part-2-updated

October 14, 2023

[ ]:
```python
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
import cvxpy as cp
from sklearn.linear_model import LinearRegression
```

[ ]:
```python
raw_data = pd.read_excel('Data.xlsx', sheet_name=None, index_col=0)
```

[ ]:
```python
data = {}
sheet_names = list(raw_data.keys())

for s in sheet_names:
    if s == 'FamaFrenchFactors':
        new_name = s
        col_labels = raw_data[s].columns
    else:
        new_name = s[18:]
        col_labels = range(1, 11)

    data[new_name] = raw_data.pop(s)
    data[new_name].columns = col_labels
    data[new_name].index = (pd.to_datetime(data[new_name].index, format='%Y%m')
                            .to_period('M').rename('Date'))
```

0.1 Question 3

[ ]:
```python
class Portfolio:
    def __init__(
        self, mu: pd.Series, S: pd.DataFrame,
        long_only: bool = False,
        ret: float = None, sd: float = None, sharpe: float = None,
        weights: np.ndarray = None
    ) -> None:
        self.mu = mu
        self.S = S
        self.long_only = long_only
        self.ret = ret
        self.sd = sd
```

```
        self.sharpe = sharpe
        self.weights = weights

    def optimize_weights(self, target_ret: float = None) -> None:
        w = cp.Variable(self.mu.size)
        ret = self.mu.values @ w
        var = cp.quad_form(w, self.S.values)

        if target_ret is not None:
            self.ret = target_ret
        elif self.ret is None:
            print("Error: target return not specified")
            return

        if self.long_only:
            prob = cp.Problem(cp.Minimize(var),
                              [cp.sum(w) == 1, ret == self.ret, w >= 0])
        else:
            prob = cp.Problem(cp.Minimize(var),
                              [cp.sum(w) == 1, ret == self.ret])

        var_opt = prob.solve()

        self.ret = float(self.ret)
        self.sd = np.sqrt(var_opt)
        self.weights = np.array(w.value)


    def get_sharpe(self, risk_free_rate: float) -> float:
        self.sharpe = (self.ret - risk_free_rate) / self.sd
        return self.sharpe
```

[ ]:

```
class P_Efficient_Frontier:
    def __init__(
        self,
        ret_range: np.ndarray,
        portfolios: list[Portfolio] = []
) -> None:
        self.ret_range = list(ret_range)
        self.portfolios = portfolios


    def generate_ef(self, mu: pd.Series, S: pd.DataFrame, long_only: bool =
 ↪False):
        self.portfolios = []
        for r in self.ret_range:
            p = Portfolio(mu, S, long_only=long_only)
```

```
            p.optimize_weights(target_ret=r)
            self.portfolios.append(p)


    def get_tan_p(self, risk_free_rate: float) -> Portfolio:
        list_sharpe = []
        for i in range(len(self.ret_range)):
            list_sharpe.append(self.portfolios[i].get_sharpe(risk_free_rate))

        idx_max_sharpe = np.argmax(np.array(list_sharpe))
        return self.portfolios[idx_max_sharpe]


    def get_sd_ret_arr(self):
        sd_arr = []
        ret_arr = []
        for p in self.portfolios:
            sd_arr.append(p.sd)
            ret_arr.append(p.ret)

        return (sd_arr, ret_arr)
```

[ ]:
```
rf_mu = data['FamaFrenchFactors']['RF'].mean()
rf_mu
```

[ ]: 0.36216366158113733

[ ]:
```
p_inv_mu = data['Investment'].mean()
p_inv_ex = data['Investment'] - p_inv_mu
p_inv_S = p_inv_ex.cov()
```

[ ]:
```
ef_inv = P_Efficient_Frontier(ret_range=np.linspace(0, 2, 200))
ef_inv.generate_ef(p_inv_mu, p_inv_S)
tanp_inv = ef_inv.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_inv.ret, tanp_inv.sd, tanp_inv.sharpe
```

