

Getting *free* Bits Back from Rotational Symmetries in LLMs

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University of Cambridge

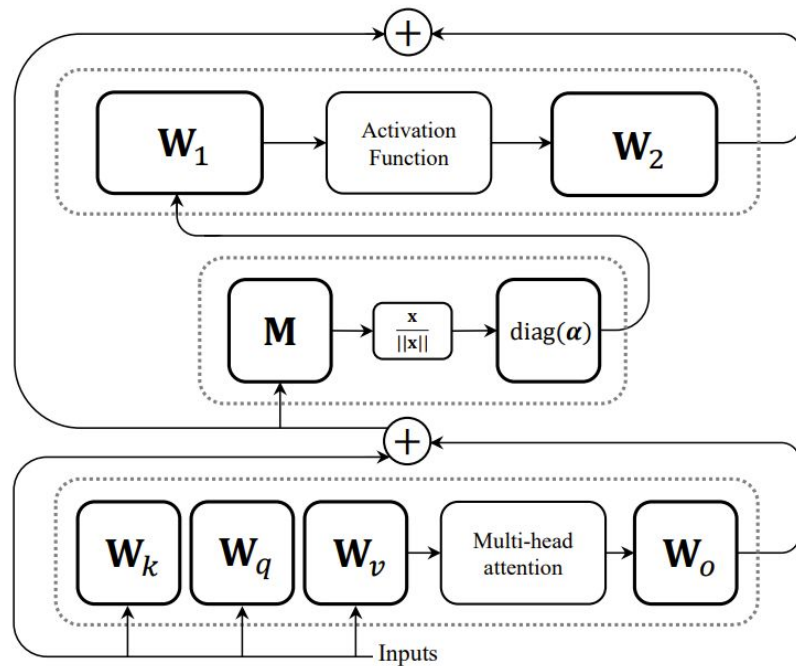
After pruning LLMs,

we can further save 3-5% additional bits *for free*

in storage and transmission

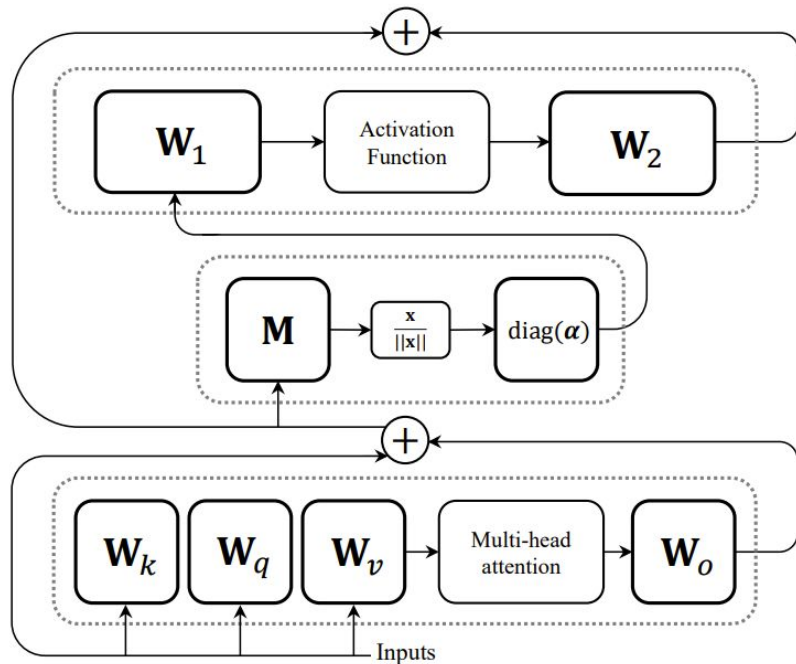
SliceGPT and bits-back coding

SliceGPT

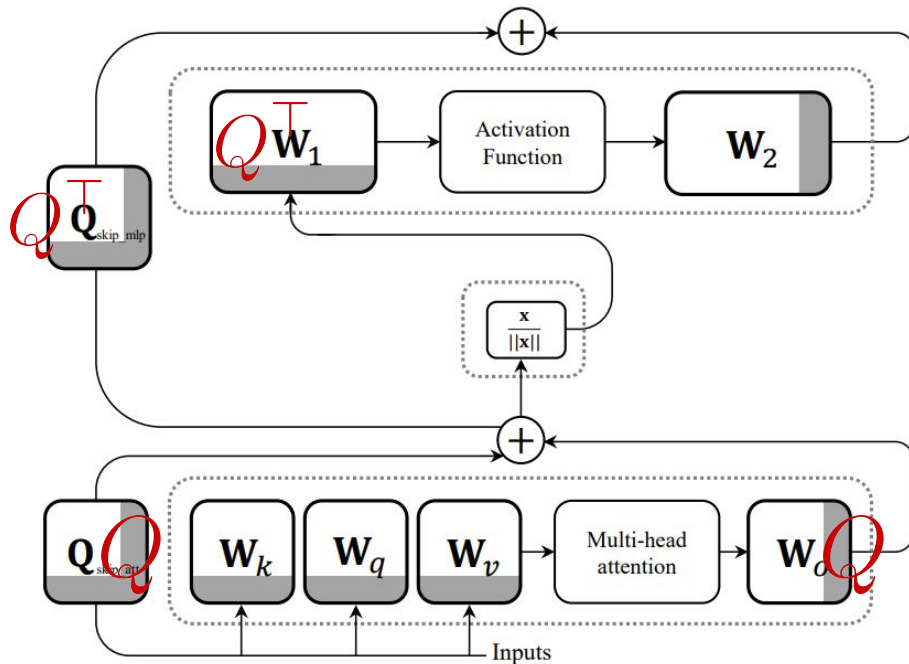


