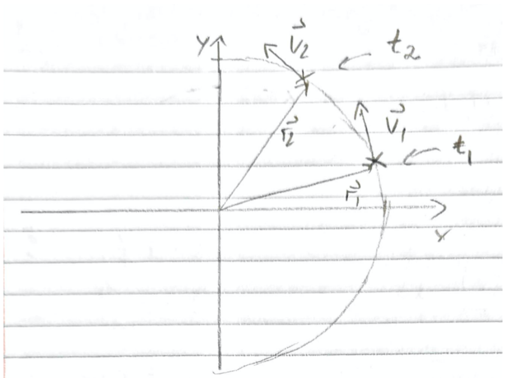
PHYS 615 – Activity 3.3: Angular Motion

1. Circular Motion at constant speed

   A block is connected to a string of length r whose other end is attached to a table. Let's call the table surface the x-y plane with its origin where the string is attached to the table. Because the block is connected to the string, it is rotating around the origin at constant speed.

   (a) Draw a sketch of the situation with the block at two times t_1 and t_2. Between these two times it moved by, say, 1/8 of a revolution.

   Solution:

   (b) Add position vectors $\vec{r}_1$ and $\vec{r}_2$, as well as velocities $\vec{v}_1$ and $\vec{v}_2$ to your sketch (1, 2 refer to the two points in time you picked above).

   Solution: see above

   (c) Calculate angular momentum $\vec{l}_{1,2}$ for the block at times t_1 and t_2. State your result as a vector (you may have to specify your z direction.)

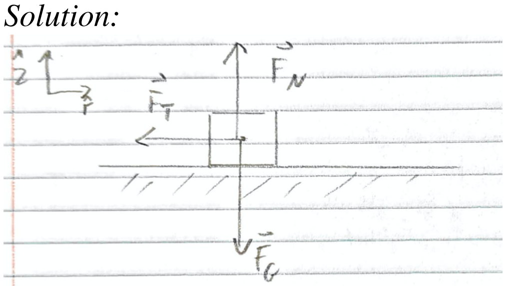
   Solution: The angle between $\vec{r}$ and $\vec{v}$ is 90 degrees at all times, so $\vec{l}_1 = \vec{l}_2 = rmv\hat{z}$.

   (d) Is $\vec{r}(t)$ constant? Is $\vec{v}(t)$ constant? How about $\vec{l}(t) = \vec{r}(t) \times m\vec{v}(t)$? Is angular momentum conserved?

   Solution: Both position and velocity are not constant (their magnitude is, but the direction is changing). Angular momentum is constant, though, as seen for the arbitrary two times above.

   (e) Draw a FBD, with all the forces acting on the block.

   Solution:

   (f) Using your FBD, find the net force and then the net torque $\vec{\Gamma}_{net}$.

   Solution: Gravity and normal force cancel out, so the only force remaining is $\vec{F}_{net} = \vec{F}_{tension} = F_{tension}(-\hat{r})$.

Therefore net torque is zero: $\vec{\Gamma}_{net} = \vec{r} \times F_{tension}(-\hat{r}) = \vec{0}$.

(g) Are your results consistent with the angular 2nd Law $\dot{\vec{l}} = \vec{\Gamma}_{net}$?

Solution: Yes – as we've seen, net torque is zero, so angular momentum should be conserved, and we have in fact seen that it did not change.

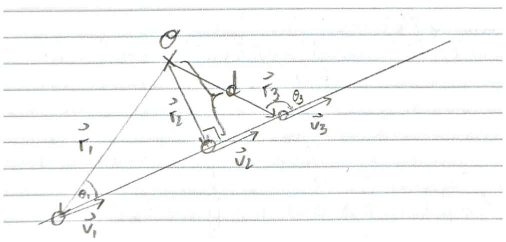
(h) (Optional – if you have time you can come back here) Let's assume there is kinetic friction between the block and the table, with a coefficient of μ_k. Do it all again.

2. Angular Momentum of straight-line motion

A particle is moving at constant velocity in the x-y plane.

(a) Sketch the situation. The particle misses the origin. Let's call the minimum distance between the particle and the origin d.

Solution:



(b) What is the net force acting on the particle? (Hint: No FBD needed)

Solution: It must be zero, since the particle's velocity is constant.

(c) What is the net torque acting on the particle?

Solution: Given that $\vec{F}_{net} = \vec{0}$, net torque is also zero.

(d) Should the particle's angular momentum be conserved?

Solution: Yes, since net torque is zero.

(e) Let's draw the particle into your sketch at 3 different times: (1) before (2) at (3) after its closest approach to the origin. Find its angular momentum $\vec{l}$ at these 3 times. Express it as a vector, in terms of the particles mass m, speed v, the distance d, and possibly some angle.

Solution: The calculation can be written the same way at each point:

$$\vec{l} = \vec{r} \times \vec{p} = rmv \sin\theta \hat{z} = (r \sin\theta) mv\hat{z} = dmv\hat{z}$$

(f) Are the three values you found for $\vec{l}$ consistent with your expectation on whether $\vec{l}$ should be conserved?

Solution: Yes, $\vec{l}$ is the same at every point along the trajectory, so it is conserved as it should be.