University of St Andrews

EC5230 Class Test (Lectures 1-5) - Guideline Solutions

Industrial Organisation

Time: 50 minutes.

These are guideline solutions. Equivalent correct methods receive full credit. Fractions or decimals are both acceptable.

Section A (10 marks)

Question 1: Cournot with asymmetric costs (4 marks)

Given

$$P = 24 - (q_1 + q_2), \qquad c_1 = 3,; c_2 = 6.$$

Firm 1 profit:

$$\pi_1 = (P - c_1)q_1 = (24 - q_1 - q_2 - 3)q_1 = (21 - q_1 - q_2)q_1.$$

FOC:

$$\frac{\partial \pi_1}{\partial q_1} = 21 - 2q_1 - q_2 = 0; \Rightarrow; BR_1(q_2) = \frac{21 - q_2}{2}.$$

Firm 2 profit:

$$\pi_2 = (P - c_2)q_2 = (24 - q_1 - q_2 - 6)q_2 = (18 - q_1 - q_2)q_2.$$

FOC:

$$\frac{\partial \pi_2}{\partial q_2} = 18 - q_1 - 2q_2 = 0; \Rightarrow; BR_2(q_1) = \frac{18 - q_1}{2}.$$

Solve:

$$q_1 = \frac{21 - q_2}{2}, \qquad q_2 = \frac{18 - q_1}{2}; \Rightarrow; q_2^* = 5, \quad q_1^* = 8.$$

Hence

$$Q^* = 13, \qquad P^* = 24 - 13 = 11.$$

Profits:

$$\pi_1^* = (11 - 3) \cdot 8 = 64, \qquad \pi_2^* = (11 - 6) \cdot 5 = 25.$$

Firm 1 has lower marginal cost, so it produces more and earns a larger margin on each unit.

Question 2: Hotelling price competition (3 marks)

Endpoints: firm 1 at 0, firm 2 at 1, with $t = 2$ and $c_1 = c_2 = 0$.

Utilities for consumer at x:

$$U_1 = v - p_1 - 2x, \qquad U_2 = v - p_2 - 2(1-x).$$

Marginal consumer $\hat{x}$ solves $U_1 = U_2$:

$$v - p_1 - 2x = v - p_2 - 2 + 2x \Rightarrow 4x = p_2 - p_1 + 2 \Rightarrow \hat{x} = \frac{p_2 - p_1 + 2}{4}.$$

Demands:

$$D_1 = \hat{x} = \frac{p_2 - p_1 + 2}{4}, \qquad D_2 = 1 - \hat{x} = \frac{p_1 - p_2 + 2}{4}.$$

Profits:

$$\pi_1 = p_1 D_1 = p_1 \frac{p_2 - p_1 + 2}{4}, \qquad \pi_2 = p_2 D_2 = p_2 \frac{p_1 - p_2 + 2}{4}.$$

FOCs:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{p_2 - 2p_1 + 2}{4} = 0 \Rightarrow BR_1(p_2) = \frac{p_2 + 2}{2},$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{p_1 - 2p_2 + 2}{4} = 0 \Rightarrow BR_2(p_1) = \frac{p_1 + 2}{2}.$$

Symmetric Nash equilibrium:

$$p_1^* = p_2^* = 2.$$

Then

$$\hat{x} = \frac{2 - 2 + 2}{4} = \frac{1}{2},$$

so each market share is $\frac{1}{2}$ and each profit is

$$\pi_i^* = p_i^* D_i^* = 2 \cdot \frac{1}{2} = 1.$$

If t increases, products become less substitutable (consumers are less willing to switch), so equilibrium prices increase.

Question 3: Innovation incentives and replacement effect (3 marks)

Demand:

$$P = 20 - Q, \qquad c_0 = 10, \qquad c_1 = 4.$$

For monopoly with cost c:

$$\pi(Q) = (20 - Q - c)Q \Rightarrow 20 - c - 2Q = 0 \Rightarrow Q^m(c) = \frac{20-c}{2},$$

$$P^m(c) = 20 - Q^m(c) = \frac{20+c}{2}, \qquad \pi^m(c) = (P^m(c) - c)Q^m(c) = \frac{(20-c)^2}{4}.$$

Before innovation ($c_0 = 10$):

$$Q_0^m = 5, \quad P_0^m = 15, \quad \pi_0^m = 25.$$

After innovation ($c_1 = 4$):

$$Q_1^m = 8, \quad P_1^m = 12, \quad \pi_1^m = 64.$$

Monopoly WTP:

$$WTP_M = \pi_1^m - \pi_0^m = 64 - 25 = 39.$$

Under perfect competition before innovation, pre-innovation profit is zero. After innovation, the innovator has exclusive access to cost c_1 while a competitive fringe remains available at cost c_0.

