PHYS 615 – Quiz 5: Lagrangian Mechanics

Name: ____________________________________

Instructions: You have 40 minutes to work on this quiz.
If you get stuck on one part, still try the other parts. Some parts are independent; for dependent parts, I'll give you full credit if your process is correct, even in your input from another part is incorrect.
Possibly useful physics equations:

$$\vec{F}_{net} = m\vec{a} \qquad F_{fk} = \mu_k F_N \qquad F_{fs} \leq \mu_s F_N \qquad F_G = mg \qquad \vec{F}_{kind,A\,on\,B} = -\vec{F}_{kind,B\,on\,A}$$

$$\vec{F}_{D,quad} = -cv^2\hat{v} \qquad \vec{F}_{D,lin} = -bv\hat{v}$$

$$\vec{p} = m\vec{v} \qquad \dot{\vec{p}} = \vec{F} \qquad \vec{l} = I\vec{\omega} = \vec{r} \times \vec{p} \qquad \dot{\vec{l}} = \vec{\Gamma} = \vec{r} \times \vec{F}$$

$$\vec{R}_{CM} = \frac{1}{M}\int_M \vec{r}dm$$

$$\mathcal{L} = T - U \qquad T = \frac{1}{2}mv^2 \qquad T_{rot} = \frac{1}{2}I\omega^2$$

Possibly useful math equations:

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} \qquad \text{(for any } n \neq 0\text{, including fractions.)}$$

$S = \int_{t_1}^{t_2} \mathcal{L}(q, \dot{q}, t)dt$ is stationary with respect to variations of the path $q(t)$ iff $\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}}$
Polar coordinates: $x = r\cos\phi$, $y = r\sin\phi$.

1. *A bead on a helical wire*

   A smooth (frictionless) wire is bent into the shape of a helix (a.k.a. spiral), with cylindrical coordinates $\rho = R$ and $z = \lambda\phi$, where R and λ are constants.

   [Note: cylindrical coordinates use polar coordinates in the x-y plane, except that we usually use ρ instead of r for the distance to the z axis. The third coordinate is just the Cartesian coordinate z.]

   (a) (20 points) Using z as your generalized coordinate, write down the Lagrangian for a bead of mass m threaded on the wire.

   (b) (20 points) Find the Lagrange Equation of Motion and hence the bead's acceleration $\ddot{z}$.

(c) (10 points) In the limit $R \to 0$, what is $\ddot{z}$. [If you do not have a $\ddot{z}$, use $\ddot{z} = -g/(1 + R^2/C^2)$.]

(d) (10 points) Does this make sense? Explain.

2. *40 points*

   A mass m is suspended from a massless string, the other end of which is wrapped around a horizontal cylinder of radius R and moment of inertia I, which is free to rotate about a fixed horizontal axis. Using suitable coordinates, set up a Lagrangian and the Lagrange equation of motion. Find the acceleration of the mass m. [If you did not find a Lagrangian, use $\mathcal{L} = \frac{1}{2}(m + M/2)\dot{x}^2 + mgx$.]