EC5230 - Industrial Organisation

Lecture 5 - Multi-stage Games

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Multi-stage Games

Multi-stage games: overview

> **i Note**
>
> How commitment in earlier stages changes equilibrium behaviour in later stages.

- ▶ So far:
  - → Mostly simultaneous-move games
  - → Briefly: two-stage game (Shy (1995) model in Lecture 4)
- ▶ Today:
  - → Stackelberg quantity leadership
  - → Strategic delegation (Vickers 1985)

Learning objectives

By the end of this lecture you should be able to:

1. Define subgame perfect Nash equilibrium and apply backward induction in finite games
2. Derive and interpret the Stackelberg equilibrium under quantity leadership
3. Set up and solve a two-stage delegation game (contract stage + product-market subgame)
4. Compare equilibrium outcomes and welfare across Cournot, Stackelberg, and delegation

Example: Ultimatum game

▸ Setup
  → Stage 1: Player 1 proposes a share x for Player 2 in splitting \$10 (with $0 \leq x \leq 10$)
  → Stage 2: Player 2 observes x and accepts the proposal (A) or rejects it (R)
  → Players' payoffs:

$$u_1(x, A) = 10 - x \qquad u_2(x, A) = x \qquad u_1(x, R) = u_2(x, R) = 0$$

Ultimatum game: Solution

▶ Solution by backward induction
- $\rightarrow$ Player 2 accepts any offer $x > 0$; for $x = 0$, they are indifferent between accepting and rejecting
- $\rightarrow$ Player 1 offers Player 2 as little as possible and keeps the largest possible remainder
- $\rightarrow$ Backward-induction outcome: $(0, A)$

Solution concept: subgame perfect equilibrium

- ▶ A strategy profile is a subgame perfect Nash equilibrium (SPNE) if it induces a Nash equilibrium in every subgame

- ▶ Operationally (finite horizon):

    $\rightarrow$ Solve from the last stage backward (backward induction)

> **! Commitment logic**
>
> A credible first-stage choice is observed by later movers and shifts their best responses. The payoff function is unchanged — only equilibrium strategies change.

Extensive-form foundations

Games with complete and perfect information: extensive form

▶ Dynamic games can be represented in extensive form

> **ℹ Extensive form (definition)**
>
> An extensive-form game specifies:
>
> 1. which player moves at each decision point (and when)
> 2. each player's payoff for every possible terminal history (outcome)
> 3. the available actions at each decision node
> 4. what the acting player knows about previous moves when choosing

Game tree: basic components

- Simple dynamic games can be illustrated with a game tree

- Components:

  $\rightarrow$ Decision nodes: points where a player chooses
  $\rightarrow$ Branches: feasible actions
  $\rightarrow$ Terminal nodes: outcomes/payoffs

- Example structure (generic two-stage game):

```
Player 1
|- L -> Player 2: l -> (a1,a2), r -> (b1,b2)
`- R -> Player 2: l -> (c1,c2), r -> (d1,d2)
```

Game tree: validity conditions

- A valid game tree has:
    - $\rightarrow$ a unique starting point (root)
    - $\rightarrow$ a unique predecessor for each non-root node
    - $\rightarrow$ no cycles

> ⚠ **Why this matters**
>
> If there are cycles or nodes with multiple predecessors, the object is not a tree and standard backward-induction logic is not directly applicable.

Strategy in a dynamic game (pure strategy)

> **ⓘ Definition (pure strategy)**
>
> A (pure) strategy is a complete contingent plan: it specifies an action at every decision node (or, with imperfect information, at every information set) where the player may move.

- ▶ Key implication:
    - → A strategy must specify actions even at nodes that may not be reached on the equilibrium path

Example: strategies induced by a game tree

▶ Consider:

```
Player 1 chooses L or R
If L: Player 2 chooses l or r -> (a1,a2) or (b1,b2)
If R: Player 2 chooses l or r -> (c1,c2) or (d1,d2)
```

  ▶ Player 1 has one decision node:
  $\rightarrow \; s_1^1 = (L)$
  $\rightarrow \; s_1^2 = (R)$
  ▶ Player 2 has two decision nodes, so 4 pure strategies:
  $\rightarrow \; s_2^1 = (l \mid L, l \mid R)$
  $\rightarrow \; s_2^2 = (l \mid L, r \mid R)$
  $\rightarrow \; s_2^3 = (r \mid L, l \mid R)$
  $\rightarrow \; s_2^4 = (r \mid L, r \mid R)$

