

# LXe scintillation model

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The number of photons emitted at time  $i$ ,

$$n_i^{ph} \sim \text{Poisson}(NY_i), \quad (1)$$

where  $N$  is the total number of photon emitted and  $Y$  is the model (assuming constant  $N$  for all events).

The number of PEs created in the PMT in time  $i$ ,

$$n_i^{pe} \sim \text{Binom}(Q, \text{Poisson}(NY_i)). \quad (2)$$

The number of PEs that need to be resolved in time  $i$ ,

$$\begin{aligned} \tilde{n}_i &= \sum_j m_{ij} \\ m_{ij} &\sim \text{Binom}(Q, \text{Poisson}(NY_j)) \text{Norm}(i - j | \sigma_t) \end{aligned} \quad (3)$$

Assume that the sum of the independent variables  $m_{ij}$  is distributed normally, so the number of PEs that need to be resolved in time  $i$ ,

$$\begin{aligned}\tilde{n}_i &\sim Norm\left(\sum_j \mu_j \middle| \sqrt{\sum_j V_j}\right) \\ \mu_j &= QNY_j Norm(i-j|\sigma_t) \\ V_j &= \mu_j\end{aligned}\tag{4}$$

The number of PEs that actually resolved in time  $i$ ,

$$n_i \sim Norm(\tilde{n}_i|\sigma_{PE})\tag{5}$$

We have a 2D histogram  $H(i, n)$  which holds the number of events in which  $n$  PEs were resolved at time  $i$ . The expected number for  $H(i, n)$  is,

$$\begin{aligned}
 E(H(i, n)) &= N_{ev} Norm(n - \tilde{n}_i | \sigma_{PE}) = \\
 &\sum_{m_i} \sum_{ev \rightarrow \tilde{n}_i = m_i} Norm(n - m_i | \sigma_{PE}) = \\
 &N_{ev} \sum_{m_i} Norm(n - m_i | \sigma_{PE}) Norm(m_i - \sum_j \mu_j | \sqrt{\sum_j V_j})
 \end{aligned}
 \tag{6}$$

# Model

$$E\left(\frac{H(i,n)}{N_{ev}}\right) = \sqrt{\frac{\pi}{2(v_i^2 + \sigma_{PE}^2)}} \exp\left(-\frac{(n - \mu_i)^2}{2(v_i^2 + \sigma_{PE}^2)}\right) \left(1 - \operatorname{Erf}\left(\frac{n_i v_i^2 + \mu_i \sigma_{PE}^2}{\sqrt{2v_i^2 \sigma_{PE}^2 (v_i^2 + \sigma_{PE}^2)}}\right)\right) \quad (7)$$

For  $Y_i = \frac{1-F}{\tau_s} e^{-t/\tau_s}$

$$\mu_i = \frac{QN(1-F)}{2\sqrt{\pi}\tau_s} \text{Exp} \left( \frac{\sigma_t^2}{2\tau_s^2} - \frac{t_i - T}{\tau_s} \right) \left( 1 + \text{Erf} \left( \frac{t_i - T}{\sqrt{2}\sigma_t - \frac{\sigma_t}{\sqrt{2}\tau_s}} \right) \right) \quad (8)$$