LXe scintillation model

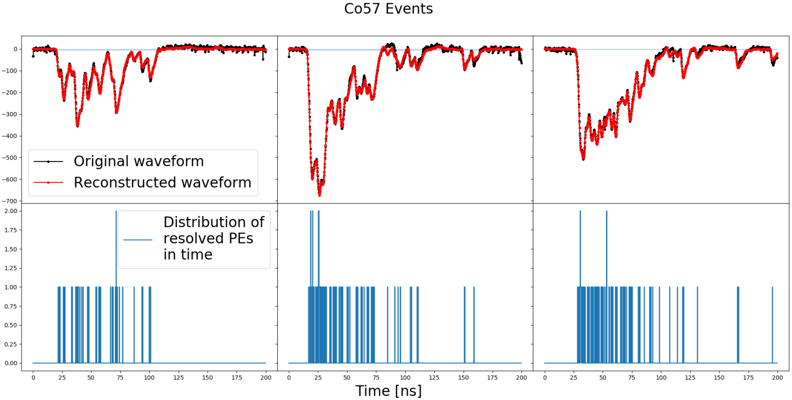
April 23, 2020

Objective

The goal is is build a scintillation model for LXe: $Y_a(t, \hat{\theta})$. $Y_a(t, \hat{\theta})$ is the probability to emit a photon at time t to the direction $\hat{\theta}$. The subscript a indicates the different type of interactions (γ, N, α and maybe different energies in the same interaction type).

Signal Reconstruction

To study the temporal structure of the photon emission a processing algorithm uses a template of the average SPE signal to reconstruct the temporal PE pattern in each event for each PMT separately. The output of this process is the number of PEs created in the PMT in each digitization point.



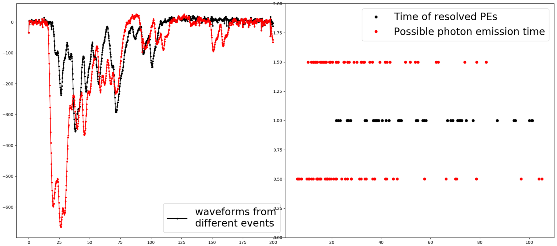
The Dataset D_{ni}

From the collection of PE times we build a dataset for each PMT - D_{ni}. This is a 2D table which holds the number of times n PEs were resolved at time t_i.
This invokes a time alignment problem between different events.

Time Alignment Problem

We do not know the ”time zero” for each event due to two reasons:

- The trigger time is ”random” so alignment by the trigger time would not help.
- Alignment by the first resolved PE in the events: the resolved PE time distribution is a random sample of the real emission times. We do not know the delay between the first resolved PE and the first photon that been emitted.



Time Alignment Problem

Solution: Align all PMTs by the first resolved PE in the event and adjust the model from probabilities of photon emission times to probabilities of time difference between photons.

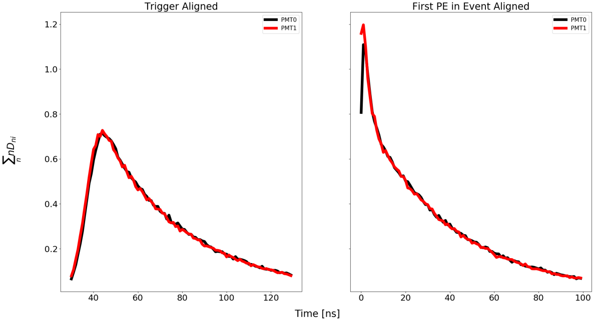
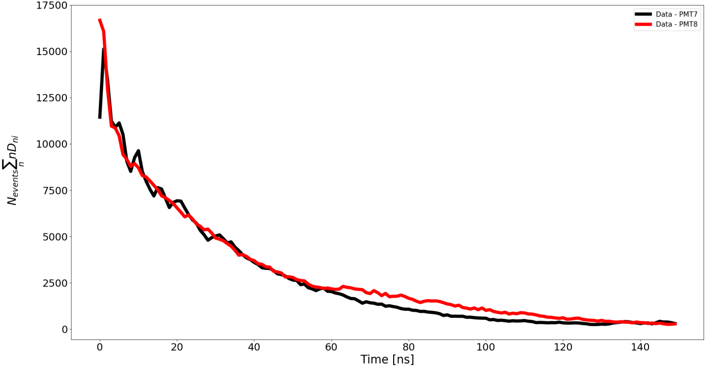


