LXe scintillation model

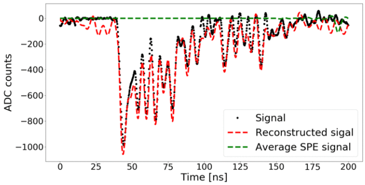
January 27, 2020

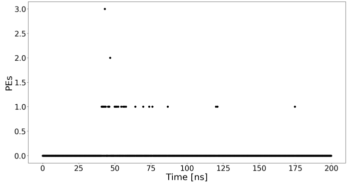
Objective

We want to use the data collected by the Cs and Co runs to estimate the parameters for the scintillation model.
In the following we show that there is a $\delta(t)$ component in the model with high statistical significance and propose how to go on to claim a discovery.

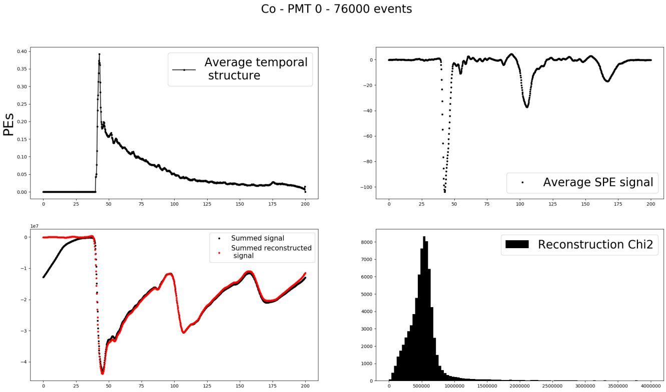
Signal Reconstruction

To study the temporal structure of the photon emission a processing algorithm uses a template of the average SPE signal to reconstruct the temporal PE pattern in each event for each PMT separately. The output of this process is the number of PEs created in the PMT in each digitization point.

