Lab 11: Archimedes' Principle
San Diego State University
Department of Physics
Physics 182A/195L

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Theory

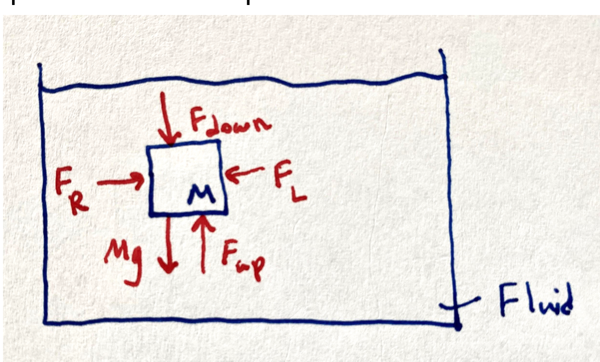
In this lab we will use the physics of floatation to determine the density of some solids that sink. Why do some objects sink and others float? This question can be answered with the concept of buoyancy, which is the force caused by differential pressure on a solid submerged in a fluid. Buoyancy is better described by **Archimedes' Principle**, which states:

When a body is partially or completely immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

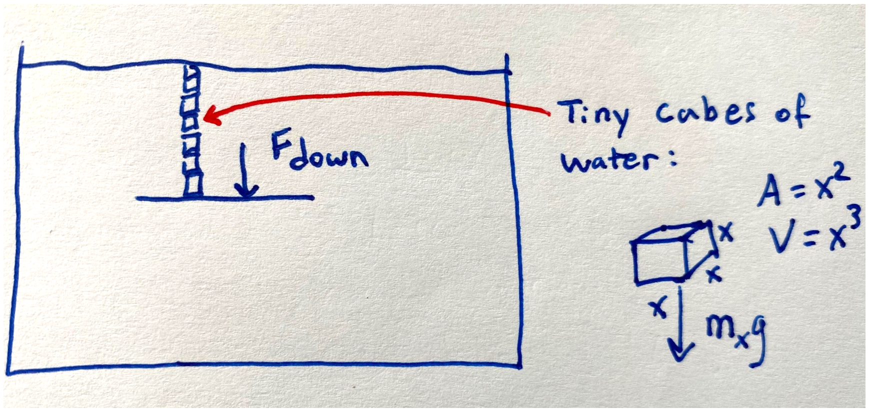
In the next section we will see how this upward force results from the pressure of a fluid pressing on all sides of the body/solid.

Buoyant force from geometry

Imagine a cube of some solid material with side lengths L and mass M, submerged in a fluid such as water, with the top of the cube a depth D from the surface.



There are forces on all sides of the solid from the water, as well as the usual force due to gravity. To understand what force water applies to the sides of the cube, let's take a closer look at the top surface of the solid:



Between the top of the solid and the surface of the water, we can imagine many tiny cubes of water all stacked on top of each other. Let's say these tiny cubes of water have side lengths x, and that each one weighs a small amount m_x. What force do all of these tiny masses apply to the top of the solid? Each one applies a force:

$$F_x = m_x g$$

So the total force is just $F_{down} = n \times m_x g$, where n is the number of small water cubes that fit on the surface of our solid. How can we find n? The solid has a surface area of L^2, so we can arrange L^2/x^2 tiny cubes on it. The distance to the surface of the water we said is D, so we can also stack D/x tiny cubes between the solid and the surface of the water. Multiplying these two numbers, we can see that the number of tiny water cubes pressing down on the solid is:

$$n = \frac{L^2 D}{x^3}.$$

Therefore, the total force of all that water pressing down on the solid cube from above is

$$F_{down} = \frac{L^2 D}{x^3} m_x g.$$

If we notice that $m_x/x^3 = m_x/V_x$, which is just the mass of a tiny cube of water divided by its own volume, we can replace this factor by a new symbol, ρ_w, the density of water. This is useful since the density of water does not depend on the size of our tiny cubes of water! Now we have $F_{down} = \rho_w L^2 D g$. We can also identify L^2 as the surface area A of the solid cube, so:

$$F_{down} = \rho_w A D g$$

We can define the pressure on the surface as the force-per-unit-area: $P = F/A$, which in this case is:

$$P = \frac{F}{A} = \rho_w D g$$.

