Simple versus average inflation targeting: a comparison

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1 Introduction

Following years of underachievements of a 2% yearly inflation target, the US FED and the ECB recently changed their inflation targets. The FED now seeks inflation that averages 2% over time, whereas the ECB aims for a symmetric 2% inflation target over the medium term[1]. What does this change imply for the evolution of the output gap and inflation? How desirable is the new average inflation target, compared to the old paradigm? In this short investigation, I set up a basic New Keynesian model with a cost-push shock that hits at $t = 0$ to answer these questions. In this environment, I find that (*i*) average targeting reduces the volatility of the welfare-relevant output gap on impact, but the response of inflation on impact may be either accentuated by the smaller response of the gap, or cushioned via inflation expectations; (*ii*) average targeting prevents interest rates from falling below zero more easily than simple targeting when the cost-push shock is small, while large shocks cause the opposite; and (*iii*) a policymaker with no commitment using a textbook calibration of the model prefers targeting two- or three-year average inflation than shorter horizons.

2 The model

I use the textbook New Keynesian model in which inflation is subject to an exogenous cosh-push shock. I incorporate the latter to avoid the divine coincidence by which the central bank faces no trade-off between inflation and output gap stabilization (Blanchard and Galí (2007)). I also assume that technology is constant. The model boils down to three equations – a dynamic IS curve, a New Keynesian Phillips curve (NKPC), and an interest rate rule. The first two are:

[1] The original speech by the Chair Powell can be found here. For the ECB, see here.

$$x_t = E_t\left(x_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t(\pi_{t+1}) - r_t^e\right) \tag{1}$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t, \tag{2}$$

where $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap, i_t is the nominal interest rate set by the central bank, π_t is the inflation rate, $r_t^e \equiv \rho + \sigma \Delta E_t(y_{t+1}^e)$ is the efficient real interest rate, and $u_t \equiv \kappa\left(y_t^e - y_t^n\right)$ is an exogenous cost-push shock assumed to follow an AR(1) like $u_t = \rho_u u_{t-1} + \epsilon_t$, where ϵ_t is white noise with zero mean and variance σ_u^2. σ, κ and β are parameters of the model, and the reader may find their exact meaning in the full derivation of the model located in the appendix.

An interest rate rule set by the central bank is needed to close the model, and two different alternatives are analyzed in what follows.

2.1 Equilibrium with simple inflation targeting

As a first possibility, I consider the case in which the central bank adjusts i_t in response to deviations from a yearly inflation target. Henceforth, I refer to this rule as *simple* inflation targeting (as opposed to *average* inflation targeting). A very simple way to model simple targeting is by establishing that the central bank follows a rule like:

$$i_t = r_t^e + \overline{\pi} + \phi_s\left(\pi_t - \overline{\pi}\right), \tag{3}$$

where $\overline{\pi}$ is an inflation target assumed to be close to 0, and $\phi_s > 0$. Intuitively, the steady state of the model becomes such that $\pi_t = \overline{\pi}$, and the central bank raises (lowers) interest rates whenever $\pi_t > \overline{\pi}$ ($\pi_t < \overline{\pi}$).

An equilibrium here is defined as the paths $\{x_t^s, \pi_t^s, i_t^s\}_{t=0}^{\infty}$ that satisfy equations 1, 2, and 3 $\forall t \in [0, \infty)$. In the case of simple inflation targeting, the model has an easy pencil-and-paper solution as long as $\kappa\left(\phi_s - 1\right) > 0$ (see the appendix for the derivation of this equilibrium), and the equilibrium values of x_t, π_t, and i_t look like:

$$x_t^s = \frac{\rho_u - \phi_s}{\sigma\left(1 - \rho_u \beta\right)\left(1 - \rho_u\right) - \left(\rho_u - \phi_s\right)\kappa} u_t + \frac{1 - \beta}{\kappa}\overline{\pi} \tag{4}$$

$$\pi_t^s = \frac{\sigma(1-\rho_u)}{\sigma(1-\rho_u\beta)(1-\rho_u)-(\rho_u-\phi_s)\kappa}u_t+\overline{\pi} \tag{5}$$

$$i_t^s = \frac{\phi_s\sigma(1-\rho_u)}{\sigma(1-\rho_u\beta)(1-\rho_u)-(\rho_u-\phi_s)\kappa}u_t+r_t^e+\overline{\pi}. \tag{6}$$

In words, a positive cost-push shock rises inflation (see equation 2), which immediately triggers a tightening by the central bank (see equation 3) that lowers x_t (see equation 1). The lower x_t triggers a round of smaller second-/third-/... order effects, with the final equilibrium values being shown in equations 4, 5, and 6. Also, note that additional effects will kick in via the forward-looking nature of the system as long as the cost push-shock is persistent.

