

# **DEFINING A FAILURE SURFACE FOR FFF PARTS USING A NOVEL FAILURE CRITERION**

**Gerardo A. Mazzei Capote**

A thesis submitted in partial fulfillment of  
the requirements for the degree of

**Master of Science  
(Mechanical Engineering)**

at the

**UNIVERSITY OF WISCONSIN-MADISON**

2018

Final Oral Examination: August 11th, 2018



# Approval

The following thesis, **Defining a failure surface for FFF parts using a novel failure criterion**, developed at the **University of Wisconsin-Madison** has been approved by:

---

*Signature*

---

*Date*

**Professor Tim A. Osswald**  
Department of Mechanical Engineering  
College of Engineering  
University of Wisconsin-Madison



# Abstract

Yada Yada Yada

# Acknowledgments

Thank you Prof. Rudolph and Prof. Osswald for your trust and patience. I couldn't hope for better advisors.

Thank you to Prof. Turng and Prof. Prabhakar for your valuable input and time.

Thank you Tom for your extrusion expertise and your polymer knowledge. The sass was OK too I suppose.

Thank you Luke for building the tools necessary for this whole project to work. That big beard hides a sharp mind -and a lot of patience to answer all my random grammar questions.

Thank you Alec for being an ever-present helping hand, a great classmate, and a good friend. Hope you make ND proud.

Thank you José for putting up with my constant interruptions *en español* from across the desk, and for sharing lab managing duties with me. *Suerte y paciencia para el doctorado.*

Thank you to Thibaut, Colby, Johannes, Peilin, Varun, Ahmet and Brendan. This thesis would look very empty without your contributions!

Thank you to the entire PEC family for creating a great learning environment. I've grown so much with all of you. I hope that you've learned a thing or two with me as well.

Thank you to Ben, Josh, Diego, Iván, Pfeifer, Ian, Abrahan, Maria, Sianna, Conny and Sara for your friendship. Madison was a lot of fun because you were in it.

*Gracias a mi familia, a quienes les debo todo.*

*Gracias a Sonya por tu cariño y paciencia. Que este tiempo que pasamos tan lejos nos traiga en creces momentos juntos. Te amo.*

# Table of Contents

|   |           |
|---|-----------|
| <b>Front Matter</b>   | <b>i</b>  |
| Abstract . . . . .  | i         |
| Acknowledgments . . . . .                                     | ii        |
| Nomenclature . . . . .  | v         |
| List of Figures . . . . .                                     | vi        |
| List of Tables . . . . .                                      | vii       |
| <b>Introduction</b>   | <b>1</b>  |
| <b>1 Background</b>   | <b>3</b>  |
| 1.1 Additive Manufacturing . . . . .                          | 3         |
| 1.1.1 Advantages, Disadvantages and Success Stories . . . . . | 4         |
| 1.2 Fused Filament Fabrication . . . . .                      | 6         |
| 1.2.1 The FFF process . . . . .                               | 7         |
| 1.2.2 Mechanical Properties of FFF parts . . . . .            | 9         |
| 1.3 Failure Criteria . . . . .                                | 10        |
| <b>2 A Novel Failure Criterion</b>                            | <b>12</b> |
| 2.1 The Gol'denblat-Kopnov Model . . . . .                    | 12        |
| 2.2 The Osswald-Osswald Criterion . . . . .                   | 15        |
| <b>3 Experimental Methods</b>                                 | <b>19</b> |
| 3.1 Material . . . . .  | 19        |
| 3.2 Sample Preparation . . . . .                              | 20        |
| 3.2.1 Printing Equipment . . . . .                            | 20        |
| 3.2.2 Tensile Specimens . . . . .                             | 22        |
| 3.2.3 Compressive Specimens . . . . .                         | 23        |
| 3.2.4 Torsion and Combined Loading Specimens . . . . .        | 24        |
| 3.3 Mechanical Testing . . . . .                              | 26        |
| 3.3.1 Tension and Compression tests . . . . .                 | 26        |
| 3.3.2 Torsion and Combined loading tests . . . . .            | 27        |

|  |           |
|--|-----------|
| <b>4 Results</b>                                 | <b>30</b> |
| 4.1 Tensile Tests . . . . .                      | 30        |
| 4.2 Compression Tests . . . . .                  | 32        |
| 4.3 Torsion Tests . . . . .                      | 34        |
| 4.3.1 45° orientation . . . . .                  | 34        |
| 4.3.2 0° orientation . . . . .                   | 34        |
| 4.3.3 90° orientation . . . . .                  | 34        |
| 4.4 Combined Loading Tests . . . . .             | 34        |
| 4.5 Development of the Failure Surface . . . . . | 34        |
| <b>Bibliography</b>                              | <b>39</b> |
| <b>A Failure Surface Code</b>                    | <b>42</b> |

# Symbols and Acronyms

## Acronyms

- $\mu$ CT Micro Computer Tomography  
 ABS Acrylonitrile Butadiene Styrene  
 ADT Anderson-Darling Test  
 AM Additive Manufacturing  
 CAD Computer Aided Design  
 EF Extrusion Factor  
 FDM Fused Deposition Modeling™  
 FFF Fused Filament Fabrication  
 GKC Gol'denblat-Kopnov Criterion  
 OOC Osswald-Osswald Criterion  
 RP Rapid Prototyping  
 SLA Stereolithography  
 SLS Selective Laser Sintering

## Symbols

|                  |   |            |
|------------------|---|------------|
| $\dot{\epsilon}$ | Engineering Strain rate   | $min^{-1}$ |
| $\dot{\gamma}$   | Shear Strain rate   | $min^{-1}$ |
| $\epsilon$       | Engineering Strain  | —          |
| $\gamma$         | Shear Strain  | —          |
| $\mu^{1112}$     | OOC parameter- slope at pure shear failure in the $\sigma_{11} - \tau_{12}$ plane | —          |

|               |   |     |
|---------------|---|-----|
| $\mu^{2212}$  | OOC parameter- slope at pure shear failure in the $\sigma_{22} - \tau_{12}$ plane | —   |
| $\sigma$      | Axial stress  | MPa |
| $\sigma_{11}$ | Axial stress in the 1-1 direction   | MPa |
| $\sigma_{22}$ | Axial stress in the 2-2 direction   | MPa |
| $\sigma_{33}$ | Axial stress in the 3-3 direction   | MPa |
| $\tau$        | Shear stress  | MPa |
| $\tau_{12}$   | Shear stress in the 1-2 plane   | MPa |
| $\tau_{13}$   | Shear stress in the 1-3 plane   | MPa |
| $\tau_{23}$   | Shear stress in the 2-3 plane   | MPa |
| $S$           | Shear strength in the 1-2 plane   | MPa |
| $S_{45n}$     | Negative shear strength for 45° specimen  | MPa |
| $S_{45p}$     | Positive shear strength for 45° specimen  | MPa |
| $X_c$         | Compressive strength in the 1-1 direction   | MPa |
| $X_t$         | Tensile strength in the 1-1 direction   | MPa |
| $Y_c$         | Compressive strength in the 2-2 direction   | MPa |
| $Y_t$         | Tensile strength in the 2-2 direction   | MPa |

# List of Figures

|      |   |    |
|------|---|----|
| 1.1  | Process flow of AM . . . . .  | 3  |
| 1.2  | Shoes with AM midsoles . . . . .  | 6  |
| 1.3  | The basic FFF machine configuration . . . . .   | 7  |
| 1.4  | Model, toolpath and final part in the FFF process . . . . .                                 | 8  |
| 1.5  | Typical FFF part mesostructure and its origin . . . . .                                     | 8  |
| 1.6  | Results from Koch <i>et al.</i> [6] . . . . .   | 10 |
| 1.7  | Comparison of different Failure criteria. [23] . . . . .                                    | 11 |
| 2.1  | Different load directions in an anisotropic material . . . . .                              | 13 |
| 2.2  | GKC failure surface developed using data from the WWFE-1 [21] . . . . .                     | 15 |
| 2.3  | $\mu^{2212}$ parameter in the $\tau_{12} - \sigma_{22}$ plane . . . . .                     | 16 |
| 2.4  | $\lambda^{ijjj}$ parameters in a generic $\sigma_{ii} - \sigma_{jj}$ stress plane . . . . . | 17 |
| 2.5  | Comparison of GKC and OOC failure envelopes [21] . . . . .                                  | 18 |
| 2.6  | Failure surface for SLS developed through the OOC [25] . . . . .                            | 18 |
| 3.1  | Extrusion line setup . . . . .  | 20 |
| 3.2  | Printing equipment . . . . .  | 21 |
| 3.3  | Print process in <i>Otto</i> . . . . .  | 21 |
| 3.4  | ASTM D-638 Type I coupon . . . . .  | 23 |
| 3.5  | Toolpath considerations for the tensile coupons . . . . .                                   | 23 |
| 3.6  | Compression specimen geometry . . . . .   | 24 |
| 3.7  | Representation of compression samples . . . . .   | 24 |
| 3.8  | $S_{45}$ Torsion specimen geometry . . . . .  | 25 |
| 3.9  | $S_{45}$ Representation of the 45° specimen . . . . .                                       | 25 |
| 3.10 | Torsion specimen used for 0° and 90° tests . . . . .  | 26 |
| 3.11 | Mechanical testing setup . . . . .  | 27 |
| 3.12 | Stress and strain caused by torque on a tubular geometry [25] . . . . .                     | 28 |
| 3.13 | Machine setup for combined loading scenarios . . . . .                                      | 29 |
| 4.1  | Comparison of tensile results . . . . .   | 31 |
| 4.2  | $X_t$ and $Y_t$ tested samples . . . . .  | 31 |
| 4.3  | ADT results for tensile data . . . . .  | 32 |
| 4.4  | Comparison of compression results . . . . .   | 33 |

|     |   |    |
|-----|---|----|
| 4.5 | $X_c$ and $Y_c$ tested samples . . . . .                              | 33 |
| 4.6 | Comparison of 45° torsion results . . . . .                           | 35 |
| 4.7 | Comparison of 45° torsion results . . . . .                           | 35 |
| 4.8 | failure envelope in the $\sigma_{11}$ - $\sigma_{22}$ plane . . . . . | 36 |

# List of Tables

|     |   |    |
|-----|---|----|
| 1.1 | Advantages and Disadvantages of Additive Manufacturing . . . . .                              | 5  |
| 2.1 | Nomenclature of the GKC parameters . . . . .  | 14 |
| 2.2 | Tensorial components of the GKC . . . . .   | 14 |
| 2.3 | Interaction components attainable through the OOC [21] . . . . .                              | 17 |
| 3.1 | Mechanical tests required for the OOC . . . . .   | 20 |
| 3.2 | Constant printing parameters . . . . .  | 22 |
| 4.1 | Summary of Tensile tests . . . . .  | 30 |
| 4.2 | Summary of Compression tests . . . . .  | 32 |
| 4.3 | Summary of $45^\circ$ torsion tests . . . . .   | 34 |
| 4.4 | Tensorial components obtained from tests in the $\sigma_{11}$ - $\sigma_{22}$ plane . . . . . | 37 |



# Introduction

*Additive Manufacturing* (AM) is an umbrella term that encompasses all fabrication techniques where the final geometry of the part is obtained through superposition of material in a layer-by-layer basis [1]. Developed in the 1980s, this manufacturing technique permits immensely shorter part development cycles, since the transition from a 3D *Computer Aided Design* (CAD) to part fabrication only requires one intermediate step: the use of a slicing engine that converts the geometry of the object into machine instructions [1]. For this reason, AM technologies were initially employed exclusively for prototype development and were referred to as *Rapid Prototyping techniques* (RP). However, recent innovations in the field have caused AM to be considered as a legitimate manufacturing technology since it is also capable of reproducing complex geometries unattainable through traditional methods [1].

While offering great advantages over traditional part fabrication methods, AM comes with its own set of limitations and disadvantages: First and foremost, the use of a stratified build approach tends to produce extremely anisotropic parts. Secondly, the geometric accuracy of the object produced is highly dependent of process parameters, particularly, the thickness of the layers. Finally, as of the time of this writing, AM lacks the standardization and scrutiny that are associated to most traditional manufacturing techniques [1].

*Fused Filament Fabrication* (FFF), also known under the trademark *Fused Deposition Modeling* (FDM™), represents perhaps the most prevalent AM technique in the market due to the advent of low-cost, desktop 3D printers in the early 2010s [2]. Due to the broad availability of machines and relatively low costs of material, there is a surging interest in optimizing FFF to produce small batches of end-user grade parts. Success stories are varied, but examples include vacuum form molds, fixtures, jigs, and tools used to aid assembly lines in the automotive industry [3, 4, 5]. However, this technology still faces the challenges and limitations that currently affect the field of AM as a whole. Namely, anisotropy introduced through the layer-by-layer build approach makes it difficult to assess the expected mechanical behavior of FFF parts when subjected to important mechanical stresses [2]. For these reasons, multiple attempts have been made to characterize the anisotropy of FFF manufactured objects. Recent studies performed by Koch *et al.* [6] and Rankouhi *et al.* [7] show that the ultimate tensile strength of FFF coupons is sensitive to process parameters such as the layer thickness and, in particular, the orientation in which the plastic strands are laid during

the build process -henceforth referred to as the bead orientation. However, literature related to preventing failure through design is scarce, given the difficulty of using commercially available FFF machines to produce test coupons with unconventional bead orientations, as well as the limitations inherent to commonly used failure criteria that make it difficult to develop an accurate failure surface.

