EC5230 - Industrial Organisation

Lecture 5 - Multi-stage Games

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Multi-stage Games

Multi-stage games: overview

> **i Object of analysis**
>
> How commitment in earlier stages changes equilibrium behaviour in later stages.

- ▶ So far:
  - → Mostly simultaneous-move games
  - → Briefly: two-stage game (Shy (1995) model in Lecture 4)
- ▶ Today:
  - → Stackelberg quantity leadership
  - → Strategic delegation (Vickers 1985)

Learning objectives

By the end of this lecture you should be able to:

1. Define subgame perfect Nash equilibrium and apply backward induction in finite games
2. Derive and interpret the Stackelberg equilibrium under quantity leadership
3. Set up and solve a two-stage delegation game (contract stage + product-market subgame)
4. Compare equilibrium outcomes and welfare across Cournot, Stackelberg, and delegation

Solution concept: subgame perfect equilibrium

▶ A strategy profile is a subgame perfect Nash equilibrium (SPNE) if it induces a Nash equilibrium in every subgame

▶ Operationally (finite horizon):

$\rightarrow$ Solve from the last stage backward (backward induction)

> **! Commitment logic**
>
> A credible first-stage choice is observed by later movers and shifts their best responses. The payoff function is unchanged — only equilibrium strategies change.

> **i Discussion question**
>
> Why does commitment require the first-stage action to be observable? What happens to the Stackelberg outcome if the follower cannot verify q_1 before choosing q_2?

Stackelberg competition

Stackelberg model: structure

- Two stages, two firms:

  1. Leader chooses q_1
  2. Follower observes q_1 and chooses q_2

- Demand and costs:

  $\rightarrow$ Inverse demand: $p(Q) = A - Q$ with $Q = q_1 + q_2$
  $\rightarrow$ Marginal cost: c (parameter)

Stackelberg model: follower best response

- ▶ Follower profit given q_1:

$$\pi_2(q_2; q_1) = (A - q_1 - q_2 - c)q_2$$

- ▶ Best response:

$$BR_2(q_1) = \frac{A - c - q_1}{2}$$

> **i Strategic substitutes (quantities)**
>
> Quantities are strategic substitutes: when the rival increases output, my best-response output decreases (downward-sloping best response).

Stackelberg model: leader choice

▶ Substitute follower response into leader profit:

$$\pi_1(q_1) = (A - q_1 - BR_2(q_1) - c)q_1$$

▶ Equilibrium (identical costs):
$\rightarrow q_1^S = \frac{A-c}{2}$
$\rightarrow q_2^S = \frac{A-c}{4}$

> **! First-mover advantage**
>
> The leader commits to a higher q_1, anticipating that the follower reduces q_2.

Full FOC derivation (leader):

Substitute $BR_2(q_1) = \frac{A-c-q_1}{2}$ into leader profit:

Stackelberg model: leader choice

$$\pi_1(q_1) = \left(A - q_1 - \frac{A - c - q_1}{2} - c\right) q_1 = \left(\frac{A - c}{2} - \frac{q_1}{2}\right) q_1 = \frac{A - c}{2} q_1 - \frac{1}{2} q_1^2$$

First-order condition:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{A - c}{2} - q_1 = 0 \quad \Rightarrow \quad q_1^S = \frac{A - c}{2}$$

Then follower output:

$$q_2^S = BR_2(q_1^S) = \frac{A - c - \frac{A - c}{2}}{2} = \frac{A - c}{4}$$

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- Identical marginal costs (c) imply:

Outcome	Stackelberg	Cournot
Firm 1 output	$q_1^S = \frac{A-c}{2}$	$q_1^C = \frac{A-c}{3}$
Firm 2 output	$q_2^S = \frac{A-c}{4}$	$q_2^C = \frac{A-c}{3}$
Aggregate output	$Q^S = \frac{3(A-c)}{4}$	$Q^C = \frac{2(A-c)}{3}$
Price	$p^S = \frac{A+3c}{4}$	$p^C = \frac{A+2c}{3}$
Firm 1 profit	$\pi_1^S = \frac{(A-c)^2}{8}$	$\pi_1^C = \frac{(A-c)^2}{9}$
Firm 2 profit	$\pi_2^S = \frac{(A-c)^2}{16}$	$\pi_2^C = \frac{(A-c)^2}{9}$

ℹ Discussion question

The leader earns more than in Cournot and the follower earns less. Is this because the leader is more capable, or because the structure of sequential moves creates an advantage? Could the follower also benefit from moving first?

Stackelberg model: remarks

▶ Comparative statics questions:

→ What changes when marginal costs are asymmetric?
→ What changes if the timing is endogenous (announcement/commitment issues)?

Stackelberg: numerical example ($A = 10, c = 1$)

- ▶ Parameters: $A = 10$ (demand intercept), $c = 1$ (marginal cost)

- ▶ Stackelberg outcomes:

  $\rightarrow \quad q_1^S = \frac{A-c}{2} = 4.5; q_2^S = \frac{A-c}{4} = 2.25$

  $\rightarrow \quad Q^S = 6.75; p^S = A - Q^S = 3.25$

  $\rightarrow \quad \pi_1^S = (p^S - c)q_1^S = 10.125$

- ▶ Cournot benchmark:

  $\rightarrow \quad \pi_1^C = \frac{(A-c)^2}{9} = 9$

> **! Concrete comparison**
>
> With $A = 10, c = 1$: $\pi_1^S = 10.125 > \pi_1^C = 9$ (first-mover advantage).

Stackelberg: welfare comparison (to Cournot)

- ▶ With linear demand $p = A - Q$ and constant marginal cost c:

  - $\rightarrow$ Higher total output implies lower price $\Rightarrow$ higher consumer surplus (CS)
  - $\rightarrow$ Stackelberg yields $Q^S > Q^C$ (hence $CS^S > CS^C$)

- ▶ Total surplus changes because:

  - $\rightarrow$ CS rises with Q
  - $\rightarrow$ profits can rise for the leader and fall for the follower relative to Cournot

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ⓘ Exam-useful

Stackelberg leadership typically increases output relative to Cournot, so it is (weakly) better for consumers.

Strategic delegation (Vickers)

Delegation: economic idea

▶ Firms may delegate output choice to managers with objectives that differ from pure profit maximisation

▶ Delegation instrument:

$\rightarrow$ an incentive parameter θ_i written into the manager's contract

Delegation (Vickers): model

▶ Cournot oligopoly with n firms: $i = 1, ..., n$

▶ Manager of firm i maximises:

$$M_i = \pi_i + \theta_i q_i$$

where $\pi_i = p(Q)q_i - cq_i$ and $Q = \sum_i q_i$

▶ Equivalently: $M_i = p(Q)q_i - (c - \theta_i)q_i$ – so θ_i acts like a reduction in the manager's effective marginal cost

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⚠ **Commitment assumption**

For delegation to affect rivals' behaviour, contracts must be:

▶ observable (rivals see θ_i)
▶ binding (owners cannot override managers after θ_i is set)

Delegation: triopoly exercise (setup)

- ▶ Cournot triopoly: $i = 1, 2, 3$

- ▶ Demand: $p(Q) = A - Q$

- ▶ Identical marginal costs: c

- ▶ Managers: $\theta_i \geq 0$

- ▶ Task:

    $\rightarrow$ Solve the θ-setting game given the induced Cournot equilibrium in (q_1, q_2, q_3)

Delegation: triopoly solution steps

- ▶ Step 1 (product-market subgame): given $(\theta_1, \theta_2, \theta_3)$, managers play Cournot and determine $q_i^*(\theta)$

- ▶ Step 2 (contract stage): each owner chooses θ_i to maximise own profit $\pi_i(\theta_i; \theta_{-i})$

- ▶ Step 3 (equilibrium incentives): solve the system of best responses in θ

    $\rightarrow$ in symmetry: $\theta_1 = \theta_2 = \theta_3 = \hat{\theta}$

> **! Exam template**
>
> Two-stage delegation problems are solved exactly like Stackelberg: solve the quantity subgame first, then solve the incentive-setting stage.

