International Trade

Lecture 1: Introduction

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Sciences Po

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Housekeeping issues

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Housekeeping issues

- No office hours (send me an email to meet)
- No TA, no problem set
- Class material:
  `https://sites.google.com/site/thomaschaney/teaching`

Introduction

- This class introduces modern models of international trade.
- We will study in details key theoretical papers, and a few empirical ones.
- It is essential that you master techniques from these papers.

Grading

- Class participaption: 10%
- Weekly reports: 30%
- Oral presentations: 30%
- Research proposal: 30%

Roadmap

1. Basic techniques: Spence-Dixit-Stiglitz (1976-1977)
2. The Gravity Equation in International Trade
3. New Trade Theory: Krugman (1980)
4. Heterogeneous Firms: Melitz (2003)
5. Ricardian Trade: Dornbusch-Fischer-Samuelson (1977) Eaton-Kortum (2002)
6. Welfare: Arkolakis, Costinot, Rodriguez-Clare (2012)
7. Factor endowments: Heckscher-Ohlin (1933)
8. Firm sizes.
9. Firm growth and variance.

What we *won't* do in this class

1. Trade policy (see Bagwell in Stanford and Staiger in UW-Madison)
2. Open economy macro and international finance (other Sciences Po faculty: Coeurdacier, Martin)

Road map for this class: Spence-Dixit-Stiglitz

- CES preferences and demand
- Monopolistic competition and pricing
- Beyond Dixit-Stiglitz:
    - few firms
    - nested CES
    - ideal variety models
    - discrete choice models,

CES preferences

- Consumption of different products, $i = 1, \cdots, I$
- Different varieties of product i, $\omega \in \Omega_i$

$$U = U\left[u_1(\cdot), \cdots, u_I(\cdot)\right]$$

$$u_i = \begin{cases} \left(\sum_{\omega \in \Omega_i} q_{i\omega}^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i - 1}} \\ \left(\int_{\omega \in \Omega_i} q_{i\omega}^{\frac{\sigma_i - 1}{\sigma_i}} d\omega\right)^{\frac{\sigma_i}{\sigma_i - 1}} \end{cases}$$

$$\text{with } \sigma_i > 1$$

Note: We will now focus on one particular i, and drop the i subscript.

Love for variety (drop the i subscript)

- n varieties available.
- Total expenditure on differentiated goods E.
- Special case $p_\omega = p$, $\forall \omega \Rightarrow q_\omega = \frac{E}{np} \forall \omega$

$$
\begin{aligned}
u(q_1, \cdots, q_n) &= \left(\sum_{\omega=1}^{n} q_\omega^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\sum_{\omega=1}^{n} \left(\frac{E}{np} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\
&= n^{\frac{1}{\sigma-1}} \frac{E}{p}
\end{aligned}
$$

- More goods, even at the same price, is better.
- σ higher, weaker love for variety.

Consumption choice

$$\begin{cases} \max_{q_\omega} u = \left(\sum_\omega q_\omega^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ s.t. \sum_\omega p_\omega q_\omega \leq E \end{cases}$$

$$\mathcal{L} = \left(\sum_\omega q_\omega^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \lambda \left(\sum_\omega p_\omega q_\omega - E \right)$$

$$\frac{\partial \mathcal{L}}{\partial q_\omega} = 0 \Leftrightarrow q_\omega^{-1/\sigma} u^{1/\sigma} = \lambda p_\omega$$

Constant Elasticity of Substitution

$$\frac{q_\omega}{q_{\omega'}} = \left(\frac{p_\omega}{p_{\omega'}}\right)^{-\sigma}$$

$$\frac{d \ln \left(q_\omega / q_{\omega'}\right)}{d \ln \left(p_\omega / p_{\omega'}\right)} = -\sigma$$

- When the p_ω increases relative to $p_{\omega'}$, the demand for q_ω relative to $q_{\omega'}$ falls.
- The higher σ, the more sensitive relative demand is to relative prices.