This research applies a novel criterion, tailored for anisotropic materials, to develop a failure surface for FFF parts through mechanical testing of coupons under various types of loading conditions. Certain test specimens were produced using a unique off-axis 3D printer developed in-house that allows production of coupons in unconventional configurations. Such a surface can be an invaluable tool in part design, since catastrophic failure can be prevented in the early stages of part development. This could potentially allow a broader embrace of FFF as a legitimate manufacturing technique in highly demanding engineering fields, such as the aerospace or automotive industries where part failure is to be avoided at all costs.

This work offers a comprehensive overview of AM technologies, FFF and shortcomings of current failure criteria in Chapter 1. Chapter 2 details the failure criterion used throughout this work, as well as outlining its advantages over similar models. Chapters 3 through 4 detail the experimental setup followed, as well as outlining noteworthy results. Finally, Conclusions and Recommendations are given in Chapter ?? in the hopes of guiding future work on the topic.

# 1 Background

## 1.1 Additive Manufacturing

*Additive Manufacturing* (AM) technologies had their beginnings in the decade of the 1980s. During this time, various independently developed patents were filed across the globe, describing a process that would construct an object by selectively adding layers of material -as opposed to removing excess matter or deforming mass to obtain a desired shape. This represents the core definition of AM: any technology where the final geometry of the manufactured object is obtained through controlled addition of material qualifies as an Additive Manufacturing technique [1].

Advancements in the fields of computing, *Computer Aided Design* (CAD), and controllers, among other technological developments, were necessary to translate the patents into working prototypes, with some eventually becoming the foundations of commercially successful companies -such as 3D Systems in 1986 and Stratasys in 1989 [1, 8, 9]. The basic process of AM has remained largely unchanged from its first iteration in the late 80s: First, a computer model of the object is made using CAD software and exported under the *.stl* file format. Afterwards, the part geometry is stratified, or “sliced”, and translated into machine instructions using a specialized software called *slicing engine*. An AM machine then follows said instructions, commonly referred to as the *toolpath*, to build the object in layers. Finally, the part is available to the user. Depending on either the requirements of the part, or the specifics of the AM technique used, some post-processing may be required [1]. A visual representation of the process is shown in Figure 1.1.

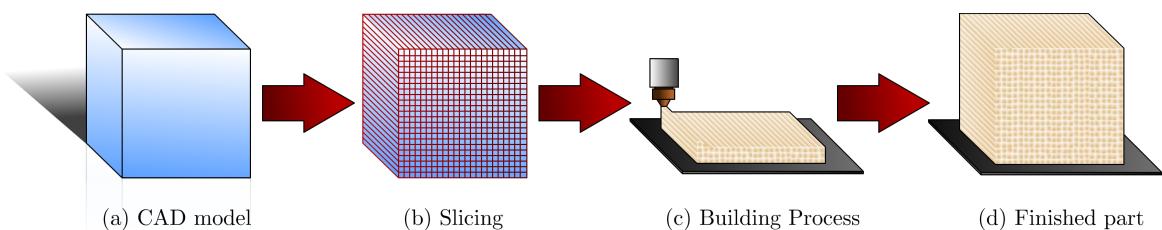


Figure 1.1: Process flow of AM

While all AM technologies operate on the same basic process flow described above, the specifics of each AM technique vary substantially, ranging from processes that use paper and binder, all the way through metal-based, laser tracing technologies. Since this is a rapidly evolving field, no general consensus exists for classifying the multiple AM processes available as of the time of this writing. However, the classification system proposed under the ASTM/ISO 52900 standard [10], has been somewhat accepted by the field and divides AM technologies as follows:

1. **Binder Jetting:** AM techniques where a binding agent is used to selectively promote cohesion in powder materials -generally gypsum, sand or metallic powders [10, 11].
2. **Directed Energy Deposition:** AM processes where a focused thermal energy source (i.e. laser, electron beam, plasma arc) is used to fuse materials as they are being deposited in the build volume. Materials are almost exclusively metals [10, 11].
3. **Material Extrusion:** In this type of AM technology, material is dispensed through a nozzle or orifice. Fused Filament Fabrication belongs to this classification. Materials are almost exclusively thermoplastics [10, 11].
4. **Material Jetting:** AM techniques where build material is deposited selectively in droplets. Materials are usually wax or thermoplastics, but there are examples of metal-based, material jetting techniques [10, 11].
5. **Powder Bed Fusion:** AM processes where portions of a powder bed are selectively fused through application of thermal energy. *Selective Laser Sintering* (SLS) belongs to this category. Materials are usually thermoplastic polymers or metals [10, 11].
6. **Sheet Lamination:** In this type of AM technology, the final part is formed by bonding sheets of material -usually paper or composites [10, 11].
7. **Vat Photopolymerization:** In this AM process, a liquid photopolymer is selectively cured by a light source. *Stereolithography* (SLA), arguably the first AM technology, belongs to this category. Due to the nature of this technique, the only materials used are photopolymers [10, 11].

### 1.1.1 Advantages, Disadvantages and Success Stories

Since AM processes allow a relatively direct conversion of a CAD model into a constructed object, they were originally exclusively used for prototype development. For this reason, they were initially classified as “*Rapid Prototyping*” (RP) technologies. This terminology is still used today, however, it is being superseded by *Additive Manufacturing* since its potential to become a proper fabrication technique exists [1]. While

being capable of quickly jumping from part design to manufacturing is a great advantage, AM has its own set of drawbacks. Table 1.1 summarizes the most noteworthy set of advantages and disadvantages typical of most AM technologies.

Table 1.1: Advantages and Disadvantages of Additive Manufacturing

| Advantages   | Disadvantages   |
|--|---|
| Faster product development cycles [1]                  | Part quality highly dependent on process parameters [1]           |
| No additional tools needed for part fabrication[1]     | Stratified build generally results in anisotropic parts [1, 2]    |
| Cost effective for small batches of parts [12, 13, 14] | Costly for production of more than hundreds of parts [12, 13, 14] |

Out of all the described advantages and disadvantages, the high anisotropy of AM parts is responsible for the slow embrace of AM in highly demanding engineering fields -such as the aerospace and automotive industries. The highly anisotropic mechanical behavior makes it extremely difficult to predict part failure, therefore, it cannot be implemented in engineering applications where catastrophic failure is to be avoided at all costs. Even so, success stories of implementation of AM in industrial environments are abundant. Below is a number of relatively recent examples:

- **Volkswagen Autoeuropa:** This automotive assembly plant implemented the use of FFF machines to manufacture tools, jigs and fixtures used in their assembly line. They now produce 93% of the tools that were historically externally sourced, and have reportedly cut their tool development time and costs by 95% and 91% respectively [5].
- **General Electric:** GE is currently producing a complex fuel nozzle injector for the LEAP jet engine, using powder based, metal AM. The complex geometry of this component could not be manufactured by any other manufacturing technique. The production plant in Alabama is expected to have 50 AM machines producing 35,000 fuel nozzle injectors annually by 2020 [15].
- **Adidas and New Balance:** Both shoe companies have developed separate approaches to constructing highly optimized, 3D printed midsoles for high performance running sneakers. New Balance makes use of SLS technology to build the intricate geometry of their “*Zante Generate*” sneaker, using powdered TPU elastomer as the parent material. The designed honeycomb structure of the midsole, combined with the flexible material used, is supposed to improve the comfort and support brought by the shoe [16]. Adidas on the other hand chose to develop the “*AlphaEDGE 4D LTD*” running shoe using the CLIP technology by Carbon3D. While the cell geometry in the midsole is also supposed to bring performance and comfort improvements, the final ambition of Adidas is to perfect the technology to a point where a customer can simply go to a shoe store, have their feet

scanned, and receive a fully customized shoe with a 3D printed midsole that fits their particular needs [17, 18]. In both cases, the geometry of the midsole can only be produced by AM. The intricate structures in the midsoles can be seen in Figure 1.2.



(a) New Balance Zante Generate [16]



(b) Adidas AlphaEdge 4D LTD [18]

Figure 1.2: Shoes with AM midsoles

Note that in the cases presented, the main reason behind the usage of AM was either reduction of expenses associated with producing small batches of parts, or the capability of reproducing a unique and complex geometry. This is a trend that is observed in most of the literature describing implementation of AM into industrial scenarios.

While the advantages and disadvantages described here cover the field of AM as a whole, each technique comes with its own set of pros and cons that may make it the preferred method to reproduce a particular product or geometry. This work, however, focuses solely on FFF. The specifics of this process are described in detail in Section 1.2.

## 1.2 Fused Filament Fabrication

*Fused Filament Fabrication* (FFF) is an AM technology where the final geometry of the part is obtained through controlled extrusion of a liquid, self-hardening material-usually a thermoplastic polymer in molten state [1]. Originally developed by Stratasys in the 1980s under the still trademarked *Fused Deposition Modeling* (FDM<sup>TM</sup>) moniker, it has recently become one of the most widely used AM techniques due to the advent of low-cost, desktop FFF machines in the early 2010s caused by the expiration of key patents from Stratasys [1, 2].

### 1.2.1 The FFF process

At its core, the typical FFF machine consists of a heated build surface commonly referred to as a *build plate*, a specialized tool known as a *printhead*, and the fabrication material -supplied in the form of spools of thermoplastic polymer filament. The printhead is itself composed of a heating element, a nozzle, and some form of driving mechanism that pushes the filament downward. As the thermoplastic material is moved through the heated chamber, polymer melt is formed and extruded through the opening at the tip of the nozzle, producing a *bead*. The molten polymer can then be deposited upon the build plate, where controlled movements of the printhead and the fabrication surface gradually construct the final geometry of the part in a layer-by-layer build approach [1]. The typical setup of an FFF machine can be seen in Figure 1.3. In this example, the printhead moves in the *x-y* plane, while the build plate moves in the *z* direction.



Figure 1.3: The basic FFF machine configuration

Like all AM technologies, the FFF process starts in a computer with a CAD model converted to the *.stl* file format. The geometry is then translated to machine instructions through a *slicing engine*, where the user inputs a plethora of process parameters that include nozzle and build plate temperatures, print speed, layer thickness, and build orientation. Finally the *toolpath* is executed by the FFF printer, building the object in a layer-by-layer basis – sometimes referred to as *2.5D* printing [1, 4]. Figure 1.4 shows an abridged version of the process. The *z* axis indicates the intended build direction. Note how some of the finer details in the original CAD file are lost in the printed part – due in part to the layer height and build orientation selected.



Figure 1.4: Model, toolpath and final part in the FFF process

The process is capable of producing complex geometries that would be otherwise hard to reproduce through other polymer processing techniques, such as injection molding. However, it is bound by the disadvantages described in Section 1.1.1, as well its own unique set of drawbacks. Namely:

- The circular orifice in the nozzle makes FFF incapable of reproducing sharp corners, limits the size of the smallest reproducible feature, and causes the final part to be filled with voids –originating in the junction of round beads. These problems can be seen in Figure 1.5: On the left, a comparison of a  $90^\circ$  corner planned in the toolpath and the final geometry of the printed bead is shown. Note the rounded nature of the turn. On the right, a cross section of an FFF part obtained through *Micro Computer Tomography* ( $\mu$ CT) shows the voids that form during the printing process.

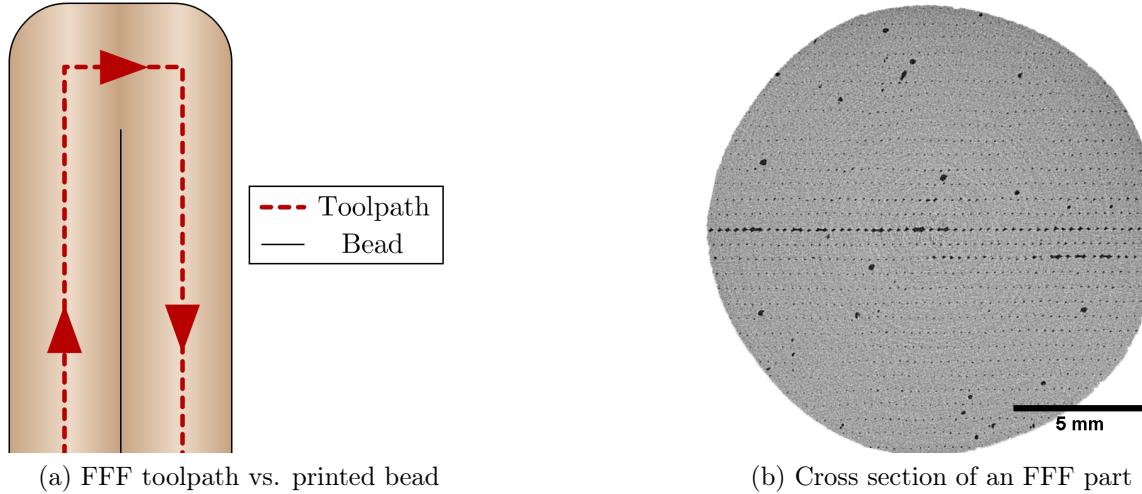


Figure 1.5: Typical FFF part mesostructure and its origin

- The junction of adjacent beads behaves akin to a polymeric weld, and has inferior mechanical properties than the bulk material [2]. This, coupled with the aforementioned voids which can act as stress concentrators, causes FFF parts

to behave in extremely anisotropic manner with diminished mechanical performance when compared to analogous parts obtained through traditional polymer processing technologies – such as injection molding [2].

