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# EC5230 - Industrial Organisation

## Lecture 2 – Product Differentiation

Gerhard Riener

# Differentiation types

- ▶ Horizontal: consumers disagree on which variety is best (ideal-point / mismatch preferences)
- ▶ Vertical: products differ in quality; at equal prices all consumers (weakly) prefer the higher-quality good

## Today: roadmap

- ▶ Focus: horizontal differentiation
- ▶ Hotelling line: 2 firms, pricing with fixed locations
- ▶ Salop circular city: free entry and welfare

# Hotelling Model

# Setup: Fixed locations, price competition

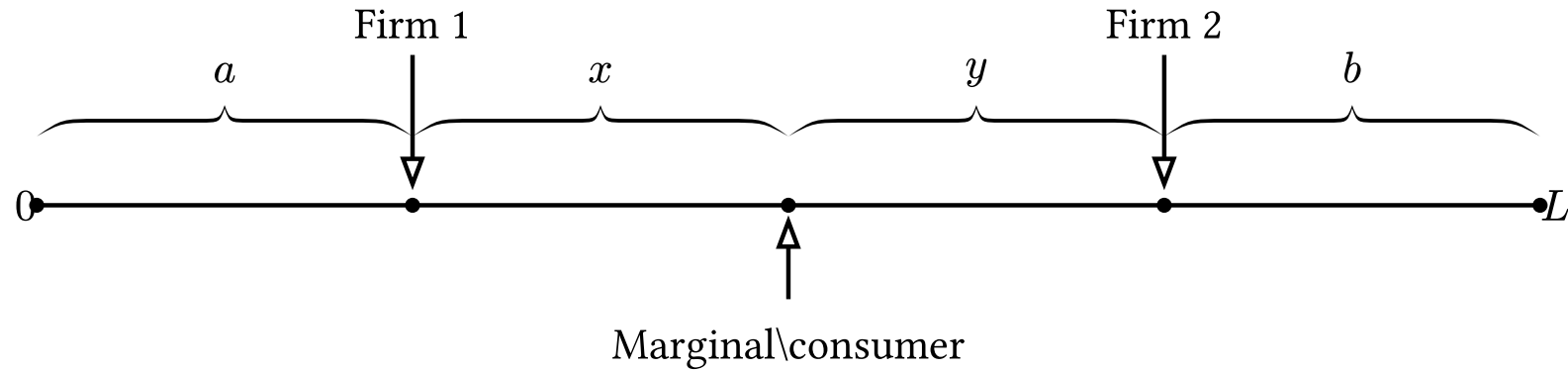


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- ▶ Duopoly, marginal cost 0
- ▶ Consumers ( $\ell$ ) uniformly distributed on  $[0, L]$  (Hotelling 1929)
- ▶ Firm 1 at  $a$  and Firm 2 at  $L - b$
- ▶ Mismatch (transport) cost  $C(d) = cd$  with  $c > 0$ ,  $d$  extent of mismatch
- ▶ Unit demand; assume full market coverage in equilibrium

## i Utility (full coverage)

$U_1(\ell) = \bar{u} - p_1 - c | \ell - a |$ ,  $U_2(\ell) = \bar{u} - p_2 - c | \ell - (L - b) |$ , outside option 0;  
assume  $\bar{u}$  is large enough that all consumers buy.



- ▶ Choice variables:  $p_1, p_2$
- ▶ Parameters:  $L, a, b, c$
- ▶ Distance between firms:  $L - a - b$
- ▶ Marginal consumer distances:  $x + y = L - a - b$
- ▶ Demands:  $q_1 = a + x$  and  $q_2 = b + y$

# Marginal consumer and demands



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- ▶ Indifference of the marginal consumer:  $p_1 + cx = p_2 + cy$
- ▶ Geometry:  $x + y = L - a - b$

## Demands by firm

- ▶  $q_1 = a + x = \frac{p_2 - p_1 + c(L + a - b)}{2c}$
- ▶  $q_2 = b + y = \frac{p_1 - p_2 + c(L + b - a)}{2c}$

# Best responses



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- ▶  $BR_1(p_2) = \frac{p_2}{2} + \frac{c}{2}(L + a - b)$
- ▶  $BR_2(p_1) = \frac{p_1}{2} + \frac{c}{2}(L + b - a)$

