Using Queueing Network Modelling to Assess the Impact of the OPICP

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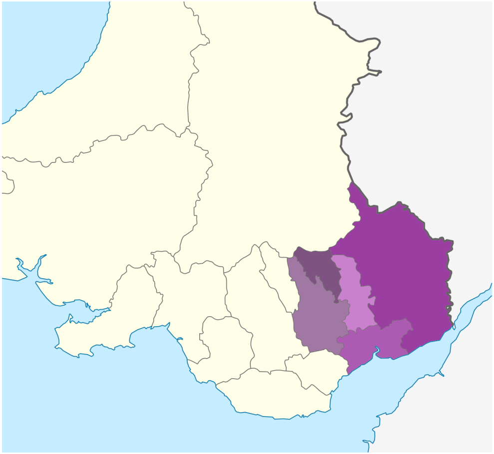
8th IMA International Conference on Quantitative Modelling in the Management of Health and Social Care

Aneurin Bevan University Health Board

GIG CYMRU NHS WALES

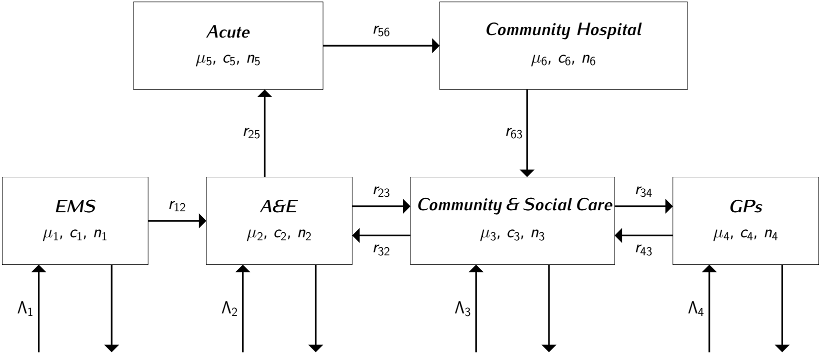
Bwrdd Iechyd Prifysgol Aneurin Bevan University Health Board



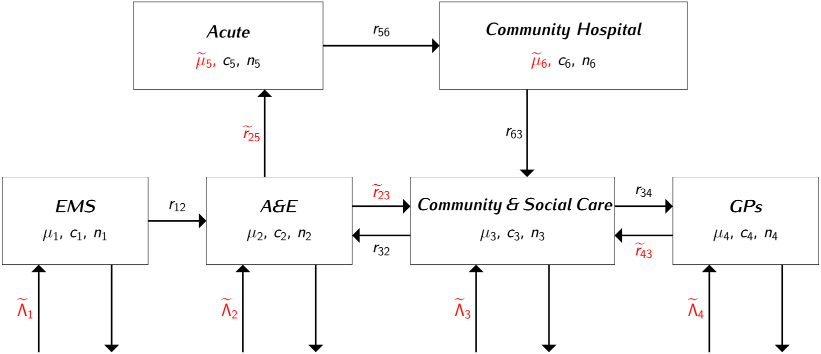
Older People's Integrated Care Pathway

- Pathway focused around pro-active patient centred coordinated care
- Individuals identified through ***risk stratification*** as being at risk of admission to institutionalised care or becoming frequent users of high cost care
- Develop holistic personal ***Stay Well Plans*** for these individuals, utilising ***low and no cost services***
- Aim to keep individuals and carers as well and as independent as possible