2.2 Equilibrium with average inflation targeting

Another possibility is that the central bank adjusts i_t in response to *average* inflation. In this case, the homolog rule of equation 3 would be:

$$i_t = r_t^e+\overline{\pi}+\phi_a(\tilde{\pi}_t-\overline{\pi}), \tag{7}$$

where $\tilde{\pi}_t \equiv \frac{1}{N+1}\sum_{n=0}^{N}\pi_{t-n}$ is the average inflation in the past N periods + present period. The interpretation of the rule is the same as in 3, but now the central bank reacts to deviations of $\tilde{\pi}_t$ from the target.

An equilibrium here is defined as the paths $\{x_t^a,\pi_t^a,i_t^a\}_{t=0}^{\infty}$ that satisfy equations 1, 2, and 7 $\forall t \in [0,\infty)$, for given initial lags π_{-p}, $p=1,...N$. I assume an equilibrium exists[2], and I compare the equilibrium paths $\{x_t^s,\pi_t^s,i_t^s\}_{t=0}^{\infty}$ and $\{x_t^a,\pi_t^a,i_t^a\}_{t=0}^{\infty}$ in the section that comes below.

3 Simulations

I now simulate the equilibrium paths of $\{x_t,\pi_t,i_t\}_{t=0}^{\infty}$ for either interest rate rule, assuming that a system that comes from the steady state is shocked at $t=0$. The values I use for the default calibration can be found in the appendix. The simulation is done in MATLAB, and, in the case of average inflation targeting, it features the search of a fixed point π_t^a such that $\pi_t(\tilde{\pi}_t(\pi_t^a)) = \pi_t^a$, where $\tilde{\pi}_t(\pi_t)$ is the average inflation path produced by the vector π_t. The simulation for the simple targeting rule is straightforward

[2] I leave the proof of this for future work.

given the analytical expressions shown above.

Figure 1 shows the plot for a one-standard-deviation negative cost-push shock (i.e. a decline of 1% in u_0) when the shock is short-lived ($\rho_u = 0$, unlike in the default calibration), which helps gain some intuition. I plot $\phi_s = \phi_a = 1.5$ for the sake of comparison; and I choose $N = 3$, which means that the central bank considers the realizations of inflation for the four most recent quarters when constructing $\tilde{\pi}_t$.

