# Supplementary Materials
# Combinatory Chemistry: Towards a Simple Model of Emergent Evolution

Germán Kruszewski[1*] Tomas Mikolov[2]

[1]Naver Labs Europe, Grenoble, France
[2]CIIRC CTU, Prague, Czech Republic
german.kruszewski@naverlabs.com

## A Appendix

The following derivations show one of the possible pathways that each of the described structures can undertake as they develop. They demonstrate how can these structures preserve their function in time. Note that every expression written as $(fX)Y$ can also be written simply as $fXY$. This "trick" is known as Uncurrying.

### A.1 Metabolic cycle of a simple autopoietic pattern

Let $A = (SII)$. Then,

$$\begin{aligned} (\underline{AA}) + A &\rightarrow ((IA)(IA)) + S \\ (\underline{IA}(IA)) &\rightarrow (A(IA)) + I \\ (A(\underline{IA})) &\rightarrow (AA) + I \end{aligned}$$

### A.2 Metabolic cycle of a ternary autopoietic pattern

Let $A = (SSK)$. Then,

* Work done while the author was at Facebook AI.

$$\begin{aligned}(\underline{AA}A)+A &\rightarrow (SA(KA)A)+S\\ (\underline{SA(KA)A})+A &\rightarrow (AA(KAA))+S\\ (AA(\underline{KAA})) &\rightarrow (AAA)+A+K\end{aligned}$$

## A.3 Metabolic cycle of a tail recursive structure

Let $A=(S(SI)I)$. Then,

$$\begin{aligned}(\underline{AA})+A &\rightarrow (SIA(IA))+S\\ (SIA(\underline{IA})) &\rightarrow SIAA+I\\ (\underline{SIAA})+A &\rightarrow (IA(AA))+S\\ (IA(AA)) &\rightarrow (A(AA))+I\end{aligned}$$

## A.4 Metabolic cycle of a binary-branching structure

Let $A=(S(SSI)K)$. Then $(AA)$ can follow the metabolic pathway:

$$\begin{aligned}(\underline{AA})+A &\rightarrow (SSIA(KA))+S\\ \underline{SSIA}(KA)+A &\rightarrow (SA(IA)(KA))+S\\ (SA(\underline{IA})(KA)) &\rightarrow (SAA(KA))+I\\ (\underline{SAA(KA)})+(KA) &\rightarrow (A(KA)(A(KA)))+S\end{aligned}$$

Then each copy of $(A(KA))$ can be reduced as follows

$$\begin{aligned}(\underline{A(KA)})+(KA) &\rightarrow SSI(KA)(K(KA))+S\\ (\underline{SSI(KA)}(K(KA))+(KA) &\rightarrow (S(KA)(I(KA))(K(KA)))+S\\ (S(KA)(\underline{I(KA)})(K(KA))) &\rightarrow (S(KA)(KA)(K(KA)))+I\\ (\underline{S(KA)(KA)(K(KA))})+(K(KA)) &\rightarrow (KA(K(KA))(KA(K(KA))))+S\\ (\underline{KA(K(KA))}(KA(K(KA)))) &\rightarrow (A(KA(K(KA))))+(K(KA))+K\\ (A(\underline{KA(K(KA))})) &\rightarrow (AA)+(K(KA))+K\end{aligned}$$

Thus, the complete pathway can be summarized as $(AA)+2A+5(KA)+(K(KA)) \twoheadrightarrow AA(AA)+4(K(KA))+2\phi(A)$.

## A.5 Metabolic cycle of a self-reproducing expression

Let $A = (SI(S(SK)I))$

Then,

$$\begin{aligned}
(\underline{AA}) + A &\rightarrow (IA(S(SK)IA)) + S \\
(\underline{IA}(S(SK)IA)) &\rightarrow (A(S(SK)IA)) + I \\
(A(\underline{S(SK)IA})) + A &\rightarrow (A(SKA(IA))) + S \\
(A(SKA(\underline{IA}))) &\rightarrow (A(SKAA)) + I \\
(A(\underline{SKAA})) &\rightarrow (A(KA(AA))) + S \\
(A(\underline{KA(AA)})) &\rightarrow (AA) + (AA) + K
\end{aligned}$$