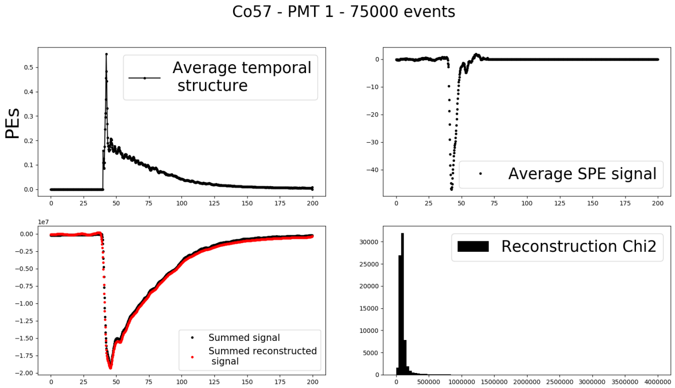




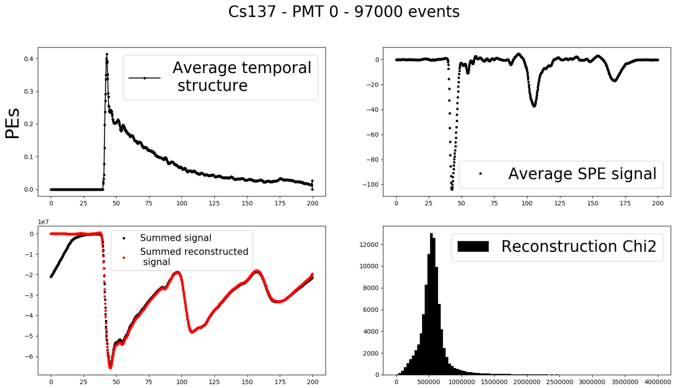
Reconstruction results



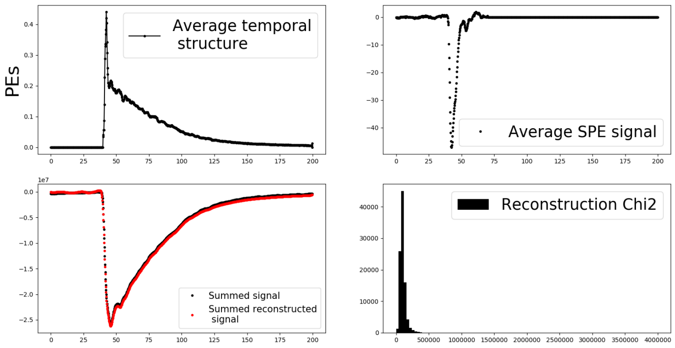
Reconstruction results



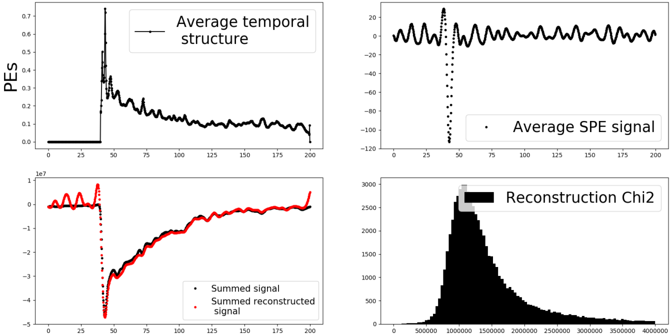
Reconstruction results



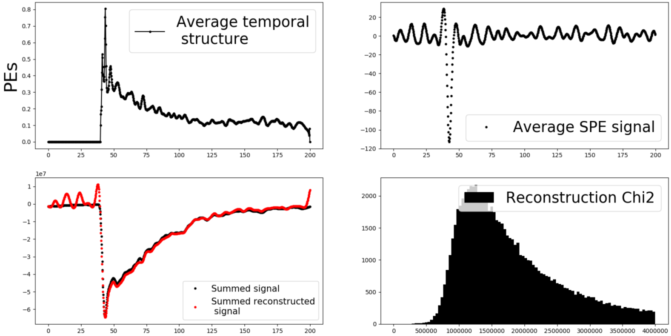
Cs137 - PMT 1 - 96000 events



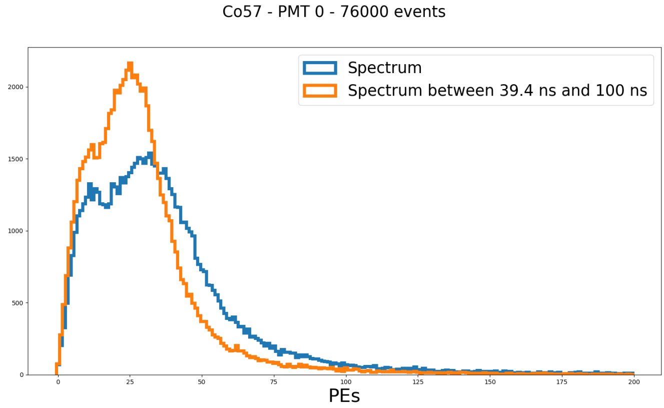
Co57 - PMT 3 - 71000 events



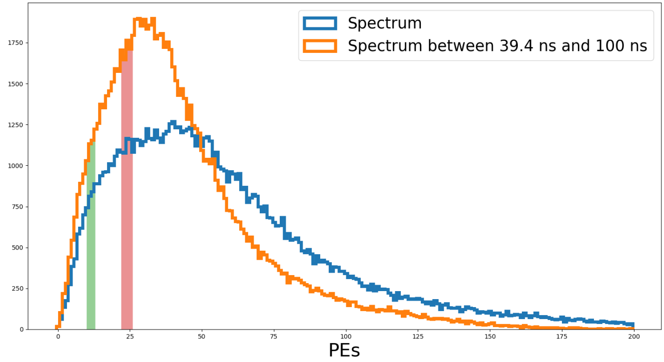
Cs137 - PMT 3 - 86000 events



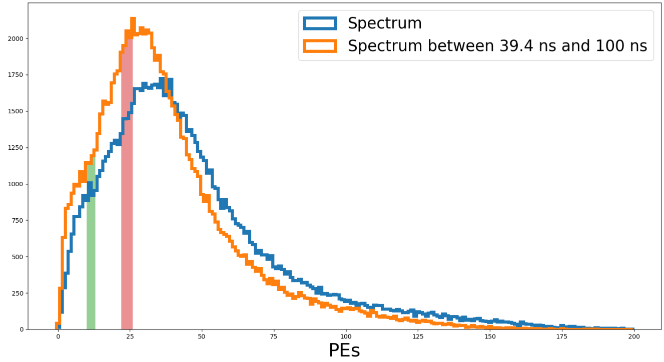
1 PMT Spectrum

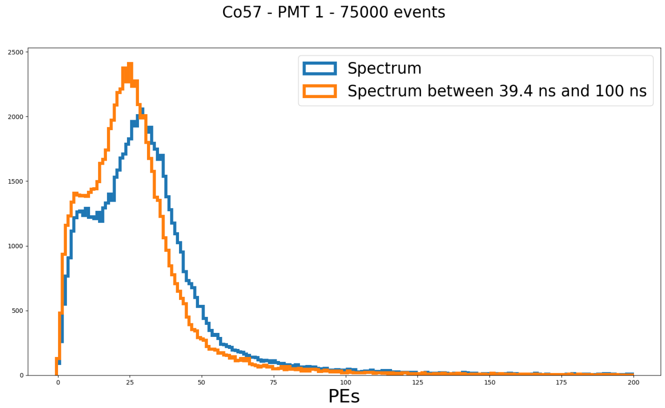


Cs137 - PMT 0 - 97000 events



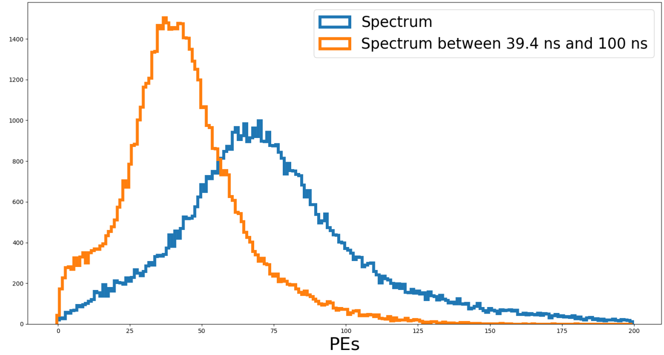
Cs137 - PMT 1 - 96000 events



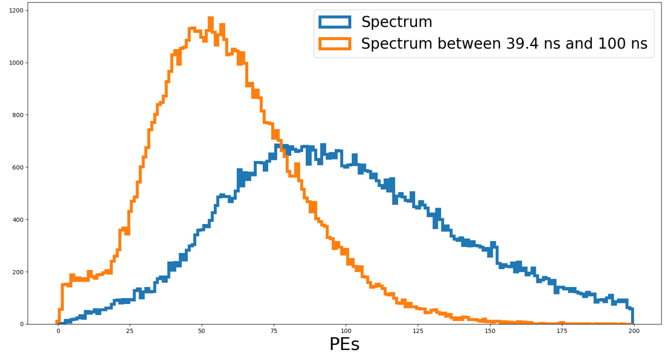


Co57 - PMT 1 - 75000 events
Spectrum
Spectrum between 39.4 ns and 100 ns
0
500
1000
1500
2000
2500
0
25
50
75
100
125
150
175
200
PEs

