Université Nice-Sophia Antipolis
SMEMP302 - ECUE Probabilitic computational methods, 2022-2023
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ANSWERS FOR EXERCISES (3)

EXERCISE 3

(1) ...

(2) Set $g(x) = \mathbb{1}_{x>0} e^{\beta x}$, $f(x) = e^{-x^2/2}/\sqrt{2\pi}$. We observe that

$$g(x)f(x) = \frac{\mathbb{1}_{x>0}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-\beta)^2 + \frac{\beta^2}{2}\right).$$

We take (same notation as in the course)

$$g(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-\beta)^2 + \frac{\beta^2}{2}\right).$$

We get

$$\frac{\widetilde{f}(x)}{\int_{\mathbb{R}} \widetilde{f}(y)dy} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-\beta)^2\right)$$

(this is the density $\mathcal{N}(\beta, 1)$, we can simulate this).

(3) We have

$$\begin{aligned} I &= \mathbb{E}(g(X)) \\ &= \mathbb{E}(g(X) - h(X)) + \mathbb{E}(h(X)) \end{aligned}$$

with $h(x) = e^{\beta x}$ and

$$\begin{aligned} \mathbb{E}(h(X)) &= \int_{\mathbb{R}} \frac{\exp\left(-\frac{x^2}{2} + \beta x\right)}{\sqrt{2\pi}} dx \\ &= \int_{\mathbb{R}} \frac{\exp\left(-\frac{1}{2}(x-\beta)^2 + \frac{\beta^2}{2}\right)}{\sqrt{2\pi}} dx \\ &= \exp\left(\frac{\beta^2}{2}\right). \end{aligned}$$

(4) If $X \sim \mathcal{N}(0,1)$, then $-X \sim \mathcal{N}(0,1)$. So

$$\mathbb{E}(g(X)) = \mathbb{E}(g(-X)) = \mathbb{E}\left(\frac{g(X) + g(-X)}{2}\right).$$

This gives us a new M-C method.