4. $\ddot{x} = \frac{g}{\mu_s}$

(a) This equation can be integrated directly, or one could see this as already separated variables:

$$\int \ddot{x}\, dt = \int \frac{g}{\mu_s}\, dt$$

$$\dot{x} = \frac{g}{\mu_s} t + C_0$$

again: $$\int \dot{x}\, dt = \int \left(\frac{g}{\mu_s} t + C_0\right) dt$$

$$\underline{\underline{x(t) = \frac{1}{2}\frac{g}{\mu_s} t^2 + C_0 t + C_1}}$$

(b) Calculate derivatives, plug into ODE

(c) Because the r.h.s doesn't depend on x (nor on t, though after the first integration it does, but that's still not a problem)

(d) 2 constants of integration, consistent with 2nd order ODE.

5. (a) Calculate derivatives, plug into ODE

(b) $\omega(t) = \frac{d}{dt}(A\cos\omega_0 t + B\sin\omega_0 t)$

$= -A\omega_0 \sin\omega_0 t + B\omega_0 \cos\omega_0 t$

(c) $\phi(0) = \phi_0$

$A \underbrace{\cos\omega_0 0}_{1} + B \cancel{\sin\omega_0 0} = \phi_0$

$\Rightarrow A = \phi_0$

$\omega(0) = 0$

$-A\omega_0 \cancel{\sin\omega_0 0} + B\omega_0 \cos\omega_0 0 = 0$

$\Rightarrow B = 0$

So $\omega(t) = \phi_0 \cos(\omega_0 t)$