In the select algorithm, the runtime is represented with the recurrence relation

$$T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right).$$

Here,  $T(\frac{n}{5})$  is for selecting the pivot, and  $T(\frac{7n}{10})$  is for the recursive call to select the k-th element.

Consider the modified version of the select algorithm, where we split our array into  $\lceil \frac{n}{7} \rceil$  groups of size  $\leq 7$  instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as  $T(n) = O(n) + T(\frac{n}{a}) + T(\frac{bn}{c})$ , where a, b, and c are non-negative integers, what are the smallest possible values of a, b, and c?

What is the smallest exponent x such that the modified version of the select described above on an array of size n always takes time  $O(n^x)$ ?

1 Correct

Now assume that the O(n) work per recursive step takes exactly n units of time on our machine. In other words, suppose that the recurrence relation for the runtime is

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right).$$

What is the smallest coefficient C such that we can use the substitution method to prove that the recurrence relation for the modified select algorithm is  $T(n) \leq Cn$ 

7 Correct

Now consider another modified version of the select algorithm, where we split our array into  $\lceil n/3 \rceil$  groups of size  $\leq 3$  instead. What would be the recurrence relation for this modified version? Specifically, if we write the recurrence relation as

$$T(n) = n + T\left(\frac{n}{a}\right) + T\left(\frac{bn}{c}\right),$$

where a, b, and c are non-negative integers, what are the smallest possible values of a, b, and c?

 $a = \frac{3}{3}$  Correct  $b = \frac{2}{3}$  Correct  $c = \frac{3}{3}$ 

Which one is true for the modified select recurrence relation that you came up with in the last part?

- O  $T(n) = \Theta(n)$
- $T(n) = \Theta(n \log n)$
- O  $T(n) = \Theta(n^2)$

Correct