

EXERCISE 4

SGN-1156 Signal Processing Techniques
<http://www.cs.tut.fi/courses/SGN-1156>
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PROBLEM 1 (problem 5.21 from the book): Let $x[n]$, $0 \leq n \leq N-1$ be a length- N sequence with an N -point DFT $X[k]$, $0 \leq k \leq N-1$. Determine the N -point DFTs of the following length- N sequences in terms of $X[k]$:

- (a) $w[n] = \alpha x[\langle n - m_1 \rangle_N] + \beta x[\langle n - m_2 \rangle_N]$, where m_1 and m_2 are positive integers less than N .
- (b) $g[n] = \begin{cases} x[n] & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases}$
- (c) $y[n] = x[n] \overset{N}{\otimes} x[n]$.

PROBLEM 2 (problem 5.23 from the book): Let $x[n]$, $0 \leq n \leq N-1$ be a length- N sequence with an N -point DFT $X[k]$, $0 \leq k \leq N-1$. Determine the N -point inverse DFTs of the following length- N DFTs in terms of $x[n]$:

- (a) $W[k] = \alpha X[\langle k - m_1 \rangle_N] + \beta X[\langle k - m_2 \rangle_N]$, where m_1 and m_2 are positive integers less than N .
- (b) $G[k] = \begin{cases} X[k] & \text{for } k \text{ even} \\ 0 & \text{for } k \text{ odd} \end{cases}$
- (c) $Y[k] = X[k] \overset{N}{\otimes} X[k]$.

PROBLEM 3 (problem 5.42 from the book): A 126-point DFT $X[k]$ of a real-valued sequence $x[n]$ has the following DFT samples: $X[0] = 12.8 + j\alpha$, $X[13] = -3.7 + j2.2$, $X[k_1] = 9.1 - j5.4$, $X[k_2] = 6.3 + j2.3$, $X[51] = -j1.7$, $X[63] = 13 + j\beta$, $X[k_3] = \gamma + j1.7$, $X[79] = 6.3 + j\delta$, $X[108] = \epsilon + j5.4$, $X[k_4] = -3.7 - j2.2$. The remaining DFT samples are assumed to be equal to zero.

- (a) Determine the values of the indices k_1 , k_2 , k_3 and k_4 .
- (b) Determine the values of α , β , δ , and ϵ .
- (c) What is the DC value of $\{x[n]\}$?
- (d) Determine the expression for $\{x[n]\}$ without computing the IDFT.
- (e) What is the energy of $\{x[n]\}$?