Case of two point incident on a parabola

Equation Of Reflective Curve In Parametric and Cartesian Form

In[1]:=
```
curveparaeqn = {8t, -2t^2+6}
curvecarteqn = Last[List @@ Reduce[Eliminate[{curveparaeqn[[1]]==x,curveparaeqn[[2]]==y}, t],y]]
```

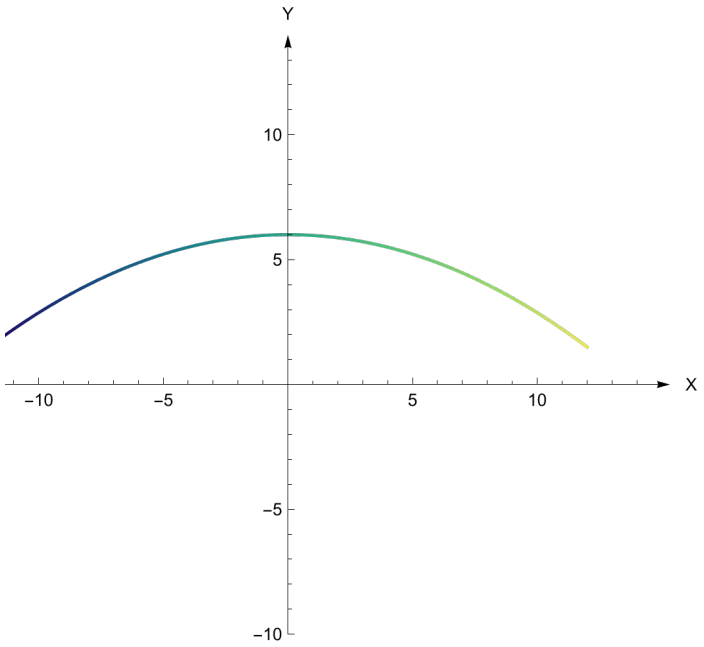
Out[1]= $\{8\,t,\ 6-2\,t^2\}$

Out[2]= $\frac{1}{32}\times\left(192-x^2\right)$

Plot of the equation

In[17]:=
```
curve = ParametricPlot[curveparaeqn, {t, -1.5, 1.5},PlotRange→{-10, 14},AxesLabel→{"X", "Y"},
```

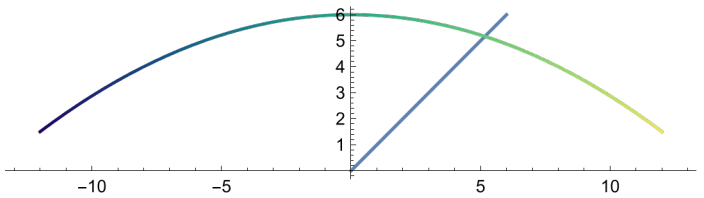
Out[17]=



Incident ray of light at 45 degrees to x axis with source at origin

In[4]:=
```
sourceparaeqn = {t, t};
sourcecarteqn = x;
sourceline = ParametricPlot[sourceparaeqn, {t, 0, 6}, PlotRange→All];
Show[sourceline, curve]
a1incipt = N[Solve[sourcecarteqn==curvecarteqn, x]][[2]]
```

Out[7]=



Out[8]= $\{x \to 5.16601\}$

Tangent line at point of incident and reflected ray

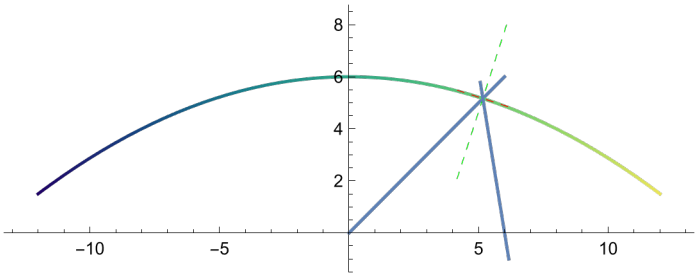
In[18]:=
```
a1taneqn = Reduce[y-(curvecarteqn /. a1incipt)== (D[curvecarteqn, x] /. a1incipt)(x-({x} /.a1
a1noreqn = Reduce[y-(curvecarteqn /. a1incipt)== (-1/(D[curvecarteqn, x] /. a1incipt))(x-({x}
a1norplot = Plot[Last[List @@ a1noreqn], {x,({x} /.a1incipt)[[1]]-1,({x} /.a1incipt)[[1]]+1}, Pl
a1tanplot = Plot[Last[List @@ a1taneqn], {x,({x} /.a1incipt)[[1]]-1,({x} /.a1incipt)[[1]]+1}, Pl
a1refangle = Reduce[(1-(-1/(D[curvecarteqn, x] /. a1incipt)))/(1+1*(-1/(D[curvecarteqn, x] /
a1refeqn = Reduce[y-(curvecarteqn /. a1incipt)== Last[List @@ a1refangle](x-({x} /.a1incipt)[[
a1refplot =  Plot[Last[List @@ a1refeqn], {x,({x} /.a1incipt)[[1]]-0.1,({x} /.a1incipt)[[1]]+1}]
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot]
```