Co57 - PMT 3 - 71000 events

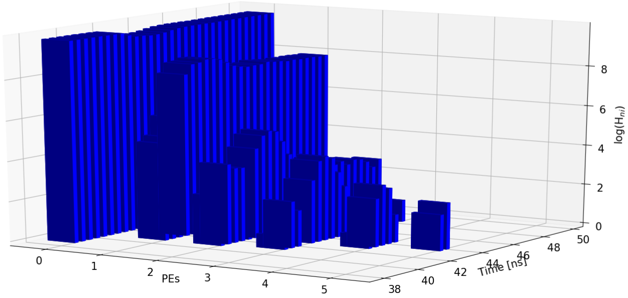


Cs137 - PMT 3 - 86000 events



PE Histogram

The basis for analysis is a 2D histogram (for each PMT and each source) H_{ni} which holds the number of events in which n PEs were resolved at time i,



The average temporal structure which shown in the Reconstruction result slide is the average on each column.

Model for H_{ni}

The number of photons emitted at time i,

$$n_i^{ph} \sim \text{Poisson}(Y_i N), \tag{1}$$

where N is the average number of photon emitted in the events, and Y_i is the scintillation model (the probability to emit a photon at time i).
The number of PEs created in the PMT at time i,

$$n_i^{pe} \sim \text{Binom}(Q, \text{Poisson}(Y_i N)), \tag{2}$$

where Q is the photon detection efficiency (quantum efficiency, collection efficiency and double PE probability).

