Mixed Traffic Flow of Human-Driven and Autonomous Vehicles: Self-organized Clustering and Lane Formation

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ABSTRACT

Add any numbers if possible and talk about opportunistic behavior leading to clustering

This paper reveals the existence of self-organized clustering (platooning) and lane formation in mixed traffic flow of autonomous vehicles (AVs) and human-driven vehicles (HVs) based on simulation experiments. We propose a parsimonious Cellular Automata model to capture the different characters of AVs and HVs as well as their interactions. AVs are endowed with opportunistic behaviors, reflected through gap seeking and awareness of neighbor vehicle types. We compare clustering and lane formation properties and mixed flow flux in three scenarios, accounting for the impact of two distinct AV behavior types and a control case of homogeneous flow. We observe that, intriguingly, even with this relatively simple model, AVs demonstrate self-organized properties in the mixed traffic flow. AVs form into clusters (i.e. platoons) and even lanes on their own in the mixed flow, without centralized control. Such phenomena seem to relate to the intrinsic incentives that AVs perceive and their ability to tell neighbor vehicle types. This finding suggests the possibility of regulating mixed AV-HV flow through distributed incentives, rather than centralized coordination.

INTRODUCTION

Driver-less cars (i.e. autonomous vehicles, henceforth called AVs) have already appeared on the road and shared right of way with human-driven vehicles (henceforth called HVs). An appealing feature of AVs is their capability to form platoons, which is anticipated to boost efficiency of traffic flow substantially. A number of recent research have delved into modeling and operations of the mixed flow of AVs and HVs, most of them attempted to capture or optimize the platooning feature of AVs.

Much attention in literature (see e.g. [2][28][22][29][12][31]) has focused on problems along the longitudinal dimension and adopt a microscopic approach, which embraces simulation of car-following behaviors, platoon control, and stability analysis. While the understanding of microscopic longitudinal behaviors of mixed AV-HV flow has become relatively mature, the collective behaviors to be anticipated in a multi-lane setting remain elusive, where the interplay of AVs and HVs can be substantially more complicated, due to the possibility of lane choice and lane changing. The lateral behaviors are coupled with longitudinal dynamics, and the two together determine dynamics of mixed AV-HV flow. Discussions along this line are still primitive, mostly focused on the estimation of mixed flow capacity (see e.g. [5][11]) and aggregate flow-density relation (see e.g. [25][16]). These works all assume the mixed flow is in perfect steady state does not explicitly consider lateral interactions in general multilane setting. Concerning the impacts of lanes on traffic flow, several empirical studies [18][9][3][4] have been conducted towards understanding the impacts of HOV lanes on traffic in general purpose (GP) lanes, which revealed the existence of a smoothing effect caused by restriction of lane-changing. Past research also examines lane-to-lane traffic interactions on multilane highways, which include lane flow distribution [19][27], as well as impacts of microscopic lane-changes [15][1]. On the theoretical side, it is only in limited cases that the connection between lane policy and traffic flow characteristics has been investigated, and in almost all studies, the policy considered is static. [8] and [14] both considered traffic consisting of regular and priority vehicles, and a number of special lanes are accessible only to the priority vehicles. Both regular vehicles and priority vehicles are endowed with the identical fundamental diagram. Nonetheless, these studies are focused on human-driven traffic flow and do not consider any AV-specific behaviors. [5] discussed capacity of mixed flow of AVs and HVs on multi-lane freeway under different combination of static lane access policies, assuming traffic flow is steady. [11] derived capacity of mixed AV-HV flow in a similar context, assuming the spatial distribution of mixed flow follows a Markov process. An interesting lateral policy considered in literature is intermittent bus lane (IBL), which offers priority to buses, but also allows other regular vehicles to use the lane when space is available. [10] and [6] conducted queuing analysis using the LWR model and derived corresponding roadway capacity as well as conditions when IBL will benefit the entire system.

