LXe scintillation model

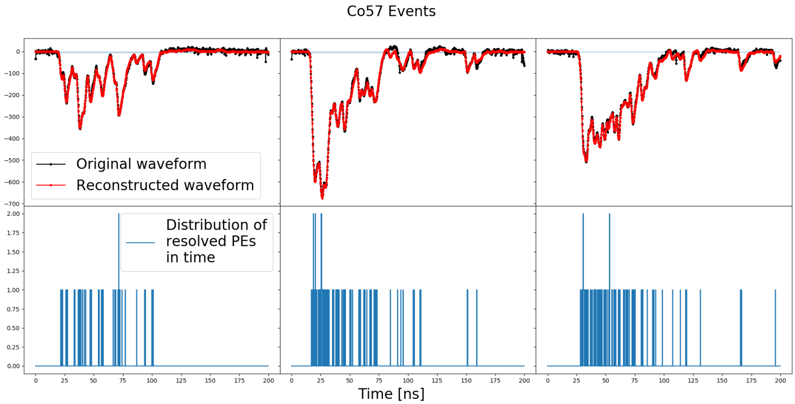
April 20, 2020

Objective

The goal is is build a scintillation model for LXe: $Y_a(t, \hat{\theta})$. $Y_a(t, \hat{\theta})$ is the probability to emit a photon at time t to the direction $\hat{\theta}$. The subscript a indicates the different type of interactions (γ, N, α and maybe different energies in the same interaction type).

Signal Reconstruction

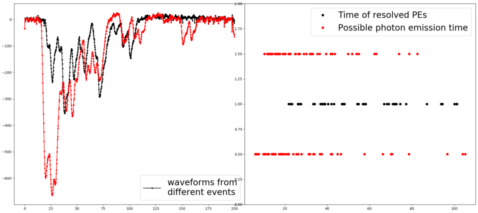
To study the temporal structure of the photon emission a processing algorithm uses a template of the average SPE signal to reconstruct the temporal PE pattern in each event for each PMT separately. The output of this process is the number of PEs created in the PMT in each digitization point.



Time Alignment Problem

We do not know the "time zero" for each event due to two reasons:

- The trigger time is "random" so alignment by the trigger time would not help.
- Alignment by the first resolved PE in the events: the resolved PEs time distribution is a random sample of the real emission times. We do not know the delay between the first resolved PE and the first photon that been emitted.



Time Alignment Problem

Solution: Align by the first resolved photon and adjust the model from probabilities of photon emission times to probabilities of time difference between photons.

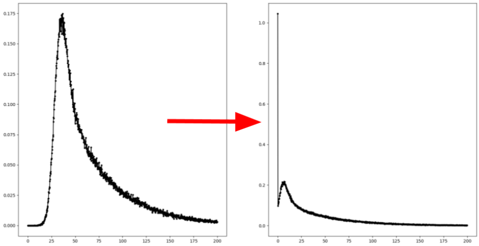
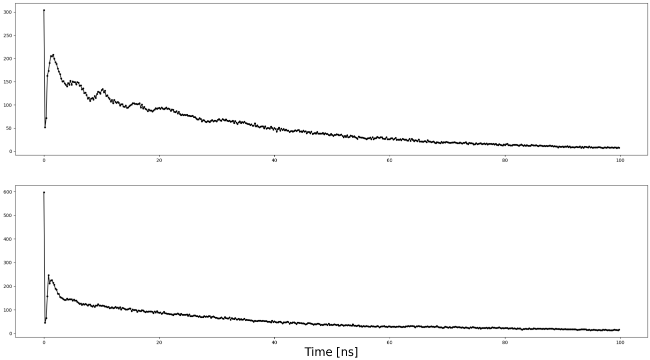


Figure 1: Mean temporal distribution of 10K simulated events (two exponential decay model). Left is no aligned at all so the fast component is smeared by the jitter of the trigger. Right is all the events aligned by the first PE in the events. The pole at 0 represents the fact that in each event the number of PEs resolved at time 0 is greater then 0 (by definition). More about the simulation in next slides.

