Teória automatického riadenia III.

Cvičenie IX, Ohraničenia a kvadratický problém

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Na dnešnom cvičení

- Zostavenie matíc ohraničenia pre MPC
- Simulácia MPC regulátora pomocou kvadratického programovania

Z prednášky

Minimalizujeme:

$$\vec{\boldsymbol{u}}_k^* = arg \min_{\vec{\boldsymbol{u}}_k} J(\boldsymbol{x}_k, \vec{\boldsymbol{u}}_k)$$

pričom:

$$\underline{u} \leq u_k \leq \bar{u}$$
$$\underline{x} \leq x_k \leq \bar{x}$$

Z prednášky

Celkový problém:

$$\min_{\vec{\boldsymbol{u}}_k} J(\boldsymbol{x}_k, \vec{\boldsymbol{u}}_k) = \sum_{i=0}^{n_p-1} (\boldsymbol{x}_{k+i}^T \boldsymbol{Q} \boldsymbol{x}_{k+i} + \boldsymbol{u}_{k+i}^T \boldsymbol{R} \boldsymbol{u}_{k+i}) + \boldsymbol{x}_{k+n_p}^T \boldsymbol{P}_f \boldsymbol{x}_{k+n_p}$$

pri

$$\begin{aligned}
\underline{u} \leq u_{k+i} \leq \bar{u} \quad & , i = 0, 1, 2, ..., n_p - 1 \\
\underline{x} \leq x_{k+i} \leq \bar{x} \quad & , i = 1, 2, 3, ..., n_p \\
x_{k+0} = x_k \quad & \\
x_{k+i+1} = \boldsymbol{A} x_{k+i} + \boldsymbol{B} u_{k+i} \quad & , i \geq 0 \\
y_{k+i} = \boldsymbol{C} x_{k+i} \quad & , i \geq 0 \\
u_{k+i} = \boldsymbol{K} x_{k+i} \quad & , i \geq n_p
\end{aligned}$$

Zostavenie matíc ohraničenia

Transformácia matematickej podoby kriteriálnej funkcie z:

$$J_k = \boldsymbol{u}_k^T \boldsymbol{H} \boldsymbol{u}_k + 2x_k^T \boldsymbol{G}^T \boldsymbol{u}_k$$

do:

$$f(\boldsymbol{u}) = \frac{1}{2} \boldsymbol{u}^T \boldsymbol{H} \boldsymbol{u} + \boldsymbol{G}^T \boldsymbol{u}$$

teda, $\boldsymbol{H} \longrightarrow \boldsymbol{H}$ a $\boldsymbol{G} \longrightarrow \boldsymbol{G} x_k$

Zostavenie matíc ohraničenia

A ohraničení:

$$\underline{u} \leq u_{k+i} \leq \bar{u}$$
$$\underline{x} \leq x_{k+i} \leq \bar{x}$$

do

$$\boldsymbol{A}_c \boldsymbol{u}_k \leq \boldsymbol{b}_0 + \boldsymbol{B}_0 x_k$$

Dá sa napísať,

$$\begin{aligned} \underline{u} \leq \quad u_k & \leq \bar{u} \\ u_k & \leq \bar{u} \\ -u_k & \leq -\underline{u} \end{aligned} \qquad\qquad \begin{aligned} \underline{x} \leq \quad x_k & \leq \bar{x} \\ x_k & \leq \bar{x} \\ -x_k & \leq -\underline{x} \end{aligned}$$

Zostavenie matíc ohraničenia

Matice ohraničenia vstupu:

$$
\begin{aligned}
u_k &\leq \bar{u} \\
-u_k &\leq -\underline{u} \\
&\Downarrow \\
\begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{bmatrix} \boldsymbol{u}_k &\leq \begin{bmatrix} \mathbf{1}\bar{u} \\ -\mathbf{1}\underline{u} \end{bmatrix}
\end{aligned}
$$

Matice ohraničenia stavov:

$$
\begin{aligned}
x_k &\leq \bar{x} \\
-x_k &\leq -\underline{x} \\
&\Downarrow \\
x_{k+1} &= \boldsymbol{M}_i x_k + \boldsymbol{N}_i \boldsymbol{u}_k \\
&\Downarrow \\
\begin{bmatrix} \boldsymbol{N}_i \\ -\boldsymbol{N}_i \end{bmatrix} \boldsymbol{u}_k &\leq \begin{bmatrix} \bar{x} \\ -\underline{x} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{M}_i \\ \boldsymbol{M}_i \end{bmatrix} x_k
\end{aligned}
$$

Zostavenie matíc ohraničenia

Toto skombinujeme:

$$
\begin{aligned}
\begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{bmatrix} \boldsymbol{u}_k &\leq \begin{bmatrix} \mathbf{1}\bar{u} \\ -\mathbf{1}\underline{u} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} x_k \\
&+ \\
\begin{bmatrix} \boldsymbol{N}_i \\ -\boldsymbol{N}_i \end{bmatrix} \boldsymbol{u}_k &\leq \begin{bmatrix} \bar{x} \\ -\underline{x} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{M}_i \\ \boldsymbol{M}_i \end{bmatrix} x_k \\
&\Downarrow \\
\boldsymbol{A}_c \boldsymbol{u}_k &\leq \boldsymbol{b}_0 + \boldsymbol{B}_0 x_k
\end{aligned}
$$

Kvadratické programovanie

Toto potom nakŕmime do riešiča kvadratického problému:

$$
\begin{aligned}
f(\boldsymbol{u}_k) &= \frac{1}{2}\boldsymbol{u}_k^T \boldsymbol{H}\boldsymbol{u}_k + \boldsymbol{G}^T\boldsymbol{u}_k \\
\boldsymbol{A}_c\boldsymbol{u}_k &\leq \boldsymbol{b}_0 + \boldsymbol{B}_0 x_k
\end{aligned}
$$

```
[u_opt, f] = quadprog(H,G,Ac,b0)
[u_opt, f] = quadprog(H,G*x,Ac,b0+B0*xk)
```

Zadanie

- načítajte váš identifikovaný model nosníka
- vytvorte LQ regulátor
- vytvorte matice **M**,**N**,**H**,**G** na základe **Q** a **R** a pre počet krokov $n_p = 10$
- urobte simuláciu LQ riadenia nosníka so saturáciou
- urobte simuláciu MPC riadenia s obmedzeniami (bez obmedzení stavov)
- porovnajte LQ a MPC