

Lecture 2 – Product Differentiation

Gerhard Riener

Differentiation types

- ▶ Horizontal: consumers disagree on which variety is best (ideal-point / mismatch preferences)
- ▶ Vertical: products differ in quality; at equal prices all consumers (weakly) prefer the higher-quality good

Today: roadmap

- ▶ Focus: horizontal differentiation
- ▶ Hotelling line: 2 firms, pricing with fixed locations
- ▶ Salop circular city: free entry and welfare

Hotelling Model

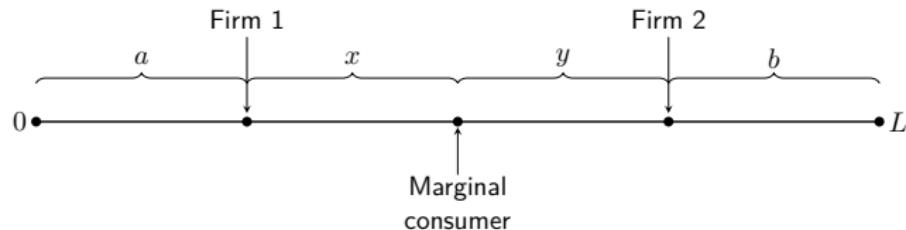
Setup: Fixed locations, price competition

- ▶ Duopoly, marginal cost 0
- ▶ Consumers (ℓ) uniformly distributed on $[0, L]$ (Hotelling 1929)
- ▶ Firm 1 at a and Firm 2 at $L - b$
- ▶ Mismatch (transport) cost $C(d) = cd$ with $c > 0$, d extent of mismatch
- ▶ Unit demand; assume full market coverage in equilibrium

i Utility (full coverage)

$U_1(\ell) = \bar{u} - p_1 - c|\ell - a|$, $U_2(\ell) = \bar{u} - p_2 - c|\ell - (L - b)|$,
outside option 0; assume \bar{u} is large enough that all consumers buy.

Geometry



- ▶ Choice variables: p_1, p_2
- ▶ Parameters: L, a, b, c
- ▶ Distance between firms: $L - a - b$
- ▶ Marginal consumer distances: $x + y = L - a - b$
- ▶ Demands: $q_1 = a + x$ and $q_2 = b + y$

Marginal consumer and demands

- ▶ Indifference of the marginal consumer: $p_1 + cx = p_2 + cy$
- ▶ Geometry: $x + y = L - a - b$

Demands by firm

- ▶ $q_1 = a + x = \frac{p_2 - p_1 + c(L + a - b)}{2c}$
- ▶ $q_2 = b + y = \frac{p_1 - p_2 + c(L + b - a)}{2c}$

Best responses

- ▶ $BR_1(p_2) = \frac{p_2}{2} + \frac{c}{2}(L + a - b)$
- ▶ $BR_2(p_1) = \frac{p_1}{2} + \frac{c}{2}(L + b - a)$



Warning

Prices are strategic complements; if $a = b$ then $p_1^* = p_2^*$

Price equilibrium fixed locations

- ▶ Prices: $p_1^* = c \left(L + \frac{a-b}{3} \right)$ and $p_2^* = c \left(L + \frac{b-a}{3} \right)$
- ▶ Profits: $\pi_1^* = \frac{c}{18}(3L + a - b)^2$ and $\pi_2^* = \frac{c}{18}(3L + b - a)^2$
- ▶ If $a = b$, symmetry implies $p_1^* = p_2^*$
- ▶ Higher c lowers substitutability and raises equilibrium markups and profits

Hotelling: co-location

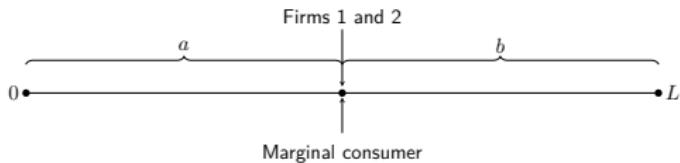
- ▶ Co-location implies $a + b = L$ and $x = y = 0$
- ▶ Products become homogeneous \Rightarrow Bertrand undercutting
- ▶ Equilibrium: $p_1^B = p_2^B = 0$ and $\pi_1^B = \pi_2^B = 0$



Warning

Co-location and price competition

Co-location removes captive consumers; any $p > 0$ can be profitably undercut.