Out[18]= $y == 6.83399 - 0.322876\, x$

Out[19]= $y == -10.834 + 3.09717\, x$

Out[22]= $m == -6.16601$

Out[25]=

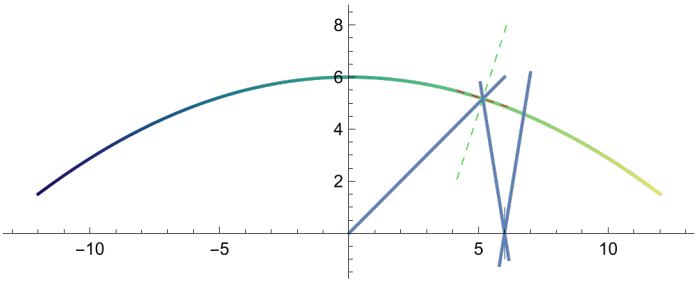


Now, on 2nd reflection on the axis

In[26]:=
```
a2incipt = Last[List @@ Reduce[Last[List @@ a1refeqn]==0,x]];
a2noreqn = {a2incipt, y};
a2norplot = ParametricPlot[a2noreqn,{x, -1, 1}, {y, -1, 1}];
a2refangle = (Tan[π]-Last[List @@ a1refangle])/(1+Tan[π]*Last[List @@ a1refangle]);
a2refeqn = Reduce[y-0==a2refangle(x-a2incipt),y]
a2refplot = Plot[Last[List @@ a2refeqn], {x,a2incipt - 0.2,a2incipt+1}];
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refplot]
```

Out[30]= $y == -37.0197 + 6.16601\, x$

Out[32]=



Now on 3rd Reflection

In[42]:=
```
a3incipt = N[Solve[Last[List @@ a2refeqn]==curvecarteqn, x]][[2]]
a3taneqn = Reduce[y-(curvecarteqn /. a3incipt)== (D[curvecarteqn, x] /. a3incipt)(x-({x} /.a3
a3noreqn = Reduce[y-(curvecarteqn /. a3incipt)== (-1/(D[curvecarteqn, x] /. a3incipt))(x-({x}
a3norplot = Plot[Last[List @@ a3noreqn], {x,({x} /.a3incipt)[[1]]-1,({x} /.a3incipt)[[1]]+1}, Pl
a3tanplot = Plot[Last[List @@ a3taneqn], {x,({x} /.a3incipt)[[1]]-1,({x} /.a3incipt)[[1]]+1}, Pl
a3refangle = Reduce[(a2refangle-(-1/(D[curvecarteqn, x] /. a3incipt)))/(1+a2refangle*(-1/(D[
a3refeqn = Reduce[y-(curvecarteqn /. a3incipt)== Last[List @@ a3refangle](x-({x} /.a3incipt)[[
a3refplot =  Plot[Last[List @@ a3refeqn], {x,({x} /.a3incipt)[[1]]-4,({x} /.a3incipt)[[1]]+1}];
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refplot, a3tanplot,a3norplo
```