## ⚠ Warning

Prices are strategic complements; if  $a = b$  then  $p_1^* = p_2^*$

# Price equilibrium fixed locations



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- ▶ Prices:  $p_1^* = c\left(L + \frac{a-b}{3}\right)$  and  $p_2^* = c\left(L + \frac{b-a}{3}\right)$
- ▶ Profits:  $\pi_1^* = \frac{c}{18}(3L + a - b)^2$  and  $\pi_2^* = \frac{c}{18}(3L + b - a)^2$
- ▶ If  $a = b$ , symmetry implies  $p_1^* = p_2^*$
- ▶ Higher  $c$  lowers substitutability and raises equilibrium markups and profits



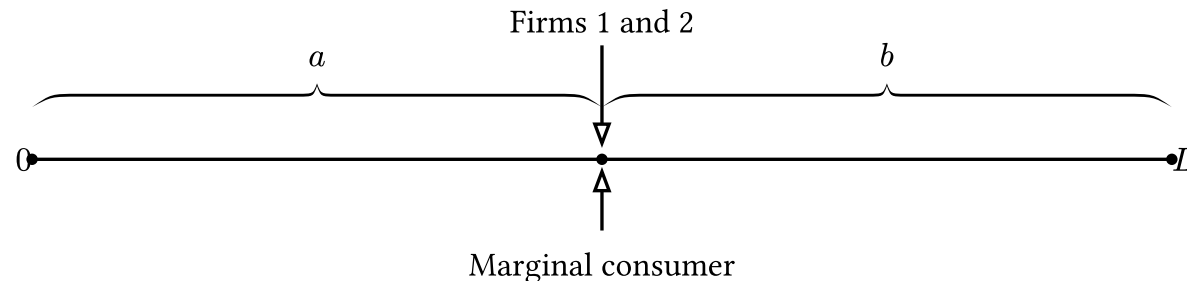
# Hotelling: co-location

- ▶ Co-location implies  $a + b = L$  and  $x = y = 0$
- ▶ Products become homogeneous  $\Rightarrow$  Bertrand undercutting
- ▶ Equilibrium:  $p_1^B = p_2^B = 0$  and  $\pi_1^B = \pi_2^B = 0$

## ⚠ Warning

Co-location and price competition

Co-location removes captive consumers; any  $p > 0$  can be profitably undercut.



# Move incentives at fixed prices

- ▶  $\frac{\partial \pi_1^*}{\partial a} = \frac{c}{9}(3L + a - b) > 0$  and  $\frac{\partial \pi_2^*}{\partial b} = \frac{c}{9}(3L + b - a) > 0$
- ▶ In the pricing subgame, increasing  $a$  (resp.  $b$ ) moves Firm 1 (resp. Firm 2) inward (toward the centre)
- ▶ These are comparative statics holding the rival location fixed
- ▶ Under linear transport costs, pricing incentives can pull locations toward the centre; firms will “go where the demand is”

# Identifying clustering motives

Question: Why are so many businesses clustered together?

- ▶ Going where the demand is?
- ▶ Possible collusion story? Being next to the competitor facilitates monitoring.

Next: the formal location game clarifies when this clustering logic breaks down under linear transport costs.

# Location game (d'Aspremont et al. 1979)



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- ▶ Two-stage game: choose locations, then prices
- ▶ Linear transport costs ( $C(d) = cd$ ): pure-strategy location equilibrium may fail to exist (market-capture regions; non-concavity)
- ▶ Quadratic transport costs ( $C(d) = td^2$ ): pure-strategy location equilibrium exists (baseline: maximum differentiation)
- ▶ Principle of minimum differentiation need not hold

## **i** Note

Why convexity matters

More convex mismatch costs make small location changes less effective at stealing marginal consumers through pricing.