Move incentives at fixed prices

- ▶ $\frac{\partial \pi_1^*}{\partial a} = \frac{c}{9}(3L + a - b) > 0$ and $\frac{\partial \pi_2^*}{\partial b} = \frac{c}{9}(3L + b - a) > 0$
- ▶ In the pricing subgame, increasing a (resp. b) moves Firm 1 (resp. Firm 2) inward (toward the centre)
- ▶ These are comparative statics holding the rival location fixed
- ▶ Under linear transport costs, pricing incentives can pull locations toward the centre; firms will “go where the demand is”

Identifying clustering motives

Question: Why are so many businesses clustered together?

- ▶ Going where the demand is?
- ▶ Possible collusion story? Being next to the competitor facilitates monitoring.

Next: the formal location game clarifies when this clustering logic breaks down under linear transport costs.

Location game (d'Aspremont et al. 1979)

- ▶ **Two-stage game:** choose locations, then prices
- ▶ Linear transport costs ($C(d) = cd$): pure-strategy location equilibrium may fail to exist (market-capture regions; non-concavity)
- ▶ Quadratic transport costs ($C(d) = td^2$): pure-strategy location equilibrium exists (baseline: maximum differentiation)
- ▶ Principle of minimum differentiation need not hold

i Note

Why convexity matters

More convex mismatch costs make small location changes less effective at stealing marginal consumers through pricing.

Profit function: interior region

- ▶ Let $\Delta \equiv L - a - b$ denote the distance between firms
- ▶ Interior-demand region: $|p_1 - p_2| \leq c\Delta$
- ▶ In this region, Firm 1's profit is:

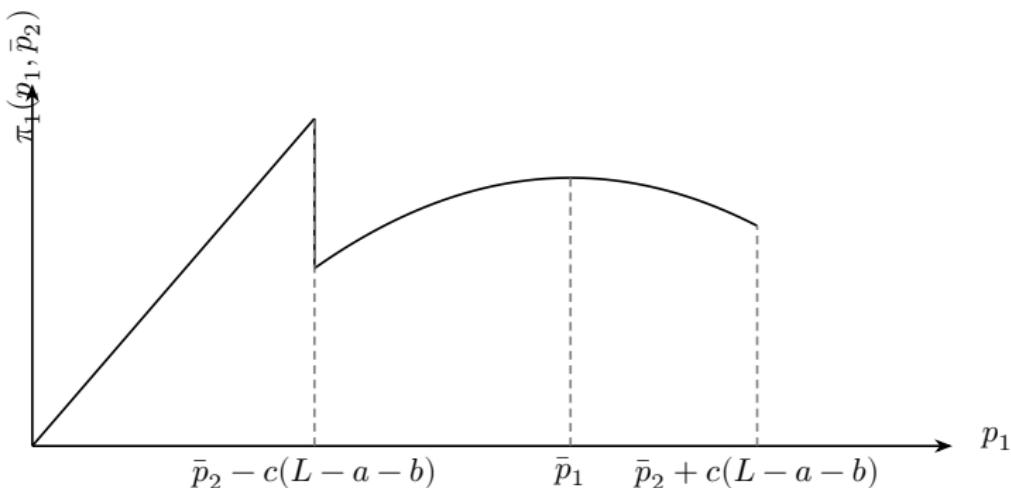
$$\pi_1(p_1, p_2) = \frac{p_1 p_2}{2c} - \frac{p_1^2}{2c} + \frac{(L + a - b)p_1}{2}$$

Profit function: corner regions

- ▶ Market capture: if $p_1 < p_2 - c\Delta$, then $\pi_1 = Lp_1$
- ▶ No demand: if $p_1 > p_2 + c\Delta$, then $\pi_1 = 0$
- ▶ Together with the interior expression, these define the piecewise payoff in (d'Aspremont et al. 1979)

