$$I = \int \frac{1}{2 + \cos(x)} \, dx$$

$$I = \frac{1}{2} \int \left(\frac{1}{2} + \frac{1 + \cos(x)}{2} \right)^{-1} dx$$

$$I = \frac{1}{2} \int \left(\frac{1}{2} + \cos^2\left(\frac{x}{2}\right) \right)^{-1} dx$$

$$I = \frac{1}{2} \int \left(\frac{1 + 2\cos^2\left(\frac{x}{2}\right)}{2} \right)^{-1} dx$$

$$I = \int \frac{1}{1 + 2\cos^2\left(\frac{x}{2}\right)} \, dx$$

$$I = \int \frac{1}{1 + \dfrac{2}{\sec^2\left(\frac{x}{2}\right)}} \, dx$$

$$I = \int \frac{\sec^2\left(\frac{x}{2}\right)}{\sec^2\left(\frac{x}{2}\right) + 2} \, dx$$

$$I = \int \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 3} \, dx$$

$$t = \tan\left(\frac{x}{2}\right), dt = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) dx$$

$$I = 2\int \frac{1}{t^2 + 3} \, dt$$

$$t = \sqrt{3}\, v$$

$$dt = \sqrt{3}\, dv$$

$$I = 2\sqrt{3} \cdot \tan^{-1}(v)$$

$$\boxed{I = 2\sqrt{3} \cdot \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}$$