Lab 8: Ballistic Pendulum
San Diego State University
Department of Physics
Physics 182A/195L

TA:	
Lab partner 1:	
Lab partner 2:	
Date:	
Score:	

Theory

A ballistic pendulum is a simple system used to measure the speed of a projectile. A projectile is launched with some initial speed, and then collides with the mass at the end of a simple pendulum and is captured. See the left side of Figure 1. The energy left over from this collision causes the pendulum to swing upward until all of its kinetic energy is converted into potential energy. See the right side of Figure 1.

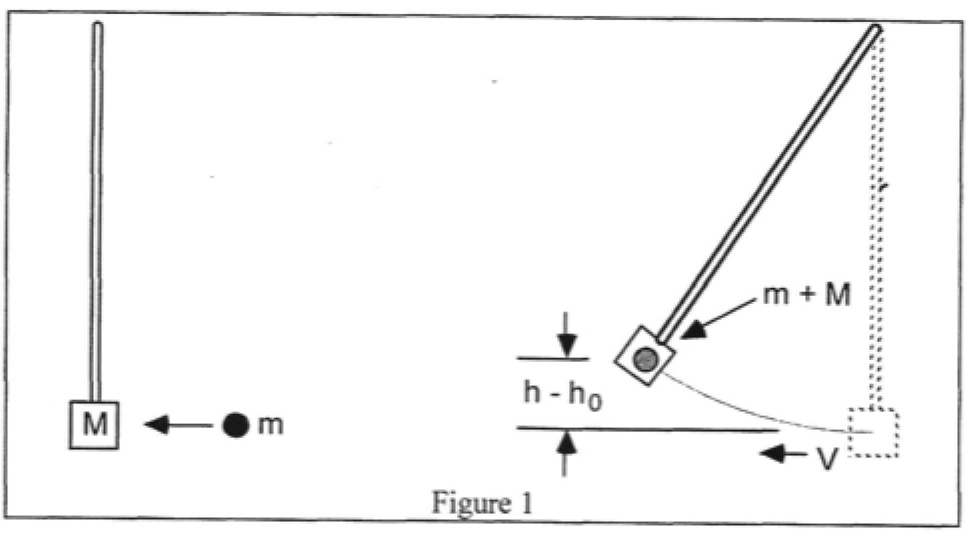


Figure 1: Schematic for the Ballistic Pendulum

In this lab, we will measure the maximum height h that the pendulum mass swings to, and using that information, calculate the speed v_{bi} that the ball had before it collided with the pendulum. You can probably imagine that increasing the speed of the ball will increase how far the pendulum will swing. In the next section we will derive a formula that tells us exactly how these two things are related.

The Collision: conservation of momentum

First let's see what conservation of momentum can tell about the initial speed of the ball.

Let's label the mass of the projectile (a metal ball) as m_b, and the mass of the pendulum m_p. Before the collision, the ball is traveling with some speed v_{bi}. Before the collision, the projectile is traveling with some speed v_{bi}. The pendulum mass is at rest, so its speed is zero, $v_{pi} = 0$. The law of conservation of momentum relates the speeds of the masses before and after the collision:

$$m_b v_{bi} + m_p v_{pi} = m_b v_{bf} + m_p v_{pf}$$

The experimental setup is designed so that the projectile is captured by the pendulum after the collision. Recall from the conservation of momentum lab that this means the collision is perfectly inelastic, and that $v_{bf} = v_{pf}$. Plugging in $v_{pi} = 0$ and $v_{bf} = v_{pf}$, we can write a simplified equation for our system:

$$m_b v_{bi} + m_p(0) = (m_b + m_p) v_{bf}$$

Or, in other words, conservation of momentum relates the speed of the ball before the collision v_{bi} (this is what we want to find), to its speed after the collision v_{bf} :

$$v_{bi} = \frac{(m_b + m_p)}{m_b} v_{bf}.$$

If we can determine v_{bf}, the speed of the ball (and pendulum) right after the collision, then we can find the initial speed of the ball.

The Swing: conservation of energy

Immediately after the collision, the ball is inside of the pendulum, and the pendulum with combined mass $m_p + m_b$ is about to swing upward. Let's see what conservation of energy can tell us about the system. Before the pendulum has swung, it has a kinetic energy ($KE = mv^2/2$) of:

$$KE_{bottom} = \frac{1}{2}(m_b + m_p) v_{bf}^2.$$

If the pendulum is starting from a height of 0, then it has a potential energy ($PE = mgh$) of:

$$PE_{bottom} = 0.$$

At the point where the pendulum swings to its maximum height, let's call that h, the pendulum will momentarily be at rest (just like at the highest point of projectile motion), and so

$$KE_{top} = 0.$$

At that same instant, the pendulum will have a potential energy of:

$$PE_{top} = (m_b + m_p)gh.$$

Conservation of energy tells us that

$$KE_{bottom} + PE_{bottom} = KE_{top} + PE_{top}.$$

Plugging in the values we found above, we have

$$\frac{1}{2}(m_b + m_p)v_{bf}^2 = (m_b + m_p)gh$$

If we solve for v_{bf}, we get:

$$v_{bf} = \sqrt{2gh}.$$

This is the speed of the ball, after the collision, in terms of the maximum height h that the pendulum swings to. We are almost done!

Combining the two systems

From conservation of momentum during the collision of the ball with the pendulum, we found the initial speed of the ball must be

$$v_{bi} = \frac{(m_b + m_p)}{m_b}v_{bf}.$$

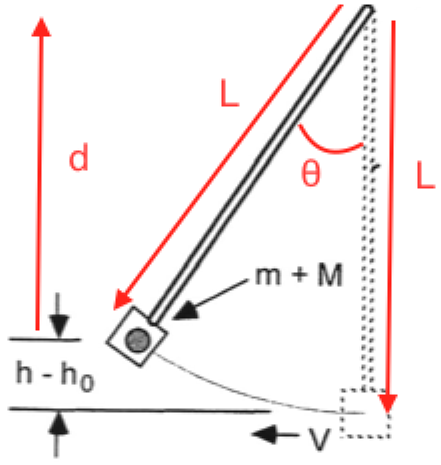
And from conservation of energy during the swing of the pendulum, we found that the speed of the ball after the collision is related to h:

$$v_{bf} = \sqrt{2gh}$$

If we combine these two equations (plug v_{bf} into the first equation):

$$v_{bi} = \frac{m_b + b_p}{m_b}\sqrt{2gh}$$

This is the speed of the ball in terms of the maximum height that the pendulum swings to. For one final convenience, let's try to write this equation in terms of an angle θ that the pendulum arm swings through. This will be useful during the experiment because it's hard to measure the height that the pendulum mass reaches, since it is also moving in the x-direction. But the angle θ it swings to is easy to measure using a protractor built into the ballistic pendulum device.



Looking at the figure above, we can write down a few relations:

$$L = h + d,$$
$$h = L - d$$
$$\cos(\theta) = \frac{d}{L}$$
$$d = L\cos(\theta)$$

Solving for h in terms of θ:

$$h = L(1 - \cos(\theta)).$$

The final result is:

$$\boxed{v_{bi} = \frac{m_b + m_p}{m_b}\sqrt{2gL(1-\cos\theta)}}$$

Procedure

Part A: Ballistic Pendulum Experiment

Setup

1. Attach the Projectile Launcher to the Ballistic Pendulum and make sure the launcher is level using the thread and bead. Fasten the launcher to the bracket using the two thumb screws through the two holes.
2. Screw in the ballistic pendulum into the pendulum stand using the pendulum screw toward the top of the stand.
3. When the pendulum and projectile launcher are attached properly, the launcher should almost (but not quite) touch the pendulum catcher, shown in **Figure 2**. Again, make sure the launcher is set for a launch angle of zero degrees.
4. Before you use the projectile launcher, make sure the area is clear of others and nothing obstructs the path of the pendulum.

