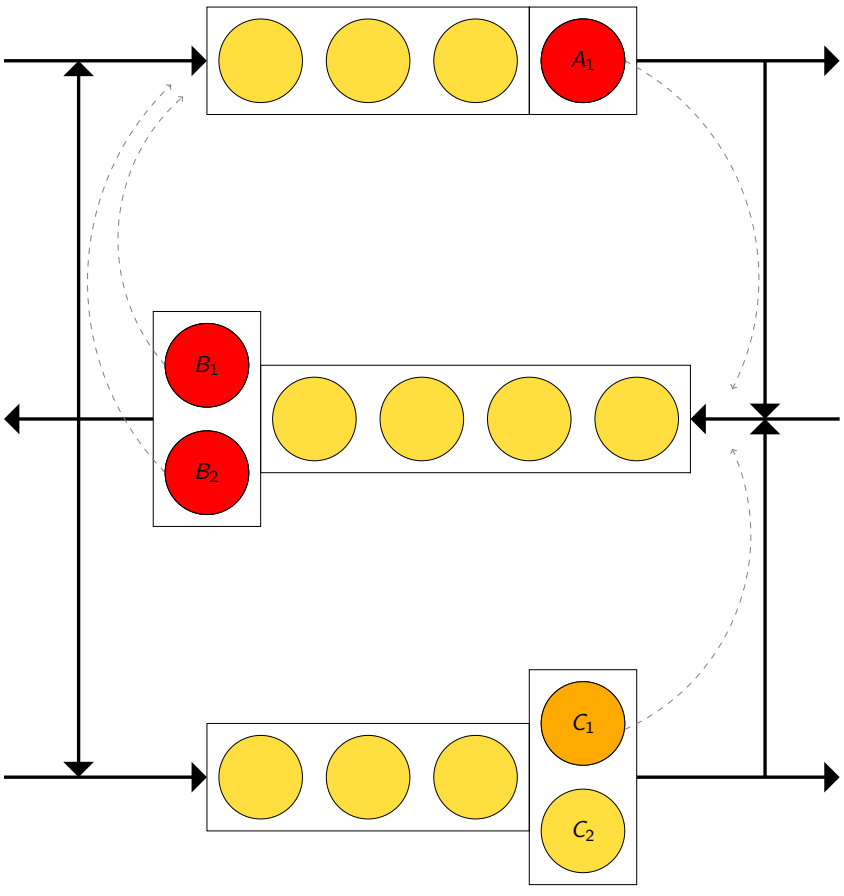
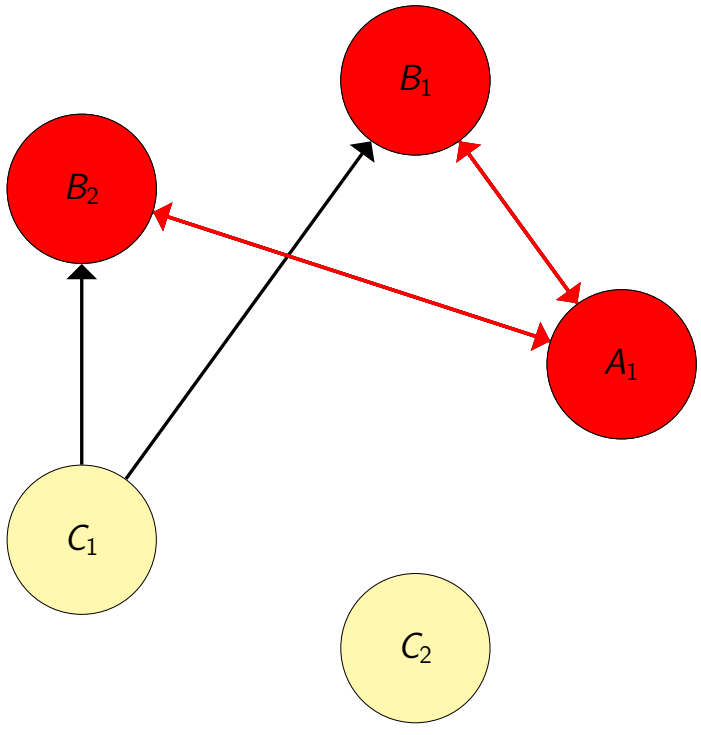
