Lecture 2 – Product Differentiation

Gerhard Riener

Differentiation types

- Horizontal: consumers disagree on which variety is best (ideal-point / mismatch preferences)
- Vertical: products differ in quality; at equal prices all consumers (weakly) prefer the higher-quality good

Today: roadmap

- Focus: horizontal differentiation
- Hotelling line: 2 firms, pricing with fixed locations
- Salop circular city: free entry and welfare

Hotelling Model

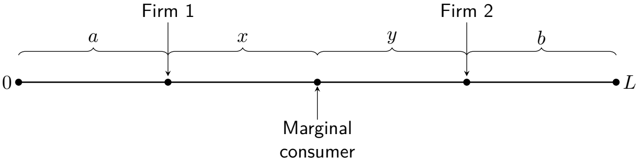
Setup: Fixed locations, price competition

- Duopoly, marginal cost 0
- Consumers (ℓ) uniformly distributed on $[0, L]$ (Hotelling 1929)
- Firm 1 at a and Firm 2 at $L - b$
- Mismatch (transport) cost $C(d) = cd$ with $c > 0$, d extent of mismatch
- Unit demand; assume full market coverage in equilibrium

Utility (full coverage)

$U_1(\ell) = \bar{u} - p_1 - c|\ell - a|$, $U_2(\ell) = \bar{u} - p_2 - c|\ell - (L - b)|$, outside option 0; assume $\bar{u}$ is large enough that all consumers buy.

Geometry



- Choice variables: p_1, p_2
- Parameters: L, a, b, c
- Distance between firms: $L - a - b$
- Marginal consumer distances: $x + y = L - a - b$
- Demands: $q_1 = a + x$ and $q_2 = b + y$

Marginal consumer and demands

- Indifference of the marginal consumer: $p_1 + cx = p_2 + cy$
- Geometry: $x + y = L - a - b$

Demands by firm

- $q_1 = a + x = \frac{p_2 - p_1 + c(L + a - b)}{2c}$
- $q_2 = b + y = \frac{p_1 - p_2 + c(L + b - a)}{2c}$

Best responses

- $BR_1(p_2) = \frac{p_2}{2} + \frac{c}{2}(L + a - b)$
- $BR_2(p_1) = \frac{p_1}{2} + \frac{c}{2}(L + b - a)$

Warning

Prices are strategic complements; if $a = b$ then $p_1^* = p_2^*$

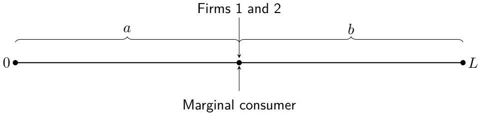
Price equilibrium fixed locations

- Prices: $p_1^* = c\left(L + \frac{a-b}{3}\right)$ and $p_2^* = c\left(L + \frac{b-a}{3}\right)$
- Profits: $\pi_1^* = \frac{c}{18}(3L + a - b)^2$ and $\pi_2^* = \frac{c}{18}(3L + b - a)^2$
- If $a = b$, symmetry implies $p_1^* = p_2^*$
- Higher c lowers substitutability and raises equilibrium markups and profits

Hotelling: co-location

- Co-location implies $a + b = L$ and $x = y = 0$
- Products become homogeneous $\Rightarrow$ Bertrand undercutting
- Equilibrium: $p_1^B = p_2^B = 0$ and $\pi_1^B = \pi_2^B = 0$

> **Warning**
>
> **Co-location and price competition**
> Co-location removes captive consumers; any $p > 0$ can be profitably undercut.



Move incentives at fixed prices

- $\frac{\partial \pi_1^*}{\partial a} = \frac{c}{9}(3L + a - b) > 0$ and $\frac{\partial \pi_2^*}{\partial b} = \frac{c}{9}(3L + b - a) > 0$
- In the pricing subgame, increasing a (resp. b) moves Firm 1 (resp. Firm 2) inward (toward the centre)
- These are comparative statics holding the rival location fixed
- Under linear transport costs, pricing incentives can pull locations toward the centre; firms will "go where the demand is"

Identifying clustering motives

Question: Why are so many businesses clustered together?

- Going where the demand is?
- Possible collusion story? Being next to the competitor facilitates monitoring.

Next: the formal location game clarifies when this clustering logic breaks down under linear transport costs.

Location game (d'Aspremont et al. 1979)

- **Two-stage game:** choose locations, then prices
- Linear transport costs ($C(d) = cd$): pure-strategy location equilibrium may fail to exist (market-capture regions; non-concavity)
- Quadratic transport costs ($C(d) = td^2$): pure-strategy location equilibrium exists (baseline: maximum differentiation)
- Principle of minimum differentiation need not hold

> **Note**
>
> **Why convexity matters**
> More convex mismatch costs make small location changes less effective at stealing marginal consumers through pricing.