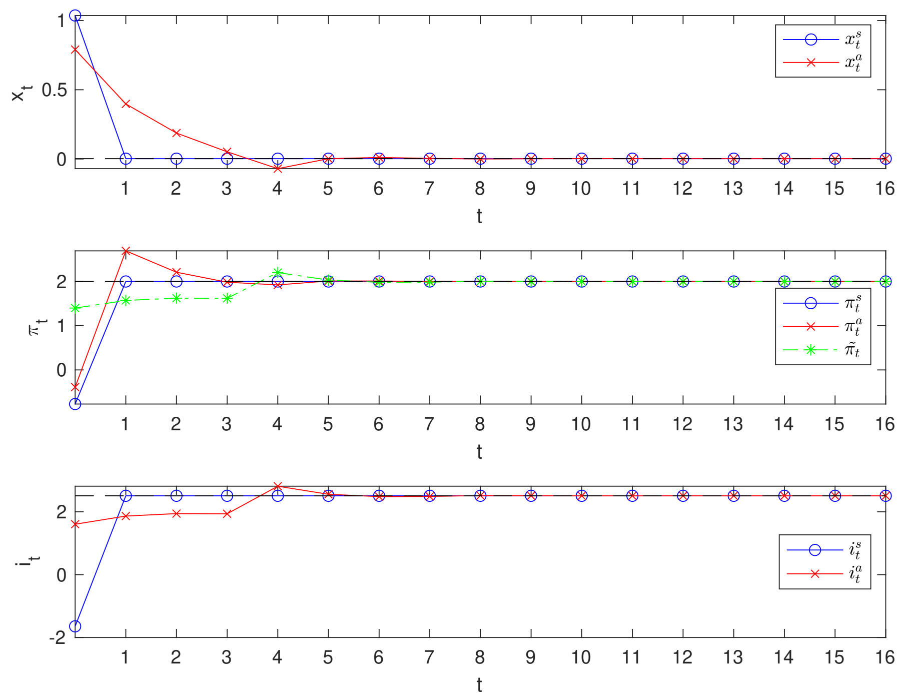

Figure 1: Equilibrium paths when $\rho_u = 0$ for simple targeting (blue) and average targeting (red). Average inflation in green. Inflation and interest rates are annualized, and expressed in %.

First, and qualitatively speaking, the negative cost-push shock has the same contemporaneous effects in both rules, causing a drop in the rate and inflation, and a rise in the welfare-relevant output gap. However, the quantitative shocks on impact are not the same. As proved in lemma 1 (see appendix), i_0^a always drops by less than i_0^s, which causes x_0^a to rise by less than x_0^s. Furthermore, the exogenous deflationary pressure is gone at $t = 1$, but $\tilde{\pi}_1$ will still be below target as long as $N > 0$. Consumers and firms must consequently anticipate a low i_1^a, which in a system with no exogenous deflationary pressures results into a welfare-relevant output gap above its steady state that puts an (endogenous) upward pressure to inflation at $t = 1$. The final effect on π_0^a is then ambiguous; on the one hand, a lower x_0^a pushes π_0^a downwards relative to the fall in π_0^s; on the other hand, a higher $E_0\{\pi_1^a\}$ pushes π_0^a upwards relative to the fall in π_0^s. In other words, a policymaker considering to adopting average targeting may not face a trade-off "lower volatility of x_0 versus higher volatility of π_0" if inflation expectations are sufficiently large to cushion the deflationary pressure. As for the periods after the shock; x_t^s, π_t^s, and i_t^s return to their steady states immediately, but their homologs in the average targeting rule remain above/below their steady states given the memory introduced in the interest-rate rule. It is worth noting, however, that the influence of π_0^a via $\tilde{\pi}_t$ disappears at $t = 4$, which places $\tilde{\pi}_t$ above target for a while at $t \geq 4$, causing i_t^a to be above its steady state, which in turn exerts a downward pressure to x_t^a and π_t^a.

Figure 2 shows the same as figure 1, but for a persistent cost-push shock with a half-life of 2 quarters.

