# Lecture 2 – Product Differentiation

Gerhard Riener

# Differentiation types

▶ Horizontal: consumers disagree on which variety is best (ideal-point / mismatch preferences)
▶ Vertical: products differ in quality; at equal prices all consumers (weakly) prefer the higher-quality good

**Today: roadmap**

▶ Focus: horizontal differentiation
▶ Hotelling line: 2 firms, pricing with fixed locations
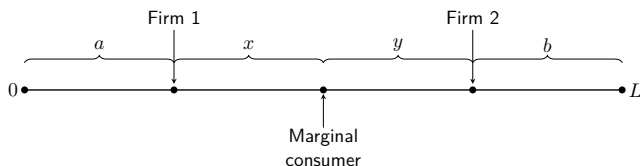▶ Salop circular city: free entry and welfare

# Hotelling Model

# Setup: Fixed locations, price competition

▶ Duopoly, marginal cost $0$
▶ Consumers ($\ell$) uniformly distributed on $[0, L]$ (Hotelling 1929)
▶ Firm 1 at $a$ and Firm 2 at $L - b$
▶ Mismatch (transport) cost $C(d) = cd$ with $c > 0$, $d$ extent of mismatch
▶ Unit demand; assume full market coverage in equilibrium

> **i** Utility (full coverage)
>
> $U_1(\ell) = \bar{u} - p_1 - c|\ell - a|$, $U_2(\ell) = \bar{u} - p_2 - c|\ell - (L - b)|$, outside option $0$; assume $\bar{u}$ is large enough that all consumers buy.

# Geometry



- Choice variables: $p_1, p_2$
- Parameters: $L, a, b, c$
- Distance between firms: $L - a - b$
- Marginal consumer distances: $x + y = L - a - b$
- Demands: $q_1 = a + x$ and $q_2 = b + y$

# Marginal consumer and demands

- Indifference of the marginal consumer: $p_1 + cx = p_2 + cy$
- Geometry: $x + y = L - a - b$

**Demands by firm**

- $q_1 = a + x = \frac{p_2 - p_1 + c(L + a - b)}{2c}$
- $q_2 = b + y = \frac{p_1 - p_2 + c(L + b - a)}{2c}$

# Best responses

- $BR_1(p_2) = \frac{p_2}{2} + \frac{c}{2}(L + a - b)$
- $BR_2(p_1) = \frac{p_1}{2} + \frac{c}{2}(L + b - a)$

> ⚠️ **Warning**
>
> Prices are strategic complements; if $a = b$ then $p_1^* = p_2^*$

# Price equilibrium fixed locations

▶ Prices: $p_1^* = c\left(L + \frac{a-b}{3}\right)$ and $p_2^* = c\left(L + \frac{b-a}{3}\right)$

▶ Profits: $\pi_1^* = \frac{c}{18}(3L + a - b)^2$ and $\pi_2^* = \frac{c}{18}(3L + b - a)^2$

▶ If $a = b$, symmetry implies $p_1^* = p_2^*$

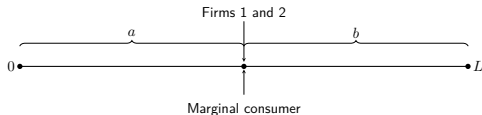▶ Higher $c$ lowers substitutability and raises equilibrium markups and profits

# Hotelling: co-location

▶ Co-location implies $a + b = L$ and $x = y = 0$
▶ Products become homogeneous $\Rightarrow$ Bertrand undercutting
▶ Equilibrium: $p_1^B = p_2^B = 0$ and $\pi_1^B = \pi_2^B = 0$

> ⚠ **Warning**
>
> **Co-location and price competition**
> Co-location removes captive consumers; any $p > 0$ can be profitably undercut.



Firms 1 and 2

$a$        $b$

$0$             $L$

Marginal consumer

# Move incentives at fixed prices

▶ $\frac{\partial \pi_1^*}{\partial a} = \frac{c}{9}(3L + a - b) > 0$ and $\frac{\partial \pi_2^*}{\partial b} = \frac{c}{9}(3L + b - a) > 0$

▶ In the pricing subgame, increasing $a$ (resp. $b$) moves Firm 1 (resp. Firm 2) inward (toward the centre)

▶ These are comparative statics holding the rival location fixed

▶ Under linear transport costs, pricing incentives can pull locations toward the centre; firms will "go where the demand is"

# Identifying clustering motives

**Question:** Why are so many businesses clustered together?

▶ Going where the demand is?

▶ Possible collusion story? Being next to the competitor facilitates monitoring.

Next: the formal location game clarifies when this clustering logic breaks down under linear transport costs.