Spoiler

This is how the temporal distribution from ^{57}Co events looks like on two PMTs:



PE Histogram

The observable that we want to build a model for is a 2D histogram (for each PMT and each interaction type) H_{ni} which holds the number of events in which n PEs where resolved at time t_i after the first resolved PE in that events.

$\sum_n nH_{ni}$ is the right panel in figure 1.

Scintillation Model

The number of photons emitted at the time window $[t_i, t_i + dt_i]$ is distributed Poisson,

$$n_i^{ph} \sim \text{Poisson}(Y_a(t_i)N_a dt_i) \tag{1}$$

where N_a is the average number of photons generated by event type a and $Y_a(t_i)$ is the probability to emit a photon at time t_i. The number of PEs created in the time window is

$$n_i^{pe} \sim \text{Binom}(Q, \text{Poisson}(Y_a(t_i)N_a dt_i)), \tag{2}$$

where Q is the photon detection efficiency (quantum efficiency, collection efficiency, double PE probability..., all the mechanisms that takes m photons and convert them to $n < m$ PEs).

Scintillation Model

Each PMT has its temporal uncertainty combined with the code's temporal uncertainty. $\tilde{n}_i$ - the number of PEs that resolved at time window t_i is a sum of a random variables m_{ij} that represents the number of PEs that were created at time t_j but were resolved at time t_i:

$$\begin{aligned} m_{ij} &\sim n_j^{pe} dt_i \mathrm{Norm}(t_i | T_0 + t_j, \sigma_t), \\ \tilde{n}_i &\sim \sum_{j \in \text{all digi points}} m_{ij}, \end{aligned} \tag{3}$$

where T_0 is the time when the events started in the digitization window and σ_t is the temporal uncertainty of each PMT.

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m_{ij} is kind of ill-defined with the normal distribution which gives non-integer values, but its need to be though in the sense that as the temporal uncertainty is larger there is a higher probability to resolve a bigger fraction of n_j^{pe} at a different time.

Scintillation Model

Since $\tilde{n}_i$ is a sum of independent random variables its distribution can be approximated by a different distribution with mean and variance which are the sum of the means and variances of m_{ij},

$$\begin{aligned} \langle \tilde{n}_i \rangle &= \sum_j \langle m_{ij} \rangle \\ \mathrm{Var}(\tilde{n}_i) &= \sum_j \mathrm{Var}(m_{ij}) \end{aligned} \tag{4}$$

Plug in the given n_j^{pe} and compute the mean and the variance of m_{ij} you get

$$\langle m_{ij} \rangle = \mathrm{Var}(m_{ij}) = Q N_a dt_i dt_j Y_a(t_j) \mathrm{Norm}(t_i | T_0 + t_j, \sigma_t). \tag{5}$$

Scintillation Model

The average and the variance of $\tilde{n}$ are equal so we assume its distributed Poisson

$$\begin{aligned}\tilde{n}_i \sim & \text{Poisson}\left(QN_a dt_i dt_j \sum_j Y_a(t_j)\text{Norm}(t_i|T_0+t_j,\sigma_t)\right) = \\ & \text{Poisson}\left(QN_a dt_i \int Y_a(t_j)\text{Norm}(t_i|T_0+t_j,\sigma_t)dt_j\right).\end{aligned} \tag{6}$$

Scintillation Model

If we will take $Y_a(t_i)$ as a sum exponential decaying components

$$Y_a(t_j) = \sum_c \frac{F_c}{\tau_c} e^{-t_j/\tau_c} \quad (\sum_c F_c = 1) \tag{7}$$

we will get

$$\begin{aligned} &\int Y_a(t_j) \text{Norm}(t_i | T_0 + t_j, \sigma_t) dt = \\ &\sum_c \frac{F_c K_c}{\tau_c} e^{-t_i \tau_c} \left[1 - \text{erf}\left(\frac{\sigma_t}{\sqrt{2}\tau_c} - \frac{t_i - T_0}{\sqrt{2}\sigma_t} \right) \right] \end{aligned} \tag{8}$$

where K_c is a normalization factor

$$K = \left[1 - \text{erf}\left(\frac{\sigma_t}{\sqrt{2}\tau} + \frac{T_0}{\sqrt{2}\sigma_t} \right) + e^{-\sigma_t^2/2\tau^2 - T_0/\tau} \left(1 + \text{erf}\left(\frac{T}{\sqrt{2}\sigma_t} \right) \right) \right]^{-1} \tag{9}$$