Extensive form to normal form (example)

	$s_2^1 = $ $(l \mid L, l \mid R)$	$s_2^2 = $ $(l \mid L, r \mid R)$	$s_2^3 = $ $(r \mid L, l \mid R)$	$s_2^4 = $ $(r \mid L, r \mid R)$
$s_1^1 = (L)$	(a_1, a_2)	(a_1, a_2)	(b_1, b_2)	(b_1, b_2)
$s_1^2 = (R)$	(c_1, c_2)	(d_1, d_2)	(c_1, c_2)	(d_1, d_2)

> **! Exam tip**
>
> When converting a game tree to normal form, count strategies by counting complete plans, not just actions on the equilibrium path.

Entry game: extensive form

▶ Stage 1: entrant chooses Enter or Stay out

▶ Stage 2 (if entry): incumbent chooses Fight or Accommodate

▶ Payoffs (as in the original example):

$\to$ Stay out $\to (0, 2)$
$\to$ Enter + Fight $\to (-3, -1)$
$\to$ Enter + Accommodate $\to (2, 1)$

▶ Strategies:

$\to$ Entrant: $(\text{Stay out}), (\text{Enter})$
$\to$ Incumbent: $(\text{Fight if entry}), (\text{Accommodate if entry})$

Entry game: Nash equilibria in normal form

Entrant / Incumbent	Fight (if entry)	Accommodate (if entry)
Stay out	$(0, 2)$	$(0, 2)$
Enter	$(-3, -1)$	$(2, 1)$

- Pure-strategy Nash equilibria:
  1. (Stay out, Fight if entry)
  2. (Enter, Accommodate if entry)
- Question:
  - $\rightarrow$ Are both equilibria equally plausible in a dynamic setting?

Backward induction: credibility and equilibrium selection

- The equilibrium $(\mathrm{Stay\ out}, \mathrm{Fight\ if\ entry})$ relies on a non-credible threat

- Backward induction logic:

  $\rightarrow$ If entry occurs, the incumbent prefers Accommodate to Fight $(1 > -1)$
  $\rightarrow$ Anticipating this, the entrant chooses Enter $(2 > 0)$

> ! Backward-induction outcome (entry game)
>
> $(\mathrm{Enter}, \mathrm{Accommodate\ if\ entry})$

- Backward induction is applicable to finite dynamic games with perfect information

Backward induction: generic procedure (two-stage game)

- ▶ Stage 1: Player 1 chooses $a_1 \in A_1$

- ▶ Stage 2: Player 2 observes a_1 and chooses $a_2 \in A_2$

- ▶ Payoffs: $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$

- ▶ Procedure:

  1. Solve Player 2's problem for each history a_1:
     - $\rightarrow$ choose a_2 to maximise $u_2(a_1, a_2)$
     - $\rightarrow$ define the reaction function $R_2(a_1)$ (if unique)
  2. Substitute into Player 1's payoff and solve:
     - $\rightarrow$ choose a_1 to maximise $u_1(a_1, R_2(a_1))$
  3. The equilibrium path is $\left(a_1^*, R_2(a_1^*) \right)$

Backward induction: generic procedure (two-stage game)

> **ℹ Bridge to Stackelberg**
>
> The Stackelberg model is exactly this two-stage structure: the follower's best response is solved first, then the leader optimises anticipating that response.

Stackelberg competition

Stackelberg model: Structure

- Two stages, two firms:

  1. Leader chooses q_1
  2. Follower observes q_1 and chooses q_2

- Demand and costs:

  $\rightarrow$ Inverse demand: $p(Q) = A - Q$ with $Q = q_1 + q_2$

  $\rightarrow$ Marginal cost: c (parameter)

Follower best response

▶ Follower profit given q_1:

$$\pi_2(q_2; q_1) = (A - q_1 - q_2 - c)q_2$$

▶ Best response:

$$BR_2(q_1) = \frac{A - c - q_1}{2}$$

> ℹ **Strategic substitutes (quantities)**
>
> Quantities are strategic substitutes: when the rival increases output, my best-response output decreases (downward-sloping best response).