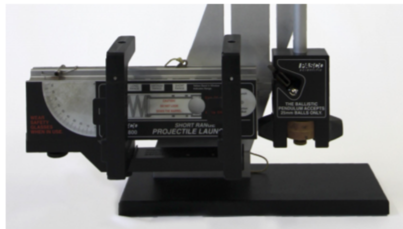


Figure 2: Pendulum and projectile launcher in their initial position.

Five trials

1. To load the launcher, swing the pendulum out of the way, place the ball in the end of the barrel and, using a push rod (not your fingers), push the ball down the barrel until the trigger catches in the second (Medium Range) position.
2. Return the pendulum to its normal hanging position and wait until it stops moving. Make sure to move the angle indicator back to the zero angle position. Note that if the angle indicator is not at zero degrees when your pendulum is in the normal hanging position that you will need to account for this difference. (See step 4.)
3. Launch the ball so that it is caught in the pendulum. You will notice that the angle measurement scale will have moved to the maximum angle the pendulum reached during its swing.
4. Record the maximum angle in *Table 1*. If your indicator does not start at zero, you must subtract the starting value from the final value!
5. Repeat Steps (3) and (4) five times.
6. Record the average angle in the data input box. Remember to reset the angle indicator each time you launch the ball.

Pendulum Arm Length

1. Remove the pendulum from the pendulum stand, and balance it on a meter stick as shown in **Figure 3**. Note: the ball must be in the pendulum!
2. Find the point at which the catcher is extended as far as possible out over the edge of the meter stick. When properly balanced, the center of mass is directly over the end of the stick.
3. Once balanced, record length in *Table 2*, the distance from the center of mass out to the pivot (screw).
4. Using your value for length, calculate the change in height, Δh. Record the answer in *Table 2*.
5. Measure the mass of the ball + pendulum system. Record your mass in *Table 2*.
6. Measure the mass of just the pendulum. Record your mass in *Table 2*.
7. Calculate the mass of the ball using Steps (5) and (6). Record your mass in *Table 2*.
8. Calculate the launch speed of the ball using the equations from the Theory section. Record your answer in *Table 2*.

Figure 3: Balance the pendulum on a meter stick to find the center of mass.

Part B: Freeflight Experiment

1. Attach Photogates to the launcher using the Photogate Bracket as shown in **Figure 4**. Move the pendulum out of the way and slide the bracket such that Photogate 1 is as close to the end of the launcher as possible. Note: the launcher should not be in the way of Photogate 1, check that the red sensor lights are off to verify.
2. Connect Photogate 1 to *Digital Input 1* and Photogate 2 to *Digital Input 2*.
3. Place the ball in the launcher and compress the spring to the second (medium range) setting as before.
4. Make sure no one is in the way of the ball! Click Record and launch the ball. There is a Stop Condition that should half data recording.
5. The measured speed is shown in the Digits Display. Record this value in *Table 3* and repeat 5 times.
6. Calculate and record your average measured speed in *Table 3*.

Figure 4: Attach the photogates to the launcher stand.

Data

Table 1: Maximum Angle Reached

Trial	Maximum Angle Reached (Degrees)
1	
2	

3	
4	
5	
Average	

Table 2: Pendulum Quantities

Length (m)	Change in Height (m)	Mass: Ball & Pendulum (kg)	Mass: Pendulum (kg)	Mass: Ball (kg)	Launch Velocity (m/s)
0.285		0.316	0.250		

Table 3: Launch Velocity Measurements

Trial	**Launch Velocity**
1	
2	
3	
4	
5	
Average	

Analysis

1. What is the percent difference between your calculated launch velocity from *Table 2* and your average launch velocity from *Table 3*?

2. Which method of finding the launch velocity do you think is more accurate? Why?

3. Calculate the percentage of the total kinetic energy that is lost during the collision with the pendulum. ($100 \times (KE_{total,i} - KE_{total,f})/KE_{total,i}$).

4. Where does the energy go and how is it transferred?

Questions

1. This question is from the Colorado State Driver's license exam: A car at 30 miles per hour skids 40 feet with locked brakes. How far will the car skid with locked brakes at 90 miles per hour? (Hint: Think about Kinetic Energy).

2. Suppose that you and two friends are discussing the design of a roller coaster. One friend says that each summit must be lower than the previous one. Your other friend says this is nonsense, for as long as the first one is the highest it doesn't matter what height the others are. Explain which one is correct and why. (Assume no energy losses due to friction or air resistance).

3. When a jet plane lands on the deck of a carrier, is this an elastic or an inelastic "collision"? In this "collision" the before is just before landing and the after is after the plane has come to a stop.

4. In the measurement of velocity by the free-flight method, what would be the effect on the measurements if (a) the front of the launcher were lower than the rear? (b) the distance between the photogates was long enough to let the ball drop 1 cm between the photogates?

Lab 9: Work Energy Theorem
San Diego State University
Department of Physics
Physics 182A/195L

TA:
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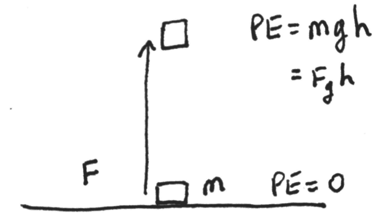
Date:
Score:

Theory

Work is the change in energy of a system (one or more objects with mass) as the result of a force applied to that system. How exactly is force related to energy? Let's think of an example. Recall that if a mass is held at a height h above the earth, then it has a potential energy $PE = mgh$. In order to move an object from the ground ($y = 0$) to a height $y = h$, then it must have at least force $F = mg$ acting on it while it moves from $y = 0$ to $y = h$ (otherwise gravity would pull it back down).



If we simply replace mg with F in the expression for energy, we get:

$$PE = Fh$$

That is, the potential energy of the object is equal to the force we had to apply to it, times the distance h, that it moved while the force was applied. More generally, any *constant force* applied F to an object over some distance x will change the energy of that object:

$$W = \Delta E = Fx$$

This change in energy could be potential energy or kinetic energy, depending on the circumstances. A change in energy of a system due to a force is called Work.

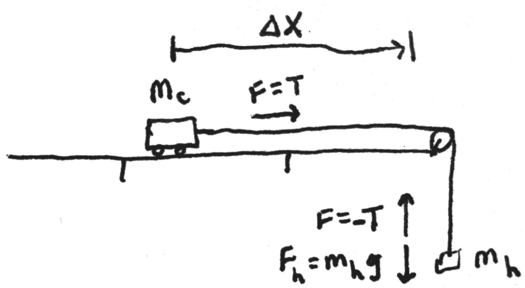
What if the force is not constant? A situation with non-constant force requires calculus. For a force that changes with x, the work done by that force is

$$W = \Delta E = \int_{x_i}^{x_f} F(x)dx$$

This is one way to write the *work-energy theorem*, which tells us exactly how to compute the work done on a system W by a changing force $F(x)$.

Constant force

Let's use the work-energy theorem to calculate the work W done on a cart with mass m_c by a constant force. To generate a constant force, we will use a hanging mass attached to the cart with a pulley system. This way, a known hanging mass m_h can be used to calculate the force $F_h = m_h g$ applied to the system.