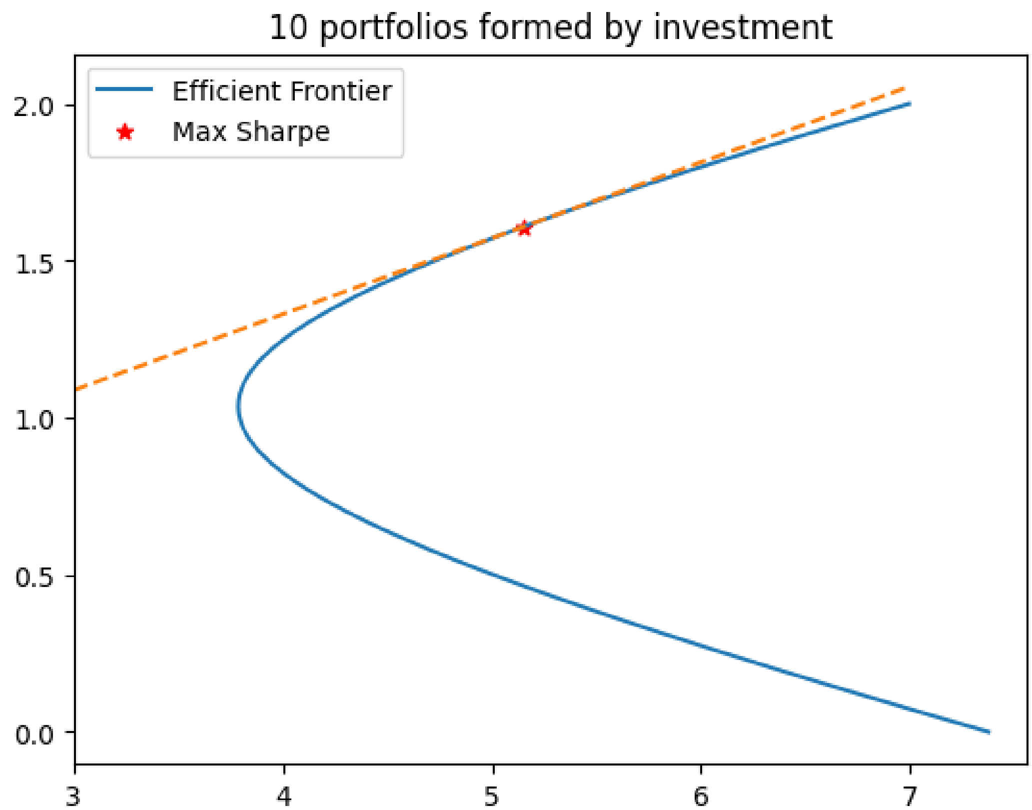
[ ]: (1.6080402010050252, 5.147896588073307, 0.2420166213731538)

[ ]:
```
tanp_inv.weights
```

[ ]:
```
array([ 0.40781556,  0.79465305,  0.52837744, -0.20908625, -0.11511586,
       -0.31282123,  0.3852204 ,  0.31157557,  0.86795987, -1.65857855])
```

[ ]:
```
_, ax = plt.subplots()

sd_ef_inv, ret_ef_inv = ef_inv.get_sd_ret_arr()
```

```
ax.plot(sd_ef_inv, ret_ef_inv, label='Efficient Frontier')

ax.scatter(tanp_inv.sd, tanp_inv.ret, marker="*", c="r", label="Max Sharpe")

ax.set_xlim(3, )
ax.plot([3, 7],
        [3 * tanp_inv.sharpe + rf_mu, 7 * tanp_inv.sharpe + rf_mu],
        '--')

ax.set_title("10 portfolios formed by investment")
ax.legend()
plt.show()
```



[ ]:
```
p_20 = pd.concat([data['Investment'], data['Profitability']], axis=1)
p_20.columns = ([f'inv_{i}' for i in data['Investment'].columns]
                + [f'prof_{i}' for i in data['Profitability'].columns])
```

[ ]:
```
p_20_mu = p_20.mean()
p_20_S = (p_20 - p_20_mu).cov()
```

[ ]:
```
ef_20 = P_Efficient_Frontier(ret_range=np.linspace(0, 4, 400))
ef_20.generate_ef(p_20_mu, p_20_S)
tanp_20 = ef_20.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_20.ret, tanp_20.sd, tanp_20.sharpe
```

[ ]:
```
(2.0350877192982457, 5.593824346035855, 0.2990662477456323)
```

[ ]:
```
tanp_20.weights
```

[ ]:
```
array([ 0.78988051,  0.99014918,  0.56167952,  0.58888532,  0.17093563,
       -0.25742778,  0.28170231,  0.31519619,  0.78938115, -0.95186396,
       -0.37081168, -0.60565736, -0.78919639, -0.54669002,  0.44796668,
       -0.75018478, -0.22358038,  0.06362291,  0.63441061, -0.13839766])
```

[ ]:
```
p_30 = pd.concat([data['Investment'], data['Profitability'], data['Momentum']],
                 axis=1)
p_30.columns = ([f'inv_{i}' for i in data['Investment'].columns]
                + [f'prof_{i}' for i in data['Profitability'].columns]
                + [f'mom_{i}' for i in data['Momentum'].columns])
```

[ ]:
```
p_30_mu = p_30.mean()
p_30_S = (p_30 - p_30_mu).cov()
```

[ ]:
```
ef_30 = P_Efficient_Frontier(ret_range=np.linspace(0, 4, 400))
ef_30.generate_ef(p_30_mu, p_30_S)
tanp_30 = ef_30.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_30.ret, tanp_30.sd, tanp_30.sharpe
```