Figure 1: Mean temporal distribution of 10K simulated events with two PMTs, double exponential decay model, 1 ns binned. Left - the fast component is smeared by the jitter of the trigger. Right - The sub-nanosecond misalignment between the two PMTs manifests itself as greater probability to resolve a PE in one PMT then in the other in the first couple of nanoseconds.

Data

This is how the temporal distribution from ^{57}Co events looks like on two PMTs (1 ns binned):



What was Wrong in the Previous Analysis

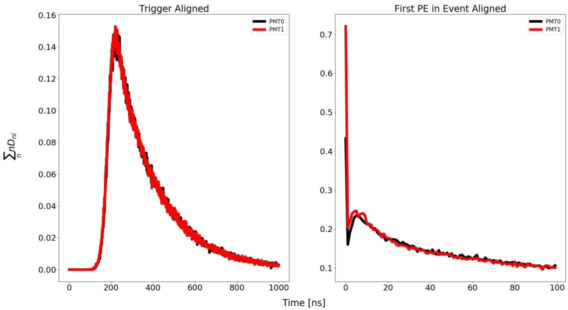


Figure 2: This is the same simulation 0.2 ns binned. The pole in the aligned temporal structure is due the alignment by first PE - by definition we have atleast one PE at time 0. In the previous analysis I aligned the events by the time the reached 10% of their height, which is like alignment by the fist PE with some smearing. This lead to a fake δ peak. I found this systematic by simulating double exponential decay events that also gave the face δ. To prevent this the development of the analyticity model should go hand-in-hand with simulation.

Event Simulation

the simulation creates the temporal structure of 10K events on two PMTs. Align the events by the first PE and finally creates a simulated 2D table for each PMT (S_{ni}) which holds the number of simulated events in which n PEs were created at time t_i after the first PE in the event. For each simulated event:

- A trigger time (t_{trig}) is randomly chosen from a normal distribution with mean 0 and variance σ^2_{trig}. This trigger is common for all PMTs.
- For each PMT a total number of PEs in event (n) is randomly chosen out of a Poisson distribution with mean NQ.
- For each PMT the n PEs are grouped in two groups n_f, n_s (three groups with n_δ if we want to simulate a $\delta(t)$ pulse). The occupancy of each group chosen randomly from distribution with probabilities $F, 1 - F$ (and R^a_δ for the δ model).

Event Simulation

- For each PMT, nf times (t_f) are randomly chosen from an exponential distribution with decay constant τ_f, and ns times (t_s) are randomly chosen from an exponential distribution with decay constant τ_s.
- For each PMT we smear the two exponential component by shifting each sampled time $t^i_{f/s} \to \mathrm{Normal}(\mathrm{mean} = t_{trig} + T^a_0 + t^i_{f/s}, \mathrm{Var} = (\sigma_t)^2)$, where σ_t is the temporal uncertainty of the PMT.
- Find the minimal time over all PMTs (global for event) and roll all times back relative to this time.

Scintillation Model

The number of photons emitted at the time window $[t_i, t_i + dt_i]$ is distributed Poisson,

$$n_i^{ph} \sim \text{Poisson}(Y_a(t_i) N_a dt_i) \tag{1}$$

where N_a is the average number of photons generated by event type a and $Y_a(t_i)$ is the probability to emit a photon at time t_i. The number of PEs created in the time window is

$$n_i^{pe} \sim \text{Binom}(Q, \text{Poisson}(Y_a(t_i) N_a dt_i)), \tag{2}$$

where Q is the photon detection efficiency (quantum efficiency, collection efficiency, double PE probability..., all the mechanisms that takes m photons and convert them to $n < m$ PEs).