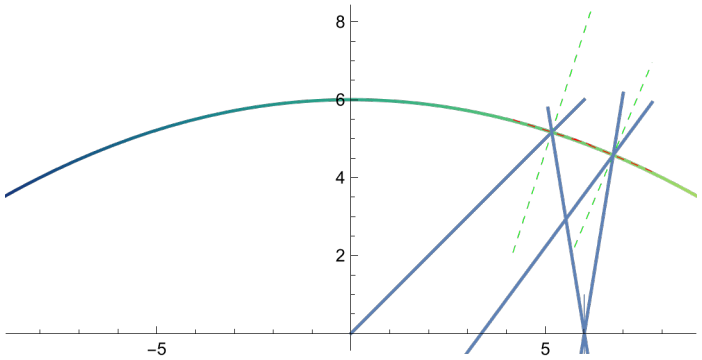
Out[42]= $\{x \to 6.74625\}$

Out[43]= $y == 7.42225 - 0.421641\, x$

Out[44]= $y == -11.4222 + 2.37169\, x$

Out[47]= $m == 1.3508$

Out[50]=

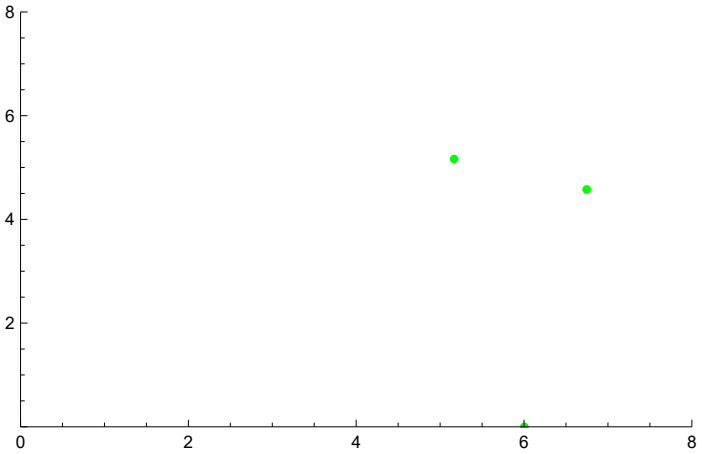


Plot of points of intersection

In[51]:=
```
interpts = {{x/.a1incipt,(curvecarteqn /. a1incipt)},{a2incipt, 0},{x /.a3incipt,(curvecarteq
ptplot = ListPlot[interpts, PlotRange→{{0, 8},{0, 8}},PlotStyle→{Green}]
```

Out[51]= $\{\{5.16601, 5.16601\}, \{6.00383, 0\}, \{6.74625, 4.57775\}, \{a4incipt, 0\}\}$

Out[52]=

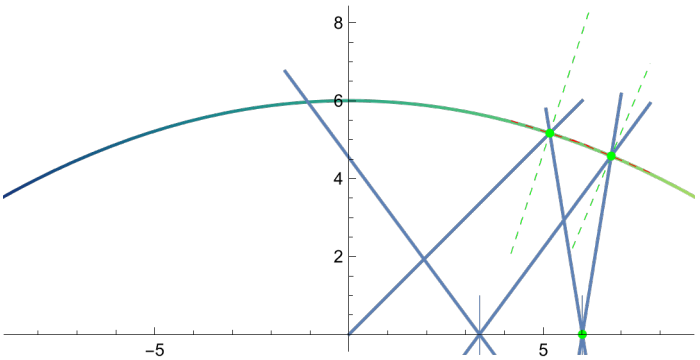


Now 4th reflection

In[53]:=
```
a4incipt = Last[List @@ Reduce[Last[List @@ a3refeqn]==0,x]];
a4noreqn = {a4incipt, y};
a4norplot = ParametricPlot[a4noreqn,{x, -1, 1}, {y, -1, 1}];
a4refangle = (Tan[π]-Last[List @@ a3refangle])/(1+Tan[π]*Last[List @@ a3refangle]);
a4refeqn = Reduce[y-0==a4refangle(x-a4incipt),y]
a4refplot = Plot[Last[List @@ a4refeqn], {x,a4incipt - 5,a4incipt+1}];
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refplot, a3tanplot,a3norplo
```

Out[57]= $y == 4.53511 - 1.3508\,x$

Out[59]=



Now on 5th Reflection

In[60]:=
```
a5incipt = N[Solve[Last[List @@ a4refeqn]==curvecarteqn, x]][[1]]
a5taneqn = Reduce[y-(curvecarteqn /. a5incipt)== (D[curvecarteqn, x] /. a5incipt)(x-({x} /.a5
a5noreqn = Reduce[y-(curvecarteqn /. a5incipt)== (-1/(D[curvecarteqn, x] /. a5incipt))(x-({x}
a5norplot = Plot[Last[List @@ a5noreqn], {x,({x} /.a5incipt)[[1]]-1,({x} /.a5incipt)[[1]]+1}, Pl
a5tanplot = Plot[Last[List @@ a5taneqn], {x,({x} /.a5incipt)[[1]]-1,({x} /.a5incipt)[[1]]+1}, Pl
a5refangle = Reduce[(a4refangle-(-1/(D[curvecarteqn, x] /. a5incipt)))/(1+a4refangle*(-1/(D[
a5refeqn = Reduce[y-(curvecarteqn /. a5incipt)== Last[List @@ a5refangle](x-({x} /.a5incipt)[[
a5refplot =  Plot[Last[List @@ a5refeqn], {x,({x} /.a5incipt)[[1]]-4,({x} /.a5incipt)[[1]]+1}];
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refplot, a3tanplot,a3norplo
```