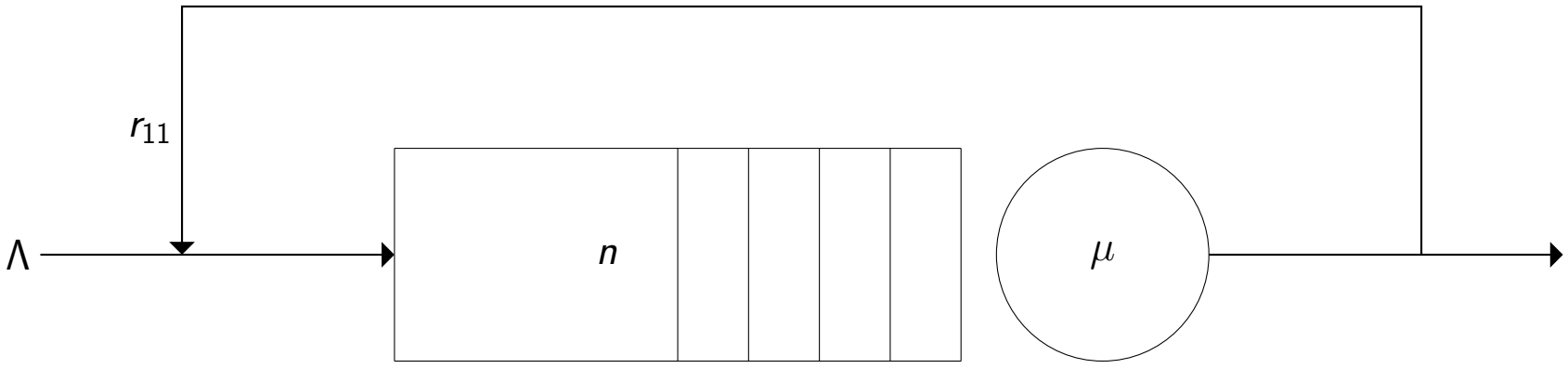