Scintillation Model

Each PMT has its temporal uncertainty combined with the code's temporal uncertainty. $\tilde{n}_i$ - the number of PEs that resolved at time window t_i is a sum of a random variables m_{ij} that represents the number of PEs that were created at time t_j but were resolved at time t_i:

$$\begin{aligned} m_{ij} &\sim n_j^{pe} dt_i \mathrm{Norm}(t_i | T_0 + t_j, \sigma_t), \\ \tilde{n}_i &\sim \sum_{j \in \text{all digi points}} m_{ij}, \end{aligned} \tag{3}$$

where T_0 is the time when the events started in the digitization window and σ_t is the temporal uncertainty of each PMT.

m_{ij} is kind of ill-defined with the normal distribution which gives non-integer values, but its need to be though in the sense that as the temporal uncertainty is larger there is a higher probability to resolve a bigger fraction of n_j^{pe} at a different time.

Scintillation Model

Since $\tilde{n}_i$ is a sum of independent random variables its distribution can be approximated by a different distribution with mean and variance which are the sum of the means and variances of m_{ij},

$$\begin{aligned} \langle \tilde{n}_i \rangle &= \sum_j \langle m_{ij} \rangle \\ \mathrm{Var}(\tilde{n}_i) &= \sum_j \mathrm{Var}(m_{ij}) \end{aligned} \tag{4}$$

Plug in the given n_j^{pe} and compute the mean and the variance of m_{ij} you get

$$\langle m_{ij} \rangle = \mathrm{Var}(m_{ij}) = QN_a dt_i dt_j Y_a(t_j) \mathrm{Norm}(t_i | T_0 + t_j, \sigma_t). \tag{5}$$

Scintillation Model

The average and the variance of $\tilde{n}$ are equal so we assume its distributed Poisson

$$\begin{aligned}\tilde{n}_i \sim & \text{Poisson}\left(QN_a dt_i dt_j \sum_j Y_a(t_j)\text{Norm}(t_i|T_0+t_j,\sigma_t)\right) = \\ & \text{Poisson}\left(QN_a dt_i \int Y_a(t_j)\text{Norm}(t_i|T_0+t_j,\sigma_t)dt_j\right).\end{aligned} \tag{6}$$

Scintillation Model

If we will take $Y_a(t_i)$ as a sum exponential decaying components

$$Y_a(t_j) = \sum_c \frac{F_c}{\tau_c} e^{-t_j/\tau_c} \quad (\sum_c F_c = 1) \tag{7}$$

we will get

$$\begin{aligned} &\int Y_a(t_j) \mathrm{Norm}(t_i | T_0 + t_j, \sigma_t) dt = \\ &\sum_c \frac{F_c K_c}{\tau_c} e^{-t_i \tau_c} \left[1 - \mathrm{erf}\left(\frac{\sigma_t}{\sqrt{2}\tau_c} - \frac{t_i - T_0}{\sqrt{2}\sigma_t} \right) \right] \end{aligned} \tag{8}$$

where K_c is a normalization factor

$$K = \left[1 - \mathrm{erf}\left(\frac{\sigma_t}{\sqrt{2}\tau} + \frac{T_0}{\sqrt{2}\sigma_t} \right) + e^{-\sigma_t^2/2\tau^2 - T_0/\tau} \left(1 + \mathrm{erf}\left(\frac{T}{\sqrt{2}\sigma_t} \right) \right) \right]^{-1} \tag{9}$$

Scintillation Model with $\delta(t)$

If we want to add a super fast component in the beginning of the model it is represented by

$$Y_a(t) = (1 - R_\delta) \sum_c \frac{F_c}{\tau_c} e^{-t/\tau_c} + R_\delta \delta(t), \tag{10}$$

So we need to add to equation 8 from the previous slide:

$$\frac{R_\delta}{\sqrt{2\pi}\sigma_t} e^{-(t-T_0)^2/(2\sigma_t^2)} \tag{11}$$

Scintillation Model - Alignment

Recall that we build a model for H_{ni} which holds the number of events in which n PEs were resolved at time t_i after the first resolved PE. Without the alignment problem, naively,

$$\tilde{H}_{ni} = N_{\text{events}}\text{Poisson}\left(n \middle| \lambda = QN_a dt \int Y_a(t_j)\text{Norm}(t_i|T_0 + t_j, \sigma_t)dt\right) \tag{12}$$

$\tilde{H}_{ni}$ is H_{ni} before the alignment. The naively comment states that we also need to account the uncertainty in the number of PEs resolved, i.e the probability to resolve n PEs where m where actually extracted. This is related to the width of the SPE area distribution and will be treated later.