Delegation: triopoly equilibrium (given)

Cournot–Nash equilibrium outcomes (given incentives θ_i):

$$q_i^* = \frac{1}{4}\left(A - c + 3\theta_i - \sum_{j \neq i} \theta_j \right)$$

$$p^* = \frac{1}{4}\left(A + 3c - \sum_i \theta_i \right)$$

▶ Interpretation:
   → Increasing θ_i shifts q_i^* up; rivals' incentives are strategic substitutes in outputs through Q

Delegation: solving for equilibrium incentives

Owner's problem (taking rivals' incentives as given):

▶ Owner maximises profit (not managerial utility):

$$\max_{\theta_i \geq 0} \pi_i^*(\theta_i; \theta_{-i}) = \left(p^*(\theta) - c \right) q_i^*(\theta)$$

First-order condition:

$$\frac{\partial \pi_i^*(\theta_i; \theta_{-i})}{\partial \theta_i} = 0$$

▶ Solving the three FOCs gives a best-response system in $(\theta_1, \theta_2, \theta_3)$
▶ In symmetry, $\theta_i = \hat{\theta}$ for all i, implying $\hat{\theta} = \frac{1}{5}(A - c)$

Full FOC derivation (owner, triopoly):

Let $\Theta_{-i} = \theta_j + \theta_k$. From the equilibrium expressions:

Delegation: solving for equilibrium incentives

$$\pi_i^* = \frac{(A - c - \theta_i - \Theta_{-i})(A - c + 3\theta_i - \Theta_{-i})}{16}$$

Differentiate with respect to θ_i:

$$\frac{\partial \pi_i^*}{\partial \theta_i} = \frac{1}{16}\left[-(A - c + 3\theta_i - \Theta_{-i}) + 3(A - c - \theta_i - \Theta_{-i}) \right] = 0$$

Expand and collect:

$$2(A - c) - 6\theta_i - 2\Theta_{-i} = 0 \quad \Rightarrow \quad \theta_i = \frac{A - c}{3} - \frac{\Theta_{-i}}{3} = \frac{A - c}{3} - \frac{2}{3}\bar{\theta}_{-i}$$

This is the best-response function shown on the next slide. Imposing symmetry $\theta_i = \bar{\theta}_{-i} = \hat{\theta}$:

Delegation: solving for equilibrium incentives

$$\hat{\theta} = \frac{A-c}{3} - \frac{2}{3}\hat{\theta} \quad \Rightarrow \quad \frac{5}{3}\hat{\theta} = \frac{A-c}{3} \quad \Rightarrow \quad \hat{\theta} = \frac{A-c}{5}$$

Delegation: best response in incentives (triopoly)

Define the rivals' average incentive:

$$\bar{\theta}_{-i} = \frac{1}{n-1} \sum_{j \neq i} \theta_j \quad (n = 3)$$

A reduced-form best response for the owner's incentive choice can be written as:

$$\theta_i = BR(\theta_{-i}) = \frac{1}{3}(A - c) - \frac{2}{3}\bar{\theta}_{-i}$$

▶ Interpretation:
  → Higher rival incentives make rivals more aggressive, reducing the marginal gain from raising θ_i

Delegation: symmetric equilibrium (fixed point)

In symmetry, $\theta_i = \bar{\theta}_{-i} = \hat{\theta}$ for all i.

$$\hat{\theta} = \frac{1}{3}(A - c) - \frac{2}{3}\hat{\theta} \quad \Rightarrow \quad \hat{\theta} = \frac{1}{5}(A - c)$$

> **i What this is doing**
>
> This is the standard "best-response fixed point" method: compute $BR(\cdot)$, then impose symmetry.