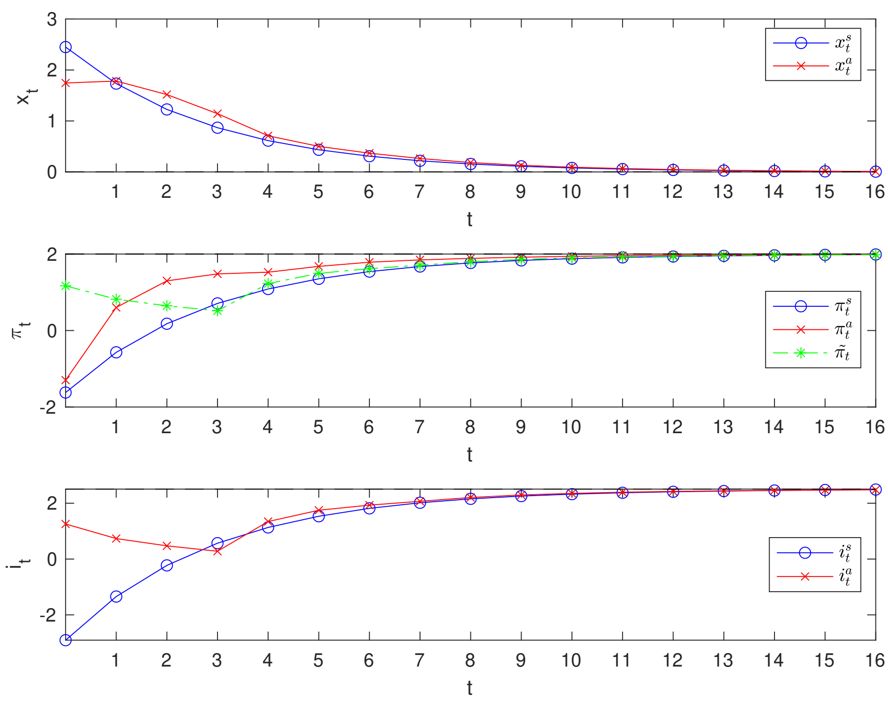

Figure 2: Equilibrium paths when $\rho_u = \frac{\sqrt{2}}{2}$ for simple targeting (blue) and average targeting (red). Average inflation in green. Inflation and interest rates are annualized, and expressed in %.

Figure 2 illustrates that i_t^s falls in negative terrain more easily than i_t^a following a moderate shock to u_t, which can be regarded as an advantage of average targeting rules. This result is intuitive given that i_t^a always reacts less on impact than i_t^s, and N plays a crucial role here. More specifically, N small brings the system close to the simple targeting rule ($N = 0$ corresponds to simple targeting by construction), which increases the risk of hitting the zero lower bound, whereas N large makes the system behave closer to the limit case where $i_t^a = \bar{i} = \rho + \bar{\pi}$[3]. Nevertheless, large cosh-push shocks flip the advantage of average targeting rules with large N for the same logic. If the cost-push shock is large enough to bring i_t^a into negative terrain, i_t^a will generally stay negative for longer than i_t^s given the memory introduced by $\tilde{\pi}_t$.

4 Welfare analysis

In this section, I analyze the welfare implications of each interest rate rule. I assume that variations in the gap and inflation rate produce losses according to the following expression:

$$\mathbb{L} = \omega \cdot var(x_t) + var(\pi_t). \tag{8}$$

Intuitively, equation 8 reflects that welfare will increase in (i) the variance of inflation (inflation above/below the target rate imply suboptimal adjustment costs in the case of Rotemberg pricing), and in (ii) the variance of the welfare-relevant output gap (ideally, the gap is always closed and output coincides with the efficient level). Also, the policymaker will attach a relative importance, reflected by ω, to each of the two components. Just for the sake of brevity, I leave the microfoundation of ω for future work[4], and also note that I am assuming that the policymaker only cares about current-period welfare losses, as it is often assumed when the policymaker acts under discretion.

I assume, for simplicity, that $\epsilon_0 \sim N\left(0, \sigma_u^2\right)$ and $\epsilon_t = 0$ for $t > 0$, i.e. the economy is subject to a single one-time shock happening at $t = 0$. I then compute welfare losses by performing montecarlo simulations for the calibration specified in the appendix, and for $\omega = 0.1$[5]. Table 1 below shows the loss incurred under the optimal ϕ for a simple targeting rule ($N = 0$), and for one-, two- and three-year

[3] This is isomorphic to having an interest rate rule with $\phi_a = 0$, *ergo* an unstable system

[4] The microfoundation of ω is treated extensively in Woodford (2003)–chapter 6 and Galí (2015)–chapter 4 for the case of Calvo's sticky prices. In these papers, the welfare loss of following a certain rule is measured as the equivalent permanent consumption decline (relative to steady state). Vestin et al. (2007) and Damjanovic and Nolan (2011) also offer similar derivations, but for Rotemberg adjustment costs. The latter shows how welfare losses can, like in Calvo pricing, be writen as a function of variances also in the case of Rotemberg pricing.

[5] Under Calvo pricing, welfare losses weight the welfare-relevant output gap by $\omega = \frac{\kappa}{\varepsilon}$, which is 0.1 under my default calibration. See Galí (2015)–chapter 5.

average targeting rules.

N	ϕ^*	$var(x_t)$	$var(\pi_t)$	$\mathbb{L}$
0	3.5	0.91	0.01	0.101
3	3.8	0.573	0.014	0.071
7	5	0.483	0.019	0.067
11	7.1	0.453	0.019	0.064

Table 1: Optimal ϕ^* and minimum welfare losses for different rules. Variances and $\mathbb{L}$ are shown in thousandths.

