PHYS 615 – Activity 4.2: Conservative Forces, Potential Energy

1. *Conditions for a force to be conservative*

   What are the two conditions for a force to be called *conservative*?

2. *Potential Energy for a spring*

   The force exerted by a spring $\vec{F}_{spr}$ is given by Hooke's Law $\vec{F}_{spr} = -k\vec{r}$, where $\vec{r}$ indicates how much the spring is stretched (or compressed) from its equilibrium length.

   (a) For each of the two conditions that $\vec{F}_{spr}$ be conservative, is it satisified / not satisfied / not known at this point?

   (b) Since a spring is usually stretched or compressed along its axis, you can assume that the only motion that happens is in the x direction. Calculate the work done by the spring on a particle attached to its end as it moves from x_1 to x_2.

   (c) Is the spring force conservative?

(d) Find the potential energy for a spring U_{spr}.

3. *Friction force*

   A block is sliding across a plane (inclined or not). Since the normal force is constant, kinetic friction $F_{fk} = \mu_k F_N$ is also constant.

   It would appear that (a) the friction force is constant and (b) the motion is 1-d, and in both of those cases we can more easily calculate the work done by the friction force than having to do full-blown line integrals. So that makes one think that the force of kinetic friction should be conservative. Is that true? What, if anything, have we missed?

4. A block is sliding down a distance of d on an inclined plane – it starts from rest. Find an expression of the block's final speed in terms of d, the angle of inclination θ, and the coefficient of kinetic friction μ_k.

   We'll do this multiple ways.

   (a) Draw a free-body diagram. Identify all forces acting on the block.

   (b) You probably want to choose your coordinate system to be parallel / perpendicular to the inclined plane, though you could choose horizontal / vertical if you're up for some more math.

Write down Newton's 2nd Law for both the x component and the y component of the net force. Express the net force in terms of the forces on your FBD, and put in what you know about the acceleration components.

Solve to find the acceleration of the block a.

(c) Give your acceleration a and distance d, find the final speed v_f of the block.

(d) So in the above, we didn't use energy at all (though the 3rd kinematics law is closely related to the work-energy theorem). So let's do it using energy.

For all three forces, figure out whether they are conservative, and if so, use their potential energy. If a force is not conservative, use the work done by that force. Write down

$$\Delta E = \Delta(T + U) = W_{nc}$$

and plug in kinetic and potential energy at the initial and final times, as well as the work done by non-conservative force(s). (You will still need to find the friction force in order to be able to calculate the work done by it. Fortunately, you've already done that above, so you can reuse that result.)

Solve for v_f. Do you expect to get the same answer? Did you?