PHYS 615 – Activity 15.1: Galilean Transform

1. *Dropping a Ball in an Elevator*

   A glass elevator is moving up at constant velocity V – it's initial height at time 0 is H_0. A person inside of the elevator drops a ball from height h_0 above the elevator floor – they hold the ball in their hand and then let go of it at time 0. How long does it take for the ball to hit the floor of the elevator?

   Find the answer in two different frames of reference: (1) in the frame of reference of the person dropping the ball, that is, the frame of reference moving together with the elevator; (2) in the frame of reference fixed to the ground outside.

   Bonus question: What changes if the elevator, initially moves at V_0, but accelerates at a rate A? Explain.

   Solution: In the frame of reference of the elevator, the ball is initially at height $y(0) = h_0$ and at rest $v(0) = 0$. The only force acting once the hand lets go is gravity, so Newton's 2nd Law tells me $-mg = F = ma$, ie., $a = -g$.

   Since that acceleration is constant, I can write down the solution of the equation of motion directly:

   $$y(t) = -\frac{1}{2}gt^2 + h_0$$

   To find how it long it takes for the ball to hit the floor, I'll use that it's height is 0 at the time, ie.,

   $$0 = -\frac{1}{2}gt^2 + h_0 \Longrightarrow t = \sqrt{\frac{2h_0}{g}}$$

   Let's do it again in the frame of the observer on the ground. Newton's 2nd Law works the same, so still $a = -g$. The initial height of the ball is now $y'(0) = h_0 + H_0$ (I'm choosing primed quantities are for the ground frame of reference.) The person, their hand, and the ball are initially moving together with the elevator at V, so $v'(0) = V$. Here's the same solution again, with the correct initial conditions:

   $$y'(t) = -\frac{1}{2}gt^2 + Vt + H_0 + h_0$$

   The ball hits the elevator floor when it's height y' equals the height of the elevator floor, which is $H_0 + Vt$ as the elevator is moving.

   $$H_0 + Vt = -\frac{1}{2}gt^2 + Vt + H_0 + h_0$$

   After some cancelations, this equation is exactly the same as above, with the same solution $t = \sqrt{\frac{2h_0}{g}}$.

   The take-away here is that we can use Newton's Laws and Kinematics just as well in one inertial frame (ground) as in the other (elevator). However, if the elevator wasn't moving at constant speed, ie., not be an inertial frame, this doesn't work anymore (we'd have to introduce a fictious force to make up for the acceleration of the frame).

2. *Newton's Laws Invariance under Galilean Transformation*

   Using arguments similar to those of the text's section 15.2, show that Newton's 1st and 3rd Laws are invariant under the Galilean transformation.

   Solution: The easy way to do 1st Law would be to just say it's a special case of the 2nd Law, where we've already shown invariance.

   But it can also be done explicitly: If the first Law holds in frame S, ie., $\vec{F} = 0$ and $\vec{v} = const.$ In some other frame moving at relative velocity $\vec{V}$, the forcee $\vec{F}'$ would also be zero, since it is invariant. and $\vec{v}' = \vec{v} - V$ would still be constant, since $\vec{v}$ is constant, as is $\vec{V}$.

   The Third Law invariance directly follows from the invariance of forces: 3rd Law in S: $\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}$. Either side doesn't change if we add a prime, so $\vec{F}'_{1\,on\,2} = -\vec{F}'_{2\,on\,1}$

3. *Inelastic Collision*

   Consider a classical inelastic collision of the form $A + B \to C + D$. (For example, this could be a collision such as $Na + Cl \to Na^+ + Cl^-$, in which two neutral atoms exchange an electron.)

   Show that the law of conservation of classical momentum is unchanged under the Galilean transformation if and only if total mass is conserved – as is certainly true in classical mechanics. (We shall find in relativity that the classical definition of momentum has to be modified and that total mass is *not* conserved.)

   Solution: Conservation of Momentum in frame S:

   $$m_A\vec{v}_A + m_B\vec{v}_B = m_C\vec{v}_D + m_C\vec{v}_D$$

   We want it to be true in frame S', too:

   $$m_A\vec{v}'_A + m_B\vec{v}'_B = m_C\vec{v}'_D + m_C\vec{v}'_D$$

   We know that velocities transform as $\vec{v}'_X = \vec{v}_X - \vec{V}$. Plugging that in

   $$m_A\vec{v}_A + m_B\vec{v}_B - (m_A + m_B)\vec{V} = m_C\vec{v}'_D + m_C\vec{v}'_D - (m_C + m_D)\vec{V}$$

   Which clearly is satisfied if $m_A + m_B = m_C + m_D$ only (unless $\vec{V} = 0$).