Note that the force acting on the cart is not F_h, it is the tension T in the string. Using the free-body diagram above, and the fact that both masses must have the same acceleration, we can determine that the tension T acting on the cart is:

$$F_c = T = \frac{m_h m_c}{m_c + m_h} g$$

(For a detailed derivation, see the appendix.) Therefore, the work done on the cart by the pulley system is $W = T\Delta X$:

$$\boxed{W = \frac{m_h m_c}{m_c + m_h} g\Delta X}$$

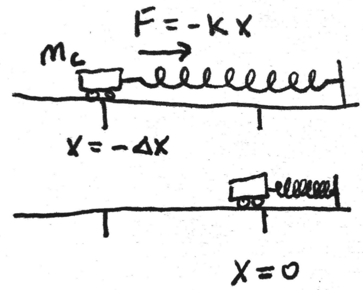
Here, Δx is the distance that the cart has moved. (It is also the distance that the hanging mass has fallen. Why?).

Linear force

Next, we'll use the work-energy theorem to calculate the work W done on a cart with mass m_c by a linear force F_s. To generate a force which changes linearly with x, we use a simple spring, which obeys Hooke's law:

$$F_s(X) = -kX$$

Here, k is the spring constant and X is the distance the spring is stretched from equilibrium.



Suppose that the cart starts from some initial displacement $x_i = -\Delta X$, where the spring is taut. The cart is then released, so that the spring pulls it to the right. The force applied by the spring is changing linearly, since as the cart changes it's x-position, the length of the spring changes and so does the force it applies. The total work W done on the cart by the spring from the carts initial position x_i to its final position $x_f = 0$ is given by the work-energy theorem:

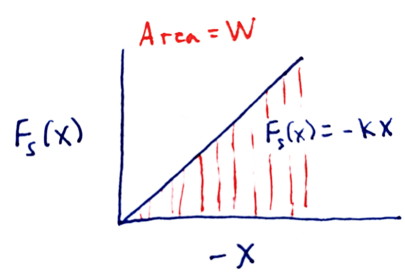
$$W = \int_{x_i=-\Delta X}^{x_f=0} -kx\ dx$$

This integral can be computed with the power-rule from calculus:

$$W = -\frac{1}{2}kx^2\Big|_{x_i=-\Delta X}^{x_f=0} = \frac{1}{2}k\Delta X^2$$

An integral of a function is nothing but the area under the curve. Therefore, the work W done by the force $F_s(x)$ can also be seen as the area under the $F_s(x)$ curve. In this case, the curve is a linear function with slope $-k$. The area is just the area of a triangle with base length ΔX and height $k\Delta X$, so

$$\boxed{W = Area = \frac{1}{2}(\Delta X)(k\Delta X)}$$



Procedure

Setup

1. Use the Balance Scale to measure the mass of the Smart Cart alone, the Smart Cart + the four 250g Stackable Masses. Record these values in Table A.1.
2. Connect the Wireless Smart Cart via Bluetooth under Hardware Setup in Pasco Capstone™.
3. Lay the cart flat on the table and make sure only the hook is attached. Go back to hardware setup and click on the smart cart force sensor settings (blue gear). Click "zero sensor now" to zero the force sensor.

Part A: Constant Force

1. Set up the equipment as shown in **Figure A.1**, but place magnetic End-Stops on both ends of the track. There is a hole in the End-Stop that you can thread the string through during the experiment. Put the 4 stackable 250g masses on the cart.
2. Cut about 1 meter of string. Tie one end of the string to the hook of the cart. Thread the string through the End-Stop and tie the other end to a Mass Hanger.
3. Hang the mass hanger over the pulley and place 45g on the mass hanger. Record the total hanging mass (hanger + weights) in Table A.1.

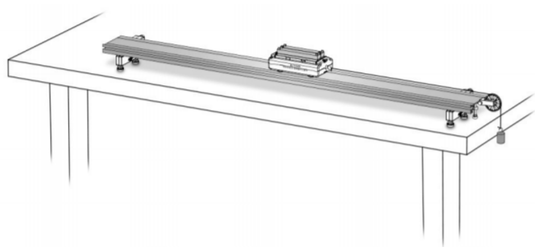


Figure A.1: The cart is placed in the center of a level track, attached to a pulley system.

4. Roll the cart as far back as you can before the mass hanger touches the pulley. Hold it in place.
5. Measure the displacement of the cart from the end of the track (pulley end) using the measuring tape on the track. Record this value in Table A.1.
6. Wait for the hanging mass to stop swinging, then hit START to begin recording.
7. Release the cart to roll freely down the track, and then catch the cart before it rolls into the End-Stops. Hit STOP to stop recording.
8. Use the "Highlight tool" to select only the data points recorded when the cart was in motion, and which have positive force values.
9. Record the area under the Force vs. Position curve. using the "Display Area tool" to calculate the area. Record the value in Table A.2.
10. Determine the average force using the "Mean tool". Record the value in Table A.2.

11. Use the “Coordinate tool” to find the final velocity of the cart on the Velocity vs. Position graph. (This should be the peak of the velocity graph.) Record the value in Table A.2.

Part B: Linear Force

1. Starting from the setup from **Part A**, remove the pulley, mass hanger, and the string.
2. Take a stiff long spring, and attach one end to the End-Stop and the other end to the hook of the cart.
3. Note the position of the cart when the spring is in equilibrium.
4. Now pull the cart backwards, allowing the spring to stretch. Move the cart away about 20cm from equilibrium.
5. Measure the exact displacement of the cart from equilibrium using the measuring tape on the track. Record this value in Table B.1.
6. Hit START to begin recording.
7. Release the cart to roll freely down the track, and then catch the cart before it changes directions. Hit STOP to stop recording.
8. Use the “Highlight tool” to select only the data points recorded when the cart was in motion, and which have positive force values.
9. Record the area under the Force vs. Position curve, using the “Display Area tool” to calculate the area. Record the value in Table B.2.
10. Determine the average force using the “Mean tool”. Record the value in Table B.2.
11. Use the linear curve fit tool to find the slope of the force curve. Record the value in Table B.2.
12. Use the “Coordinate tool” to find the final velocity of the cart on the Velocity vs. Position graph. (This should be the peak of the velocity graph.) Record the value in Table B.2.

Data

Table A.1: Masses and Displacement

Smart Cart Mass (kg)	0.25
Smart Cart + Four Stackable Masses (kg)	1.25
Hanging Mass (kg)	0.05
Displacement (ΔX) (m)	0.60

Table A.2: Constant Force Variables

Area under constant force curve (N*m)	
Average Force (N)	

Final Velocity (m/s)	

Table B.1: Cart Displacement

Displacement (ΔX)(m)	

Table B.2: Linear Force Variables

Area under linear force curve (N*m)	
Average Force (N)	
Slope (N/m)	
Final Velocity (m/s)	

Analysis

Part A: Constant Force

1. Use your area measurement in Table A.2 to find the work W done on the cart by the pulley system.

2. What do you think would happen if the cart was being pulled on both sides by equal masses?

3. Calculate the work done on the Smart Cart by the pulley system using the equation derived in the **Theory** section (show your work):

4. Calculate the percent error of the value of work you found in question 1 compared to the value predicted by the theory, which you found in question 3:

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5. Using the final velocity given in Table A.2, calculate the final kinetic energy of the Smart Cart in Part A (show your work):

6. Compare the value of the final kinetic energy you calculated in question 5 to the work done on the cart calculated in question 3. Explain your findings.

Part B: Linear Force

1. Given the fact that the force from the spring is linearly proportional to the displacement of the spring, which of the measured values in Table B.2 represents the spring constant? Explain your answer.

2. What is the value of the spring constant k?

3. Calculate the work done on the Smart Cart by the spring using the equation derived in the **Theory** section (show your work):

4. Calculate the percent error of the “Area under linear force curve” from Table B.2 compared to the value predicted by theory in the previous question.