(a) A standard transformer block.

SliceGPT introduces rotational symmetries:

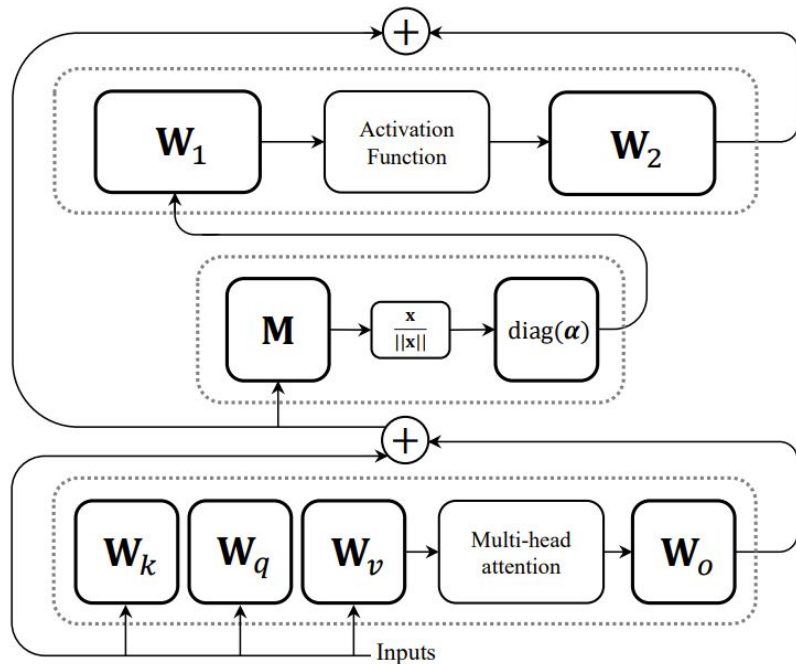


(a) A standard transformer block.

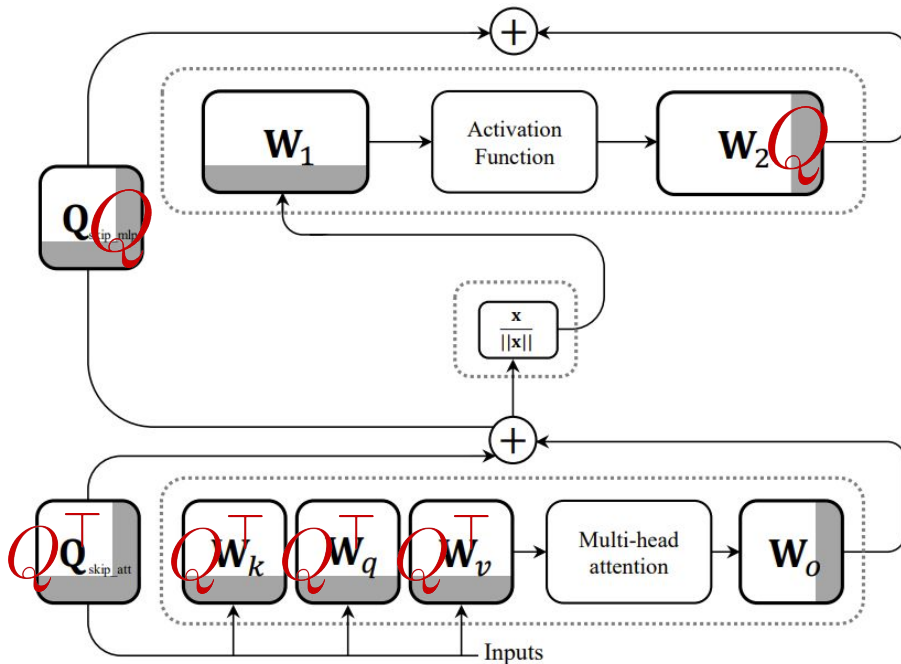


(b) A transformer block with SliceGPT.

SliceGPT introduces rotational symmetries:



(a) A standard transformer block.








(b) A transformer block with SliceGPT.