An advantage of increasing N is that, as long as an equilibrium exists, $var(x_t)$ can be diminished for any fixed ϕ. However, in my calibration this generally comes at the cost of a rising $var(\pi_t)$, which calls for a rise in ϕ by the central bank to contain inflation. Table 1 shows how this toy model prefers a two- or three-year average targeting rule coupled with an aggressive stance against inflation than a short-average targeting rule. I leave the determination of the optimal pair (ϕ^*, N^*) for future work, given the computational power needed to minimize $\mathbb{L}$ numerically by performing montecarlo simulations.

5 Conclusions

I have used a basic New Keynesian model to compare average inflation targeting to simple inflation targeting. I have found that, in response to a single cost-push shock, average targeting smooths the response of the welfare-relevant output gap over time because the nominal rate behaves stickily. Inflation in average targeting is, on impact, more volatile or less volatile than in simple targeting depending on how strong the effect of inflation expectations is. From the point of view of welfare, my calibration reveals that a policymaker with no commitment prefers to use a long-average rule (three- or four-year average) than a short-average rule or simple targeting. However, long-average targeting rules can be problematic in the event of a large cost-push deflationary shock, because interest rates get stuck in negative terrain for a long time.

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A Derivation of the model

The economy is populated by (i) a representative household; (ii) a representative producer of a final good that uses a continuum of inputs to produce, each produced by (iii) a producer of intermediate goods, indexed by $i \in [0,1]$, operating in a monopolistic competition regime; and (iv) a central bank.

Consumers

The representative household takes the prices of goods P_t, price of bonds Q_t, wages W_t, and the profits accruing from the ownership of firms Π_t as given, and chooses how much to consume C_t, work N_t, and invest in bonds B_t to maximize intertemporal (discounted by $0 < \beta < 1$) utility subject to a sequence of budget constraints. Mathematically, this is:

$$\begin{gathered} \underset{C_t,N_t,B_t}{max} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}-1}{1+\varphi} \right) \\ s.to \quad P_t C_t + Q_t B_t = W_t N_t + B_{t-1} + \Pi_t \end{gathered} \tag{M1}$$

The labor supply and Euler equations associated to this problem are, respectively:

$$\frac{W_t}{P_t} = N_t^{\varphi} C_t^{\sigma} \tag{9}$$

$$C_t^{-\sigma} = \beta Q_t^{-1} E_t \left(\frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \right) \tag{10}$$

Log-linearizing equations 9 and 10 around the steady state yields:

$$w_t - p_t = \varphi n_t + \sigma c_t \tag{11}$$

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}\left(i_t - E_t(\pi_{t+1}) - \rho\right), \tag{12}$$

where lower case letters refer to the corresponding log variable, and where I am defining $\rho \equiv -log(\beta)$, $i_t \equiv log(Q_t^{-1})$, and $\pi_{t+1} \equiv p_{t+1} - p_t$. Using the log-version of market clearing $c_t = y_t$ and defining $x_t \equiv y_t - y_t^e$ as the welfare-relevant output gap, equation 12 can be written as:

$$x_t = E_t\left(x_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t(\pi_{t+1}) - \underbrace{\left(\rho + \sigma \Delta E_t(y^e_{t+1})\right)}_{\equiv r^e_t}\right) \tag{13}$$

where $\Delta E_t(y^e_{t+1}) \equiv E_t(y^e_{t+1}) - y^e_t$. This is already equation 1.

Final good producer

On the supply side, I derive a NKPC à la Rotemberg (1982). A representative producer of a final good operating a CES technology and taking the prices $P_t(i)$ of inputs $Y_t(i)$, $i \in [0,1]$, as given minimizes its costs given a production level:

$$\begin{aligned} &\underset{\{Y(i)\}_{i=0}^{1}}{min} \int_0^1 P_t(i) Y_t(i) di \\ s.to \quad &Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned} \tag{M2}$$

The relative demand of any input j in terms of i associated to the problem above is $Y_t(j) = \left(\frac{P_t(i)}{P_t(j)}\right)^{\varepsilon} Y_t(i)\ \forall j$, which plugged into the constraint of the optimization problem yields the following sequence of Marshallian demands after some algebra:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t\ \forall i \in [0,1], \tag{14}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