5. Using the final velocity given in Table B.2, calculate the final kinetic energy of the Smart Cart in Part B (show your work):

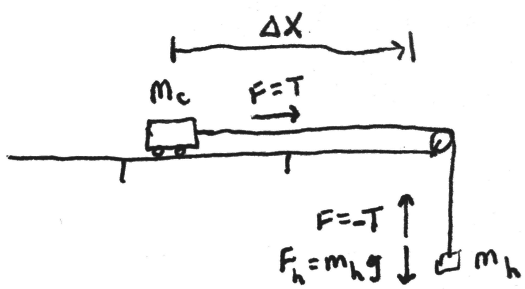
6. Compare the value of the final kinetic energy you calculated in question 5 to the work done on the Smart Cart calculated in question 3. Explain your findings.

Questions

1. Sometimes, in physics, we appear to be equating two different quantities. Unit analysis is a useful sanity check when learning a new concept. Show that the base SI units for work and kinetic energy are the same:

Appendix

In the following, we will derive the tension T pulling on the cart in the cart-pulley-hanging-mass system shown in the drawing below.



There are two free-body diagrams to consider. First, the forces acting on the hanging mass are the tension from the string (pulling in the negative direction), and the force due to gravity. Thus, Newton's second law tells us:

$$m_h a_h = m_h g - T.$$

Second, there is only one force acting on the cart, and that is the tension. Again from Newton's second law:

$$m_c a_c = T.$$

What can we say about the relationship between a_c and a_h? Since both masses are connected by a rigid string, we know that $a_c = a_h$. We can thus substitute $a_h = T/m_c$ into the first equation:

$$m_h \frac{T}{m_c} = m_h g - T.$$

If we isolate T in this equation, we obtain the result stated in the theory section:

$$T = \frac{m_c m_h}{m_c + m_h} g.$$

San Diego State University

Lab 9: Rotational Inertia
San Diego State University
Department of Physics
Physics 182A/195L

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Introduction
An object with mass resists changes in velocity. This resistance is called inertia, and it is the fundamental principle behind Newton's first law. Newton's first law states that a force is required to accelerate a massive object, whether that be to slow it down or speed it up. What if that object is simply rotating and not moving to a different point in space? Objects with mass do indeed resist changes in rotational velocity, or angular velocity, and this phenomenon is called rotational inertia. In this lab, you will investigate this property of an object to resist changes in angular velocity.

Theory
First, recall that the torque, τ, on a rod of length l is simply:

$$\tau = \vec{r} \times \vec{F},$$

where $\vec{r}$ is a vector, colinear with the rod, corresponding to the point at which the force is applied and $\vec{F}$ is that applied force. The cross product,

$$\vec{r} \times \vec{F} = |\vec{r}| \cdot |\vec{F}| sin\, \theta,$$

becomes

$$l \cdot F,$$

as long as the magnitude of the force is applied perpendicularly to the end of the rod. Now, imagine this rod is massless except at the point where the force is being applied, and it is also fixed at the other end. The rod would then sweep a circle with radius l, with the mass at one end creating a ring. Let's consider this ring of mass that was sweeped by the circulating rod, rename the radius r, and then analyze the applied force:

$$F = ma.$$

The acceleration here is actually a tangential acceleration, because the acceleration at any point on the edge of the circle is a vector tangent to the circle. Then, multiply both sides by the radius,

$$r \cdot F = r \cdot ma,$$

and we see a familiar term, the torque:

$$\tau = m(r)a.$$

We can then substitute the tangential acceleration with something more helpful when dealing with rotation:

$$a = r \cdot \alpha,$$

where α is the object's angular acceleration, a measure of how fast the angular velocity is changing. Looking at the torque again,

$$\tau = mr^2\alpha,$$

we can see that the torque is equal to *something* times the angular acceleration. This *something* is the Rotational Inertia we are worried about:

$$mr^2 = I$$

Additionally, the total inertia of a system is the sum of all the inertias:

$$I_{tot} = \Sigma I_i.$$

This implies that if our ring of mass was actually a disc, we can integrate (or sum every infinitesimal chunk) the mass along every point of the radius.

$$I_{disc} = \int_0^R \quad dm\, r^2.$$

Here, we will call the full radius of the disc R. There needs to be a substitution for the infinitesimal chunks of mass, dm, so we can get a proper integral. Assuming the disc is uniform, then ratios of the circular area covered at the point of integration to the total area and the mass at the point of integration to the total mass, are equal:

$$\frac{2\pi r dr}{\pi R^2} = \frac{dm}{M}.$$

This leads us to an expression for the Rotational Inertia of a disc:

$$I_{disc} = \frac{1}{2}MR^2.$$

Similarly, if it is not a disc, but a thick ring with an inner and outer radius, then the logic is the same and you would subtract the inertia of the 'disc' not present in the center, leading to an expression for the Inertia of a thick ring of mass:

$$I_{ring} = \frac{1}{2}M({R_1}^2 + {R_2}^2),$$

with R_1 being the inner radius and R_2 the outer radius.

Experiment

Now that we know the inertia of the disc and ring, we will test this experimentally. Based on the free-body diagram of the experimental setup, the inertia is a function of the gravitational force pulling on the hanger, the tension in the string, the radius of the pulley that the string wraps around, and the angular acceleration of that pulley:

$$ma = mg - T \qquad \textit{From the Free-body diagram}$$

$$I = m_h\, r^2(\frac{g}{r\,\alpha} - 1) \qquad \textit{Using Torque derivation from above}$$

A few **IMPORTANT** things to note here:

- m is your hanging mass (don't forget the 5g of hanger itself)
- r is the radius of the rotating disc
- α is the angular acceleration you will find experimentally
- Pasco can measure the angular acceleration, but it has a lot of noise. The best method for obtaining angular acceleration is to find the slope on a graph of angular velocity versus time.

USE THESE NUMBERS	r	m_h	M_{ring}	R_1
Large Setup	0.228 m	0.105 kg	1.4 kg	0.127 m
Small Setup	0.0950 m	0.0100 kg	0.472 kg	0.0767 m

Procedure

Large Setup

- Make sure your Pasco Interface is turned on
- In Hardware Setup, add the photogate on the image to the port you see it physically plugged into
- Timer Setup should appear on the far left, click that for Timer Setup
    - Click Pre-Configured Timer
    - Then click next twice, and for 'select timer' make sure you select 'photogate with pulley'
    - Click next
    - Verify the next options say Spoke Arc Length 0.015 m and Spoke Angle 36□
    - Click next one last time and then click finish
- Drag down a graph to the workbook
- For y-axis measurement, select Angular Speed
- Add a 100g mass to your hanger
- Start with the large disc only, no black ring on top
- Spin the disc to begin spooling the string and lifting the hanging mass. **There is a correct direction to spin the disc. If you spin the disc and the string begins spooling off-center on the clamp-pulley, spin the other direction. If the string is off-center on the clamp pulley, this will add unnecessary friction.**
- Click record, wait for Pasco to catch up, then let the hanging mass drop
- After the hanging mass has reached its final length and begun to head back up, stop recording
- There should now be a neat and clean linear function on your Angular Speed vs Time graph
- Use the highlighter and the best fit tool to get a linear fit to a large portion of this linear function (**you don't need to fit the entire line, especially at the ends where the function begins to flatten out**)
- The slope of the line is your Angular Acceleration, and you need this value for the analysis later on (record it, don't lose/forget it)

Small Setup

- Make sure your Pasco Interface is turned on
- In Hardware Setup, make sure the Rotary Motion Sensor is connected
- Drag down a graph to the workbook
- For y-axis measurement, select Angular Velocity
- Add a 5 g mass to your hanger
- Start with the disc only, no black ring on top
- Spin the disc to begin spooling the string and lifting the hanging mass. **There is a correct direction to spin the disc. If you spin the disc and the string begins spooling off-center on the pulley, spin the other direction. If the string is off-center on the pulley, this will add unnecessary friction.**
- Click record, wait for Pasco to catch up, then let the hanging mass drop
- After the hanging mass has reached its final length and begun to head back up, stop recording
- There should now be a neat and clean linear function on your Angular Velocity vs Time graph
- Use the highlighter and the best fit tool to get a linear fit to a large portion of this linear function (**you don't need to fit the entire line, especially at the ends where the function begins to flatten out**)
- The slope of the line is your Angular Acceleration, and you need this value for the analysis later on (record it, don't lose/forget it)

What was the Angular Acceleration you obtained (slope of your graph) for the DISC ONLY?

