Fundamental limits of three-dimensional sensing (or: nature makes no presents)

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About fundamental limits of three-dimensional sensing
(or: nature makes no presents)

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ABSTRACT

We will discuss some physical limits of 3D-sensors that work on 'rough' surfaces. An important result is that the most frequently used principles (laser triangulation and focus sensing with projected light spots) cannot achieve a better depth resolution than given by the Rayleigh depth of field. However, we can achieve 'superresolution' in depth by sacrificing lateral resolution: There is an 'uncertainty relation' between lateral and longitudinal resolution.

1. INTRODUCTION

3D-sensors supply data about the shape of the object surface in 3D space. A very useful property of 3D-data is their large dynamical range: The most commercial 3D-sensors supply 1000-4000 resolvable depth steps. Why not more? Where are the fundamental limits of the depth resolution of 3D-sensors?

About 35 years ago Ingelstam[1] and others derived uncertainty relations for certain imaging systems (multiple beam interferometry, phase contrast) that connect the lateral resolution $1/\delta x$ and the depth resolution $1/\delta z$. The origin of those uncertainty relations is quantum mechanics, so one should expect the existence of uncertainty relations as well for macroscopic 3D-sensors that work on rough objects.

We consider a system where our object surface is illuminated with coherent light. The surface be optically 'rough'. The complex field emerging from the object is measured at a plane. The task of a 3D-sensor is to find the distance of each point on the object surface, from the measured field. It is well known that it is not possible, generally, to retrieve the field at the location of the object: free space only transmits spatial frequencies on the Ewald sphere, all the other frequencies of the object get lost. Furthermore, the interaction of the illuminating wave with the object is not exactly known, specifically for rough objects.

To get a 3D-sensor, we must design a more specific system, utilizing additional a priori information. This can be achieved very effectively, by illuminating the object with a projected light spot. Many 3D-sensors are based on this kind of illumination. Utilizing the a priori knowledge about the illumination, it is possible to locate the spot by several methods: we will discuss the measurement of the phase in the sensor plane z=0,as shown in Fig. 1, and later, the more commonly used method of triangulation.

2. DEPTH UNCERTAINTY IS INTRODUCED BY SPECKLE NOISE

With modern heterodyne techniqes it is possible to determine the phase of a wave with an accuracy better than $\lambda/10.000$. Hence, it should be possible to determine the distance of the object spot better than 1/2.500 of the Rayleigh depth of field δz_R. (For the latter, Rayleigh assumed a minimum detectable wave aberration of $\lambda/4$). However, our experiments[2] and theoretical investigations[3] show that the depth resolution of this method is limited, in essence, by the Rayleigh depth of the observation aperture. The uncertainty of the measured depth is caused by the statistical phase errors that are unavoidably connected with the speckles occuring in the wave that is scattered from the rough object.

The depth resolution can be improved by averaging statistically independent speckles. Those can be generated by moving the light spot over the object by a distance as large as the spot diameter. An equivalent result is achieved by (spatially) partial coherent illumination with a larger object spot. A simple analysis yields the following uncertainty relation (factors of the order of π are dropped):

$$\delta z/\delta z_R \approx \delta x_R/\delta x \quad \text{or} \quad \delta z \cdot \delta x \approx \delta z_R \cdot \delta x_R \approx \lambda^2/\sin^3\alpha_o \qquad (1)$$

Therein $1/\delta z$, $1/\delta x$ denote the achievable depth resolution and lateral resolution. δz_R and δx_R are the corresponding limits given by Rayleigh: $\delta z_R=\lambda/\sin^2\alpha_o$, $\delta x_R=\lambda/\sin\alpha_o$, with α_o = aperture of observation. Relation (1) corresponds to the relation that was derived by Ingelstam[1] for interferometry at non rough surfaces. Equation (1) means that it is possible to reduce the depth uncertainty δz, by sacrificing lateral resolution $1/\delta x$. In other words, we can achieve superresolution in depth, compared to the Rayleigh limit. Experimentally[3], we achieved a depth resolution about 25 times better than the Rayleigh depth. The relation Eq. (1) is completely different from the usual coupling between lateral and longitudinal resolution which is given by $\delta x_R/\delta z_R = \sin\alpha_o$.

