$$f(x) = \begin{cases} 0, & x < -1 \\ 1 - |x|, & |x| \leq 1 \\ 0, & x > 1 \end{cases} \qquad L = 2$$

$$f(x) = \frac{a_0}{2} + \sum_{K=1}^{\infty} \left[a_K \cos\left(\frac{2\pi K}{L}\right) + b_K \sin\left(\frac{2\pi K}{L}\right) \right]$$

where

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\, dx$$

$$a_K = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{\pi K}{L} x\right) dx$$

$$b_K = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{\pi K}{L} x\right) dx$$

since $f(x)=0$ for $|x|>1$

By symmetry of f

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\int_{-1}^{1} f(x)\,dx = \cancel{\frac{1}{2}}\cdot\cancel{2}\int_0^1 (1-x)\,dx$$

$$= x - \frac{x^2}{2}\Big]_0^1 = \frac{1}{2}$$

$$a_K = \frac{1}{2}\int_{-2}^{2} f(x)\cos\left(\frac{\pi K}{2}x\right)dx = \frac{1}{2}\left[\int_0^1 (1-x)\cos\left(\frac{\pi K}{2}x\right)dx + \int_{-1}^{0}(1+x)\cos\left(\frac{\pi K}{2}x\right)\right.$$

$$= \frac{1}{2}\left[\int_0^1 \cos\left(\frac{\pi K}{2}x\right)dx - \int_0^1 x\cos\left(\frac{\pi K}{2}x\right)dx + \int_{-1}^{0}\cos\left(\frac{\pi K}{2}x\right)dx + \int_{-1}^{0} x\cos\left(\frac{\pi K}{2}x\right)dx\right]$$

$$= \frac{1}{2}\left[\cancel{\frac{2}{\pi K}\sin\left(\frac{\pi K}{2}x\right)\Big|_0^1} - \cancel{\frac{2}{\pi K}x\sin\left(\frac{\pi K}{2}x\right)\Big|_0^1} + \int_0^1 \frac{2}{\pi K}\sin\left(\frac{\pi K}{2}x\right)dx\right.$$

$$\left.\cancel{\frac{2}{\pi K}\sin\left(\frac{\pi K}{2}x\right)\Big|_{-1}^{0}} + \cancel{\frac{2}{\pi K}x\sin\left(\frac{\pi K}{2}x\right)\Big|_{-1}^{0}} - \int_{-1}^{0}\frac{2}{\pi K}\sin\left(\frac{\pi K}{2}x\right)dx\right]$$

$$= \cancel{\frac{1}{2}}\left[-\left(\frac{2}{\pi K}\right)^2\cos\left(\frac{\pi K}{2}x\right)\right]_0^1 \cdot \cancel{2}$$ by symmetry

$$= \frac{4}{\pi^2 K^2}\left[1 - \cos\left(\frac{\pi K}{2}\right)\right]$$

$$b_k = \frac{1}{2}\int_{-2}^{2} f(x)\sin\left(\frac{\pi k}{2}x\right)dx$$

$$= \frac{1}{2}\int_{-1}^{1} f(x)\sin\left(\frac{\pi k}{2}x\right)dx = \frac{1}{2}\left[\int_{0}^{1}(1-x)\sin\left(\frac{\pi k}{2}x\right)dx + \int_{-1}^{0}(1+x)\sin\left(\frac{\pi k x}{2}\right)dx\right]$$

$$\Rightarrow \int_{0}^{1}(1-x)\sin\left(\frac{\pi k x}{2}\right)dx = \int_{0}^{1}\sin\left(\frac{\pi k x}{2}\right)dx - \int_{0}^{1}\overset{u}{x}\,\overset{dv}{\sin\left(\frac{\pi k}{2}x\right)dx} =$$

$$= -\frac{2}{\pi k}\cos\left(\frac{\pi k}{2}x\right)\Big]_{0}^{1} + \frac{2}{\pi k}x\cos\left(\frac{\pi k}{2}x\right)\Big]_{0}^{1} - \frac{2}{\pi k}\int_{0}^{1}\cos\left(\frac{\pi k}{2}x\right)dx$$

$$= +\frac{2}{\pi k} - \left(\frac{2}{\pi k}\right)^{2}\sin\left(\frac{\pi k}{2}x\right)\Big|_{0}^{1} = \frac{2}{\pi k} - \frac{4}{\pi^{2}k^{2}}\sin\left(\frac{\pi k}{2}\right)$$

$$\Rightarrow \int_{-1}^{0}(1+x)\sin\left(\frac{\pi k}{2}x\right)dx = \int_{-1}^{0}\sin\left(\frac{\pi k x}{2}\right)dx + \int_{-1}^{0}x\sin\left(\frac{\pi k}{2}x\right)dx =$$

$$= -\frac{2}{\pi k}\cos\left(\frac{\pi k}{2}x\right)\Big]_{-1}^{0} - \frac{2}{\pi k}x\cos\left(\frac{\pi k}{2}x\right)\Big]_{-1}^{0} + \frac{2}{\pi k}\int_{-1}^{0}\cos\left(\frac{\pi k}{2}x\right)dx$$

$$= -\frac{2}{\pi k}\left(1-\cos\left(\frac{\pi k}{2}\right)\right) - \frac{2}{\pi k}\left(0+\cos\frac{\pi k}{2}\right) + \left(\frac{2}{\pi k}\right)^{2}\sin\left(\frac{\pi k}{2}x\right)\Big|_{-1}^{0}$$

$$= -\frac{2}{\pi k} + \frac{4}{\pi^{2}k^{2}}\sin\left(\frac{\pi k}{2}\right)$$

$$\therefore b_k = 0 \quad \forall k \in \mathbb{N}$$