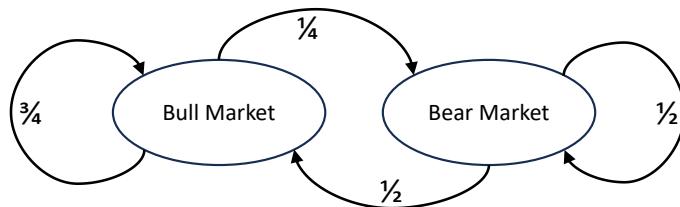


# Problem Sheet 2

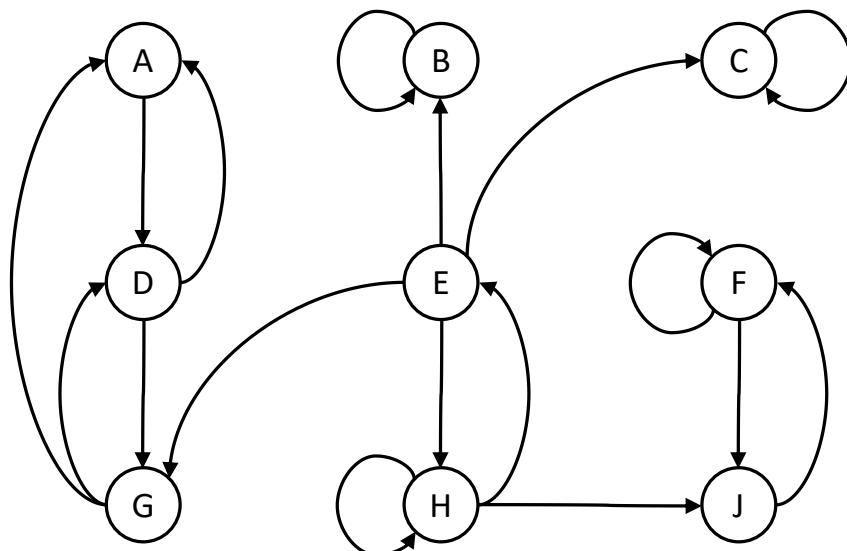
1. A country's economy can be described as either a Bull market (where stock prices rise and things are going well), or a Bear market (where stock prices fall and things are not going so well). The economy is categorised as such each quarter. This process can be described as a discrete-time Markov chain, with probabilities of being in each state in the next quarter:



If the country is currently in a Bull market, what is the probability of being in either a Bull or a Bear market in three quarters times?

2. Consider the discrete-time Markov chain below with nine states. An arrow indicates that the probability of transitioning from one state to another is greater than zero.

- (a) Identify all the irreducible classes and state whether they are closed or not.
- (b) Classify each state as either Recurrent, Transient, or Absorbing.



3. A mental health doctor is trying to understand a patient's mental state. They ask the patient to record daily whether they feel Calm, Mildly Anxious, or Very Anxious. Crunching the data the doctor finds:

- On a calm day,  $\frac{1}{3}$  of the time they will remain calm tomorrow, and  $\frac{2}{3}$  of the time they will become mildly anxious tomorrow;
  - On a mildly anxious day,  $\frac{1}{4}$  of the time they will become calm tomorrow,  $\frac{1}{2}$  the time they remain mildly anxious tomorrow, while  $\frac{1}{4}$  of the time they become very anxious tomorrow;
  - On a very anxious day, only  $\frac{1}{5}$  of the time will they become calm tomorrow,  $\frac{2}{5}$  of the time they will become mildly anxious, however  $\frac{2}{5}$  of the time they remain very anxious tomorrow.
- (a) Draw the discrete-time Markov chain describing the patient's mental state.
- (b) Find the steady-state probabilities.
- (c) The doctor devises a medication plan: on calm days the patient should not take any medication; on mildly anxious days they should take a pill of type A, costing 1p per pill; and on very anxious days they should take a pill of type B, costing 23p per pill. What is the expected yearly cost for this medication plan?

4. A printing shop owns four printers. Each printer breaks down at a rate of  $\beta$ . Once broken down, it is sent for repair. The rate at which printers get repaired is  $\alpha$ .

Letting  $i$  be the state that there are  $i$  printers in operation, draw the Markov chain for this system, and find the steady-state probabilities for general  $\alpha$  and  $\beta$ , and for when  $\alpha = \beta$ .