Each PMT has its temporal uncertainty combined with the code's temporal uncertainty. The number of PEs that need to be resolved at time i,

$$\tilde{n}_i \sim \sum_j m_j^i, \tag{3}$$

where m_j^i is a random variable that represents the number of PEs that were created at time j but resolved at time i.

$$m_j^i \sim n_j^{pe} \mathrm{Norm}(i - j - T|\sigma_t) \tag{4}$$

where σ_t is the temporal resolution of the PMT and the code and T is the average delay of the PMT.

Since $\tilde{n}_i$ is a sum of independent random variables its distribution can be approximated by a distribution with mean and variance $\langle \tilde{n}_i \rangle = \sum_j \langle m \rangle_j^i$, $\mathrm{Var}_i = \sum_j \mathrm{Var}_j^i$, where,

$$\langle m \rangle_j^i = \mathrm{Var}_j = QN_0 Y_j \mathrm{Norm}(i - j - T | \sigma_t) \tag{5}$$

Poisson has an equal mean and variance, so

$$\tilde{n}_i \sim \mathrm{Poisson}\left(QN \sum_j Y_j \mathrm{Norm}(i - j - T | \sigma_t) \right) \tag{6}$$

Scintillation model

Y_j is the probability to emit a photon at time $t_j \Delta t$.

$$Y_j = \Delta t F \delta(t_j - T) + \Delta t \frac{1-F}{\tau_s} e^{-t_j/\tau_s} \tag{7}$$

$$\langle \tilde{n}_i \rangle = \frac{QN\Delta t}{\sqrt{2\pi}\sigma_t} \int_0^\infty Y(\tilde{t}) e^{-(\tilde{t}-t_i-T)^2/2\sigma_t^2} =$$
$$NQ\Delta t \left[\frac{e^{-(t_i-T)^2/2\sigma_t^2}}{\sqrt{2\pi}\sigma_t} + \frac{K}{\tau} e^{-t/\tau} \left(1 - \operatorname{erf}\left(\frac{\sigma}{\sqrt{2}\tau} - \frac{t-T}{\sqrt{2}\sigma} \right) \right) \right] \tag{8}$$

Where K is a normalization constant,

$$K = \left[1 - \operatorname{erf}\left(\frac{\sigma}{\sqrt{2}\tau} + \frac{T}{\sqrt{2}\sigma} \right) + e^{-\sigma^2/2\tau^2 - T/\tau} \left(1 + \operatorname{erf}\left(\frac{T}{\sqrt{2}\sigma} \right) \right) \right]^{-1} \tag{9}$$

SPE resolution

Finally, the code has its resolution for SPEs (its accuracy to resolve the correct number of PEs from the signal at a given time). The probability to resolve n PEs given $\tilde{n}$ is the correct number,

$$n_i \sim \mathrm{P}_n(\tilde{n}|\sigma_{pe}) = \sum_m \mathrm{Norm}(m-1|\sigma_{pe})\mathrm{P}_{n-m}(\tilde{n}-1|\sigma_{pe}) \quad (10)$$

Expectation for H_{ni}

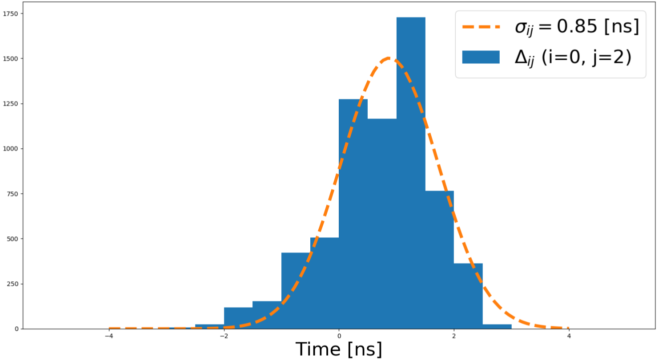
The expected number of events in which n PEs were resolved at time i is,

$$h_{ni} = N_{ev} \sum_m \mathrm{P}_n(m|\sigma_{pe}) \mathrm{Poisson}(m|\langle\tilde{n}_i\rangle) \tag{11}$$

