EC5230 Class Test (Lectures 1-5)

Industrial Organisation

Time: 50 minutes. **No writing for the first 5 minutes.**

Instructions

- Answer all questions.
- Show clear intermediate steps for calculations.
- Unsupported final answers may receive limited credit.
- Total marks: **20**.
- **Section A (Q1–Q3): 10 marks** (core material).
- **Section B (Q4): 7 marks** (application).
- **Section C (Q5): 3 marks** (understanding).

Section A (10 marks)

Question 1: Cournot with asymmetric costs (4 marks)

Consider a Cournot duopoly with inverse demand

$$P = 24 - Q, \qquad Q = q_1 + q_2,$$

and constant marginal costs

$$c_1 = 3, \qquad c_2 = 6.$$

1. Derive each firm's best-response function.
2. Compute the Nash equilibrium quantities (q_1^*, q_2^*), total output Q^*, and price P^*.
3. Compute equilibrium profits (π_1^*, π_2^*) and briefly explain why they differ.

Question 2: Hotelling price competition (3 marks)

Two firms are located at the endpoints of a unit interval: $x_1 = 0$, $x_2 = 1$. Consumers are uniformly distributed on $[0, 1]$ and each buys one unit. Utility from buying from firm i is

$$U_i = v - p_i - t, |x - x_i|,$$

with $t = 2$ and marginal costs $c_1 = c_2 = 0$.

Assume that (v) is sufficiently large so that all consumers buy one unit (full market coverage), and that the equilibrium is interior.

1. Find the location of the marginal consumer $\hat{x}$ and the demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$. **[5]**
2. Derive both price best responses and solve for the symmetric Nash equilibrium prices.
3. Compute each firm's equilibrium market share and profit.
4. Briefly explain the effect of an increase in t on equilibrium prices.

Question 3: Innovation incentives and replacement effect (3 marks)

Consider inverse demand

$$P = 20 - Q.$$

A process innovation reduces marginal cost from $c_0 = 10$ to $c_1 = 4$.

1. Under monopoly both before and after innovation, compute profits before and after, and the monopolist's maximum willingness to pay (WTP) for the innovation.
2. Under perfect competition before innovation, assume the innovator obtains exclusive use of the new technology after innovation while other firms remain available at cost (c_0) (competitive fringe). Compute the innovator's WTP and state whether the innovation is drastic or non-drastic.
3. Compare your two WTP values and explain the replacement effect in this example.

Section B (7 marks)

Question 4: Investment game with and without patents (7 marks)

Consider a Cournot duopoly. If firms have marginal costs (c_i, c_j), firm i's gross profit is

$$\pi_i(c_i, c_j) = \frac{(A - 2c_i + c_j)^2}{9}, \qquad i \neq j.$$

Let

$$A = 12, \quad c_0 = 4, \quad c' = 1, \quad \tilde{c} = 2, \quad F = 5.$$

Each firm simultaneously chooses whether to invest (I) or not (N).

- If a firm invests, it pays F.
- Investing reduces own cost from c_0 to c'.

Policy regimes:

1. **Patents (P):**
   - If only one firm invests, it obtains the patent with probability 1.
   - If both invest, each wins the patent with probability $(\frac{1}{2})$; payoffs are expected values over this patent lottery.
   - A non-patent-holder remains at cost c_0.
2. **No patents (NP):**
   - If one firm invests, the rival can imitate and reduce cost to $\tilde{c}$ without paying F.

Tasks:

1. Compute all gross profit terms needed for the game (for example $\pi(c', c_0)$, $\pi(c_0, c')$, $\pi(c', c')$, $\pi(c', \tilde{c})$, $\pi(\tilde{c}, c')$, $\pi(c_0, c_0)$).
2. Construct the 2×2 payoff matrix for each regime (P and NP), with **net** payoffs.
3. Find all pure-strategy Nash equilibria under each regime.
4. Evaluate the statement: “Without patents, imitation necessarily destroys incentives to invest.” Use your results.

Section C (3 marks, advanced)

Question 5: Commitment stage before quantity competition (3 marks)

Demand is

$$P = 18 - Q, \qquad Q = q_1 + q_2,$$

with identical constant marginal costs (c=2).

Stage 0 (commitment):

- Each firm simultaneously chooses (C) (commit to move first) or (N) (no commitment).
- If exactly one firm chooses (C), that firm becomes the Stackelberg leader and pays commitment cost (K).
- If both choose the same action ((C,C) or (N,N)), firms play Cournot in the quantity stage; under ((C,C)) each still pays (K).

Tasks:

1. Solve the quantity-stage outcomes and profits for:
   - Cournot benchmark
   - Stackelberg leader and follower
2. Build the stage-0 payoff matrix as a function of (K).
3. Derive pure-strategy Nash equilibria as a function of (K) (identify threshold values).
4. Suppose one firm can be assigned commitment rights (so the market outcome is Stackelberg with one commitment cost). For which values of (K) is this welfare-improving relative to Cournot?