PHYS 615 – HW 4

Types of homework questions

- RQ (Reading questions): prompt you to go back to the text and read and think about the text more carefully and explain in your own words. While not directly tested in quizzes, can help you think more deeply about quiz questions.
- BF (Building foundations): gives you an opportunity to build and practice foundational skills that you have, presumably, seen before.
- TQQ (typical quiz questions): Similar questions (though perhaps longer or shorter) will be asked on quizzes. But the difficulty level and skills tested will be similar.
- Design (D): These are questions in which you are given a desired outcome and asked to figure out how to make it happen. These will often also be TQQ's, but always starting with desired motion/behavior as the given.
- COMP (Computing): computing questions often related to TQQ but will never be asked on a quiz (since debugging can take so long). You will need to do at least four computing questions over the semester
- FC (free choice): allows you to decide where to put your time. Any of the following are possible: work through a section of the text or a lecture in detail; redo a problem from before; do an unassigned problem in the text; extend a computing project; try a problem using a different analytical approach (e.g. forces instead of conservation of energy).
- ACT (in-class activity): These questions are repeats of questions (or similar to) that occurred in a previous in-class activity.
- **Standard Reading Questions**: How does the reading connect with what you already know? What was something new? Ask an "I wonder" question OR give an example applying the idea in the reading.

Please remember to say something about the "Check/Learn" part at the end of solving a problem!

Note that while this homework does not have designated "RQ" questions, in particular the activity-based questions can certainly benefit from reading the corresponding sections in the text.

Full credit will be given at 75% of the total points possible, so you can choose a subset of problems (you can do more / all, but the score is capped at 75%)

1. TQQ / ACT (25 points) Hand in Activity 2.3.

2. TQQ / ACT (25 points) Someone (not you, obviously) is trying to shoot down a balloon that's up at 60,000 feet.

   (a) Let's presume you have a 9mm hand gun. That is to say, the diameter of the projectile is 9 mm, and its mass is 8.5 g. Let's assume a drag coefficient of $C_D = 0.2$, and the initial speed of the bullet is $450\mathrm{m/s}$, and an air density of $1.225\mathrm{kg/m}$. Even though now I've given you all those numbers, don't plug them in until the very end!

   (b) Assuming there's no drag at all, how high can the bullet reach? (You can do this any way you want, whatever seems most convenient.)

(c) Now solve the problem with drag $F_D = \frac{1}{2}\rho A C_D v^2$, where A is the cross section of the projectile. In order to do so, follow the steps from Activity 2.4 – but now the projectile is moving upward.

Some notes:

- The ODE will be similar but not the same, as some signs change. You might want to pick the up direction to be positive.
- Even though there is no terminal velocity while moving upward, one can still define a v_{ter} as before and use it to simplify the equation. (Not required, but makes it look more familiar.)
- Obviously, there needs to be a non-zero initial condition for the velocity, rather than zero.
- Since the ODE isn't quite the same, the guess I gave you won't work. Since the integral that comes up when solving this by separation of variables is not trivial, you might want to ask Wolfram Alpha or some integration tables for help (e.g., https://en.wikipedia.org/wiki/List_of_integrals_of_rational_functions)
- If the hand gun isn't good enough, you might try again with a more powerful higher caliber gun.

3. TQQ / BF (10 points) More on hyperbolic functions (see last HW for some intro)

   (a) Show that $\cosh z = \cos(iz)$. What is the corresponding relation for $\sinh$?

   (b) Show that $\cosh^2 z - \sinh^2 z = 1$.

   (c) What is the derivative of $\tanh z$?

   (d) Show that $\int \tanh z \, dz = \ln \cosh z$.

   (e) Prove that $1 - \tanh^2 z = \operatorname{sech}^2 z$, where $\operatorname{sech} z = 1/\cosh z$.

   (f) Show that $\int dx/(1 - x^2) = \tanh^{-1} x$. ($\tanh^{-1}$ is also called $\operatorname{arctanh}$.)

4. TQQ (5 points)

   The transverse velocity of the gyrating particle (see Taylor 2.5, 2.7) is contained in Taylor Eq. (2.77), since $\eta = v_x + iv_y$. By taking the real and imaginary parts, find expressions for v_x and v_y separately. Based on these expressions, describe the time dependence of the transverse velocity.

5. TQQ (10 points)

   In activity 2.5 (and Taylor 2.5), we solved the motion of a gyrating particle using the trick $\eta = v_x + iv_y$. Instead, use the way that I briefly pointed out in class: Take the equation of motion for $\dot{v}_x$ and differentiate it one more time with respect to time. Then use the equation of motion for $\dot{v}_y$ to substitute into the r.h.s. You should obtain an uncoupled 2nd order ODE for v_x that we have solved before (by making an educated guess, for the small angle approximation of the skateboard problem.) Use this known solution for v_x, and you can then find v_y from your original ODEs.

   Show that the solution you obtain is equivalent to the one in Taylor Eq. (2.77).

6. FC (10 points) (free choice): allows you to decide where to put your time. Any of the following are possible: work through a section of the text or a lecture in detail; polish up a group work assignment from class; redo a problem from before; do an unassigned problem in the text; extend a computing project; try a problem using a different analytical approach (e.g. forces instead of conservation of energy).