Profit regions

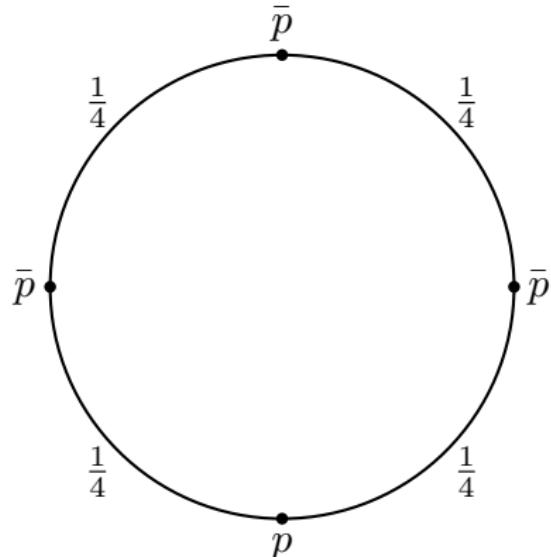
- ▶ **Region 1 (interior)**: both firms serve positive mass of consumers; demand is linear in p_1
- ▶ **Region 2 (market capture)**: Firm 1 undercuts enough to serve the whole line segment
- ▶ **Region 3 (no demand)**: Firm 1 sets too high a price; demand is zero



Salop Circular City

Setup

- ▶ Outside good yields utility u^*
- ▶ Differentiated varieties lie on a circle of circumference 1; consumers are uniformly distributed (Salop 1979)
- ▶ n firms are symmetrically located; adjacent distance is $\frac{1}{n}$
- ▶ Each consumer buys at most one unit: from some firm or the outside good



Objects

- ▶ Choice variable: p_i
- ▶ Parameters: n, c, u, u^*
- ▶ Outcomes: market boundaries / cutoffs, quantities
- ▶ Symmetric benchmark: if neighbours set $\tilde{p} = p$, the boundary consumer is at $x = \frac{1}{2n}$
- ▶ Baseline local demand before participation truncation:
$$q^C = 2x = \frac{1}{n}$$

Consumers

- ▶ Each consumer has an ideal variety ℓ^* (location on the circle)
- ▶ Arc distance d generates mismatch cost cd
- ▶ Utility from firm j : $U_j = u - cd_j - p_j$
- ▶ Outside option: $U^O = u^*$
- ▶ Let $v = u - u^*$; participation requires $v \geq p_j + cd_j$

Monopoly region

- ▶ Outside option truncates the market (participation binds), not neighbours
- ▶ Farthest served consumer: $d_m = \frac{v-p}{c}$
- ▶ Monopoly demand: $q^M = 2d_m = 2\left(\frac{v-p}{c}\right)$

Monopoly inverse demand

- ▶ Inverse demand: $p = v - \frac{c}{2}q^M$
- ▶ Slope is $-\frac{c}{2}$ because expanding quantity extends the served interval on both sides
- ▶ Participation binds at the marginal consumer: $v = p + cd_m$

Competitive region

- ▶ Boundary consumer (indifference):

$$v - cx - p = v - c \left(\frac{1}{n} - x \right) - \tilde{p}$$

- ▶ Boundary location: $x = \frac{\frac{c}{n} - p + \tilde{p}}{2c}$

- ▶ Competitive quantity: $q^C = 2x$

Competitive demand

- ▶ **Competitive demand:** $q^C = 2x = \frac{\frac{c}{n} - p + \tilde{p}}{c}$
- ▶ **Inverse demand:** $p = \frac{c}{n} + \tilde{p} - cq^C$

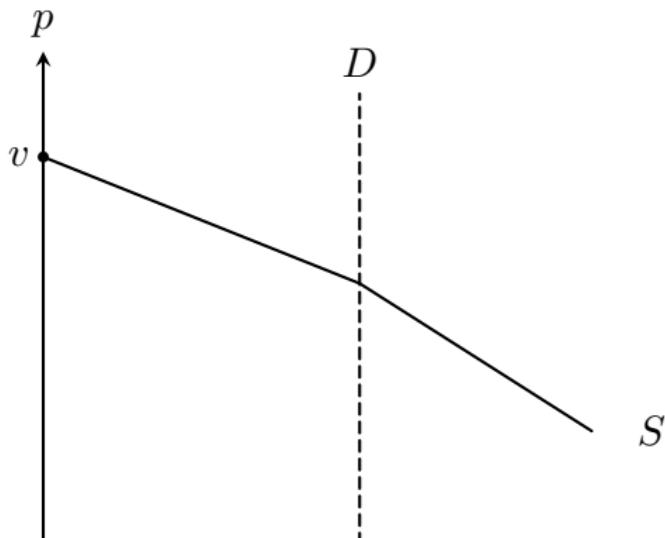
! Important

Slope comparison

Competitive-region inverse demand is steeper than monopoly-region inverse demand: $-c$ vs $-\frac{c}{2}$.