The key challenge to model mixed flow dynamics of AVs and HVs lies in capturing agents' longitudinal and lateral behaviors simultaneously, which should account for not only agents' dynamical characters (e.g. their longitudinal speed and acceleration as functions of headway), but also agents' decision-making, as well as how the decisions of individual agents aggregate. Such interactions in mixed flow of AVs and HVs is conceivably more complicated than mixed human-driving traffic due to the rich possibilities of AV behaviors. For instance, with the advanced sensing and communication capabilities, AVs may behave more opportunistically than HVs in seeking peers in its neighborhood, because this potentially allow them to form platoons with peers and benefit from more smooth driving and better traveling speed.

In this paper, we are interested in understanding the collective dynamics of AV-HV flow, as a result of the interactions between heterogeneous agents. In particular, we seek to answer the following research questions: when AVs are self-interested and individually controlled by themselves (as opposed to coordinated by system operator), will they be able to self-organize into platoons? And if so, what are the conditions for the self-organization to initiate and sustain? This research is partly inspired from the self-organized phenomena widely existing in biological systems, such as army ants [7], where orders emerge when agents interact with each other locally. In traffic flow literature, similar self-organized phenomena were also reported [13].

To answer the above questions, we propose a Cellular Automaton (CA) model to encapsulate the essential behavioral factors of AVs and HVs, and conduct simulation experiments to characterize the impact of these behavior factors on the formation of clusters in mixed traffic flow. The simulation experiments were conducted on a circular road with three lanes. We consider two alternatives of AV behaviors. In the first scenario, AVs behave like HVs, except the difference in free flow speed and braking probability. In the second scenario, AVs are assumed to be fully aware of vehicle types in neighborhood and opportunistic in gap seeking.

The major finding of this research is the self-organized formation of AV clusters and lanes without any centralized control. This finding may suggest clustering as an intrinsic property of mixed flow, when AVs and HVs interact, and AVs are endowed with opportunistic behaviors. The existence of self-organization, if turns out to exist in real mixed flow of AVs and HVs, may suggest the possibility of regulating their interactions through a decentralized approach.

The remaining part of the paper is organized as follows. We first introduce how we capture the different car-following and gap seeking behaviors of AVs and HVs through a Cellular Automata (CA) model. Then we describe the setup of simulation experiments along with results and discussion. In this section, we focus on the flux of mixed flow as well as the formation process of AV clusters in different behavioral scenarios. Self-organized behaviors are observed in both models of AV behavior. We summarize the findings and remark on future works in the last section.

CELLULAR AUTOMATA MODEL

Cellular Automata (CA) is a simple yet effective framework to model agent interactions over cells, which have found wide and successful applications modeling traffic flow. In CA, each agent occupies a cell, and its behavior is determined by its own state and the state of other agents in its neighborhood. Simple functions can be applied to these cell objects to change its state and as a result it can change the states of neighboring cells [30]. Nagel and Schreckenberg (1992) were amongst the first to use this computational technique to model traffic flow [21].

The CA model in traffic flow literature comprises a system of vehicles that evolve over linear time in accordance to rules (see e.g. [20][26]) summarized in Fig.1; where, v_j represents the speed of the car j, v_{max} is the speed limit of the road, d_j is the number of empty cells in front of car j, x_j is the cell that car j occupies, $v_{l'}$ is the number of empty cells in front of car j if it were on lane, l', and d_{back} is the number of empty cells behind in the car in lane l'. Each update of the system, requires all the vehicles to follow these rules simultaneously, with each vehicle first deciding whether to change lane before deciding how much to move forward; this order of decision making is fundamental to the CA model.

