PHYS 615 – Quiz 2: Velocity-Dependent Forces

Name: ____________________________________

Instructions: You have 40 minutes to work on this quiz.
Write your answer on your own paper and, if possible, upload to Gradescope when you are done. If you get stuck on one part, still try the other parts. Some parts are independent; for dependent parts, I'll give you full credit if your process is correct, even in your input from another part is incorrect.
Possibly useful physics equations:

$$\vec{F}_{net} = m\vec{a} \qquad F_{fk} = \mu_k F_N \qquad F_{fs} \leq \mu_s F_N \qquad F_G = mg \qquad \vec{F}_{kind,A\,on\,B} = -\vec{F}_{kind,B\,on\,A}$$

$$\vec{F}_{D,quad} = -cv^2\hat{v} \qquad \vec{F}_{D,lin} = -bv\hat{v}$$

Possibly useful math equations:

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} \qquad \text{(for any } n \neq 0\text{, including fractions.)}$$

Problem: Consider a train car moving (coasting) horizontally at an initial speed v_0, subject to a novel drag force given by

$$\vec{F}_{D,cubic} = -\alpha v^3\hat{v}$$

You can neglect friction between the train car and the rails.

1. Qualitative solution.

   (a) (5 points) Sketch the free body diagram for the train car. You can limit yourself just horizontal forces. In addition, draw vectors that show the train car's velocity and acceleration, and choose a coordinate system (in particular, indicate which direction is positive).

   (b) (5 points) Sketch (qualitatively) your expectation for $v(t)$, assuming an initial positive velocity v_0. Also sketch, with a dashed or dotted line, your expectation for $v(t)$ if there were no drag ($\alpha = 0$).

   (c) (5 points) Sketch (qualitatively) your expectation for $x(t)$. You can assume an initial position of $x(0) = 0$. Also sketch, with a dashed or dotted line, your expectation for $x(t)$ if there were no drag ($\alpha = 0$).

   (d) (5 points) What are the units for α? You have enough information to determine this exactly.

2. Calculate the velocity as a function of time.

   (a) (5 points) Write the differential equation for velocity.

   (b) (15 points) Solve the differential equation for $v(t)$ (be sure actually put your solution into the form $v(t) = \ldots$).

3. Check your solution for the velocity. If you did not find a solution in the previous part, use this function instead: $v(t) = v_0 \left(1 + \frac{\alpha v_0^2 t}{m}\right)^{-1/2}$.

   (a) (5 points) Check that your solution has units of velocity. Write out the units of every term for full credit. If your units are incorrect, note that. (This would also mean that your solution is incorrect. Keep going anyway. You will get credit for correct process even if your answers are incorrect.)

   (b) (5 points) Tidy up your solution. You may have found in your unit check a set of constants with units of time. Call that a ”typical time” τ and write your solution in terms of τ.

   (c) (5 points) Check that your solution gives $v(0) = v_0$. If your solution does not give this, note that.

   (d) (5 points) Does velocity reach zero in finite time?

4. Calculate position as a function of time.

   (a) (10 points) Write the differential equation for position $x(t)$, and show how you would go about solving it, but don't actually evaluate any integrals. (If you have extra time in the end, you can solve it all the way for extra credit, using the initial condition $x(0) = 0$.)

   The solution is

$$x(t) = 2v_0\tau\left(\sqrt{1 + t/\tau} - 1\right)$$

5. Check the solution for the position.

   (a) (5 points) Check that the solution has units of position.

   (b) (5 points) Check that the solution gives $x(0) = 0$.

   (c) (20 points) Check that your solution gives the expected behavior for short times. It's not enough to note that $x(0) = 0$, you want the behavior of $x(t)$ for $t \geq 0$, but $t \ll \tau$, which probably means expanding your solution (think Taylor series) to at least the linear (1st order) term is a good idea.