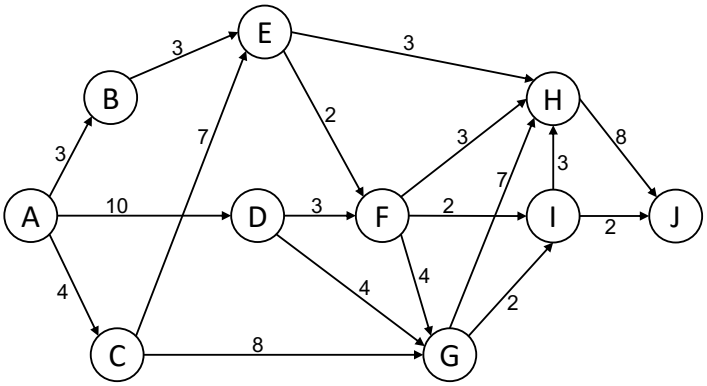
Problem Sheet 7

1. Consider the network below with road lengths indicating the distance between each city. Use dynamic programming to find the shortest path from A to J.



2. You have eight cardboard boxes in your house with the following dimensions:

Box	Length	Width	Height
α	10	10	10
β	15	6	6
γ	3	3	3
δ	4	5	7
ϵ	9	9	11
ζ	11	4	4
η	5	3	4
θ	5	10	6

They are fragile and cannot be rotated of flipped. A box can be stacked on top of another box if its length is shorter and its width is shorter than the other box. Use dynamic programming to find the tallest possible stack of boxes.

3. A mobile burger truck has been contracted to supply burgers for a music festival. The festival is to last 5 days, however the burger truck only has 3 boxes of frozen meat, and once defrosted each box of frozen meat only lasts one day. It costs €$1k$ per box per day to keep frozen. The burger truck must decide how many boxes to defrost and sell each day of the festival. They anticipate that they will take in €$1.5k$ per day if they defrost one box, €$3k$ if they defrost two boxes on the same day, and €$3.5k$ if they defrost 3 boxes on the same day. However, due to a broken contract, they will incur a fine of €$2k$ each day they do not offer burgers. Use dynamic programming to devise a strategy that will maximise the burger truck's intake.

4. Finding the longest common subsequence between to sequences is a useful problem to solve, for example it can tell us how similar two strands of DNA are. Use dynamic programming to find the longest common subsequence between ACTAGCTA and TCAGGTAT.

   (Hint: consider the triples (i, j, k) as states, corresponding to the i^{th} element of the first sequence, and the j^{th} element of the second sequence. You only need to consider states where these letters, k, are identical. E.g. the state (1, 3, A) denoted the letter A i shared by the first letter of the first sequence, and the third letter of the second sequence.)