Alignment for $i > 0$

The model $\tilde{H}_{ni}$ tells us what is the probability that n PEs will be resolved at time t_i. So

$$H_{ni} = \sum_j [\text{Non of the PMTs resolved a PE untill time } t_j] \times [\text{Some PMT resolved a PE (or more) at time } t_j] \times \tilde{H}_{ni+j} \tag{13}$$

Alignment for $i > 0$

$$[\text{Non of the PMTs resolved a PE untill time } t_j] = \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}_{0k}^{\text{pmt}} \tag{14}$$

$$[\text{Some PMT resolved a PE (or more) at time } t_j] = \sum_{\text{pmt}} [1 - \tilde{H}_{0j}^{\text{pmt}}] \tag{15}$$

The superscript pmt indicates the product on all PMTs (each pmt has a different model).

Alignment for $i > 0$

$$H_{ni} = \sum_j \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}^{\text{pmt}}_{0k} \sum_{\text{pmt}} [1 - \tilde{H}^{\text{pmt}}_{0j}] \tilde{H}_{ni+j} \tag{16}$$

Alignment for $i = 0, n > 0$

In this case we dont need the middle term in the previous slide (which represents the probability that the first PE was resolved at time t_j), So for $n > 0$

$$H_{n0} = \sum_j \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}^{\text{pmt}}_{0k} \tilde{H}_{nj} \tag{17}$$

Alignment for $i = 0, n = 0$

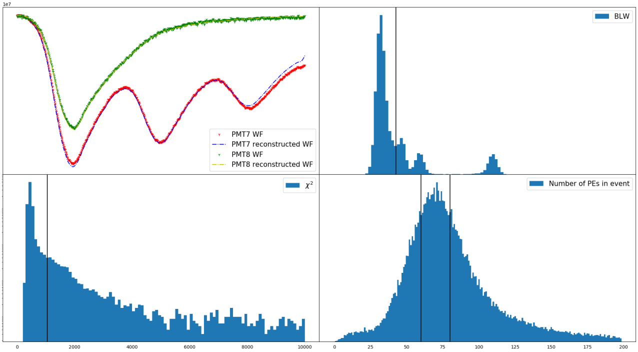
Here we do need the middle term but we dont want to sum on the pmt of interest. So

$$H_{00}^{\mathrm{pmt}_0} = \sum_{j} \prod_{\text{all PMTs}} \prod_{k<j} \tilde{H}_{0k}^{\mathrm{pmt}} \sum_{\mathrm{pmt} \neq \mathrm{pmt}_0} [1 - \tilde{H}_{0j}^{\mathrm{pmt}}] \tilde{H}_{0j}^{\mathrm{pmt}_0} \qquad (18)$$

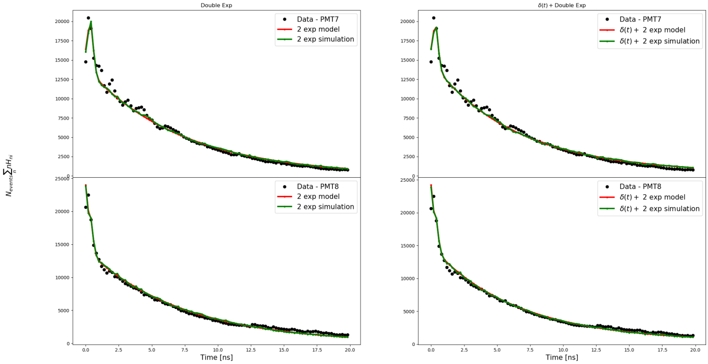
First Look at Data

I ran the reconstruction algorithm on the ^{57}Co dataset with PMTs 7 and 8. In each event the two signals were aligned by the delay that was measured by the pulser data. After reconstruction the temporal pattern of the resolved PEs was aligned ones more relative to the first PE resolved (in PMT 7 or 8). The events in which the χ^2 of the reconstructed signal relative to the signal was too big were cut out. Also events with large baseline width were cut out. A range in the energy spectrum (number of resolved PEs) of each PMT was chosen and from these events a 2D table was made for each PMT (D_{ni}) which holds the number of events in which n PEs was resolved at time t_i after the first resolved PE.