# Profit function: interior region



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- ▶ Let  $\Delta \equiv L - a - b$  denote the distance between firms
- ▶ Interior-demand region:  $|p_1 - p_2| \leq c\Delta$
- ▶ In this region, Firm 1's profit is:

$$\pi_1(p_1, p_2) = \frac{p_1 p_2}{2c} - \frac{p_1^2}{2c} + \frac{(L + a - b)p_1}{2}$$

# Profit function: corner regions



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- ▶ Market capture: if  $p_1 < p_2 - c\Delta$ , then  $\pi_1 = Lp_1$
- ▶ No demand: if  $p_1 > p_2 + c\Delta$ , then  $\pi_1 = 0$
- ▶ Together with the interior expression, these define the piecewise payoff in (d'Aspremont et al. 1979)

# Profit regions



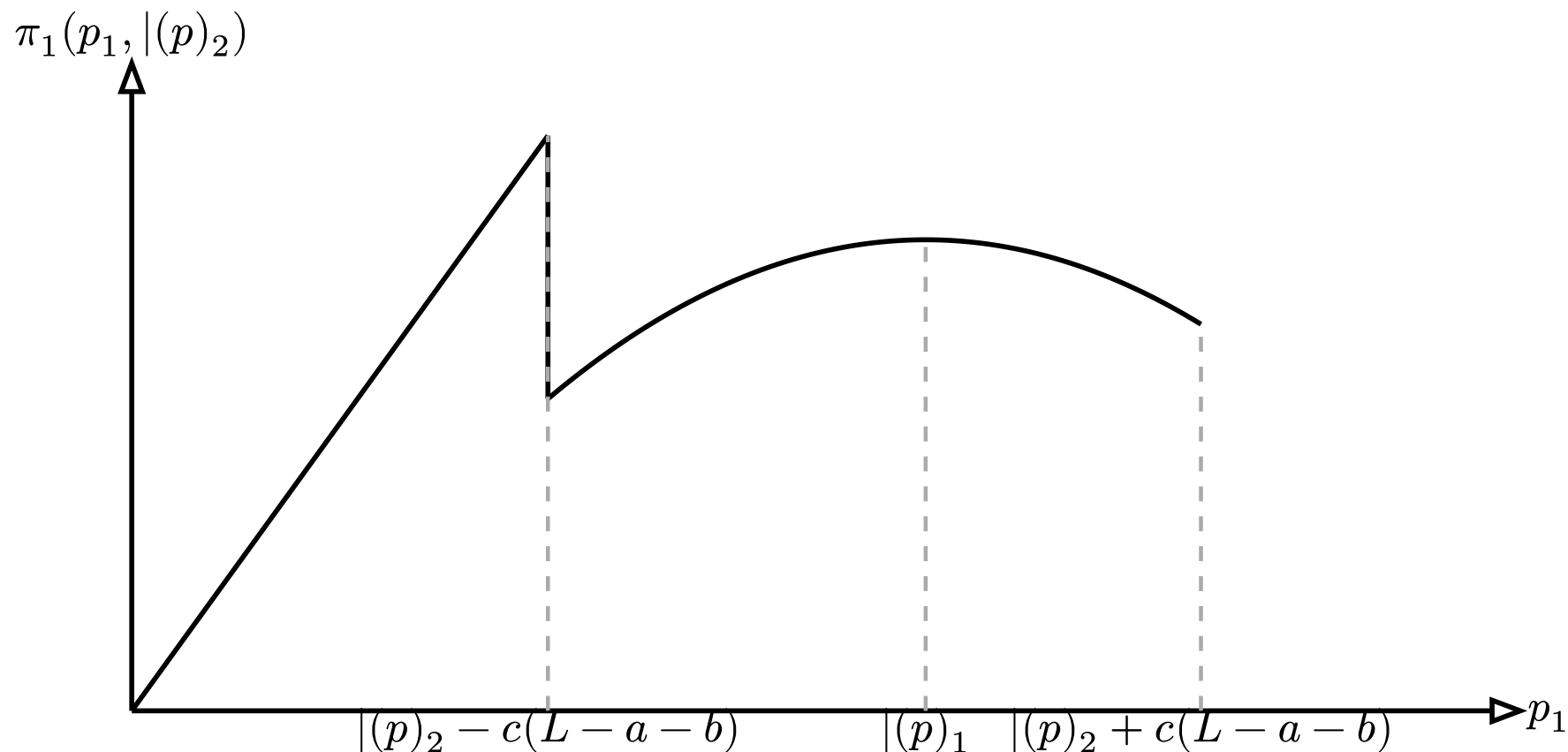
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- ▶ Region 1 (interior): both firms serve positive mass of consumers; demand is linear in  $p_1$
- ▶ Region 2 (market capture): Firm 1 undercuts enough to serve the whole line segment
- ▶ Region 3 (no demand): Firm 1 sets too high a price; demand is zero

