

$x_0 \in \{0, 1, 2\}$,
 $\Phi_0(t) \simeq 0.8907$.

	0.09
19	0.5359
14	0.5753
33	0.6141
30	0.6517
14	0.6879
30	0.7224
17	0.7549
23	0.7852
36	0.8133
55	0.8389
29	0.8621
10	0.8830
77	0.9015
52	0.9177
36	0.9319
29	0.9441
15	0.9545
25	0.9633
29	0.9706
51	0.9767
22	0.9817
54	0.9857
77	0.9890
33	0.9916
34	0.9936
51	0.9952
53	0.9964
73	0.9974
30	0.9981
36	0.9986

he fact that the
rval $[-3, 3]$ as
we observe that

12.2 Black–Scholes Formula(s) (To Compute Reference Prices)

In a risk-neutral Black–Scholes model, the quoted price of a risky asset is a solution to the SDE $dX_t = X_t(rdt + \sigma dW_t)$, $X_0 = x_0 > 0$, where r is the interest rate and $\sigma > 0$ is the volatility and W is a standard Brownian motion. Itô's formula (see Sect. 12.8) yields that

$$X_t^{x_0} = x_0 e^{(r - \frac{\sigma^2}{2})t + \sigma W_t}, \quad W_t \stackrel{d}{=} \mathcal{N}(0; 1).$$

A vanilla (European) payoff of maturity $T > 0$ is of the form $h_T = \varphi(X_T)$. A European option contract written on the payoff h_T is the right to receive h_T at the maturity T . Its price – or premium – at time $t = 0$ is given by $e^{-rt}\mathbb{E}[\varphi(X_T^{x_0})]$ and, more generally at time $t \in [0, T]$, it is given by $e^{-r(T-t)}\mathbb{E}[\varphi(X_T^{x_0}) | X_t^{x_0}] = e^{-r(T-t)}\mathbb{E}[\varphi(X_{T-t}^{x_0})]$. In the case where $\varphi(x) = (x - K)_+$ (call with strike price K) this premium at time t has a closed form given by

$$\text{Call}_t(x_0, K, R, \sigma, T) = \text{Call}_0(x_0, K, R, \sigma, T - t),$$

where

$$\text{Call}_0(x_0, K, r, \sigma, \tau) = x_0 \Phi_0(d_1) - e^{-r\tau} K \Phi_0(d_2), \quad \tau > 0, \quad (12.1)$$

with

$$d_1 = \frac{\log(\frac{x_0}{K}) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau}. \quad (12.2)$$

As for the put option written on the payoff $h_T = (K - X_T^{x_0})_+$, the premium is

$$\text{Put}_t(x_0, K, r, \sigma, T) = \text{Put}_0(x_0, K, r, \sigma, T - t),$$

where

$$\text{Put}_0(x_0, K, r, \sigma, \tau) = e^{-r\tau} K \Phi_0(-d_2) - x_0 \Phi_0(-d_1). \quad (12.3)$$

The avatars of the regular Black–Scholes formulas can be obtained as follows:

- Stock without dividend (Black–Scholes): the risky asset is X .
- Stock with continuous yield $\lambda > 0$ of dividends: the risky asset is $e^{\lambda t} X$, and one has to replace x_0 by $e^{-\lambda t} x_0$ in the right-hand sides of (12.1), (12.2) and (12.3).

12.2 Black–Scholes

- Foreign exchange: $e^{rf t} X_t$ where i is $e^{-rf t} x_0$ in the right-hand side
- Future contracts (with maturity T): the right-hand side

12.3 Measures

Theorem 12.1 (F)

where $C(S, R)$ denotes the space of compact (i.e. is $C(S, R)$ by the space

Theorem 12.2 (F)
space. Let V be a vector space. Let V be a vector space. Let C be a subspace of V . Furthermore, let V

- (i) $1 \in V$,
- (ii) V is closed under addition,
- (iii) V is closed under scalar multiplication: $\varphi_n \leq \varphi_{n+1}$, $\varphi \in V$.

Then H contains the

We refer to [216] for

12.4 Uniform

In this section, we consider variables taking values in \mathbb{R} . These variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The random variable X_i is a function from Ω to \mathbb{R} . It is straightforward to adapt the definitions of the previous sections to this setting.