First Look at Data



First Look at Fit



First of all we see that the model and the simulation gives the same results so we can assume that the analytical model models the process we think happens.

First Look at Fit

- Double exp model:

PMT	NQ	T_0 [ns]	σ_t [ns]	F	τ_f [ns]	τ_s [ns]
7	34	39.9	1.01	0.07	0.19	36.5
8	35	39.5	1.08	–	–	–

  It seems that the fit needs a sub-nanosecond component, but a sub-nanosecond exponential with a nanosecond smearing is identical to a δ signal.

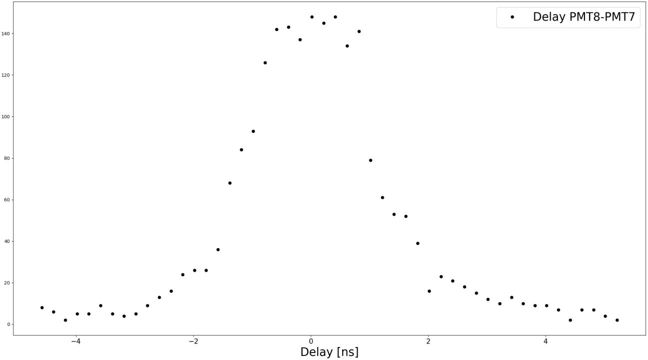
- $\delta(t)+$ Double exp model:

PMT	NQ	T_0 [ns]	σ_t [ns]	R	F	τ_f [ns]	τ_s [ns]
7	36	45.3	0.9	0.06	0.7	30	100
8	37	45.07	1.08	0.07	–	–	–

  Here it is seems that the δ takes most of the fast component and the τ_f represents the slow component (notice the difference of F in the two models).

Constraints on σ_t and T_0

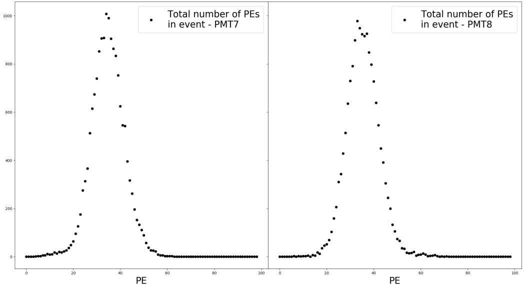
We can use the delay distribution from the pulser data to help the fit.



$$\text{Delay Distribution}^{ij} = a^{ij}_{\text{delay}} e^{-\frac{\left(\text{Delay}-(T_0^i - T_0^j)\right)^2}{2\left((\sigma_t^i)^2+(\sigma_t^j)^2\right)}} \tag{19}$$

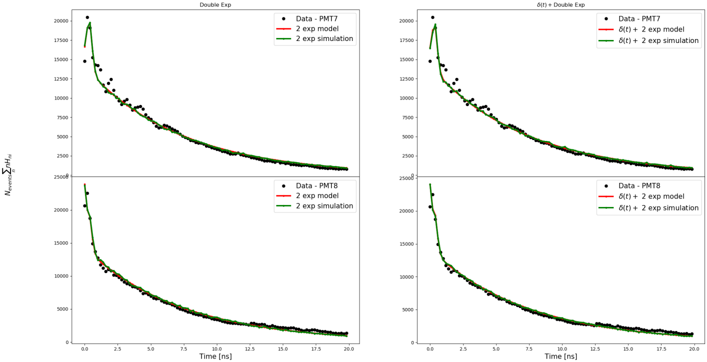
Constraints on NQ

We can use the number of PEs resolved in each event ("energy spectrum") to help the fit.

