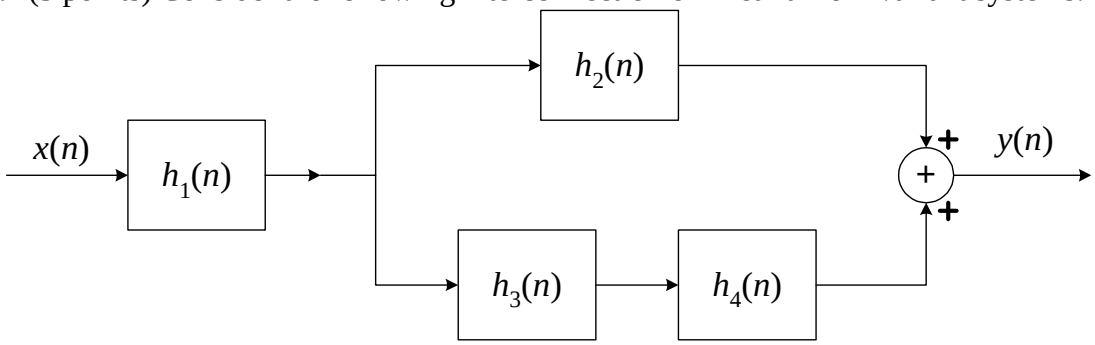
**SGN-1156 SIGNAL PROCESSING TECHNIQUES**
**Toy Exam 4-12-2009**

*Instructions*: Write your name on **every** page in CAPITAL LETTERS and your student number as well. Number pages consecutively. Please, be clean.

*Note*: You have to solve five problems. Maximum total grade is 20 bonus points.

**1.** (3 points) Consider the following interconnection of linear time-invariant systems:




a) Express the frequency response of the overall system in terms of $H_1(e^{jw})$, $H_2(e^{j\omega})$, $H_3(e^{j\omega})$, and $H_4(e^{j\omega})$.
b) Find the frequency response if

$$h_1[n] = \delta[n] + 2\delta[n-2] + \delta[n-4]$$
$$h_2[n] = h_3[n] = (0.2)^n \mu[n]$$
$$h_4[n] = \delta[n-4]$$

**2.** (3 points) Consider the discrete time sequence

$$x[n] = \cos\left(\frac{n\pi}{8}\right).$$

Find two different continuous-time signals $x(t)$ that would produce this sequence when sampled at a frequency of $f_s$ = 10 Hz (*Hint*: consider the cases of sampling above and below the Nyquist rate).

**3.** (3 points) Find the $z$-transform and region of convergence of each of the following sequences

a) $x_a[n] = \alpha^n \mu[-n-1]$, b) $x_b[n] = (n+1)\alpha^n \mu[-n-1]$.

**4.** (5 points) Consider an LTI system which is causal and whose transfer function is

$$H(z) = \frac{3 - 7z^{-1} + 5z^{-2}}{3 - \frac{5}{2}z^{-1} + z^{-2}}$$

Determine the output of the system $y[n]$ when the input is unit step sequence $x[n] = \mu[n]$

**5.** (6 points) Find the system function $H(z)$ of a minimum phase system that has a magnitude response given by

$$\left|H\left(e^{j\omega}\right)\right|^2 = \frac{\frac{5}{4} - \cos\omega}{\frac{10}{9} - \frac{2}{3}\cos\omega}$$

(*Hint*: express the cosines by complex exponentials and then the exponentials – by the $z$-variable).