Elderly People's Flows Through Health System



Elderly People's Flows Through Health System

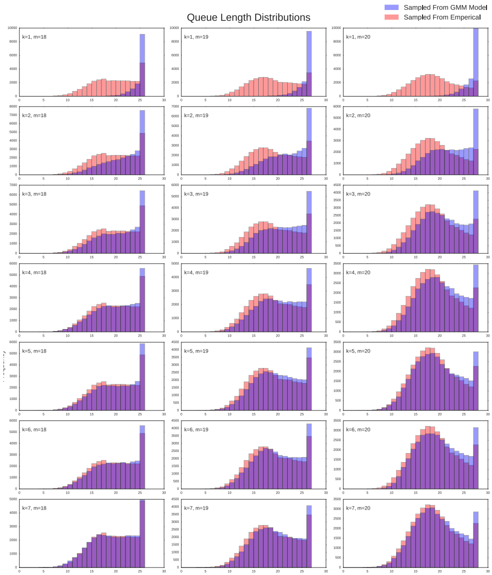


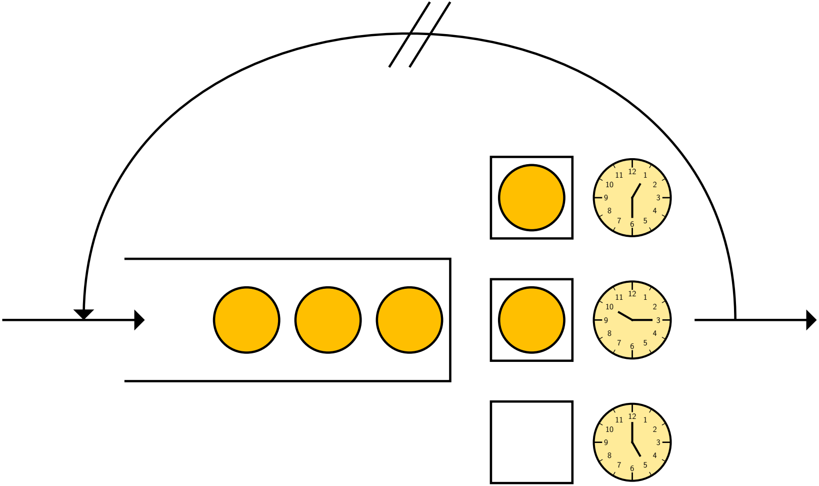
Simulation with Ciw

Open Source Python Library
Three-Phase Simulation

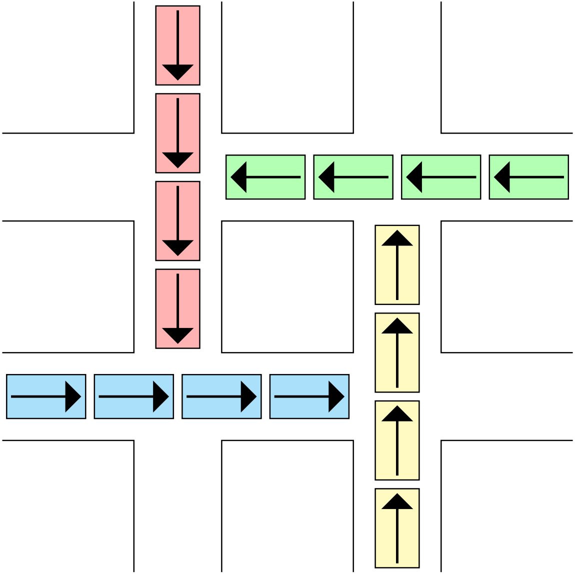
https://github.com/geraintpalmer/Ciw
https://pypi.python.org/pypi/Ciw
http://ciw.readthedocs.org

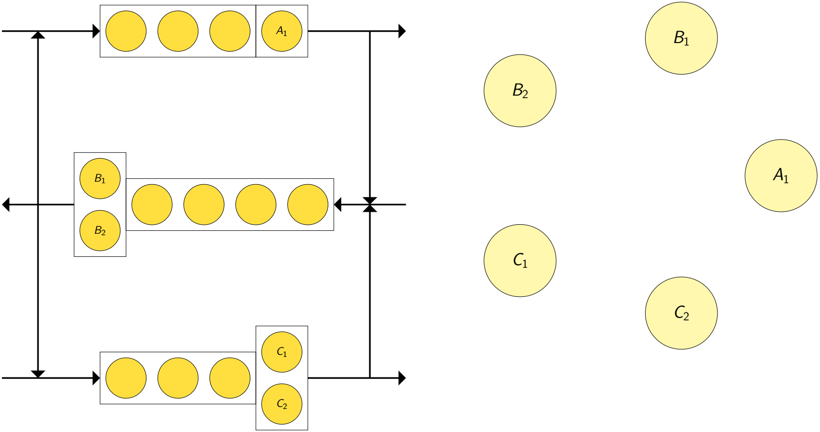
```
params = {
  'Number_of_nodes': 2,
  'Number_of_classes': 2,
  'Number_of_servers': [3, 1],
  'Simulation_time': 2000,

  'Arrival_distributions':{
    'Class 0': [['Exponential', 6.0],
                ['Uniform', 0.2, 0.3]],
    'Class 1': [['Lognormal', 0.4, 0.3],
                ['Deterministic', 0.25]]},

  'Service_distributions':{
    'Class 0': [['Exponential', 7.0],
                ['Deterministic', 0.3]],
    'Class 1': [['Exponential', 7.0],
                ['Exponential', 4.5]]},

  'Transition_matrices':{
    'Class 0': [[0.0, 0.5],
                [0.0, 0.8]],
    'Class 1': [[0.2, 0.0],
                [0.2, 0.4]]}
}
```