Longitudinal Update Rule:

$*$Rule 1: Acceleration, $v_j \to min(v_j + 1, v_{max})$
Rule 2: Braking, $v_j \to min(v_j, d_j)$
Rule 3: Randomization, $v_j \to v_j - 1$
(with probability p_s)
Rule 4: Motion, $x_j \to x_j + v_j$

(a) rules for single-lane highway

Multi-lane Update Rule:

$*$Rule A: Incentive criterion , $v_{l'} > d_j$
$*$Rule B: Safety criteria , $d_{back} > v_{max}$
Rule C: Decision, $l \to l'$
(with probability p_l)
Rule D: Longitudinal Update Rule

(b) rules for multi-lane highway

FIGURE 1 Rules for CA models for traffic flow

In this paper, we adapt the Nagel-Schreckenberg Cellular Automaton model to introduce a model of mixed traffic flow of HVs and AVs that captures two potential behaviors of AVs -*opportunistic* and *neighbor awareness*- on a three lane circular road. We distinguish between the two class of vehicles in our simulation by assigning different behavioural parameters to each vehicle type.

Opportunistic Model of AV

Autonomous Vehicles require well defined instructions in the form of algorithms and optimization functions to make decisions while driving [17]. This level of algorithmic control on the decision making of AVs imply that such vehicles can achieve idealized traffic flow parameters that cannot be attained by HVs. This is due to the fact that HVs are prone to velocity fluctuations due to human behavior or due to varying external conditions. This type of *erratic* behaviour does not apply to AVs, since they are well aware of their surroundings and make their decisions of accelerating/deceleration solely based on safety and opportunity. Similarly, when it comes to changing lanes, AVs will always change lanes given that it is both safe and rewarding to change lanes. HVs, like in previous CA models, behave more "humanly" in this aspect, because it depends on the *aggressiveness* of the human driver. In order to capture such opportunistic behavior of AV, we propose the following model.

Theorem 1 *Let $p_l(X)$ be the probability that represents the willingness of a vehicle of type X to change lanes after the Incentive and Safety criteria are met, and let $p_s(X)$ be the probability of that same vehicle to brake randomly. Then, if AV is purely opportunistic, the following must be true.*

$$p_l(AV) = 1$$

$$p_s(AV) = 0$$

From Theorem 1, we can conclude that:

$$p_l(HV) < p_l(AV) \tag{1}$$

$$p_s(HV) > p_s(AV) \tag{2}$$

In our model, the $p_l(HV)$ and $p_s(HV)$ values are such that HV behaviour in our simulation is stochastic and hence realistic, whereas the values for $p_l(AV)$ and $p_s(AV)$ for AVs were chosen to reflect opportunistic behaviour. These rules and parameters, however, do not capture the *neighbor awareness* behavior of AVs. To account for such behavior, we introduce some new rules to the existing rule base of the standard CA model, which we discuss in the following subsection.

Neighbor Aware Model of AV

Autonomous Vehicles can communicate with each other and the surrounding using V2X technology [24]. This means that is is possible for AVs to be aware of its spatial orientation and location. Furthermore, AVs can also be aware of the location of other nearby AVs and can be in constant communication with each other [23]. We hypothesize that these *"neighbor aware"* AVs would behave differently depending on the type of vehicle they are trailing. An AV trailing another AV can maintain a shorter headway due to their inter-connectivity and, hence, can maintain a higher speed; such phenomenon would not be seen if the leading vehicle is a HV. We assume HVs to not have any distinct behavior differences that depend on the class of vehicle they follow. In order to model such behaviour, we need to introduce a new definition of v_{max}, which traditionally represents the speed limit of the road in CA models. Since, the car is now aware of the type of vehicle it is trailing, we can redefine the maximum speed of the car, v_{max}, as follows:

Theorem 2 *Let v_{aa} be the maximum speed attainable if an AV trails an AV, v_{ah} the maximum speed attainable if an AV trails an HV, and v_h be the maximum speed for a HV independent of the class of vehicle it is following.*

$$v_{max} = \begin{cases} v_{aa} & \text{if } AV - AV \\ v_{ah} & \text{if } AV - HV \\ v_h & \text{if } HV \end{cases} \tag{3}$$

