

Manipulating Eq. 9-23 by substituting for \vec{p} from Eq. 9-22 gives, for constant mass m ,

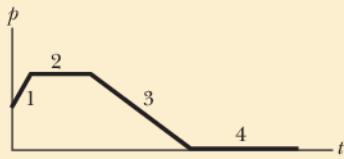
$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m \frac{d\vec{v}}{dt} = m\vec{a}.$$

Thus, the relations $\vec{F}_{\text{net}} = d\vec{p}/dt$ and $\vec{F}_{\text{net}} = m\vec{a}$ are equivalent expressions of Newton's second law of motion for a particle.



Checkpoint 3

The figure gives the magnitude p of the linear momentum versus time t for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the magnitude of the force, greatest first. (b) In which region is the particle slowing?



The Linear Momentum of a System of Particles

Let's extend the definition of linear momentum to a system of particles. Consider a system of n particles, each with its own mass, velocity, and linear momentum. The particles may interact with each other, and external forces may act on them. The system as a whole has a total linear momentum \vec{P} , which is defined to be the vector sum of the individual particles' linear momenta. Thus,

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n \\ &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.\end{aligned}\quad (9-24)$$

If we compare this equation with Eq. 9-17, we see that

$$\vec{P} = M\vec{v}_{\text{com}} \quad (\text{linear momentum, system of particles}), \quad (9-25)$$

which is another way to define the linear momentum of a system of particles:



The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

Force and Momentum. If we take the time derivative of Eq. 9-25 (the velocity can change but not the mass), we find

$$\frac{d\vec{P}}{dt} = M \frac{d\vec{v}_{\text{com}}}{dt} = M\vec{a}_{\text{com}}. \quad (9-26)$$

Comparing Eqs. 9-14 and 9-26 allows us to write Newton's second law for a system of particles in the equivalent form

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \quad (\text{system of particles}), \quad (9-27)$$

where \vec{F}_{net} is the net external force acting on the system. This equation is the generalization of the single-particle equation $\vec{F}_{\text{net}} = d\vec{p}/dt$ to a system of many particles. In words, the equation says that the net external force \vec{F}_{net} on a system of particles changes the linear momentum \vec{P} of the system. Conversely, the linear momentum can be changed only by a net external force. If there is no net external force, \vec{P} cannot change. Again, this fact gives us an extremely powerful tool for solving problems.

9-4 COLLISION AND IMPULSE

Learning Objectives

After reading this module, you should be able to...

- 9.19 Identify that impulse is a vector quantity and thus has both magnitude and direction and also components.
- 9.20 Apply the relationship between impulse and momentum change.
- 9.21 Apply the relationship between impulse, average force, and the time interval taken by the impulse.
- 9.22 Apply the constant-acceleration equations to relate impulse to average force.

Key Ideas

- Applying Newton's second law in momentum form to a particle-like body involved in a collision leads to the impulse–linear momentum theorem:

$$\vec{p}_f - \vec{p}_i = \Delta \vec{p} = \vec{J},$$

where $\vec{p}_f - \vec{p}_i = \Delta \vec{p}$ is the change in the body's linear momentum, and \vec{J} is the impulse due to the force $\vec{F}(t)$ exerted on the body by the other body in the collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t) dt.$$

- If F_{avg} is the average magnitude of $\vec{F}(t)$ during the collision and Δt is the duration of the collision, then for one-dimensional motion

$$J = F_{\text{avg}} \Delta t.$$

- 9.23 Given force as a function of time, calculate the impulse (and thus also the momentum change) by integrating the function.

- 9.24 Given a graph of force versus time, calculate the impulse (and thus also the momentum change) by graphical integration.

- 9.25 In a continuous series of collisions by projectiles, calculate the average force on the target by relating it to the rate at which mass collides and to the velocity change experienced by each projectile.

- When a steady stream of bodies, each with mass m and speed v , collides with a body whose position is fixed, the average force on the fixed body is

$$F_{\text{avg}} = -\frac{n}{\Delta t} \Delta p = -\frac{n}{\Delta t} m \Delta v,$$

where $n/\Delta t$ is the rate at which the bodies collide with the fixed body, and Δv is the change in velocity of each colliding body. This average force can also be written as

$$F_{\text{avg}} = -\frac{\Delta m}{\Delta t} \Delta v,$$

where $\Delta m/\Delta t$ is the rate at which mass collides with the fixed body. The change in velocity is $\Delta v = -v$ if the bodies stop upon impact and $\Delta v = -2v$ if they bounce directly backward with no change in their speed.