This last disadvantage is responsible for the slow embrace of FFF as a proper manufacturing technique: the high anisotropy of FFF parts imply that predicting part failure becomes extremely difficult and thus, proper part design that guarantees safe operation of the object under important loads is hard to achieve. For this reason, efforts to characterize the mechanical behavior of FFF parts have existed since as early as the 1990s. Recent examples are presented in Section 1.2.2.

### 1.2.2 Mechanical Properties of FFF parts

Efforts have been made to characterize the mechanical anisotropy of FFF parts. However, due to the lack of testing standards and problems during toolpath planning, most studies focus solely in the tensile mechanical performance of FFF coupons.

Recent studies performed by Koch *et al.* [6] and Rankouhi *et al.* [7] indicate that the final tensile properties of FFF coupons are particularly sensitive to bead orientation and proper mass output through the nozzle. Other process parameters, such as the layer thickness, have varying degrees of impact upon the final tensile strength of the part. In both studies, tensile coupons were printed with bead orientations of  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  in the  $x$ - $y$  plane. Results showed that in all the experimental conditions selected, a  $0^\circ$  orientation always behaved closer to the bulk material, whereas a  $90^\circ$  sample always had significantly lower tensile strengths. The  $45^\circ$  samples sat between both extremes. It is important to note that in both studies, toolpath manipulation was necessary to avoid premature failure of the coupons due to stress concentrators originating in void formation due to the elliptical nature of the beads. Figure 1.6 shows some of the results by Koch *et al.* The geometry corresponds to an ASTM Type I Tensile coupon. Injection molded results are denoted *IM* for comparison. Note that the  $90^\circ$  orientation had a tensile strength that was 25% inferior to the IM counterpart, and 20% worse than the  $0^\circ$  oriented FFF coupon. This is a prevalent trend in the consulted bibliography.

Literature for other types of mechanical testing of FFF parts is relatively scarce when compared to tension experiments. Research indicates that the compressive strength of FFF parts tends to be higher than the tensile strength, as well as being less sensitive to process parameters —the bead orientation in particular seems to have a significantly diminished impact upon the compressive strength when compared to its effect upon tensile tests [19, 20]. Shear strength results are virtually non-existent.

Figure 1.6: Results from Koch *et al.* [6]

### 1.3 Failure Criteria

The increased use of advanced materials in industry has brought upon a necessity to properly characterize their strengths and failure modes. Composites in particular are commonly used in highly demanding engineering fields given that they excel in mechanical properties. However, due to their nature, their behavior is extremely anisotropic. For this reason, it has been of great interest to develop a proper way to model the behavior of anisotropic materials under mechanical stresses as a way to predict part failure – a practice from here on referred to as developing a *failure criterion*.

Early attempts to properly predict failure of anisotropic materials go as far back as 1948 with the Hill model [21]. Further developments led to a plethora of failure criteria, such as the Tsai-Hill, Malmeister, Tsai-Wu, Gol'denblat-Kopnov, Puck, and Cuntze to name a few [21, 22]. A wide variety of criteria exists because a model will rarely capture the complete failure behavior of an anisotropic material. To illustrate this point, refer to Figure 1.7, reproduced from work by Sun *et al.* [23] where a composite glass fiber and epoxy laminate was loaded biaxially, in a direction that was either parallel ( $\sigma_{11}$ ), perpendicular ( $\sigma_{22}$ ) to the fiber, or a combination of both. Positive stresses indicate tensile load, while negative values point to compressive forces. The data, represented by the white squares, does not agree with any of the used models in the fourth quadrant of the graph. This type of behavior is common throughout the literature: Puck's model is great at predicting shear strengthening effects, but doesn't perform well when dealing with combined axial loading scenarios; the Gol'denblat-Kopnov model by contrast is great at predicting axial stress interactions, but falls short when dealing with shear strengthening effects caused by combined shear-axial loadings. These trends point to the limitations of each model: in order to either facilitate calculations, or due to the difficulty of performing combined loading tests, interaction effects are neglected either

by mathematical choice, or indirectly through the inner workings of the failure criterion [21].

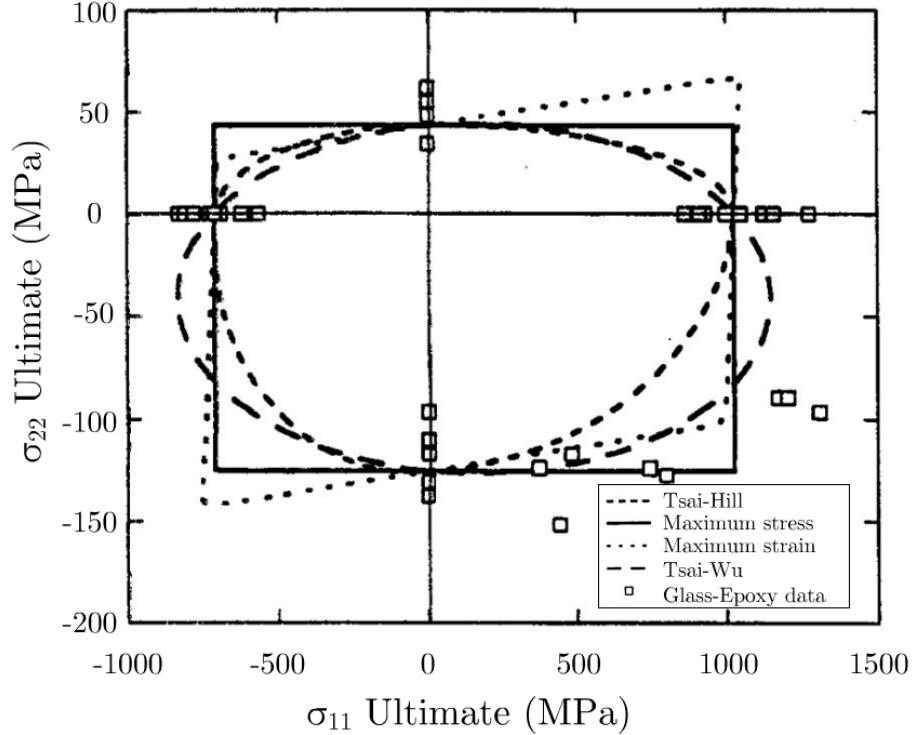


Figure 1.7: Comparison of different Failure criteria. [23]

Properly mapping a failure surface through a criterion proves to be an invaluable tool for design, since it allows engineers to assess if a part will perform safely under its intended loading conditions. Such tool could in theory help overcome the main shortcoming of FFF since a properly tailored failure envelope would allow proper part design considerations. However, since no present criterion completely captures the behavior of anisotropic materials, a novel technique is needed. Chapter 2 describes in detail a novel approach, based on the Gol'denblat-Kopnov model, that includes interaction effects to properly describe the failure behavior of anisotropic parts.

## 2 A Novel Failure Criterion

---

*The Osswald<sup>2</sup> criterion*

---

As described during Section 1.3 of Chapter 1, currently available failure criteria fail to completely integrate interaction effects into the modeled failure behavior of anisotropic materials. In 2017, Paul and Tim Osswald proposed a model that attempts to overcome these limitations [21]. This recent failure criterion has the following characteristics:

- **Tensor based and purely mathematical:** as opposed to phenomenological or mechanistic models such as the Puck or Cuntze failure criteria.
- **Based on the Gol'denblat-Kopnov model.**
- **Includes stress interactions that other models neglect.**

Originally titled “A Strength Tensor Based Failure Criterion with Stress Interactions”, it will be referred in this work as the Osswald-Osswald Criterion (OOC). This chapter will describe the Gol'denblat-Kopnov model upon which the OOC is based, followed by a proper description of how this novel model implements stress interactions.

### 2.1 The Gol'denblat-Kopnov Model

The Gol'denblat-Kopnov Criterion (GKC) describes a mathematical function that depends on the stress state of an anisotropic material. Should the computation of this expression exceed a threshold, part failure is to be expected. To that end, a scalar function that depends on stress tensors that completely characterize the state of the material was developed [24]. This function is shown in Equation 2.1, where stresses are denoted  $\sigma$ , and the subindices  $i,j,k,l$  denote a particular load direction.

$$f = (F_{ij}\sigma_{ij})^\alpha + (F_{ijkl}\sigma_{ij}\sigma_{kl})^\beta + (F_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn})^\gamma + \dots \quad (2.1)$$

The terms  $F_{ij}$ ,  $F_{ijkl}$  and  $F_{ijklmn}$  represent second, fourth and sixth order tensors respectively. These terms of the equation depend on engineering strength parameters, such as the ultimate tensile and compressive strengths of the material in a particular load direction [21]. Due to the complexity associated with using higher order tensors, Gol'denblat and Kopnov limited their approach to using only the second and fourth order terms. Thus Equation 2.1 is reduced to:

$$f = (F_{ij}\sigma_{ij})^\alpha + (F_{ijkl}\sigma_{ij}\sigma_{kl})^\beta \quad (2.2)$$

In order to attain a linear criterion scalar function, the exponents  $\alpha$  and  $\beta$  were assigned values of 1 and  $1/2$  respectively. Finally, in plane stress scenarios, the GKC becomes:

$$\begin{aligned} f = F_{11}\sigma_{11} + F_{22}\sigma_{22} + F_{12}\tau_{12} + & (F_{1111}\sigma_{11}^2 + F_{2222}\sigma_{22}^2 + F_{1212}\tau_{12}^2 \\ & + 2F_{1122}\sigma_{11}\sigma_{22} + 2F_{1112}\sigma_{11}\tau_{12} + 2F_{2212}\sigma_{22}\tau_{12})^{1/2} \end{aligned} \quad (2.3)$$

Note that in Equation 2.3  $\sigma$  and  $\tau$  denote normal and shear stresses respectively. Figure 2.1 depicts an anisotropic material and all the possible loading directions for reference.

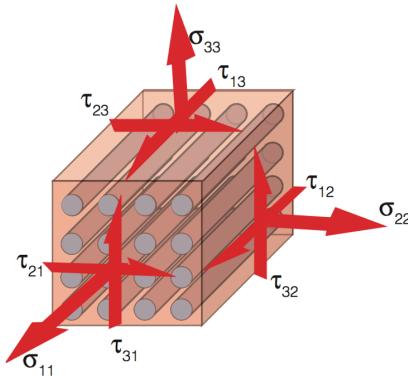


Figure 2.1: Different load directions in an anisotropic material

Per Gol'denblat and Kopnov's design, should the computation of  $f$  in Equation 2.3 be greater or equal to 1, part failure is to be expected. However, to simplify calculations, they deliberately assumed the interaction terms  $F_{1112}$  and  $F_{2212}$  to be zero. This is an important consideration that will come into play when describing the OOC.

Most of the terms in the GKC are obtained through mechanical testing of coupons under pure uniaxial loads in the 1 or 2 direction, or pure shear in the 1-2 plane [21]. In these scenarios,  $f$  will be equal to 1 at failure, and the stress state will be known to the user, allowing some of the unknown tensorial parameters to be easily calculated. Using  $F_{11}$  and  $F_{1111}$  as examples, the process would be as follows:

1. The tensile and compressive strength in the 1-1 direction would be obtained through mechanical testing. These values are named  $X_t$  and  $X_c$  respectively.
2. Under these failure conditions, Equation 2.3 is reduced to the following system of equations:

$$\begin{cases} +1 = F_{11}X_t + (F_{1111}X_t^2)^{1/2} \\ +1 = -F_{11}X_c + (F_{1111}X_c^2)^{1/2} \end{cases}$$

3.  $F_{11}$  and  $F_{1111}$  can be obtained, yielding  $F_{11} = \frac{1}{2}(\frac{1}{X_t} - \frac{1}{X_c})$  and  $F_{1111} = \frac{1}{4}(\frac{1}{X_t} + \frac{1}{X_c})^2$ .

The only exception to this procedure would be the  $F_{1122}$  component, which requires measuring the positive and negative shear strengths of a coupon with reinforcement oriented in  $45^\circ$ . These parameters are named  $S_{45p}$  and  $S_{45n}$  respectively. Table 2.1 summarizes the nomenclature used for the strength parameters required to completely populate the failure function of the GKC. Table 2.2 summarizes all the tensorial component calculations.