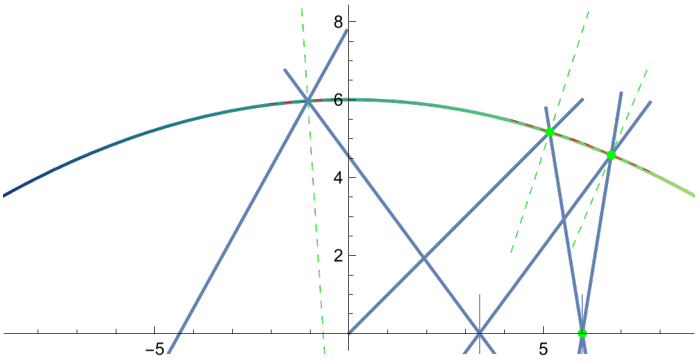
Out[60]= $\{x \to -1.05854\}$

Out[61]= $y == 6.03502 + 0.0661585\,x$

Out[62]= $y == -10.035 - 15.1152\,x$

Out[65]= $m == 1.80834$

Out[68]=

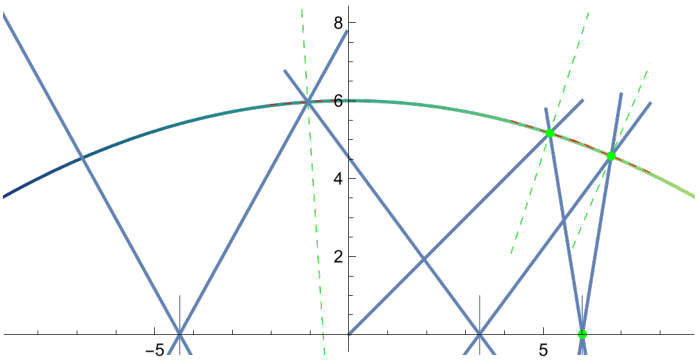


Now 6th reflection

In[69]:=
```
a6incipt = Last[List @@ Reduce[Last[List @@ a5refeqn]==0,x]];
a6noreqn = {a6incipt, y};
a6norplot = ParametricPlot[a6noreqn,{x, -1, 1}, {y, -1, 1}];
a6refangle = (Tan[π]-Last[List @@ a5refangle])/(1+Tan[π]*Last[List @@ a5refangle]);
a6refeqn = Reduce[y-0==a6refangle(x-a6incipt),y]
a6refplot = Plot[Last[List @@ a6refeqn], {x,a6incipt - 5,a6incipt+1}];
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refplot, a3tanplot,a3norplo
```

Out[73]= $y == -7.87917 - 1.80834\, x$

Out[75]=



Now on 7th Reflection

In[76]:=
```
a7incipt = N[Solve[Last[List @@ a6refeqn]==curvecarteqn, x]][[1]]
a7taneqn = Reduce[y-(curvecarteqn /. a7incipt)== (D[curvecarteqn, x] /. a7incipt)(x-({x} /.a7
a7noreqn = Reduce[y-(curvecarteqn /. a7incipt)== (-1/(D[curvecarteqn, x] /. a7incipt))(x-({x}
a7norplot = Plot[Last[List @@ a7noreqn], {x,({x} /.a7incipt)[[1]]-1,({x} /.a7incipt)[[1]]+1}, Pl
a7tanplot = Plot[Last[List @@ a7taneqn], {x,({x} /.a7incipt)[[1]]-1,({x} /.a7incipt)[[1]]+1}, Pl
a7refangle = Reduce[(a6refangle-(-1/(D[curvecarteqn, x] /. a7incipt)))/(1+a6refangle*(-1/(D[
a7refeqn = Reduce[y-(curvecarteqn /. a7incipt)== Last[List @@ a7refangle](x-({x} /.a7incipt)[[
a7refplot =  Plot[Last[List @@ a7refeqn], {x,({x} /.a7incipt)[[1]]-4,({x} /.a7incipt)[[1]]+1}];
Show[sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refplot, a3tanplot,a3norplo
```