A_1
B_1
B_2
C_1
C_2


B_1
B_2
A_1
C_1
C_2

Markovian Model of Deadlock



(i)

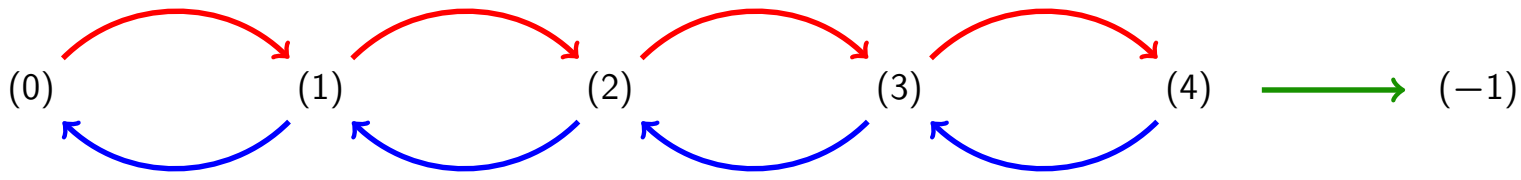
$$S = \{i \in \mathbb{N} \mid 0 \leq i \leq n+1\} \cup \{-1\}$$

Define $\delta = i_2 - i_1$

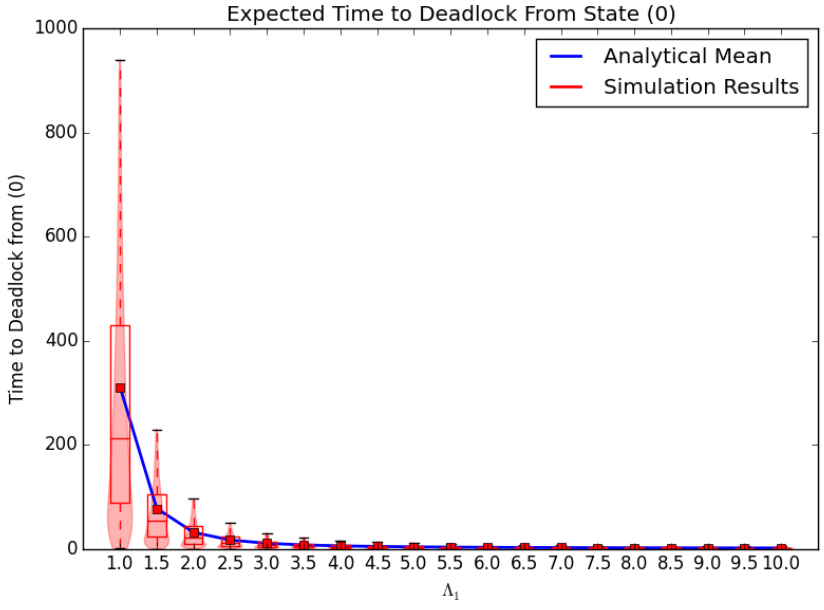
$$q_{i_1,i_2} = \begin{cases} \left.\begin{array}{ll} \Lambda & \text{if } i < n+1 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = 1 \\ (1 - r_{11})\mu & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{cases}$$

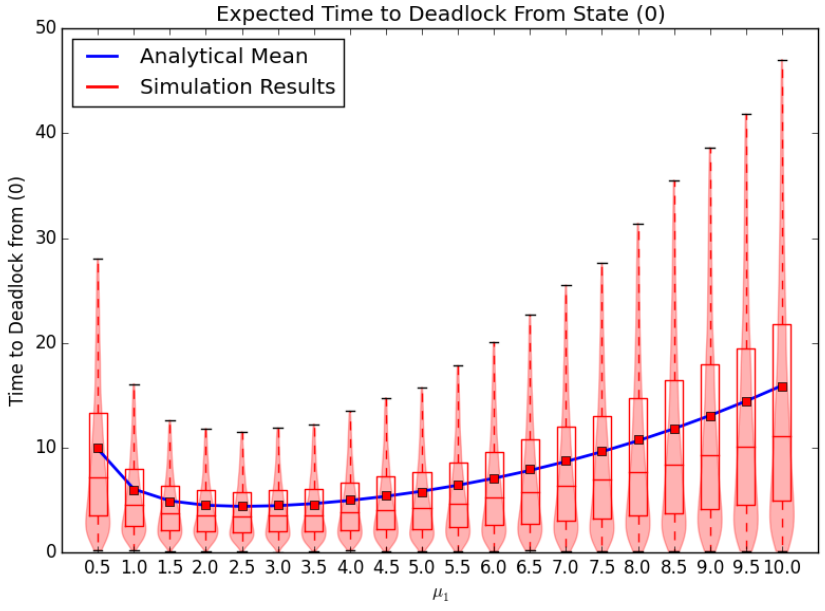
$$q_{i,-1} = \begin{cases} r_{11}\mu & \text{if } i = n+1 \\ 0 & \text{otherwise} \end{cases}$$

