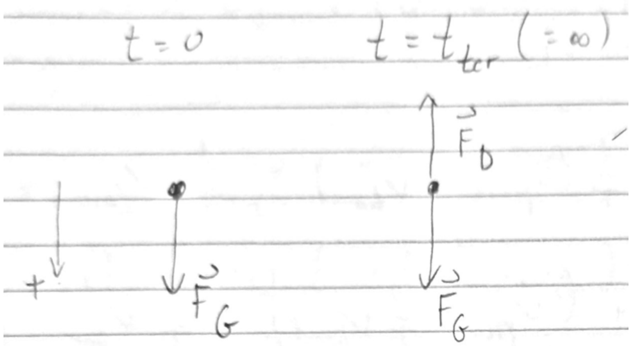
PHYS 615 – Activity 1.4: Intro to Drag

Air resistance and Newton's 2nd Law. Suppose that you took a small rubber ball to the top of a very tall building and dropped it from rest at $t = 0$. At a later time, the ball moves with *constant speed*. (That speed is called *terminal speed*.)

1. Draw separate free body diagrams for the ball (i) at time $t = 0$, and (ii) after it has reached terminal speed. Clearly label all forces.

   Solution:

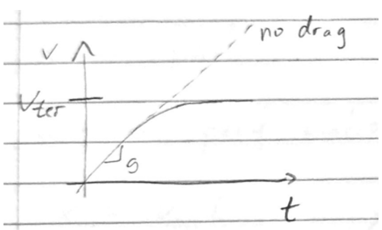
2. What can be said about the acceleration of the ball (i) at time $t = 0$, and (ii) after it has reached terminal speed. Discuss both magnitude and direction. How are your answers related to the FBDs above?

   Solution: Since the initial speed of the ball is zero, there is no drag force yet, so acceleration is the acceleration due to gravity g (downward). This is consistent with the FBDs above.

   After it has reached terminal speed, the speed doesn't change anymore (hence "terminal"), so acceleration is zero, ie., drag cancels gravity.

3. Sketch a qualitatively correct graph of velocity vs time for the ball. Since the ball is falling down, let's take the down direction to be positive.

   Solution:

4. On the same graph, show the v vs t graph for the case of no air resistance. Make sure it is otherwise consistent with the first graph you drew.

   Solution: See above.

Calculating terminal speed

5. Let's still keep downward to be the positive direction.

   Starting with Newton's 2nd Law, write an equation that includes the acceleration $\dot{v}$ of the ball and all relevant force terms (mg, linear drag bv, quadratic drag cv^2 – see Taylor 2.1).

   Solution:
   $$m\dot{v} = mg - bv - cv^2$$

6. How would your equation be different if the ball was instead moving upward?

   Solution: In this case, with the ball moving up, the drag forces would point downward. Given that downward is positive, and that v would be negative in this case:
   $$m\dot{v} = mg - bv + cv^2$$

   (This looks admittedly quite confusing, and might be clearer if written as $m\dot{v} = g + b|v| + c|v|^2$.)

7. Back to dropping the ball: If the force of air resistance were purely linear with respect to velocity (ie., $b \neq 0, c = 0$), use the appropriate equation to express the terminal speed v_t of the object in terms of b, m and g.

   Solution: Since now $v = v_{ter} = const$, $\dot{v} = 0$, So
   $$0 = mg - bv_{ter} \qquad \Longrightarrow \qquad v_{ter} = \frac{mg}{b}$$

   Check that your expression for v_t has the correct units. That is, determine the appropriate units for b and confirm that your expression for v_{ter} does have the appropriate units for a speed.

   Solution: From $F_D = -bv$: N $= \frac{\text{kgm}}{\text{s}^2} = [b]\text{m/s}$, so $[b] = \frac{\text{kg}}{\text{s}}$.

   Therefore, $[v_{ter}] = \frac{\text{kgm/s}^2}{\text{kg/s}} = \frac{\text{m}}{\text{s}}$, as it should be.