Intermediate goods producers

The producer of the intermediate good i maximizes the following constrained problem:

$$\max_{\{P_t(i),Y_t(i),N_t(i)\}_{t=0}^{\infty}} \quad E_0\left\{\sum_{t=0}^{\infty} Q_t\left[P_t(i)Y_t(i)-W_tN_t(i)-\Lambda\left(P_t(i),P_{t-1}(i)\right)\right]\right\}$$

$$s.to \quad \begin{cases} \Lambda\left(P_t(i),P_{t-1}(i)\right)=\frac{\theta}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 P_t(i)Y_t(i) \\ Q_t=\beta E_t\left(\frac{P_t}{P_{t+1}}\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\right) \\ Y_t(i)=A_tN_t(i)^{\alpha} \\ Y_t(i)=\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}Y_t \end{cases} \quad \forall t, \tag{M3}$$

i.e. chooses a sequence of outputs $Y_t(i)$, inputs $N_t(i)$, and prices $P_t(i)$, $\forall t$, to maximize an intertemporal (discounted) stream of profits subject to a sequence of adjustment costs, Cobb-Douglas production technologies, and the demand schedules in equation 14, whilst taking the wage W_t as given, and given an initial value for $P_{t-1}(i)$. Note that profits are expressed in nominal terms, and are thus discounted using the corresponding value of a nominal unit of resources t periods ahead. The associated Bellman equation to the maximization problem M3 is:

$$V_t\left(P_{t-1}(i),A_t\right)=\max_{P_t(i),Y_t(i),N_t(i)}\left\{\begin{array}{c} P_t(i)Y_t(i)-W_tN_t(i)-\dfrac{\theta}{2}\left(\dfrac{P_t(i)}{P_{t-1}(i)}-1\right)^2 P_t(i)Y_t(i)+\dots \\ \beta E_t\left(\dfrac{P_t}{P_{t+1}}\dfrac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}V_{t+1}\left(P_t(i),A_{t+1}\right)\right) \\ s.to\begin{cases} Y_t(i)=A_tN_t(i)^{\alpha} \\ Y_t(i)=\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}Y_t \end{cases} \end{array}\right\}, \tag{15}$$

whose $P_t(i), Y_t(i), N_t(i)$ first-order and envelope conditions are, respectively:

$$\begin{array}{c} Y_t(i)-\theta\left(\dfrac{P_t(i)}{P_{t-1}(i)}-1\right)\dfrac{P_t(i)}{P_{t-1}(i)}Y_t(i)-\dfrac{\theta}{2}\left(\dfrac{P_t(i)}{P_{t-1}(i)}-1\right)^2 Y_t(i)+\dots \\ \beta E_t\left\{\dfrac{P_t}{P_{t+1}}\dfrac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}V'_{t+1,P_t(i)}\left(P_t(i),A_{t+1}\right)\right\}-\lambda_t\varepsilon\left(\dfrac{P_t(i)}{P_t}\right)^{-\varepsilon-1}\dfrac{Y_t}{P_t}=0 \end{array} \forall t \tag{16}$$

$$P_t(i)-\frac{\theta}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 P_t(i)-\mu_t-\lambda_t=0\,\forall t \tag{17}$$

$$-W_t + \mu_t \alpha A_t N_t(i)^{\alpha-1} = 0 \,\forall t \tag{18}$$

$$V'_{t,P_{t-1}(i)}\left(P_{t-1}(i), A_t\right) = \theta\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)\left(\frac{P_t(i)}{P_{t-1}(i)}\right)^2 Y_t(i) \,\forall t, \tag{19}$$

where $V'_{t+1,P_t(i)}(\cdot)$ denotes the derivative of the value function with respect to $P_t(i)$ (idem for $V'_{t,P_{t-1}(i)}(\cdot)$).

Plugging conditions 17, 18, and 19 into condition 16 obtains a single optimality condition:

$$\begin{gathered}Y_t(i) - \theta\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)\frac{P_t(i)}{P_{t-1}(i)} Y_t(i) - \frac{\theta}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t(i) + \beta E_t\left\{\frac{P_t}{P_{t+1}}\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\theta\left(\frac{P_{t+1}(i)}{P_t(i)} - 1\right)\dots\right. \\ \left.\left(\frac{P_{t+1}(i)}{P_t(i)}\right)^2 Y_{t+1}(i)\right\} - \left(P_t(i) - \frac{\theta}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t(i) - \frac{W_t}{\alpha A_t N_t(i)^{\alpha-1}}\right)\varepsilon\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1}\frac{Y_t}{P_t}\end{gathered} = 0 \,\forall t \tag{20}$$

