SGN-1156 SIGNAL PROCESSING TECHNIQUES
Exam
December 7, 2009

Instructions: Write your name on **every** page in CAPITAL LETTERS and your student number as well. Number pages consecutively. You have to solve six problems. The total number points is 30 points. The correspondence between number of points and exam grade is: 0-14 points = grade 0; 15-17 points = grade 1; 18-20 points = grade 2; 21-23 points = grade 3; 24-26 points = grade 4; 27-30 points = grade 5.

1. (5 points) An L-th order moving average filter is a system that, for an input $x[n]$ produces the output:

$$y[n] = \frac{1}{1+L}\sum_{k=0}^{L} x[n-k]$$

where L is a finite non-negative integer. Is this system linear? Is it time-invariant? Justify your answers. Find the system's impulse response $h[n]$ and frequency response $H(e^{j\omega})$.

2. (5 points) Consider the system defined by the difference equation

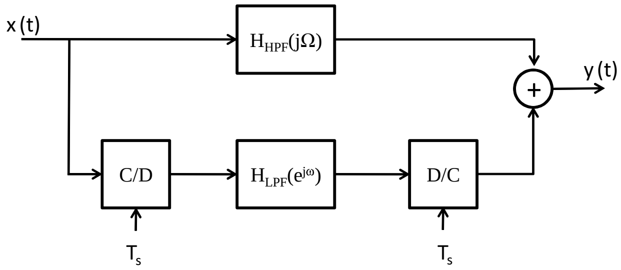
$$y[n] = ay[n-1] + bx[n] + x[n-1]$$

where a and b are real, and $|a| < 1$.

(a) Find the relationship between a and b that must exist if the frequency response is to have a constant magnitude for all ω, that is $|H(e^{j\omega})| = 1$.

(b) Assuming that this relationship is satisfied, find the output $y[n]$ of the system when $a = \frac{1}{2}$ and $x[n] = \left(\frac{1}{2}\right)^n \mu[n]$.

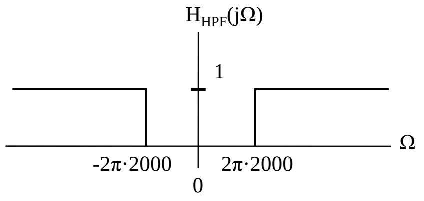
3. (5 points) Diagrammed below is a hybrid digital-analog system.



The discrete-time system $H_{LPF}(e^{j\omega})$ is a low-pass filter:

$$H_{LPF}(e^{j\omega}) = \begin{cases} A & |\omega| \leq \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

and the analog system is a filter with a frequency response as shown below:



The input analog signal is bandlimited to $\Omega_0 = 2\pi \cdot 4000$, and the sampling period of the ideal C/D and D/C converters is $T_s = 10^{-4}$ seconds. Find values for A and ω_0 that will result in perfect reconstruction of $x(t)$, i.e. values for which $y(t) = x(t)$.

4. (5 points) Use the Z-transform to perform the convolution of the following two sequences:

$$\begin{aligned} h[n] &= \left(\tfrac{1}{2}\right)^n (\mu[n] - \mu[n-2]) \\ x[n] &= \delta[n] + \delta[n-1] + 4\delta[n-2] \end{aligned}$$

5. (4 points) Find the region of convergence of the Z-transform of these sequences:

(a) (1 point) $x_a[n] = \left(\frac{1}{4}\right)^{-n} \mu[n-5]$

(b) (1 point) $x_b[n] = \begin{cases} 1 & 5 \leq n \leq 15 \\ 0 & \text{otherwise} \end{cases}$

(c) (1 point) $x_c[n] = 2^n (\mu[n-5] - \mu[n-10])$

(d) (1 point) $x_d[n] = 4^{-n}\mu[-n-2] + 2^{-2n}\mu[n-5]$

6. (6 points) When the input of a linear time-invariant system is $x[n] = (0.1)^n \mu[n]$, the output is $y[n] = \left(\left(\frac{-1}{3}\right)^n + 2\left(\frac{1}{4}\right)^n\right)\mu[n-2]$.

(a) (4 points) Find the system function $H(z)$ of the system. Plot the poles and zeroes of $H(z)$ and indicate the region of convergence.

(b) (2 points) Is the system causal? Is it stable? Is it minimum phase? Justify your responses.