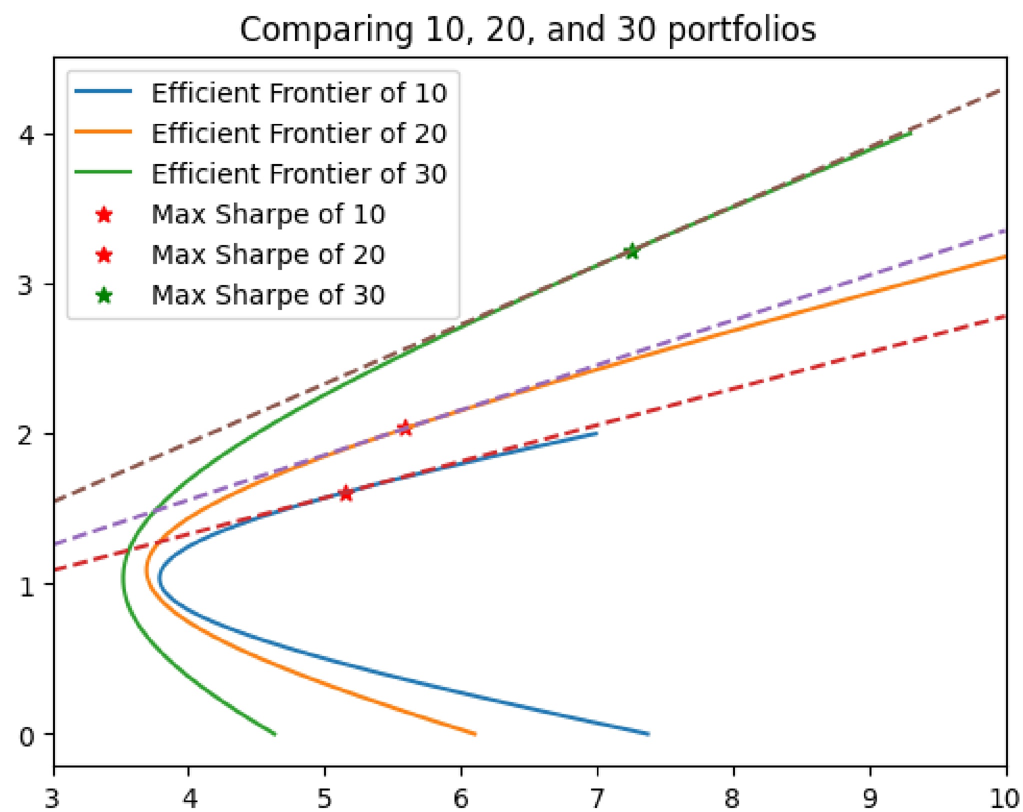
[ ]:
```
(3.2180451127819545, 7.250488713684098, 0.3938881314042753)
```

[ ]:
```
tanp_30.weights
```

[ ]:
```
array([ 0.47510953,  0.78206432,  0.63087101,  0.36542712, -0.0805739 ,
       -0.36666614, -0.02724849, -0.11491686,  0.29314728, -1.49698766,
       -0.5103277 , -0.55616111, -0.64796704, -0.55649414,  0.67394582,
       -0.79013437,  0.19087547,  0.39121704,  0.45853068,  0.09833374,
       -0.63605413,  0.43168549,  0.9173083 ,  0.78970808, -0.45781271,
       -0.04901819, -0.52625056, -0.03100014, -0.46699115,  1.81638042])
```

[ ]:
```
_, ax = plt.subplots()

ax.plot(sd_ef_inv, ret_ef_inv, label='Efficient Frontier of 10')

sd_ef_20, ret_ef_20 = ef_20.get_sd_ret_arr()
ax.plot(sd_ef_20, ret_ef_20, label='Efficient Frontier of 20')
```

```
sd_ef_30, ret_ef_30 = ef_30.get_sd_ret_arr()
ax.plot(sd_ef_30, ret_ef_30, label='Efficient Frontier of 30')

ax.scatter(tanp_inv.sd, tanp_inv.ret, marker="*", c="r", label="Max Sharpe of␣
↪10")
ax.scatter(tanp_20.sd, tanp_20.ret, marker="*", c="r", label="Max Sharpe of 20")
ax.scatter(tanp_30.sd, tanp_30.ret, marker="*", c="g", label="Max Sharpe of 30")

ax.set_xlim(3, )
ax.plot([3, 10],
        [3 * tanp_inv.sharpe + rf_mu, 10 * tanp_inv.sharpe + rf_mu],
        '--')
ax.set_xlim(3, 10)
ax.plot([3, 10],
        [3 * tanp_20.sharpe + rf_mu, 10 * tanp_20.sharpe + rf_mu],
        '--')
ax.plot([3, 10],
        [3 * tanp_30.sharpe + rf_mu, 10 * tanp_30.sharpe + rf_mu],
        '--')

ax.set_title("Comparing 10, 20, and 30 portfolios")

ax.legend()
plt.show()
```



[ ]:
```
df_tanp_30 = pd.DataFrame({
    'mean_return': p_30_mu,
    'sd_return': np.sqrt(np.diag(p_30_S))
})

df_tanp_30['sharpe'] = (df_tanp_30['mean_return'] - rf_mu) /␣
 ↪df_tanp_30['sd_return']
df_tanp_30['weights'] = tanp_30.weights

df_tanp_30 = df_tanp_30.sort_values(by='weights', ascending=False)
df_tanp_30
```