To classify drastic/non-drastic, compare the unconstrained monopoly price after innovation with the old cost:

$$P_1^m = 12 > c_0 = 10.$$

So the innovation is non-drastic. The innovator limit-prices at (approximately) $p = c_0 = 10$.

Then

$$Q = 20 - 10 = 10, \qquad \pi = (10 - 4) \cdot 10 = 60.$$

Hence

$$WTP_{PC} = 60 - 0 = 60.$$

Comparison:

$$WTP_{PC} = 60 > WTP_M = 39.$$

This is the replacement effect: a monopolist replaces existing rents, while a competitive-industry innovator mostly creates new rents.

Section B (7 marks)

Question 4: Investment game with and without patents (7 marks)

Gross profit function:

$$\pi_i(c_i, c_j) = \frac{(12 - 2c_i + c_j)^2}{9}.$$

Parameters:

$$c_0 = 4, \quad c' = 1, \quad \tilde{c} = 2, \quad F = 5.$$

Required gross profits:

$$\pi(c', c_0) = \pi(1, 4) = \frac{(12 - 2 + 4)^2}{9} = \frac{196}{9},$$

$$\pi(c_0, c') = \pi(4, 1) = \frac{(12 - 8 + 1)^2}{9} = \frac{25}{9},$$

$$\pi(c', c') = \pi(1, 1) = \frac{(12 - 2 + 1)^2}{9} = \frac{121}{9},$$

$$\pi(c', \tilde{c}) = \pi(1, 2) = \frac{(12 - 2 + 2)^2}{9} = 16,$$

$$\pi(\tilde{c}, c') = \pi(2, 1) = \frac{(12 - 4 + 1)^2}{9} = 9,$$

$$\pi(c_0, c_0) = \pi(4, 4) = \frac{(12 - 8 + 4)^2}{9} = \frac{64}{9}.$$

Patents regime (P)
If both invest, each expected net payoff is

$$\frac{1}{2}\pi(c', c_0) + \frac{1}{2}\pi(c_0, c') - F = \frac{1}{2} \cdot \frac{196}{9} + \frac{1}{2} \cdot \frac{25}{9} - 5 = \frac{131}{18}.$$

Net payoff matrix (row firm, column firm):

$$\begin{array}{ccc} & I & N \\ I & \left(\frac{131}{18}, \frac{131}{18}\right) & \left(\frac{151}{9}, \frac{25}{9}\right) \\ N & \left(\frac{25}{9}, \frac{151}{9}\right) & \left(\frac{64}{9}, \frac{64}{9}\right) \end{array}$$

where

$$\frac{151}{9} = \frac{196}{9} - 5.$$

Best responses:

- Against N: I yields $\frac{151}{9} > \frac{64}{9}$.

- Against I: I yields $\frac{131}{18} > \frac{25}{9}$.

So I is dominant. Unique pure-strategy Nash equilibrium:

$$(I, I).$$

No-patents regime (NP)
If one firm invests and the other does not, the non-investor imitates and has cost $\tilde{c}$ while the investor has cost c'. Thus the relevant gross profits are $\pi(c', \tilde{c}) = 16$ for the investor and $\pi(\tilde{c}, c') = 9$ for the imitator.

Net payoff matrix:

$$\begin{array}{ccc} & I & N \\ I & \left(\dfrac{76}{9}, \dfrac{76}{9}\right) & (11, 9) \\ N & (9, 11) & \left(\dfrac{64}{9}, \dfrac{64}{9}\right) \end{array}$$

where

$$\frac{76}{9} = \frac{121}{9} - 5, \qquad 11 = 16 - 5.$$

Pure-strategy best responses:

- Against N: invest ($11 > \frac{64}{9}$).
- Against I: do not invest ($9 > \frac{76}{9}$).

So the pure-strategy Nash equilibria are

$$(I, N) \quad \text{and} \quad (N, I).$$

Remark (mixed equilibrium): This is a chicken/hawk–dove game, so there is also a symmetric mixed-strategy equilibrium. Let p be the probability the opponent invests. Indifference between I and N requires

$$p \cdot \frac{76}{9} + (1-p) \cdot 11 = p \cdot 9 + (1-p) \cdot \frac{64}{9}.$$

Solving:

$$p = \frac{35}{40} = \frac{7}{8}.$$

So in the mixed equilibrium each firm invests with probability $\frac{7}{8}$.

Evaluation of statement: "Without patents, imitation necessarily destroys incentives to invest."