Table 2.1: Nomenclature of the GKC parameters

| Parameter | Description                                     |
|-----------|---|
| $X_t$     | Tensile strength in the 1-1 direction           |
| $X_c$     | Compressive strength in the 1-1 direction       |
| $Y_t$     | Tensile strength in the 2-2 direction           |
| $Y_c$     | Compressive strength in the 2-2 direction       |
| $S_{45p}$ | Positive shear strength for $45^\circ$ specimen |
| $S_{45n}$ | Negative shear strength for $45^\circ$ specimen |
| $S$       | Shear strength in the 1-2 plane                 |

Table 2.2: Tensorial components of the GKC

| Component  | Formula  |
|------------|--|
| $F_{11}$   | $\frac{1}{2}(\frac{1}{X_t} - \frac{1}{X_c})$   |
| $F_{1111}$ | $\frac{1}{4}(\frac{1}{X_t} + \frac{1}{X_c})^2$   |
| $F_{22}$   | $\frac{1}{2}(\frac{1}{Y_t} - \frac{1}{Y_c})$   |
| $F_{2222}$ | $\frac{1}{4}(\frac{1}{Y_t} + \frac{1}{Y_c})^2$   |
| $F_{12}$   | 0  |
| $F_{1212}$ | $\frac{1}{S^2}$  |
| $F_{1122}$ | $\frac{1}{8}[(\frac{1}{X_t} + \frac{1}{X_c})^2 + (\frac{1}{Y_t} + \frac{1}{Y_c})^2 - (\frac{1}{S_{45p}} + \frac{1}{S_{45n}})^2]$ |

## 2.2 The Osswald-Osswald Criterion

One of the assumptions made in the GKC is that the components  $F_{1112}$  and  $F_{2212}$  in Equation 2.3 are null. While this simplifies the model, it essentially neglects any interactions between axial loads and shear stresses, namely, the  $\sigma_{11}$  -  $\tau_{12}$  and  $\sigma_{22}$  -  $\tau_{12}$  interactions. Practically, this causes the failure surface developed through the GKC to under predict shear strengthening effects exhibited by anisotropic materials loaded in combined axial and shear conditions. Figure 2.2 shows a comparison between experimental data obtained from the first World Wide Failure Exercise (WWFE-I) and the GKC envelope developed for this material. In this example, a unidirectional glass reinforced epoxy composite was tested in multiple loading conditions in the  $\sigma_{22}$  and  $\tau_{12}$  stress plane. White circles indicate average values for pure uniaxial or shear scenarios. Note how on the first quadrant, the model follows the data closely. However, in the second quadrant, the criterion is relatively conservative, failing to capture the strengthening that occurs when loading the material in compression and shear.



Figure 2.2: GKC failure surface developed using data from the WWFE-1 [21]

The Osswald-Osswald Criterion (OOC) attempts to overcome these limitations by building upon the GKC. For the OOC, the interaction effects are captured through the use of the slopes of the failure surface at any of the points where the engineering strength is known within a particular stress plane [21]. In this failure scenario, the stress state of the coupon is known and easy to implement into Equation 2.3, where  $f = 1$ . The resulting expression can then be derived with respect to one of the stresses, allowing for the interaction components to be calculated. This is better illustrated through an example. Assuming the component of interest is  $F_{2212}$ , the procedure to calculate it through the OOC would be as follows:

1. Obtain all the tensorial components possible through the GKC.
2. Using the  $\sigma_{22}-\tau_{12}$  stress plane, derive Equation 2.3 as a function of  $\sigma_{22}$  in the scenario of failure under pure shear ( $f = 1$ ). This yields the expression:

$$0 = F_{22} + [F_{1212}S(\frac{d\tau_{12}}{d\sigma_{22}}) + F_{2212}S] \quad (2.4)$$

where  $\frac{d\tau_{12}}{d\sigma_{22}}$  is the slope of the graph at failure under shear. This term is named  $\mu^{2212}$  in the OOC and can be obtained by performing combined loading tests. Refer to Figure 2.3 for a visual representation.

3. Rearranging Equation 2.4 to solve for the unknown  $F_{2212}$  gives the following expression:

$$F_{2212} = -\frac{F_{22}}{S} - F_{1212}\mu^{2212} \quad (2.5)$$

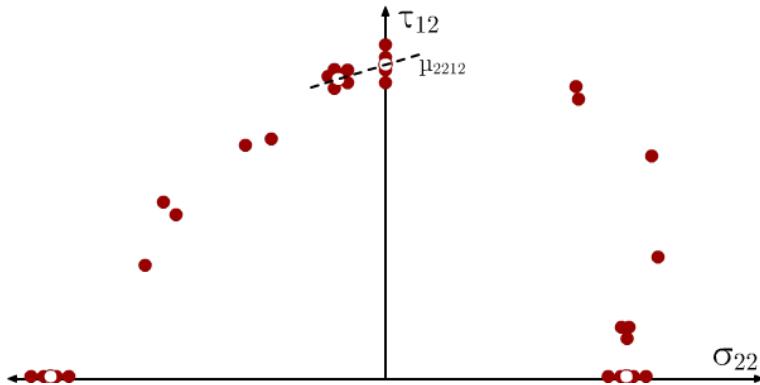


Figure 2.3:  $\mu^{2212}$  parameter in the  $\tau_{12}$  -  $\sigma_{22}$  plane

A similar procedure can be followed for any  $\sigma_{ii}-\tau_{ij}$  interaction, or even any  $\sigma_{ii}-\sigma_{jj}$  components. For this last scenario, the user has four potential choices of slopes to determine the tensorial component of interest. In the OOC, any slope obtained from a  $\sigma_{ii}-\sigma_{jj}$  stress plane is named  $\lambda^{iijj}$ , as opposed to  $\mu^{iiji}$  for slopes in a  $\sigma_{ii}-\tau_{ij}$  reference. A schematic of all possible  $\lambda^{iijj}$  is shown in Figure 2.4, while Table 2.3 summarizes all the possible interaction factors available through the OOC, where  $\tau_{ij}^u$  denotes ultimate shear strength in a particular shear plane.



Figure 2.4:  $\lambda^{iijj}$  parameters in a generic  $\sigma_{ii}$  -  $\sigma_{jj}$  stress plane

Table 2.3: Interaction components attainable through the OOC [21]

| Component                             | Formula  |
|---------------------------------------|--|
| $F_{iiji}$                            | $-\frac{F_{ii}}{\tau_{ij}^u} - F_{ijij}\mu^{iiji}$                                     |
| $F_{iiji}$ through $\lambda_1^{iijj}$ | $-\frac{(F_{ii} + F_{jj}\lambda_1^{iijj})F_{iiji}^{1/2} + F_{iiji}}{\lambda_1^{iijj}}$ |
| $F_{iiji}$ through $\lambda_2^{iijj}$ | $-(F_{ii} + F_{jj}\lambda_2^{iijj})F_{jiji}^{1/2} - F_{jiji}\lambda_2^{iijj}$          |
| $F_{iiji}$ through $\lambda_3^{iijj}$ | $\frac{(F_{ii} + F_{jj}\lambda_3^{iijj})F_{iiji}^{1/2} - F_{iiji}}{\lambda_3^{iijj}}$  |
| $F_{iiji}$ through $\lambda_4^{iijj}$ | $(F_{ii} + F_{jj}\lambda_4^{iijj})F_{jiji}^{1/2} - F_{jiji}\lambda_4^{iijj}$           |

Applying the OOC to the data shown in Figure 2.2 demonstrates how the failure surface developed through this new criterion better reflects the failure behavior than its GKC counterpart. A comparison between the two can be seen in Figure 2.5. Note how the OOC envelope captures the shear strengthening effect that the GKC under-predicts.

The OOC offers a way of capturing in a more accurate manner the different failure modes of parts produced through AM technologies. As an example, the model has been

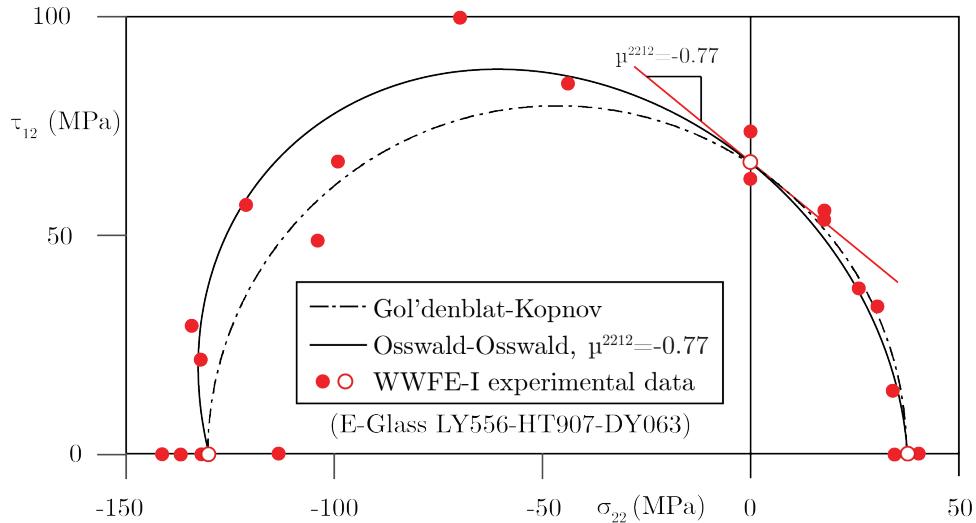


Figure 2.5: Comparison of GKC and OOC failure envelopes [21]

successfully implemented by Obst *et al.* in 2018 for SLS manufactured parts produced with PA12 [25, 26]. Their results show how the model was able to capture the  $\tau_{12}$ - $\sigma_{22}$  and  $\sigma_{11}$ - $\sigma_{22}$  interactions. The failure surface obtained is shown in Figure 2.6.

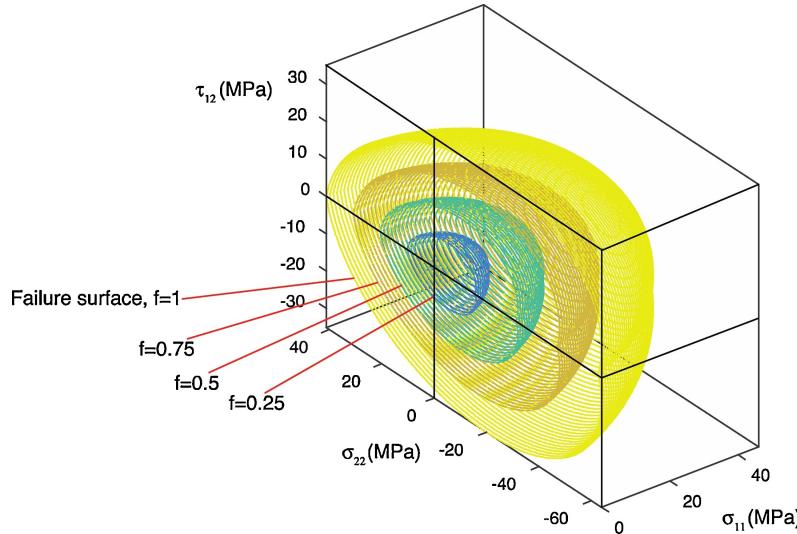


Figure 2.6: Failure surface for SLS developed through the OOC [25]

This work will apply the OOC to FFF, in the hopes that it becomes a tool for safely designing parts intended to be manufactured through this process. Chapter 3 will detail the material and experimental methods used to achieve this goal.

# 3 Experimental Methods

This chapter explains all the experimental procedures followed to obtain all the parameters necessary to compute the failure surface described by the OOC. This includes detailed information pertaining to the material used, sample geometries and toolpath considerations, as well as describing the equipment used to produce and test the coupons.

## 3.1 Material

The first step of the experimental work involved development of a custom thermoplastic filament for the FFF process. The reasoning behind this decision was two-fold. First, the use of an off the shelf, commercial thermoplastic filament generally does not guarantee that two different spools were produced under the same processing conditions—or even using the same parent material. Secondly, the results from Koch *et al.* [6] show that fluctuations in the filament diameter have an impact in the mechanical properties of FFF parts due to improper volumetric output at the nozzle.

The Cycloc® MG94 material produced by SABIC was chosen for this work. This is an Acrylonitrile Butadiene Styrene (ABS) based material traditionally used for injection molding thin walled parts, as well as extrusion of FFF filament. With a reported Melt Flow Index of 11.7 g/10 min, it is an ideal material for both the FFF and extrusion processes. The MG94 was extruded in a setup that consisted of a single screw extruder (Extrudex EDN 45X30D, Germany) with 45 mm screw diameter and L/D ratio of 30D. The hot melt was extruded at 205 °C through a circular die with a 5.8 mm diameter, and then guided through a pre-skinner into a vacuum-assisted, heated water bath (Conair, USA) to cool the extrudate whilst minimizing void formation. The solidified filament was then passed through a 3-axis laser micrometer (LaserLinc, USA) and a belt puller (Conair, USA) configured in a control loop. The dimensions of the filament were controlled by automatically adjusting the speed of the puller if the readings from the micrometer were out of specification—in this case a diameter of 2.85 mm with a tolerance of  $\pm 0.02$  mm. Finally, the product was wound onto spools using a filament winder. A schematic of the extrusion setup can be seen in Figure 3.1. Prior to any usage in a printer, the filament was dried in a silo (Novatec, USA) at 82 °C for 3 hours.



Figure 3.1: Extrusion line setup

## 3.2 Sample Preparation

As explained in Chapter 2, the OOC requires mechanical tests to obtain the multiple tensorial components of the mathematical function that describes part failure. Table 3.1 summarizes the tests required.

Table 3.1: Mechanical tests required for the OOC

| Mechanical Test  | OOC parameters obtained  |
|------------------|--------------------------|
| Tensile          | $X_t, Y_t$               |
| Compressive      | $X_c, Y_c$               |
| Torsion          | $S, S_{45p}, S_{45n}$    |
| Combined loading | $\mu^{1112}, \mu^{2212}$ |

Given that at the moment of this writing AM testing standards are still in development, custom specimen geometries had to be created in order to test certain loading conditions. Additionally, some bead orientations required are difficult or impossible to reproduce through 2.5-D FFF. Therefore, the use of a customized robotic, off-axis FFF printer was necessary. This section will detail the coupon geometries used to perform all the mechanical tests described in Table 3.1, as well as the printing equipment, and toolpath considerations necessary to properly arrange the printed beads in the desired orientation for each condition.