[ ]:

	mean_return	sd_return	sharpe	weights
mom_10	1.475284	6.118831	0.181917	1.816380
mom_3	0.913426	5.449894	0.101151	0.917308
mom_4	0.942635	4.899604	0.118473	0.789708
inv_2	1.151318	4.750456	0.166122	0.782064
prof_5	0.997906	4.675570	0.135971	0.673946
inv_3	1.067323	4.356696	0.161857	0.630871
inv_1	1.163939	5.358714	0.149621	0.475110
prof_9	1.075284	4.547001	0.156833	0.458531

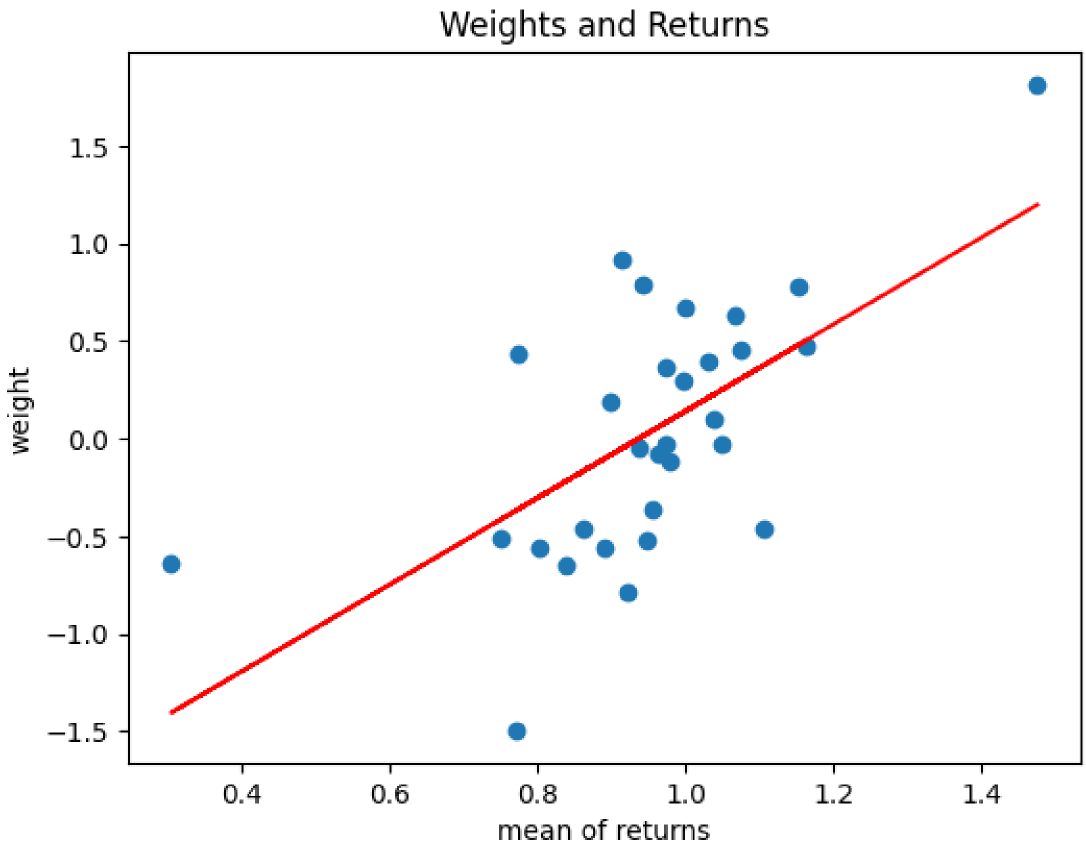
```
mom_2       0.774466   6.488377  0.063545  0.431685
prof_8      1.030042   4.526771  0.147540  0.391217
inv_4       0.973245   4.230571  0.144444  0.365427
inv_9       0.997379   5.392302  0.117800  0.293147
prof_7      0.898044   4.600458  0.116484  0.190875
prof_10     1.038655   4.644635  0.145650  0.098334
inv_7       0.972663   4.400360  0.138738 -0.027248
mom_8       1.049528   4.456146  0.154251 -0.031000
mom_6       0.937004   4.558031  0.126116 -0.049018
inv_5       0.962386   4.198568  0.142959 -0.080574
inv_8       0.979251   4.771906  0.129317 -0.114917
inv_6       0.954660   4.390712  0.134943 -0.366666
mom_5       0.860333   4.498224  0.110748 -0.457813
mom_9       1.106463   4.756176  0.156491 -0.466991
prof_1      0.748890   6.578407  0.058787 -0.510328
mom_7       0.946117   4.391049  0.132987 -0.526251
prof_2      0.801942   5.189462  0.084744 -0.556161
prof_4      0.889695   4.616743  0.114265 -0.556494
mom_1       0.304411   8.392381 -0.006882 -0.636054
prof_3      0.837323   4.939107  0.096204 -0.647967
prof_6      0.922080   4.554809  0.122929 -0.790134
inv_10      0.770569   6.083457  0.067134 -1.496988
```

