# part-2

October 6, 2023

[ ]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
import cvxpy as cp
```

[ ]:
```
raw_data = pd.read_excel('Data.xlsx', sheet_name=None, index_col=0)
```

[ ]:
```
data = {}
sheet_names = list(raw_data.keys())

for s in sheet_names:
    if s == 'FamaFrenchFactors':
        new_name = s
        col_labels = raw_data[s].columns
    else:
        new_name = s[18:]
        col_labels = range(1, 11)

    data[new_name] = raw_data.pop(s)
    data[new_name].columns = col_labels
    data[new_name].index = (pd.to_datetime(data[new_name].index, format='%Y%m')
                            .to_period('M').rename('Date'))
```

## 0.1 Question 3

[ ]:
```
class Portfolio:
    def __init__(
        self, mu: pd.Series, S: pd.DataFrame,
        long_only: bool = False,
        ret: float = None, sd: float = None, sharpe: float = None,
        weights: np.ndarray = None
    ) -> None:
        self.mu = mu
        self.S = S
        self.long_only = long_only
        self.ret = ret
        self.sd = sd
        self.sharpe = sharpe
```

```
        self.weights = weights

    def optimize_weights(self, target_ret: float = None) -> None:
        w = cp.Variable(self.mu.size)
        ret = self.mu.values @ w
        var = cp.quad_form(w, self.S.values)

        if target_ret is not None:
            self.ret = target_ret
        elif self.ret is None:
            print("Error: target return not specified")
            return

        if self.long_only:
            prob = cp.Problem(cp.Minimize(var),
                              [cp.sum(w) == 1, ret == self.ret, w >= 0])
        else:
            prob = cp.Problem(cp.Minimize(var),
                              [cp.sum(w) == 1, ret == self.ret])

        var_opt = prob.solve()

        self.ret = float(self.ret)
        self.sd = np.sqrt(var_opt)
        self.weights = np.array(w.value)


    def get_sharpe(self, risk_free_rate: float) -> float:
        self.sharpe = (self.ret - risk_free_rate) / self.sd
        return self.sharpe
```

[ ]:
```
class P_Efficient_Frontier:
    def __init__(
        self,
        ret_range: np.ndarray,
        portfolios: list[Portfolio] = []
) -> None:
        self.ret_range = list(ret_range)
        self.portfolios = portfolios


    def generate_ef(self, mu: pd.Series, S: pd.DataFrame, long_only: bool =␣
 ↪False):
        self.portfolios = []
        for r in self.ret_range:
            p = Portfolio(mu, S, long_only=long_only)
            p.optimize_weights(target_ret=r)
```

```
            self.portfolios.append(p)


    def get_tan_p(self, risk_free_rate: float) -> Portfolio:
        list_sharpe = []
        for i in range(len(self.ret_range)):
            list_sharpe.append(self.portfolios[i].get_sharpe(risk_free_rate))

        idx_max_sharpe = np.argmax(np.array(list_sharpe))
        return self.portfolios[idx_max_sharpe]


    def get_sd_ret_arr(self):
        sd_arr = []
        ret_arr = []
        for p in self.portfolios:
            sd_arr.append(p.sd)
            ret_arr.append(p.ret)