### 3.2.1 Printing Equipment

Specimens were produced using either a commercially available desktop FFF printer (Lulzbot TAZ5, USA), or a customized 6-axis robotic printing solution whenever the bead orientation was hard to achieve using a 2.5-D machine. The robotic printer, developed in the Polymer Engineering Center and nicknamed *Otto* [4], was based on a 6-axis robot (ABB IRB-120, Switzerland) and fitted with a stationary printhead mounted on an aluminum frame, chosen to be the same extruder from the traditional printer (LulzBot TAZ Single Extruder Tool Head v2, 0.5 mm nozzle, USA) to minimize machine influence on the results. The printing equipment can be seen in Figure 3.2.

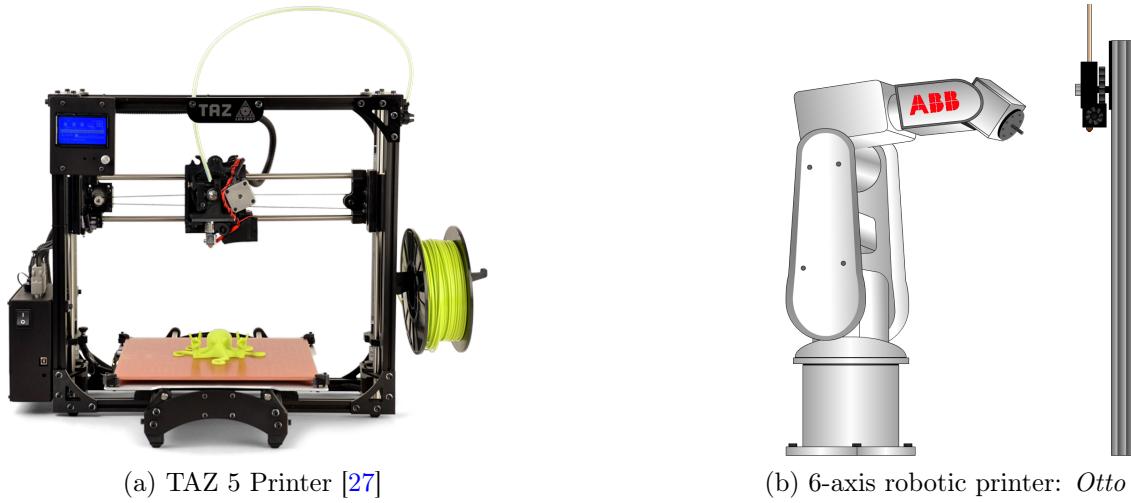


Figure 3.2: Printing equipment

The 6-axis robotic printer's layout is optimized to produce objects of cylindrical nature. A specialized base plate is attached to the sixth axis of the robot, where a threaded rod allows the attachment of disposable plastic cylinders that acts as a build surface. A cylindrical core can then be built by *Otto*, upon which beads in any orientation can be deposited after the robot reorients its joints. Refer to Figure 3.3 for a representation of the process. The left side shows the core being produced atop the disposable build surface, followed by the right hand side, where beads are being laid upon the core in a  $45^\circ$  angle.

Figure 3.3: Print process in *Otto*

Since printing parameters in FFF are known to impact in varying degrees the mechanical performance of objects manufactured through this technology, a conscious effort was made to keep as many processing parameters as possible constant. Table 3.2 summarizes these values.

Table 3.2: Constant printing parameters

| Printing Parameter           | Value                 |
|------------------------------|-----------------------|
| Nozzle temperature           | 220°C                 |
| Bed temperature <sup>a</sup> | 100°C                 |
| Printing Speed               | 2000 $\frac{mm}{min}$ |
| Layer height                 | 0.2 mm                |
| Path width                   | 0.5 mm                |
| Extrusion Factor             | 1                     |

Extrusion Factor (EF) refers to a ratio of the area occupied by the cross section of a bead ( $A_{bead}$ ) divided by the product of the bead width ( $W_{bead}$ ) with the layer height ( $H_{layer}$ ). This factor is used to calculate the length of filament ( $L_{fil}$ ) necessary to produce a bead of known length ( $L_{bead}$ ) as shown in Equations 3.1 and 3.2. The cross sectional area of the filament ( $A_{fil}$ ) is known through the filament diameter used by the printer.

$$EF = \frac{A_{bead}}{W_{bead} \times H_{layer}} \quad (3.1)$$

$$\frac{L_{fil}}{L_{bead}} \times A_{fil} = EF \times W_{bead} \times H_{layer} \quad (3.2)$$

### 3.2.2 Tensile Specimens

Tensile coupons were manufactured on the TAZ5, with bead orientations of 0° and 90° with respect to the load direction in order to test  $X_t$  and  $Y_t$  respectively. The chosen geometry was the ASTM D-638 Type I coupon [28] which can be seen in Figure 3.4. The toolpath required to produce these samples was developed through *SciSlice*, a customized slicing engine created in the Polymer Engineering Center that allows layer-by-layer and part-by-part controls of crucial process parameters —such as the bead orientation—making it ideal for research in the field of FFF [29].

---

<sup>a</sup>Applicable only to prints performed on the traditional FFF printer.

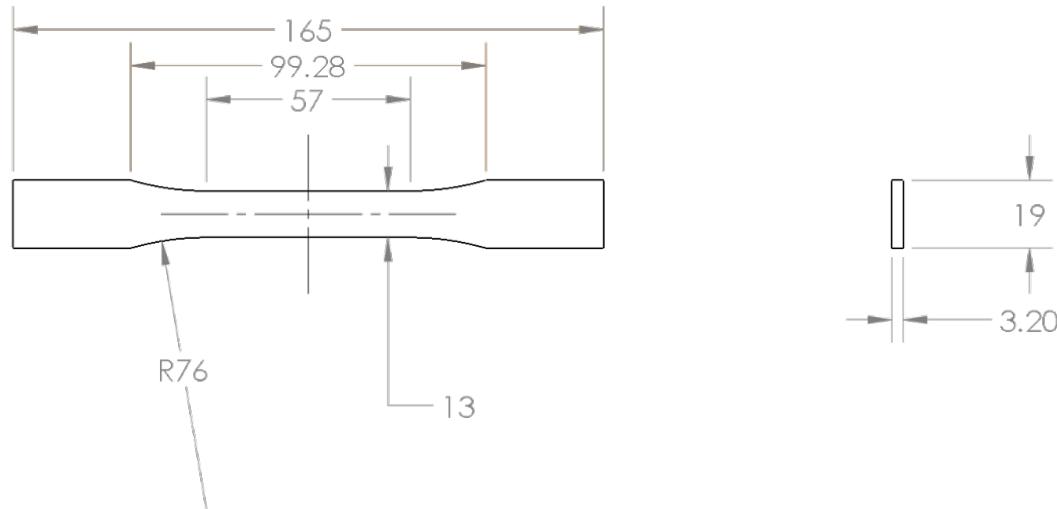


Figure 3.4: ASTM D-638 Type I coupon

In order to minimize stress concentrators due to printing discontinuities, the  $0^\circ$  specimens were printed using 13 perimeters, also known as *shells*. This produced a continuous toolpath with beads oriented in the loading direction at the neck section of the specimen. No shells were added to the  $90^\circ$  samples. Refer to Figure 3.5 for a visual representation of the coupons.



Figure 3.5: Toolpath considerations for the tensile coupons

### 3.2.3 Compressive Specimens

Compressive samples were designed with a tubular cross section. This geometry was chosen to mitigate hydrostatic stresses that could artificially increase the compressive strength of the sample if it were made as a completely solid object. Additionally, the cylindrical geometry allowed production of the  $0^\circ$  samples in *Otto*. The geometry can be seen in Figure 3.6.

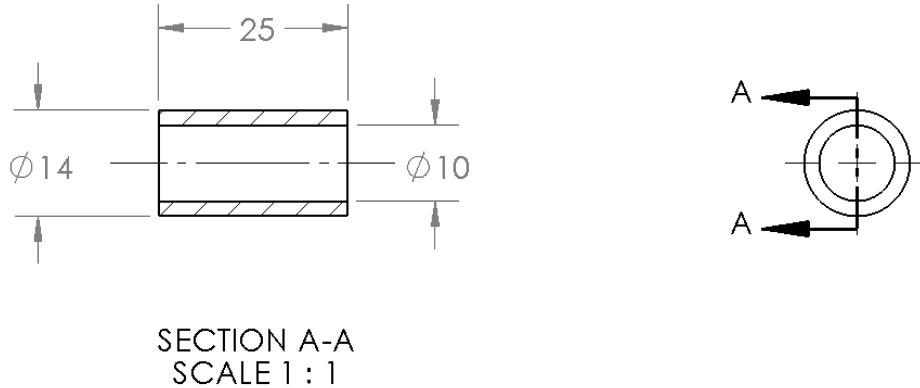


Figure 3.6: Compression specimen geometry

Compression samples with a bead orientation of  $90^\circ$  with respect to the loading direction were produced using the TAZ5 and *SciSlice*. The coupons with a bead orientation of  $0^\circ$  with respect to the loading direction were made using *Otto* and a customized *Python* script that converted process parameters into instructions for the robotic arm through ABB's *RAPID* toolpathing language. A visual representation of the specimens can be seen in Figure 3.7.

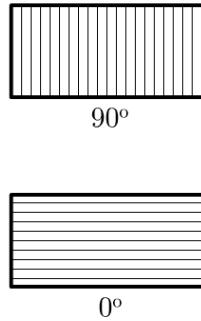


Figure 3.7: Representation of compression samples

### 3.2.4 Torsion and Combined Loading Specimens

The geometry of the torsion specimens was loosely based in the EN ISO 3167, Type A specimens used by Obst *et al.* [25]. The cross sectional area of the testing area was chosen to be the same as for the compression specimens. This geometry was chosen since its tubular arrangement allows easy integration with the 6-axis robotic printer, as well as offering axisymmetry and reduction of the risk of artificially increasing the strength of the sample due to yielding restrictions associated with using a completely

solid coupon [25]. However, due to toolpathing complications, more than one torsion geometry was necessary. Each one is described in detail below.

### 45° torsion samples

To test the  $\sigma_{11}-\sigma_{22}$  interaction, a torsion sample with beads oriented in 45° was necessary. The geometry, which can be seen in Figure 3.8, consists of a specimen of cylindrical nature, with a filleted, widening change in cross sectional area that culminates in a gripping section. In this portion, three flat surfaces are added to ensure proper contact with the grips of the torsion machine.



Figure 3.8:  $S_{45}$  Torsion specimen geometry

The manufacturing of this type of specimen through *Otto* involved laying 10 layers of beads in a 45° angle atop a cylindrical core of 10 mm in diameter and 95 mm in height. Finally, the gripping section is added using beads in alternating ±45° orientations, culminating in a wider area with a diameter of 24 mm. The flat areas had to be machined in post processing by producing cuts of 1.5 mm in depth in the grips, separated by 120°. A visual representation of the finished sample can be seen in Figure 3.9. Note that the disposable build surface (visible on the left side of the figure) becomes a part of the coupon without impacting the gage section of the sample. All toolpath was generated using a customized *Python* script that converted process parameters into instructions for the robotic arm.

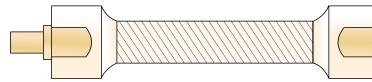


Figure 3.9:  $S_{45}$  Representation of the 45° specimen

### 0° and 90° torsion samples

For the 0° and 90° bead orientations, a specimen redesign was necessary due to tool-pathing problems. Specifically, the length of the 45° torsion specimens proved problematic to print using 0° and 90° bead orientations, each for different reasons. In the case of the 0° orientation, the long and unidirectional travel distance of each bead introduced considerable tool pressure on the core, causing bowing. This was significant enough to cause poor adhesion of the beads towards the end of the specimen. By contrast, production of 90° samples using the original length caused the sixth axis of the robot to heat up considerably and to almost completely maximize its working range.

The torsion specimen was redesigned to have the same cross sectional area as the original 45° torsion samples, but shorter length. An optimal distance was devised through multiple print trials where the length of the specimen was varied between 95 and 50 mm. The chosen geometry is shown below in Figure 3.10.

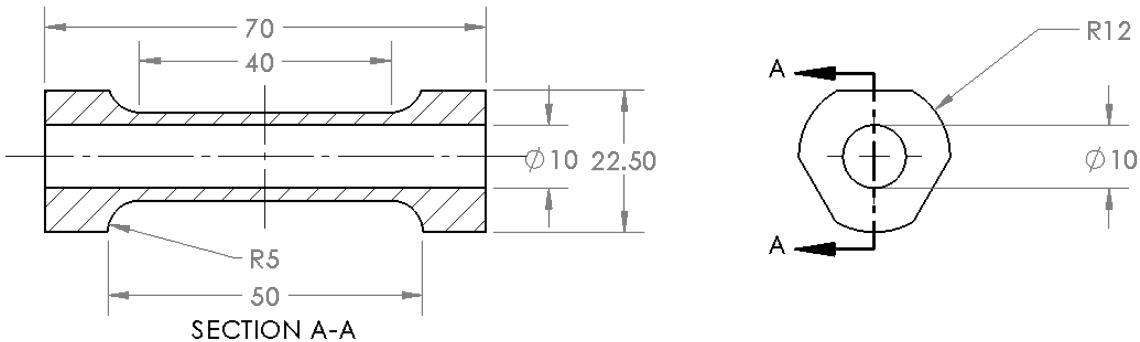


Figure 3.10: Torsion specimen used for 0° and 90° tests

## 3.3 Mechanical Testing

### 3.3.1 Tension and Compression tests

Tensile and compressive tests were performed with an Instron 5967 Dual Column Universal testing machine, using a 30 kN load cell. All data acquisition was handled through the accompanying Instron Bluehill 3 software. A movement speed of 5 mm/min was used to deform the 50 mm gage section of the tensile specimens, while a testing speed of 2.5 mm/min was used to deform the 25 mm of height of the compression samples. These testing speeds ensured a strain rate of  $0.1 \text{ min}^{-1}$  across both specimen types.