which can be rewritten, after some algebra, as follows:

$$\pi_t\left(1+\pi_t\right) = \beta E_t\left\{\left(\frac{Y_{t+1}}{Y_t}\right)^{1-\sigma}\pi_{t+1}\left(1+\pi_{t+1}\right)\right\} + \frac{\varepsilon}{\theta}\left[\frac{Y_t^\sigma N_t^\varphi}{\alpha A_t N_t^{\alpha-1}} - \frac{\varepsilon-1}{\varepsilon}\right] + \frac{\varepsilon-1}{2}\pi_t^2 \,\forall t, \tag{21}$$

where I used the fact that $P_t(i) = P_t, Y_t(i) = Y_t, N_t(i) = N_t$, $\forall i,t$, following the symmetry of the problem, $\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi$ following the labor supply in 9, $C_t = Y_t \,\forall t$ following a market-clearing condition for the market of final goods, and $\frac{P_{t+1}}{P_t} - 1 \equiv \pi_t \,\forall t$ is defined as the inflation rate. Finally, a log-linearization around a zero-inflation steady state yields, assuming that $\sigma \approx 1$ (the household has approximately log-utility), the standard NKPC:

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa \tilde{y}_t, \tag{22}$$

where $\kappa \equiv \frac{\varepsilon-1}{\theta}\left(\sigma - 1 + \frac{\varphi+1}{\alpha}\right)$. Note that the conditions $N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{\alpha}}$ (production function + symmetry) and $\frac{1-\varphi}{\alpha}a_t = \left(\sigma - 1 + \frac{\varphi-1}{\alpha}\right)y_t^n$ (equilibrium output when prices are flexible) need to be used to arrive to this final expression.

Equilibrium with simple inflation targeting

Substituting the interest-rate rule 3 into the dynamic IS curve 1 obtains a forward-looking system of two equations:

$$x_t = E_t\left(x_{t+1}\right) - \frac{1}{\sigma}\left(\bar{\pi} + \phi_s\left(\pi_t - \bar{\pi}\right) - E_t(\pi_{t+1})\right) \tag{23}$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t, \tag{24}$$

or, in matrix form:

$$\begin{bmatrix} 1 & \frac{\phi_s}{\sigma} \\ -\kappa & 1 \end{bmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} E_t \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} + \begin{pmatrix} -\frac{1-\phi_s}{\sigma}\bar{\pi} \\ u_t \end{pmatrix},$$

which requires that both eigenvalues of $\begin{bmatrix} 1 & \frac{\phi_s}{\sigma} \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix}$ lie within the unit circle for stationarity[6], i.e. $\kappa(\phi_\pi - 1) > 0$ after some algebra.

Now I guess that the solution looks like $x_t = \psi_x u_t + \lambda_x$ and $\pi_t = \psi_\pi u_t + \lambda_\pi$, where ψ_x , λ_x, ψ_π, λ_π are four *a priori* unknown parameters. This guess allows me to rewrite equations 23 and 24 as:

$$\psi_x u_t + \lambda_x = \psi_x \rho_u u_t + \lambda_x - \frac{1}{\sigma}\left(\bar{\pi} + \phi_s\left(\psi_\pi u_t + \lambda_\pi - \bar{\pi}\right) - \psi_\pi \rho_u u_t - \lambda_\pi\right)$$

$$\psi_\pi u_t + \lambda_\pi = \beta\left(\psi_\pi \rho_u u_t + \lambda_\pi\right) + \kappa\left(\psi_x u_t + \lambda_x\right) + u_t,$$

where I used the fact that, for a given parameter Ψ, $E_t(\Psi u_{t+1}) = E_t(\Psi(\rho_u u_t + \epsilon_t)) = \Psi \rho_u u_t$ since ϵ_t is white noise. In order for these last two equations to hold $\forall t$, it needs to be the case that:

$$\begin{cases} \psi_x = \psi_x \rho_u - \frac{\psi_\pi}{\sigma}\left(\phi_s - \rho_u\right) \\ \lambda_x = \lambda_x - \frac{1}{\sigma}\left(\bar{\pi} + \phi_s\left(\lambda_\pi - \bar{\pi}\right) - \lambda_\pi\right) \\ \psi_\pi = \beta \psi_\pi \rho_u + \kappa \psi_x + 1 \\ \lambda_\pi = \beta \lambda_\pi + \kappa \lambda_x \end{cases},$$

which obtains $\psi_x^* = \frac{\rho_u - \phi_s}{\sigma(1-\rho_u\beta)(1-\rho_u)-(\rho_u-\phi_s)\kappa}$, $\psi_\pi^* = \frac{\sigma(1-\rho_u)}{\sigma(1-\rho_u\beta)(1-\rho_u)-(\rho_u-\phi_s)\kappa}$, $\lambda_x^* = \frac{1-\beta}{\kappa}\bar{\pi}$, $\lambda_\pi^* = \bar{\pi}$, and verifies the initial linear-solution guess.

B Default calibration

Unless otherwise specified, I use the following choice of parameters to perform the simulations and the welfare analysis.

[6]This is an *if and only if* condition as long as u_t is stationary

Parameter	Value	Justification
t	$\frac{1}{4}$	Implies that the default period is a quarter.
ρ	$\sqrt[4]{1.005}-1$	Implies an annualized real rate of return of 0.5% in the steady state. I deliberately target such low rate to reflect the current low-rates phenomenon (King and Low (2014); or more recently, Mian et al. (2020)).
$\Delta E_t(y^e_{t+1})$	0	For simplicity. Implies that $r^e_t = \rho$.
σ	1	Implies log-utility, commonly used in the literature − see for instance Clarida et al. (2000).
$\overline{\pi}$	$\sqrt[4]{1.02}-1$	Implies an annualized 2% inflation target.
α	$\frac{2}{3}$	Standard labor share in the literature.
ε	3	I follow Galí (2015). See also some discussion in Hsieh and Klenow (2009).
φ	1	I follow Galí (2015). Implies a unitary Frisch elasticity of labor supply.
θ	20	Jointly with the last three parameter values, I target $\kappa = 0.3$ (see discussion in Clarida et al. (2000)).
ρ_u	$\frac{\sqrt{2}}{2}$	Implies a half-life of 2 quarters for the cost-push shock, a moderately persistent shock.
σ_u	0.01	Implies that a one-standard deviation shock to ϵ_t changes u_t by approximately 1%.

C Lemmas and proofs

This section intuitively proves some lemmas invoked during the paper. Its purpose is to give some intuitions, and the development of more rigorous proofs is left for future work.

Lemma 1. i^a_t always reacts by less than i^s_t on impact for the same parametrization of the economy.

The proof is easy to illustrate in the case of a short-lived cost-push shock, i.e. $\rho_u = 0$. I assume this is the case to develop the proof, although this is not a crucial assumption.

Let's assume an equilibrium exists and proceed by contradiction. Imagine that a shock happens at

$t = 0$ and that, as a consequence, i_0^a falls by more than i_0^s. From equations 3 and 7 it is straightforward to see that this requires $\tilde{\pi}_0$ to fall by more than π_0^s, *ergo* π_0^a has to fall by more than π_0^s. Given that $u_1 = 0$ and given that agents know that i_1^a will be below its steady state as long as $N > 0$ (*ergo* π_1^a and x_1^a will be above their respective steady states if an equilibrium exists and the system is not unstable), π_0^a lower than π_0^s can only happen if x_0^a rises by less than x_0^s. Now, x_0^a rising less than x_0^s cannot come via expectations (again, $u_1 = 0$ and $i_1^a < i^{ss}$ implies π_1^a and x_1^a will be above their respective steady states, *ergo* above π_1^s and x_1^s if an equilibrium exists), which implies that this can only happen via i_0^a falling less than i_0^s. The contradiction is then achieved.

Note that I invoked the result that "$u_1 = 0$ and $i_1^a < i^{ss}$ implies π_1^a and x_1^a will be above their respective steady states". This needs proof, but intuitively $i_1^a < i^{ss}$ can only imply π_1^a and x_1^a below their steady states if a large tightening is expected at $t = 2$, which can only come via a large spike in inflation at $t = 2$, and so on. This will not happen if the system is stable.