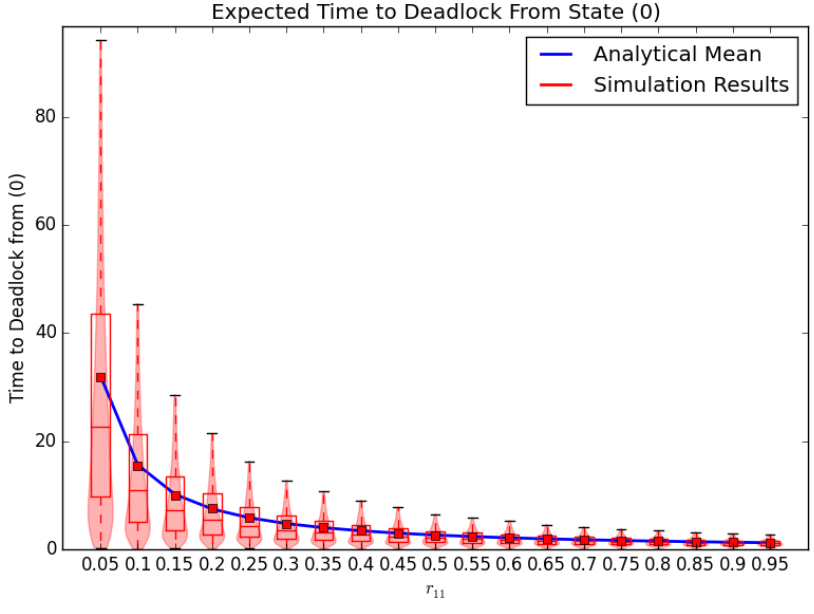
$$q_{-1,s} = 0$$

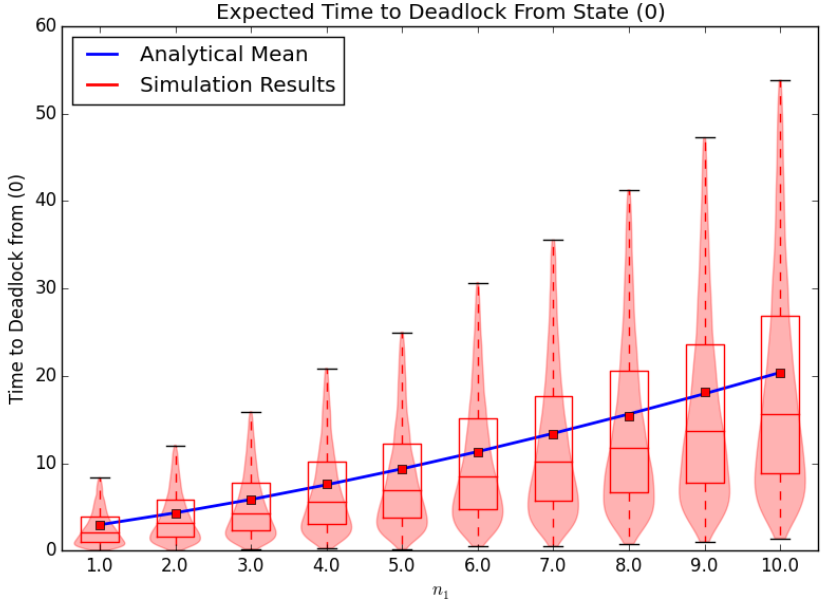

(0)
(1)
(2)
(3)
(4)
(−1)

