# Exercise 1: Oligopoly and Product Differentiation

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**Instructions**

- Answer all questions.
- Show intermediate steps.
- Solutions are included only when `params.solution: true`.

## Problem 1: Welfare analysis

Consider the example exercise in Lecture 1, where

$P = 20 - Q$

and there are three firms with cost function $c_i(q_i) = c_i q_i$ with

$c_1 = 0$, $c_2 = 5$, and $c_3 = 7$.

**(a)**

Compute the Nash equilibrium of the Cournot duopoly game between firm 1 and firm 2.

**(b)**

Given your answer to (a), compute the equilibrium profits, consumer surplus, and welfare loss.

**(c)**

Compare the values in (b) to those of the other three market structures (perfect competition, monopoly, Cournot triopoly) in Lecture 1. Discuss any policy implications for a social planner who maximises total welfare.

**Solution**

**(a)**

The best response functions are

$$q_1 = \frac{20 - q_2}{2} \qquad \text{and} \qquad q_2 = \frac{15 - q_1}{2}.$$

Solving this system yields the equilibrium quantities

$$q_1^D = \frac{25}{3}, \qquad q_2^D = \frac{10}{3}.$$

**(b)**

The equilibrium price is

$$P(Q^D) = 20 - \frac{35}{3} = \frac{25}{3}.$$

Consumer surplus is therefore

$$CS^D = \frac{1}{2}\left(\frac{35}{3}\right)^2 = \frac{1225}{18},$$

and producer surplus is

$$PS^D = \Pi_1 + \Pi_2 = \left(\frac{25}{3}\right)^2 + \left(\frac{10}{3}\right)^2 = \frac{725}{9}.$$

The welfare loss is

$$WL^D = 200 - \frac{1225}{18} - \frac{725}{9} = \frac{925}{18}.$$

**(c)**

The values for all three welfare components fall between those of monopoly and Cournot triopoly:

$$CS^M < CS^D < CS^T, \qquad PS^M > PS^D > PS^T, \qquad WL^M < WL^D < WL^T.$$

As the number of producing firms increases (monopoly $\rightarrow$ Cournot duopoly $\rightarrow$ Cournot triopoly), prices fall and consumer surplus increases, but production is reallocated toward higher-cost firms (reducing productive efficiency).

## Problem 2: Linear Hotelling model with monopoly

Consider the Hotelling linear-city model. Suppose there is only one restaurant, located at the centre of a street of length 1 km. The restaurant's cost is zero.

Consumers are uniformly distributed on the interval $[0, 1]$, with one consumer at each point. Transportation cost is £1 per kilometre.

The utility of a consumer located at distance $a$ from the restaurant is

$$U = B - a - p,$$

where $p$ is the price of a meal and $B$ is a constant.

If the consumer does not eat at the restaurant, utility is

$$U^* = 0.$$

### (a)

Suppose $0 < B < 1$. Find the number of consumers eating at the restaurant. Calculate the monopoly price and profit.

### (b)

Answer the previous question assuming $B > 1$.

### Solution

### (a)

Let $d_m$ denote the distance of the marginal consumer from the restaurant. Indifference implies

$$B - d_m - p = 0 \quad \Rightarrow \quad d_m = B - p.$$

Demand is

$$q = 2d_m = 2(B - p).$$

The monopolist solves

$$\max_p \ \Pi(p) = p \cdot 2(B - p).$$

FOC implies $p^M = \frac{B}{2}$, hence $q^M = B$ and $\Pi^M = \frac{B^2}{2}$.

**(b)**

For $B > 1$, the unconstrained solution implies $d_m = \frac{B}{2} > \frac{1}{2}$, but demand cannot exceed 1, so $q = 1$.

The monopolist sets price so the marginal consumers are exactly at the endpoints $(d = \frac{1}{2})$:

$$B - \frac{1}{2} - p = 0 \quad \Rightarrow \quad p^* = B - \frac{1}{2}.$$

Then $\Pi^* = p^* \cdot 1 = B - \frac{1}{2}$.