PHYS 615 – Activity 8.2: Kepler Orbits

1. *Effective Potential*

   (a) By examining the effective potential energy

   $$U_{eff} = -\frac{\gamma}{r} + \frac{l^2}{2\mu r^2}$$

   find the radius r_0 at which a planet with angular momentum l can orbit the Sun in a circular orbit with fixed radius. (Look at dU_{eff}/dr)

   (b) Show that this circular orbit is stable, in the sense that a small nudge will only cause small radial oscillations. (Look at d^2U_{eff}/dr^2). Show that the period of these oscillations is equal to the planet's orbital period.

2. *Ellipses*

We have proved that any Kepler orbit can be written in the form

$$r(\phi) = \frac{c}{1 + \epsilon \cos \phi}$$

where $c > 0$ and $\epsilon \geq 0$. For the case that $0 \leq \epsilon < 1$, rewrite this equation in Cartesian coordinates (x, y) and prove that the equation can be cast in the form

$$\frac{(x+d)^2}{a^2} + \frac{y^2}{b^2} = 1$$

which the equation of an ellipse. Verify these expressions:

$$a = \frac{c}{1-\epsilon^2}, \qquad b = \frac{c}{\sqrt{1-\epsilon^2}}, \qquad d = a\epsilon$$

Draw a picture of the ellipse and indicate all the lengths (a, b, c, d).