# Location game (d'Aspremont et al. 1979)

▶ **Two-stage game:** choose locations, then prices
▶ Linear transport costs ($C(d) = cd$): pure-strategy location equilibrium may fail to exist (market-capture regions; non-concavity)
▶ Quadratic transport costs ($C(d) = td^2$): pure-strategy location equilibrium exists (baseline: maximum differentiation)
▶ Principle of minimum differentiation need not hold

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**i** Note

**Why convexity matters**
More convex mismatch costs make small location changes less effective at stealing marginal consumers through pricing.

# Profit function: interior region

▶ Let $\Delta \equiv L - a - b$ denote the distance between firms
▶ Interior-demand region: $|p_1 - p_2| \leq c\Delta$
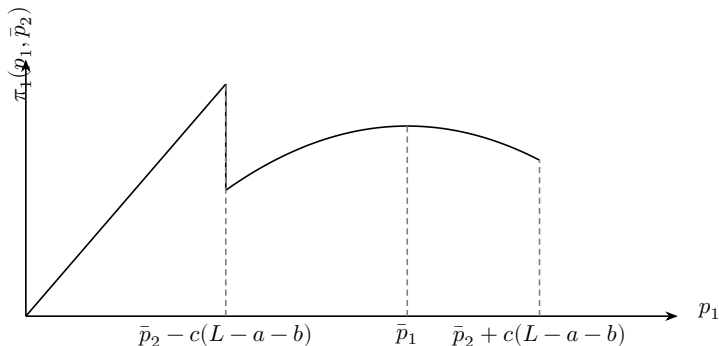▶ In this region, Firm 1's profit is:

$$\pi_1(p_1, p_2) = \frac{p_1 p_2}{2c} - \frac{p_1^2}{2c} + \frac{(L + a - b)p_1}{2}$$

# Profit function: corner regions

▶ Market capture: if $p_1 < p_2 - c\Delta$, then $\pi_1 = Lp_1$
▶ No demand: if $p_1 > p_2 + c\Delta$, then $\pi_1 = 0$
▶ Together with the interior expression, these define the piecewise payoff in (d'Aspremont et al. 1979)
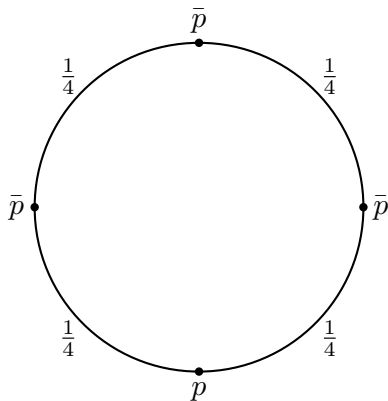
# Profit regions

▶ **Region 1 (interior)**: both firms serve positive mass of consumers; demand is linear in $p_1$

▶ **Region 2 (market capture)**: Firm 1 undercuts enough to serve the whole line segment

▶ **Region 3 (no demand)**: Firm 1 sets too high a price; demand is zero

# Salop Circular City

# Setup

- ▶ Outside good yields utility $u^*$
- ▶ Differentiated varieties lie on a circle of circumference 1; consumers are uniformly distributed (Salop 1979)
- ▶ $n$ firms are symmetrically located; adjacent distance is $\frac{1}{n}$
- ▶ Each consumer buys at most one unit: from some firm or the outside good

# Objects

▶ Choice variable: $p_i$

▶ Parameters: $n, c, u, u^*$

▶ Outcomes: market boundaries / cutoffs, quantities

▶ Symmetric benchmark: if neighbours set $\tilde{p} = p$, the boundary consumer is at $x = \frac{1}{2n}$

▶ Baseline local demand before participation truncation: $q^C = 2x = \frac{1}{n}$

# Consumers

▶ Each consumer has an ideal variety $\ell^*$ (location on the circle)

▶ Arc distance $d$ generates mismatch cost $cd$

▶ Utility from firm $j$: $U_j = u - cd_j - p_j$

▶ Outside option: $U^O = u^*$

▶ Let $v = u - u^*$; participation requires $v \geq p_j + cd_j$

# Monopoly region

- Outside option truncates the market (participation binds), not neighbours
- Farthest served consumer: $d_m = \frac{v-p}{c}$
- Monopoly demand: $q^M = 2d_m = 2\left(\frac{v-p}{c}\right)$

# Monopoly inverse demand

▶ Inverse demand: $p = v - \frac{c}{2}q^M$
▶ Slope is $-\frac{c}{2}$ because expanding quantity extends the served interval on both sides
▶ Participation binds at the marginal consumer: $v = p + cd_m$

## Competitive region

▶ Boundary consumer (indifference):
$v - cx - p = v - c \left( \frac{1}{n} - x \right) - \tilde{p}$