This is false here. Under no patents, incentives are reduced (because imitation erodes the advantage), but not eliminated: there are pure equilibria in which exactly one firm invests, and there is also a mixed equilibrium in which firms invest with positive probability.

Section C (3 marks, advanced)

Question 5: Commitment stage before quantity competition (3 marks)

Demand:

$$P = 18 - Q, \qquad c = 2, \qquad A - c = 16.$$

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1) Quantity-stage outcomes

Cournot benchmark:

$$q_i^C = \frac{A-c}{3} = \frac{16}{3}, \qquad Q^C = \frac{32}{3}, \qquad P^C = 18 - \frac{32}{3} = \frac{22}{3}.$$

Each profit:

$$\pi_i^C = (P^C - c)q_i^C = \left(\frac{22}{3} - 2\right)\frac{16}{3} = \frac{256}{9}.$$

Stackelberg (leader L, follower F): Follower BR:

$$q_F = \frac{A - c - q_L}{2} = \frac{16 - q_L}{2}.$$

Leader chooses q_L to maximize

$$\pi_L = (16 - q_L - q_F)q_L = \left(16 - q_L - \frac{16 - q_L}{2}\right)q_L = \frac{16 - q_L}{2}q_L.$$

FOC gives

$$q_L^* = 8, \qquad q_F^* = 4, \qquad Q^S = 12, \qquad P^S = 6.$$

Profits:

$$\pi_L^S = (6-2)\cdot 8 = 32, \qquad \pi_F^S = (6-2)\cdot 4 = 16.$$

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2) Stage-0 payoff matrix

- If exactly one firm chooses C, it becomes the Stackelberg leader and pays K.
- If both choose the same action, the quantity-stage outcome is Cournot; under (C, C) both also pay K.

Matrix:

$$\begin{array}{ccc} & C & N \\ C & \left(\frac{256}{9}-K, \frac{256}{9}-K\right) & (32-K, 16) \\ N & (16, 32-K) & \left(\frac{256}{9}, \frac{256}{9}\right) \end{array}$$

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3) Pure-strategy Nash equilibria as function of K
Best response to opponent choosing N:

$$C \quad \text{preferred iff} \quad 32-K \geq \frac{256}{9} \Leftrightarrow K \leq \frac{32}{9}.$$

Best response to opponent choosing C:

$$C \quad \text{preferred iff} \quad \frac{256}{9}-K \geq 16 \Leftrightarrow K \leq \frac{112}{9}.$$

Now check the candidate action profiles:

- ((C,C)) is a Nash equilibrium iff (C) is a best response to (C), i.e. $K \leq \frac{112}{9}$.
- ((N,N)) is a Nash equilibrium iff (N) is a best response to (N), i.e. $K \geq \frac{32}{9}$.

Important note (why no asymmetric pure NE): Profiles like ((C,N)) or ((N,C)) are not Nash equilibria here because when the opponent plays (N), committing yields leader status and payoff (32-K), but when the opponent plays (C), committing does not yield leader status; it yields Cournot and payoff $\frac{256}{9}-K$. Because leadership requires being the unique committer, unilateral commitment does not generate the usual stable "leader–follower" pure equilibrium in this stage-0 game.

Therefore:

- If $K < \frac{32}{9}$: unique pure NE is ((C,C)).
- If $\frac{32}{9} \leq K \leq \frac{112}{9}$: two pure NE, ((C,C)) and ((N,N)).
- If $K > \frac{112}{9}$: unique pure NE is ((N,N)).

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4) Welfare comparison: assigned commitment rights
If one firm is assigned commitment rights, the outcome is Stackelberg with one commitment cost (K).

Cournot welfare:

$$CS^C = \frac{1}{2}(Q^C)^2 = \frac{1}{2}\left(\frac{32}{3}\right)^2 = \frac{512}{9},$$

$$PS^C = 2 \cdot \frac{256}{9} = \frac{512}{9},$$

$$W^C = CS^C + PS^C = \frac{1024}{9}.$$

Stackelberg welfare (one commitment payment):

$$CS^S = \frac{1}{2}(Q^S)^2 = \frac{1}{2} \cdot 12^2 = 72,$$

$$PS^S = \pi_L^S + \pi_F^S - K = 32 + 16 - K = 48 - K,$$

$$W^S = 72 + (48 - K) = 120 - K.$$

Stackelberg with assigned rights improves welfare iff

$$120 - K > \frac{1024}{9} \Leftrightarrow K < \frac{56}{9} \approx 6.22.$$

So it is welfare-improving for

$$K < \frac{56}{9},$$

indifferent at $K = \frac{56}{9}$, and worse for larger K.

Bibliography