Let L be the likelihood (which previously was the object of our optimisation).

Taking into account Jeffrey prior we now need to optimise (maximise) the following:

$$L\sqrt{\sum_{t_i:x_{t_i}=k}\frac{1}{(N_k-n_{k,t_i})^2}} \tag{1}$$

and that's it.. (notice the sum is a bit different than before, due to the "squaring", but not very much so!).
Moreover, if we want something very very fast and dirty: since we need it to bound away from infinity, we can assume that N is very large (compared to n) and therefor the expression above simplify (too?) violently to

$$L\frac{\sqrt{n_{k,\tau}}}{N_k} \tag{2}$$

this, however, might be horribly wrong and I'm not sure it is really any good..

By differentiating (the *log* of) eq. (1) w.r.t. N_k:

$$\begin{aligned}\sum_{t_i:x_{t_i}=k}\frac{1}{N_k-n_{k,t_i}}-\frac{\sum_{t_i:x_{t_i}=k}\frac{1}{(N_k-n_{k,t_i})^3}}{\sum_{t_i:x_{t_i}=k}\frac{1}{(N_k-n_{k,t_i})^2}}&=A_k\beta_k\\ A_k&:=\sum_{t_i}I_{t_i}\Delta_i\end{aligned} \tag{3}$$

Solving for β_k (as before) and plugging in

$$\left(N_k-\frac{B_k}{A_k}\right)\left(\sum_{t_i:x_{t_i}=k}\frac{1}{N_k-n_{k,t_i}}-\frac{\sum_{t_i:x_{t_i}=k}\frac{1}{(N_k-n_{k,t_i})^3}}{\sum_{t_i:x_{t_i}=k}\frac{1}{(N_k-n_{k,t_i})^2}}\right)-n_k=0 \tag{4}$$

It thus remains to find the root(s) of Eq.4 to find $\hat{N}_k$

PREVIOUSLY WAS:
The log likelihood:

$$l(\boldsymbol{\beta},\boldsymbol{N};\boldsymbol{t})=\sum_{t_i\in\boldsymbol{t}}\sum_{j=1}^{\infty}\left[I\{x_{t_i}=j\}\log(\lambda_{j,t_i})-I\{x_{t_i}\neq j\}\lambda_{j,t_i}\Delta_i\right]$$

where

$$\lambda_{j,t_i} := \beta_j I_{t_i}(N_j - n_{j,t_i})$$
$$\Delta_i := t_i - t_{i-1}.$$

By differentiating w.r.t. N_k:

$$\begin{aligned} \sum_{t_i : x_{t_i} = k} \frac{1}{N_k - n_{k,t_i}} = A_k \beta_k \\ A_k := \sum_{t_i : x_{t_i} \neq k} I_{t_i} \Delta_i \end{aligned} \tag{5}$$

By differentiating w.r.t. β_k

$$\begin{aligned} n_k &= \beta_k [N_k A - B] \\ n_k &:= \sum_{t_i : x_{t_i} = k} 1 \\ B_k &:= \sum_{t_i : x_{t_i} \neq k} I_{t_i} \, n_{k,t_i} \, \Delta_i. \end{aligned} \tag{6}$$

Solving for β_k and plugging in Eq.5

$$\left(N_k - \frac{B_k}{A_k} \right) \sum_{t_i : x_{t_i} = k} \frac{1}{N_k - n_{k,t_i}} - n_k = 0 \tag{7}$$

It thus remains to find the root(s) of Eq.7 to find $\hat{N}_k$ and plug it in Eq 6 to find $\hat{\beta}_k$.