[ ]:
```python
df_tanp_30.corr()
```

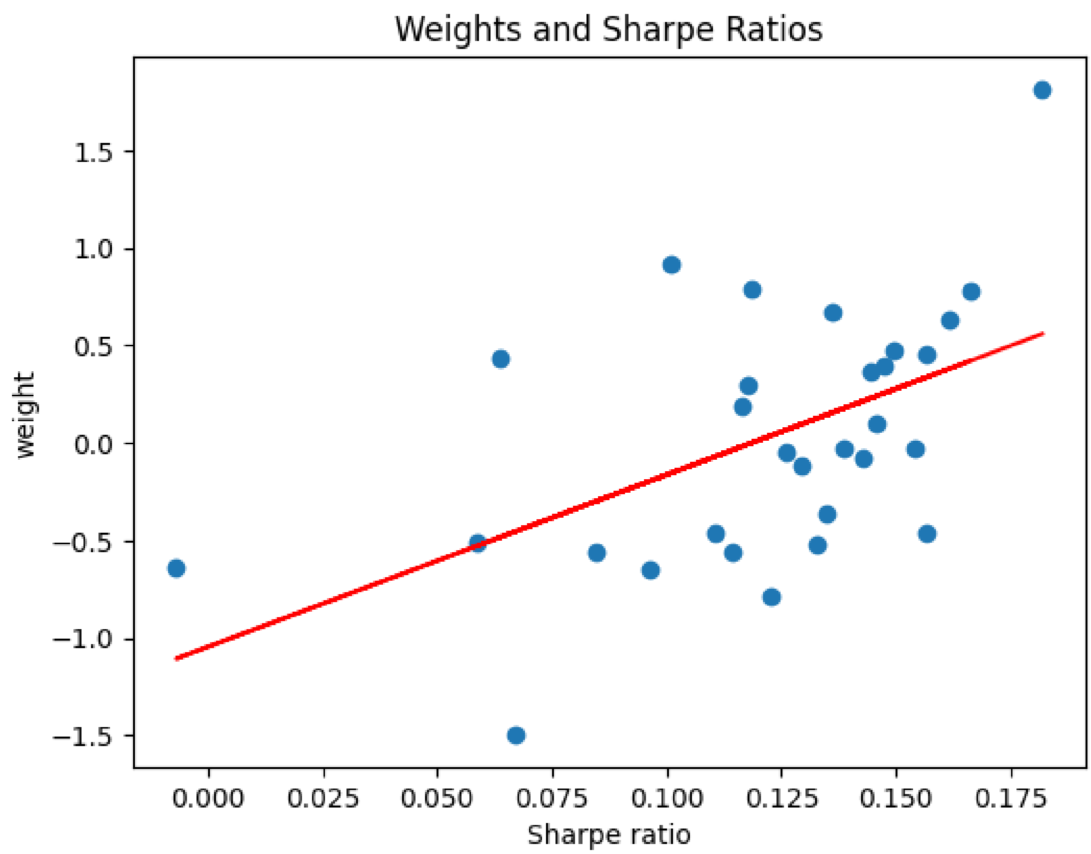
[ ]:

	mean_return	sd_return	sharpe	weights
mean_return	1.000000	-0.529972	0.928828	0.631035
sd_return	-0.529972	1.000000	-0.756718	-0.079825
sharpe	0.928828	-0.756718	1.000000	0.521095
weights	0.631035	-0.079825	0.521095	1.000000