# Profit regions



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# Salop Circular City

# Setup



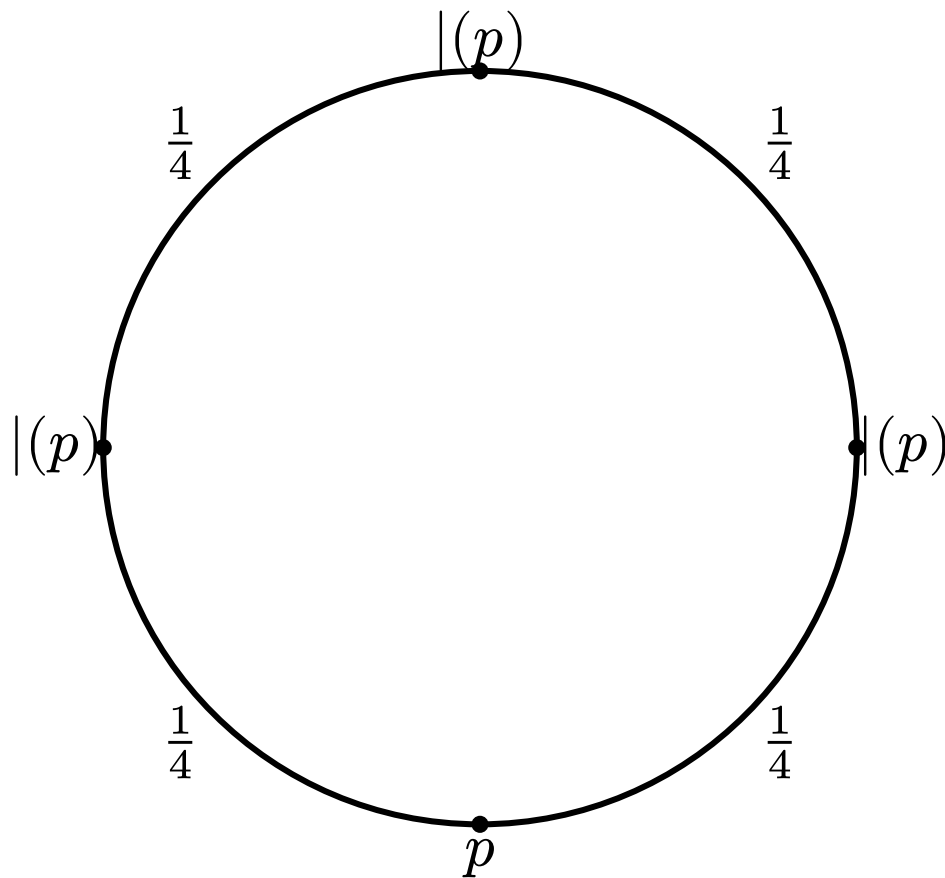
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- ▶ Outside good yields utility  $u^*$
- ▶ Differentiated varieties lie on a circle of circumference 1; consumers are uniformly distributed (Salop 1979)
- ▶  $n$  firms are symmetrically located; adjacent distance is  $\frac{1}{n}$
- ▶ Each consumer buys at most one unit: from some firm or the outside good

# Setup



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# Objects



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- ▶ Choice variable:  $p_i$
- ▶ Parameters:  $n, c, u, u^*$
- ▶ Outcomes: market boundaries / cutoffs, quantities
- ▶ Symmetric benchmark: if neighbours set  $\tilde{p} = p$ , the boundary consumer is at  $x = \frac{1}{2n}$
- ▶ Baseline local demand before participation truncation:  $q^C = 2x = \frac{1}{n}$

# Consumers



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- ▶ Each consumer has an ideal variety  $\ell^*$  (location on the circle)
- ▶ Arc distance  $d$  generates mismatch cost  $cd$
- ▶ Utility from firm  $j$ :  $U_j = u - cd_j - p_j$
- ▶ Outside option:  $U^O = u^*$
- ▶ Let  $v = u - u^*$ ; participation requires  $v \geq p_j + cd_j$

# Monopoly region



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- ▶ Outside option truncates the market (participation binds), not neighbours
- ▶ Farthest served consumer:  $d_m = \frac{v-p}{c}$
- ▶ Monopoly demand:  $q^M = 2d_m = 2\left(\frac{v-p}{c}\right)$