        return (sd_arr, ret_arr)
```

[ ]:
```
rf_mu = data['FamaFrenchFactors']['RF'].mean()
rf_mu
```

[ ]:
```
0.36216366158113733
```

[ ]:
```
p_inv_mu = data['Investment'].mean()
p_inv_ex = data['Investment'] - p_inv_mu
p_inv_S = p_inv_ex.cov()
```

[ ]:
```
ef_inv = P_Efficient_Frontier(ret_range=np.linspace(0, 2, 200))
ef_inv.generate_ef(p_inv_mu, p_inv_S)
tanp_inv = ef_inv.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_inv.ret, tanp_inv.sd, tanp_inv.sharpe
```

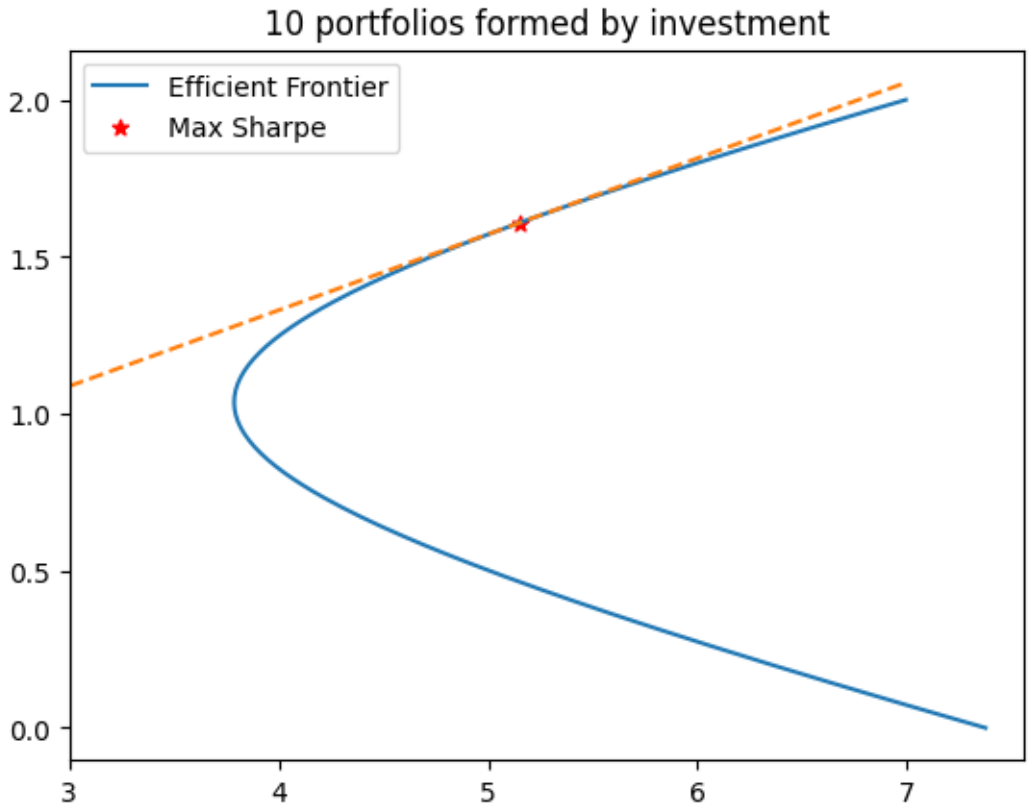
[ ]:
```
(1.6080402010050252, 5.147896588073307, 0.2420166213731538)
```

[ ]:
```
tanp_inv.weights
```

[ ]:
```
array([ 0.40781556,  0.79465305,  0.52837744, -0.20908625, -0.11511586,
       -0.31282123,  0.3852204 ,  0.31157557,  0.86795987, -1.65857855])
```

[ ]:
```
_, ax = plt.subplots()

sd_ef_inv, ret_ef_inv = ef_inv.get_sd_ret_arr()

ax.plot(sd_ef_inv, ret_ef_inv, label='Efficient Frontier')
```

```
ax.scatter(tanp_inv.sd, tanp_inv.ret, marker="*", c="r", label="Max Sharpe")

ax.set_xlim(3, )
ax.plot([3, 7],
        [3 * tanp_inv.sharpe + rf_mu, 7 * tanp_inv.sharpe + rf_mu],
        '--')

