Quantum Algorithms for the Steiner Tree Problem

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ABSTRACT

- The **Steiner Tree problem** generalises the idea of a *minimum spanning tree*.
- Given an undirected and unweighted graph G = (V, E) and a subset S ⊆ V of vertices (called **terminal vertices**), what is the minimum number of edges required in a subtree of G that connects all of the vertices in S?
- The decision problem is NP-complete.

Our result: Parameterised in the number of vertices in S, we have a quantum algorithm that solves Steiner Tree with running time $O^*(1.728^k)$, where k = |S|.

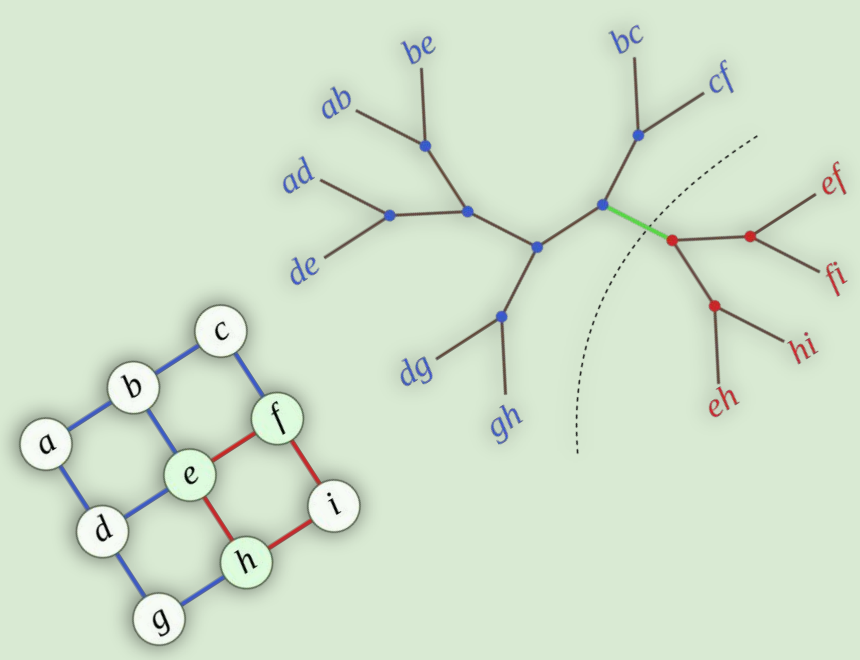
IMPORTANCE/ MOTIVATION

- The Steiner Tree problem pops up in telecommunications.
- Has connections to many other hard problems.
  - Set Cover
  - (Connected) Vertex Cover
  - 3SAT

METHODS

- Classically solve the problem on small instances, using the $O^*(2^k)$ Inclusion-Exclusion algorithm.
- For larger instances, we apply Grover's search and a tree decomposition.
  - Tree decomposition decomposes the tree into two roughly even sized* trees.
- Grover's search solves the problem of unstructured search quantumly with $O(\sqrt{n})$ many queries, with high probability.

* The subtrees are approximately even in the number of terminal vertices.



FINDINGS

- Previously, the best classical algorithm to solve the Steiner Tree problem has running time $O^*(2^k)$.
  - Conjectured that Set Cover cannot be solved any faster than $O^*(2^n)$ which strongly believes that the best classical algorithm for Steiner Tree is $O^*(2^k)$.
- Assuming we allow quantum computation, we improved the running time to $O^*(1.728^k)$ via an application of Grover's search.
  - Aligns with the best known quantum algorithm for Set Cover, via Ambainis et al. (2019).

FUTURE WORKS

- The **Set Cover Conjecture** asserts that there is no subexponential-time algorithm that solves Set Cover.
  - This implies that there is no subexponential-time algorithm that solves Steiner Tree, parameterised in the size of S.
  - The reduction from **Set Cover** to **Steiner Tree** implies that a quantum algorithm for Steiner Tree implies a quantum algorithm for Set Cover.
- We plan to extend this work further to discuss related problems that have reductions that scale linearly in the parameters.
  - This gives rise to quantum algorithms for other problems as well.