

Lecture 5 – Multi-stage Games

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Multi-stage Games

Multi-stage games: overview

Object of analysis

How commitment in earlier stages changes equilibrium behaviour in later stages.

- ▶ So far:
 - ▶ Mostly simultaneous-move games
 - ▶ Briefly: two-stage game (Shy (1995) model in Lecture 4)
- ▶ Today:
 - ▶ Stackelberg quantity leadership
 - ▶ Strategic delegation (Vickers 1985)

Solution concept: subgame perfect equilibrium

- ▶ A strategy profile is a **subgame perfect Nash equilibrium (SPNE)** if it induces a Nash equilibrium in every subgame
- ▶ Operationally (finite horizon):
 - ▶ Solve from the last stage backward (backward induction)

Stackelberg competition

Stackelberg model: structure

- ▶ Two stages, two firms:
 1. Leader chooses q_1
 2. Follower observes q_1 and chooses q_2
- ▶ Demand and costs:
 - ▶ Inverse demand: $p(Q) = A - Q$ with $Q = q_1 + q_2$
 - ▶ Marginal cost: c (parameter)

Stackelberg model: follower best response

- ▶ Follower profit given q_1 :

$$\pi_2(q_2; q_1) = (A - q_1 - q_2 - c)q_2$$

- ▶ Best response:

$$BR_2(q_1) = \frac{A - c - q_1}{2}$$

i Strategic substitutes (quantities)

Stackelberg model: leader choice

- ▶ Substitute follower response into leader profit:

$$\pi_1(q_1) = (A - q_1 - BR_2(q_1) - c)q_1$$

- ▶ Equilibrium (identical costs):

- ▶ $q_1^S = \frac{A-c}{2}$

- ▶ $q_2^S = \frac{A-c}{4}$

! First-mover advantage

The leader commits to a higher q_1 , anticipating that the follower reduces q_2 .

Stackelberg v Cournot: equilibrium outcomes

- ▶ Identical marginal costs (c) imply:

Outcome	Stackelberg	Cournot
Firm 1 output	$q_1^S = \frac{A-c}{2}$	$q_1^C = \frac{A-c}{3}$

Strategic delegation (Vickers)

Delegation: economic idea

- ▶ Firms may delegate output choice to managers with objectives that differ from pure profit maximisation
- ▶ Delegation instrument:
 - ▶ an incentive parameter θ_i written into the manager's contract

Delegation (Vickers): model

- ▶ Cournot oligopoly with n firms: $i = 1, \dots, n$
- ▶ Manager of firm i maximises:

$$M_i = \pi_i + \theta_i q_i$$

where $\pi_i = p(Q)q_i - cq_i$ and $Q = \sum_i q_i$

i Equivalent formulation

$M_i = p(Q)q_i - (c - \theta_i)q_i$ so θ_i shifts the manager's objective like a reduction in marginal cost.