Operators

- An operator is a mapping between functions in its domain (input) and functions (output) in its range
  - Examples:
  - Multiplication operator
  - Differentiation operator
  - Integral operator

  - Classes:
  - Linear operators: If A and B are linear operators, f and g functions, and k a constant, then (A+B)f=Af+Bf, A(f+g)=Af+Ag, Ak=kA
  - Differential Operators: include differentiation of the functions to which they are applied

$$\hat{A}\ (\psi) = \varphi$$

Figure 1. Â is an operator, acting on a function (ψ). Therefore, Â will map one state vector into another one.
Source: Quantum Mechanical Operators - Quantum Chemistry - PSIBERG.

Classical observables have an associated quantum mechanical operator. In other words, for every parameter that we can measure in physical systems, a quantum mechanical operator exists.

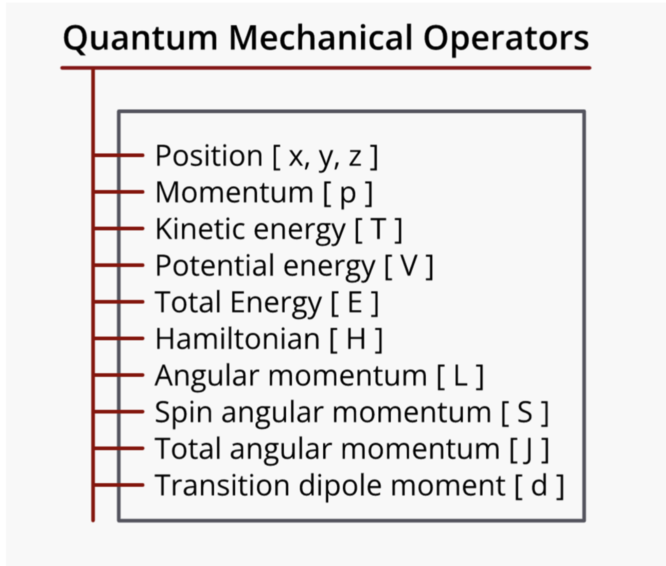

Figure. Examples of quantum mechanical operators. Source: [Quantum Mechanical Operators - Quantum Chemistry - PSIBERG]

Commutation of Operators

Differential operators act on the function(s) to their right, they do not necessarily commute with other operators containing the same independent variable. The commutator of operators A and B is therefore useful: $[A, B] = AB - BA$. Let's do an example: We will evaluate the commutator $[x, p]$ where $p = -id/dx$. We apply $[x, p]$ to an arbitrary function $f(x)$.

$$[x, p]f(x) = (xp - px)f(x)$$
$$= -ixdf(x)/dx - (-id/dx)(xf(x))$$
$$= -ixf'(x) + i(f(x) + xf'(x)) = if(x)$$

Therefore, we see that $[x, p] = i$. This also indicates that $[x, p]f(x) = if(x)$ for all f.

In general, if A, B, and C are operators and k is a constant, we have the following properties:

$[A,\ B] =- [B,\ A]$
$[A,\ B + C] = [A,\ B] + [A, C]$
$k[A,\ B] = [kA, B] = [A, kB]$

Identity, Inverse, Adjoint

The identity is an operator that leaves a function unchanged. It is generally denoted by I.
Some operators have an inverse, which is an operator that will "undo" its effect. Let A denote an operator, then A^{-1} will donte the inverse of A. Furthermore, the inverse will have the following property: $A^{-1}A = AA^{-1} = I$.
Associated with many operators will be another one called its adjoint. This operator is denoted: A^{+}. For all functions f and g in Hilbert space, we have the following: $< f|Ag >=< A^{+}f|g >$. We can see that if we apply A to the right of any scalar product produces the same result as if we apply A^{+} to the left of any scalar product. If we have the case that $A = A^{+}$, then the operator A is defined as self-adjoint or Hermitian. If we have that
$A^{+} =- A$ then A is called aint-Hermitian.
In addition, if the adjoint of an operator is equal to its inverse, $A^{-1} = A^{+}$, then the operator A is called unitary.

Sources:

Mathematical Methods for Physicists by Arfken, George B. & Webber
[Quantum Mechanical Operators - Quantum Chemistry - PSIBERG]