Thus, if we take into account the lower headways of AVs trailing other AVs. It follows that:

$$v_{aa} > v_{ah} \geq v_h \tag{4}$$

From Theorem 2, it can be seen that an AV following another AV can attain the highest possible speed, v_{aa}, whereas the maximum speed for a HV is the lowest, v_h and is independent of the class of the vehicle it trails. Throughout the rest of the paper when we refer to v_{max} we imply the meaning of v_{max} as described in Eqn. 3. Our model follows the general rules outlined in Fig 1, with the some changes to certain rules marked by "$*$ " in Fig.1. These new rules are explained below:

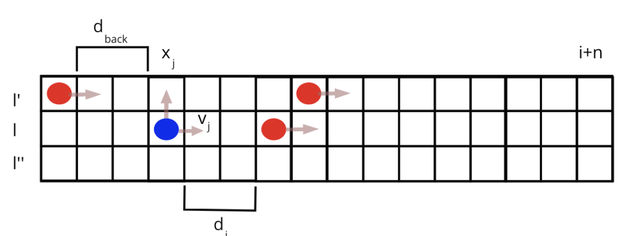

FIGURE 2 Road - Car grid in Cellular Automata

- **Rule A:** In our model $v_{l'}$ is defined as: $v_{l'} = min(v_{max}, d_{back})$. Here, $v_{l'}$ represents the potential speed the vehicle can attain if it switches lane from lane l to lane l'. If d_j is the number of empty cells in front of car in lane l. The incentive criterion dictates that the vehicle will switch lanes only and only if:

$$v_{l'} > d_j \tag{5}$$

- **Rule B:**

$$d_{back} > v_{prev} \tag{6}$$

Once an incentive to switch lanes have been established, the safety criterion implies that the vehicle looks for the car that would be behind it in the target lane, l' Then, it is required that the distance to the previous car, d_{back}, is greater than the speed of the previous car, v_{prev} in the target lane to avoid any collision while switching lanes.

- **Rule 1 and 2:** If $v_j < v_{max}$ and the distance to the next vehicle, d_j, is greater than $v_j + 1$, the vehicle accelerates and the new speed, v_j, is:

$$v_j = min(min(v_j + 1, v_{max}), d_j) \tag{7}$$

Rule 1 ensures that the vehicle accelerates linearly till it reaches its maximum speed, v_{max}, while Rule 2 also makes sure that there is no collision.

To summarize, when an AV exhibits *neighbor aware* behaviour, it is aware of the type of vehicle in front of it and changes its velocity as a response limiting its maximum speed, v_{max}, while HV behaves indifferently. A consequence of this model is the next corollary.

Corollary 1 *The velocity of a car, v_j is a function of the type of vehicle under consideration, t_1 and the type of vehicle it precedes, t_2.*

$$v_j = f(t_1, t_2)$$

It should be also be noted that both behaviors- *opportunistic and "neighbor aware"*- satisfy the safety constraints and correspond to pure opportunistic and pure random nature. Moreover, since, real-world AV behaviors are not well known as of yet due to the lack of large scale deployment of such vehicles, we cannot be sure of the accuracy of our models.

Model Implementation

Both our simulation and analysis program were written in Python3. The graphics for the simulation were made using the Pygame 1.9.6 module. To create our model, we used principles of Object Oriented Programming with the main objects being two Abstract Data Structures: the Road and Car. The implementation of the NaSch CA algorithms was done through passing functions to these data structures and manipulating the control flow of the program.

EXPERIMENTS AND ANALYSIS

In this section, we discuss about the two simulation experiments that we conducted and the results we obtained from these experiments. The purpose of these experiments was to study the impact of the two models of AV - *"neighbor aware"* and *"opportunistic"* - on the overall traffic flow and better understand any emergent patterns of collective behavior. Mention our results here briefly?.