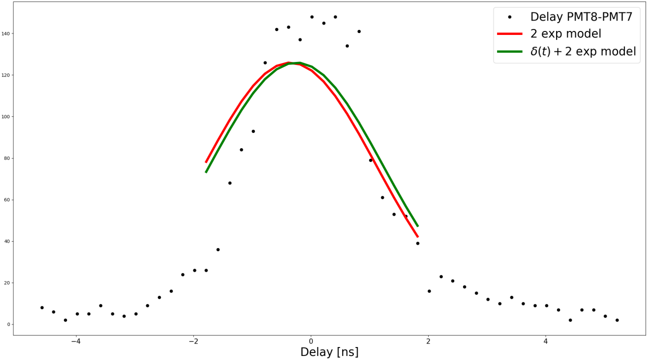


$$\text{Total number of PEs in event resolved in a PMT} = N_{\text{events}} \text{Poisson} \left(PE | \lambda = \sum_{ni} n H_{ni} \right) \tag{20}$$

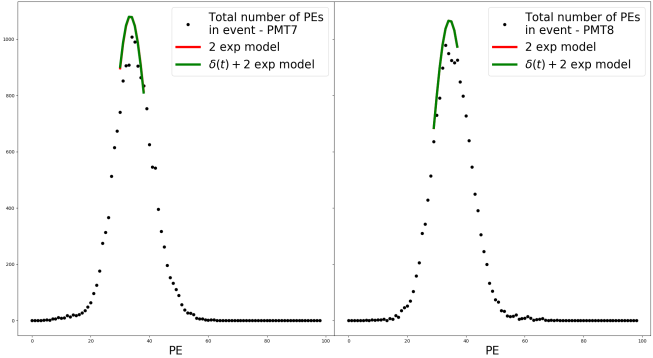
Global Fit for the Temporal Structure the Spectra and the Delay Distribution



Global Fit for the Temporal Structure the Spectra and the Delay Distribution



Global Fit for the Temporal Structure the Spectra and the Delay Distribution



Global Fit for the Temporal Structure the Spectra and the Delay Distribution

- Double exp model:

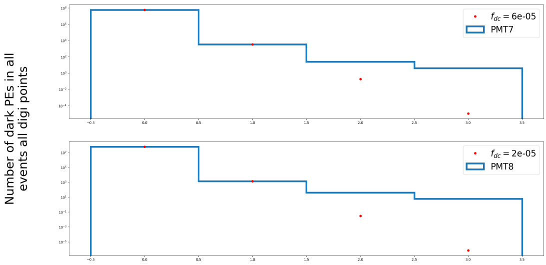
PMT	NQ	T_0 [ns]	σ_t [ns]	F	τ_f [ns]	τ_s [ns]
7	34	40	1.01	0.07	0.2	36.5
8	35	39.5	1.08	–	–	–

- $\delta(t)+$ double exp model:

PMT	NQ	T_0 [ns]	σ_t [ns]	R	F	τ_f [ns]	τ_s [ns]
7	36	45.3	0.97	0.06	0.7	30	100
8	37	35.1	1.1	0.07	–	–	–

Dark Count Correction

The reconstruction algorithm some time reconstruct a PE where it should not be. The rate of this falls reconstructions is the dark count (f_{dc}). The probability to have $n > 0$ dark PEs at a digi point is f_{dc}^n and the probability to have 0 dark PEs in a digi point is $1 - \frac{f_{dc}}{1-f_{dc}}$. This parameter can be calibrated from the pulser data by applying the reconstruction algorithm out of the time window where the SPE is expected.



Dark Count Correction to the Model

$$
\begin{aligned}
\tilde{H}_{0i} &\to \left(1 - \frac{f_{dc}}{1 - f_{dc}}\right) \tilde{H}_{0i} \\
\tilde{H}_{0i} &\to \left(1 - \frac{f_{dc}}{1 - f_{dc}}\right) \tilde{H}_{ni} + \sum_{m=1}^{n} f_{dc}^{m} \tilde{H}_{n-mi}
\end{aligned}
\tag{21}
$$

This should be corrected before the alignment.