Profit function: interior region

- Let $\Delta \equiv L - a - b$ denote the distance between firms
- Interior-demand region: $|p_1 - p_2| \leq c\Delta$
- In this region, Firm 1's profit is:

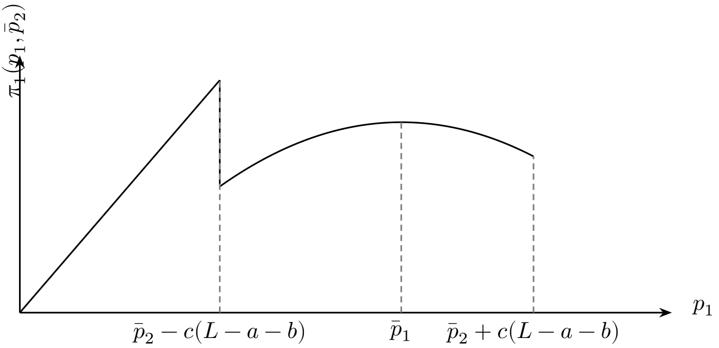
$$\pi_1(p_1, p_2) = \frac{p_1 p_2}{2c} - \frac{p_1^2}{2c} + \frac{(L + a - b)p_1}{2}$$

Profit function: corner regions

- Market capture: if $p_1 < p_2 - c\Delta$, then $\pi_1 = Lp_1$
- No demand: if $p_1 > p_2 + c\Delta$, then $\pi_1 = 0$
- Together with the interior expression, these define the piecewise payoff in (d'Aspremont et al. 1979)

Profit regions

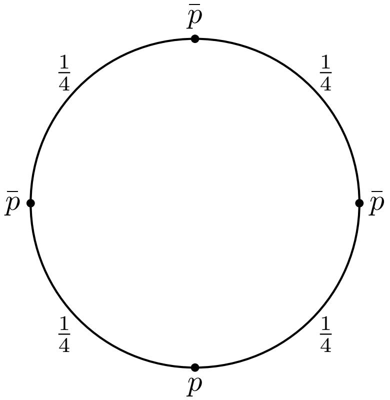
- **Region 1 (interior)**: both firms serve positive mass of consumers; demand is linear in p_1
- **Region 2 (market capture)**: Firm 1 undercuts enough to serve the whole line segment
- **Region 3 (no demand)**: Firm 1 sets too high a price; demand is zero



Salop Circular City

Setup

- Outside good yields utility u^*
- Differentiated varieties lie on a circle of circumference 1; consumers are uniformly distributed (Salop 1979)
- n firms are symmetrically located; adjacent distance is $\frac{1}{n}$
- Each consumer buys at most one unit: from some firm or the outside good



Objects

- Choice variable: p_i
- Parameters: n, c, u, u^*
- Outcomes: market boundaries / cutoffs, quantities
- Symmetric benchmark: if neighbours set $\tilde{p} = p$, the boundary consumer is at $x = \frac{1}{2n}$
- Baseline local demand before participation truncation: $q^C = 2x = \frac{1}{n}$

Consumers

- Each consumer has an ideal variety ℓ^* (location on the circle)
- Arc distance d generates mismatch cost cd
- Utility from firm j: $U_j = u - cd_j - p_j$
- Outside option: $U^O = u^*$
- Let $v = u - u^*$; participation requires $v \geq p_j + cd_j$

Monopoly region

- Outside option truncates the market (participation binds), not neighbours
- Farthest served consumer: $d_m = \frac{v-p}{c}$
- Monopoly demand: $q^M = 2d_m = 2\left(\frac{v-p}{c}\right)$

Monopoly inverse demand

- Inverse demand: $p = v - \frac{c}{2}q^M$
- Slope is $-\frac{c}{2}$ because expanding quantity extends the served interval on both sides
- Participation binds at the marginal consumer: $v = p + cd_m$

Competitive region

- Boundary consumer (indifference): $v - cx - p = v - c\left(\frac{1}{n} - x\right) - \tilde{p}$
- Boundary location: $x = \frac{\frac{c}{n} - p + \tilde{p}}{2c}$
- Competitive quantity: $q^C = 2x$

Competitive demand

- **Competitive demand**: $q^C = 2x = \frac{\frac{c}{n} - p + \tilde{p}}{c}$
- **Inverse demand**: $p = \frac{c}{n} + \tilde{p} - cq^C$