ax.set_title("10 portfolios formed by investment")
ax.legend()
plt.show()
```




[ ]:
```
p_20 = pd.concat([data['Investment'], data['Profitability']], axis=1)
p_20.columns = ([f'inv_{i}' for i in data['Investment'].columns]
                + [f'prof_{i}' for i in data['Profitability'].columns])
```

[ ]:
```
p_20_mu = p_20.mean()
p_20_S = (p_20 - p_20_mu).cov()
```

[ ]:
```
ef_20 = P_Efficient_Frontier(ret_range=np.linspace(0, 4, 400))
ef_20.generate_ef(p_20_mu, p_20_S)
tanp_20 = ef_20.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_20.ret, tanp_20.sd, tanp_20.sharpe
```

[ ]:
```
(2.0350877192982457, 5.593824346035855, 0.2990662477456323)
```

[ ]:
```
tanp_20.weights
```

[ ]:
```
array([ 0.78988051,  0.99014918,  0.56167952,  0.58888532,  0.17093563,
       -0.25742778,  0.28170231,  0.31519619,  0.78938115, -0.95186396,
       -0.37081168, -0.60565736, -0.78919639, -0.54669002,  0.44796668,
       -0.75018478, -0.22358038,  0.06362291,  0.63441061, -0.13839766])
```

[ ]:
```
p_30 = pd.concat([data['Investment'], data['Profitability'], data['Momentum']],
                 axis=1)
p_30.columns = ([f'inv_{i}' for i in data['Investment'].columns]
                + [f'prof_{i}' for i in data['Profitability'].columns]
                + [f'mom_{i}' for i in data['Momentum'].columns])
```

[ ]:
```
p_30_mu = p_30.mean()
p_30_S = (p_30 - p_30_mu).cov()
```

[ ]:
```
ef_30 = P_Efficient_Frontier(ret_range=np.linspace(0, 4, 400))
ef_30.generate_ef(p_30_mu, p_30_S)
tanp_30 = ef_30.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_30.ret, tanp_30.sd, tanp_30.sharpe
```

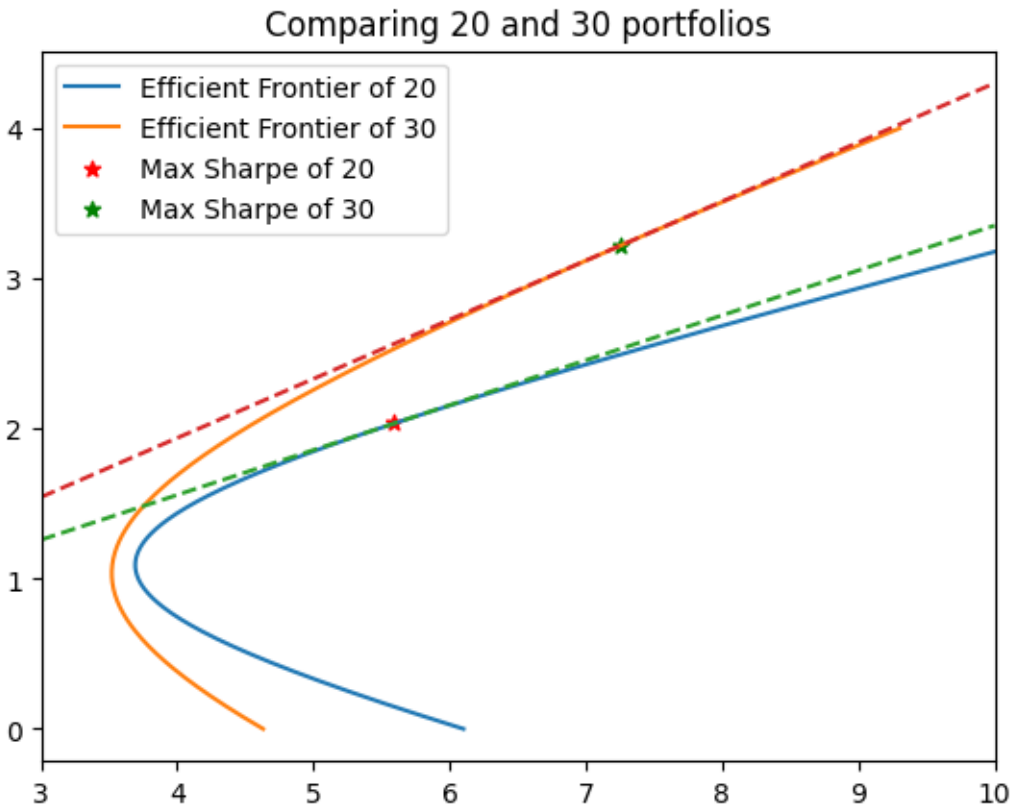
[ ]:
```
(3.2180451127819545, 7.250488713684098, 0.3938881314042753)
```

[ ]:
```
tanp_30.weights
```

[ ]:
```
array([ 0.47510953,  0.78206432,  0.63087101,  0.36542712, -0.0805739 ,
       -0.36666614, -0.02724849, -0.11491686,  0.29314728, -1.49698766,
       -0.5103277 , -0.55616111, -0.64796704, -0.55649414,  0.67394582,
       -0.79013437,  0.19087547,  0.39121704,  0.45853068,  0.09833374,
       -0.63605413,  0.43168549,  0.9173083 ,  0.78970808, -0.45781271,
       -0.04901819, -0.52625056, -0.03100014, -0.46699115,  1.81638042])
```

[ ]:
```
_, ax = plt.subplots()

sd_ef_20, ret_ef_20 = ef_20.get_sd_ret_arr()
ax.plot(sd_ef_20, ret_ef_20, label='Efficient Frontier of 20')

sd_ef_30, ret_ef_30 = ef_30.get_sd_ret_arr()
ax.plot(sd_ef_30, ret_ef_30, label='Efficient Frontier of 30')
```

