Teďż˝ria automatickďż˝ho riadenia III.

Cviďż˝enie II, Diskrďż˝tne stavovďż˝ modely

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Odvodenie spojitďż˝ho stavovďż˝ho modelu

Dynamickďż˝ systďż˝m n-tďż˝ho rďż˝du v tvare:

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + ... + a_0 y(t) = u(t)$$

Mďż˝ďż˝eme rozbiďż˝ na n diferenciďż˝lnych rovnďż˝c prvďż˝ho rďż˝du, kde:

$$\begin{aligned}
x_1(t) &= y(t) \\
x_2(t) &= \frac{dy(t)}{dt^{n-1}} \\
&\dots \\
x_n(t) &= \frac{d^{n-1}y(t)}{dt}
\end{aligned}$$

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= \dot{x}_1 = \frac{dy(t)}{dt} = x_2(t) \\
\frac{dx_2(t)}{dt} &= \dot{x}_2 = \frac{d^2 y(t)}{dt^2} = x_3(t) \\
&\dots \\
\frac{dx_n(t)}{dt} &= \dot{x}_n = \frac{d^n y(t)}{dt^n}
\end{aligned}$$

Odvodenie spojitď ż˝ho stavovď ż˝ho modelu

Z toho sa dď ż˝ poskladaď ż˝ stavovď ż˝ model v maticovom tvare:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t)$$

Diskretizďż˝cia stavovďż˝ho modelu

Spojitďż˝ stavovďż˝ model v tvare:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)\end{aligned}$$

Transformujeme do diskrďż˝tneho tvaru:

$$\begin{aligned}\boldsymbol{x}(k+1) &= \boldsymbol{A_d}\boldsymbol{x}(k) + \boldsymbol{B_d}\boldsymbol{u}(k)\\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k)\end{aligned}$$

kde,

$$\begin{aligned}\boldsymbol{A_d} &= e^{\boldsymbol{A}T}\\ \boldsymbol{B_d} &= \int_0^T e^{\boldsymbol{A}\lambda} d\lambda \boldsymbol{B}\\ \lambda &\in [0, kT]\end{aligned}$$

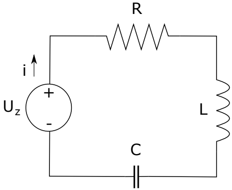
Simulďż˝cia diskrďż˝tneho modelu v Matlabe

- definovanie matďż˝c stavovďż˝ho modelu ***A***, ***B***, ***C***, ***D***
- vytvorenie objektu stavovďż˝ho modelu prďż˝kazom "**ss**"
- definovanie periďż˝dy vzorkovania T
- diskretizďż˝cia stavovďż˝ho modelu pomocou "**c2d**"
- definovanie poďż˝iatoďż˝nďż˝ho stavu $x0$
- rekurentnďż˝ simulďż˝cia diskrďż˝tneho systďż˝mu (presne tak ako ste to robili v TAR II)

Zadanie

Vytvorte diskrďż˝tny stavovďż˝ model, a urobte simulďż˝ciu systďż˝mu:

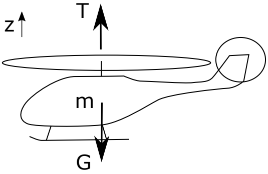
a) RLC obvod



$$\frac{di(t)}{dt} = -\frac{R}{L}i(t) - \frac{1}{LC}\int i(t)dt + \frac{1}{L}u_Z$$

$$u_C = \frac{1}{C}\int i(t)dt \rightarrow \frac{du_C(t)}{dt} = \frac{1}{C}i(t)$$

b) Vertikďż˝lna dynamika vrtulnďż˝ku



$$\frac{d^2z}{dt^2} = \frac{1}{m}F$$

$$F = T - G$$

Zadanie

a) RLC obvod

- odvodenie stavovď ż˝ho modelu
- stavovď ż˝ vektor vo forme: $\begin{bmatrix} u_C \\ i \end{bmatrix}$
- periď ż˝da vzorkovania $T_s = 1x10^{-6}s$
- parametre $R = 100\Omega$, $L = 2x10^{-3}H$, $C = 1x10^{-8}F$
- urobte diskrď ż˝tnu simulď ż˝ciu skokovej zmeny napď ż˝tia $u_Z = 10V$
- urobte diskrď ż˝tnu simulď ż˝ciu z poď ż˝iatoď ż˝nď ż˝ho stavu $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$ pri $u_Z = 0V$
- simulujte 400 krokov / 400 krokov
- do grafu vykreslite vď ż˝etky stavy (2)

b) Vertikď ż˝lna dynamika vrtulnď ż˝ku

- odvodenie stavovď ż˝ho modelu
- stavovď ż˝ vektor vo forme: $\begin{bmatrix} z \\ \dot{z} \end{bmatrix}$
- periď ż˝da vzorkovania $T_s = 0.05s$
- parametre $m = 2200kg$, $g = 9.81m/s^2$, $G = mg$
- urobte diskrď ż˝tnu simulď ż˝ciu skokovej zmeny ď ż˝ahu rotora $T = 30000N$
- urobte diskrď ż˝tnu simulď ż˝ciu z poď ż˝iatoď ż˝nď ż˝ho stavu $\begin{bmatrix} 0 \\ 10 \end{bmatrix}$ pri $T = 0N$
- simulujte 40 krokov / 40 krokov
- do grafu vykreslite vď ż˝etky stavy (2)