EXERCISE 2

SGN-1156 Signal Processing Techniques
http://www.cs.tut.fi/courses/SGN-1156
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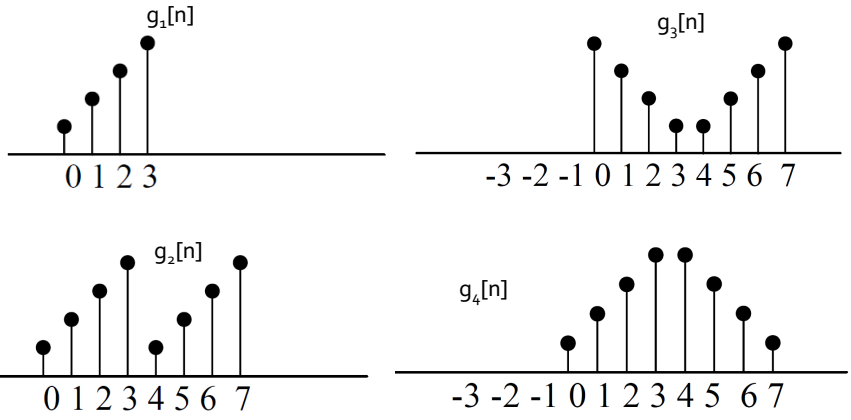
PROBLEM 1: Determine the DTFT of each of the following sequences:

(a) $x_a[n] = \mu[n] - \mu[n-5]$

(b) $x_b[n] = \alpha^n \left(\mu[n] - \mu[n-8]\right) \qquad |\alpha| < 1$

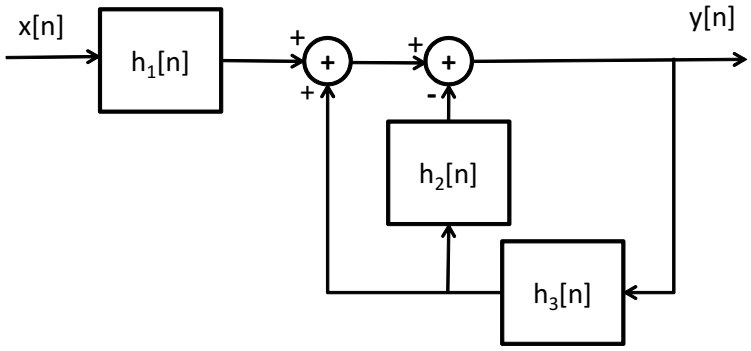
(c) $x_c[n] = (n+1)\alpha^n \mu[n] \qquad |\alpha| < 1$

PROBLEM 2: (problem 3.41 from the book) Let $G_1(e^{j\omega})$ denote the discrete-time Fourier transform of the sequence $g_1[n]$ shown in the figure below. Express the DTFTs of $g_2[n]$, $g_3[n]$ and $g_4[n]$ in terms of $G_1(e^{j\omega})$. Do not evaluate $G_1(e^{j\omega})$.



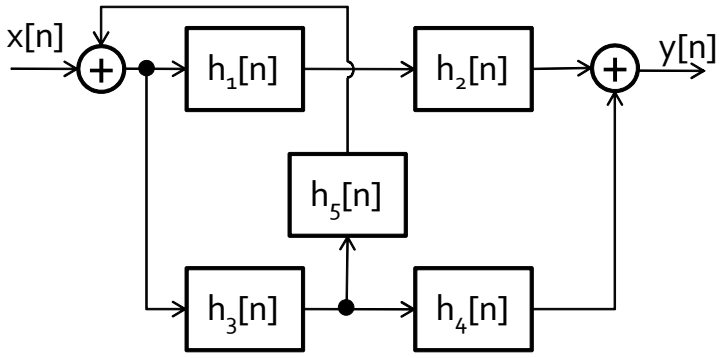
PROBLEM 3: (problem 3.34 from the book) Let $X(e^{j\omega})$ denote the DTFT of a complex sequence $x[n]$. Determine the DTFT $Y(e^{j\omega})$ of the sequence $y[n] = x[n] * x^*[-n]$ in terms of $X(e^{j\omega})$, and show that it is a real-valued function of ω.

PROBLEM 4: Consider the following interconnection of linear shift-invariant systems:



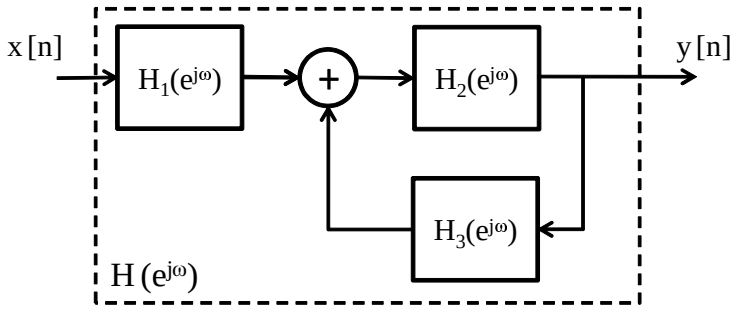
Express the frequency response of the overall system $H(e^{j\omega})$ in terms of the frequency responses of the subsystems $H_1(e^{j\omega})$, $H_2(e^{j\omega})$, and $H_3(e^{j\omega})$.

PROBLEM 5. Consider the following interconnection of LTI systems:



Express the frequency response of the overall system $H(e^{j\omega})$ in terms of the frequency responses of the subsystems depicted in the diagram.

PROBLEM 6. Consider the interconnection of linear shift-invariant systems in the figure below:



(a) Express the frequency response of the overall system $H(e^{j\omega})$ in terms of the frequency responses of the subsystems $H_1(e^{j\omega})$, $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$.

(b) Determine the frequency response $H(e^{j\omega})$ of the overall system if:

$$\begin{aligned} h_1[n] &= \frac{\sin(\frac{\pi}{3}n)}{\pi n} \\ h_2[n] &= (0.3)^n \mu[n] \\ h_3[n] &= \delta[n-2] \end{aligned}$$

PROBLEM 7 (problem 3.59 from the book): An LTI IIR discrete-time system is described by the difference equation

$$y[n] + a_1 y[n-1] = b_0 x[n] + b_1 x[n-1]$$

where the input is $x[n]$, the output is $y[n]$, and the constants a_1, b_0 and b_1 are real. Determine the expression for its frequency response. For what values of b_0 and b_1 will the magnitude response be a constant for all values of ω?.

PROBLEM 8: Consider the system defined by the difference equation

$$y[n] = ay[n-1] + bx[n] + x[n-1]$$

where a and b are real, and $|a| < 1$. Find the relationship between a and b that must exist if the frequency response is to have a constant magnitude for all ω, that is $|H(e^{j\omega})| = 1$.