Scintillation Model with $\delta(t)$

If we want to add a super fast component in the beginning of the model it is represented by

$$Y_a(t) = (1 - R_\delta) \sum_c \frac{F_c}{\tau_c} e^{-t/\tau_c} + R_\delta \delta(t), \tag{10}$$

So we need to add to equation 8 from the previous slide:

$$\frac{R_\delta}{\sqrt{2\pi}\sigma_t} e^{-(t-T_0)^2/(2\sigma_t^2)} \tag{11}$$

Scintillation Model - Alignment

Recall that we build a model for H_{ni} which holds the number of events in which n PEs were resolved at time t_i after the first resolved PE. Without the alignment problem, naively,

$$\tilde{H}_{ni} = N_{\text{events}} \text{Poisson}\left(n \middle| \lambda = QN_a dt \int Y_a(t_j) \text{Norm}(t_i | T_0 + t_j, \sigma_t) dt\right) \tag{12}$$

$\tilde{H}_{ni}$ is H_{ni} before the alignment. The naively comment states that we also need to account the uncertainty in the number of PEs resolved, i.e the probability to resolve n PEs where m where actually extracted. This is related to the width of the SPE area distribution and will be treated later.

Alignment for $i > 0$

The model $\tilde{H}_{ni}$ tells us what is the probability that n PEs will be resolved at time t_i. So

$$\begin{aligned} H_{ni} = \sum_j &[\text{Non of the PMTs resolved a PE untill time } t_j]\times \\ &[\text{Some PMT resolved a PE (or more) at time } t_j]\times \\ &\tilde{H}_{ni+j} \end{aligned} \tag{13}$$

Alignment for $i > 0$

$$[\text{Non of the PMTs resolved a PE untill time } t_j] = \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}_{0k}^{\text{pmt}} \tag{14}$$

$$[\text{Some PMT resolved a PE (or more) at time } t_j] = \sum_{\text{pmt}} [1 - \tilde{H}_{0j}^{\text{pmt}}] \tag{15}$$

The superscript pmt indicates the product on all PMTs (each pmt has a different model).

Alignment for $i > 0$

$$H_{ni} = \sum_j \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}_{0k}^{\text{pmt}} \sum_{\text{pmt}} [1 - \tilde{H}_{0j}^{\text{pmt}}] \tilde{H}_{ni+j} \qquad (16)$$

Alignment for $i = 0, n > 0$

In this case we dont need the middle term in the previous slide (which represents the probability that the first PE was resolved at time t_j), So for $n > 0$

$$H_{n0} = \sum_j \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}^{\text{pmt}}_{0k} \tilde{H}_{nj} \tag{17}$$

Alignment for $i = 0, n = 0$

Here we do need the middle term but we dont want to sum on the pmt of interest. So

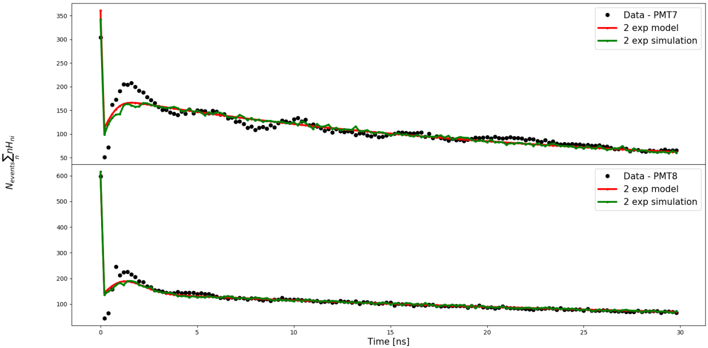
$$H_{00}^{\mathrm{pmt}_0} = \sum_j \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}_{0k}^{\mathrm{pmt}} \sum_{\mathrm{pmt} \neq \mathrm{pmt}_0} [1 - \tilde{H}_{0j}^{\mathrm{pmt}}] \tilde{H}_{0j}^{\mathrm{pmt}_0} \tag{18}$$