The tensile specimens were tested using clamp grips and an extensometer. To protect the samples from excessive gripping force, emery cloth tabs were used, as described

by Mazzei Capote *et al* [2]. Figure 3.11 shows a photograph of the setup. Compressive tests were performed using standard compression platens with the universal testing machine.

### 3.3.2 Torsion and Combined loading tests

All torsion tests were performed using an ADMET eXpert 9618 torsion machine fitted with the MTEST Quattro controller and software suite. The machine sits on a rail system that allows the sample to deform freely during testing. The equipment has two adjustable jaws, with three contact surfaces that holds the sample in place, as shown in Figure 3.11. Of the two jaws, one rotates as controlled by a servomotor, while the other is fixed in terms of angular movement.

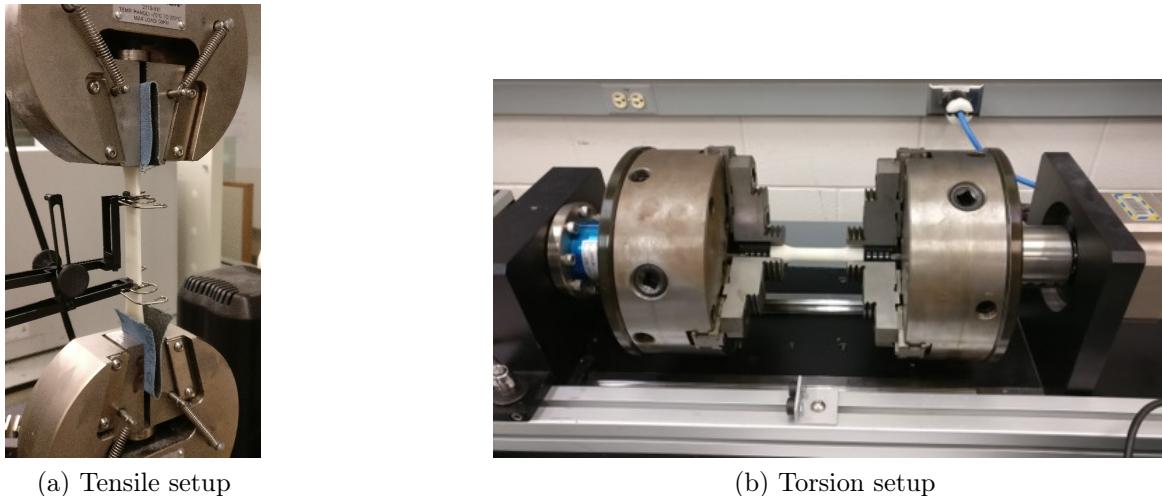


Figure 3.11: Mechanical testing setup

A conscious effort was made to maintain the same strain rate of  $0.1 \text{ min}^{-1}$  used for the tensile and compression tests. The approach used was the same described by Obst *et al.* [25]. The initial assumption is that the shear strain rate ( $\dot{\gamma}$ ) is twice the engineering strain rate ( $\dot{\epsilon}$ ), as shown in Equation 3.3.

$$\dot{\gamma} = 2 \times \dot{\epsilon} \quad (3.3)$$

Figure 3.12 represents a tubular element of outer radius  $\rho$  and length  $L$ , subjected to a torque  $T$ . Using this image as reference, one can see that if point B is fixed in space —as is the case of the stationary jaw of the torsion machine used throughout this work—point A will deform to position A'. The angle of twist  $\Phi$  can then be approximated using the arc length  $AA'$  as  $AA' = \rho \times \Phi$  for small values of shear strain  $\gamma$ . It can also be seen in Figure 3.12 that the relationship  $AA' = L \times \gamma$  also exists. From this, Equation 3.4 can be obtained.

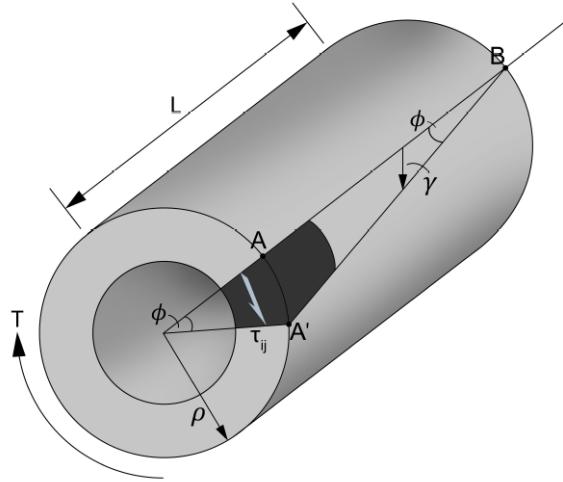


Figure 3.12: Stress and strain caused by torque on a tubular geometry [25]

$$L \times \gamma = \rho \times \Phi \quad (3.4)$$

Finally, after rearranging and deriving with respect to time, Equation 3.5 is obtained.

$$\dot{\Phi} = \frac{L \times \dot{\gamma}}{\rho} \quad (3.5)$$

For this work,  $\dot{\gamma}$  is known to be  $2 \times 0.1 \text{ min}^{-1} = 0.2 \text{ min}^{-1}$ , as dictated by Equation 3.3 and the testing conditions chosen for the tensile and compressive tests. In the case of the  $45^\circ$  specimens, preliminary tests performed in positive torsion showed an average outer radius of 6.91 mm and a mean length of failure propagation of 54 mm. Using these values in Equation 3.5 yields a rotational speed of 1.56 rad/min, or  $89.4^\circ/\text{min}$ . Following a similar procedure for the  $0^\circ$  and  $90^\circ$  samples, the angular speed of the torsion test was calculated, using  $L$  as 40 mm, to be  $63.7^\circ/\text{min}$ .

In the case of combined loading scenarios, the setup was fitted with a pulley system that allowed a weight to pull the sample in a nominal direction, while the torsion machine applied shear stresses. Refer to Figure 3.13 for a visual representation of the compressive and tensile force setup of the torsion machine.

The shear stress from the torsion tests was obtained through Equation 3.6, where  $\tau_{ij}$ ,  $R$ ,  $T$ , and  $J_z$  represent the shear stress in the  $ij$  plane, the outer radius of the test section of the sample, the Torque applied, and the second moment of area respectively.

$$\tau_{ij} = \frac{T \times R}{J_z} \quad (3.6)$$

For this case, the second moment of area is defined as described in Equation 3.7, where  $r$  represents the inner radius of the tubular specimen.

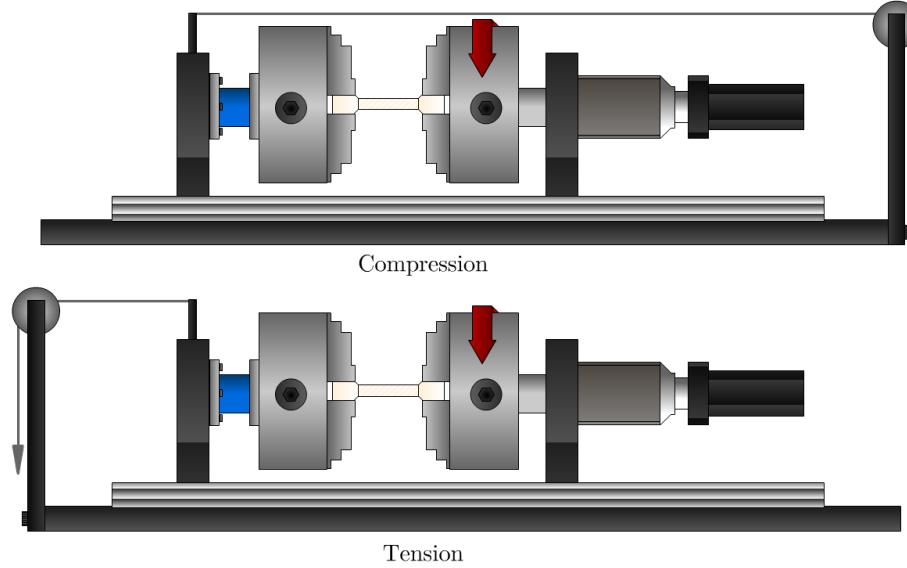


Figure 3.13: Machine setup for combined loading scenarios

$$J_z = \frac{\pi \times (R^4 - r^4)}{2} \quad (3.7)$$

Finally, combining Equations 3.6 and 3.7 yields Equation 3.8

$$\tau_{ij} = \frac{2 \times T \times R}{\pi \times (R^4 - r^4)} \quad (3.8)$$

Chapter 4 will discuss in detail all results that stem from the experimental methods described. Finally, the failure envelope will be calculated using the mathematical relations described in Chapter 2.

## 4 Results

This chapter presents all the results stemming from the experimental procedures described in Chapter 3. Additionally, details are provided regarding proper data processing, and the calculations that lead to the definition of the desired failure surface.

### 4.1 Tensile Tests

Performing tensile tests showed that the values of  $X_t$  and  $Y_t$  were significantly different, in accordance to the literature review presented in Section 1.2.2. An initial number of 20 samples per orientation was produced, however, multiple specimens failed outside the gage section of the coupon and thus, data originating from these coupons was considered invalid and promptly discarded. The valid results are summarized in Table 4.1. Note that  $X_t$  was on average 9.16 MPa higher than  $Y_t$ , a difference of 22.7%.

Table 4.1: Summary of Tensile tests

| Information               | $X_t$ | $Y_t$ |
|---------------------------|-------|-------|
| Average [MPa]             | 40.29 | 31.13 |
| Standard Deviation        | 0.75  | 0.58  |
| Number of samples         | 19    | 12    |
| Lowest measurement [MPa]  | 39.37 | 29.68 |
| Highest measurement [MPa] | 40.90 | 32.08 |

The behavior of both sets of samples was completely different. Coupons used to measure  $X_t$  clearly showed whitening of the gage section, indicating plastic deformation. Specimens used for  $Y_t$  on the other hand generally failed between beads and rarely showed any change in color. Figure 4.1 clearly shows the difference in mechanical behavior. Note how the  $X_t$  specimen shows a ductile failure, as opposed to brittle breakage for the  $Y_t$  sample. Figure 4.2 shows both tested samples side by side.

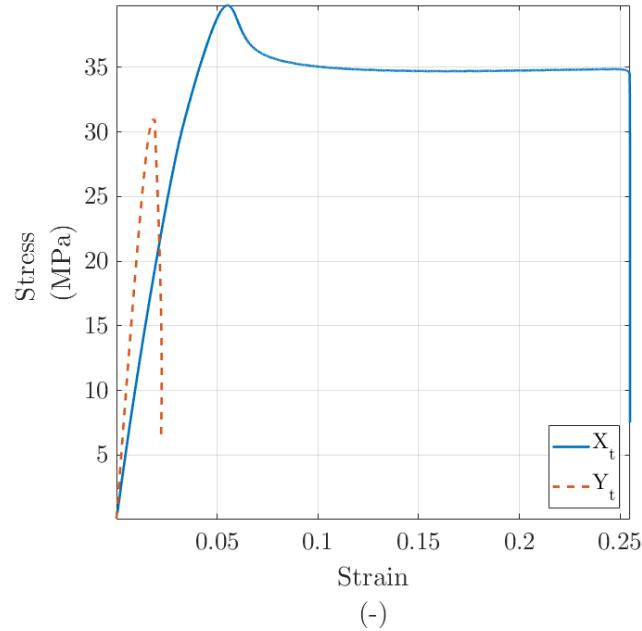
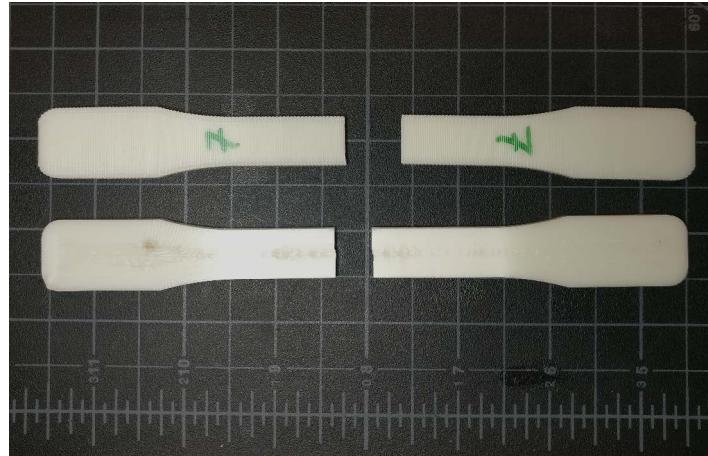


Figure 4.1: Comparison of tensile results

Figure 4.2: Y<sub>t</sub> (top) and X<sub>t</sub> (bottom) tested samples. Scale in inches. Note whitening in gage section for X<sub>t</sub> specimen.

Anderson-Darling tests (ADT) were performed on the results to validate if the data follows a normalized distribution. In the case of X<sub>t</sub>, the ADT yields a p-value of 0.023. Thus, on a 95% confidence interval, a normalized distribution can be discarded. This can be seen graphically in the goodness of fit plot, where data points fail to align with the slope that corresponds to a theoretical Gaussian distribution. By contrast, performing the ADT on the Y<sub>t</sub> samples fails to reject a normalized distribution using

the same confidence interval, offering a p-value of 0.109. The results from ADT can be seen in Figure 4.3.