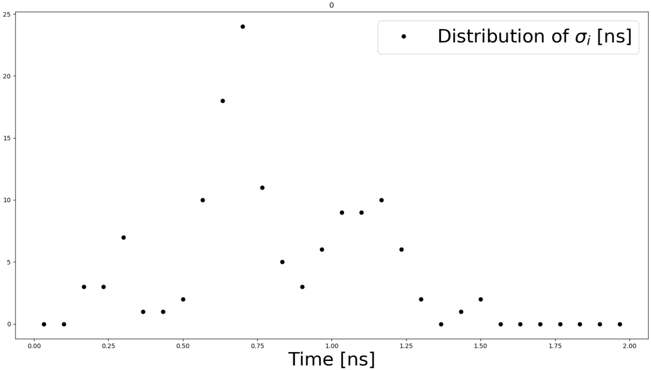
This is a model with 7 parameters: $N, Q, F, \tau_s, T, \sigma_t, \sigma_{PE}$. Some of them can be constrained by calibration data.

Calibration σ_t

We can calibrate σ_t for each PMT by the pulser data. Consider PMT_i that we want to calibrate and a any other PMT_j. For each pulser event in which both PMTs saw SPE we measure Δ_{ij}, the time difference between the two signals.
$\Delta_{ij} \sim \mathrm{Norm}(T_{ij}, \sigma_{ij})$.



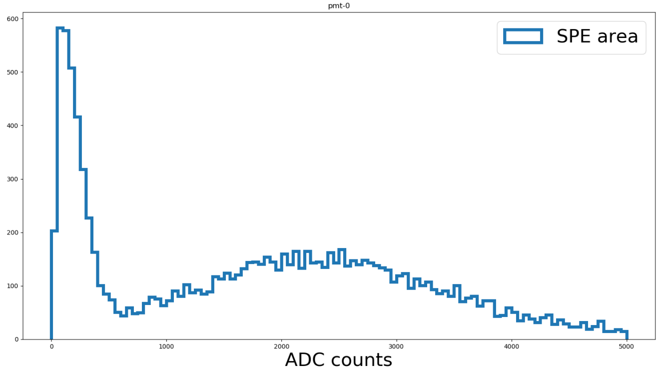
$\sigma_i = \sqrt{\frac{1}{2}(\sigma_{ij}^2 + \sigma_{ik}^2 - \sigma_{jk}^2)}$ for any $j \neq k$.
So a pair of PMTs (k, j) is used to estimate the σ_i. Each PMT$_i$ has $\frac{1}{2}\binom{19}{2}$ estimates for σ_i and their mean is σ_t for that PMT.



Calibration of σ_{PE}

σ_{PE} is the standard deviation of the distribution of the number of PEs that are resolved from a SPE created in the PMT. This is calibrated from the SPE area distribution.

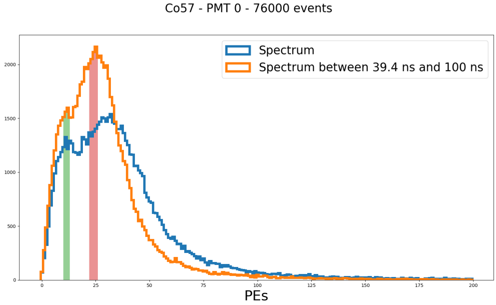
$$\text{Area}_{spe} \sim \text{Norm}(\bar{A}, \bar{A} \cdot \sigma_{PE}) \quad \rightarrow \quad n \sim \text{Norm}(1, \sigma_{PE}) \qquad (12)$$



Constrain on NQ

We choose the data to fit our model to from a narrow PE bands in the spectrum, where it can be estimated that the origin of the events in these bands is from the same energy and their variation is due to statistical fluctuation and no physical. Thus we expect that

$$N_{PE} \sim \text{Poisson}(\sum_{39.4ns}^{100ns} \langle h_{ni} \rangle_n) \tag{13}$$



Fit

To find the parameters which give h_{ni} as close to H_{ni} we maximize

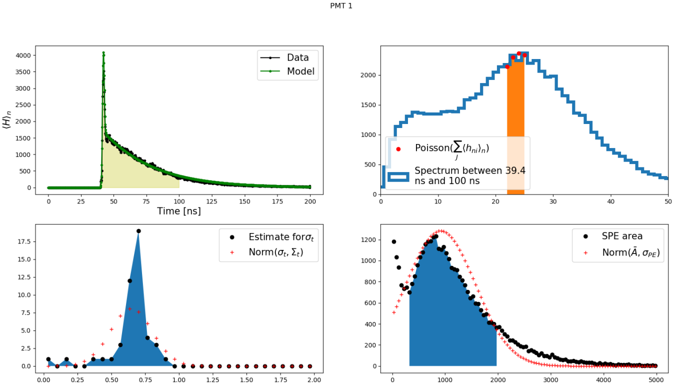
$$L(\text{data}, \text{model}(\theta)) = \sum_{\text{dataset}} \sum_{j \in J} \text{Poisson}(\text{data}_j | \text{model}_j)/\text{len}(J) \tag{14}$$

