PHYS 615 – HW 8

Types of homework questions

- RQ (Reading questions): prompt you to go back to the text and read and think about the text more carefully and explain in your own words. While not directly tested in quizzes, can help you think more deeply about quiz questions.
- BF (Building foundations): gives you an opportunity to build and practice foundational skills that you have, presumably, seen before.
- TQQ (typical quiz questions): Similar questions (though perhaps longer or shorter) will be asked on quizzes. But the difficulty level and skills tested will be similar.
- Design (D): These are questions in which you are given a desired outcome and asked to figure out how to make it happen. These will often also be TQQ's, but always starting with desired motion/behavior as the given.
- COMP (Computing): computing questions often related to TQQ but will never be asked on a quiz (since debugging can take so long). You will need to do at least four computing questions over the semester
- FC (free choice): allows you to decide where to put your time. Any of the following are possible: work through a section of the text or a lecture in detail; redo a problem from before; do an unassigned problem in the text; extend a computing project; try a problem using a different analytical approach (e.g. forces instead of conservation of energy).
- ACT (in-class activity): These questions are repeats of questions (or similar to) that occurred in a previous in-class activity.
- **Standard Reading Questions**: How does the reading connect with what you already know? What was something new? Ask an "I wonder" question OR give an example applying the idea in the reading.

Please remember to say something about the "Check/Learn" part at the end of solving a problem!

Full credit will be given at 75% of the total points possible, so you can choose a subset of problems (you can do more / all, but the score is capped at 75%)

This homework contains some previous group activities. I'm including them here in order to try to help gradescope, but you can of course hand in the original paper version I handed out in class.

1. COMP (20 points – required) Runge-Kutta integration

   Back an even longer while ago, we solved the skateboard in a half pipe problem numerically, and we noticed that even in the linear (small-angle) approximation, the amplitude seemed to grow over time quite substantially unless we used a tiny timestep: https://github.com/germasch/hw/blob/main/notebooks/euler-skateboard.ipynb

   Our next goal is going to be solving projectile motion with quadratic drag, but before we try to do so (down the road), let's try to get our code to give us a more accurate numerical approximate solution.

   Follow the tutorial at https://lpsa.swarthmore.edu/NumInt/NumIntSecond.html through "Example 1". As a first step, implement the 2nd order Runge-Kutta method to solve the 1st order ODE $\dot{y} = -2y$. (As usual, you can do so in Matlab or Python).

   Once it is working, you can now apply it to the skateboard problem, which we have also written as 1st order ODE previously. Compare the solution we got previously from the Euler method to this hopefully improved method for a timestep of 0.1 and 0.01.

   Please hand in your code on Canvas, and include a brief write-up on the results you got (here or on canvas).

   Solution: See https://github.com/germasch/hw/blob/main/notebooks/rk2.ipynb

2. RQ/TQQ (20 points) *Two unconstrained particles*

   Consider two particles moving unconstrained in three-dimensional space, with potential energy $U(\vec{r}_1, \vec{r}_2)$.

   (a) Write down the six equations of motion obtained by applying Newton's 2nd Law to either particle.
   Solution:

$$F_{1,x} = m_1 \dot{x}_1$$
$$F_{1,y} = m_1 \dot{y}_1$$
$$F_{1,z} = m_1 \dot{z}_1$$
$$F_{2,x} = m_2 \dot{x}_2$$
$$F_{2,y} = m_2 \dot{y}_2$$
$$F_{2,z} = m_2 \dot{z}_2$$

   (b) Write down the Lagrangian $\mathcal{L} = T - U$ and show that the six Euler-Lagrange equations are the same as the six Newtonian equations obtained from part (a).
   Solution:

$$\mathcal{L} = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2) - U(x_1, y_1, z_1, x_2, y_2, z_2)$$

   Fortunately, I can copy & paste things, because this is pretty repetitive:

$$\begin{aligned}
F_{1x} = -(\nabla_1 U)_x = -\frac{\partial U}{\partial x_1} = \frac{\partial \mathcal{L}}{\partial x_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = \frac{d}{dt} m\dot{x}_1 = m\ddot{x}_1 \\
F_{1y} = -(\nabla_1 U)_y = -\frac{\partial U}{\partial y_1} = \frac{\partial \mathcal{L}}{\partial y_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}_1} = \frac{d}{dt} m\dot{y}_1 = m\ddot{y}_1 \\
F_{1z} = -(\nabla_1 U)_z = -\frac{\partial U}{\partial z_1} = \frac{\partial \mathcal{L}}{\partial z_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{z}_1} = \frac{d}{dt} m\dot{z}_1 = m\ddot{z}_1 \\
F_{2x} = -(\nabla_2 U)_x = -\frac{\partial U}{\partial x_2} = \frac{\partial \mathcal{L}}{\partial x_2} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = \frac{d}{dt} m\dot{x}_2 = m\ddot{x}_2 \\
F_{2y} = -(\nabla_2 U)_y = -\frac{\partial U}{\partial y_2} = \frac{\partial \mathcal{L}}{\partial y_2} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}_2} = \frac{d}{dt} m\dot{y}_2 = m\ddot{y}_2 \\
F_{2z} = -(\nabla_2 U)_z = -\frac{\partial U}{\partial z_2} = \frac{\partial \mathcal{L}}{\partial z_2} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{z}_2} = \frac{d}{dt} m\dot{z}_2 = m\ddot{z}_2
\end{aligned}$$

   And so, yeah, that's exactly the same as Newton's 2nd Law.

3. FC (10 points) (free choice): allows you to decide where to put your time. Any of the following are possible: work through a section of the text or a lecture in detail; polish up a group work assignment from class; redo a problem from before; do an unassigned problem in the text; extend a computing project; try a problem using a different analytical approach (e.g. forces instead of conservation of energy).

4. TQQ / ACT (30 points) Hand in Activity 7.1

   Solution: See Activity 7.1 solution.

5. TQQ / ACT (20 points) Hand in Activity 7.2

   Solution: See Activity 7.2 solution.