Units of rad/s^2

BOTH Setups

After you have obtained the angular acceleration of the disc, add the black ring on top. There should be grooves so the ring fits snugly.

Repeat the process from above and obtain the angular acceleration of the disc plus ring system.

What angular acceleration (slope of graph) did you measure with the DISC + RING?

Units of rad/s^2

Using the radius given in the table above for your specific setup, the hanging mass, and the angular acceleration you measured for the different materials, calculate the Rotational Inertias:

What is the calculated Rotational Inertia of your Disc (no ring)?

Units of kg*m^2

What is the calculated Rotational Inertia of your Disc + Ring?

Units of kg*m^2

Knowing that total inertia is simply the sum of all inertias in the system, what is the Rotational Inertia of the Ring?

Units of kg*m^2

Using the data given at the top of page 3, and the equation for Inertia of a Ring on page 2, find R_2, the inner radius of your setup's black ring.

Find the percent error for your calculated R_2 versus the actual. (use ${R_1}^{large} = 10.7\ cm$ and ${R_1}^{small} = 5.37\ cm$)

Further Analysis

If Rotational Inertia is a function of radius, why does an ice skater spin FASTER when they bring their arms inward?

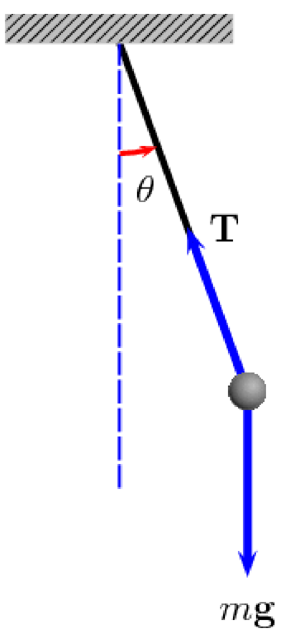
Calculate the total Rotational Inertia of a Ring sandwiched between two Discs. Use a mass of 1.50 kg for all three objects. The two discs have a radius of 0.333 m, and the ring has inner and outer radii of 0.125 m and 0.225 m respectively.

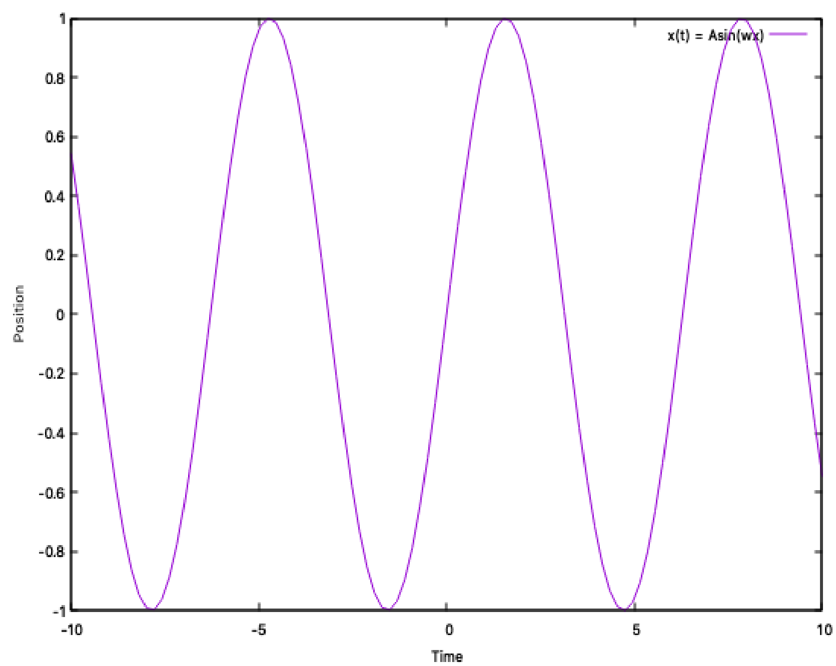
Lab 10: Simple Pendulum
San Diego State University
Department of Physics
Physics 182A/195L

TA:
Lab partner 1:
Lab partner 2:
Date:
Score:

Theory

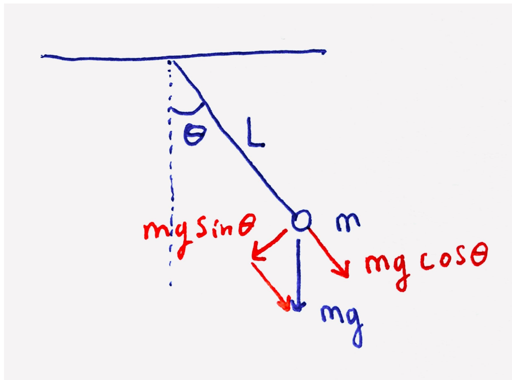
Oscillations can occur whenever a system is pulled away from a stable equilibrium. For example, Hooke's law $F = -kx$ describes a spring system with an equilibrium at $x = 0$. A spring will oscillate if you pull it away from that equilibrium and then release it. In this lab we will investigate the oscillations of the simple pendulum system: a mass m hangs from a fixed point by a string of length L and negligible mass. We will see that this system exhibits *simple harmonic motion*, a kind of oscillation that can be described by a simple sine or cosine function.





Force diagram

To figure out how to describe this motion, we start by writing down the forces acting on the pendulum mass m . Looking at the free-body diagram above, we can see that there are two forces acting on the mass: there is gravity pulling downward on the mass with a force $\vec{f}_1 = -mg\hat{y}$ (mg in the negative y-direction), and there is the force of the pendulum arm creating a tension $\vec{f}_2 = \vec{Tension}$ keeping the mass a fixed length from the pivot.



Newton's second law for this system reads $\vec{f}_1 + \vec{f}_2 = m\vec{a}$, a vector equation. Let's decompose each vector into its components parallel to and perpendicular to the direction of the pendulum arm. Along the direction parallel to the pendulum arm:

$$Tension - mg\cos(\theta) = ma_{parallel}$$

The cosine function comes from the fact that the gravitational force is shifted θ degrees away from being parallel to the pendulum arm. Something convenient happens here: since we know the length of the pendulum arm will not change, we can safely assume that the tension in the string and the force $-mg\cos(\theta)$ will cancel, leaving $a_{parallel} = 0.$

Along the direction perpendicular to the pendulum arm, or in other words, along the direction of motion:

$$-mg\sin(\theta) = ma_{motion}$$

Again, the sine function appears here due to the right-triangle geometry depicted above. Since only a_{motion} is non-zero, we will just call it 'a' from here on.

The position equation

We will briefly describe some math that you may have seen in your more advanced calculus courses. If you don't want to see any calculus, skip ahead to the calculus free conclusion.

