Exercise 1: Oligopoly and Product Differentiation

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Instructions

- Answer all questions.
- Show intermediate steps.
- Solutions are included only when `params.solution: true`.

Problem 1: Welfare analysis

Consider the example exercise in Lecture 1, where

$P = 20 - Q$

and there are three firms with cost function $c_i(q_i) = c_i q_i$ with

$c_1 = 0$, $c_2 = 5$, and $c_3 = 7$.

(a)

Compute the Nash equilibrium of the Cournot duopoly game between firm 1 and firm 2.

(b)

Given your answer to (a), compute the equilibrium profits, consumer surplus, and welfare loss.

(c)

Compare the values in (b) to those of the other three market structures (perfect competition, monopoly, Cournot triopoly) in Lecture 1. Discuss any policy implications for a social planner who maximises total welfare.

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Solution

(a)

The best response functions are

$$q_1 = \frac{20 - q_2}{2} \qquad \text{and} \qquad q_2 = \frac{15 - q_1}{2}.$$

Solving this system yields the equilibrium quantities

$$q_1^D = \frac{25}{3}, \qquad q_2^D = \frac{10}{3}.$$

(b)

The equilibrium price is

$$P(Q^D) = 20 - \frac{35}{3} = \frac{25}{3}.$$

Consumer surplus is therefore

$$CS^D = \frac{1}{2}\left(\frac{35}{3}\right)^2 = \frac{1225}{18},$$

and producer surplus is

$$PS^D = \Pi_1 + \Pi_2 = \left(\frac{25}{3}\right)^2 + \left(\frac{10}{3}\right)^2 = \frac{725}{9}.$$

The welfare loss is

$$WL^D = 200 - \frac{1225}{18} - \frac{725}{9} = \frac{925}{18}.$$

(c)

The values for all three welfare components fall between those of monopoly and Cournot triopoly:

$$CS^M < CS^D < CS^T, \qquad PS^M > PS^D > PS^T, \qquad WL^M < WL^D < WL^T.$$

As the number of producing firms increases (monopoly → Cournot duopoly → Cournot triopoly), prices fall and consumer surplus increases, but production is reallocated toward higher-cost firms (reducing productive efficiency). ::: < endif >

Problem 2: Linear Hotelling model with monopoly

Consider the Hotelling linear-city model. Suppose there is only one restaurant, located at the centre of a street of length 1 km. The restaurant's cost is zero.

Consumers are uniformly distributed on the interval $[0, 1]$, with one consumer at each point. Transportation cost is £1 per kilometre.

The utility of a consumer located at distance a from the restaurant is

$$U = B - a - p,$$

where p is the price of a meal and B is a constant.

If the consumer does not eat at the restaurant, utility is

$$U^* = 0.$$

(a)

Suppose $0 < B < 1$. Find the number of consumers eating at the restaurant. Calculate the monopoly price and profit.

(b)

Answer the previous question assuming $B > 1$.

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Solution

(a)

Let d_m denote the distance of the marginal consumer from the restaurant. Indifference implies

$$B - d_m - p = 0 \quad \Rightarrow \quad d_m = B - p.$$

Demand is

$$q = 2d_m = 2(B - p).$$

The monopolist solves

$$\max_p \ \Pi(p) = p \cdot 2(B - p).$$

FOC implies $p^M = \frac{B}{2}$, hence $q^M = B$ and $\Pi^M = \frac{B^2}{2}$.

(b)

For $B > 1$, the unconstrained solution implies $d_m = \frac{B}{2} > \frac{1}{2}$, but demand cannot exceed 1, so $q = 1$. The monopolist sets price so the marginal consumers are exactly at the endpoints ($d = \frac{1}{2}$):

$$B - \frac{1}{2} - p = 0 \quad \Rightarrow \quad p^* = B - \frac{1}{2}.$$

Then $\Pi^* = p^* \cdot 1 = B - \frac{1}{2}$. ::: < endif >