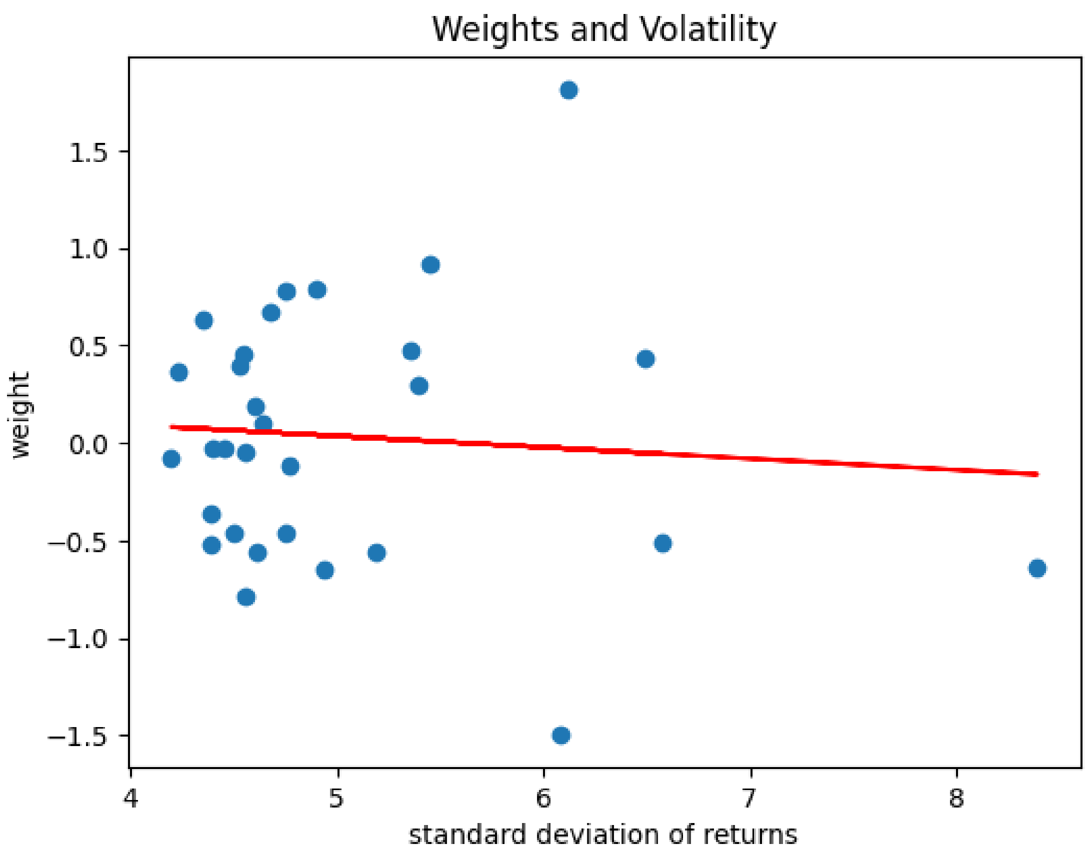
[ ]:
```python
lm_ret = LinearRegression().fit(df_tanp_30['mean_return'].values.reshape(-1, 1),
                                df_tanp_30['weights'].values)
w_pred_ret = lm_ret.predict(df_tanp_30['mean_return'].values.reshape(-1, 1))

plt.scatter(df_tanp_30['mean_return'], df_tanp_30['weights'])
plt.plot(df_tanp_30['mean_return'], w_pred_ret, c="r")
plt.title("Weights and Returns")
plt.xlabel("mean of returns")
plt.ylabel("weight")
plt.show()
```



[ ]:
```
lm_sharpe = LinearRegression().fit(df_tanp_30['sharpe'].values.reshape(-1, 1),
                                   df_tanp_30['weights'].values)
w_pred_sharpe = lm_sharpe.predict(df_tanp_30['sharpe'].values.reshape(-1, 1))

plt.scatter(df_tanp_30['sharpe'], df_tanp_30['weights'])
plt.plot(df_tanp_30['sharpe'], w_pred_sharpe, c="r")
plt.title("Weights and Sharpe Ratios")
plt.xlabel("Sharpe ratio")
plt.ylabel("weight")
plt.show()
```



[ ]:
```
lm_sd = LinearRegression().fit(df_tanp_30['sd_return'].values.reshape(-1, 1),
                                df_tanp_30['weights'].values)
w_pred_sd = lm_sd.predict(df_tanp_30['sd_return'].values.reshape(-1, 1))

plt.scatter(df_tanp_30['sd_return'], df_tanp_30['weights'])
plt.plot(df_tanp_30['sd_return'], w_pred_sd, c="r")
plt.title("Weights and Volatility")
plt.xlabel("standard deviation of returns")
plt.ylabel("weight")
plt.show()
```



[ ]:
```
ef_30_long = P_Efficient_Frontier(ret_range=np.linspace(0, 2, 200))
ef_30_long.generate_ef(p_30_mu, p_30_S, long_only=True)
tanp_30_long = ef_30_long.get_tan_p(risk_free_rate=rf_mu)
```