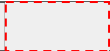

SliceGPT introduces rotational symmetries:

we can write each transformer block as:

$$f(x, W) = f(x, QW)$$

source coding:  + x = 

“AC way”:  + x =  + 
=  

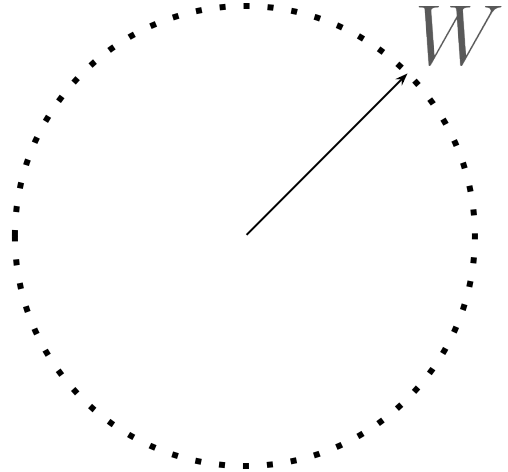
“bits-back way”:  + x =   + 
=   

**Getting free bits back from rotational
symmetries**

$$f(x, W) = f(x, QW)$$

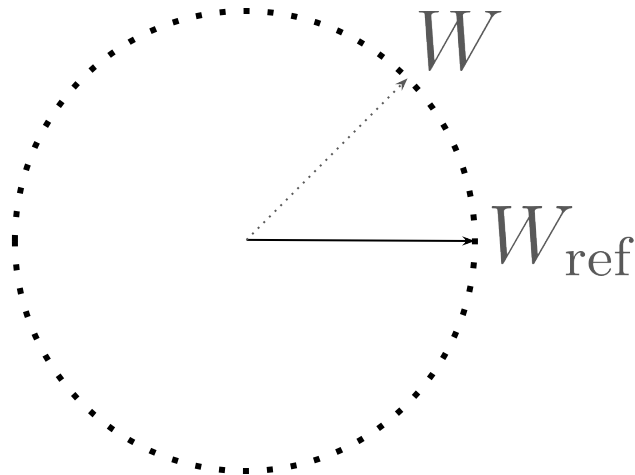
encode:

encode: start with a weight matrix and some initial bits



encode step 1: rotate weight matrix to a “canonical” direction

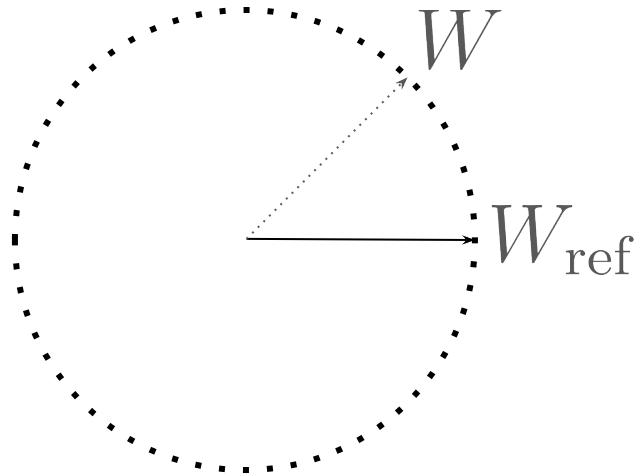
Initial bits



encode step 1: rotate weight matrix to a “canonical” direction

$$\text{svd}(W) = U \underbrace{[\Sigma V^T]}_{\bar{W}_{\text{ref}}}$$

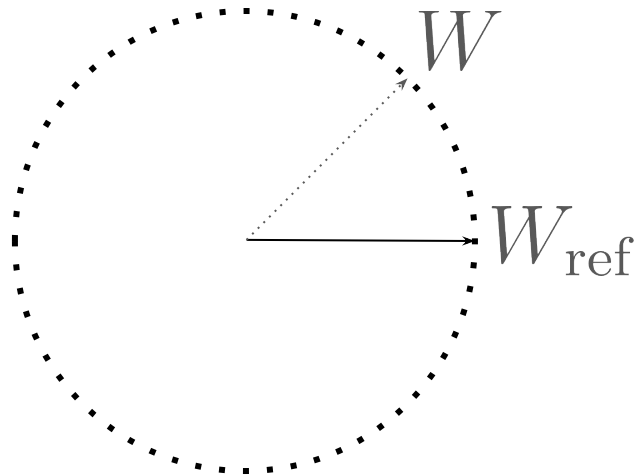
Initial bits



encode step 1: rotate weight matrix to a “canonical” direction

$$\text{svd}(W) = U \underbrace{(\Sigma V^\top)}_{\bar{W}_{\text{ref}}} \quad \text{i.e., define } W_{\text{ref}} W_{\text{ref}}^\top \text{ to be diagonal}$$