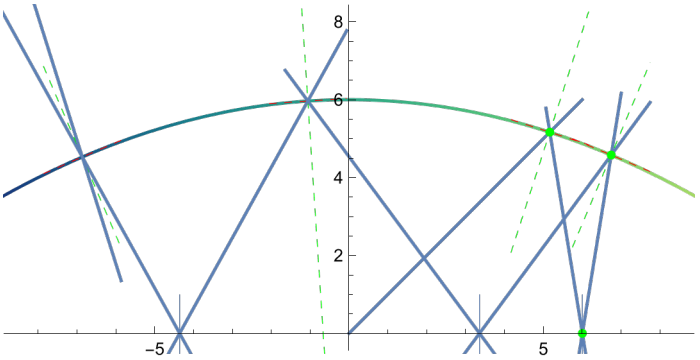
Out[76]= $\{x \to -6.86151\}$

Out[77]= $y == 7.47126 + 0.428844\, x$

Out[78]= $y == -11.4713 - 2.33185\, x$

Out[81]= $m == -3.17522$

Out[84]=

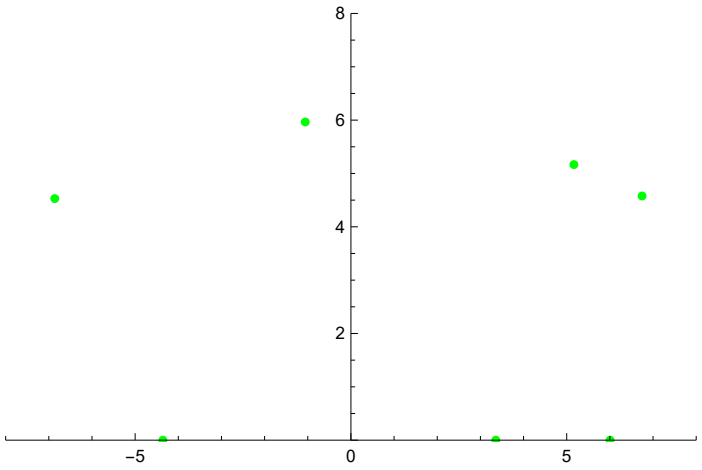


Plot of points of intersection

In[85]:=
```
interpts = {{x/.a1incipt,(curvecarteqn /. a1incipt)},{a2incipt, 0},{x /.a3incipt,(curvecarteq
ptplot = ListPlot[interpts, PlotRange→{{-8, 8},{0, 8}},PlotStyle→{Green}]
```

Out[85]=
```
{{5.16601, 5.16601}, {6.00383, 0}, {6.74625, 4.57775},
 {3.35734, 0}, {-1.05854, 5.96498}, {-4.35714, 0}, {-6.86151, 4.52874}}
```

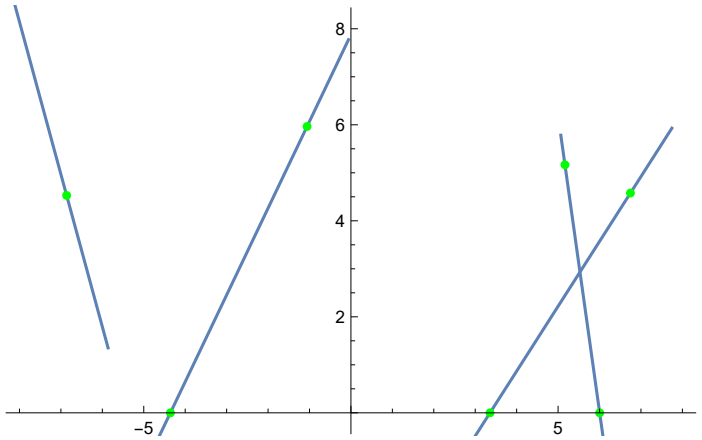
Out[86]=



Plot of reflected rays

In[87]:=
```
Show[a1refplot,a3refplot,a5refplot, a7refplot,ptplot, PlotRange→{{-8, 8},{0, 8}}, AxesOrigin→
```

Out[87]=



Now we have to consider the incident points on the plane mirrors as source if they produce any further reflection. Each point source will lead to a different orthotomic curve. (If the light rays from the sources were parallel we have a relation among them in this case.)

Plotting orthotomic points for each incident point on curve with the respective source

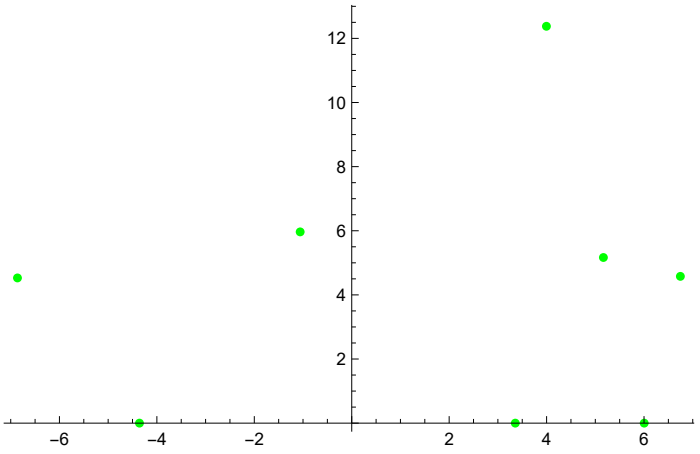
In[88]:=
```
a = 0.322876; b=1; c=-6.83399;
source = {0, 0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points]
ptplot = ListPlot[interpts, PlotRange→Full,PlotStyle→{Green}]
```