In both the experiments, we consider a circular road with three lanes (periodic boundary conditions), each lane comprising of 100 cells. The vehicle objects follow the set of update rules we defined in Section 2. For each simulation, the road starts from empty state. N Vehicles are then distributed randomly on the road with zero initial velocity; the vehicle type is determined stochastically upon allocation, following a binomial distribution with $P(AV) = 0.3$, where $P(AV)$ is the probability of a vehicle being an AV (i.e. percentage of AVs).

Experiment 1:

Answer to find: is clustering a normal phenomenon? can we design AV behavior that can form self organized clustering?

For experiment 1, we consider three different models of AV: *"neighbor aware", opportunistic* and *base scenario*. Here, the *"neighbor aware"* model of AV behaves according to rules described in Section 2.2; the *opportunistic* model of AV behaves similar to HV's but are modeled to be *purely* opportunistic by being incentivized to seek gaps. The *base scenario* model of AV is where an AV behaves *exactly* like a HV, with their behavior being indistinguishable from one another. Table 1 summarizes the differences between the three models.

Parameter	Neighbor Aware	Opportunistic	Base Scenario
$p_l(HV)$	0.6	0.6	0.6
$p_l(AV)$	1	1	0.6
$p_s(HV)$	0.4	0.4	0.4
$p_s(AV)$	0	0	0.4
v_{aa}	5	5	5
v_{ah}	4	5	5
v_h	3	4	5

TABLE 1 Table of parameter values used in the three test cases of Experiment 1 and 2

We simulate the traffic flow for each of these models of AV for the same number of simulation time steps for three different densities: $0.08, 0.2$ and 0.6- these three densities correspond to a low occupancy state, critical state and high occupancy state for the road respectively. The proportion of AV, throughout the experiment, was kept constant at $P(AV) = 0.3$. We record all the key traffic flow parameters and present our results in the following sections.

Experiment 2:

Answer to find: does clustering improve traffic flow parameters?

Similar to experiment 1, we again consider three different models of AV: *"neighbor aware", opportunistic* and *base scenario*, but unlike experiment 1, in experiment 2 we increase our system density linearly for each simulation till it reaches the jam density. The trajectory of the system density evolution in the density phase space is determined by $P(AV)$, where state trajectory is a straight line with slope $\frac{\rho_{AV}}{1-\rho_{AV}}$ (see Figure 3). Throughout the simulation, we track this evolution of system density and system throughput (i.e. flux).

We keep the simulation period at each density constant and discard the data from the first fifth of the period to account for system transition from empty to steady state.

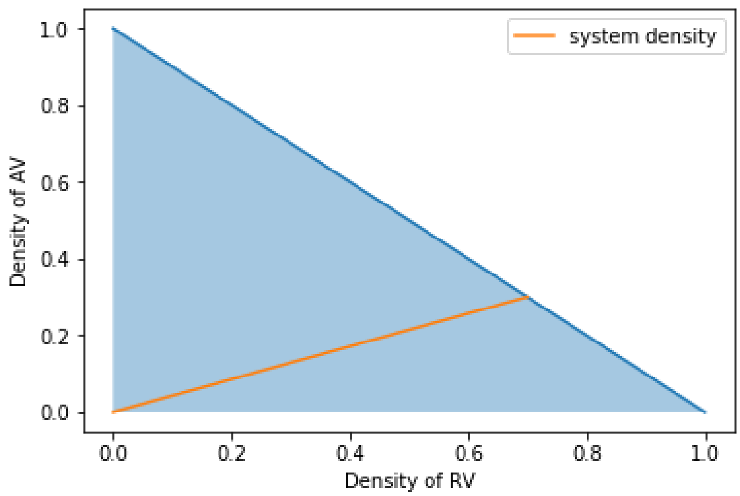

FIGURE 3 Example of system density (orange) evolution in allowed phase space (blue)

Clustering and Lane Formation

Structure:

what is clustering? how did we quantify?

is clustering a regular phenomenon? if not why is it not? what are the factors behind clustering?

which AV behavior results in clustering and how significant is it compared to other models?

How do our results prove our explanation?