Sampled From GMM Model
Sampled From Emperical
Queue Length Distributions
k=1, m=18
k=1, m=19
k=1, m=20
k=2, m=18
k=2, m=19
k=2, m=20
k=3, m=18
k=3, m=19
k=3, m=20
k=4, m=18
k=4, m=19
k=4, m=20
k=5, m=18
k=5, m=19
k=5, m=20
k=6, m=18
k=6, m=19
k=6, m=20
k=7, m=18
k=7, m=19
k=7, m=20

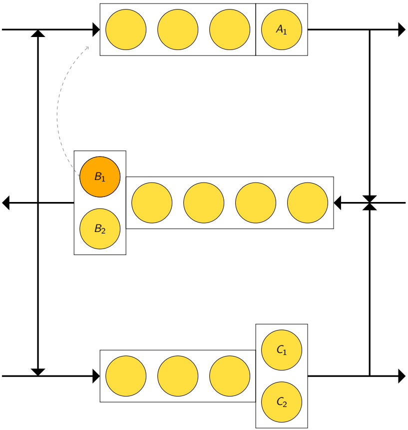


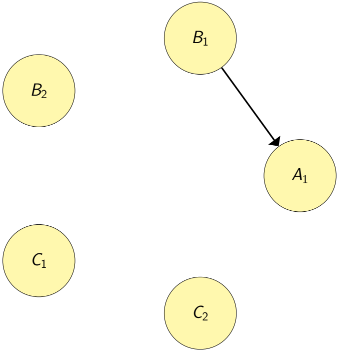
Deadlock

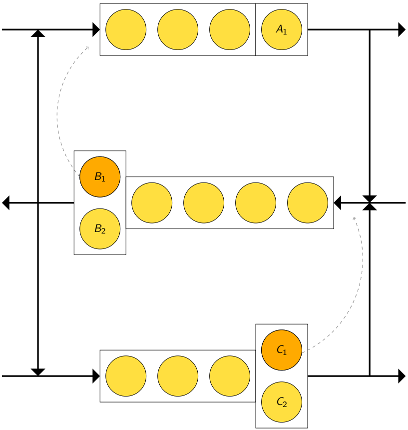


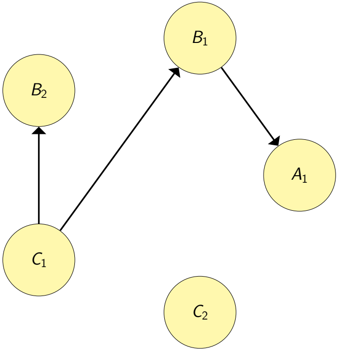


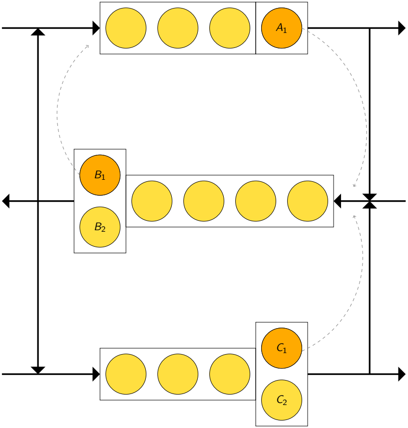
A_1
B_1
B_2
C_1
C_2
B_1
B_2
A_1
C_1
C_2


A_1
B_1
B_2
C_1
C_2

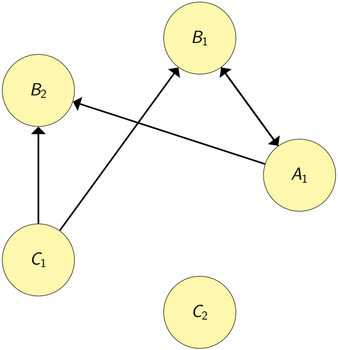

B_1
B_2
A_1
C_1
C_2


A_1
B_1
B_2
C_1
C_2

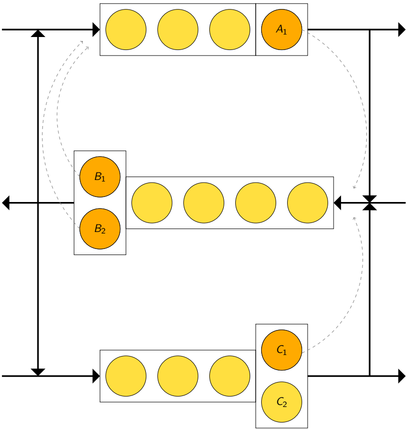

B_1
B_2
A_1
C_1
C_2

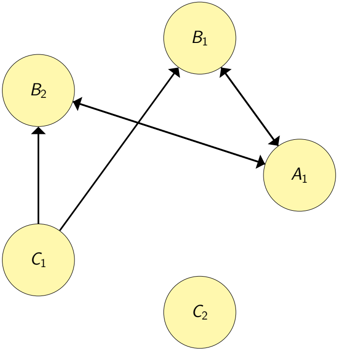


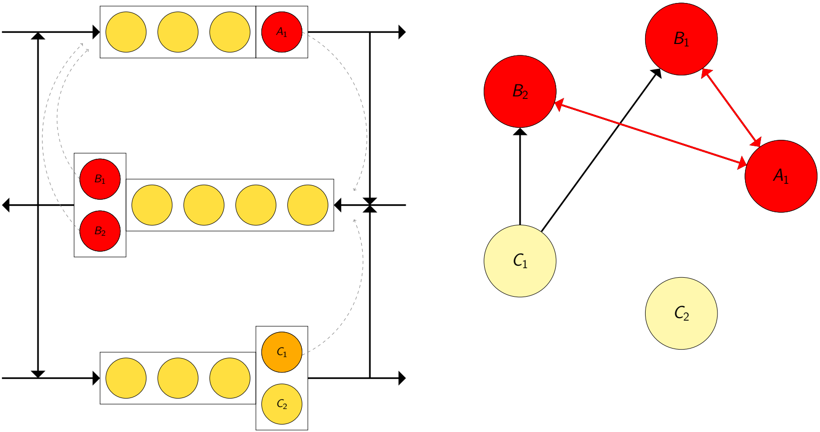