Figure 4.3: ADT results for tensile data

## 4.2 Compression Tests

For the compression tests, a total of 25 samples were produced for each orientation. However, a number of coupons were discarded due to manufacturing defects. Table 4.2 summarizes the test results.

Table 4.2: Summary of Compression tests

| Information               | $X_c$ | $Y_c$ |
|---------------------------|-------|-------|
| Average [MPa]             | 43.91 | 57.96 |
| Standard Deviation        | 3.23  | 1.81  |
| Number of samples         | 25    | 22    |
| Lowest measurement [MPa]  | 37.95 | 54.93 |
| Highest measurement [MPa] | 48.87 | 61.39 |

Surprisingly, both sets of specimens had different failure behavior. The  $Y_c$  samples displayed pure ductile behavior through testing. A clear maximum stress can be observed at the yield point in a stress-strain graph, and all samples showed localized whitening and deformation along the center of the specimen. By contrast, the  $X_c$  samples showed a considerably lower yield point, and specimens deformed in a way that formed petal-like structures due to contiguous bead delamination. This difference in mechanical behavior can be seen in Figure 4.4, where stress-strain curves for  $X_c$  and  $Y_c$  are compared. Note the erratic behavior of the  $X_t$  sample, caused by delamination of adjacent beads. Figure 4.5 shows  $X_c$  and  $Y_c$  samples side by side post testing, where the different failure behavior becomes evident.

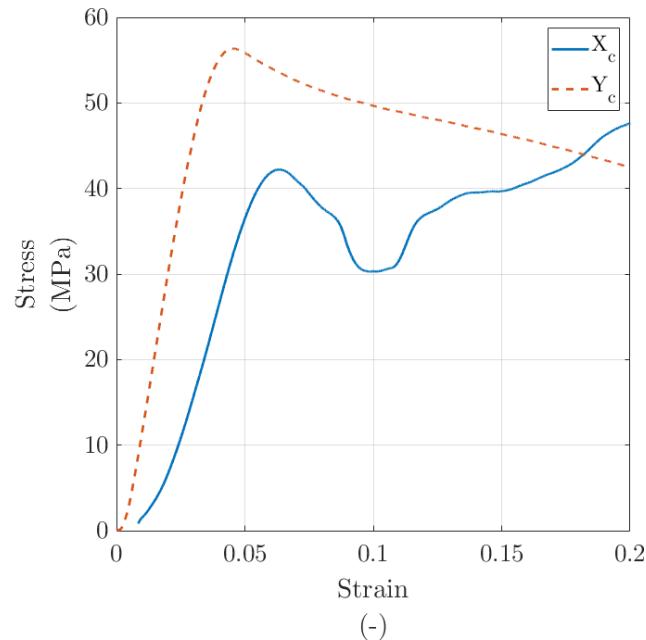


Figure 4.4: Comparison of compression results



Figure 4.5:  $X_c$  (left) and  $Y_c$  (right) tested samples. Scale in inches. Note petal-like structure of  $X_c$  sample.

Performing ADT on the data reveals a p-value of 0.072 for  $Y_c$ , and 0.056 for  $X_c$ , thus, normalized distributions cannot be discarded on a 95% confidence interval.

## 4.3 Torsion Tests

### 4.3.1 45° orientation

A total of 30 samples were produced with a 45° bead orientation, divided evenly for tests in positive and negative shear. As was the case for tensile and compressive tests, a number of specimens had to be discarded due to undesired behavior during testing. A common problem was delamination of the grips, thus requiring the data to be discarded.

Results showed significant difference in behavior depending on the direction of the applied torque. The  $S_{45p}$  samples showed a completely brittle behavior, with fracture occurring between beads, as opposed to ductile failure in the center of the specimen for the  $S_{45n}$  coupons. This resulted in an average difference of 6.3 MPa between both sets of data. Results are summarized in Table 4.3 and a graph comparing the behavior of both sets of samples can be seen in Figure 4.6. Note how the positive shear sample fails at low angles in a completely brittle manner.

Table 4.3: Summary of 45° torsion tests

| Information               | $S_{45p}$ | $S_{45n}$ |
|---------------------------|-----------|-----------|
| Average [MPa]             | 20.80     | 27.13     |
| Standard Deviation        | 2.50      | 0.50      |
| Number of samples         | 9         | 9         |
| Lowest measurement [MPa]  | 17.21     | 26.59     |
| Highest measurement [MPa] | 24.61     | 28.17     |

### 4.3.2 0° orientation

### 4.3.3 90° orientation

## 4.4 Combined Loading Tests

## 4.5 Development of the Failure Surface

The failure surface calculations were performed using MATLAB® code based on previous work by Obst *et al.* [25] and the mathematical relations shown in Chapter 2. This code can be found in Appendix A for reference.

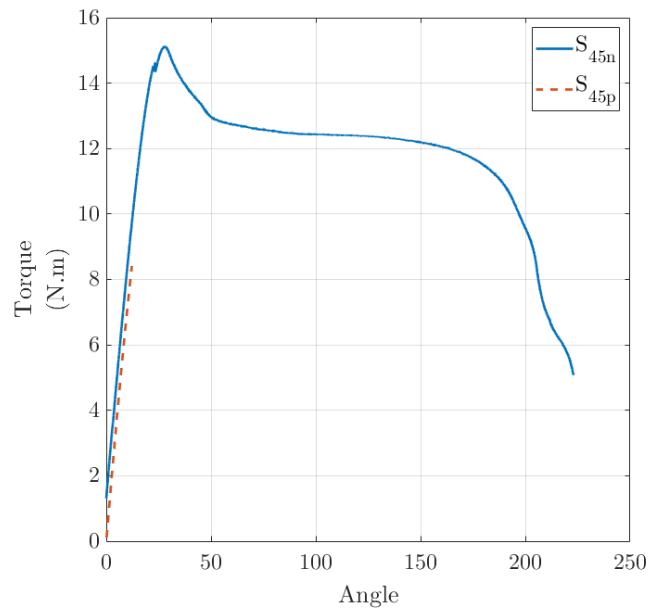


Figure 4.6: Comparison of  $45^\circ$  torsion results

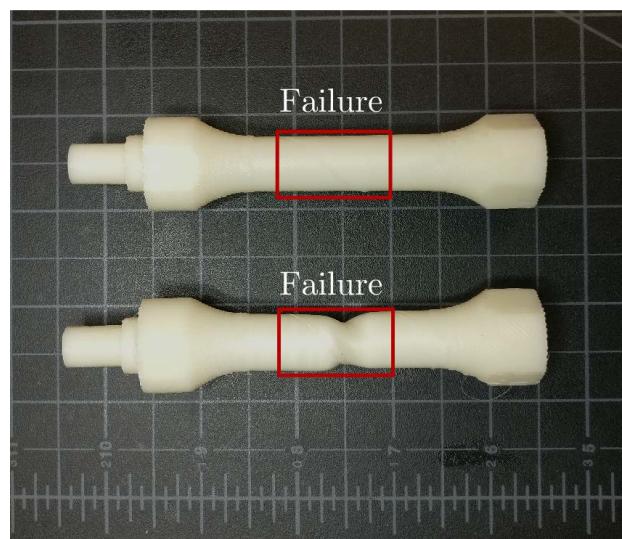


Figure 4.7: Comparison of  $45^\circ$  torsion samples. Positive shear (top) caused bead delamination and brittle failure, whereas negative shear (bottom) produced plastic deformation of the gage section

Calculations were based on the average values obtained from the mechanical tests in order to incorporate a probabilistic approach as developed by Zaitsev, Pashkov and Strelyaeve [30]. In practice, this signifies that the condition  $f = 1$  in Equation 2.3 is equivalent to a probability of part failure equal to 50%.

Starting with the  $\sigma_{11}$ - $\sigma_{22}$  plane, it can be seen that the failure envelope has a notable tilt, agreeing with the difference observed between compressive and tensile strengths in both directions, as well as the difference in behavior for the  $S_{45}$  tests. Refer to Figure 4.8 for a graph showing the calculated failure envelope, including the experimental data for reference.

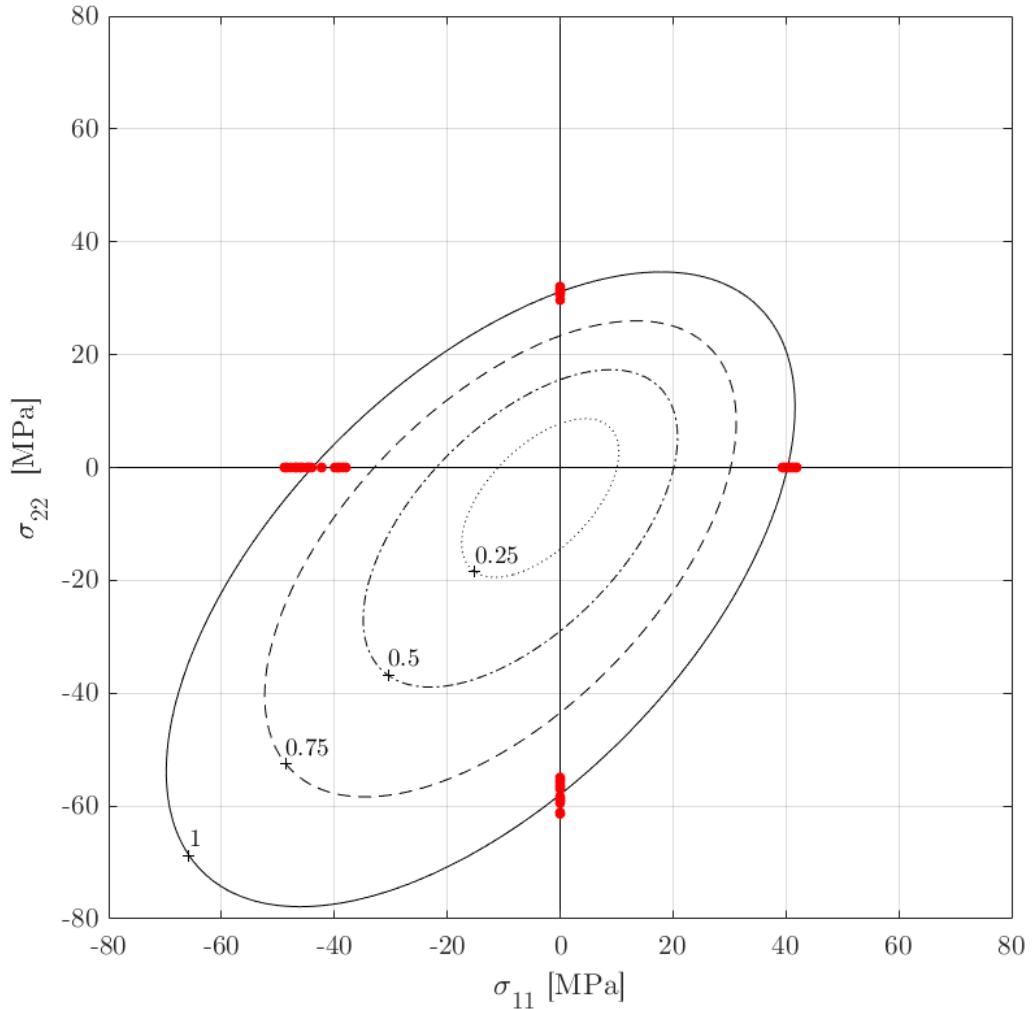


Figure 4.8:  $\sigma_{11}$ - $\sigma_{22}$  plane including data of  $X_t$ ,  $X_c$ ,  $Y_t$  and  $Y_c$  tests. Graph shows calculated  $f$  values of 1, 0.75, 0.50 and 0.25 to illustrate safety factors of 1,  $4/3$ , 2 and 4 respectively

It can be seen from the tilt of the envelope that there's a strong interaction between the transverse and longitudinal stresses. Computing all tensorial components associated with the  $\sigma_{11}$ - $\sigma_{22}$  plane results in Table 4.4. Note how  $F_{1122}$  is negative and in the same order of magnitude as  $F_{1111}$  and  $F_{2222}$ .

Table 4.4: Tensorial components obtained from tests in the  $\sigma_{11}$ - $\sigma_{22}$  plane

| Component  | Value                   |
|------------|-------------------------|
| $F_{11}$   | $1.023 \times 10^{-3}$  |
| $F_{1111}$ | $5.663 \times 10^{-4}$  |
| $F_{22}$   | $7.435 \times 10^{-3}$  |
| $F_{2222}$ | $6.095 \times 10^{-4}$  |
| $F_{1122}$ | $-3.139 \times 10^{-4}$ |

Figure 4.8 indicates that FFF parts produced with the print parameters used should show considerable strengthening when loaded bi-axially in compression. Further experimental work would be of interest to compare bi-axial compression test data to this result.