Out[89]= {0, 0}

Out[91]= {3.99644, 12.3776}

Out[92]= {{5.16601, 5.16601}, {6.00383, 0}, {6.74625, 4.57775}, {3.35734, 0},
{-1.05854, 5.96498}, {-4.35714, 0}, {-6.86151, 4.52874}, {3.99644, 12.3776}}

Out[93]=

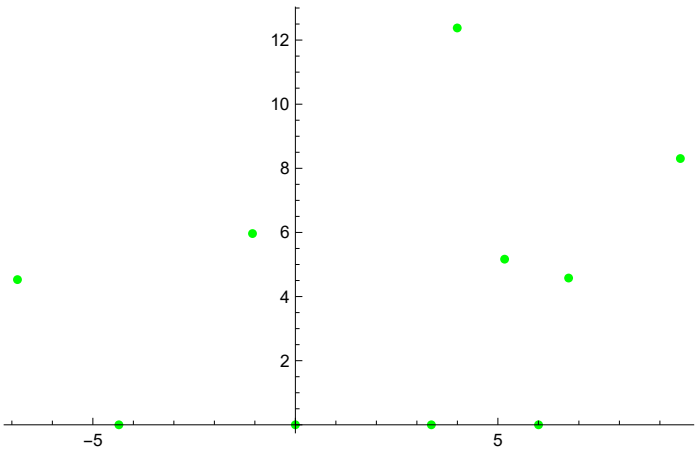


In[94]:=
```
a = 0.4216405353733948; b=1; c=-7.422245928559704;
source = {6.003831069285709`,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points2 = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points2];
AppendTo[interpts, {0, 0}];
ptplot = ListPlot[interpts, PlotRange→Full,PlotStyle→{Green}]
```

Out[95]= {6.00383, 0}

Out[97]= {9.50559, 8.30509}

Out[100]=

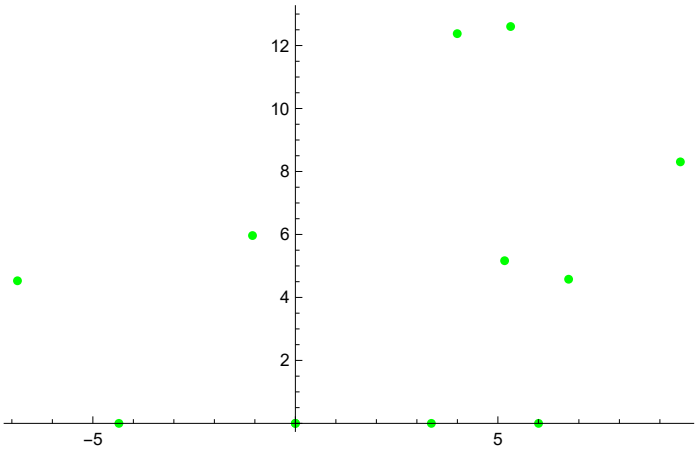


In[101]:=
```
a = 0.4216405353733948; b=1; c=-7.422245928559704;
source = {0,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points3 = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points3];
AppendTo[interpts, {0, 0}];
ptplot = ListPlot[interpts, PlotRange→Full,PlotStyle→{Green}]
```

Out[102]= {0, 0}

Out[104]= {5.31427, 12.6038}

Out[107]=



In[108]:=
```
a = -0.0661585; b=1; c=-6.03502;
source = {0,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points3 = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points3];
AppendTo[interpts, {0, 0}];


a = -0.0661585; b=1; c=-6.03502;
source = {6.003831069285709`,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points3 = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points3]

a = -0.428844; b=1; c=-7.47126;
source = {0,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points3 = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points3];
AppendTo[interpts, {0, 0}];


a = -0.428844; b=1; c=-7.47126;
source = {6.003831069285709`,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points3 = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points3]

ptplot = ListPlot[interpts, PlotRange→Full,PlotStyle→{Green}]
```

Out[109]= {0, 0}

Out[111]= {−0.795056, 12.0174}

Out[115]= {6.00383, 0}

Out[117]= {5.15645, 12.8084}

Out[118]= {{5.16601, 5.16601}, {6.00383, 0}, {6.74625, 4.57775}, {3.35734, 0}, {−1.05854, 5.96498}, {−4.35714, 0}, {−6.86151, 4.52874}, {3.99644, 12.3776}, {9.50559, 8.30509}, {0, 0}, {5.31427, 12.6038}, {0, 0}, {−0.795056, 12.0174}, {0, 0}, {5.15645, 12.8084}}