A_1
B_1
B_2
C_1
C_2



B_1
B_2
A_1
C_1
C_2


A_1
B_1
B_2
C_1
C_2


B_1
B_2
A_1
C_1
C_2

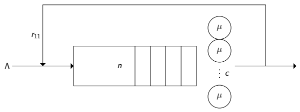


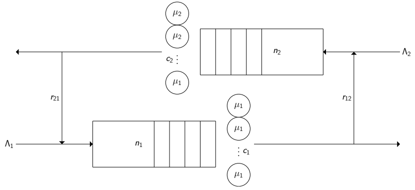
A_1
B_1
B_2
C_1
C_2
B_1
B_2
A_1
C_1
C_2

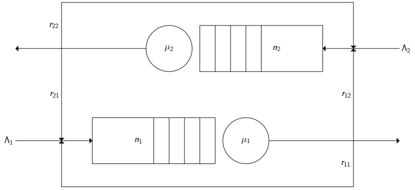
Deadlock Detection in Ciw

```
>>> import ciw
>>> params = {'Number_of_nodes': 1,
...           'Arrival_distributions': [['Exponential', 6.0]],
...           'Service_distributions': [['Exponential', 5.0]],
...           'Transition_matrices': [[0.5]],
...           'Number_of_servers': [1],
...           'Queue_capacities': [3],
...           'Detect_deadlock': True}
>>> Q = ciw.Simulation(params)
>>> times_to_deadlock = Q.simulate_until_deadlock()
>>> times_to_deadlock[((0, 0),)]
1.1707879982560288
```