# Monopoly inverse demand



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- ▶ Inverse demand:  $p = v - \frac{c}{2}q^M$
- ▶ Slope is  $-\frac{c}{2}$  because expanding quantity extends the served interval on both sides
- ▶ Participation binds at the marginal consumer:  $v = p + cd_m$

# Competitive region



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- ▶ Boundary consumer (indifference):  $v - cx - p = v - c\left(\frac{1}{n} - x\right) - \tilde{p}$
- ▶ Boundary location:  $x = \frac{\frac{c}{n} - p + \tilde{p}}{2c}$
- ▶ Competitive quantity:  $q^C = 2x$



# Competitive demand



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- ▶ Competitive demand:  $q^C = 2x = \frac{\frac{c}{n} - p + \tilde{p}}{c}$
- ▶ Inverse demand:  $p = \frac{c}{n} + \tilde{p} - cq^C$

! Important

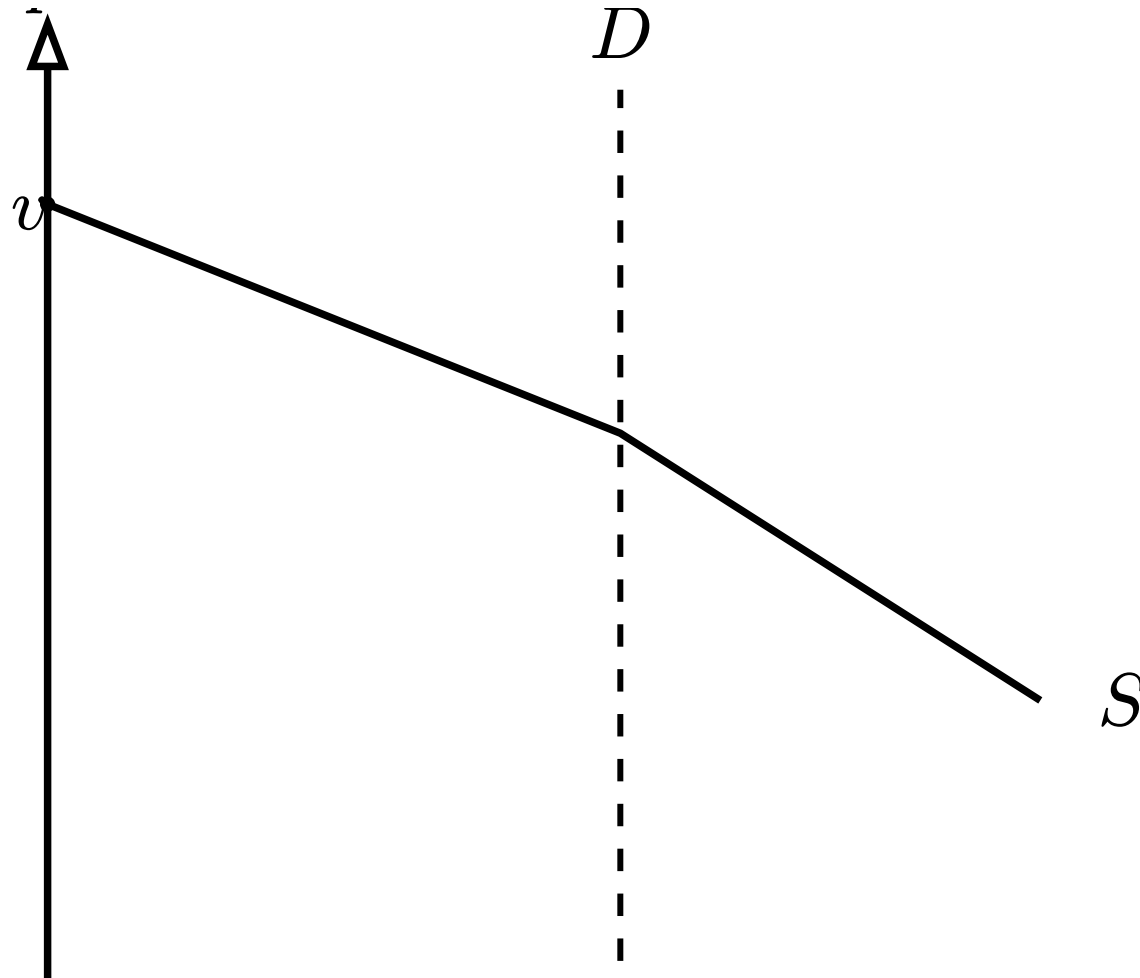
Slope comparison

Competitive-region inverse demand is steeper than monopoly-region inverse demand:  
 $-c$  VS  $-\frac{c}{2}$ .

