3, CALCULUS

3.1, $$\sum_{i=0}^{\infty}\left(\frac{1}{8^i}+0{,}5^i\right)=\sum_{i=0}^{\infty}\frac{1}{8^i}+\sum_{i=0}^{\infty}\frac{1}{2^i}=\frac{1}{1-\frac{1}{8}}+\frac{1}{1-\frac{1}{2}}=$$

$$=\frac{1}{\frac{7}{8}}+\frac{1}{\frac{1}{2}}=\frac{8}{7}+2=\underline{\underline{\frac{22}{7}}}$$

3.2, $$\lim_{x\to 3}\frac{x-3}{2}\qquad \frac{x-3}{2}=\frac{x}{2}-\frac{3}{2}$$

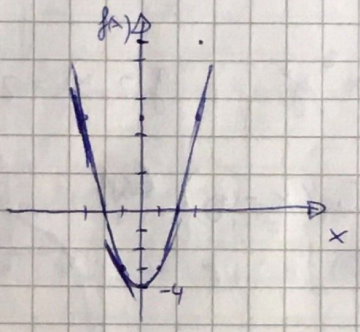
$$x=3 \Rightarrow \frac{3}{2}-\frac{3}{2}=\underline{\underline{0}}$$

3.3, $f(x)=x^2-4$

$f'(x)=2x$

$f'(-1)=-2$

$f'(-3)=-6$



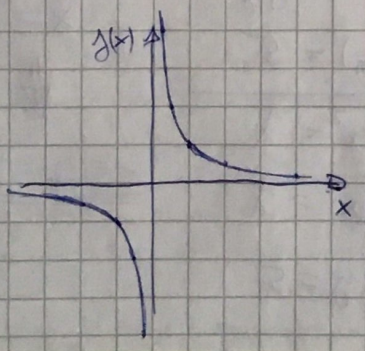
3.4, $$\frac{d}{dx}\frac{x^2+3}{x+2}=\frac{\left(\frac{d}{dx}x^2+3\right)(x+2)-\left(x^2+3\right)\left(\frac{d}{dx}x+2\right)}{(x+2)^2}=$$

$$=\frac{2x(x+2)-(x^2+3)}{x^2+4x+4}=\frac{2x^2+4x-x^2+3}{(x+2)^2}=$$

$$=\frac{x^2+4x+3}{(x+2)^2}=\underline{\underline{\frac{(x+1)(x+3)}{(x+2)^2}}}$$

3.5, $$\frac{d^2}{dx^2}4x^3+4=\frac{d}{dx}12x^2=24x$$

3.6, $f(x)=\frac{1}{x}$



$\lim_{x\to-\infty}\frac{1}{x}=-\infty$

$\lim_{x\to+\infty}\frac{1}{x}=\infty$

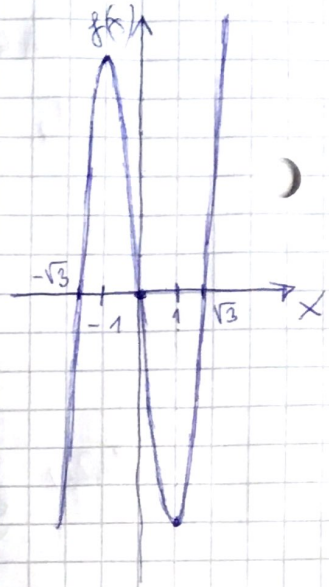
$\Rightarrow$ cannot be continuous at $\emptyset$

3.7) $3x^3 - 9x = f(x)$

$f(x) = 3x(x^2-3) = 3x(x-\sqrt{3})(x+\sqrt{3})$

$f'(x) = 9x^2 - 9 = 9(x-1)(x+1)$

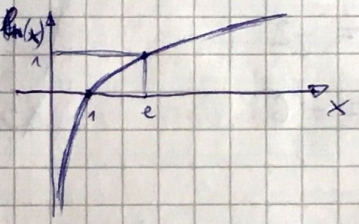
$f''(x) = 18x \Rightarrow$ 1 inflection point @ $\emptyset$



		$-\sqrt{3}$		-1		$\emptyset$		1		$\sqrt{3}$	
$f(x)$	−	$\emptyset$	+	+	+	$\emptyset$	−	−	−	$\emptyset$	+
$f'(x)$	+	+	+	$\emptyset$	−	−	−	$\emptyset$	+	+	+
$f''(x)$	−	−	−	−	−	$\emptyset$	+	+	+	+	+
		concave						convex			

3.8) $f(x,y) = x^2 y^3$; $f(2,3) = 2^2 \cdot 3^3 = 4 \cdot 27 = \underline{\underline{108}}$

3.9) $f(x,y) = \ln(x-y)$



$x - y > \emptyset$

$\underline{\underline{x > y}}$

3.10) $\frac{\partial^2}{\partial x^2} x^5 + x y^3 = \frac{\partial}{\partial x} 5x^4 + y^3 = \underline{\underline{20x^3}}$

3.11) $f(x,y) = \sqrt{xy} - 0{,}5x - 0{,}5y = (xy)^{\frac{1}{2}} - \frac{1}{2}x - \frac{1}{2}y$

$f'_x = \frac{1}{2} y \cdot x^{-\frac{1}{2}} - \frac{1}{2} = \frac{y}{2\sqrt{x}} - \frac{1}{2} \Rightarrow \frac{y}{2\sqrt{x}} - \frac{1}{2} = \emptyset$

$f'_y = \frac{1}{2} x y^{-\frac{1}{2}} \frac{1}{2} = \frac{x}{2\sqrt{y}} - \frac{1}{2}$ $\quad \frac{y}{2\sqrt{x}} = \frac{1}{2}$

$\hookrightarrow \underline{\underline{x = \sqrt{y}}}$ $\quad \underline{\underline{y = \sqrt{x}}}$

3.12) $f(x,y) = x^2 y^2$

$g(x,y) = (x+y) = 5$

$f'_x = 2xy^2$ $\quad g'_x = 1$

$f'_y = 2yx^2$ $\quad g'_y = 1$

$f_x = \lambda g_x$

$f_y = \lambda g_y$

$g(x,y) = 5$

$2xy^2 = \lambda$

$2yx^2 = \lambda$

$x + y = 5$

$2yx^2 = 2xy^2$

$x = y$

$\Downarrow$

$\underline{\underline{x = y = 2{,}5}}$