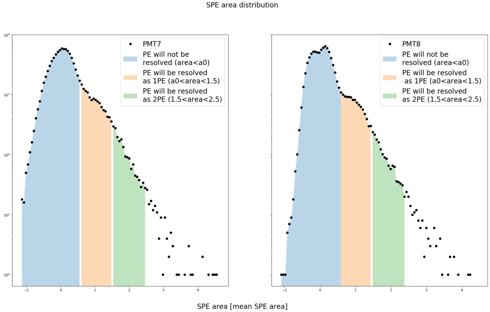
Dark Count Correction to the Simulation

For each digi point add a random number sampled with the probability

$$
\begin{aligned}
P(0) &= 1 - \frac{1}{1 - f_{dc}} \\
P(n > 0) &= f_{dc}^{n}
\end{aligned}
\tag{22}
$$

Correction due the SPE Area Resolution

When n PEs created at some time in the PMT there is some probability that $m \neq n$ PEs will be resolved. This is related to the SPE area resolution.



μ_{pad} and σ_{pad} are the mean and width of the padestial distribution, σ_{spe} is the width of the SPE area distribution (around 1) and a_0 is some threshold area which under it the PE will not be resolved.

Correction to the Simulation due the SPE Area Resolution

For each non-dark PE assign an area chosen randomly from Normal $\left(\text{mean} = \mu_{pad} + 1, \sigma^2 = \sigma^2_{pad} + \sigma^2_{spe}\right)$.

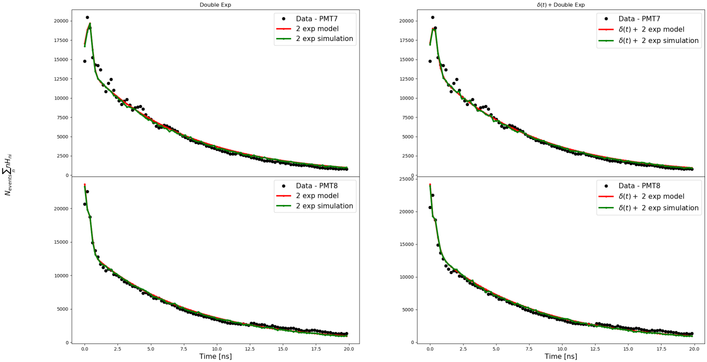
- If the area$< a_0$ eraser this PE.
- If the a_0 <area<1.5 leave the PE.
- If the $n - 0.5$ <area$< n + 0.5$ PE$\rightarrow$ nPEs.

Correction to the Model due the SPE Area Resolution

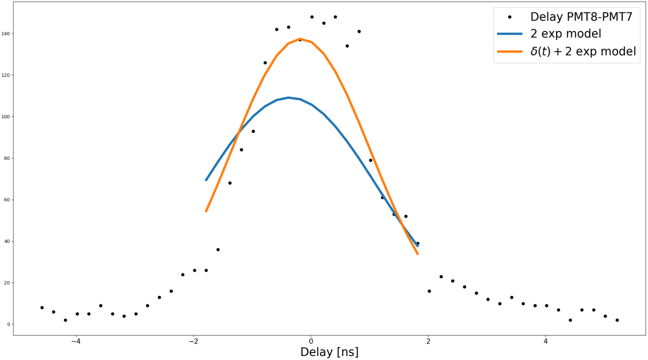
Recall that $\tilde{H}_{ni}$ is the number of times n PEs should be resolved at time t_i. $H_{ni} = \sum_m P_{nm}\tilde{H}_{mi}$ is the probability to resolve n PEs at time t_i, where P_{nm} is the probability to resolve n PEs from m real PEs.