Kinked demand

- ▶ Demand is limited either by the outside option (participation) or by neighbours (local competition)
- ▶ Effective demand is $q(p) = \min\{q^M(p), q^C(p)\}$
 - ▶ **Participation-boundary (monopoly region):** $q^M = 2 \left(\frac{v-p}{c} \right)$
 - ▶ **Neighbour-boundary (competitive region):** $q^C = \frac{\frac{c}{n}-p+\tilde{p}}{c}$
- ▶ The switch in the binding constraint creates a kink in inverse demand and a discrete change in marginal revenue



Cost functions

- ▶ Cost function: $C(q) = F + mq$ (fixed cost F , marginal cost m)
- ▶ Average cost: $AC(q) = m + \frac{F}{q}$ (decreasing in q)

Pricing (competitive region)

- ▶ Inverse demand: $p = \frac{c}{n} + \tilde{p} - cq$
- ▶ Marginal revenue: $MR = \frac{c}{n} + \tilde{p} - 2cq$
- ▶ Symmetry: $\tilde{p} = p$

i Competitive-region condition (outside option not binding)

- ▶ In a symmetric profile, the farthest consumer from a firm is at distance $\frac{1}{2n}$.
- ▶ A sufficient condition for the competitive region to apply at the symmetric equilibrium is $v \geq p^C + \frac{c}{2n}$.

Zero profit

- ▶ Symmetry implies $q^C = \frac{1}{n}$
- ▶ Zero profit requires $p^C = AC = m + \frac{F}{q^C}$
- ▶ Therefore $p^C = m + nF$

Pricing condition

- ▶ **Competitive-region inverse demand:** $p = \frac{c}{n} + \tilde{p} - cq$
- ▶ **First-order condition:** $MR = MC$ with $MR = \frac{c}{n} + \tilde{p} - 2cq$ and $MC = m$
- ▶ **Symmetric equilibrium implication:** with $\tilde{p} = p$ and $q^C = \frac{1}{n}$, we get $p^C = m + \frac{c}{n}$

Free-entry equilibrium

- ▶ **Zero profit condition:** $p^C = m + nF$
- ▶ **Solve:** $m + nF = m + \frac{c}{n}$ so $n^C = \sqrt{\frac{c}{F}}$ and $p^C = m + \sqrt{cF}$
- ▶ **Comparative statics:** n^C increases in c and decreases in F

Welfare

- ▶ Average distance to the nearest firm: $\bar{d} = \frac{1}{4n}$
- ▶ Resource cost of variety: nF
- ▶ With quasilinear utility, prices are transfers
- ▶ Per-consumer welfare: $W(n) = v - \frac{c}{4n} - nF$ (where $v = u - u^*$)

Social planner solution

Planner problem: Choose n to maximise $W(n)$

- ▶ FOC: $\frac{dW}{dn} = \frac{c}{4n^2} - F = 0$
- ▶ Social optimum: $n^S = \frac{1}{2}\sqrt{\frac{c}{F}}$
- ▶ Free entry: $n^C = \sqrt{\frac{c}{F}}$ so $n^C = 2n^S$

Excess entry due to the business-stealing externality.

Summary and next week

- ▶ Mismatch costs soften price competition (markups increase in c)
- ▶ Hotelling: demand, best responses, and equilibrium prices with fixed locations
- ▶ Salop: free entry gives $n^C = \sqrt{\frac{c}{F}}$ and implies excess entry relative to the planner

Next week:

- ▶ Innovation under uncertainty
- ▶ Oligopoly incentives to innovate (replacement effect, entry threat)

References

- d'Aspremont, C., J. Jaskold Gabszewicz, and J.-F. Thisse. 1979. 'On Hotelling's "Stability in Competition"'. *Econometrica* 47 (5): 1145–50. <https://doi.org/10.2307/1911955>.
- Hotelling, Harold. 1929. 'Stability in Competition'. *The Economic Journal* 39 (153): 41–57. <https://doi.org/10.2307/2224214>.
- Salop, Steven C. 1979. 'Monopolistic Competition with Outside Goods'. *The Bell Journal of Economics* 10 (1): 141. <https://doi.org/10.2307/3003323>.