The equation of motion above is really a differential equation for the displacement $x(t)$ of the pendulum mass:

$$-mg\sin(\theta) = ma = m\frac{d^2x(t)}{dt^2}$$

$$\frac{d^2x(t)}{dt^2} = -g\sin(\theta)$$

This says that the second derivative of $x(t)$ is related to $-g\sin(\theta(t))$. In general, this is a difficult equation to solve. In fact, it can't be solved with normal calculus methods. We can solve this equation with an approximation**.** We will apply the **small angle approximation**, which says that as long as the angle θ is small,

$$\sin(\theta(t)) \approx \theta(t).$$

By applying this small angle approximation, our equation for $x(t)$ becomes much simpler:

$$\frac{d^2x(t)}{dt^2} = -g\theta(t).$$

This will make more sense if we replace θ with something that depends on x and t. If we remember from geometry that arc length is related to the radius and angle, we can see for this situation that:

$$x(t) = L\theta(t)$$

$$\theta(t) = \frac{x(t)}{L}$$

If we plug this into the equation above:

$$\frac{d^2x(t)}{dt^2} = -\frac{g}{L}x(t)$$

Notice that this looks a lot like Hooke's law! The force (ma) is equal to a negative constant times displacement: $F = -(gm/L)x$. A function that satisfies this equation is

$$\boxed{x(t) = A\sin(\sqrt{\frac{g}{L}}t)}$$

Calculus free conclusion: *a simple pendulum will oscillate in a way according to a sine function, as long as the angle it swings to is small.*

Period of motion

There's more that we can learn given the boxed equation for x(t). One question we might like to answer is: how long will it take the pendulum to swing back and forth? In other words, what is the period of the motion? Recalling our knowledge of sine waves, we know that they repeat every 2π radians, and if we write the sine wave in the form:

$$x(t) = A\sin(\frac{2\pi}{T}t)$$

Then we can identify the constant T as the period of the motion. If we set these two forms of $x(t)$ equal to each other:

$$x(t) = A\sin(\frac{2\pi}{T}t) = A\sin(\sqrt{\frac{g}{L}}t),$$

We can recognize that

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

The period of motion T (the time it takes to complete one cycle of oscillation) of a simple pendulum is given by:

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Procedure

In this procedure we will measure the period of motion of a simple pendulum. We will try to answer questions about the period T, to see how it is affected by different properties of the pendulum system such as the length of the pendulum arm L, the mass of the pendulum bob m, the acceleration due to gravity g, and the amplitude of the motion A. You can probably guess from the equation for T which of these will have an impact and which will not.

Setup

1. Run the string through the hole in the brass cylinder and put the ends of the string on the inner and outer clips of the clamp (**Figure 1**), so the string forms a ’V’ shape as it hangs.

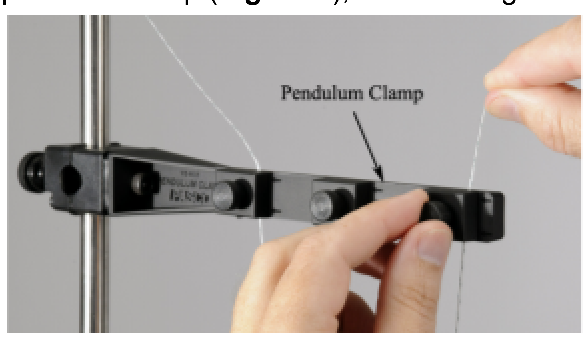


Figure 1: Pull the string between the inner and outer clips of the ramp

2. Adjust the string so the vertical distance from the bottom edge of the Pendulum Clamp the middle of the pendulum bob is 0.70 m.

3. Position the Motion Sensor in front of the pendulum so the brass colored disk is vertical and facing the bob, aimed along the direction that the pendulum will swing (**Figure 2**).
4. Make sure that the range switch on the Motion Sensor is positioned on top. Adjust the Motion Sensor up or down so that it is at the same height as the pendulum bob.
5. Adjust the position of the Rod Base so that the Motion Sensor is 0.25 m from the pendulum bob.

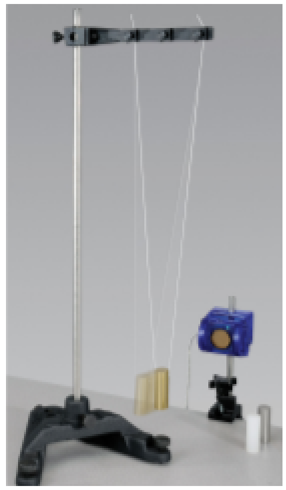


Figure 2: Suspend the bob in a “V” shape, with the motion sensor facing the bob.

Measuring the Period

There are three methods available for measuring the period of motion. Ask your TA which method or methods to use.

Method 1: The most straightforward way to measure the period of oscillation is to set the pendulum into motion and to simply time N complete cycles. If the pendulum swings back and forth, returning to its original position N times in t seconds, then the period is $T = t/N$. It’s important to choose N>10 so that random errors from the timing method will average out.

The other two methods use PASCO Capstone™ software to find your period. Start by recording trials of your oscillations:

- Click on Record to start recording data, data should appear on the graph. After 15 seconds, stop recording. Make sure the amplitude of the swing is relatively small (such as 0.05 m). If readings are not being recorded, make sure the objects are more than 0.15 m away from the sensor.
- If the resulting data is not a smooth sine curve, you can try changing the position of the range switch on top of the Motion Sensor. In general, the position with the cart icon is for smaller, closer objects and the position with the person icon is for larger objects further away. You should position the switch for whichever works the best.
- Select the rate that data is recorded by changing the frequency of the motion sensor at the bottom of the screen. Make sure there are enough points to create a smooth sine wave. A rate of 50 Hz will work well for this experiment.

Once these steps have been performed, you can find your period with one of the two following methods:

Method 2: PASCO Capstone™ can find the period using a recording of the position versus time of the pendulum bob. Begin by using the Coordinates Tool (from the Graph Tool palette), select a peak in your data recovered from oscillating a pendulum. Right click on the graph box and select Show Delta. Find an adjacent peak where $\Delta x \sim 0m$ to view the period.

Method 3: Another method involves using various Curve Fit functions available. Select the Sine Fit from the Graph Tool palette. The general form of the sine wave is: $Asin(\frac{2\pi}{T}t + \phi) + C$, where T is the period. The values from your Curve Fit can be used to calculate the period of the oscillation. (Note: ω in the Sine Fit option is equal to $2\pi/T$).

Part A: Period of Oscillation as a Function of Length

We will test how changing L, the length of the pendulum, affects the period of motion.

1. Adjust the length of the pendulum to be 0.7 meters by lengthening or shortening the string. Measure from the bottom edge of the pendulum clamp down to the middle of the cylinder.
2. Pull the pendulum bob about 0.05 m away from equilibrium and then release. Allow the bob to oscillate for a few seconds until the oscillations are smooth.
3. Measure the period of oscillation using the methods from **Measuring the Period**.
4. Record your measured period in Table A.1.
5. Stop the pendulum from swinging and then shorten the pendulum length by 0.05 m.
6. Repeat steps 2-5, recording data until you have a range of lengths from 0.70 m to 0.15 m according to Table A.1.

Part B: Period of Oscillation as a Function of Amplitude

1. Keep the pendulum length fixed at 0.35 m from the bottom of the clamp to the middle of the bob (last length used from **Part A**).
2. Pull the pendulum bob about 0.10 m away from equilibrium and then release. Allow the bob to oscillate for a few seconds until the oscillations are smooth.
3. Measure the period of oscillation using the methods from **Measuring the Period**.
4. Record the measured period in Table B.1.
5. Stop the pendulum from swinging.
6. Repeat steps 2-5 for amplitudes of 0.09 m, 0.08 m, 0.07 m, 0.06 m and 0.05 m, recording each measured period in Table B.1.