Stackelberg model: leader choice

▶ Substitute follower response into leader profit:

$$\pi_1(q_1) = (A - q_1 - BR_2(q_1) - c)q_1$$

▶ Equilibrium (identical costs):
$\rightarrow q_1^S = \frac{A-c}{2}$
$\rightarrow q_2^S = \frac{A-c}{4}$

> **! First-mover advantage**
>
> The leader commits to a higher q_1, anticipating that the follower reduces q_2.

Full FOC derivation (leader):

Substitute $BR_2(q_1) = \frac{A-c-q_1}{2}$ into leader profit:

Stackelberg model: leader choice

$$\pi_1(q_1) = \left(A - q_1 - \frac{A - c - q_1}{2} - c\right)q_1 = \left(\frac{A - c}{2} - \frac{q_1}{2}\right)q_1 = \frac{A - c}{2}q_1 - \frac{1}{2}q_1^2$$

First-order condition:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{A - c}{2} - q_1 = 0 \quad \Rightarrow \quad q_1^S = \frac{A - c}{2}$$

Then follower output:

$$q_2^S = BR_2(q_1^S) = \frac{A - c - \frac{A - c}{2}}{2} = \frac{A - c}{4}$$

Stackelberg v Cournot: equilibrium outcomes

Outcome	Stackelberg	Cournot
Firm 1 output	$q_1^S = \frac{A-c}{2}$	$q_1^C = \frac{A-c}{3}$
Firm 2 output	$q_2^S = \frac{A-c}{4}$	$q_2^C = \frac{A-c}{3}$
Aggregate output	$Q^S = \frac{3(A-c)}{4}$	$Q^C = \frac{2(A-c)}{3}$
Price	$p^S = \frac{A+3c}{4}$	$p^C = \frac{A+2c}{3}$
Firm 1 profit	$\pi_1^S = \frac{(A-c)^2}{8}$	$\pi_1^C = \frac{(A-c)^2}{9}$
Firm 2 profit	$\pi_2^S = \frac{(A-c)^2}{16}$	$\pi_2^C = \frac{(A-c)^2}{9}$

Stackelberg model: remarks

▶ Comparative statics questions:

$\rightarrow$ What changes when marginal costs are asymmetric?

$\rightarrow$ What changes if the timing is endogenous (announcement/commitment issues)?

Numerical example ($A = 10, c = 1$)

- ▶ Parameters: $A = 10$ (demand intercept), $c = 1$ (marginal cost)

- ▶ Stackelberg outcomes:

$$\rightarrow q_1^S = \frac{A-c}{2} = 4.5; q_2^S = \frac{A-c}{4} = 2.25$$
$$\rightarrow Q^S = 6.75; p^S = A - Q^S = 3.25$$
$$\rightarrow \pi_1^S = (p^S - c)q_1^S = 10.125$$

- ▶ Cournot benchmark:

$$\rightarrow \pi_1^C = \frac{(A-c)^2}{9} = 9$$

> **! Concrete comparison**
>
> With $A = 10, c = 1$: $\pi_1^S = 10.125 > \pi_1^C = 9$ (first-mover advantage).

Welfare comparison (to Cournot)

- With linear demand $p = A - Q$ and constant marginal cost c:

  $\rightarrow$ Higher total output implies lower price $\Rightarrow$ higher consumer surplus (CS)
  $\rightarrow$ Stackelberg yields $Q^S > Q^C$ (hence $CS^S > CS^C$)

- Total surplus changes because:

  $\rightarrow$ CS rises with Q
  $\rightarrow$ profits can rise for the leader and fall for the follower relative to Cournot

Vickers (1985) : Strategic delegation

Delegation: Economic idea

- Firms may delegate output choice to managers with objectives that differ from pure profit maximisation

- Delegation instrument:

    $\rightarrow$ an incentive parameter θ_i written into the manager's contract

Delegation (Vickers): model

- Cournot oligopoly with n firms: $i = 1, ..., n$

- Manager of firm i maximises:

$$M_i = \pi_i + \theta_i q_i$$

where $\pi_i = p(Q)q_i - cq_i$ and $Q = \sum_i q_i$

- Equivalently: $M_i = p(Q)q_i - (c - \theta_i)q_i$ − so θ_i acts like a reduction in the manager's effective marginal cost