[ ]:
```
tanp_30_long.ret, tanp_30_long.sd, tanp_30_long.sharpe
```

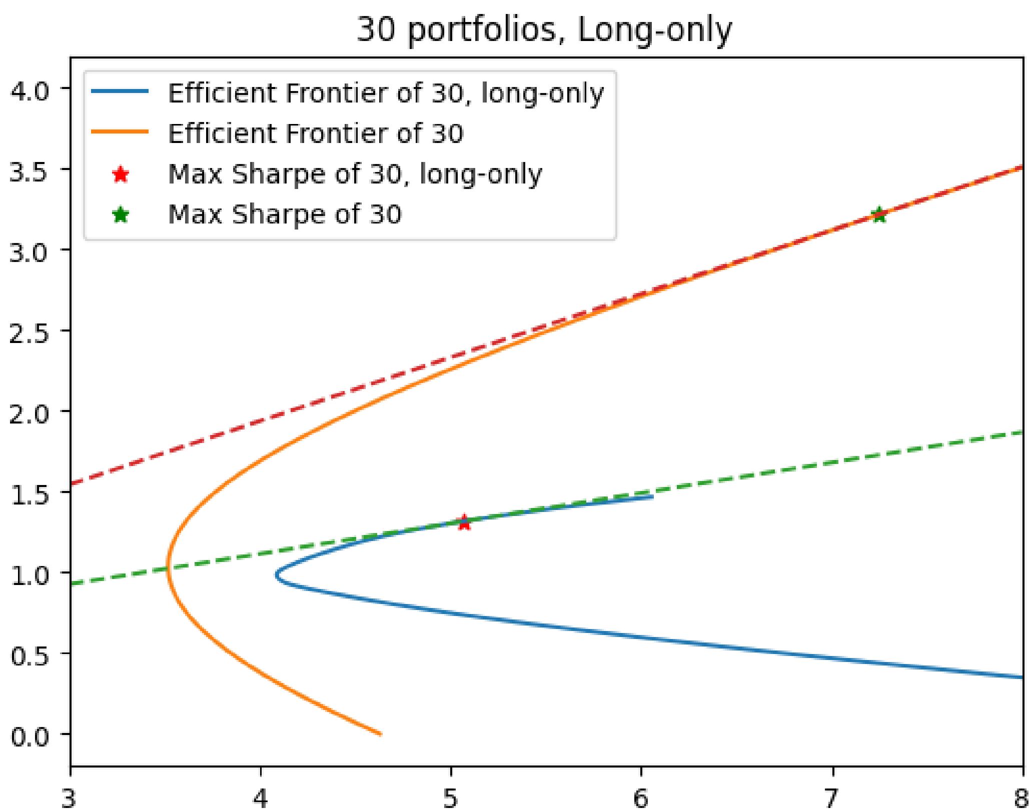
[ ]:
```
(1.3165829145728642, 5.065741491751002, 0.18840662409360856)
```

[ ]:
```
tanp_30_long.weights.round(decimals=8)
```

[ ]:
```
array([-0.        ,  0.25682331,  0.18506469, -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        , -0.        ,
       -0.        , -0.        , -0.        , -0.        ,  0.55811199])
```

[ ]:
```
_, ax = plt.subplots()

sd_ef_30_long, ret_ef_30_long = ef_30_long.get_sd_ret_arr()
```

```
ax.plot(sd_ef_30_long, ret_ef_30_long,
        label='Efficient Frontier of 30, long-only')

sd_ef_30, ret_ef_30 = ef_30.get_sd_ret_arr()
ax.plot(sd_ef_30, ret_ef_30,
        label='Efficient Frontier of 30')

ax.scatter(tanp_30_long.sd, tanp_30_long.ret, marker="*", c="r",
           label="Max Sharpe of 30, long-only")
ax.scatter(tanp_30.sd, tanp_30.ret, marker="*", c="g",
           label="Max Sharpe of 30")

ax.set_xlim(3, 8)
ax.plot([3, 8],
        [3 * tanp_30_long.sharpe + rf_mu, 8 * tanp_30_long.sharpe + rf_mu],
        '--')
ax.plot([3, 8],
        [3 * tanp_30.sharpe + rf_mu, 8 * tanp_30.sharpe + rf_mu],
        '--')

ax.set_title("30 portfolios, Long-only")

ax.legend()
plt.show()
```



[ ]:
```
df_tanp_30_long = pd.DataFrame({
    'mean_return': p_30_mu,
    'sd_return': np.sqrt(np.diag(p_30_S))
})

df_tanp_30_long['sharpe'] = (df_tanp_30_long['mean_return'] - rf_mu) /
 ↪df_tanp_30_long['sd_return']
df_tanp_30_long['weights'] = tanp_30_long.weights.round(decimals=8)

df_tanp_30_long = df_tanp_30_long.sort_values(by='sharpe', ascending=False)
df_tanp_30_long
```

[ ]:

	mean_return	sd_return	sharpe	weights
mom_10	1.475284	6.118831	0.181917	0.558112
inv_2	1.151318	4.750456	0.166122	0.256823
inv_3	1.067323	4.356696	0.161857	0.185065
prof_9	1.075284	4.547001	0.156833	-0.000000
mom_9	1.106463	4.756176	0.156491	-0.000000
mom_8	1.049528	4.456146	0.154251	-0.000000
inv_1	1.163939	5.358714	0.149621	-0.000000
prof_8	1.030042	4.526771	0.147540	-0.000000

```
prof_10     1.038655   4.644635  0.145650 -0.000000
inv_4       0.973245   4.230571  0.144444 -0.000000
inv_5       0.962386   4.198568  0.142959 -0.000000
inv_7       0.972663   4.400360  0.138738 -0.000000
prof_5      0.997906   4.675570  0.135971 -0.000000
inv_6       0.954660   4.390712  0.134943 -0.000000
mom_7       0.946117   4.391049  0.132987 -0.000000
inv_8       0.979251   4.771906  0.129317 -0.000000
mom_6       0.937004   4.558031  0.126116 -0.000000
prof_6      0.922080   4.554809  0.122929 -0.000000
mom_4       0.942635   4.899604  0.118473 -0.000000
inv_9       0.997379   5.392302  0.117800 -0.000000
prof_7      0.898044   4.600458  0.116484 -0.000000
prof_4      0.889695   4.616743  0.114265 -0.000000
mom_5       0.860333   4.498224  0.110748 -0.000000
mom_3       0.913426   5.449894  0.101151 -0.000000
prof_3      0.837323   4.939107  0.096204 -0.000000
prof_2      0.801942   5.189462  0.084744 -0.000000
inv_10      0.770569   6.083457  0.067134 -0.000000
mom_2       0.774466   6.488377  0.063545 -0.000000
prof_1      0.748890   6.578407  0.058787 -0.000000
mom_1       0.304411   8.392381 -0.006882 -0.000000
```

[ ]:
```
def get_beta(ret: pd.Series, mkt: pd.Series) -> float:
    return ret.cov(mkt) / mkt.var()

betas = np.empty(p_30.shape[1])
mkt = data['FamaFrenchFactors']['Mkt-RF'] + data['FamaFrenchFactors']['RF']

for i in range(p_30.shape[1]):
    betas[i] = get_beta(p_30.iloc[:, i], mkt)

betas
```

[ ]:
```
array([1.0730364 , 0.96392217, 0.88169068, 0.87582824, 0.87952881,
       0.92888586, 0.92715182, 1.01874518, 1.13051673, 1.27741701,
       1.29757414, 1.0748537 , 1.03310631, 0.96464766, 0.97529246,
       0.96364074, 0.98268772, 0.96360922, 0.96551035, 0.96853943,
       1.48538278, 1.22822955, 1.0460064 , 0.98329692, 0.92500163,
       0.94744666, 0.90416274, 0.9161091 , 0.96024434, 1.15474595])
```

[ ]:
```
p_30_ex = p_30 - rf_mu
treynors = p_30_ex.mean() / betas
```

[ ]:
```
df_treynor = pd.DataFrame({
    'beta': betas,
    'treynor': treynors,
```

```python
    'sharpe': df_tanp_30_long['sharpe'],
    'weights': df_tanp_30['weights'],
    'long_weights': df_tanp_30_long['weights']
})

df_treynor.sort_values(by='long_weights', ascending=False)
```

[ ]:

	beta	treynor	sharpe	weights	long_weights
mom_10	1.074854	0.963953	0.181917	1.816380	0.558112
inv_2	0.881691	0.818691	0.166122	0.782064	0.256823
inv_3	0.875828	0.799781	0.161857	0.630871	0.185065
inv_1	1.073036	0.747202	0.149621	0.475110	-0.000000
mom_7	0.963609	0.645849	0.132987	-0.526251	-0.000000
prof_8	0.960244	0.693100	0.147540	0.391217	-0.000000
prof_7	0.916109	0.545321	0.116484	0.190875	-0.000000
prof_6	0.904163	0.581043	0.122929	-0.790134	-0.000000
prof_5	0.947447	0.651848	0.135971	0.673946	-0.000000
prof_4	0.925002	0.546864	0.114265	-0.556494	-0.000000
prof_3	0.983297	0.459933	0.096204	-0.647967	-0.000000
prof_2	1.046006	0.409152	0.084744	-0.556161	-0.000000
prof_10	1.228230	0.698465	0.145650	0.098334	-0.000000
prof_1	1.485383	0.298038	0.058787	-0.510328	-0.000000
mom_9	0.968539	0.775115	0.156491	-0.466991	-0.000000
mom_8	0.965510	0.750309	0.154251	-0.031000	-0.000000
mom_5	0.963641	0.538560	0.110748	-0.457813	-0.000000
mom_6	0.982688	0.606726	0.126116	-0.049018	-0.000000
inv_10	0.963922	0.319712	0.067134	-1.496988	-0.000000
mom_4	0.975292	0.590332	0.118473	0.789708	-0.000000
mom_3	0.964648	0.527016	0.101151	0.917308	-0.000000
mom_2	1.033106	0.335688	0.063545	0.431685	-0.000000
mom_1	1.297574	-0.038881	-0.006882	-0.636054	-0.000000
inv_9	1.277417	0.561880	0.117800	0.293147	-0.000000
inv_8	1.130517	0.605733	0.129317	-0.114917	-0.000000
inv_7	1.018745	0.658467	0.138738	-0.027248	-0.000000
inv_6	0.927152	0.637857	0.134943	-0.366666	-0.000000
inv_5	0.928886	0.682436	0.142959	-0.080574	-0.000000
inv_4	0.879529	0.697719	0.144444	0.365427	-0.000000
prof_9	1.154746	0.738595	0.156833	0.458531	-0.000000