Mock Coursework

1. A nationwise programme has began to test for a rare, symptomless, disease. We know that around 3% the the population has the disease. Let D represent the event of a patient having the disease, then $\mathbb{P}(D) = 0.03$. The test returns either a positive or negative result, let T be the event that the test is positive. The test is good, but not always accurate: when tested on a patient who has the disease, the test returns a positive result 95% of the time, that is $\mathbb{P}(T \mid D) = 0.95$; when tested on a patient without the disease, it returns negative 85% of the time, that is $\mathbb{P}(\bar{T} \mid \bar{D}) = 0.85$.

   Use Monte Carlo simulation with 10,000 trials to estimate the probability of $\mathbb{P}(D \mid T)$. **[35%]**

2. A computer repair shop has three engineers, each specialising in a different aspect of the computer: software, hardware, or connectivity. Customers arrive at the shop according to a Poisson distribution with rate 10 per hour. Computers first arrive and queue at the inspection desk, manned by four apprentice engineers. Inspection takes exactly 15 minutes. Computers are then labelled according to which specialist they need to see: 50% require the software specialist, 30% require the hardware specialist, and 20% require the connectivity specialist. Computers are sent to separate queues for each specialist.

   The repair times of the software specialist are Exponentially distributed with rate 8 per hour. The repair times for the hardware specialist are Uniformly distributed between 5 and 20 minutes. The repair times for the connectivity specialist are Uniformly distributed between 5 and 25 minutes.

   The computer repair shop is open for 10 hours a day, and any jobs incomplete at the end of the say are lost.

   By running a discrete-event simulation of the system, find the probability that a computer spends more than 30 minutes waiting for both the inspection and repair combined. You will need to show the conceptual model. **[65%]**