CSCI-398 Longest Common Subsequence Dynamic Programming Problem

Given two strings x and y, design a dynamic programming algorithm to find and print the longest common subsequence of x and y.

Input: 2 strings (one per line), both have all characters lowercase.

string1
string2

Output: Two lines (no quotes):

"LCS:"<space><lcs_string><endl>

"Length:"<space><int_length_of_lcs><endl>

If no LCS exists (because two strings do not share any characters), then print out the following two lines:

"LCS:"<space><endl>

`"No LCS was found."`<endl>

Input Sample:

springtime
pioneer

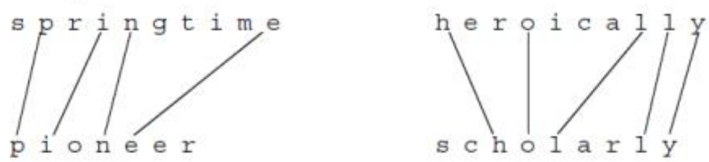
Output Sample:

LCS: pine
Length: 4

Longest common subsequence

Problem: Given 2 sequences, $X = \langle x_1, \ldots, x_m \rangle$ and $Y = \langle y_1, \ldots, y_n \rangle$. Find a subsequence common to both whose length is longest. A subsequence doesn't have to be consecutive, but it has to be in order.

Examples



Brute-force algorithm:

For every subsequence of X, check whether it's a subsequence of Y.

Time: $\Theta(n2^m)$.

- 2^m subsequences of X to check.
- Each subsequence takes $\Theta(n)$ time to check: scan Y for first letter, from there scan for second, and so on.

Optimal substructure

Notation:

X_i = prefix $\langle x_1, \ldots, x_i \rangle$

Y_i = prefix $\langle y_1, \ldots, y_i \rangle$

Theorem

Let $Z = \langle z_1, \ldots, z_k \rangle$ be any LCS of X and Y.

1. If $x_m = y_n$, then $z_k = x_m = y_n$ and Z_{k-1} is an LCS of X_{m-1} and Y_{n-1}.
2. If $x_m \neq y_n$, then $z_k \neq x_m \Rightarrow Z$ is an LCS of X_{m-1} and Y.
3. If $x_m \neq y_n$, then $z_k \neq y_n \Rightarrow Z$ is an LCS of X and Y_{n-1}.

Recursive formulation

Define $c[i, j]$ = length of LCS of X_i and Y_j. We want $c[m, n]$.

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0\,, \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j\,, \\ \max(c[i-1, j], c[i, j-1]) & \text{if } i, j > 0 \text{ and } x_i \neq y_j\,. \end{cases}$$