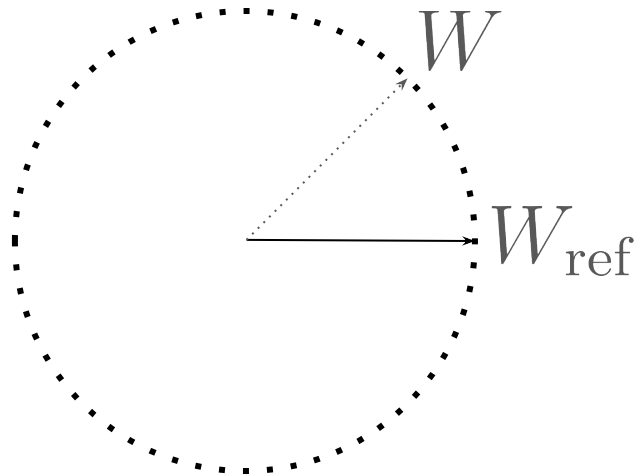
Initial bits



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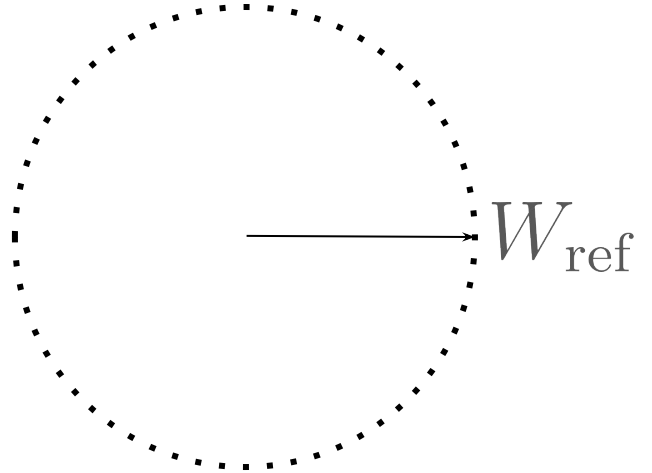
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Initial bits

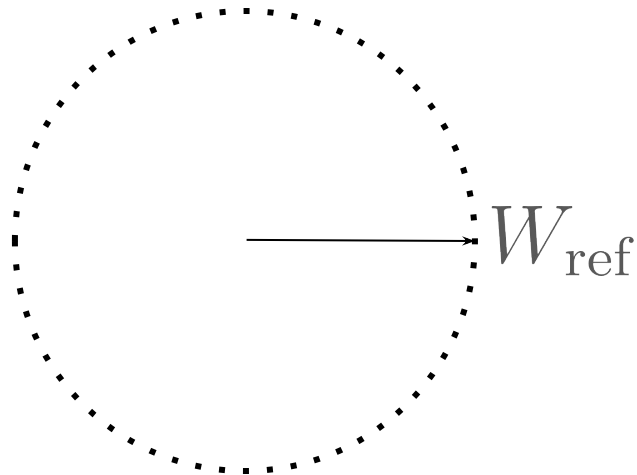


encode step 2: decode a rotation from the bitstream

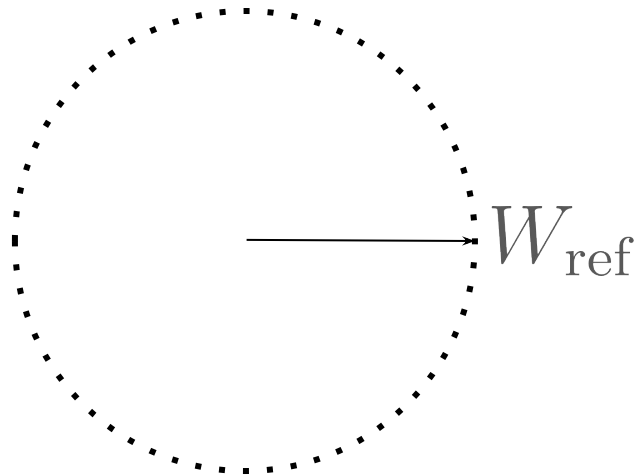
Initial bits



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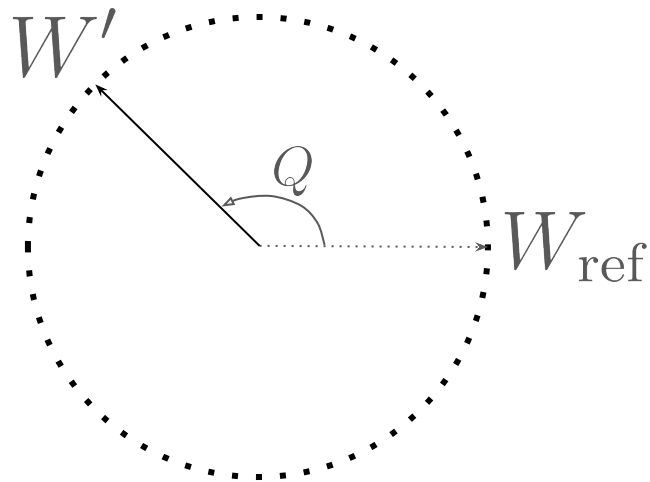
encode step 3: rotate weight by the decoded rotation matrix