Commitment assumption

For delegation to affect rivals' behaviour, contracts must be:

- observable (rivals see θ_i)

- binding (owners cannot override managers after θ_i is set)

Delegation: Triopoly exercise (setup)

- Cournot triopoly: $i = 1, 2, 3$

- Demand: $p(Q) = A - Q$

- Identical marginal costs: c

- Managers: $\theta_i \geq 0$

- Task:

  $\rightarrow$ Solve the θ-setting game given the induced Cournot equilibrium in (q_1, q_2, q_3)

Delegation: triopoly solution steps

▸ Step 1 (product-market subgame): given $(\theta_1, \theta_2, \theta_3)$, managers play Cournot and determine $q_i^*(\theta)$

▸ Step 2 (contract stage): each owner chooses θ_i to maximise own profit $\pi_i(\theta_i; \theta_{-i})$

▸ Step 3 (equilibrium incentives): solve the system of best responses in θ

$\rightarrow$ in symmetry: $\theta_1 = \theta_2 = \theta_3 = \hat{\theta}$

!Exam template

Two-stage delegation problems are solved exactly like Stackelberg: solve the quantity subgame first, then solve the incentive-setting stage.

Triopoly equilibrium (given)

Cournot–Nash equilibrium outcomes (given incentives θ_i):

$$q_i^* = \frac{1}{4}\left(A - c + 3\theta_i - \sum_{j \neq i} \theta_j \right)$$

$$p^* = \frac{1}{4}\left(A + 3c - \sum_{i} \theta_i \right)$$

▶ Interpretation:
  → Increasing θ_i shifts q_i^* up; rivals' incentives are strategic substitutes in outputs through Q

Delegation: solving for equilibrium incentives

Owner's problem (taking rivals' incentives as given):

▶ Owner maximises profit (not managerial utility):

$$\max_{\theta_i \geq 0} \pi_i^*(\theta_i; \theta_{-i}) = \left(\, p^*(\theta) - c \,\right) q_i^*(\theta)$$

First-order condition:

$$\frac{\partial \pi_i^*(\theta_i; \theta_{-i})}{\partial \theta_i} = 0$$

▶ Solving the three FOCs gives a best-response system in $(\theta_1, \theta_2, \theta_3)$
▶ In symmetry, $\theta_i = \hat{\theta}$ for all i, implying $\hat{\theta} = \frac{1}{5}(A - c)$

Full FOC derivation (owner, triopoly):

Let $\Theta_{-i} = \theta_j + \theta_k$. From the equilibrium expressions:

Delegation: solving for equilibrium incentives

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$$\pi_i^* = \frac{(A - c - \theta_i - \Theta_{-i})(A - c + 3\theta_i - \Theta_{-i})}{16}$$

Differentiate with respect to θ_i:

$$\frac{\partial \pi_i^*}{\partial \theta_i} = \frac{1}{16} \left[-(A - c + 3\theta_i - \Theta_{-i}) + 3(A - c - \theta_i - \Theta_{-i}) \right] = 0$$

Expand and collect:

$$2(A - c) - 6\theta_i - 2\Theta_{-i} = 0 \quad \Rightarrow \quad \theta_i = \frac{A - c}{3} - \frac{\Theta_{-i}}{3} = \frac{A - c}{3} - \frac{2}{3}\bar{\theta}_{-i}$$

This is the best-response function shown on the next slide. Imposing symmetry $\theta_i = \bar{\theta}_{-i} = \hat{\theta}$:

Delegation: solving for equilibrium incentives

$$\hat{\theta} = \frac{A-c}{3} - \frac{2}{3}\hat{\theta} \quad \Rightarrow \quad \frac{5}{3}\hat{\theta} = \frac{A-c}{3} \quad \Rightarrow \quad \hat{\theta} = \frac{A-c}{5}$$

Delegation: best response in incentives (triopoly)

Define the rivals' average incentive:

$$\bar{\theta}_{-i} = \frac{1}{n-1} \sum_{j \neq i} \theta_j \quad (n = 3)$$

A reduced-form best response for the owner's incentive choice can be written as:

$$\theta_i = BR(\theta_{-i}) = \frac{1}{3}(A - c) - \frac{2}{3}\bar{\theta}_{-i}$$

