

ATIVIDADE 1 - MÉTODOS COMPUTACIONAIS PARA FÍSICA

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3) Eq. (14): $x_{n+1} = 2x_n - x_{n-1} + a_n \Delta t^2$

Eq. (15): $v_n = \frac{x_{n+1} - x_{n-1}}{2\Delta t}$

Eq. (16): $x_{n+1} = x_n + v_n \Delta t + \frac{1}{2} a_n \Delta t^2$

Eq. (17): $v_{n+1} = v_n + \frac{1}{2} (a_{n+1} + a_n) \Delta t$

i) Mostrando a equivalência de (16):

$$x_{n+1} = 2x_n - x_{n-1} + a_n \Delta t^2$$

Usando (15), substituímos x_{n-1}

$$x_{n-1} = 2v_n \Delta t - x_{n+1}$$

$$x_{n+1} = 2x_n + 2v_n \Delta t - x_{n+1} + a_n \Delta t^2$$

$$2x_{n+1} = 2x_n + 2v_n \Delta t + a_n \Delta t^2 \quad \quad \quad \times 2$$

$$x_{n+1} = x_n + v_n \Delta t + \frac{1}{2} a_n \Delta t^2$$

ii) Mostrando a equivalência de (17):

Partindo de (15)

$$v_n = \frac{x_{n+1} - x_{n-1}}{2\Delta t} ; \text{ substituindo (16), (14) em (15):}$$

$$v_n = (x_n + v_n \Delta t + \frac{1}{2} a_n \Delta t^2 + x_{n+1} - 2x_n - a_n \Delta t^2) / 2\Delta t$$

$$\text{Usando (15) para } x_{n+2}: \quad x_{n+2} = x_{n+1} + v_{n+1} \Delta t + \frac{1}{2} a_{n+1} \Delta t^2$$
$$x_{n+1} = x_{n+2} - v_{n+1} \Delta t - \frac{1}{2} a_{n+1} \Delta t^2$$

$$v_n = (\underbrace{-x_n}_{+} + v_n \Delta t - \frac{1}{2} a_n \Delta t^2 + \underbrace{x_{n+2}}_{+} - v_{n+1} \Delta t - \frac{1}{2} a_{n+1} \Delta t^2) / 2\Delta t$$

$$v_n = v_{n+1} + \frac{v_n \Delta t - \frac{1}{2} (a_n + a_{n+1}) \Delta t^2 - v_{n+1} \Delta t}{2\Delta t}$$

$$v_{n+1} - \frac{v_{n+1}}{2} = v_n - \frac{v_n}{2} + \frac{1}{4} (a_n + a_{n+1}) \Delta t$$

$$v_{n+1} = v_n + \frac{1}{2} (a_n + a_{n+1}) \Delta t$$