The purpose of experiment 1 is to understand self-organization phenomenon of AV into clusters and lanes (which we call lane formation). We define them as follows:

- **Cluster:** Has 4 or more AVs each at most 3 cells apart

- **Lane Formation:** Has 4 or more AVs trailing each other on the same lane with no HVs in between them

In both cases, we observe interesting self-organization phenomena: as time passes by, AVs organize them into clusters and form lanes without any centralized command. Figure 4 and 5 provide snapshots of this phenomenon for both cases. In the case $R1M1$ (AVs are type-aware), we find the number and size of such clusters and lanes are both larger than the case $R2M2$. This may be verified from the distribution of cluster sizes in Figure **??** and **??** .

We hypothesize that the formation of clusters and lane is due to the opportunistic nature of AVs, modeled through both its gap seeking and braking behaviors as we described above. In the type aware model of AVs, it is actually rewarding to the AV to trail behind another AV as they can achieve smaller headways. This leads to the phenomenon of lane formation as AVs are essentially incentivized to change lanes aggressively and overtake HVs to seek and follow a peer. The fact that $R1M1$ case has stronger clustering to certain extent verify this hypothesis, because AVs have better incentives in $R1M1$ due to their awareness of vehicle types.

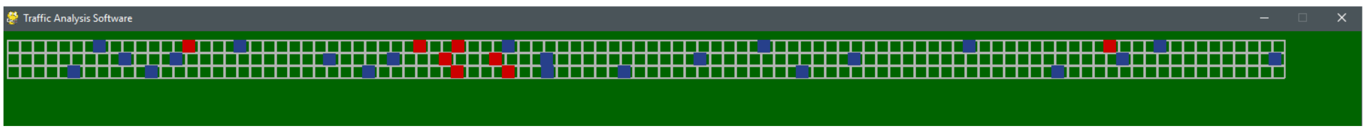

FIGURE 4 **A cluster of 6 AVs (red)**

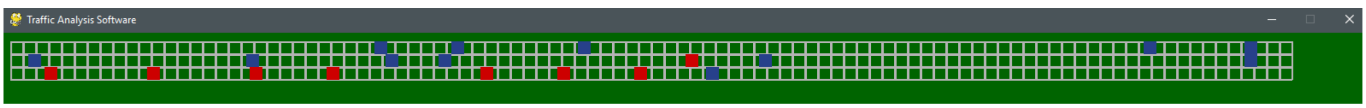

FIGURE 5 **A self-organized lane of 7 AVs (red)**

Flux of Mixed Flow

Structure:
Introduce this section as results from experiment 2
experimental data
talk about how we got the plots
Show FD's for three cases
show individual FDs for AV and RV
Analyze the plots
Answer whether clustering improved traffic flow or not?
Explain why or why not

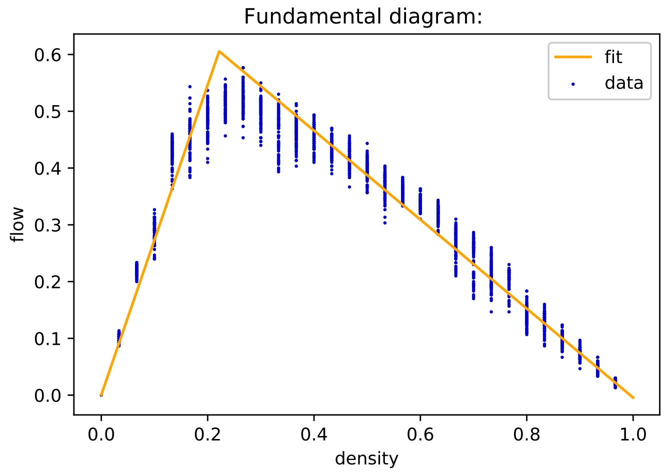

FIGURE 6 Piecewise linear fit of fundamental diagram

We also compare the flux of mixed traffic flow in all scenarios (see Figure 7 through Figure 9). Once the simulation is done, we use a modified Linear Regression method to fit the plot, where the fit is specified to be a piecewise linear function. Figure 6 shows an example of the fit.