Where θ is the parameter array and J is some sub-range of the dataset. The datasets in the summation:

- Dataset - H_{ni}, model - h_{ni}.
- Dataset - distribution of estimates for σ_t, model - $\text{Norm}(\sigma_t, \Sigma_t)$.
- Dataset - distribution of SPE areas, model - $\text{Norm}(\bar{A}, \bar{A} \cdot \sigma_{PE})$.
- Dataset - PE spectrum, model - $\text{Poisson}(\sum_{39.4ns}^{100ns} \langle h_{ni} \rangle_n)$.

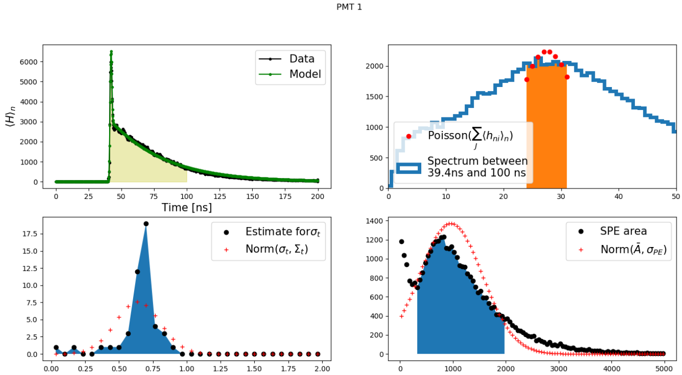
This adds two dummy parameters, Σ_t is the standard deviation of the estimates of σ_t and $\bar{A}$ is the mean SPE area.

Results (^{57}Co peak 2)



NQ	F	$\sigma_t[ns]$	$\sigma_t[ns]$	σ_{PE}
33	0.1	34	0.6	0.7

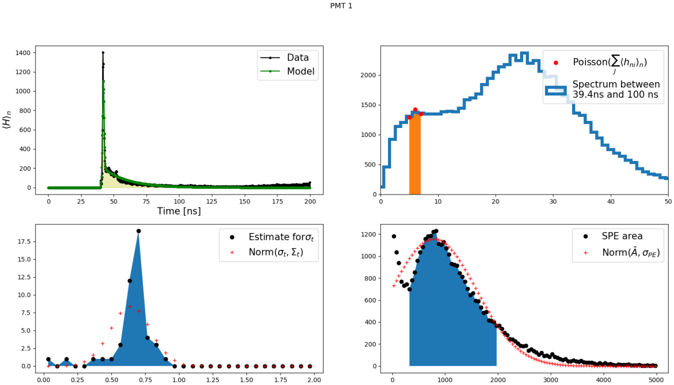
Results (^{137}Cs)



NQ	F	$\sigma_t[ns]$	$\sigma_t[ns]$	σ_{PE}
39	0.09	38	0.6	0.6

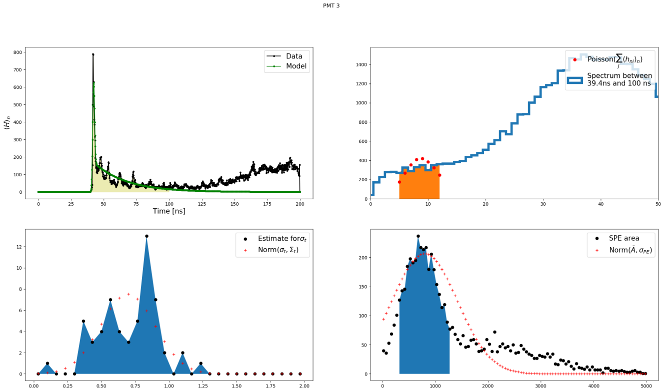
Results (^{57}Co peak 1)

Notice that for this peak F is much greater and τ_s is much smaller.