Times to Deadlock

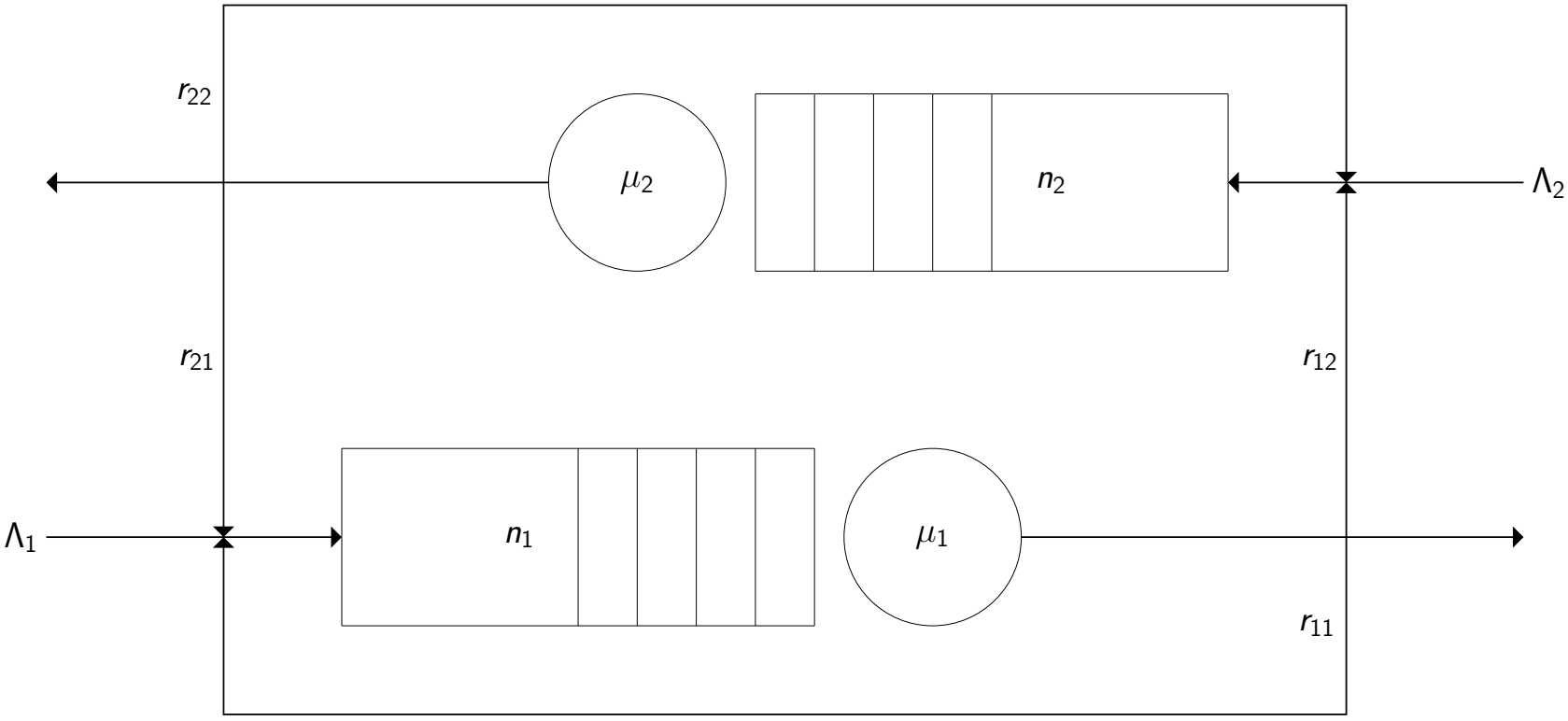








Markovian Model of Deadlock



(i, j)

$$S = \{(i,j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} \mid 0 \leq i+j \leq n_1+n_2+2\} \cup \{(-1)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