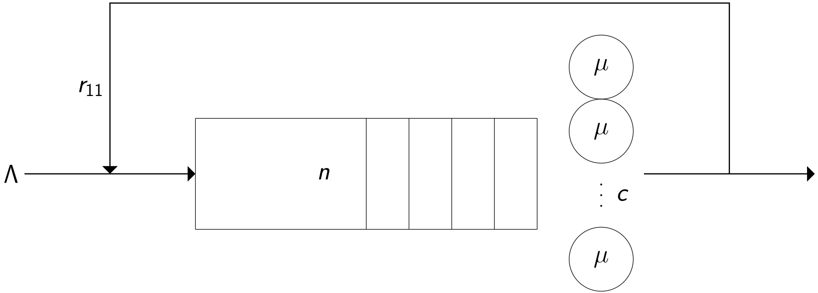
Three Deadlocking Queueing Networks







Markovian Model of Deadlock



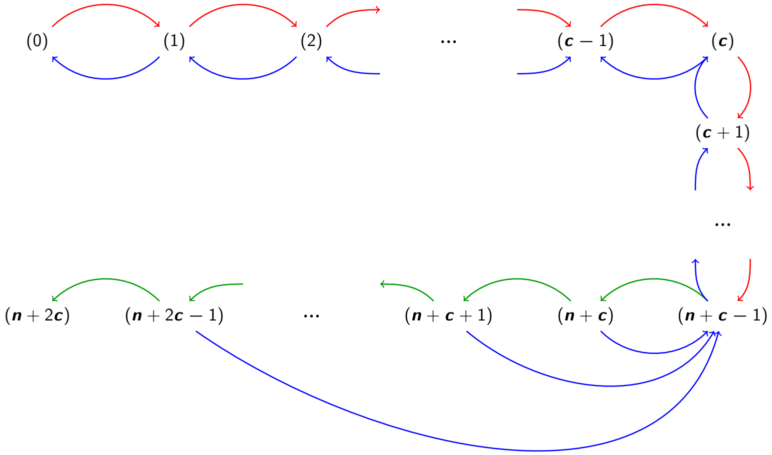
(i)

$$S = \{i \in \mathbb{N} \mid 0 \leq i \leq n + 2c\}$$

Define $\delta = i_2 - i_1$

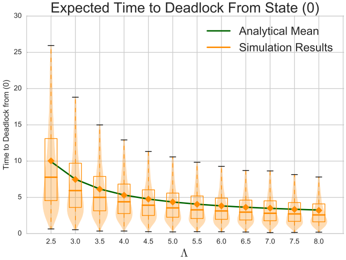
$$q_{i_1,i_2} = \left\{ \begin{array}{rr} \Lambda & \text{if } \delta = 1 \\ (1 - r_{11})\mu \min(i, c) & \text{if } \delta = -1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 < n + c$$

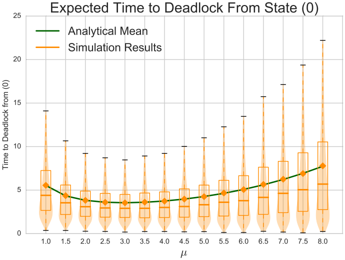
$$q_{i_1,i_2} = \left\{ \begin{array}{rr} (c - b)r_{11}\mu & \text{if } \delta = 1 \\ (1 - r_{11})(c - b)\mu & \text{if } \delta = -b - 1 \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{if } i_1 = n + c + b \quad \forall \quad 0 \leq b \leq c$$

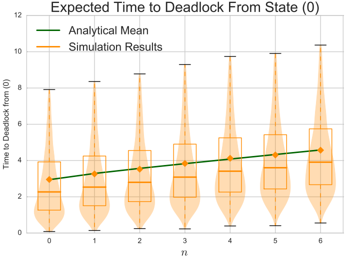


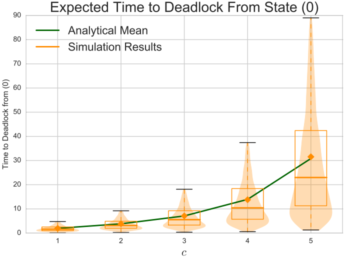
(0)
(1)
(2)
...
$(\boldsymbol{c}-1)$
$(\boldsymbol{c})$
$(\boldsymbol{c}+1)$
...
$(\boldsymbol{n}+2\boldsymbol{c})$
$(\boldsymbol{n}+2\boldsymbol{c}-1)$
...
$(\boldsymbol{n}+\boldsymbol{c}+1)$
$(\boldsymbol{n}+\boldsymbol{c})$
$(\boldsymbol{n}+\boldsymbol{c}-1)$

