Application limits of runaway electron modeling based on analytical formulas of generation and loss rates

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Introduction Runaway electrons pose one of the most critical problems of reactor-size tokamaks and a serious challenge for modeling at the same time. Description of runaway electrons is fundamentally a kinetic problem, however, self-consistent plasma simulations demand less computationally expensive models than the full kinetic description. This has brought to life an approach of modeling the evolution of the first moments of the runaway electron distribution function (namely density and current) by using analytical estimates of runaway generation and loss rates derived as approximative solutions of the kinetic problem. This allows for a self-consistent simulation of tokamak disruptions including energy balance affected by impurity penetration and electric field diffusion along with the dynamics of runaway electron current [1].

Recently a three-level modeling approach was adopted to runaway electron simulation within the European Integrated Modeling (EU-IM) framework [2]. The first level of modeling is limited to the indication if runaway electron generation is possible or likely. The second level adopts a similar approach as described above, using analytical formulas to estimate changes in the runaway electron density and current. The third level is foreseen to be based on the solution of the full electron kinetics using the LUKE code [3].

This paper analyses the limitations of the first two levels of modeling in view of recent theoretical and numerical results in runaway electron theory.

Models for indication of runaway electrons Within the EU-IM framework [2], a "Runaway Indicator" module ("*actor*" in a Kepler workflow) was developed to provide an indication for when to expect run-away tail formation. This *actor* is integrated into the "Instantaneous events" *composite actor* of the European Transport Simulator (ETS) [4] to provide warning messages of possible runaway electron generation during simulation-time. Another possibility is to do the check as post-processing of simulations in a separate workflow to provide confirmation that no runaway electrons were generated when their generation was unexpected.

The most obvious parameter to check is if the toroidal electric field exceeds the critical electric field (E_c) needed for the existence of runaway electrons by definition [5]:

*See http://www.euro-fusionscipub.org/eu-im.

$$E_c = n_e \frac{e^3 \ln\Lambda}{4\pi\varepsilon_0^2 m_0 c^2}, \tag{1}$$

which only depends on the electron density n_e and constants - there is only a weak dependence on temperature through the Coulomb logarithm $\ln\Lambda$.

This criterion can be used as a conservative check, however we do not tend to see runaway electrons in experiments just above this threshold, which is mostly explained by the runaway electron generation rate being limited to very low values for near-critical electric field [6]. For this reason a second check was implemented to verify if the primary Dreicer generation rate is above a pre-defined threshold. A convenient choice for the formula for the Dreicer generation rate is the final (67) formula in the classical paper by Connor and Hastie [5], which is also used in the GO code [1]. However, one should be aware that this formula is the non-relativistic approximation of formula (66) of the same paper, which is in turn a high electric field approximation of formula (63). It is evident from Figure 1 that there is a region of the parameter space where these formulas give significantly different results. This high temperature ($T > 500$ eV and low electric field ($E/E_c < 10$) region is not particularly relevant in disruptions, that is the typical application of GO [1], however, it is just the region that might occur in the present ETS simulations [4]. In this case, the applied formula (67) is a conservative approximation for distribution functions close to Maxwellian.

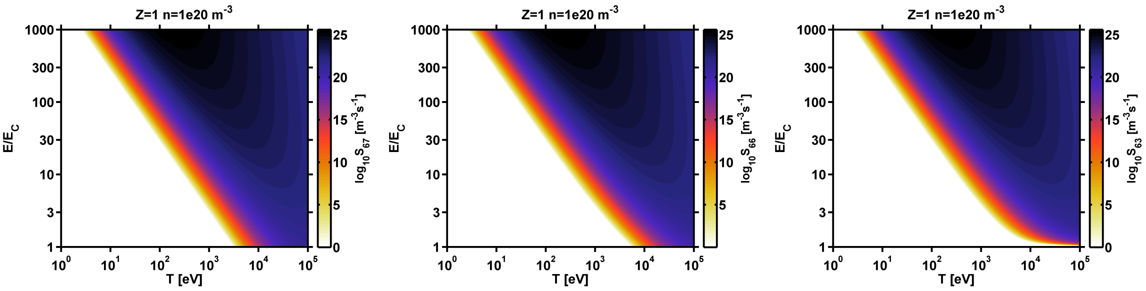

Figure 1: *Comparison of Dreicer-generation growth rates for the different approximations of formulas (67), (66) and (63) of the classical paper by Connor and Hastie [5].*

It was also shown that the critical electric field of equation 1 and the Dreicer growth rate can be significantly modified by the momentum loss due to synchrotron emission [6, 7]. Taking this into account is not necessary for the runaway electron warning, but would be desired for the actual modeling of runaway electron current.