Demand function

$$q_\omega = \frac{\lambda^{-\sigma}}{u} p_\omega^{-\sigma}$$

$$p_\omega q_\omega = \frac{\lambda^{-\sigma}}{u} p_\omega^{1-\sigma}$$

$$\sum_\omega p_\omega q_\omega = \frac{\lambda^{-\sigma}}{u} \sum_\omega p_\omega^{1-\sigma}$$

$$E = \frac{\lambda^{-\sigma}}{u} \sum_\omega p_\omega^{1-\sigma}$$

$$\boxed{p_\omega q_\omega = s_\omega E = \frac{p_\omega^{1-\sigma}}{\sum_{\omega'} p_{\omega'}^{1-\sigma}} E}$$

- Total spending on variety ω $(p_\omega q_\omega)$, and the share of spending on variety ω (s_ω), depend on its price (p_ω) relative to the other prices (the $p'_{\omega'}s$).

Indices and composites

$$\boxed{p_\omega q_\omega = \left(\frac{p_\omega}{P}\right)^{1-\sigma} E}$$

$$\boxed{q_\omega = \left(\frac{p_\omega}{P}\right)^{-\sigma} Q}$$

$$\boxed{E = \frac{Q}{P}}$$

- E: expenditure, Q: quantity composite, P: ideal price index,

$$E = \sum_\omega p_\omega q_\omega$$

$$\boxed{Q \equiv \left(\sum_\omega q_\omega^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} (= U)}$$

$$\boxed{P \equiv \left(\sum_\omega p_\omega^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}$$

Ideal price index

Definition

A function $P(\mathrm{p})$ of the vector of prices $\mathrm{p} = (p_1, \cdots, p_n)$ such that the indirect utility $u(\mathrm{p}, E) = u\left(\mathrm{p}', \frac{EP(\mathrm{p}')}{P(\mathrm{p})}\right) \forall (\mathrm{p}, \mathrm{p}')$.

Ideal price index

- With homothetic preferences, the ideal price index P is the minimum cost of buying one unit of u.

$$\begin{cases} \min_{q_\omega} & \sum_\omega p_\omega q_\omega \\ s.t. & u\left(q_1, \cdots\right) \geq \bar{u} \end{cases}$$

$$\mathcal{L} = \sum_\omega p_\omega q_\omega - \lambda \left(\left(\sum_\omega q_\omega^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \bar{u} \right)$$

$$\frac{\partial \mathcal{L}}{\partial q_\omega} = 0 \Leftrightarrow p_\omega = \lambda q_\omega^{-1/\sigma} u^{1/\sigma} \Leftrightarrow q_\omega^{\frac{\sigma-1}{\sigma}} = \lambda^{\sigma-1} u^{\frac{\sigma-1}{\sigma}} p_\omega^{1-\sigma}$$

$$\sum_\omega q_\omega^{\frac{\sigma-1}{\sigma}} = \lambda^{\sigma-1} u^{\frac{\sigma-1}{\sigma}} \sum_\omega p_\omega^{1-\sigma} \Leftrightarrow \left(\sum_\omega q_\omega^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \lambda^\sigma u \left(\sum_\omega p_\omega^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}$$

$$\boxed{\lambda = P = \left(\sum_\omega p_\omega^{1-\sigma} \right)^{\frac{1}{1-\sigma}}}$$

Demand elasticity

$$\varepsilon_\omega \equiv \frac{d\ln q_\omega}{d\ln p_\omega} = \frac{dq_\omega}{dp_\omega}\frac{p_\omega}{q_\omega} = \sigma + \frac{p_\omega^{1-\sigma}}{\sum_{\omega'} p_{\omega'}^{1-\sigma}}(1-\sigma)$$

$$\boxed{\varepsilon_\omega = (1-s_\omega)\,\sigma + s_\omega \underset{s_\omega\to 0}{\approx} \sigma}$$