Times to Deadlock

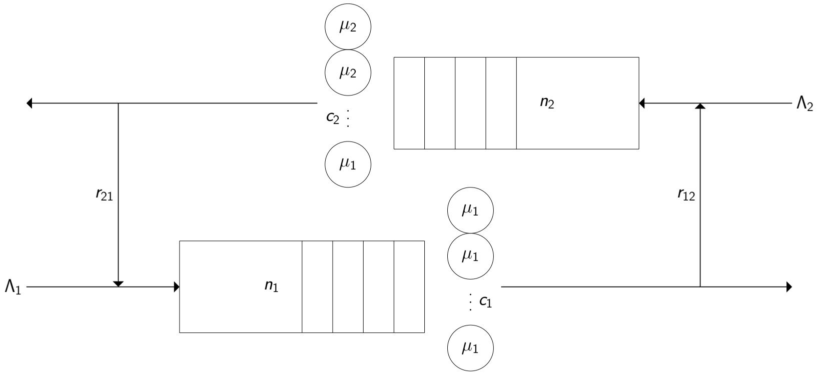








Markovian Model of Deadlock

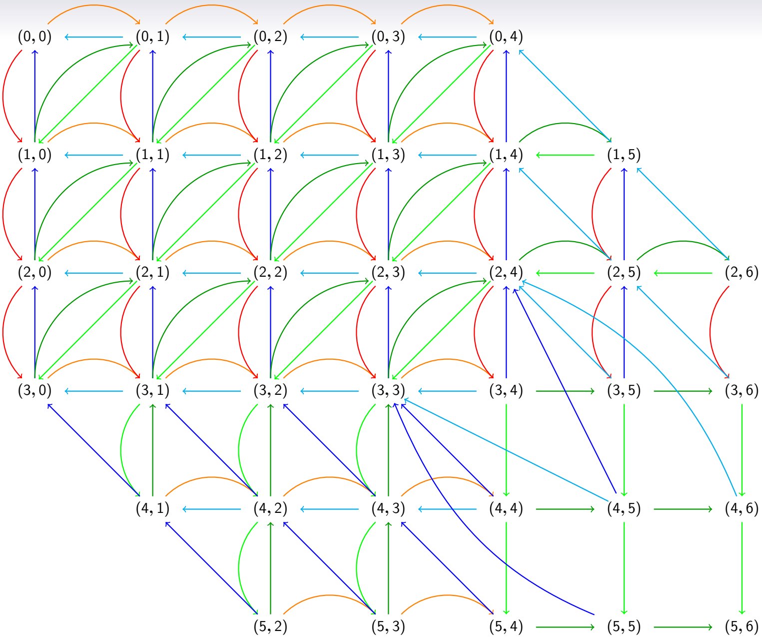


$$(i, j)$$

$$S = \{(i,j) \in \mathbb{N}^{(n_1+c_1+c_2)\times(n_2+c_2+c_1)} \mid i \leq n_1 + c_1 + j,\ j \leq n_2 + c_2 + i\}$$

$$\begin{aligned}
\delta &= (i_2, j_2) - (i_1, j_1) \\
b_1 &= \max(0, i_1 - n_1 - c_1) \\
b_2 &= \max(0, i_2 - n_2 - c_2) \\
s_1 &= \min(i_1, c_1) - b_2 \\
s_2 &= \min(i_2, c_2) - b_1
\end{aligned}$$