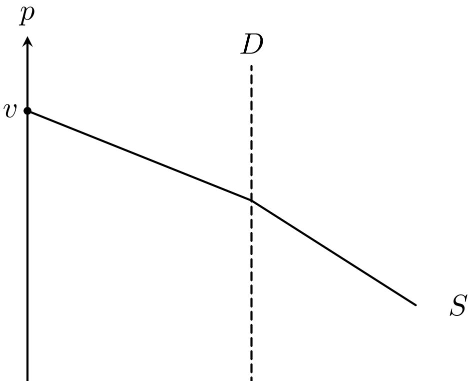
! Important

Slope comparison

Competitive-region inverse demand is steeper than monopoly-region inverse demand: $-c$ vs $-\frac{c}{2}$.

Kinked demand

- Demand is limited either by the outside option (participation) or by neighbours (local competition)
- Effective demand is $q(p) = \min\{q^M(p), q^C(p)\}$
  - **Participation-boundary (monopoly region)**: $q^M = 2\left(\frac{v-p}{c}\right)$
  - **Neighbour-boundary (competitive region)**: $q^C = \frac{\frac{c}{n} - p + \tilde{p}}{c}$
- The switch in the binding constraint creates a kink in inverse demand and a discrete change in marginal revenue



Cost functions

- Cost function: $C(q) = F + mq$ (fixed cost F, marginal cost m)
- Average cost: $AC(q) = m + \frac{F}{q}$ (decreasing in q)

Pricing (competitive region)

- Inverse demand: $p = \frac{c}{n} + \tilde{p} - cq$
- Marginal revenue: $MR = \frac{c}{n} + \tilde{p} - 2cq$
- Symmetry: $\tilde{p} = p$

Competitive-region condition (outside option not binding)

- In a symmetric profile, the farthest consumer from a firm is at distance $\frac{1}{2n}$.
- A sufficient condition for the competitive region to apply at the symmetric equilibrium is $v \geq p^C + \frac{c}{2n}$.

Zero profit

- Symmetry implies $q^C = \frac{1}{n}$
- Zero profit requires $p^C = AC = m + \frac{F}{q^C}$
- Therefore $p^C = m + nF$

Pricing condition

- **Competitive-region inverse demand**: $p = \frac{c}{n} + \tilde{p} - cq$
- **First-order condition**: $MR = MC$ with $MR = \frac{c}{n} + \tilde{p} - 2cq$ and $MC = m$
- **Symmetric equilibrium implication**: with $\tilde{p} = p$ and $q^C = \frac{1}{n}$, we get $p^C = m + \frac{c}{n}$

Free-entry equilibrium

- **Zero profit condition**: $p^C = m + nF$
- **Solve**: $m + nF = m + \frac{c}{n}$ so $n^C = \sqrt{\frac{c}{F}}$ and $p^C = m + \sqrt{cF}$
- **Comparative statics**: n^C increases in c and decreases in F

Welfare

- Average distance to the nearest firm: $\bar{d} = \frac{1}{4n}$
- Resource cost of variety: nF
- With quasilinear utility, prices are transfers
- Per-consumer welfare: $W(n) = v - \frac{c}{4n} - nF$ (where $v = u - u^*$)

Social planner solution

Planner problem: Choose n to maximise $W(n)$

- FOC: $\frac{dW}{dn} = \frac{c}{4n^2} - F = 0$
- Social optimum: $n^S = \frac{1}{2}\sqrt{\frac{c}{F}}$
- Free entry: $n^C = \sqrt{\frac{c}{F}}$ so $n^C = 2n^S$

Excess entry due to the business-stealing externality.

Summary and next week

- Mismatch costs soften price competition (markups increase in c)
- Hotelling: demand, best responses, and equilibrium prices with fixed locations
- Salop: free entry gives $n^C = \sqrt{\frac{c}{F}}$ and implies excess entry relative to the planner

Next week:

- Innovation under uncertainty
- Oligopoly incentives to innovate (replacement effect, entry threat)

References

d'Aspremont, C., J. Jaskold Gabszewicz, and J.-F. Thisse. 1979. 'On {H}Otelling's "{S}Tability in Competition"'. *Econometrica* 47 (5): 1145–50. https://doi.org/10.2307/1911955.

Hotelling, Harold. 1929. 'Stability in Competition'. *The Economic Journal* 39 (153): 41–57. https://doi.org/10.2307/2224214.

Salop, Steven C. 1979. 'Monopolistic Competition with Outside Goods'. *The Bell Journal of Economics* 10 (1): 141. https://doi.org/10.2307/3003323.