```
ax.scatter(tanp_20.sd, tanp_20.ret, marker="*", c="r", label="Max Sharpe of 20")
ax.scatter(tanp_30.sd, tanp_30.ret, marker="*", c="g", label="Max Sharpe of 30")

ax.set_xlim(3, 10)
ax.plot([3, 10],
        [3 * tanp_20.sharpe + rf_mu, 10 * tanp_20.sharpe + rf_mu],
        '--')
ax.plot([3, 10],
        [3 * tanp_30.sharpe + rf_mu, 10 * tanp_30.sharpe + rf_mu],
        '--')

ax.set_title("Comparing 20 and 30 portfolios")

ax.legend()
plt.show()
```




[ ]:
```
df_tanp_30 = pd.DataFrame({
    'mean_return': p_30_mu,
    'sd_return': np.sqrt(np.diag(p_30_S))
```

```
})

df_tanp_30['sharpe'] = (df_tanp_30['mean_return'] - rf_mu) /␣
 ↪df_tanp_30['sd_return']
df_tanp_30['weights'] = tanp_30.weights

df_tanp_30 = df_tanp_30.sort_values(by='weights', ascending=False)
df_tanp_30
```

[ ]:

| | mean_return | sd_return | sharpe | weights |
|---|---|---|---|---|
| mom_10 | 1.475284 | 6.118831 | 0.181917 | 1.816380 |
| mom_3 | 0.913426 | 5.449894 | 0.101151 | 0.917308 |
| mom_4 | 0.942635 | 4.899604 | 0.118473 | 0.789708 |
| inv_2 | 1.151318 | 4.750456 | 0.166122 | 0.782064 |
| prof_5 | 0.997906 | 4.675570 | 0.135971 | 0.673946 |
| inv_3 | 1.067323 | 4.356696 | 0.161857 | 0.630871 |
| inv_1 | 1.163939 | 5.358714 | 0.149621 | 0.475110 |
| prof_9 | 1.075284 | 4.547001 | 0.156833 | 0.458531 |
| mom_2 | 0.774466 | 6.488377 | 0.063545 | 0.431685 |
| prof_8 | 1.030042 | 4.526771 | 0.147540 | 0.391217 |
| inv_4 | 0.973245 | 4.230571 | 0.144444 | 0.365427 |
| inv_9 | 0.997379 | 5.392302 | 0.117800 | 0.293147 |
| prof_7 | 0.898044 | 4.600458 | 0.116484 | 0.190875 |
| prof_10 | 1.038655 | 4.644635 | 0.145650 | 0.098334 |
| inv_7 | 0.972663 | 4.400360 | 0.138738 | -0.027248 |
| mom_8 | 1.049528 | 4.456146 | 0.154251 | -0.031000 |
| mom_6 | 0.937004 | 4.558031 | 0.126116 | -0.049018 |
| inv_5 | 0.962386 | 4.198568 | 0.142959 | -0.080574 |
| inv_8 | 0.979251 | 4.771906 | 0.129317 | -0.114917 |
| inv_6 | 0.954660 | 4.390712 | 0.134943 | -0.366666 |
| mom_5 | 0.860333 | 4.498224 | 0.110748 | -0.457813 |
| mom_9 | 1.106463 | 4.756176 | 0.156491 | -0.466991 |
| prof_1 | 0.748890 | 6.578407 | 0.058787 | -0.510328 |
| mom_7 | 0.946117 | 4.391049 | 0.132987 | -0.526251 |
| prof_2 | 0.801942 | 5.189462 | 0.084744 | -0.556161 |
| prof_4 | 0.889695 | 4.616743 | 0.114265 | -0.556494 |
| mom_1 | 0.304411 | 8.392381 | -0.006882 | -0.636054 |
| prof_3 | 0.837323 | 4.939107 | 0.096204 | -0.647967 |
| prof_6 | 0.922080 | 4.554809 | 0.122929 | -0.790134 |
| inv_10 | 0.770569 | 6.083457 | 0.067134 | -1.496988 |

[ ]:
```
df_tanp_30.corr()
```

[ ]:

| | mean_return | sd_return | sharpe | weights |
|---|---|---|---|---|
| mean_return | 1.000000 | -0.529972 | 0.928828 | 0.631035 |
| sd_return | -0.529972 | 1.000000 | -0.756718 | -0.079825 |
| sharpe | 0.928828 | -0.756718 | 1.000000 | 0.521095 |