	$j_1 < n_2 + c_2$	$j_1 = n_2 + c_2$	$j_1 > n_2 + c_2$
$i_1 < n_1 + c_1$	Λ_1 if $\delta = (1,0)$ Λ_2 if $\delta = (0,1)$ $r_{12}s_1\mu_1$ if $\delta = (-1,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,-1)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	Λ_1 if $\delta = (1,0)$ $r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,-1)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	Λ_1 if $\delta = (1,0)$ $r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (0,-1)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (-1,-1)$
$i_1 = n_1 + c_1$	Λ_2 if $\delta = (0,1)$ $r_{12}s_1\mu_1$ if $\delta = (-1,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,0)$ $(1-r_{21})s_2\mu_2$ if $\delta = (-1,-1)$
$i_1 > n_1 + c_1$	Λ_2 if $\delta = (0,1)$ $r_{12}s_1\mu_1$ if $\delta = (-1,0)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,-1)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-1,-1)$ $(1-r_{21})s_2\mu_2$ if $\delta = (0,-1)$	$r_{12}s_1\mu_1$ if $\delta = (0,1)$ $r_{21}s_2\mu_2$ if $\delta = (1,0)$ $(1-r_{12})s_1\mu_1$ if $\delta = (-\min(b_1+1, b_2+1), -\min(b_1, b_2+1))$ $(1-r_{21})s_2\mu_2$ if $\delta = (-\min(b_1+1, b_2), -\min(b_1+1, b_2+1))$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(3,6)
(4,1)
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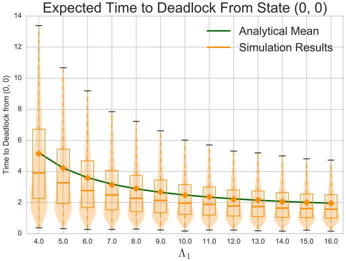
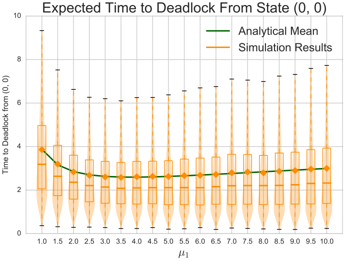
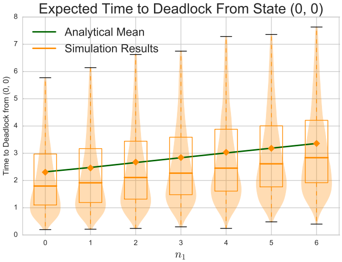
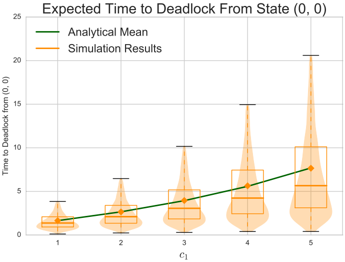
(5,6)

Times to Deadlock

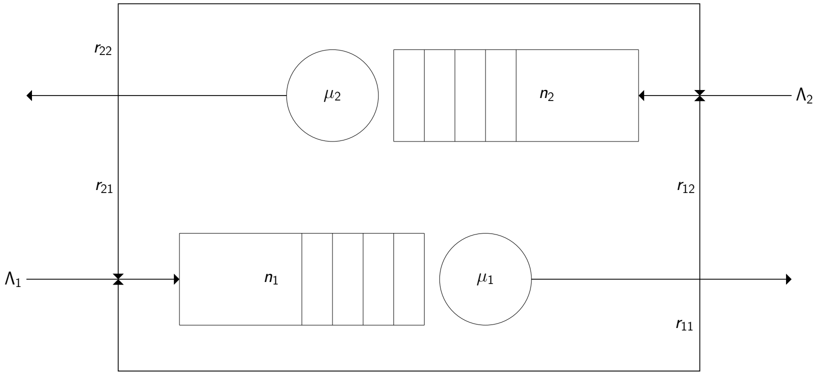








Markovian Model of Deadlock



$$(i, j)$$

$$S = \{(i,j) \in \mathbb{N}^{(n_1+2 \times n_2+2)} \mid 0 \leq i+j \leq n_1+n_2+2\} \cup \{(-1)\}$$