Collision and Impulse

The momentum \vec{p} of any particle-like body cannot change unless a net external force changes it. For example, we could push on the body to change its momentum. More dramatically, we could arrange for the body to collide with a baseball bat. In such a *collision* (or *crash*), the external force on the body is brief, has large magnitude, and suddenly changes the body's momentum. Collisions occur commonly in our world, but before we get to them, we need to consider a simple collision in which a moving particle-like body (*a projectile*) collides with some other body (*a target*).

Single Collision

Let the projectile be a ball and the target be a bat. The collision is brief, and the ball experiences a force that is great enough to slow, stop, or even reverse its motion. Figure 9-8 depicts the collision at one instant. The ball experiences a force $\vec{F}(t)$ that varies during the collision and changes the linear momentum \vec{p} of the ball. That change is related to the force by Newton's second law written in the form $\vec{F} = d\vec{p}/dt$. By rearranging this second-law expression, we see that, in time interval dt , the change in the ball's momentum is

$$d\vec{p} = \vec{F}(t) dt. \quad (9-28)$$

Photo by Harold E. Edgerton, © The Harold and Esther Edgerton Family Trust, courtesy of Palm Press, Inc.



The collision of a ball with a bat collapses part of the ball.

We can find the net change in the ball's momentum due to the collision if we integrate both sides of Eq. 9-28 from a time t_i just before the collision to a time t_f just after the collision:

$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt. \quad (9-29)$$

The left side of this equation gives us the change in momentum: $\vec{p}_f - \vec{p}_i = \Delta\vec{p}$. The right side, which is a measure of both the magnitude and the duration of the collision force, is called the **impulse** \vec{J} of the collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t) dt \quad (\text{impulse defined}). \quad (9-30)$$

Thus, the change in an object's momentum is equal to the impulse on the object:

$$\Delta\vec{p} = \vec{J} \quad (\text{linear momentum-impulse theorem}). \quad (9-31)$$

This expression can also be written in the vector form

$$\vec{p}_f - \vec{p}_i = \vec{J} \quad (9-32)$$

and in such component forms as

$$\Delta p_x = J_x \quad (9-33)$$

and $p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x dt. \quad (9-34)$

Integrating the Force. If we have a function for $\vec{F}(t)$, we can evaluate \vec{J} (and thus the change in momentum) by integrating the function. If we have a plot of \vec{F} versus time t , we can evaluate \vec{J} by finding the area between the curve and the t axis, such as in Fig. 9-9a. In many situations we do not know how the force varies with time but we do know the average magnitude F_{avg} of the force and the duration Δt ($= t_f - t_i$) of the collision. Then we can write the magnitude of the impulse as

$$J = F_{\text{avg}} \Delta t. \quad (9-35)$$

The average force is plotted versus time as in Fig. 9-9b. The area under that curve is equal to the area under the curve for the actual force $F(t)$ in Fig. 9-9a because both areas are equal to impulse magnitude J .

Instead of the ball, we could have focused on the bat in Fig. 9-8. At any instant, Newton's third law tells us that the force on the bat has the same magnitude but the opposite direction as the force on the ball. From Eq. 9-30, this means that the impulse on the bat has the same magnitude but the opposite direction as the impulse on the ball.



Checkpoint 4

A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

Series of Collisions

Now let's consider the force on a body when it undergoes a series of identical, repeated collisions. For example, as a prank, we might adjust one of those machines that fire tennis balls to fire them at a rapid rate directly at a wall. Each collision would produce a force on the wall, but that is not the force we are seeking. We

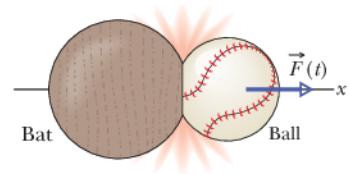
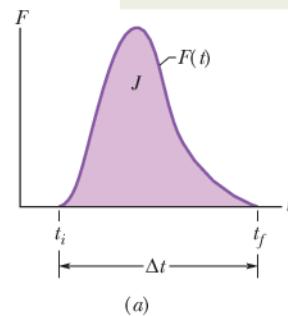


Figure 9-8 Force $\vec{F}(t)$ acts on a ball as the ball and a bat collide.

The impulse in the collision is equal to the area under the curve.



The average force gives the same area under the curve.

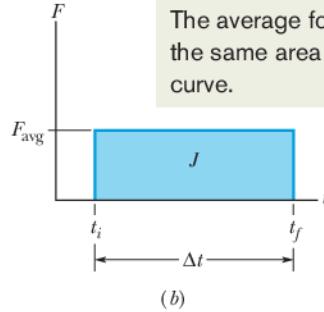


Figure 9-9 (a) The curve shows the magnitude of the time-varying force $F(t)$ that acts on the ball in the collision of Fig. 9-8. The area under the curve is equal to the magnitude of the impulse \vec{J} on the ball in the collision. (b) The height of the rectangle represents the average force F_{avg} acting on the ball over the time interval Δt . The area within the rectangle is equal to the area under the curve in (a) and thus is also equal to the magnitude of the impulse \vec{J} in the collision.