So far depth resolution. What about the measuring range? To guarantee a small spot diameter on the object surface, the depth of field δz_I of the illumination must be as large as the desired measuring range Δz. For a system with an aperture of illumination and observation $\sin\alpha_i$ and $\sin\alpha_o$, respectively, and with resolvable depth $\delta z_R = \lambda/\sin^2\alpha_o$ (without speckle reduction) we get the number N of resolvable depth steps: $N = \sin^2\alpha_i/\sin^2\alpha_o$. One instructive example ($\lambda = .5\mu m$): With $\sin\alpha_i = .001 \rightarrow \Delta z = 500mm$ and $\delta x_i = .5mm$. With $\sin\alpha_o = .1 \rightarrow \delta z_R = .05mm$ and hence, follows N=10.000.

Now we proceed to triangulation, see Fig. 2. What about the depth resolution? If we assume a resolvable lateral shift of $\delta x_R = \lambda/\sin\alpha_o$, we find from Fig. 2 the resolvable depth $\delta z_R = \lambda/(\sin\theta \cdot \sin\alpha_o)$. For $\theta = \alpha_o$ we get the Rayleigh depth of conventional imaging.

If the image of the light spot on the camera target is well shaped, its lateral position can be localized much better than given by δx_R, and therefore, much better depth resolution can be achieved. However, the image of the light spot on the camera target is speckled, and can be localized only with a statistical error[3,4]. It is remarkable that even if the aperture of observation is that small that the spot image displays no speckles, its location suffers from statistical uncertainty. Hence, the achievable depth resolution is again limited to the (modified) Rayleigh depth $\delta z_R = \lambda/(\sin\theta \cdot \sin\alpha_o)$. Again, we can achieve improvement of the depth resolution by speckle reduction, as described above. However then again, we must sacrifice lateral resolution. Very similar results have been found in all our experiments based on spot illumination and it appears that the relation Eq. (1) is fundamental for these sensors.

What about the measuring range ? The depth of field problem for the camera can be solved by the Scheimpflug condition. Large depth of field in the projection step can be achieved by use of an axicon[4]. Now the number N of resolvable depth steps is given essentially by the observable object field $\Delta x \approx \Delta z \cdot \sin\theta$ and by δz_R: $N = \Delta z/\delta z_R = \Delta x/\delta x_R$. With $\sin\theta = .2$ and $\sin\alpha_o = .05 \rightarrow \delta z_R = .05mm$ and $\delta x_R = .01$ mm. With a field $\Delta x = 40mm$ of the camera follows N=4000.

4. SUMMARY AND CONCLUSIONS

The speckle that occurs in the light scattered from rough objects limits the depth resolution of 3D-sensors to the Rayleigh limit. However, superresolution in depth is possible, sacrificing lateral resolution, by reducing spatial coherence. We found that - dependent on the surface - some temporal incoherence can reduce speckle noise too and increase depth resolution without sacrificing lateral resolution. A preliminary experiment displayed a depth resolution of 1/100.000 of the measuring distance, by illumination with a semiconductor laser working below its threshold. These results, as well as sensors working with spatially incoherent illumination are discussed in a further paper.

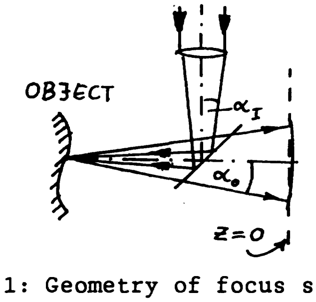

Fig. 1: Geometry of focus sensing

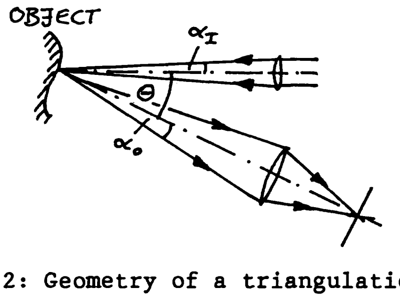

Fig. 2: Geometry of a triangulation system

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