PHYS 615 – HW 11

Types of homework questions

- RQ (Reading questions): prompt you to go back to the text and read and think about the text more carefully and explain in your own words. While not directly tested in quizzes, can help you think more deeply about quiz questions.
- BF (Building foundations): gives you an opportunity to build and practice foundational skills that you have, presumably, seen before.
- TQQ (typical quiz questions): Similar questions (though perhaps longer or shorter) will be asked on quizzes. But the difficulty level and skills tested will be similar.
- Design (D): These are questions in which you are given a desired outcome and asked to figure out how to make it happen. These will often also be TQQ's, but always starting with desired motion/behavior as the given.
- COMP (Computing): computing questions often related to TQQ but will never be asked on a quiz (since debugging can take so long). You will need to do at least four computing questions over the semester
- FC (free choice): allows you to decide where to put your time. Any of the following are possible: work through a section of the text or a lecture in detail; redo a problem from before; do an unassigned problem in the text; extend a computing project; try a problem using a different analytical approach (e.g. forces instead of conservation of energy).
- ACT (in-class activity): These questions are repeats of questions (or similar to) that occurred in a previous in-class activity.
- **Standard Reading Questions**: How does the reading connect with what you already know? What was something new? Ask an ”I wonder” question OR give an example applying the idea in the reading.

Please remember to say something about the ”Check/Learn” part at the end of solving a problem!

Full credit will be given at 75% of the total points possible, so you can choose a subset of problems (you can do more / all, but the score is capped at 75%)

This homework contains some previous group activities. I'm including them here in order to try to help gradescope, but you can of course hand in the original paper version I handed out in class.

1. COMP (20 points – required) Runge-Kutta integration for projectile motion with drag

   Back in the day, we looked at motion with linear drag, which we could solve analytically, and with quadratic drag, which we could only solve analytically in 1-d (either horizontal or vertical, by itself).

   But now that we got a working RK2 integrator, we can solve projectile motion with drag numerically. Specifically, you should be able to do same calculation as Taylor in Example 2.6 (Trajectory of a Baseball).

   You've come across all the pieces you need, and the initial conditions and equations are given / referred to in Example 2.6, so, well, go for it!

   (Probably a good place to start is to write down what you know and what tools you have, and make a plan).

   Solution: See https://github.com/germasch/hw/blob/main/notebooks/projectile_with_drag.ipynb

2. TQQ (10 points) *Halley's Comet*

   Halley's Comet follows a very eccentric orbit with $\epsilon = 0.967$. At closest approach (the perihelion) the comet is 0.59 AU from the sun, fairly close to the orbit of Mercury. (The AU or astronomical unit is the mean distance between Earth and Sun, about 1.5×10^8 km.)

   What is the comet's greatest distance from the Sun, ie., the distance of the aphelion?

   Solution: See Example 8.4 in the text.

3. TQQ (10 points) *A Satellite*

   An Earth satellite is observed at perigee to be 250 km above Earth's surface and traveling at about 8500 m/s. Find the eccentricity of its orbit and its height above Earth at perigee. [Hint: Earth's radius is $R_E \approx 6400 km$. You will also need to know GM_E, but you can find this if you remember $GM_E/R_E^2 = g$.]

   Solution: The closest approach for the satellite is $r_{min} = (6400 + 250)$ km. It's speed is $v_{max} = 8500$ m/s.

   We can then find its angular momentum as $l = mr^2\dot{\phi} = r_{min}mv_{max}$, since at perigee, the velocity is perpendicular to the satellite - Earth line. I'm using the mass m of the satellite instead of the reduced mass μ, since the satellite is presumably much less than Earth's mass, so that's a good approximation.

   We can then find the parameter c of the orbit equation as $c = l^2/\gamma\mu = mr_{min}^2 v_{max}^2/\gamma$, where again I used $m \approx \mu$ to cancel one m. Using $\gamma = GMm$ (M_E being Earth's mass)

   $$c = \frac{r_{min}^2 v_{max}^2}{GM_E} = 7960\,\text{km}$$

   where I used $GM_E = gR_E^2$ with $g = 9.8$ m/s. Since $r_{min} = c/(1+\epsilon)$ and $r_{max} = c/(1-\epsilon)$, I can find $\epsilon = 0.197$ and $r_{max} = 9910$ km, which corresponds to a height of the Earth's surface of $h_{max} = 3510$ km.

4. TQQ (10 points) *A Comet*

   Consider a comet which passes through its aphelion at distance r_{max} from the Sun. Imagine that, keeping r_{max} fixed, we somehow make the angular momentum l smaller and smaller, though not actually zero; that is, we let $l \to 0$. Use Taylor's equation (8.48) and (8.50) to show that in this limit, the eccentricity ϵ of the elliptical orbit approaches 1 and that the distance of smalles approach r_{min} approaches 0. Describe the orbit with r_{max} fixed but l very small. What is the semi-major axis a?

   Solution: Since $c = l^2/\gamma\mu$, as $l \to 0$, c becomes smaller and smaller. Given that $r_{max} = c/(1-\epsilon)$, if we want to keep r_{max} constant as $c \to 0$, that means $\epsilon \to 1$. The orbit becomes very elongated in the direction of aphelion, and very narrow perpendicular to it (cigar-shaped).

   We know $r_{min} = c/(1+\epsilon)$, so $r_{min} \to 0$. Perihelion moves closer and closer to the sun. Since $2a = r_{min} + r_{max} \to r_{max}$, the semi-major axis $a \to r_{max}/2$.

5. FC (10 points) (free choice): allows you to decide where to put your time. Any of the following are possible: work through a section of the text or a lecture in detail; polish up a group work assignment from class; redo a problem from before; do an unassigned problem in the text; extend a computing project; try a problem using a different analytical approach (e.g. forces instead of conservation of energy).

   Solution:

6. TQQ / ACT (40 points) Hand in Activity 8.2

   Solution: See Activity 8.2 solution.