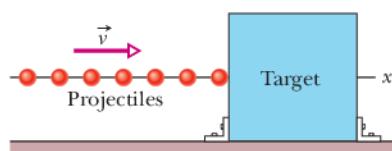


Figure 9-10 A steady stream of projectiles, with identical linear momenta, collides with a target, which is fixed in place. The average force F_{avg} on the wall during the bombardment—that is, the average force during a large number of collisions.

want the average force F_{avg} on the wall during the bombardment—that is, the average force during a large number of collisions.

In Fig. 9-10, a steady stream of projectile bodies, with identical mass m and linear momenta $m\vec{v}$, moves along an x axis and collides with a target body that is fixed in place. Let n be the number of projectiles that collide in a time interval Δt . Because the motion is along only the x axis, we can use the components of the momenta along that axis. Thus, each projectile has initial momentum mv and undergoes a change Δp in linear momentum because of the collision. The total change in linear momentum for n projectiles during interval Δt is $n \Delta p$. The resulting impulse \vec{J} on the target during Δt is along the x axis and has the same magnitude of $n \Delta p$ but is in the opposite direction. We can write this relation in component form as

$$J = -n \Delta p, \quad (9-36)$$

where the minus sign indicates that J and Δp have opposite directions.

Average Force. By rearranging Eq. 9-35 and substituting Eq. 9-36, we find the average force F_{avg} acting on the target during the collisions:

$$F_{\text{avg}} = \frac{J}{\Delta t} = -\frac{n}{\Delta t} \Delta p = -\frac{n}{\Delta t} m \Delta v. \quad (9-37)$$

This equation gives us F_{avg} in terms of $n/\Delta t$, the rate at which the projectiles collide with the target, and Δv , the change in the velocity of those projectiles.

Velocity Change. If the projectiles stop upon impact, then in Eq. 9-37 we can substitute, for Δv ,

$$\Delta v = v_f - v_i = 0 - v = -v, \quad (9-38)$$

where v_i ($= v$) and v_f ($= 0$) are the velocities before and after the collision, respectively. If, instead, the projectiles bounce (rebound) directly backward from the target with no change in speed, then $v_f = -v$ and we can substitute

$$\Delta v = v_f - v_i = -v - v = -2v. \quad (9-39)$$

In time interval Δt , an amount of mass $\Delta m = nm$ collides with the target. With this result, we can rewrite Eq. 9-37 as

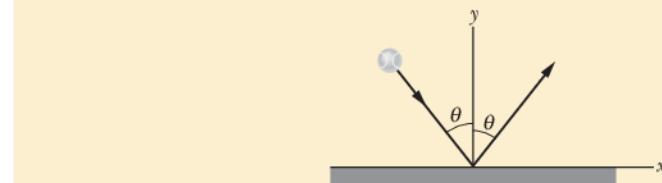
$$F_{\text{avg}} = -\frac{\Delta m}{\Delta t} \Delta v. \quad (9-40)$$

This equation gives the average force F_{avg} in terms of $\Delta m/\Delta t$, the rate at which mass collides with the target. Here again we can substitute for Δv from Eq. 9-38 or 9-39 depending on what the projectiles do.



Checkpoint 5

The figure shows an overhead view of a ball bouncing from a vertical wall without any change in its speed. Consider the change $\Delta \vec{p}$ in the ball's linear momentum. (a) Is Δp_x positive, negative, or zero? (b) Is Δp_y positive, negative, or zero? (c) What is the direction of $\Delta \vec{p}$?





Sample Problem 9.04 Two-dimensional impulse, race car-wall collision

Race car-wall collision. Figure 9-11a is an overhead view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed $v_i = 70 \text{ m/s}$ along a straight line at 30° from the wall. Just after the collision, he is traveling at speed $v_f = 50 \text{ m/s}$ along a straight line at 10° from the wall. His mass m is 80 kg .

(a) What is the impulse \vec{J} on the driver due to the collision?

KEY IDEAS

We can treat the driver as a particle-like body and thus apply the physics of this module. However, we cannot calculate \vec{J} directly from Eq. 9-30 because we do not know anything about the force $\vec{F}(t)$ on the driver during the collision. That is, we do not have a function of $\vec{F}(t)$ or a plot for it and thus cannot integrate to find \vec{J} . However, we can find \vec{J} from the change in the driver's linear momentum \vec{p} via Eq. 9-32 ($\vec{J} = \vec{p}_f - \vec{p}_i$).