First Look at Data

I ran the reconstruction algorithm on the ^{57}Co dataset with PMTs 7 and 8. In each event the two signals were aligned by the delay that was measured by the pulser data. After reconstruction the temporal pattern of the resolved PEs was aligned ones more relative to the first PE resolved (in PMT 7 or 8). The events in which the χ^2 of the reconstructed signal relative to the signal was too big were cut out. Also events with large baseline width were cut out. A range in the energy spectrum (number of resolved PEs) of each PMT was chosen and from these events a 2D histogram was made for each PMT (D_{ni}^{pmt}) which holds the number of events in which n PEs was resolved at time t_i after the first resolved PE.

Maybe it is better to choose events from the combined distribution, i.e if an events is cut in one PMT it should be cutted out of all the PMTs.

First Look at Fit - Double Exp Model



PMT	NQ	T_0 [ns]	σ_t [ns]	F	τ_f [ns]	τ_s [ns]
7	34.5	39.9	0.4	0.2	15.6	44
8	34.8	40.5	1	0.05	0.15	39.6

First Look at Fit - Double Exp Model

We see that the model doesn't fit perfectly the data but at list the simulation and the model give the same temporal structure which indicates that the analytical model represents the physics we try to model. Its a good point to talk about the simulation.

Simulation 1

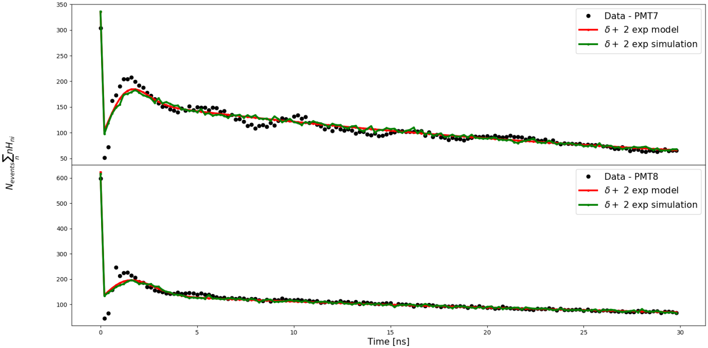
the simulation creates the temporal structure of 10K events on two PMTs. Align the events by the first PE and finally creates a simulated 2D Histogram for each PMT (S_{ni}) which holds the number of simulated events in which n PEs were created at time t_i after the first PE. For each simulated event:

- A trigger time (t_{trig}) is sampled out of a normal distribution with mean 0 and variance σ^2_{trig}. This trigger is common for all PMTs.
- For each PMT (a) a total number of PEs in event (n^a) is sampled out of Poisson distribution with mean NQ^a.
- For each PMT the n^a PEs are grouped in two groups n_f, n_s (three groups with n_δ if we want to simulate a $\delta(t)$ pulse). The occupancy of each group sampled from distribution with probabilities $F, 1-F$ (and R^a_δ for the δ model).

Simulation 1

- For each PMT, nf times (t_f) are sampled from an exponential distribution with decay constant τ_f, and ns times (t_s) are sampled from an exponential distribution with decay constant τ_s.
- For each PMT smear the two exponential component by sampling for each sampled time $t^i_{f/s} \rightarrow \text{Normal}(\text{mean} = t_{trig} + T^a_0 + t^i_{f/s}, \text{Var} = (\sigma^a_t)^2)$
- Find the minimal time over all PMTs (global for event) and roll all times back relative to this time.