- $P_{n0} = \delta_{n0}$ because the probability to resolve n PEs from 0 PEs is accounted by the dark count.
- $P_{0m} = \frac{1}{\sqrt{2\pi\left(\sigma_{pad}^2 + m\sigma_{spe}^2\right)}} \int_{-\infty}^{a_0} e^{-\frac{\left((x-(\mu_{pad}+m)\right)^2}{2\left(\sigma_{pad}^2 + m\sigma_{spe}^2\right)}} dx$
- $P_{1m} = \frac{1}{\sqrt{2\pi\left(\sigma_{pad}^2 + m\sigma_{spe}^2\right)}} \int_{a_0}^{1.5} e^{-\frac{\left((x-(\mu_{pad}+m)\right)^2}{2\left(\sigma_{pad}^2 + m\sigma_{spe}^2\right)}} dx$
- $P_{nm} = \frac{1}{\sqrt{2\pi\left(\sigma_{pad}^2 + m\sigma_{spe}^2\right)}} \int_{n-0.5}^{n+0.5} e^{-\frac{\left((x-(\mu_{pad}+m)\right)^2}{2\left(\sigma_{pad}^2 + m\sigma_{spe}^2\right)}} dx$

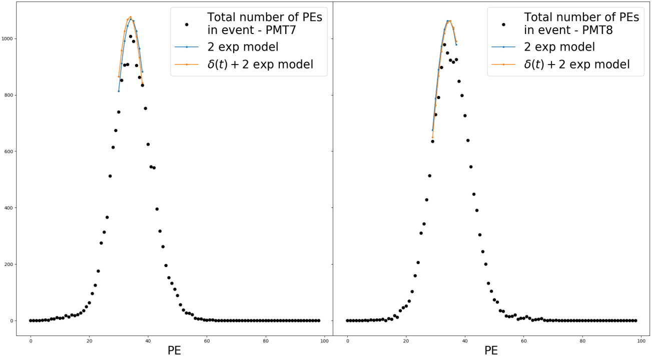
Global Fit of the Temporal Structure, Delay, Spectra, Dark Count rate and SPE Area Distribution



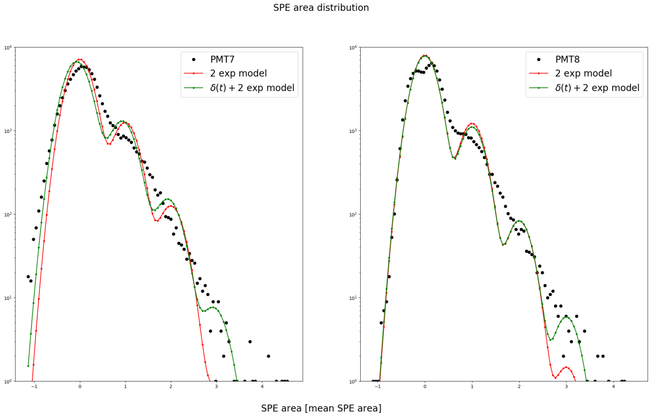
Global Fit of the Temporal Structure, Delay, Spectra, Dark Count rate and SPE Area Distribution



Global Fit of the Temporal Structure, Delay, Spectra, Dark Count rate and SPE Area Distribution

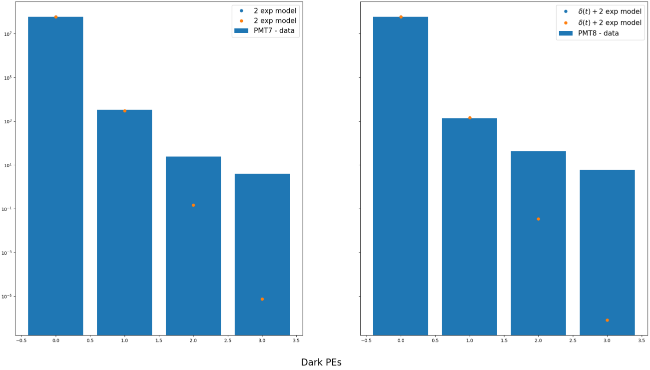


Global Fit of the Temporal Structure, Delay, Spectra, Dark Count rate and SPE Area Distribution



The global fit prefers a more narrow area distribution than the actual pulser data.

Global Fit of the Temporal Structure, Delay, Spectra, Dark Count rate and SPE Area Distribution



Global Fit for the Temporal Structure the Spectra and the Delay Distribution

model	F	τ_f [ns]	τ_s [ns]
2 exp model	0.07	0.2	36
$\delta(t)$+ 2 exp model	0.05	1.1	36