Delegation: strategic logic

- ▶ Unilateral incentive: if rivals set $\theta = 0$, choosing $\theta_i > 0$ is strictly profitable
  - $\rightarrow$ Positive θ_i commits the manager to higher output
  - $\rightarrow$ Rivals reduce output (quantities are strategic substitutes)
- ▶ Equilibrium outcome: all firms choose $\hat{\theta} > 0$
  - $\rightarrow$ The strategic advantage disappears when everyone delegates
  - $\rightarrow$ Profits fall: $\hat{\pi} < \pi^C$ (overproduction lowers price for all)

> **! Prisoners' dilemma**
>
> Each owner's dominant strategy leads to a collectively worse outcome: all firms delegate, output rises, and equilibrium profits fall below the Cournot benchmark.

Delegation: strategic logic

> **i Discussion question**
>
> If delegation always lowers industry profits in equilibrium, why would any firm ever choose to delegate? What commitment or regulatory change would allow firms to avoid this outcome?

Delegation: comparison table (triopoly)

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Scenario	Incentives	Per-firm output	Per-firm profit
All delegate	$\hat{\theta} = \frac{A-c}{5}$	$\hat{q} = \frac{3(A-c)}{10}$	$\hat{\pi} = \frac{3(A-c)^2}{100}$
None delegate	$\theta = 0$	$q^C = \frac{A-c}{4}$	$\pi^C = \frac{(A-c)^2}{16}$

With $A = 10, c = 1$:

- $\hat{\pi} = 2.43$
- $\pi^C = 5.0625$

!Takeaway

Delegation is individually rational in the incentive-setting game but reduces profits in equilibrium.

Delegation: numerical example ($A = 10, c = 1$)

- ▶ Parameters: $A = 10, c = 1$

- ▶ Delegation equilibrium:

  $\rightarrow \hat{\theta} = \frac{A-c}{5} = 1.8$

  $\rightarrow$ Per-firm profit: $\hat{\pi} = \frac{3(A-c)^2}{100} = 2.43$

- ▶ Cournot triopoly benchmark ($\theta_i = 0$):

  $\rightarrow$ Per-firm profit: $\pi^C = \frac{(A-c)^2}{16} = 5.0625$

⚠ Concrete comparison

With $A = 10, c = 1$: $\hat{\pi} = 2.43 < \pi^C = 5.0625$ (delegation intensifies competition and lowers profits).

Delegation: welfare comparison (to Cournot triopoly)

- ▶ Delegation increases effective aggressiveness in output:
  - → equilibrium price is lower than in standard Cournot triopoly
  - → therefore consumer surplus is higher under delegation than under $\theta_i = 0$
- ▶ Total surplus effect:
  - → CS rises due to lower price
  - → profits fall due to overproduction relative to joint-profit motives

> **i Exam-useful statement**
>
> Delegation can be privately attractive ex ante (as a commitment device) but can reduce profits in equilibrium while benefiting consumers.

Summary and next week

Summary

- Subgame perfect equilibrium is solved by backward induction in finite multi-stage games
- Stackelberg timing creates first-mover advantage through commitment
- Strategic delegation can raise aggressiveness but may lower industry profits in equilibrium
- Welfare effects can diverge: consumers may gain even when firms lose

Next week: cooperative R&D

- Spillovers and non-cooperative R&D incentives
- Cooperative R&D and research joint ventures
- Welfare and policy implications under different cooperation regimes

References

Shy, Oz. 1995. Industrial Organization: Theory and Applications. MIT Press.

Vickers, John. 1985. "Delegation and the Theory of the Firm." The Economic Journal 95 (Supplement): 138–47. https://doi.org/10.2307/2232877.