Part C: Period of Oscillation as a Function of Mass

1. Remove the brass cylinder from the pendulum and measure the mass of the brass, aluminum, and plastic cylinders. Record these values in Table C.1.
2. Reattach the brass cylinder to the pendulum system.
3. Adjust the length of the pendulum so that the distance from the bottom edge of the pendulum clamp to the middle of the cylinder is 0.60 m.
4. Pull the pendulum bob about 0.06 m away from equilibrium and then release. Allow the bob to oscillate for a few seconds until the oscillations are smooth.
5. Measure the period using one of the methods from the **Measuring the Period**.
6. Record the measured period in Table C.1.
7. Stop the pendulum from swinging.
8. Repeat steps 3-6 for both the aluminum and plastic cylinder. Make sure to keep the length of the pendulum, and the maximum amplitude, the same.

Data

Table A.1: Period and Varying Length

Length (m)	Cycles observed	Time, if using Method 1 (s)	Period (s)
0.70			
0.65			
0.55			
0.45			
0.35			
0.25			
0.20			
0.15			

Table B.1: Period and Varying Amplitude

Amplitude (m)	Cycles observed	Time, if using Method 1 (1)	Period (s)
0.10			
0.09			
0.08			
0.07			
0.06			
0.05			

Table C.1: Period and Varying Mass

Bob (type)	Bob Mass (kg)	Cycles observed	Time, if using Method 1 (s)	Period (s)
Brass	0.2116			
Aluminum	0.0675			
Plastic	0.0229			

Analysis

Part A: Period of Oscillation as a Function of Length

1. What happens to the period as you adjust the length of the pendulum?

2. We can solve for g, the acceleration due to gravity, using the length and period. Beginning from the equation of motion found in the **Theory** section, we can solve for g:
$$g = \frac{4\pi^2 L}{T^2}$$
Find the value of g for each of the lengths, using Table A.1 as reference:

Length (m)	**g (m/s^2)**
0.70	
0.65	
0.55	
0.45	
0.35	
0.25	
0.20	
0.15	

3. Do your values for g change significantly for different values of the pendulum length? Does this make sense? Why or why not?

Part B: Period of Oscillation as a Function of Amplitude

What happens to the period as you adjust the amplitude of the pendulum?

Part C: Period of Oscillation as a Function of Mass

1. What happens to the period as you adjust the mass of the pendulum?

2. The amplitude of a pendulum's oscillation is usually measured by the angle through which it swings, not its horizontal displacement. In **Part C** the length of the pendulum was about 60 cm and the maximum amplitude was about 6cm. Using right-triangle trigonometry, calculate the angle of the amplitude. Show your work.

3. The period of a pendulum is independent of amplitude only if the angle is small (remember the small angle approximation we made?). Was this the case?

Conclusion

Which variables (g, L, m, A) affect the period of motion T of the simple pendulum, and which do not?

Questions

1. When a body is oscillating in simple harmonic motion, is its acceleration zero at any point? If so, where and why?

Lab 11: Archimedes' Principle
San Diego State University
Department of Physics
Physics 182A/195L

TA:
Lab partner 1:
Lab partner 2:
Date:
Score:

Theory

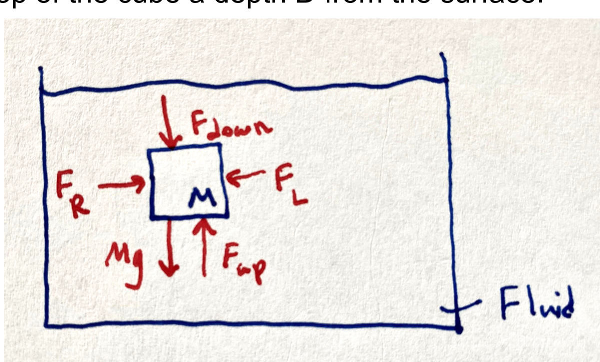
In this lab we will use the physics of floatation to determine the density of some solids that sink. Why do some objects sink and others float? This question can be answered with the concept of buoyancy, which is the force caused by differential pressure on a solid submerged in a fluid. Buoyancy is better described by **Archimedes' Principle**, which states:

When a body is partially or completely immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

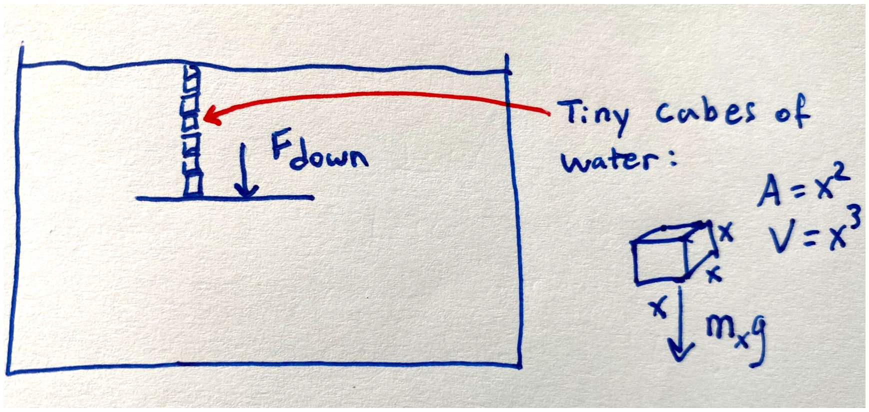
In the next section we will see how this upward force results from the pressure of a fluid pressing on all sides of the body/solid.

Buoyant force from geometry

Imagine a cube of some solid material with side lengths L and mass M, submerged in a fluid such as water, with the top of the cube a depth D from the surface.



There are forces on all sides of the solid from the water, as well as the usual force due to gravity. To understand what force water applies to the sides of the cube, let's take a closer look at the top surface of the solid:



Between the top of the solid and the surface of the water, we can imagine many tiny cubes of water all stacked on top of each other. Let's say these tiny cubes of water have side lengths x, and that each one weighs a small amount m_x. What force do all of these tiny masses apply to the top of the solid? Each one applies a force:

$$F_x = m_x g$$

So the total force is just $F_{down} = n \times m_x g$, where n is the number of small water cubes that fit on the surface of our solid. How can we find n? The solid has a surface area of L^2, so we can arrange L^2/x^2 tiny cubes on it. The distance to the surface of the water we said is D, so we can also stack D/x tiny cubes between the solid and the surface of the water. Multiplying these two numbers, we can see that the number of tiny water cubes pressing down on the solid is:

$$n = \frac{L^2 D}{x^3}\ .$$

Therefore, the total force of all that water pressing down on the solid cube from above is

$$F_{down} = \frac{L^2 D}{x^3} m_x g\ .$$

If we notice that $m_x/x^3 = m_x/V_x$, which is just the mass of a tiny cube of water divided by its own volume, we can replace this factor by a new symbol, ρ_w, the density of water. This is useful since the density of water does not depend on the size of our tiny cubes of water! Now we have $F_{down} = \rho_w L^2 D g$. We can also identify L^2 as the surface area A of the solid cube, so:

$$F_{down} = \rho_w A D g$$

We can define the pressure on the surface as the force-per-unit-area: $P = F/A$, which in this case is:

$$P = \frac{F}{A} = \rho_w D g$$.