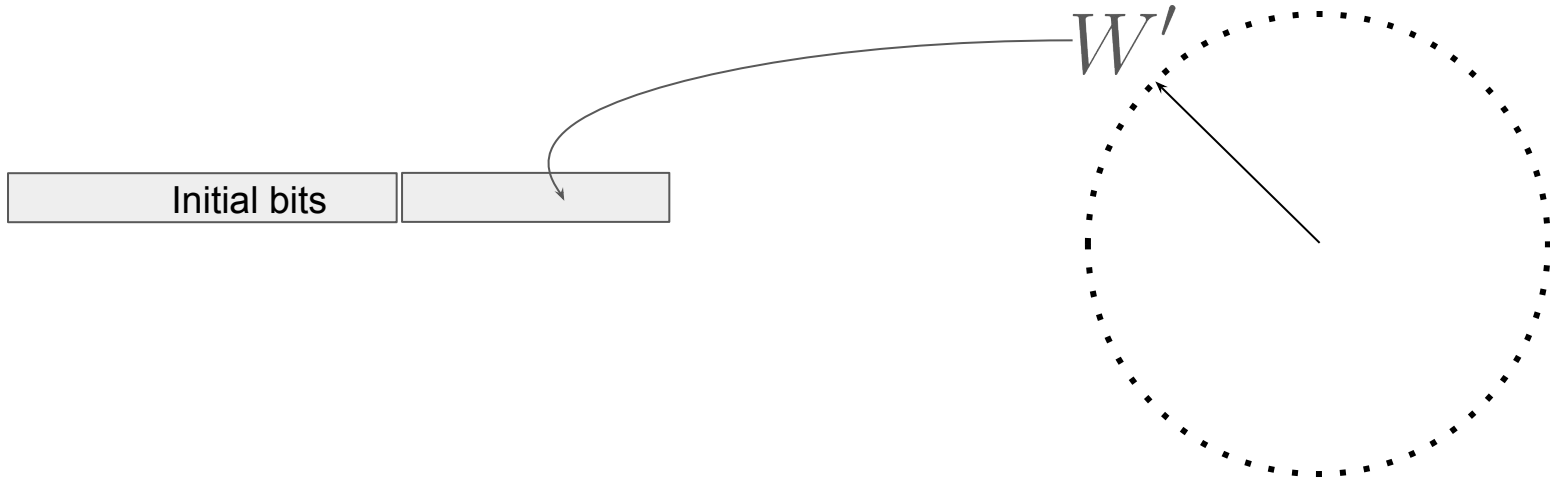
encode step 3: rotate weight by the decoded rotation matrix



$$W' = QW_{\text{ref}}$$

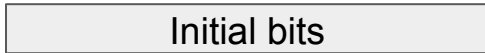


encode step 4: encode the rotated weight matrix

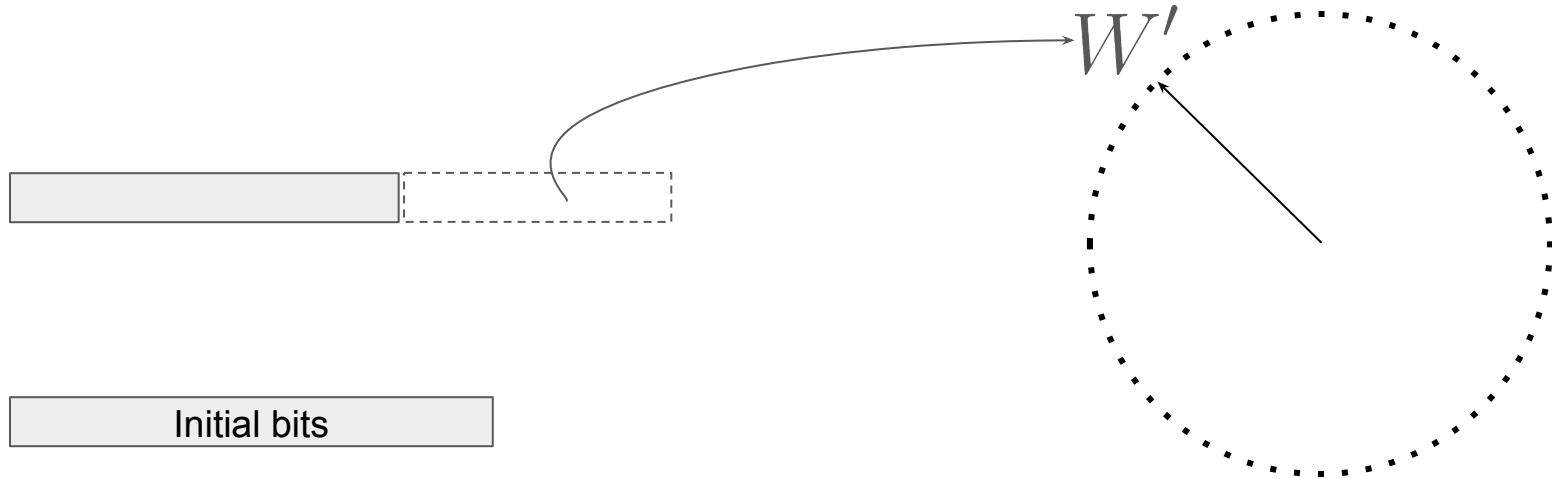


decode:

decode: start with some bits



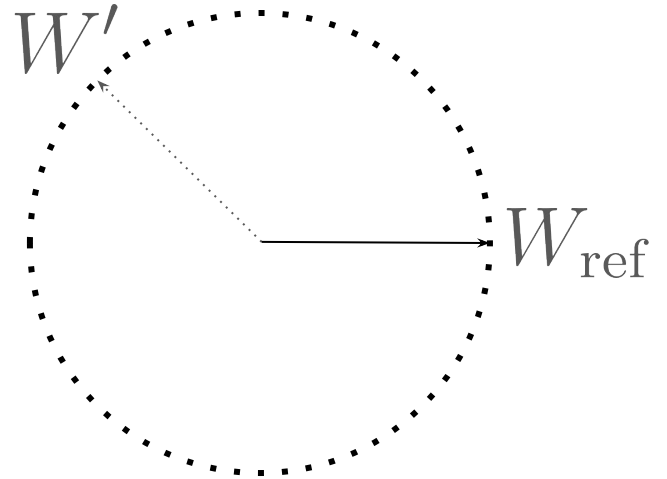
decode step 1: decode the rotated weight matrix



decode step 2: rotate it to “canonical” direction



Initial bits

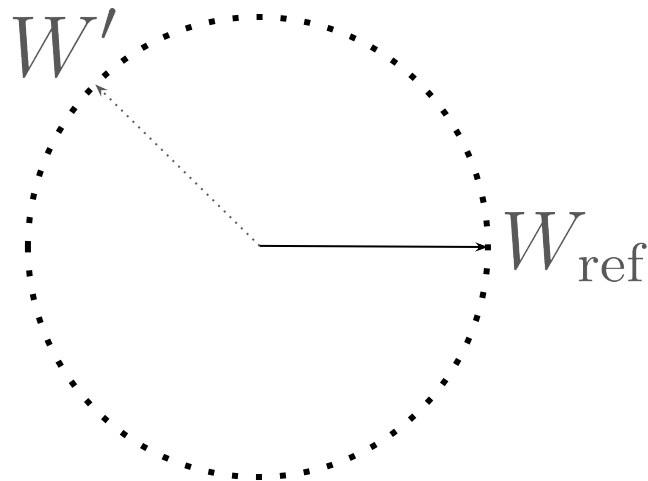


decode step 2: rotate it to “canonical” direction

recall we define $\text{svd}(W) = U \begin{bmatrix} \Sigma & V^T \\ \hline \hline \hline \end{bmatrix} \begin{matrix} \\ \\ \hline \hline \hline \end{matrix}$
 \bar{W}_{ref}



Initial bits



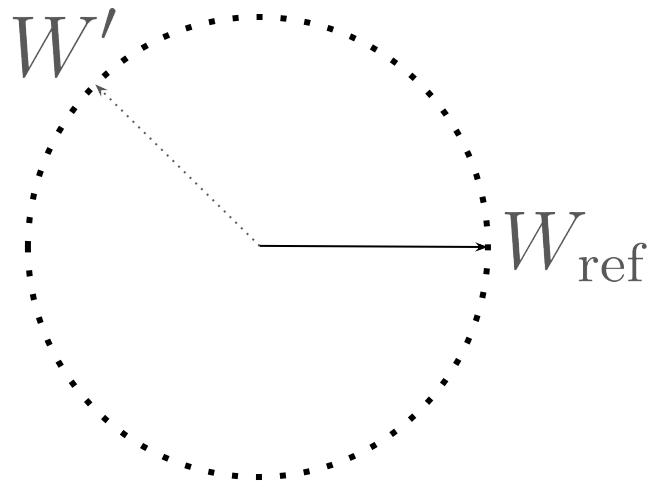
decode step 2: rotate it to “canonical” direction

recall we define $\text{svd}(W) = U \underbrace{[\Sigma V^\top]}_{\bar{W}_{\text{ref}}}$

$$\text{svd}(W') = \underbrace{[\sigma Q]}_{\hat{Q}} \Sigma V^\top \sigma, \sigma = \text{diag}(\pm 1, \dots, \pm 1)$$