Runaway electron current estimation by analytical formulas A second *actor* was developed to provide an estimation of the non-inductive current due to runaway electrons using computationally cheap analytical estimates of runaway electron growth rates and transport. This *actor* is to be integrated into the "Heating & Current Drive Workflow" which will allow simple benchmarking with more advanced kinetic models.

In order to provide a reasonable estimate for the current carried by runaway electrons, a minimum of three mechanisms are to be considered: Dreicer generation, avalanche generation and runaway electron radial transport. Dreicer generation has been discussed in the previous section, but here the high T necessitates full relativistic description that is formula (63) of Connor and Hastie [5]. This agrees well with numerical calculations by Kulsrud [8] for moderately high electric field ($E/E_c \approx 100$), but at low electric field the correction due to synchrotron radiation [6, 7] will need to be handled. This could be achieved by applying the correction factor to the growth rate as given in Figure 3 of the paper by Stahl et.al. [6]. At low temperature, where the plasma is only partially ionized, we encounter a different problem as the stopping power becomes affected by the bounded electrons. For this effect no simple corrections exist at the moment [9], however, currently this is outside the typical application range of EU-IM workflows. For low aspect ratio tokamaks a correction factor for the effect of toroidicity is to be applied as suggested by Nilsson et.al. [10, 11].

The classical way of describing the avalanche was put forward by Rosenbluth and Putvinski [12]. Their formula for the growth rate is, however, only valid for high electric fields, when the momentum loss of runaway electrons in knock-on collisions is negligible compared to the strong momentum input from the electric field. At low electric field a momentum-conserving approach to the knock-on collisions is to be applied [7]. Figure 2 (based on Figure 4 of [7]) shows the comparison of the avalanche growth rate at low electric field calculated by the Rosenbluth and Putvinski model [12], and the model put forward by Aleynikov and Breizmann [7]. The new estimate of the avalanche growth rate features a threshold (E_a), and then quickly converges to the classical expression for higher electric field values. Here we propose an approximation of estimating the threshold electric field by formula (8) by the paper by Aleynikov et.al. [7], and using the Rosenbluth and Putvinski model for higher electric field values.

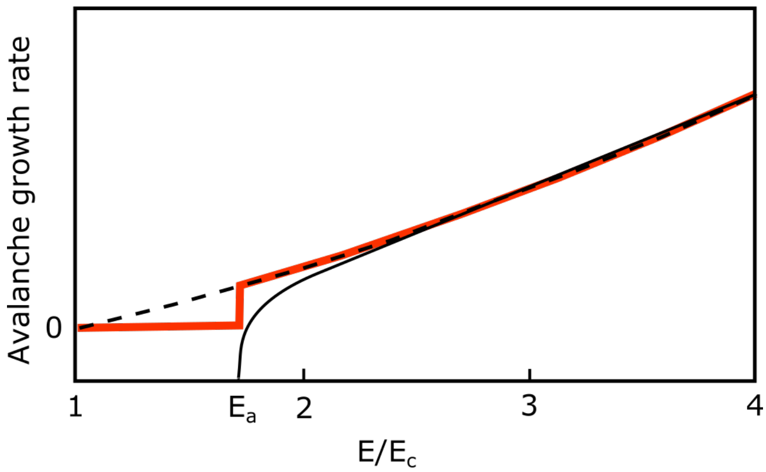

Figure 2: *Comparison of avalanche growth rate by Rosenbluth and Putvinski (dashed black line) [12] and the reduced kinetic model by Aleynikov and Breizmann (black line) as presented in [7]. Here proposed simple model is overplotted by red.*

Recent studies with LUKE showed that the avalanche growth rate can be significantly reduced at toroidal magnetic surfaces with high mirror ratio due to the trapping of the high energy electrons generated in the knock-on collisions. Formula (A.4) of the paper by Nilsson et.al. [11] (also published in [10] could be applied to have a flux-surface dependent correction factor.

Regarding the radial transport of runaway electrons the classical treatment is to use the diffusion approximation; either collisional or as derived by Rechester and Rosenbluth [13]. Recently the shortcomings of this approach were shown by a comparison with a result obtained by following the orbits of a large set of runaway electrons in a perturbed magnetic field [14]. A simple and correct treatment is still to be proposed. Meanwhile, the diffusive transport model is to be used, as it can reproduce one key aspect and that is that the radial transport is the dominating loss mechanism.

Conclusions In the present paper a modeling approach of runaway electron was put forward based on analytical kinetic formulas, which would be extremely useful in EU-IM [2]. Since at present the ETS does not address disruption conditions, a number of correction factors in the runaway model are needed, to account for high temperature and low electric field, as described in this paper. Following the implementation of the proposed model in the EU-IM framework, a benchmark within it is foreseen to full kinetic calculations, like LUKE [3].

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