Out[120]= {0, 0}

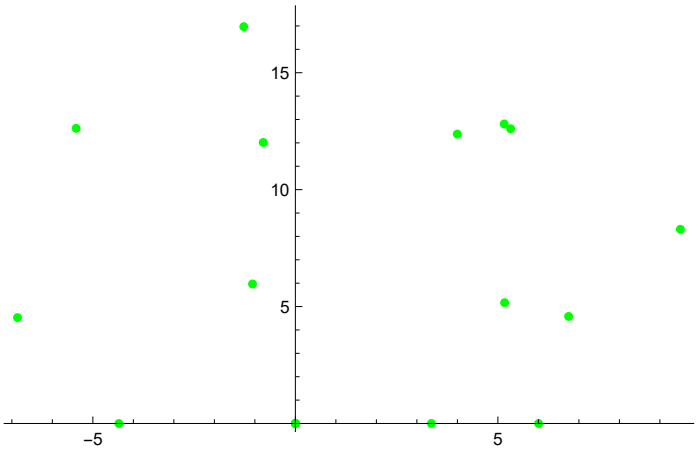
Out[122]= {−5.41259, 12.6214}

Out[126]= {6.00383, 0}

Out[128]= {−1.27402, 16.9709}

Out[129]= {{5.16601, 5.16601}, {6.00383, 0}, {6.74625, 4.57775}, {3.35734, 0}, {−1.05854, 5.96498}, {−4.35714, 0}, {−6.86151, 4.52874}, {3.99644, 12.3776}, {9.50559, 8.30509}, {0, 0}, {5.31427, 12.6038}, {0, 0}, {−0.795056, 12.0174}, {0, 0}, {5.15645, 12.8084}, {−5.41259, 12.6214}, {0, 0}, {−1.27402, 16.9709}}

Out[130]=



In[131]:=

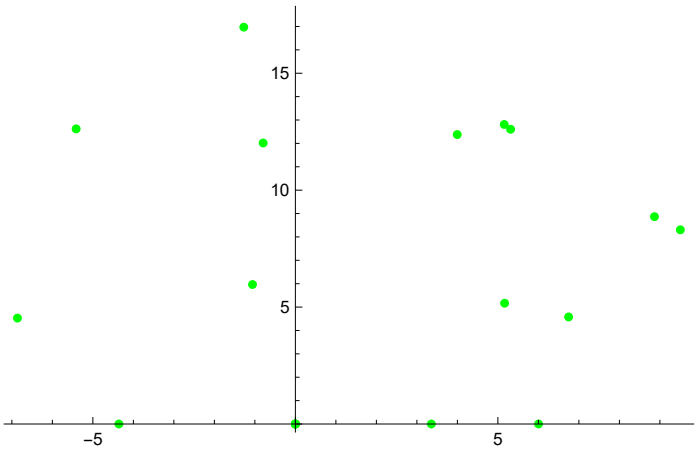
```
a = 0.322876; b=1; c=-6.83399;
source = {6.003831069285709`,0}
x = (b(b*source[[1]]-a*source[[2]])-a*c)/(a^2+b^2); y=( a(-b*source[[1]]+a*source[[2]])-b*c)/(a^2+b^2);
points = {Last[List @@ Reduce[source[[1]]-x==x-m, m]],Last[List @@ Reduce[source[[2]]-y==y-m, m]]
AppendTo[interpts, points]
ptplot = ListPlot[interpts, PlotRange→Full,PlotStyle→{Green}]
```

Out[132]= {6.00383, 0}

Out[134]= {8.86666, 8.86665}

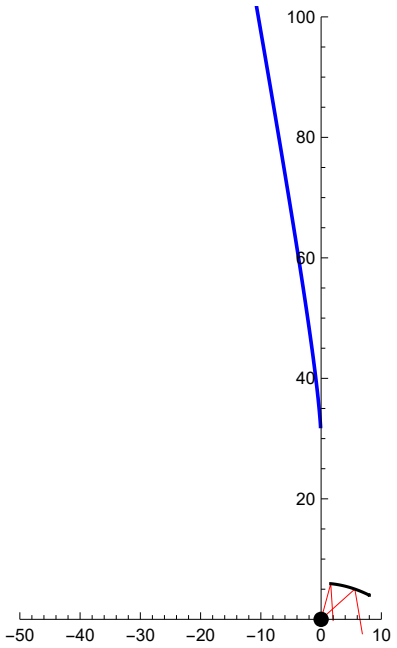
Out[135]= {{5.16601, 5.16601}, {6.00383, 0}, {6.74625, 4.57775}, {3.35734, 0}, {−1.05854, 5.96498}, {−4.35714, 0}, {−6.86151, 4.52874}, {3.99644, 12.3776}, {9.50559, 8.30509}, {0, 0}, {5.31427, 12.6038}, {0, 0}, {−0.795056, 12.0174}, {0, 0}, {5.15645, 12.8084}, {−5.41259, 12.6214}, {0, 0}, {−1.27402, 16.9709}, {8.86666, 8.86665}}