First Look at Fit - $\delta(t)$+ Double Exp Model



PMT	NQ	T_0 [ns]	σ_t [ns]	R	F	τ_f [ns]	τ_s [ns]
7	33.6	40.9	0.7	0.04	0.06	19.7	34.1
8	35.1	40.8	1.14	0.01	0.04	0.17	41.2

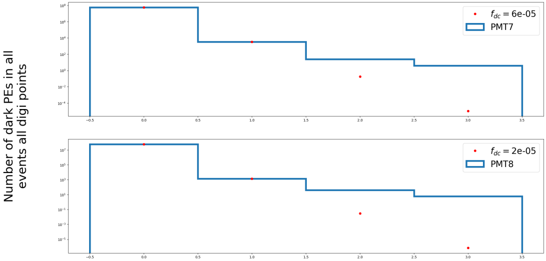
$\delta(t)$+ Double Exp Model - F, τ_f, τ_s Global

It make sense that F, τ_f and τ_s are not directional, i.e all PMTs should see the same values. When I tried to fit the data with this values globally for both PMTs I got weird results ($\tau_s \sim 200$ [ns], both $\sigma_t > 1$ [ns]). To mitigate this I fitted the data of the hit times of the PEs (D_{ni}^{pmt}) simultaneously with the delays distribution between SPE hits acquired from the calibration data and with correction for dark current. The model I used for the delay distribution is

$$\text{Delay}_{ij} = a_{delay}\text{Normal}\left(\text{mean} = T_{0,i} - T_{0,j}, \text{Var} = \left(\sigma_{t,i}^2 + \sigma_{t,j}^2\right)\right). \tag{19}$$

Dark Current

The reconstruction algorithm some time reconstruct a PE where it should not be. The rate of this falls reconstructions is the dark current (f_{dc}). The probability to have $n > 0$ dark PEs at a digi point is f_{dc}^n and the probability to 0 dark PEs in a digi point is $1 - \frac{f_{dc}}{1-f_{dc}}$. This parameter can be calibrated from the pulser data by applying the reconstruction algorithm out of the time window where the SPE is expected.



Dark Current Correction to the Model

$$\begin{aligned} \tilde{H}_{0i} &\to \left(1 - \frac{f_{dc}}{1 - f_{dc}}\right) \tilde{H}_{0i} \\ \tilde{H}_{0i} &\to \left(1 - \frac{f_{dc}}{1 - f_{dc}}\right) \tilde{H}_{ni} + \sum_{m=1}^{n} f_{dc}^{m} \tilde{H}_{n-mi} \end{aligned} \tag{20}$$

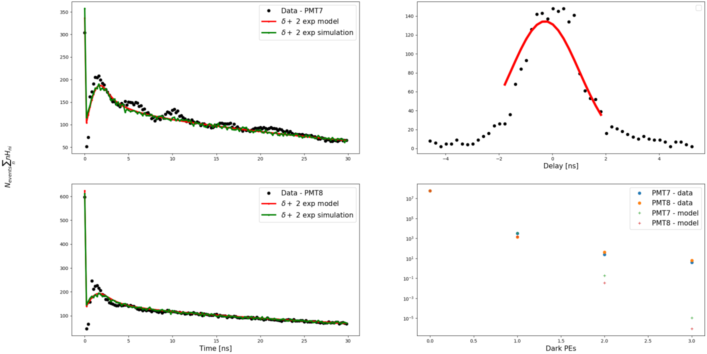
This should be corrected before the alignment.

Dark Current Correction to the Simulation

For each digi point add a random number sampled with the probability

$$
\begin{aligned}
P(0) &= 1 - \frac{1}{1 - f_{dc}} \\
P(n > 0) &= f_{dc}^{n}
\end{aligned}
\tag{21}
$$

Global Fit to Temporal structure, Delays Distribution and Dark Current Rate



Global Fit to Temporal structure, Delays Distribution and Dark Current Rate

PMT	NQ	T_0 [ns]	σ_t [ns]	R	f_{dc}
7	34	2	0.75	0.03	$6 \cdot 10^{-5}$
8	34	1.7	1	0.03	$2 \cdot 10^{-5}$

F	τ_f [ns]	τ_s [ns]
0.02	1.76	37

The T_0 parameter is weird but we don't really sensitive to it, only to the difference between the two values.

Correction due the SPE resolution

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