# Bibliography

- [1] Ian Gibson, David Rosen, and Brent Stucker. *Additive Manufacturing Technologies*. 2nd Ed. Springer, 2015. ISBN: 978-1-4939-2112-6. DOI: [10.1007/978-1-4939-2113-3](https://doi.org/10.1007/978-1-4939-2113-3). URL: <http://link.springer.com/10.1007/978-1-4939-2113-3>.
- [2] G A Mazzei Capote et al. “Towards a Robust Production of FFF End-User Parts with Improved Tensile Properties”. In: *Proceedings of the 28th Annual International Solid Freeform Fabrication Symposium*. Austin, TX, 2017, pp. 507–518. URL: <http://sffsymposium.engr.utexas.edu/sites/default/files/2017/Manuscripts/TowardsaRobustProductionofFFFEndUserParts.pdf>.
- [3] Cole Hartman and Veronica de la Rosa. *Benefits of 3D Printing Vacuum Form Molds*. 2014. URL: <http://studiofathom.com/wp-content/uploads/Vacuum-Forming-White-Paper-F001-5-1-2014.pdf> (visited on 02/06/2018).
- [4] Luke Van Hulle. “Robotic Off-Axis Fused Filament Fabrication”. Master Thesis. University of Wisconsin-Madison, 2017.
- [5] Caspar de Vries. *Volkswagen Autoeuropa: Maximizing production efficiency with 3D printed tools, jigs, and fixtures*. 2017. URL: <https://ultimaker.com/en/stories/43969-volkswagen-autoeuropa-maximizing-production-efficiency-with-3d-printed-tools-jigs-and-fixtures> (visited on 02/02/2018).
- [6] Carsten Koch, Luke Van Hulle, and Natalie Rudolph. “Investigation of mechanical anisotropy of the fused filament fabrication process via customized tool path generation”. In: *Additive Manufacturing* 16 (2017), pp. 138–145. ISSN: 22148604. DOI: [10.1016/j.addma.2017.06.003](https://doi.org/10.1016/j.addma.2017.06.003). URL: <http://dx.doi.org/10.1016/j.addma.2017.06.003>.
- [7] Behzad Rankouhi et al. “Failure Analysis and Mechanical Characterization of 3D Printed ABS With Respect to Layer Thickness and Orientation”. In: *Journal of Failure Analysis and Prevention* 16.3 (2016), pp. 467–481. ISSN: 15477029. DOI: [10.1007/s11668-016-0113-2](https://doi.org/10.1007/s11668-016-0113-2).
- [8] 3D Systems. *Our Story*. 2017. URL: <https://www.3dsystems.com/our-story> (visited on 02/15/2018).

- [9] Stratasys. *About us*. 2017. URL: <http://www.stratasys.com/corporate/about-us> (visited on 02/15/2018).
- [10] ASTM International and International Organization for Standardization. *ISO/ASTM 52900:2015-Standard Terminology for Additive Manufacturing- General Principles-Terminology*. West Conshohocken, 2015. URL: <https://www.astm.org/Standards/ISOASTM52900.htm>.
- [11] 3D Hubs. *What is 3D Printing?- The definitive guide to additive manufacturing*. 2018. URL: <https://www.3dhubs.com/what-is-3d-printing{\#}technologies> (visited on 02/26/2018).
- [12] Martin Baumers et al. “The cost of additive manufacturing: Machine productivity, economies of scale and technology-push”. In: *Technological Forecasting and Social Change* 102 (2016), pp. 193–201. ISSN: 00401625. DOI: [10.1016/j.techfore.2015.02.015](https://dx.doi.org/10.1016/j.techfore.2015.02.015). URL: <http://dx.doi.org/10.1016/j.techfore.2015.02.015>.
- [13] Brett P. Conner et al. “Making sense of 3-D printing: Creating a map of additive manufacturing products and services”. In: *Additive Manufacturing* 1 (2014), pp. 64–76. ISSN: 22148604. DOI: [10.1016/j.addma.2014.08.005](https://dx.doi.org/10.1016/j.addma.2014.08.005). URL: <http://dx.doi.org/10.1016/j.addma.2014.08.005>.
- [14] Barry Berman. “3-D printing: The new industrial revolution”. In: *Business Horizons* 55.2 (2012), pp. 155–162. ISSN: 00076813. DOI: [10.1016/j.bushor.2011.11.003](https://dx.doi.org/10.1016/j.bushor.2011.11.003). URL: <http://dx.doi.org/10.1016/j.bushor.2011.11.003>.
- [15] GE Additive. *GE Additive Manufacturing in Alabama: The Future Is Now*. 2016. URL: <https://www.ge.com/additive/press-releases/ge-additive-manufacturing-alabama-future-now> (visited on 03/01/2018).
- [16] New Balance. *The Future of Running is Here*. 2016. URL: <https://www.newbalance.com/article?id=4041> (visited on 03/07/2018).
- [17] Michelle Matisons. *Futurecraft 3D: Adidas Says It Will Turn to 3D Printed Shoes in Near Future*. 2015. URL: <https://3dprint.com/99620/futurecraft-3d-adidas-shoes/> (visited on 03/07/2018).
- [18] Sarah Saunders. *adidas and Carbon Announce Partially 3D Printed AlphaEDGE 4D LTD Shoe*. 2018. URL: <https://www.3dprint.com/203543/adidas-carbon-alphaedge-4d-ltd/> (visited on 03/07/2018).
- [19] Sung Hoon Ahn et al. “Anisotropic material properties of fused deposition modeling ABS”. In: *Rapid Prototyping Journal* 8.4 (2002), pp. 248–257. ISSN: 1355-2546. DOI: [10.1108/13552540210441166](https://doi.org/10.1108/13552540210441166). URL: <http://www.emeraldinsight.com/doi/10.1108/13552540210441166>.
- [20] C. S. Lee et al. “Measurement of anisotropic compressive strength of rapid prototyping parts”. In: *Journal of Materials Processing Technology* 187-188 (2007), pp. 627–630. ISSN: 09240136. DOI: [10.1016/j.jmatprotec.2006.11.095](https://doi.org/10.1016/j.jmatprotec.2006.11.095).

- [21] Paul V. Osswald and Tim A. Osswald. "A strength tensor based failure criterion with stress interactions". In: *Polymer Composites* (2017). ISSN: 15480569. DOI: [10.1002/pc.24275](https://doi.org/10.1002/pc.24275). arXiv: [1206.4529](https://arxiv.org/abs/1206.4529). URL: <http://onlinelibrary.wiley.com/doi/10.1002/pc.24275/full><http://doi.wiley.com/10.1002/pc.24275>.
- [22] Paul Osswald. "Comparison of Failure Criteria of Fiber Reinforced Polymer Composites". Term Thesis. Technische Universität München, 2015.
- [23] C.T. Sun, B.J. Quinn, and J. Tao. "Comparative Evaluation of Failure Analysis Methods for Composite Laminates." In: *U.S. Department of Transportation - May* (1996), p. 133. URL: <http://trid.trb.org/view.aspx?id=523207>.
- [24] I.I Gol'denblat and V.A. Kopnov. "Strength of Glass-Reinforced Plastics in the Complex Stress State". In: *Mekhanika Polimerov* 1.2 (1965), pp. 70–78.
- [25] P. Obst et al. "Failure criterion for PA12 SLS additive manufactured parts". In: *Additive Manufacturing* 21.March (2018), pp. 619–627. ISSN: 22148604. DOI: [10.1016/j.addma.2018.04.008](https://doi.org/10.1016/j.addma.2018.04.008). URL: <http://linkinghub.elsevier.com/retrieve/pii/S221486041830099X>.
- [26] P. Obst et al. "FAILURE CRITERION FOR SLS ADDITIVE MANUFACTURED PARTS". In: *SPE ANTEC® Anaheim 2017*. Anaheim, 2017, pp. 49–54.
- [27] Lulzbot. *TAZ 5*. 2018. URL: <https://www.lulzbot.com/store/printers/lulzbot-taz-5>.
- [28] ASTM International. "Standard test method for tensile properties of plastics". In: *ASTM International* 08 (2014), pp. 46–58. DOI: [10.1520/D0638-14.1](https://doi.org/10.1520/D0638-14.1). URL: [http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Standard+Test+Method+for+Tensile+Properties+of+Plastics%20](https://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Standard+Test+Method+for+Tensile+Properties+of+Plastics%20).
- [29] Luke Van Hulle. *SciSlice- The Scientific Slicer*. 2017. URL: <https://github.com/VanHulleOne/SciSlice>.
- [30] G.P. Zaitsev, V.A. Pashkov, and V.S. Strelyaeve. "Probabilistic Evaluation of the Criteria of Static Strength of Reinforced Plastics". In: *Problemy Prochnosti* 8 (1975), pp. 3–9.

# A Failure Surface Code

```

1 %
2 % FFF failure surface
3 % Developed by Gerardo A. Mazzei Capote
4 % Based on findings by T. Osswald, P.V. Osswald and P. Obst
5 % March 2018
6 %
7 %Failure Criterion proposed by P.V. Osswald and T.A. Osswald.
8 %For more information, refer to:
9 %Journal of Polymer Composites, DOI 10.1002/pc.24275
10 %The notation here follows the same notation used in this paper
11 %
12 classdef FailSurf
13     methods (Static)
14         function [F_11,F_1111,F_22,F_2222,F_12,F_1212,F_1122,F_1112,F_2212]=tens(Xt,Xc,Yt,Yc,S,S45_p,S45_n,
15             mu1112,mu2212,lambda1122) %Tensorial components calculation
16         F_1111=0.25*(1/Xt+1/Xc)^2;
17         F_2222=0.25*(1/Yt+1/Yc)^2;
18         F_1212=(1/S)^2;
19         F_11=0.5*(1/Xt-1/Xc);
20         F_22=0.5*(1/Yt-1/Yc);
21         F_12=0;
22         %Flexibility regarding which slope to use:
23         disp('0— Goldenblat-Kopnov, 1— Positive x axis, 2— Positive y axis, 3— Negative x axis, 4—
24             Negative y axis')
25         reply = input('Please enter which slope option corresponds to lambda: ');
26         if reply == 0
27             F_1122=0.125*((1/Xt+1/Xc)^2+(1/Yt+1/Yc)^2-(1/S45_p+1/S45_n)^2); %Per Gol'denblat— Kopnov
28         elseif reply == 1
29             F_1122=-((F_11+F_22*lambda1122)*F_1111^0.5+F_1111)/lambda1122; %Slope at Intercept of
30                 Positive x axis
31         elseif reply == 2
32             F_1122=-(F_11-F_22*lambda1122)*F_2222^0.5-F_2222*lambda1122; %Slope at Intercept of Positive
33                 y axis
34         elseif reply == 3
35             F_1122=((F_11+F_22*lambda1122)*F_1111^0.5-F_1111)/lambda1122; %Slope at Intercept of
36                 Negative x axis
37         elseif reply == 4
38             F_1122=(F_11+F_22*lambda1122)*F_2222^0.5-F_2222*lambda1122; %Slope at Intercept of Negative
39                 y axis
40         else
41             error('Input must be one of the options listed above. Execution ended.')
42         end
43         F_1112=-(F_11/S)-F_1212*mu1112;
44         F_2212=-(F_22/S)-F_1212*mu2212;
45     end
46     function f=criteria(s1,s2,t12,F_11,F_1111,F_22,F_2222,F_12,F_1212,F_1122,F_1112,F_2212) %Evaluating
47         the failure criteria for stress inputs
48         %s1: sigma11, s2: sigma22, t12: tau12
49         f=F_11*s1+F_22*s2+F_12*t12+...

```

```

44             (F_1111*s1.^2+F_2222*s2.^2+2*F_1122*s1.*s2+F_1212*t12.^2+2*F_2212*s2.*abs(t12)+2*F_1112*s1.*
45             abs(t12)).^0.5;
46     for x=1:size(f) %Filter any results where f is greater than 1
47         if f(x)>1
48             f(x)=1;
49         end
50     end
51 %%%
52 function g=graph(s1,s2,t12,f,s) %Used to plot the resulting failure surface
53 %s: number of equally spaced failure surfaces from 0 to 1
54 %e.g. s=5 will generate 4 surfaces: 0.25, 0.5, 0.75 and 1
55 if (nargin ==4)
56     s= 5;
57     disp('Using f from 0 to 1 in increments of 0.25 by default')
58 end
59 cvals = linspace(0,1,s);
60 Sx = []; Sy = []; Sz = -70:70;
61 g=figure(1); %Create new figure
62 contourslice(s1,s2,t12,f,Sx,Sy,Sz,cvals);
63 view(3); %View angle
64 axis tight; %Set axes
65 daspect([1,1,1]); %data aspect ratio of 1:1:1
66 box on %encloses all plot
67 set(gca,'FontSize',14) %Sets the font size for axes
68 hold on
69 xlabel('\sigma_{11} [MPa]', 'fontsize',16) % x-axis label
70 ylabel('\sigma_{22} [MPa]', 'fontsize',16) % y-axis label
71 zlabel('\tau_{12} [MPa]', 'fontsize',16) % z-axis label
72 %Allow user to save graph
73 reply1 = input('Would you like to save this graph? [Y/N]: ', 's');
74 if isempty(reply1)
75     reply1 = 'N';
76     disp('No input. Using "N" as default')
77 end
78 if (reply1=='Y')
79     disp('Available options: -dpdf, -depsc, -dpng, -djpeg')
80     reply2= input('Select file type: ', 's');
81     if isempty(reply2)
82         reply2 = '-djpeg';
83         disp('No input. Graph saved as jpeg by default.')
84     end
85     reply3= input('Choose file name : ', 's');
86     if isempty(reply3)
87         reply3 = 'fig1';
88         disp('No input. Graph named "fig1" by defalut.')
89     end
90     print(reply3,reply2) %Saves plot in selected file type on the current directory
91 elseif (reply1=='N')
92     disp('Graph not saved.')
93 else
94     error('Input must be one of the options listed above. Execution ended.')
95 end
96 end
97 end
98 end

```