Lecture 5 – Multi-stage Games

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Multi-stage Games

Multi-stage games: overview

Object of analysis

How commitment in earlier stages changes equilibrium behaviour in later stages.

- So far:
  - Mostly simultaneous-move games
  - Briefly: two-stage game (Shy (1995) model in Lecture 4)
- Today:
  - Stackelberg quantity leadership
  - Strategic delegation (Vickers 1985)

Solution concept: subgame perfect equilibrium

- A strategy profile is a **subgame perfect Nash equilibrium (SPNE)** if it induces a Nash equilibrium in every subgame
- Operationally (finite horizon):
  - Solve from the last stage backward (backward induction)

Stackelberg competition

Stackelberg model: structure

- Two stages, two firms:
  1. Leader chooses q_1
  2. Follower observes q_1 and chooses q_2
- Demand and costs:
  - Inverse demand: $p(Q) = A - Q$ with $Q = q_1 + q_2$
  - Marginal cost: c (parameter)

Stackelberg model: follower best response

- Follower profit given q_1:

$$\pi_2(q_2; q_1) = (A - q_1 - q_2 - c)q_2$$

- Best response:

$$BR_2(q_1) = \frac{A - c - q_1}{2}$$

> **Strategic substitutes (quantities)**

Stackelberg model: leader choice

- Substitute follower response into leader profit:

$$\pi_1(q_1) = (A - q_1 - BR_2(q_1) - c)q_1$$

- Equilibrium (identical costs):
  - $q_1^S = \frac{A-c}{2}$
  - $q_2^S = \frac{A-c}{4}$

> **! First-mover advantage**
>
> The leader commits to a higher q_1, anticipating that the follower reduces q_2.

Stackelberg v Cournot: equilibrium outcomes

- Identical marginal costs (c) imply:

Outcome	Stackelberg	Cournot
Firm 1 output	$q_1^S = \frac{A-c}{2}$	$q_1^C = \frac{A-c}{3}$

Strategic delegation (Vickers)

Delegation: economic idea

- Firms may delegate output choice to managers with objectives that differ from pure profit maximisation
- Delegation instrument:
  - an incentive parameter θ_i written into the manager's contract

Delegation (Vickers): model

- Cournot oligopoly with n firms: $i = 1, \dots, n$
- Manager of firm i maximises:

$$M_i = \pi_i + \theta_i q_i$$

where $\pi_i = p(Q)q_i - cq_i$ and $Q = \sum_i q_i$

> **Equivalent formulation**
>
> $M_i = p(Q)q_i - (c - \theta_i)q_i$ so θ_i shifts the manager's objective like a reduction in marginal cost.