Initial bits



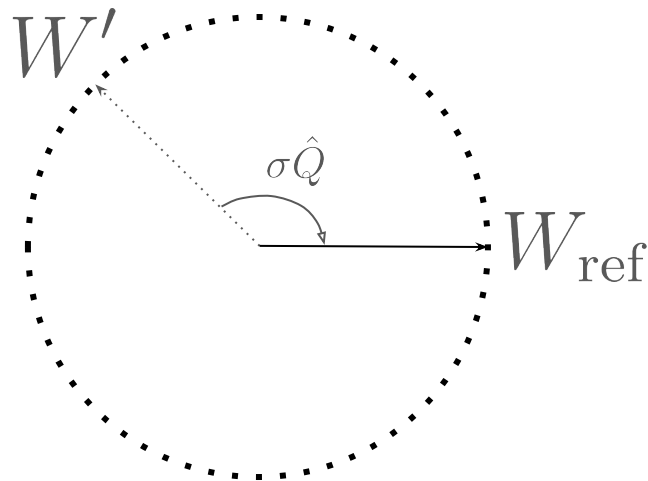
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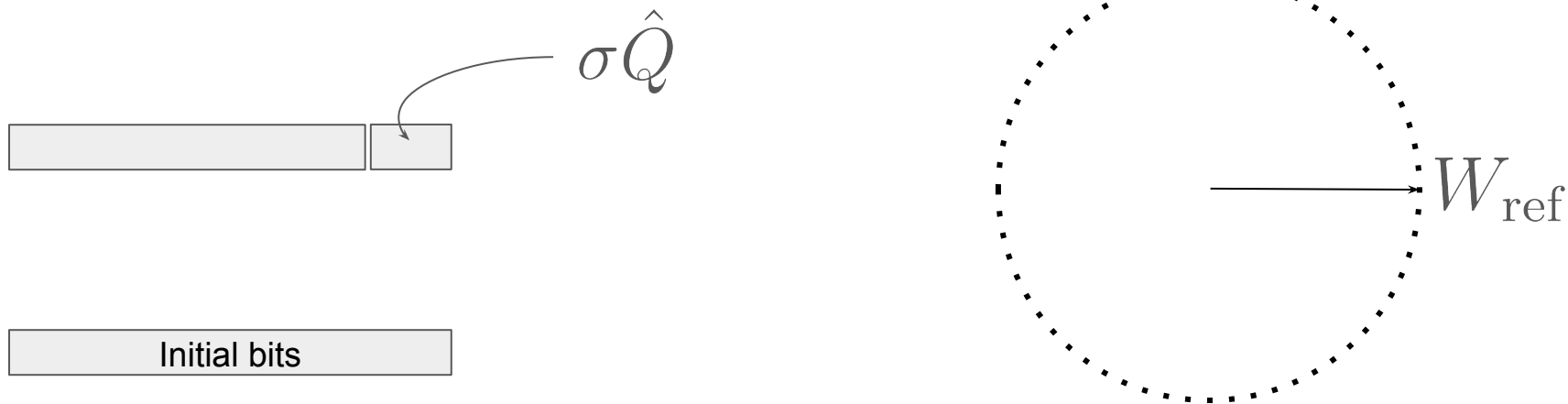
$$\text{svd}(W') = \underbrace{[\sigma \hat{Q}]}_{\hat{Q}} \Sigma V^\top \sigma, \sigma = \text{diag}(\pm 1, \dots, \pm 1)$$



Initial bits



decode step 3: encode the recovered rotation matrix



encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

decoding:

3. encode the rotation
2. rotate weight to canonical direction
1. decode the rotated matrix

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does rotation need infinite precision?

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1. rotation to canonical direction
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decoding:

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1. decode the rotated matrix

 **does rotation need infinite precision?**

 **yes. but just using float16 also works well!**

encoding:

1. rotation to canonical direction

2. decode a rotation

3. rotate weight matrix

4. encode the rotated matrix

decoding:

3. encode the rotation

2. rotate weight to canonical direction

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but there may be numerical error...

encoding:

decoding:

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\neq

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We can send correction code if the error is too large!



