

## Lecture 5 – Multi-stage Games

Gerhard Riener

# Multi-stage Games

## Multi-stage games: overview

### Object of analysis

How commitment in earlier stages changes equilibrium behaviour in later stages.

- ▶ So far:
  - ▶ Mostly simultaneous-move games
  - ▶ Briefly: two-stage game (Shy (1995) model in Lecture 4)
- ▶ Today:
  - ▶ Stackelberg quantity leadership
  - ▶ Strategic delegation (Vickers 1985)

## Solution concept: subgame perfect equilibrium

- ▶ A strategy profile is a **subgame perfect Nash equilibrium (SPNE)** if it induces a Nash equilibrium in every subgame
- ▶ Operationally (finite horizon):
  - ▶ Solve from the last stage backward (backward induction)

# Stackelberg competition

## Stackelberg model: structure

- ▶ Two stages, two firms:
  1. Leader chooses  $q_1$
  2. Follower observes  $q_1$  and chooses  $q_2$
- ▶ Demand and costs:
  - ▶ Inverse demand:  $p(Q) = A - Q$  with  $Q = q_1 + q_2$
  - ▶ Marginal cost:  $c$  (parameter)

## Stackelberg model: follower best response

- ▶ Follower profit given  $q_1$ :

$$\pi_2(q_2; q_1) = (A - q_1 - q_2 - c)q_2$$

- ▶ Best response:

$$BR_2(q_1) = \frac{A - c - q_1}{2}$$

**i** Strategic substitutes (quantities)

## Stackelberg model: leader choice

- ▶ Substitute follower response into leader profit:

$$\pi_1(q_1) = (A - q_1 - BR_2(q_1) - c)q_1$$

- ▶ Equilibrium (identical costs):

- ▶  $q_1^S = \frac{A-c}{2}$

- ▶  $q_2^S = \frac{A-c}{4}$

! First-mover advantage

The leader commits to a higher  $q_1$ , anticipating that the follower reduces  $q_2$ .

## Stackelberg v Cournot: equilibrium outcomes

- ▶ Identical marginal costs ( $c$ ) imply:

Outcome	Stackelberg	Cournot
Firm 1 output	$q_1^S = \frac{A-c}{2}$	$q_1^C = \frac{A-c}{3}$

# Strategic delegation (Vickers)

## Delegation: economic idea

- ▶ Firms may delegate output choice to managers with objectives that differ from pure profit maximisation
- ▶ Delegation instrument:
  - ▶ an incentive parameter  $\theta_i$  written into the manager's contract

## Delegation (Vickers): model

- ▶ Cournot oligopoly with  $n$  firms:  $i = 1, \dots, n$
- ▶ Manager of firm  $i$  maximises:

$$M_i = \pi_i + \theta_i q_i$$

where  $\pi_i = p(Q)q_i - cq_i$  and  $Q = \sum_i q_i$

### **i** Equivalent formulation

$M_i = p(Q)q_i - (c - \theta_i)q_i$  so  $\theta_i$  shifts the manager's objective like a reduction in marginal cost.