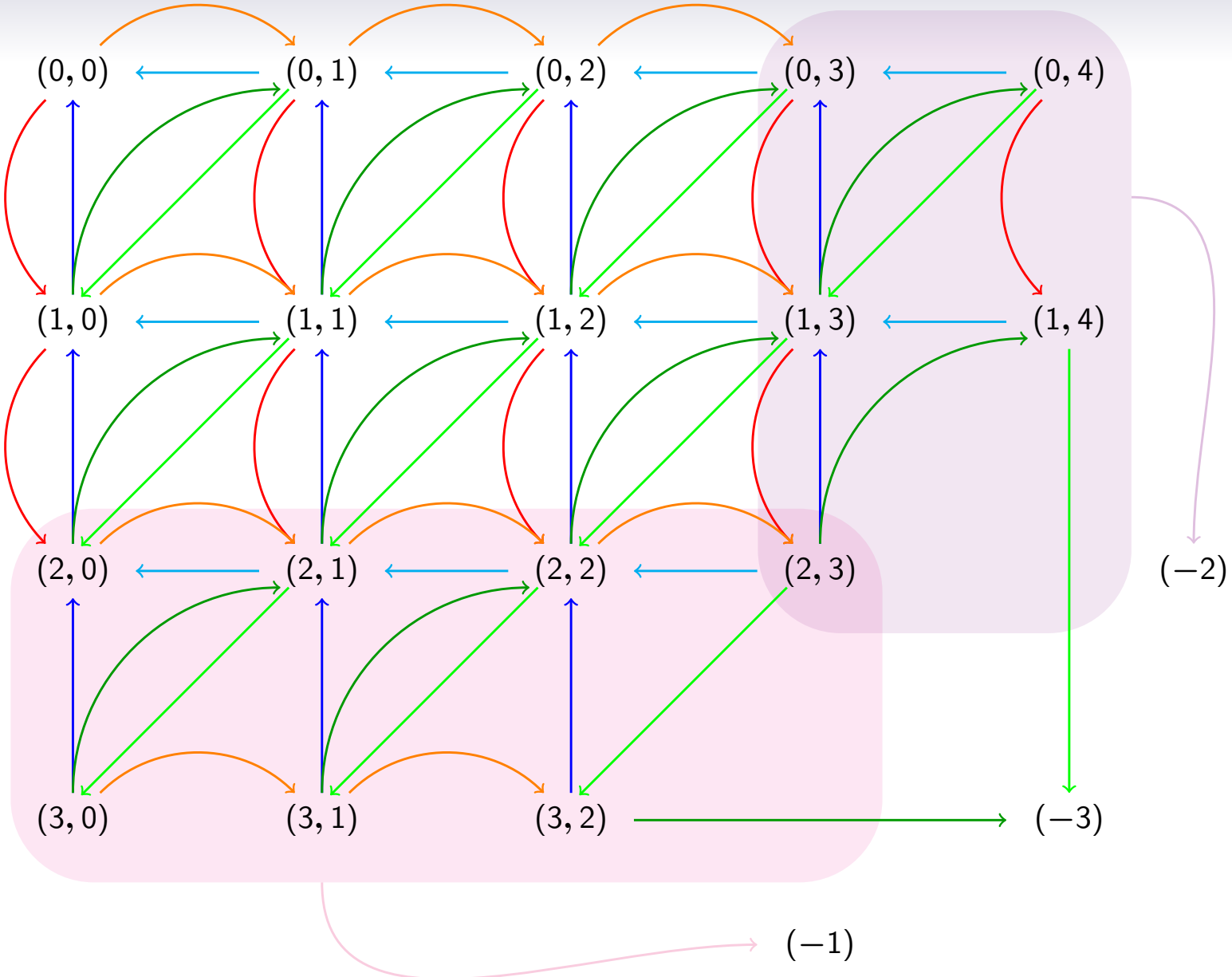
$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \begin{cases} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (1,0) \\ \begin{cases} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (0,1) \\ \begin{cases} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (-1,0) \\ \begin{cases} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (0,-1) \\ \begin{cases} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \text{ and } (i_1,j_1) \neq (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (-1,1) \\ \begin{cases} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \text{ and } (i_1,j_1) \neq (n_1, n_2+2) \\ 0 & \text{otherwise} \end{cases} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1,s} = q_{-2,s} = q_{-3,s} = 0$$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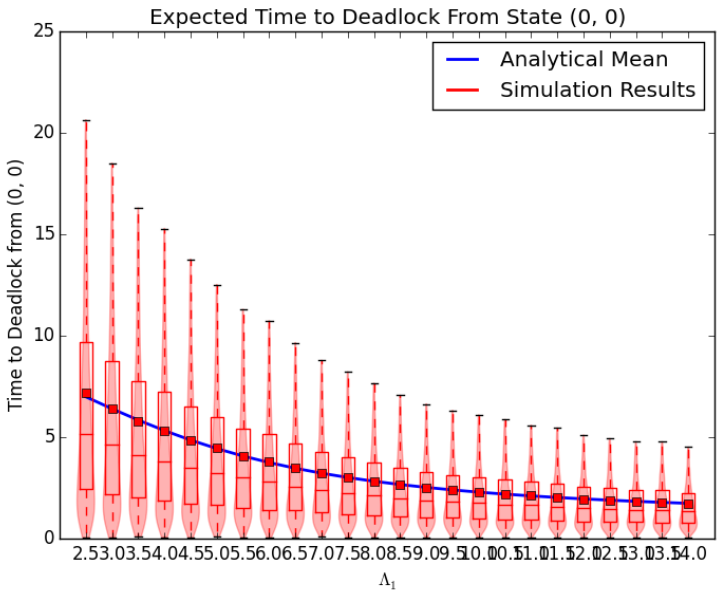
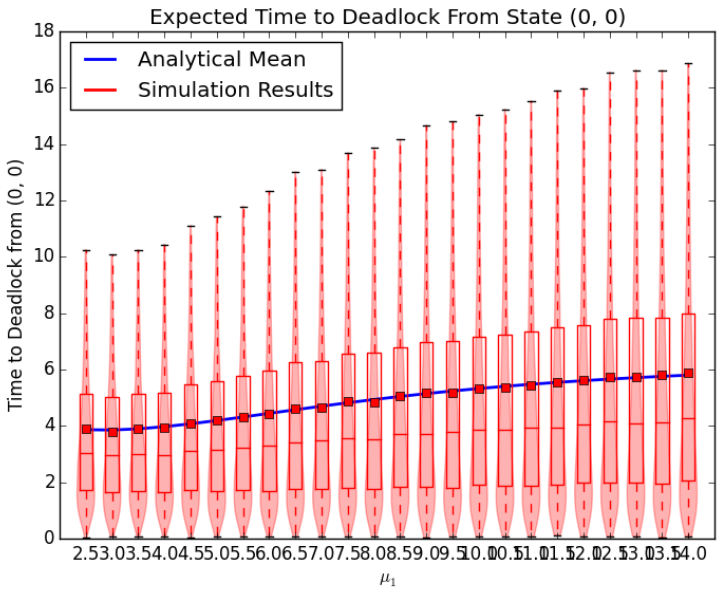
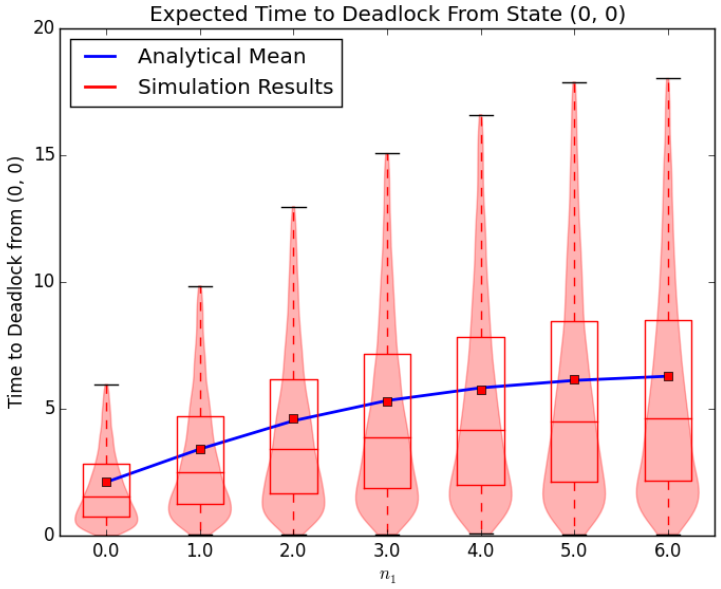
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(3, 0)
(3, 1)
(3, 2)
(−2)
(−3)
(−1)

Times to Deadlock







Diolch - Thank You

https://github.com/geraintpalmer/Presentations
palmergi1@cardiff.ac.uk