# Kinked demand

- ▶ Demand is limited either by the outside option (participation) or by neighbours (local competition)
- ▶ Effective demand is  $q(p) = \min\{q^M(p), q^C(p)\}$ 
  - Participation-boundary (monopoly region):  $q^M = 2\left(\frac{v-p}{c}\right)$
  - Neighbour-boundary (competitive region):  $q^C = \frac{\frac{c}{n}-p+\tilde{p}}{c}$
- ▶ The switch in the binding constraint creates a kink in inverse demand and a discrete change in marginal revenue

# Kinked demand



# Cost functions



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- ▶ Cost function:  $C(q) = F + mq$  (fixed cost  $F$ , marginal cost  $m$ )
- ▶ Average cost:  $AC(q) = m + \frac{F}{q}$  (decreasing in  $q$ )

# Pricing (competitive region)



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- ▶ Inverse demand:  $p = \frac{c}{n} + \tilde{p} - cq$
- ▶ Marginal revenue:  $MR = \frac{c}{n} + \tilde{p} - 2cq$
- ▶ Symmetry:  $\tilde{p} = p$

## **i** Competitive-region condition (outside option not binding)

- ▶ In a symmetric profile, the farthest consumer from a firm is at distance  $\frac{1}{2n}$ .
- ▶ A sufficient condition for the competitive region to apply at the symmetric equilibrium is  $v \geq p^C + \frac{c}{2n}$ .

# Zero profit



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- ▶ Symmetry implies  $q^C = \frac{1}{n}$
- ▶ Zero profit requires  $p^C = AC = m + \frac{F}{q^C}$
- ▶ Therefore  $p^C = m + nF$

# Pricing condition



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- ▶ Competitive-region inverse demand:  $p = \frac{c}{n} + \tilde{p} - cq$
- ▶ First-order condition:  $MR = MC$  with  $MR = \frac{c}{n} + \tilde{p} - 2cq$  and  $MC = m$
- ▶ Symmetric equilibrium implication: with  $\tilde{p} = p$  and  $q^C = \frac{1}{n}$ , we get  $p^C = m + \frac{c}{n}$

# Free-entry equilibrium



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- ▶ Zero profit condition:  $p^C = m + nF$
- ▶ Solve:  $m + nF = m + \frac{c}{n}$  so  $n^C = \sqrt{\frac{c}{F}}$  and  $p^C = m + \sqrt{cF}$
- ▶ Comparative statics:  $n^C$  increases in  $c$  and decreases in  $F$



# Welfare



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- ▶ Average distance to the nearest firm:  $\bar{d} = \frac{1}{4n}$
- ▶ Resource cost of variety:  $nF$
- ▶ With quasilinear utility, prices are transfers
- ▶ Per-consumer welfare:  $W(n) = v - \frac{c}{4n} - nF$  (where  $v = u - u^*$ )

# Social planner solution



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Planner problem: Choose  $n$  to maximise  $W(n)$

- ▶ FOC:  $\frac{dW}{dn} = \frac{c}{4n^2} - F = 0$
- ▶ Social optimum:  $n^S = \frac{1}{2} \sqrt{\frac{c}{F}}$
- ▶ Free entry:  $n^C = \sqrt{\frac{c}{F}}$  so  $n^C = 2n^S$

Excess entry due to the business-stealing externality.

# Summary and next week

- ▶ Mismatch costs soften price competition (markups increase in  $c$ )
- ▶ Hotelling: demand, best responses, and equilibrium prices with fixed locations
- ▶ Salop: free entry gives  $n^C = \sqrt{\frac{c}{F}}$  and implies excess entry relative to the planner

Next week:

- ▶ Innovation under uncertainty
- ▶ Oligopoly incentives to innovate (replacement effect, entry threat)

# References

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- Hotelling, Harold. 1929. "Stability in Competition." *The Economic Journal* 39 (153): 41–57. <https://doi.org/10.2307/2224214>.
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