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Live Exercise 2: Replacement Effect and Social Value of Innovation

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Solutions

Group exercise (≈20 minutes)

- Work in groups of 2–3.
- Show all intermediate steps — the algebra matters.
- Full market coverage assumed throughout.

Problem: Replacement effect and social value

A pharmaceutical company has discovered a new manufacturing process (Tech P) that reduces the marginal cost of producing a drug from $c_0 = 60$ to $c_1 = 30$. Market demand for the drug is

$$P(Q) = 100 - Q.$$

Assume Tech P is available exclusively to the one firm that acquires it (patent protection is perfect).

(a) Is the innovation drastic?

Use the monopoly pricing formula to determine whether Tech P is drastic or non-drastic. State the condition and evaluate it numerically.

Recall: a process innovation is drastic if $P^m(c_1) < c_0$, where $P^m(c) = \frac{A+c}{2}$ under linear demand $P = A - Q$.

(b) Monopoly and competitive WTP

Compute the maximum willingness to pay (WTP) for Tech P of:

1. a monopolist (not threatened by entry)
2. a competitive innovator — a firm that operates in a competitive market before acquiring the patent, then gains exclusive rights to Tech P

Hint for (2): use your result from (a) to determine whether the competitive innovator limit-prices at $p = c_0$ after acquiring the patent.

(c) Social planner's value

For linear demand $P = A - Q$ with efficient production (price equals marginal cost), total surplus is $W(c) = \frac{(A-c)^2}{2}$.

1. Compute the social planner's value of the innovation, $\Delta W = W(c_1) - W(c_0)$.
2. Arrange the three values — $\Delta\pi^m$, competitive WTP, ΔW — in increasing order.
3. State Arrow's replacement effect in one sentence. Using the numbers from (b), explain what drives the gap between $\Delta\pi^m$ and the competitive WTP. Why is ΔW larger than both?

Solution

(a) Drastic or non-drastic?

With $P(Q) = 100 - Q$ and $c_1 = 30$, the post-innovation monopoly price is

$$P^m(c_1) = \frac{A + c_1}{2} = \frac{100 + 30}{2} = 65.$$

Since $P^m(c_1) = 65 > c_0 = 60$, the innovation is non-drastic.

Intuitively, even with the lower cost, the monopolist would price above the competitive benchmark — so the innovation does not completely destroy the competitive fringe.

(b) Monopoly and competitive WTP

Monopoly WTP

Pre-innovation ($c_0 = 60$):

$$\max_Q (100 - Q - 60)Q = \max_Q (40 - Q)Q.$$

FOC: $40 - 2Q = 0 \Rightarrow Q_0^m = 20, P_0^m = 80,$

$$\pi_0^m = (80 - 60) \times 20 = 400.$$

Post-innovation ($c_1 = 30$):

$$\max_Q (70 - Q)Q. \quad \text{FOC:} \quad 70 - 2Q = 0 \Rightarrow Q_1^m = 35, P_1^m = 65.$$

$$\pi_1^m = (65 - 30) \times 35 = 1,225.$$

Monopoly WTP:

$$\Delta\pi^m = \pi_1^m - \pi_0^m = 1,225 - 400 = \mathbf{825}.$$

Competitive WTP

Since the innovation is non-drastic, the competitive innovator limit-prices at $p = c_0 = 60$ after acquiring the patent:

$$Q = 100 - 60 = 40, \qquad \text{profit} = (c_0 - c_1) \times Q = 30 \times 40 = \mathbf{1,200}.$$

(c) Social planner's value and the replacement effect

$$W(c_0) = \frac{(100-60)^2}{2} = \frac{1,600}{2} = 800, \qquad W(c_1) = \frac{(100-30)^2}{2} = \frac{4,900}{2} = 2,450.$$

$$\Delta W = 2,450 - 800 = \mathbf{1,650}.$$

Increasing order:

$$\underbrace{825}_{\Delta\pi^m} < \underbrace{1,200}_{\text{competitive WTP}} < \underbrace{1,650}_{\Delta W}.$$

(Check: with linear demand, $\Delta W = 2\,\Delta\pi^m$ always holds — the planner values the output expansion that the monopolist ignores.)

Replacement effect: The monopolist already earns $\pi_0^m = 400$ before innovating; innovation only adds 825 to its profit rather than yielding the full post-innovation profit of 1,225. The competitive innovator earns zero before the patent, so its incremental gain (1,200) is larger. This is Arrow's replacement effect: pre-innovation rents reduce the incumbent's marginal gain from innovation.

Why $W > $ competitive WTP: The social planner also values the consumer surplus gains from the output expansion (Q rises from 40 to 70 under efficient pricing at c_1), which no private firm captures. Neither private incentive approaches the social value of the innovation.

Bibliography