Out[136]=



In[137]:=
```
cata = ResourceFunction["CatacausticCurvePlot"][curveparaeqn, {0,0},{t, 0.2, 1,.5}, Axes→True
orthoorigin = ResourceFunction["Orthotomic"][curveparaeqn, t]
orthodiff = ResourceFunction["Orthotomic"][curveparaeqn, {6.003831069285709,0},t]
orthoplot = ParametricPlot[orthoorigin , {t, -2, 12}];
orthoplot2 = ParametricPlot[orthodiff , {t, -2,12}, ColorFunction→"Rainbow"];
Show[orthoplot,orthoplot2, sourceline, curve, a1tanplot,a1norplot,a1refplot,a2norplot, a2refpl
```

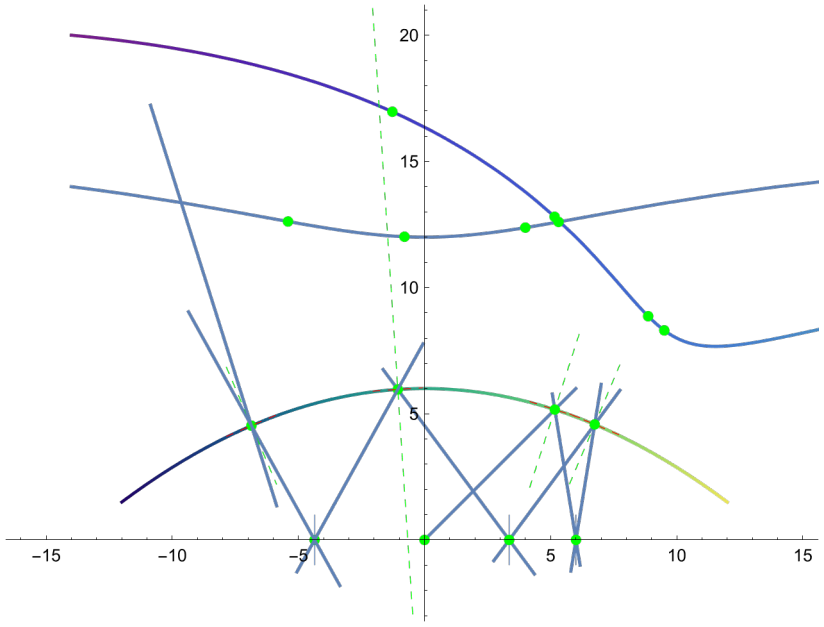
Out[137]=



Out[138]=
$$\left\{-\frac{8\,t\,\left(-32\,t^2+8\times\left(-6+2\,t^2\right)\right)}{64+16\,t^2},\ -\frac{16\times\left(-32\,t^2+8\times\left(-6+2\,t^2\right)\right)}{64+16\,t^2}\right\}$$

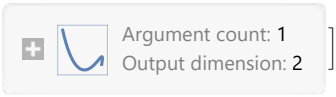
Out[139]=
$$\left\{6.00383-\frac{8\,t\,\left(4\times(6.00383-8\,t)\,t+8\times\left(-6+2\,t^2\right)\right)}{64+16\,t^2},\ -\frac{16\times\left(4\times(6.00383-8\,t)\,t+8\times\left(-6+2\,t^2\right)\right)}{64+16\,t^2}\right\}$$

Out[142]=



In[183]:=
```
orthopts = {{3.99644,12.3776},{5.31427,12.6038},{-0.795056,12.0174},{-5.41259,12.6214}}
f = BSplineFunction[orthopts]
Show[Graphics[{Red, Point[orthopts], Green, Line[orthopts]},Axes→True, ImageSize→700], Parame
```

Out[183]= {{3.99644, 12.3776}, {5.31427, 12.6038}, {−0.795056, 12.0174}, {−5.41259, 12.6214}}

Out[184]= BSplineFunction[]

Out[185]=