Experiment Data			
	Neighbor Aware	Opportunistic	Base Scenario
Critical Density	0.20	0.16	0.22
Maximum Flow	0.53	0.57	0.61
Free flow Speed	2.66	3.56	2.73
Wave Speed	-0.66	-0.68	-0.78
Critical Density, RV	0.15	0.11	0.12
Maximum Flow, RV	0.37	0.39	0.29
Critical Density, AV	0.09	0.10	0.10
Maximum Flow, AV	0.28	0.22	0.39

TABLE 2 Key traffic characteristics in each scenario

Our results show that $R2M2$ has the best overall throughput, and $R2$ overall perform better than $R1$ in our simulation setting. This is not surprising, considering that dedicated lane would suppress the conflicts between AVs and HVs, and allow AVs to move more efficiently. In this case, the opportunistic behavior of AVs seem to play a negative role in system throughput, due to the extra lane changes induced. As expected $M2$ model of AV improved overall traffic flow as compared to $M1$. Figure 7 shows the fundamental diagrams for all the scenarios.

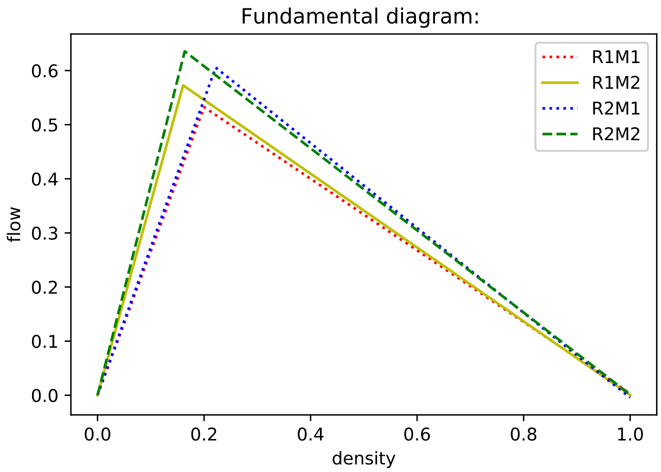

FIGURE 7 Fundamental Diagram

Fig. 8 and 9 show the fundamental diagrams for the AV class only and HV class only for each of the experimental cases. The AVs performed better under $R2$ while the RVs did better under $R1$ as expected.