This pressure will apply to all surfaces of the solid, and so we can compute the total force from the water pressing on the sides of the solid. After some calculations, or simple reasoning, we will find that all of the horizontal forces on the vertical surfaces of the solid will cancel out. This leaves only the force acting downward on the top of the solid, and the force acting upward on the bottom of the solid. The net force from the water pressure on all sides of our submerged object is known as the **Buoyant force**, which we can now compute:

$$F_{buoyant} = F_{top} + F_{bottom} + F_{sides}$$,

where F_{top} and F_{bottom} are the forces caused by the water pressure P at the top and bottom of the submerged solid. Plugging in the depths of the top and bottom of the solid, and $F_{sides} = 0$:

$$F_{buoyant} = -\rho_w A D g + \rho_w A (D + L) g$$, or

$$F_{buoyant} = \rho_w A L g$$.

In the second line we simply combine like terms. $A \times L$ is also the volume V of the solid, and so, finally, V of the solid, and so, finally,

$$\boxed{F_{buoyant} = \rho_w V g}$$.

This says that the buoyant force (the net force from the water pressure on every side of the solid) is equal to the density of water, times g, times the volume of the solid.

In conclusion, the buoyant force on a submerged solid can be calculated by knowing the volume of that solid, the density of the fluid, and g.

Buoyant force and apparent weight

We can also write the buoyant force as

$$F_{buoyant} = M_{displaced} g$$,

Where $M_{displaced} = \rho_w V$ can be interpreted as the mass of the water which was displaced by the volume of the solid. The total force acting on a submerged solid is the force of gravity plus the buoyant force:

$$F = -Mg + M_{displaced} g$$.

If we factor out g from each term, we get:

$$F = -(M - M_{displaced}) g$$.

If you were to weigh the solid while it is submerged, it would appear as if the solid has a mass $M_{apparent} = M - M_{displaced}$, since the apparent weight is the total force pulling the object downward:

$$Weight_{submerged} = F = -M_{apparent} g$$.

If we compare this to the true weight, i.e. the weight of the object when we weigh the object out of the water $Weight_{air} = Mg$, we can see that:

$$\boxed{Weight_{air} - Weight_{submerged} = F_{buoyant}}$$

This is an important result because it gives us a way to find the buoyant force acting on a solid without knowing its volume: all we need to do is weigh it in the air, weigh it in water, and take the difference.

Buoyant force and density

We have already learned that buoyant force of a solid in water is related to the density of water displaced by that solid. But we can also use our knowledge of buoyant force to determine the density of any solid. Recall that

$$F_{buoyant} = \rho_w V g.$$

If we use the method described above to find $F_{buoyant}$ using apparent weight, then we can calculate the volume V of the solid:

$$V = \frac{F_{buoyant}}{g\rho_w}$$

Then, finding the density of the solid is easy:

$$\boxed{\rho_{solid} = \frac{M}{V} = \frac{\rho_w \times Weight_{air}}{F_{buoyant}}}$$

We thus have two methods for finding the density of an object:

1. Using geometry: simply measure the mass and volume of the solid, then take the ratio.
2. Using archimedes principle: find the buoyant force using the difference between the true weight and the apparent weight of the solid, then use the formula above.

Procedure

In this lab our goal is to determine the density of different kinds of solid material using Archimedes principle. We will use both methods described in the theory section to measure the densities of our solids, and then we will compare the results to reference values.

Setup

1. Use the rod base, rods (both 45 cm and 90 cm) and the Multi-Clamp to support the Force Sensor over the beaker as shown in Figure S.1.
2. Plug the Force Sensor into the PASCO Universal Interface and Zero the sensor.
3. Tie a piece of string onto each of the masses provided in the Density Set. Tie a loop on the other end of the string so that it can be hooked onto the Force Sensor.
4. Put 800 ml of water into the beaker, but do not submerge the masses yet.

Figure S.1: Attach a force sensor to a rod base positioned over a beaker of fluid.

Part A: Density from geometry

Using Measurement Tools

1. Using the force scale, read the weight (individually) of the: brass cylinder, aluminum cylinder, brass cube, aluminum cube, and the aluminum uneven shape. Divide these values by our constant for gravity, $g = 9.81 m/s^2$, to obtain the masses. Record these masses in Table A.1.
2. Using the calipers to measure the radius and height of the brass cylinder and aluminum cylinder, record your measurements in Table A.1.
3. Using the calipers to measure the length, height and width of the brass cube and aluminum cube, record your measurements in Table A.1.
4. Calculate the volume of the brass cylinder and aluminum cylinder, recording your measurement in Table A.2.
5. Calculate the volume of the brass cube and aluminum cube, recording your measurement in Table A.2.
6. Using your calculated volumes and the measured mass, find the density of the aluminum and brass solids. Record these densities in Table A.2.
7. Calculate the volume of the unevenly shaped object, using its weight and assuming it has the same density as the other aluminum shapes. Record this volume and density in Table A.2.

Part B: Density from buoyant force

Using Force Scales

1. With nothing hanging on the Force Sensor, click ZERO on the Force Sensor. Note: the force being shown in the Digits Display below should be about 0 N.
2. Hang the brass cylinder on the Force Sensor in the air, hit Monitor and record the weight in the "Weight in Air" column of Table B.1 after roughly 5-10 seconds.
3. Move the beaker with water under the Force Sensor and completely submerge the sample. Record the weight in the "Weight in Water" column of Table B.1.

4. Repeat for the other listed samples, including the irregularly shaped aluminum piece. Make sure you hit STOP between each measurement, as well as zeroing the sensor between runs, to get an accurate average.
5. When you have made all your measurements, click on Stop.
6. Take the weight readings you recorded in Table B.1 and subtract the “Weight in Water” from the “Weight in Air” columns. Record this difference in Table B.2 as the “Buoyant Force”.
7. Using the “Buoyant Force” calculations, determine the density of each of the solids using the formula developed in the **Theory**. Be sure to convert to consistent units as used in **Part A**. Record these values in Table B.2.

Data

Table A.1: Measurements of brass and aluminum solids, using calipers.

Object (Metal)	Mass (g)	Radius (cm)	Height (cm)	Width (cm)	Length (cm)
Cylinder (Brass)	210.8			Not Applicable	
Cylinder (Aluminum)	67.0				
Cube (Brass)	67.0	Not Applicable			
Cube (Aluminum)	67.8				
Uneven Shape (Aluminum)	67.8	Not Applicable			

Table A.2: Calculations of brass and aluminum solids.

Object (Metal)	Volume (cm^3)	Density (g/cm^3)
Cylinder (Brass)		
Cylinder (Aluminum)		
Cube (Brass)		

Cube (Aluminum)		
Uneven Shape (Aluminum)		

Table B.1: Weight readings of brass and aluminum solids.

Object (Metal)	Weight in Air (N)	Weight in Water (N)
Cylinder (Brass)		
Cylinder (Aluminum)		
Cube (Brass)		
Cube (Aluminum)		
Uneven Shape (Aluminum)		

Table B.2: Buoyant force and densities from force scale measurements.

Object (Metal)	Buoyant Force (N)	Density (g/cm^3)
Cylinder (Brass)		
Cylinder (Aluminum)		
Cube (Brass)		
Cube (Aluminum)		
Uneven Shape (Aluminum)		

Analysis

Use an internet reference to find the typical values of density for aluminum and brass. Note that since brass is an alloy, it can have slightly different density depending on the exact composition.

1. Compute the percent error of your measured densities from Table A.2 and Table B.2, and fill out the table below.

Material	Brass	Aluminum
Reference density (g/cm^3)		
Average density using geometry (g/cm^3)		
Percent error using geometry (%)		
Average density using buoyancy (g/cm^3)		
Percent error using buoyancy (%)		