This pressure will apply to all surfaces of the solid, and so we can compute the total force from the water pressing on the sides of the solid. After some calculations, or simple reasoning, we will find that all of the horizontal forces on the vertical surfaces of the solid will cancel out. This leaves only the force acting downward on the top of the solid, and the force acting upward on the bottom of the solid. The net force from the water pressure on all sides of our submerged object is known as the **Buoyant force**, which we can now compute:

$$F_{buoyant} = F_{top} + F_{bottom} + F_{sides}$$,

where F_{top} and F_{bottom} are the forces caused by the water pressure P at the top and bottom of the submerged solid. Plugging in the depths of the top and bottom of the solid, and $F_{sides} = 0$:

$$F_{buoyant} = -\rho_w A D g + \rho_w A(D + L)g$$, or

$$F_{buoyant} = \rho_w A L g$$.

In the second line we simply combine like terms. $A \times L$ is also the volume V of the solid, and so, finally, V of the solid, and so, finally,

$$\boxed{F_{buoyant} = \rho_w V g}$$.

This says that the buoyant force (the net force from the water pressure on every side of the solid) is equal to the density of water, times g, times the volume of the solid.

In conclusion, the buoyant force on a submerged solid can be calculated by knowing the volume of that solid, the density of the fluid, and g.

Buoyant force and apparent weight

We can also write the buoyant force as

$$F_{buoyant} = M_{displaced} g$$,

Where $M_{displaced} = \rho_w V$ can be interpreted as the mass of the water which was displaced by the volume of the solid. The total force acting on a submerged solid is the force of gravity plus the buoyant force:

$$F = -Mg + M_{displaced} g$$.

If we factor out g from each term, we get:

$$F = -(M - M_{displaced})g$$.

If you were to weigh the solid while it is submerged, it would appear as if the solid has a mass $M_{apparent} = M - M_{displaced}$, since the apparent weight is the total force pulling the object downward:

$$Weight_{submerged} = F = -M_{apparent} g$$.

If we compare this to the true weight, i.e. the weight of the object when we weigh the object out of the water $Weight_{air} = Mg$, we can see that:

$$\boxed{Weight_{air} - Weight_{submerged} = F_{buoyant}}$$

This is an important result because it gives us a way to find the buoyant force acting on a solid without knowing its volume: all we need to do is weigh it in the air, weigh it in water, and take the difference.

Buoyant force and density

We have already learned that buoyant force of a solid in water is related to the density of water displaced by that solid. But we can also use our knowledge of buoyant force to determine the density of any solid. Recall that

$$F_{buoyant} = \rho_w V g\,.$$

If we use the method described above to find $F_{buoyant}$ using apparent weight, then we can calculate the volume V of the solid:

$$V = \frac{F_{buoyant}}{g\rho_w}$$

Then, finding the density of the solid is easy:

$$\boxed{\rho_{solid} = \frac{M}{V} = \frac{\rho_w \times Weight_{air}}{F_{buoyant}}}$$

We thus have two methods for finding the density of an object:

1. Using geometry: simply measure the mass and volume of the solid, then take the ratio.
2. Using archimedes principle: find the buoyant force using the difference between the true weight and the apparent weight of the solid, then use the formula above.

Procedure

In this lab our goal is to determine the density of different kinds of solid material using Archimedes principle. We will use both methods described in the theory section to measure the densities of our solids, and then we will compare the results to reference values.

Setup

1. Use the rod base, rods (both 45 cm and 90 cm) and the Multi-Clamp to support the Force Sensor over the beaker as shown in Figure S.1.
2. Plug the Force Sensor into the PASCO Universal Interface and Zero the sensor.
3. Tie a piece of string onto each of the masses provided in the Density Set. Tie a loop on the other end of the string so that it can be hooked onto the Force Sensor.
4. Put 800 ml of water into the beaker, but do not submerge the masses yet.

Figure S.1: Attach a force sensor to a rod base positioned over a beaker of fluid.