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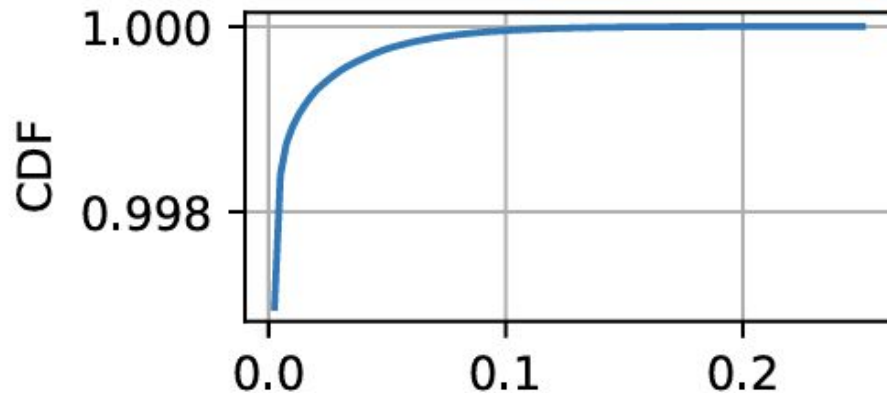
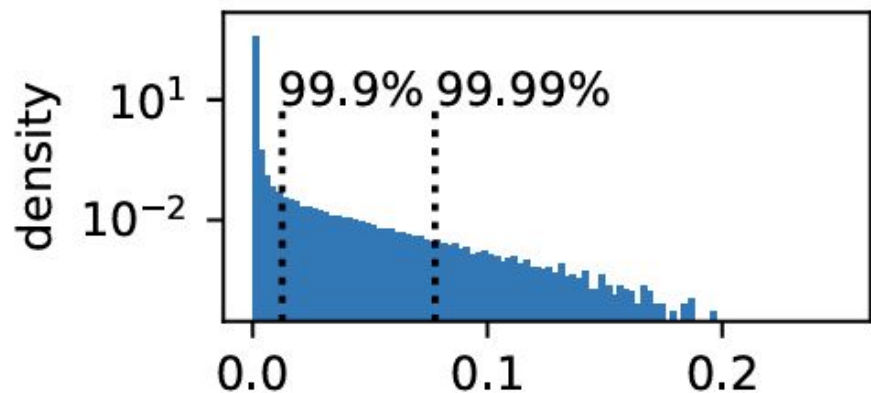


But will this correction code becomes too large?

😞 but there may be numerical error...

💡 We can send correction code if the error is too large!

😞 But will this correction code becomes too large? NO!



Reconstructure Error

Results

Results

Model	SliceGPT Slicing	Compress Rate after SliceGPT	Compress Rate after bits-back	Performance (before/after bits-back)			
				PPL (↓)	PIQA (%, ↑)	WinoGrande (%, ↑)	HellaSwag (%, ↑)
OPT-1.3B	20%	-9.53%	-13.77%	16.59 /16.60	64.91 /64.80	54.78 /54.38	45.26/ 45.32
	25%	-14.84%	-18.61%	17.78 /17.86	63.55 /63.33	52.80/ 53.28	43.20 /43.11
	30%	-20.53%	-23.81%	19.60 /19.66	60.88 /60.50	52.88/ 53.28	40.25 /40.06
OPT-2.7B	20%	-9.19%	-13.84%	13.89 /13.95	68.44 /68.12	58.88 /58.72	51.35 /51.17
	25%	-15.07%	-19.09%	14.85 /14.87	66.70 /66.76	57.30/ 57.70	48.41 /48.38
	30%	-20.88%	-24.43%	16.31 /16.33	64.64/ 64.69	55.80/ 56.04	44.52/ 44.57
OPT-6.7B	20%	-9.29%	-14.07%	11.63 /11.71	72.91/ 73.01	61.33 /61.17	60.53/ 60.55
	25%	-15.16%	-19.29%	12.12 /12.15	71.00/ 71.22	60.30/ 60.77	57.76 /57.55
	30%	-21.18%	-24.84%	12.81 /12.91	69.31/ 69.42	59.75 /59.59	53.64 /52.94
OPT-13B	20%	-9.18%	-14.01%	10.75 /10.77	74.27/74.27	64.96 /64.88	65.74/ 65.79
	25%	-15.27%	-19.51%	11.08/ 11.07	74.27 /73.72	63.46/ 63.93	63.48 /63.09
	30%	-21.29%	-24.97%	11.55 /11.59	72.69/ 73.01	61.96/ 62.43	60.12 /60.05
Llama-2-7B	20%	-9.38%	-14.13%	6.86 /6.98	69.53 /69.42	64.17/ 64.72	58.96 /58.89
	25%	-15.34%	-19.53%	7.56 /7.59	67.03/ 67.57	62.98/ 63.38	54.29 /53.93
	30%	-21.45%	-25.09%	8.63 /8.69	64.69 /64.09	62.75 /62.12	49.13 /49.07

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3-5% additional bits saving

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negligible influence on performance

Encoding and Decoding time

GPU:

Model Name	OPT-1.3B		OPT-2.7B		OPT-6.7B		OPT-13B	
Slicing	20%	30%	20%	30%	20%	30%	20%	30%
Encoding time	15 s	13 s	30 s	24 s	2.5 min	1.7 min	6.5 min	4.1 min
Decoding time	6 s	5 s	14 s	11 s	1.2 min	45 s	2.5 min	2 min

CPU:

Model Name	OPT-1.3B		OPT-2.7B		OPT-6.7B		OPT-13B	
Slicing	20%	30%	20%	30%	20%	30%	20%	30%
Encoding time	3.9 min	3.5 min	8 min	6.5 min	30 min	25 min	84 min	68 min
Decoding time	1.5 min	1.5 min	3.5 min	2.5 min	12 min	10 min	30 min	24 min

In summary:

save 3-5% additional bits,

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no influence on performance,

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save 3-5% additional bits,
no influence on performance,
a little overhead in model loading time