▸ Interpretation:
   → Higher rival incentives make rivals more aggressive, reducing the marginal gain from raising θ_i

Delegation: symmetric equilibrium (fixed point)

In symmetry, $\theta_i = \bar{\theta}_{-i} = \hat{\theta}$ for all i.

$$\hat{\theta} = \frac{1}{3}(A - c) - \frac{2}{3}\hat{\theta} \quad \Rightarrow \quad \hat{\theta} = \frac{1}{5}(A - c)$$

> **ℹ What this is doing**
>
> This is the standard "best-response fixed point" method: compute $BR(\cdot)$, then impose symmetry.

Delegation: strategic logic

- ▶ Unilateral incentive: if rivals set $\theta = 0$, choosing $\theta_i > 0$ is strictly profitable
    - $\rightarrow$ Positive θ_i commits the manager to higher output
    - $\rightarrow$ Rivals reduce output (quantities are strategic substitutes)
- ▶ Equilibrium outcome: all firms choose $\hat{\theta} > 0$
    - $\rightarrow$ The strategic advantage disappears when everyone delegates
    - $\rightarrow$ Profits fall: $\hat{\pi} < \pi^C$ (overproduction lowers price for all)

> **! Prisoners' dilemma**
>
> Each owner's dominant strategy leads to a collectively worse outcome: all firms delegate, output rises, and equilibrium profits fall below the Cournot benchmark.

Delegation: strategic logic

> **i Discussion question**
>
> If delegation always lowers industry profits in equilibrium, why would any firm ever choose to delegate? What commitment or regulatory change would allow firms to avoid this outcome?

Comparison table (triopoly)

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Scenario	Incentives	Per-firm output	Per-firm profit
All delegate	$\hat{\theta} = \frac{A-c}{5}$	$\hat{q} = \frac{3(A-c)}{10}$	$\hat{\pi} = \frac{3(A-c)^2}{100}$
None delegate	$\theta = 0$	$q^C = \frac{A-c}{4}$	$\pi^C = \frac{(A-c)^2}{16}$

With $A = 10, c = 1$:

- $\hat{\pi} = 2.43$
- $\pi^C = 5.0625$

> ! Takeaway
>
> Delegation is individually rational in the incentive-setting game but reduces profits in equilibrium.

Numerical example ($A = 10, c = 1$)

- Parameters: $A = 10, c = 1$

- Delegation equilibrium:

  $\rightarrow$ $\hat{\theta} = \frac{A-c}{5} = 1.8$

  $\rightarrow$ Per-firm profit: $\hat{\pi} = \frac{3(A-c)^2}{100} = 2.43$

- Cournot triopoly benchmark ($\theta_i = 0$):

  $\rightarrow$ Per-firm profit: $\pi^C = \frac{(A-c)^2}{16} = 5.0625$

⚠ Concrete comparison

With $A = 10, c = 1$: $\hat{\pi} = 2.43 < \pi^C = 5.0625$ (delegation intensifies competition and lowers profits).

Delegation: welfare comparison (to Cournot triopoly)

▶ Delegation increases effective aggressiveness in output:
  - $\rightarrow$ equilibrium price is lower than in standard Cournot triopoly
  - $\rightarrow$ therefore consumer surplus is higher under delegation than under $\theta_i = 0$
▶ Total surplus effect:
  - $\rightarrow$ CS rises due to lower price
  - $\rightarrow$ profits fall due to overproduction relative to joint-profit motives

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i Exam-useful statement

Delegation can be privately attractive ex ante (as a commitment device) but can reduce profits in equilibrium while benefiting consumers.

Summary and next week

Summary

- Subgame perfect equilibrium is solved by backward induction in finite multi-stage games
- Stackelberg timing creates first-mover advantage through commitment
- Strategic delegation can raise aggressiveness but may lower industry profits in equilibrium
- Welfare effects can diverge: consumers may gain even when firms lose

Next week: cooperative R&D

- Spillovers and non-cooperative R&D incentives
- Cooperative R&D and research joint ventures
- Welfare and policy implications under different cooperation regimes

References

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