# Problem Sheet 3

1. Describe in words the following queueing systems:
   (a) $M/M/5$
   (b) $D/M/1/5/$SIRO
   (c) $G/G/\infty$
   (d) $M^3/D/\infty/\infty/$PS
   (e) $M/E_2/1/\infty/$FIFO

2. At a chicken shop the deep-fat-fryer can be described as a Markovian queue. Chicken pieces are put in the fryer at a rate of $\lambda$ per time unit. There is room for only 3 pieces of chicken in the fryer, and no new chicken orders are taken when the fryer is full. When a piece of chicken if in the fryer alone it completes frying at a rate of $\mu$. When there are two pieces of chicken in the fryer they cook at a rate of $^{\mu}/_{3}$, and when there are three pieces of chicken in the fryer they cook at a rate of $^{\mu}/_{4}$.

   Find the steady-state probabilities in terms of $\lambda$ and $\mu$.

   Find the expected number of pieces of chicken in the fryer when $\lambda = 5$ and $\mu = 9$.

3. Consider an $M/M/1$ queue with arrival rate $\lambda = 10$ and $\mu = 15$. Find $\rho$, $P_0$, $L$, $W$, $W_q$, and $L_q$.

4. Consider an $M/M/\infty$ queue with arrival rate $\lambda = 6$ and $\mu = 8$. Find $P_0$, $L$, $W$, $W_q$, and $L_q$.

5. For an $M/M/\infty$ queue with arrival rate $\lambda$ and service rate $\mu$, derive the fact that $L = {^{\lambda}/_{\mu}}$ without using Little's laws.

6. A blood diagnostic centre can be described as an $M/M/1$ queue. Blood samples arrive at a rate of $\lambda$ per time unit. Once in the centre, whether being processed of waiting, the samples need to be kept cold, at a cost of $C_h$ per time unit. An automated diagnostic machine can process the blood samples one at a time, at a rate of $\mu$ per time unit, which can be controlled. It costs $\mu C_s$ per time unit to run the machine at a rate $\mu$. What should the machine's service rate be set to in order to minimise the overall cost?