PHYS 615 – Activity 3.2: Momentum, Center of Mass, Angular Momentum

1. A shell traveling with speed v_0 exactly horizontally and due north explodes into two equal mass fragments. It is observed that just after the explosion, one fragment is traveling vertically up with speed v_0. What is the velocity of the other fragment?

2. Consider a gun of mass M (when unloaded) that fires a shell of mass m with muzzle speed v (That is, the shell's speed relative to the gun is v.) Assuming that the gun is completely free to recoil (no external forces on gun or shell), use conservation of momentum to show that the shell's speed relative to the ground is $v/(1 + m/M)$.

3. The masses of the Earth and the Sun are $M_E \approx 6.0 \times 10^{24}$ kg and $M_S \approx 2.0 \times 10^{30}$ kg and their center-to-center distance is 1.5×10^8 km. Find the position of their center of mass and comment. (The radius of the Sun is $R_S \approx 7.0 \times 10^5$ km.)

4. The center of mass of a triangular plate.

   An isosceles triangular plate has a base of length b, a height of h, and a thickness of d, as well as a mass of M. It is made from a uniform material (e.g., wood).

   (a) Find the plate's volume in terms of the quantities given above. Before actually doing so exactly, think about what result you would expect from dimensional analysis.

   (b) Find an expression for the plate's density in terms of the quantities given above.

   (c) Find the position of the center of mass for the plate. It probably makes sense to orient the triangle similarly to the cone in Taylor's Example 3.2, ie., have the tip at the origin.

The angular momentum of a single particle at position $\vec{r}$ is defined as

$$\vec{l} = \vec{r} \times \vec{p}$$

5. Check that the following equality is true by evaluating explicitly the z component of both the left hand side and the right hand side.

$$\dot{\vec{l}} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}}$$

6. Show that $\dot{\vec{r}} \times \vec{p}$ is zero (think back to the definition of momentum).

7. Derive the "2nd Law of angular motion"

$$\dot{\vec{l}} = \vec{\Gamma} \equiv \vec{r} \times \vec{F}_{net}$$

   where the term on the r.h.s. is used to define "torque" $\vec{\Gamma}$. (Taylor uses capital gamma Γ for torque.) Compare to the regular 2nd Law, written in terms of momentum as $\dot{\vec{p}} = \vec{F}_{net}$.

8. Newton's Law of gravity is

$$\vec{F}_G = G\frac{mM}{r^2}(-\hat{r})$$

Sketch the situation of the Earth rotating around the Sun. Put the Sun at the origin and draw vectors $\vec{r}$ indicating the position of Earth, and $\vec{F}_G$ showing the gravitional force exerted by the Sun onto Earth.

What is the torque that the Sun exerts on Earth?