```
weights         0.631035  -0.079825  0.521095  1.000000
```

[ ]:
```
ef_30_long = P_Efficient_Frontier(ret_range=np.linspace(0, 2, 200))
ef_30_long.generate_ef(p_30_mu, p_30_S, long_only=True)
tanp_30_long = ef_30_long.get_tan_p(risk_free_rate=rf_mu)
```

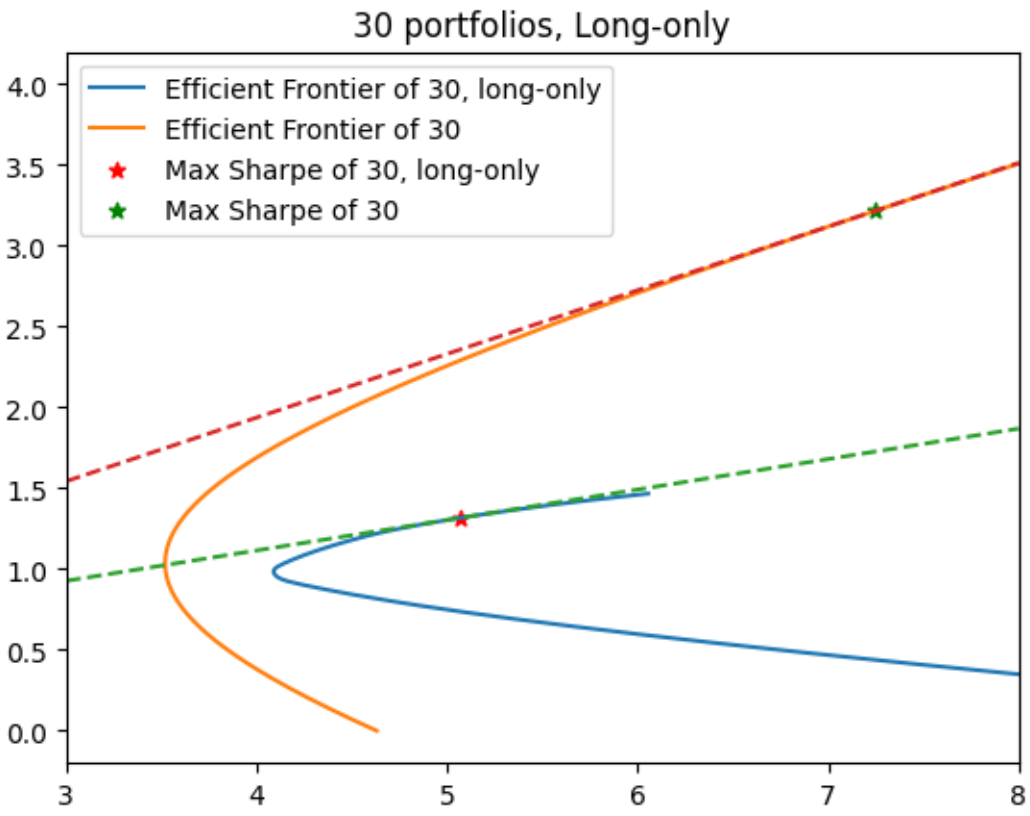
[ ]:
```
tanp_30_long.ret, tanp_30_long.sd, tanp_30_long.sharpe
```

[ ]:
```
(1.3165829145728642, 5.065741491751002, 0.18840662409360856)
```

[ ]:
```
tanp_30_long.weights.round(decimals=8)
```

[ ]:
```
array([-0.        ,  0.25682331,  0.18506469, -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        ,  0.55811199])
```

[ ]:
```
_, ax = plt.subplots()

sd_ef_30_long, ret_ef_30_long = ef_30_long.get_sd_ret_arr()
ax.plot(sd_ef_30_long, ret_ef_30_long,
        label='Efficient Frontier of 30, long-only')

sd_ef_30, ret_ef_30 = ef_30.get_sd_ret_arr()
ax.plot(sd_ef_30, ret_ef_30,
        label='Efficient Frontier of 30')

ax.scatter(tanp_30_long.sd, tanp_30_long.ret, marker="*", c="r",
           label="Max Sharpe of 30, long-only")
ax.scatter(tanp_30.sd, tanp_30.ret, marker="*", c="g",
           label="Max Sharpe of 30")

ax.set_xlim(3, 8)
ax.plot([3, 8],
        [3 * tanp_30_long.sharpe + rf_mu, 8 * tanp_30_long.sharpe + rf_mu],
        '--')
ax.plot([3, 8],
        [3 * tanp_30.sharpe + rf_mu, 8 * tanp_30.sharpe + rf_mu],
        '--')

ax.set_title("30 portfolios, Long-only")

ax.legend()
plt.show()
```


30 portfolios, Long-only
Efficient Frontier of 30, long-only
Efficient Frontier of 30
Max Sharpe of 30, long-only
Max Sharpe of 30
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
3
4
5
6
7
8