LXe scintillation model

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The number of photons emitted at time i,

$$n_i^{ph} \sim Poisson(NY_i), \tag{1}$$

where N is the total number of photon emitted and Y is the model (assuming constant N for all events).
The number of PEs created in the PMT in time i,

$$n_i^{pe} \sim Binom(Q, Poisson(NY_i)). \tag{2}$$

The number of PEs that need to be resolved in time i,

$$\begin{aligned} \tilde{n}_i &= \sum_j m_{ij} \\ m_{ij} &\sim Binom(Q, Poisson(NY_j))Norm(i-j|\sigma_t) \end{aligned} \tag{3}$$

Assume that the sum of the independent variables m_{ij} is distributed normally, so the number of PEs that need to be resolved in time i,

$$\begin{aligned}
\tilde{n}_i &\sim Norm\left(\sum_j \mu_j \middle| \sqrt{\sum_j V_j}\right) \\
\mu_j &= QNY_j Norm(i-j|\sigma_t) \\
V_j &= \mu_j
\end{aligned} \tag{4}$$

The number of PEs that actually resolved in time i,

$$n_i \sim Norm(\tilde{n}_i|\sigma_{PE}) \tag{5}$$

We have a 2D histogram $H(i,n)$ which holds the number of events in which n PEs were resolved at time i. The expected number for $H(i,n)$ is,

$$E(H(i,n)) = N_{ev} Norm(n - \tilde{n}_i|\sigma_{PE}) = \\ \sum_{m_i} \sum_{ev \to \tilde{n}_i = m_i} Norm(n - m_i|\sigma_{PE}) = \\ N_{ev} \sum_{m_i} Norm(n - m_i|\sigma_{PE}) Norm(m_i - \sum_j \mu_j | \sqrt{\sum_j V_j}) \tag{6}$$

Model

$$E\left(\frac{H(i,n)}{N_{ev}}\right) = \sqrt{\frac{\pi}{2(v_i^2+\sigma_{PE}^2)}} Exp\left(-\frac{(n-\mu_i)^2}{2(v_i^2+\sigma_{PE}^2)}\right) \left(1 - Erf\left(\frac{n_i v_i^2 + \mu_i \sigma_{PE}^2}{\sqrt{2 v_i^2 \sigma_{PE}^2 (v_i^2+\sigma_{PE}^2)}}\right)\right) \tag{7}$$

For $Y_i = \frac{1-F}{\tau_s} e^{-t/\tau_s}$

$$\mu_i = \frac{QN(1-F)}{2\sqrt{\pi}\tau_s} Exp\left(\frac{\sigma_t^2}{2\tau_s^2} - \frac{t_i - T}{\tau_s}\right) \left(1 + Erf\left(\frac{t_i - T}{\sqrt{2}\sigma_t - \frac{\sigma_t}{\sqrt{2}\tau_s}}\right)\right) \tag{8}$$