- With symmetric goods, the precision of the approximation $\varepsilon_\omega \approx \sigma$ is of the order $1/n$.

$$\frac{d\ln P}{d\ln p_\omega} = \frac{dP}{dp_\omega}\frac{p_\omega}{P} = \frac{p_\omega^{1-\sigma}}{\sum_{\omega'} p_{\omega'}^{1-\sigma}} = s_\omega$$

$$s_\omega = \frac{1}{n} \text{ if } p_\omega = p\,\forall\omega$$

Technology of production

- One factor of production, labor (wage w).
- Fixed (set-up) cost and constant marginal cost,

$$\text{Total labor requirement}\left(q_\omega\right) = f + \frac{q_\omega}{\varphi_\omega}$$

$$\text{Total cost}\left(q_\omega\right) = wf + \frac{wq_\omega}{\varphi_\omega}$$

$$\text{Marginal cost}\left(q_\omega\right) = \frac{w}{\varphi_\omega}$$

$$\text{Average cost}\left(q_\omega\right) = \frac{TC}{q_\omega} = \frac{w}{\varphi_\omega} + \frac{wf}{q_\omega}$$

Profit maximization

$$\max_{p_\omega} \pi(p_\omega) = p_\omega q_\omega(p_\omega) - \frac{w q_\omega(p_\omega)}{\varphi_\omega} - wf$$

$$\text{FOC: } \frac{\partial \pi}{\partial p_\omega} = 0 \Leftrightarrow q_\omega + p_\omega \frac{\partial q_\omega}{\partial p_\omega} = \frac{w}{\varphi_\omega}\frac{\partial q_\omega}{\partial p_\omega} \Leftrightarrow -\left(p_\omega - \frac{w}{\varphi_\omega}\right)\frac{\partial q_\omega}{\partial p_\omega} = q_\omega$$

$$\Leftrightarrow \frac{\partial q_\omega}{\partial p_\omega}\frac{p_\omega}{q_\omega} = \frac{d \ln q_\omega}{d \ln p_\omega} = \varepsilon_\omega = \frac{p_\omega}{p_\omega - w/\varphi_\omega}$$

$$\boxed{\underbrace{p_\omega}_{\text{price}} = \underbrace{\frac{\varepsilon_\omega}{\varepsilon_\omega - 1}}_{\text{mark-up}} \underbrace{\frac{w}{\varphi_\omega}}_{\text{marginal cost}}}$$

- *IF* the demand elasticity is constant $(\varepsilon_\omega = \sigma)$, *THEN* mark-ups are constant.

The determinants of firm size

- Given the optimal pricing, one gets the following value of sales,

$$p_\omega q_\omega = \left(\frac{p_\omega}{P}\right)^{1-\sigma} E = \left(\frac{\varepsilon_\omega}{\varepsilon_\omega - 1}\right)^{1-\sigma} \varphi_\omega^{\sigma-1} \left(\frac{w}{P}\right)^{1-\sigma} E$$

1. The higher a firm's mark-up $\left(\frac{\varepsilon_\omega}{\varepsilon_\omega - 1}\right)$, the lower its sales.
2. The higher a firm's productivity $\left(\varphi_\omega^{\sigma-1}\right)$, the higher its sales.
3. the higher a firm's cost (w), the lower its sales.
4. the higher the demand in a market (E), the higher its sales.
5. the tougher the competition a firm faces (lower P), the lower its sales.

