PHYS 615 – Quiz 3: Momentum, Center of Mass, Angular Momentum

Name: _____

Instructions: You have 40 minutes to work on this quiz.
Write your answer on your own paper and, if possible, upload to Gradescope when you are done. If you get stuck on one part, still try the other parts. Some parts are independent; for dependent parts, I'll give you full credit if your process is correct, even in your input from another part is incorrect.
Possibly useful physics equations:

$$\vec{F}_{net} = m\vec{a} \qquad F_{fk} = \mu_k F_N \qquad F_{fs} \leq \mu_s F_N \qquad F_G = mg \qquad \vec{F}_{kind,A\,on\,B} = -\vec{F}_{kind,B\,on\,A}$$

$$\vec{F}_{D,quad} = -cv^2\hat{v} \qquad \vec{F}_{D,lin} = -bv\hat{v}$$

$$\vec{p} = m\vec{v} \qquad \dot{\vec{p}} = \vec{F} \qquad \vec{l} = I\vec{\omega} = \vec{r} \times \vec{p} \qquad \dot{\vec{l}} = \vec{\Gamma} = \vec{r} \times \vec{F}$$

$$\vec{R}_{CM} = \frac{1}{M}\int_M \vec{r} dm$$

Possibly useful math equations:

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} \qquad \text{(for any } n \neq 0\text{, including fractions.)}$$

1. (20 points) Under which condition(s) is total angular momentum of a system of particles conserved? If there are additional conditions on the internal forces, state them.

2. (30 points) A shell traveling with velocity $\vec{v}_0$ explodes into three pieces of equal masses. Just after the explosion, one piece has velocity $\vec{v}_1 = \vec{v}_0$ and the other two have velocities $\vec{v}_2$ and $\vec{v}_3$ that are equal in magnitude ($v_2 = v_3$) but mutually perpendicular. Find $\vec{v}_2$ and $\vec{v}_3$ and sketch the three velocities.

3. (30 points) A uniform thin sheet of metal is cut into the shape of a quartercircle of radius R and lies in the x-y plane with its center at the origin and in the first quadrant (bounded by the positive x and y axes). [In this case, the center of mass integral becomes a two-dimensional integral of the form $\int \vec{r}\sigma dA$, where σ denotes the surface mass density (mass / area) and dA is the element of area $dA = r\, dr\, d\phi$.]

   Remember to check whether your result makes sense.

4. (20 points) A particle moves under the influence of a central force directed toward a fixed origin O.

   (a) Explain why the particle's angular momentum about O is constant.

   (b) Argue that the particle's orbit must lie in a single plane containing O.