▶ Boundary location: $x = \frac{\frac{c}{n} - p + \tilde{p}}{2c}$

▶ Competitive quantity: $q^C = 2x$

# Competitive demand

▶ **Competitive demand**: $q^C = 2x = \frac{\frac{c}{n} - p + \tilde{p}}{c}$

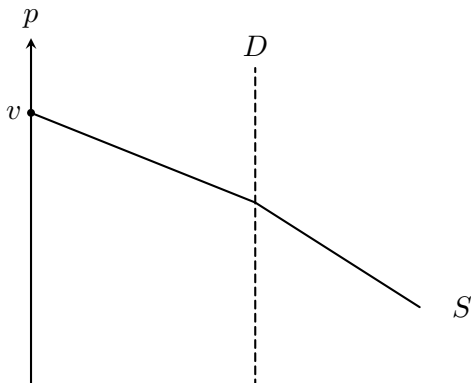▶ **Inverse demand**: $p = \frac{c}{n} + \tilde{p} - cq^C$

> **!** Important
>
> **Slope comparison**
> Competitive-region inverse demand is steeper than monopoly-region inverse demand: $-c$ vs $-\frac{c}{2}$.

# Kinked demand

▶ Demand is limited either by the outside option (participation) or by neighbours (local competition)

▶ Effective demand is $q(p) = \min\{q^M(p), q^C(p)\}$

  ▶ **Participation-boundary (monopoly region)**: $q^M = 2\left(\frac{v-p}{c}\right)$

  ▶ **Neighbour-boundary (competitive region)**: $q^C = \frac{\frac{c}{n} - p + \tilde{p}}{c}$

▶ The switch in the binding constraint creates a kink in inverse demand and a discrete change in marginal revenue

# Cost functions

▶ Cost function: $C(q) = F + mq$ (fixed cost $F$, marginal cost $m$)

▶ Average cost: $AC(q) = m + \frac{F}{q}$ (decreasing in $q$)

# Pricing (competitive region)

- Inverse demand: $p = \frac{c}{n} + \tilde{p} - cq$
- Marginal revenue: $MR = \frac{c}{n} + \tilde{p} - 2cq$
- Symmetry: $\tilde{p} = p$

> **i** Competitive-region condition (outside option not binding)
>
> - In a symmetric profile, the farthest consumer from a firm is at distance $\frac{1}{2n}$.
> - A sufficient condition for the competitive region to apply at the symmetric equilibrium is $v \geq p^C + \frac{c}{2n}$.

# Zero profit

- Symmetry implies $q^C = \frac{1}{n}$
- Zero profit requires $p^C = AC = m + \frac{F}{q^C}$
- Therefore $p^C = m + nF$

# Pricing condition

▶ **Competitive-region inverse demand**: $p = \frac{c}{n} + \tilde{p} - cq$

▶ **First-order condition**: $MR = MC$ with $MR = \frac{c}{n} + \tilde{p} - 2cq$ and $MC = m$

▶ **Symmetric equilibrium implication**: with $\tilde{p} = p$ and $q^C = \frac{1}{n}$, we get $p^C = m + \frac{c}{n}$

# Free-entry equilibrium

- **Zero profit condition**: $p^C = m + nF$
- **Solve**: $m + nF = m + \frac{c}{n}$ so $n^C = \sqrt{\frac{c}{F}}$ and $p^C = m + \sqrt{cF}$
- **Comparative statics**: $n^C$ increases in $c$ and decreases in $F$

# Welfare

- Average distance to the nearest firm: $\bar{d} = \frac{1}{4n}$
- Resource cost of variety: $nF$
- With quasilinear utility, prices are transfers
- Per-consumer welfare: $W(n) = v - \frac{c}{4n} - nF$ (where $v = u - u^*$)

# Social planner solution

Planner problem: Choose $n$ to maximise $W(n)$

▶ FOC: $\frac{dW}{dn} = \frac{c}{4n^2} - F = 0$
▶ Social optimum: $n^S = \frac{1}{2}\sqrt{\frac{c}{F}}$
▶ Free entry: $n^C = \sqrt{\frac{c}{F}}$ so $n^C = 2n^S$

Excess entry due to the business-stealing externality.

# Summary and next week

▶ Mismatch costs soften price competition (markups increase in $c$)

▶ Hotelling: demand, best responses, and equilibrium prices with fixed locations

▶ Salop: free entry gives $n^C = \sqrt{\frac{c}{F}}$ and implies excess entry relative to the planner

**Next week:**

▶ Innovation under uncertainty

▶ Oligopoly incentives to innovate (replacement effect, entry threat)

# References

d'Aspremont, C., J. Jaskold Gabszewicz, and J.-F. Thisse. 1979. 'On Hotelling's "Stability in Competition"'. *Econometrica* 47 (5): 1145–50. https://doi.org/10.2307/1911955.

Hotelling, Harold. 1929. 'Stability in Competition'. *The Economic Journal* 39 (153): 41–57. https://doi.org/10.2307/2224214.

Salop, Steven C. 1979. 'Monopolistic Competition with Outside Goods'. *The Bell Journal of Economics* 10 (1): 141. https://doi.org/10.2307/3003323.