Measuring productivity (a tricky business)

- "True" measure of productivity considers physical efficiency of a plant/firm,

$$\frac{\text{physical output}}{\text{variable workers}} = \frac{q_\omega}{q_\omega/\varphi_\omega} = \varphi_\omega$$

- But we often don't have good price data at the plant/firm level,

$$\begin{aligned}\frac{\text{value of output}}{\text{total wage bill}} &= \frac{p_\omega q_\omega}{wf + wq_\omega/\varphi_\omega} \\ &= \frac{\varepsilon_\omega}{\varepsilon_\omega - 1}\left(1 + \left(\frac{\varepsilon_\omega}{\varepsilon_\omega - 1}\right)^\sigma w^\sigma \frac{P^{1-\sigma}}{E}\varphi_\omega^{1-\sigma}\right)^{-1} = \frac{\varepsilon_\omega}{\varepsilon_\omega - 1} f\underset{+}{(\varphi_\omega)}\end{aligned}$$

- With data on production workers, interesting prediction,

$$\frac{\text{value of output}}{\text{variable wage bill}} = \frac{p_\omega q_\omega}{wq_\omega/\varphi_\omega} = \frac{p_\omega}{w/\varphi_\omega} = \frac{\text{price}}{\text{marginal cost}} = \frac{\varepsilon_\omega}{\varepsilon_\omega - 1}$$

Dixit-Stiglitz: general equilibrium

Key simplifying assumption:

- All firms are symmetric,

$$\varphi_\omega = \varphi \, \forall \omega \Rightarrow p_\omega = p \text{ and } \varepsilon_\omega = \varepsilon \, \forall \omega$$

- The number of firms in equilibrium (n) is large,

$$\varepsilon_\omega = \sigma \, \forall \omega \Rightarrow p = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}$$

Key assumption:

- There exists a perfectly elastic fringe of potential entrants $\Rightarrow$ firms enter until profits are driven down to zero.

$$\pi \left(\underset{-}{p}, \underset{+}{P} \right) \text{ with } P = n^{\frac{1}{1-\sigma}} p$$

Equilibrium conditions

$$p = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi} \tag{PM}$$

$$\pi = 0 \tag{FE}$$

$$L = n \left(f + \frac{q}{\varphi} \right) \tag{LMC}$$

Equilibrium conditions

- Profit maximization (PM) and free entry (FE) together pin down the scale of firms,

$$\pi = \left(p - \frac{w}{\varphi}\right) q - wf = \frac{1}{\sigma - 1} \frac{w}{\varphi} q - wf$$

$$\pi = 0 \Rightarrow q = (\sigma - 1) f \varphi$$

- Given the scale of each firm, (LMC) determins the number of entrants,

$$L = n\left(f + \frac{q}{\varphi}\right) \Rightarrow n = \frac{L}{\sigma f}$$

1. More firms in larger markets (L).
2. More firms when lower fixed cost (f).
3. More firms when less elastic demand (lower σ).

Welfare analysis

- Real wages give direct welfare measure,

$$u = \frac{w}{P} = \frac{w}{n^{\frac{1}{1-\sigma}} p} = \frac{w}{\left(\frac{L}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{w}{\varphi}} = \left(\frac{\sigma - 1}{\sigma}\right) \varphi \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}}$$

1. Positive impact of market size.
2. Positive impact of efficiency (high φ and low f).
3. Direct negative impact of low σ (lower mark-ups $\frac{\sigma-1}{\sigma}$).
4. Indirect positive impact of low σ (more varieties $\frac{L}{\sigma f}$).
5. Indirect positive impact of low σ (love for variety $\frac{1}{\sigma-1}$ exponent).

Welfare analysis

- Note that monopolistic competition distorts optimal allocation between varieties only if variable mark-ups,

$$\frac{p_\omega}{p_{\omega'}} = \frac{\frac{\varepsilon_\omega}{\varepsilon_\omega - 1}\frac{w}{\varphi_\omega}}{\frac{\varepsilon_{\omega'}}{\varepsilon_{\omega'} - 1}\frac{w}{\varphi_{\omega'}}} = \frac{\frac{\varepsilon_\omega}{\varepsilon_\omega - 1}}{\frac{\varepsilon_{\omega'}}{\varepsilon_{\omega'} - 1}}\frac{\varphi_{\omega'}}{\varphi_\omega}$$

  $\Rightarrow$ tax high mark-up firms and subsidize low mark-up ones.

- Typically too few varieties compared to the first best (with lump-sum transfers allowed).