Université Nice-Sophia Antipolis
SMEMP302 - ECUE Probabilitic computational methods, 2022-2023
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Exercices (2)

We want to compute the price of the *Best of Call* option with strike K. This example is developped at the end of chapter 2 in the course. We recall the following formulas.

$$\begin{aligned}&e^{-rT}h_T \overset{\text{(law)}}{=} \phi(Z^1,Z^2)\\ &=: \left(\max\left(x_0^1\exp\left(-\frac{\sigma_1^2}{2}T+\sigma_1\sqrt{T}Z^1\right), x_0^2\exp\left(-\frac{\sigma_2^2}{2}T+\sigma_2\sqrt{T}(\rho Z^1+\sqrt{1-\rho^2}Z^2)\right)\right)-Ke^{-rT}\right)_+,\end{aligned}$$

where $Z=(Z^1,Z^2)\sim\mathcal{N}(0,I_2)$.

We take a M-sample $(Z_m)_{1\leq m\leq M}$and get

$$\begin{aligned}\text{(Best-of-Call)} &= \mathbb{E}(\phi(Z^1,Z^2))\\ &\simeq \overline{\phi}_M := \frac{1}{M}\sum_{m=1}^{M}\phi(Z_m)\,.\end{aligned}$$

We can estimate the variance:

$$\overline{V}_M(\phi)=\frac{1}{M-1}\sum_{m=1}^{M}\phi(Z_m)^2-\frac{M}{M-1}\overline{\phi}_M^2\simeq\mathrm{Var}(\phi(Z))\,.$$

We can compute a confifence interval (for a level of confidence $\alpha=0.95$)

$$I_M^\alpha=\left[\overline{\phi}_M-q_\alpha\sqrt{\frac{\overline{V}_M(\phi)}{M}},\overline{\phi}_M+q_\alpha\sqrt{\frac{\overline{V}_M(\phi)}{M}}\right]$$

(with $\alpha=0.95$, we have $q_\alpha=1.96$).

We take the following numerical values:

$$T=1\,,\ K=100\,,\ r=0.1\,,\ \sigma_i=0.2=20\%\,(i=1,2)\,,\ \rho=0.5\,,\ x_0^i=100\,(i=1,2).$$

We want to draw a graph with our estimates for $M=2^m$, $m=10,11,\dots$ with the confidence intervals (it should look like Figure 0.1).

PYTHON TIPS

Start by importing packages

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.stats as sps
```

Simulation of normal random variables (in an array with two columns and a lot of lines) + convert it into a Data Frame

```
x=sps.norm.rvs(size=(2**m,2))
df=pd.DataFrame(x)
```

Apply a function (phi) to each of the lines of the above Data Frame

```
res=df.apply(phi,axis=1)
```

Mean and standard deviation of an array (or list, . . .)

```
moy=np.mean(res)
stdev=np.std(res)
```

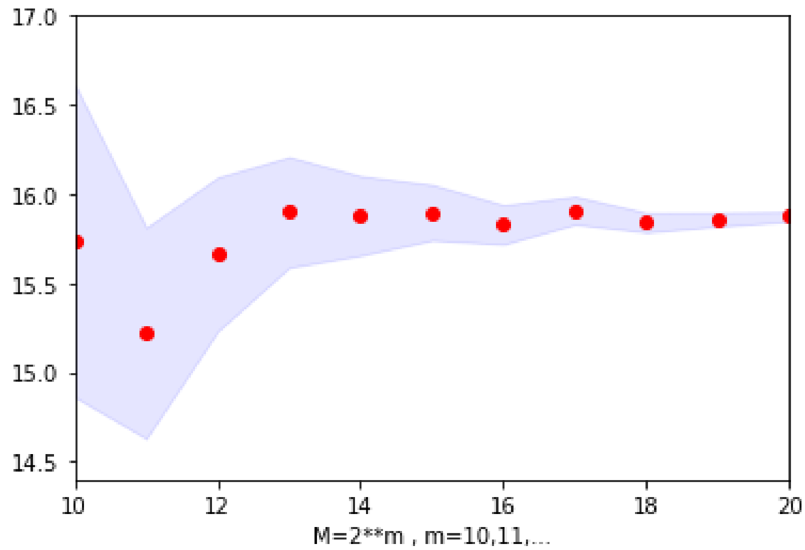

FIGURE 0.1. Monte-Carlo estimator and its confidence interval at level $\alpha = 95\%$ for sizes $M = 2^m$, $m = 10, 11, \ldots$.

Create an empty array and add numbers at the end of this array

```
gg=np.array([])
gg=np.append(gg,1)
gg=np.append(gg,3)
```

Loop on all integers between 10 and 20

```
For m in range(10,21):
...
```

Specify range in a plot

```
plt.xlim(1,2)
plt.ylim(2,10)
```

Plot a graph with red circles

```
plt.plot(x,y,'ro')
```

Plot nice shaded area between curves `y1` and `y2` (parenthesis are important)

```
plt.fill_between(x,(y1),(y2),color='b',alpha=0.1)
```

Show plot

```
plt.show()
```