NQ	F	$\sigma_t[ns]$	$\sigma_t[ns]$	σ_{PE}
7	0.4	18	0.6	1

Results (^{57}Co peak 1 - strange result)



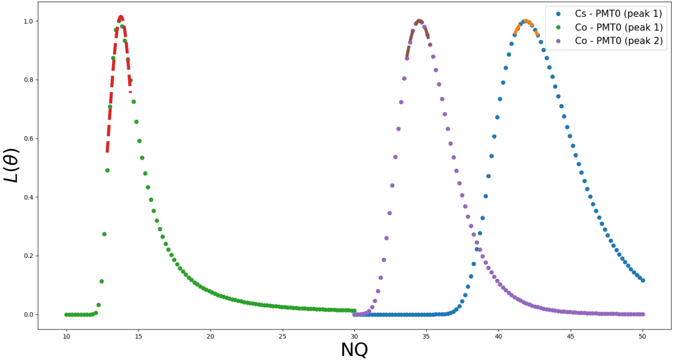
Parameter estimation

$\hat{\theta}$ is the parameter array that maximize L, so for any parameter θ_i,

$$\begin{aligned} \partial_{\theta_i} L = 0 \quad \rightarrow \quad & L(\hat{\theta} + \Delta\theta) = L_{max}(1 + \frac{\partial^2_{\theta_i} L}{2L_{max}} \Delta\theta^2) \quad \rightarrow \\ & L(\hat{\theta} + \Delta\theta) \approx L_{max} e^{-\Delta\theta^2 / 2\sigma_\theta^2} \end{aligned} \tag{15}$$

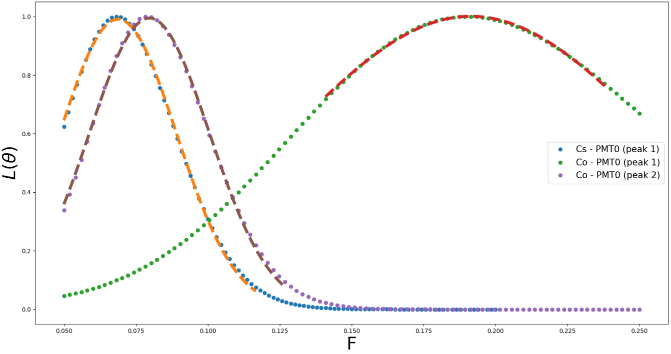
For each parameter L was maximized while holding the parameter fixed (for a range of parameters, for each PMT individually)

NQ



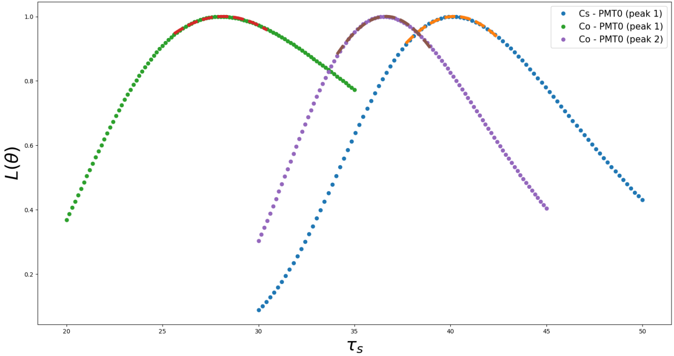
Source	$\hat{NQ}$	σ_{NQ}
Cs	42	2.5
Co (peak 1)	14	0.8
Co (peak 2)	34	1.7

F



Source	$\hat{F}$	σ_F	significance $[\sigma]$
Cs	0.07	0.02	3
Co (peak 1)	0.2	0.06	3
Co (peak 2)	0.08	0.02	4

τ_s



Source	$\hat{\tau}_s[ns]$	$\sigma_{\tau_s}[ns]$
Cs	40	6
Co (peak 1)	28	8
Co (peak 2)	37	5

How to go on

- Preform this analysis for all PMTs individually for both sources and both orientations (source to the lab / to the door).
- Study the low energy component in the Co spectrum and if it has a different scintillation regime (NR?).
- Maximize L with the constrain that N, F, τ_s is the same for all PMTs (but different between sources). This will brake the degeneracy of NQ.
- Maximize L with only N, τ_s fixed for all PMTs. This will show the anisotropy in the fast component.
- Maximize L with N, τ_s fixed and constrain a 90 deg between the symmetry axis of the anisotropy of F in the two orientations.
- Maximize L with the above with the constrain that F is isotropic in the BG data.