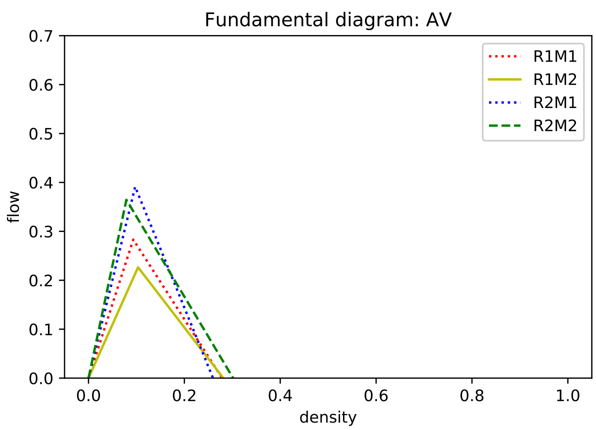

FIGURE 8 Fundamental Diagram of AV

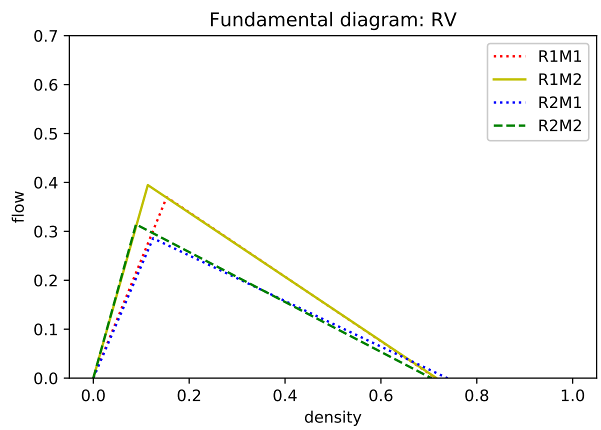

FIGURE 9 Fundamental Diagram of RV

Since overall traffic flow was better under the $R2$, this suggests that the rate at which vehicles entered the test lane in $R2$ (i.e. the dedicated lane) must have been lower than that of $R1$. This is confirmed to be to true as shown by Figure 10. $R2M2$ has the lowest flow rate while $R1M1$ has the highest rate. From Figure 7, we can see that $R2M2$ has the best flow characteristics and $R1M1$ has the worst as expected.

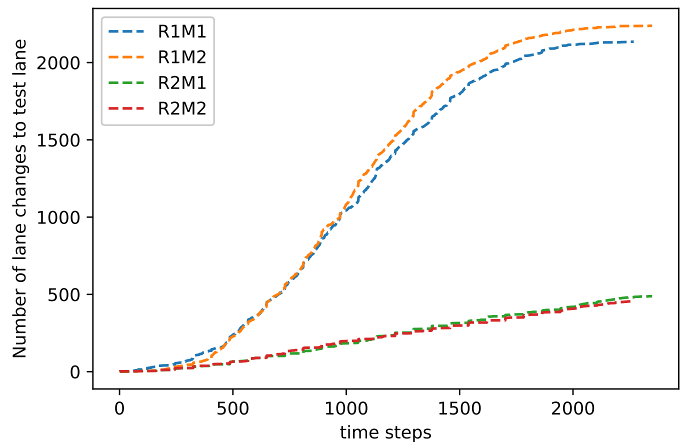

FIGURE 10 Number of lane changes to test lane over time

Table 2 summarizes the key traffic flow parameters obtained from our experiments. The values indicate that the free flow speed for both $M1$ and $M2$ remained roughly constant across all scenarios. However, the maximum flow under $R2$ was greater than that under $R1$. On the other hand, the wave speed under both $R1$ and $R2$ remained approximately the same independent of the model of AV.

CONCLUSION

Needs some change

In this paper, we investigate behaviors of mixed traffic flow of AVs and HVs through simulation experiments. The major purpose is to understand the relation between opportunistic behaviors of AV and formation of clusters and lanes without centralized control. For this purpose, we propose a new cellular automata model to account for potential behaviors difference between AVs and HVs. Notably, the opportunistic character of AVs are modeled through their gap seeking behaviors and awareness of neighboring vehicle types. Simulation experiments were conducted in four scenarios, which compare the clustering process and traffic flow performance with and without opportunistic AVs, and with and without dedicated AV lane.

One major finding of this research, which is intriguing, is the self-organized formation of AV clusters and lanes without any centralized control. Such phenomena are observed in our simulation experiments even no dedicated AV lane is in place. This may suggest clustering as an intrinsic property of mixed HV and AV flow. We postulate such self-organized phenomena is due to the incentives that AVs perceive to seek and partner with peer AVs. When AVs are opportunistic, such effect is reinforced, as seen in our experiment. If the postulation is confirmed to be true through further simulation or field experiments, it may suggest the possibility of regulating mixed HV-AV flow through designing or inducing decentralized incentives, instead of performing centralized coordination. Our research also compares the mixed flux in all scenarios, and the positive effect of dedicated lane is confirmed.

The findings of this paper may serve as initial evidence to the self-organization in mixed flow of AVs and HVs. We recognized the proposed model is by no means realistic in the quantitative sense. In addition, as AVs haven't been launched in large scale in the real world and existing data is limited, their opportunistic behavior just represent on possibility we deem likely. In future works, as more data become available, it is desirable to model the decision-making of AVs more realistically, and further verify the finding in this paper.

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