Define $\delta = (i_2, j_2) - (i_1, j_1)$

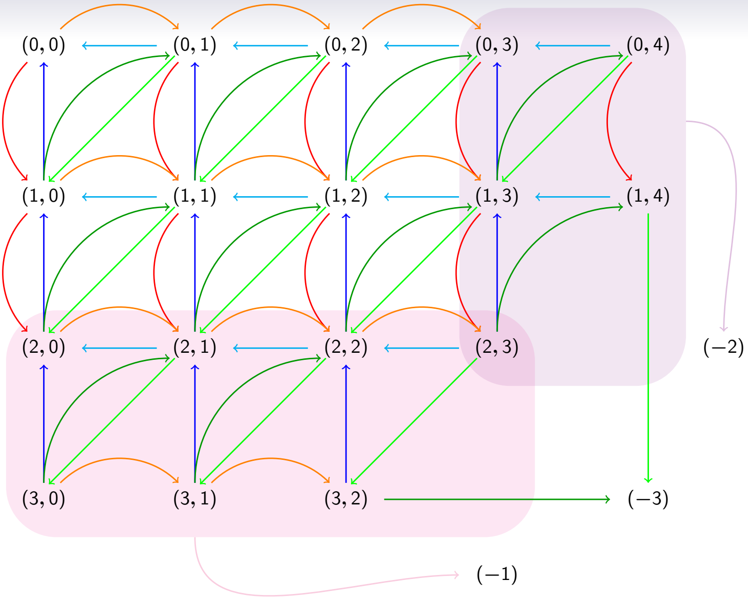
$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \left.\begin{array}{ll} \Lambda_1 & \text{if } i_1 \leq n_1 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = (1,0) \\ \left.\begin{array}{ll} \Lambda_2 & \text{if } j_1 \leq n_2 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = (0,1) \\ \left.\begin{array}{ll} (1-r_{12})\mu_1 & \text{if } j_1 < n_2+2 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = (-1,0) \\ \left.\begin{array}{ll} (1-r_{21})\mu_2 & \text{if } i_1 < n_1+2 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = (0,-1) \\ \left.\begin{array}{ll} r_{12}\mu_1 & \text{if } j_1 < n_2+2 \text{ and } (i_1,j_1) \neq (n_1+2, n_2) \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = (-1,1) \\ \left.\begin{array}{ll} r_{21}\mu_2 & \text{if } i_1 < n_1+2 \text{ and } (i_1,j_1) \neq (n_1, n_2+2) \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \delta = (1,-1) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-1)} = \begin{cases} r_{11}\mu_1 & \text{if } i > n_1 \text{ and } j < n_2+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-2)} = \begin{cases} r_{22}\mu_2 & \text{if } j > n_2 \text{ and } i < n_1+2 \\ 0 & \text{otherwise} \end{cases}$$

$$q_{(i_1,j_1),(-3)} = \begin{cases} r_{21}\mu_2 & \text{if } (i,j) = (n_1, n_2+2) \\ r_{12}\mu_1 & \text{if } (i,j) = (n_1+2, n_2) \\ 0 & \text{otherwise} \end{cases}$$

$$q_{-1,s} = q_{-2,s} = q_{-3,s} = 0$$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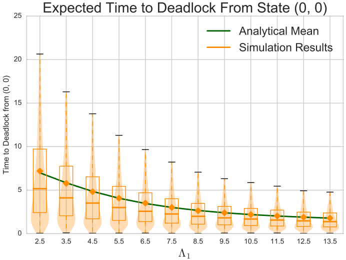
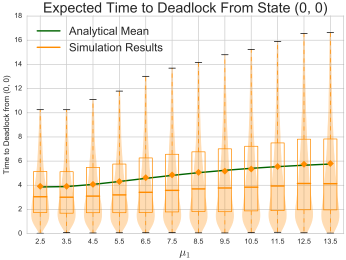
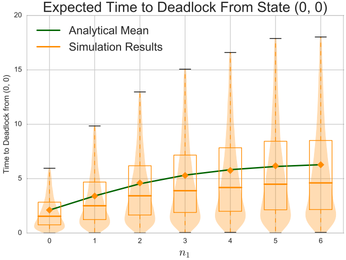
(−3)
(−1)

Times to Deadlock







Summary

Summary

- Developed a general use simulation library in Python
- Investigated potential for deadlock, automatic detection
- Modelled some deadlocking queueing networks

To Do...

- Build and parameterise patient flow networks from data
- Use queueing network analysis and simulation to investigate impact of the OPICP
- Determine the OPICP's effect on demand and workforce needs

Thank You

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