Part A: Density from geometry

Using Measurement Tools

1. Using the force scale, read the weight (individually) of the: brass cylinder, aluminum cylinder, brass cube, aluminum cube, and the aluminum uneven shape. Divide these values by our constant for gravity, $g = 9.81 m/s^2$, to obtain the masses. Record these masses in Table A.1.
2. Using the calipers to measure the radius and height of the brass cylinder and aluminum cylinder, record your measurements in Table A.1.
3. Using the calipers to measure the length, height and width of the brass cube and aluminum cube, record your measurements in Table A.1.
4. Calculate the volume of the brass cylinder and aluminum cylinder, recording your measurement in Table A.2.
5. Calculate the volume of the brass cube and aluminum cube, recording your measurement in Table A.2.
6. Using your calculated volumes and the measured mass, find the density of the aluminum and brass solids. Record these densities in Table A.2.
7. Calculate the volume of the unevenly shaped object, using its weight and assuming it has the same density as the other aluminum shapes. Record this volume and density in Table A.2.

Part B: Density from buoyant force

Using Force Scales

1. With nothing hanging on the Force Sensor, click ZERO on the Force Sensor. Note: the force being shown in the Digits Display below should be about 0 N.
2. Hang the brass cylinder on the Force Sensor in the air, hit Monitor and record the weight in the "Weight in Air" column of Table B.1 after roughly 5-10 seconds.
3. Move the beaker with water under the Force Sensor and completely submerge the sample. Record the weight in the "Weight in Water" column of Table B.1.

4. Repeat for the other listed samples, including the irregularly shaped aluminum piece. Make sure you hit STOP between each measurement, as well as zeroing the sensor between runs, to get an accurate average.
5. When you have made all your measurements, click on Stop.
6. Take the weight readings you recorded in Table B.1 and subtract the “Weight in Water” from the “Weight in Air” columns. Record this difference in Table B.2 as the “Buoyant Force”.
7. Using the “Buoyant Force” calculations, determine the density of each of the solids using the formula developed in the **Theory**. Be sure to convert to consistent units as used in **Part A**. Record these values in Table B.2.

Data

Table A.1: Measurements of brass and aluminum solids, using calipers.

<table>
<tr><th>Object (Metal)</th><th>Mass (g)</th><th>Radius (cm)</th><th>Height (cm)</th><th>Width (cm)</th><th>Length (cm)</th></tr>
<tr><td>Cylinder
(Brass)</td><td>210.8</td><td></td><td></td><td colspan="2" rowspan="2">Not
Applicable</td></tr>
<tr><td>Cylinder
(Aluminum)</td><td>67.0</td><td></td><td></td></tr>
<tr><td>Cube
(Brass)</td><td>67.0</td><td rowspan="2">Not
Applicable</td><td></td><td></td><td></td></tr>
<tr><td>Cube
(Aluminum)</td><td>67.8</td><td></td><td></td><td></td></tr>
<tr><td>Uneven Shape
(Aluminum)</td><td>67.8</td><td colspan="4">Not
Applicable</td></tr>
</table>

Table A.2: Calculations of brass and aluminum solids.

Object (Metal)	Volume (cm^3)	Density (g/cm^3)
Cylinder (Brass)		
Cylinder (Aluminum)		
Cube (Brass)		

Cube (Aluminum)		
Uneven Shape (Aluminum)		

Table B.1: Weight readings of brass and aluminum solids.

Object (Metal)	**Weight in Air (N)**	**Weight in Water (N)**
Cylinder (Brass)		
Cylinder (Aluminum)		
Cube (Brass)		
Cube (Aluminum)		
Uneven Shape (Aluminum)		

Table B.2: Buoyant force and densities from force scale measurements.

Object (Metal)	**Buoyant Force (N)**	**Density (g/cm^3)**
Cylinder (Brass)		
Cylinder (Aluminum)		
Cube (Brass)		
Cube (Aluminum)		
Uneven Shape (Aluminum)		

Analysis

Use an internet reference to find the typical values of density for aluminum and brass. Note that since brass is an alloy, it can have slightly different density depending on the exact composition.

1. Compute the percent error of your measured densities from Table A.2 and Table B.2, and fill out the table below.

Material	Brass	Aluminum
Reference density (g/cm^3)		
Average density using geometry (g/cm^3)		
Percent error using geometry (%)		
Average density using buoyancy (g/cm^3)		
Percent error using buoyancy (%)		