Calculations: Figure 9-11b shows the driver's momentum \vec{p}_i before the collision (at angle 30° from the positive x direction) and his momentum \vec{p}_f after the collision (at angle -10°). From Eqs. 9-32 and 9-22 ($\vec{p} = m\vec{v}$), we can write

$$\vec{J} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i). \quad (9-41)$$

We could evaluate the right side of this equation directly on a vector-capable calculator because we know m is 80 kg , \vec{v}_f is 50 m/s at -10° , and \vec{v}_i is 70 m/s at 30° . Instead, here we evaluate Eq. 9-41 in component form.

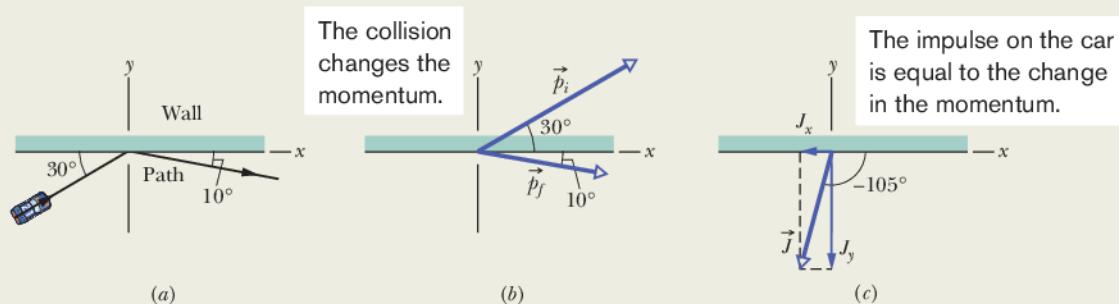
x component: Along the x axis we have

$$\begin{aligned} J_x &= m(v_{fx} - v_{ix}) \\ &= (80 \text{ kg})[(50 \text{ m/s}) \cos(-10^\circ) - (70 \text{ m/s}) \cos 30^\circ] \\ &= -910 \text{ kg} \cdot \text{m/s}. \end{aligned}$$

y component: Along the y axis,

$$\begin{aligned} J_y &= m(v_{fy} - v_{iy}) \\ &= (80 \text{ kg})[(50 \text{ m/s}) \sin(-10^\circ) - (70 \text{ m/s}) \sin 30^\circ] \\ &= -3495 \text{ kg} \cdot \text{m/s} \approx -3500 \text{ kg} \cdot \text{m/s}. \end{aligned}$$

Figure 9-11 (a) Overhead view of the path taken by a race car and its driver as the car slams into the racetrack wall. (b) The initial momentum \vec{p}_i and final momentum \vec{p}_f of the driver. (c) The impulse \vec{J} on the driver during the collision.



Impulse: The impulse is then

$$\vec{J} = (-910\hat{i} - 3500\hat{j}) \text{ kg} \cdot \text{m/s}, \quad (\text{Answer})$$

which means the impulse magnitude is

$$J = \sqrt{J_x^2 + J_y^2} = 3616 \text{ kg} \cdot \text{m/s} \approx 3600 \text{ kg} \cdot \text{m/s}.$$

The angle of \vec{J} is given by

$$\theta = \tan^{-1} \frac{J_y}{J_x}, \quad (\text{Answer})$$

which a calculator evaluates as 75.4° . Recall that the physically correct result of an inverse tangent might be the displayed answer plus 180° . We can tell which is correct here by drawing the components of \vec{J} (Fig. 9-11c). We find that θ is actually $75.4^\circ + 180^\circ = 255.4^\circ$, which we can write as

$$\theta = -105^\circ. \quad (\text{Answer})$$

(b) The collision lasts for 14 ms . What is the magnitude of the average force on the driver during the collision?

KEY IDEA

From Eq. 9-35 ($J = F_{\text{avg}} \Delta t$), the magnitude F_{avg} of the average force is the ratio of the impulse magnitude J to the duration Δt of the collision.

Calculations: We have

$$\begin{aligned} F_{\text{avg}} &= \frac{J}{\Delta t} = \frac{3616 \text{ kg} \cdot \text{m/s}}{0.014 \text{ s}} \\ &= 2.583 \times 10^5 \text{ N} \approx 2.6 \times 10^5 \text{ N}. \quad (\text{Answer}) \end{aligned}$$

Using $F = ma$ with $m = 80 \text{ kg}$, you can show that the magnitude of the driver's average acceleration during the collision is about $3.22 \times 10^3 \text{ m/s}^2 = 329g$, which is fatal.

Surviving: Mechanical engineers attempt to reduce the chances of a fatality by designing and building racetrack walls with more "give," so that a collision lasts longer. For example, if the collision here lasted 10 times longer and the other data remained the same, the magnitudes of the average force and average acceleration would be 10 times less and probably survivable.



9-5 CONSERVATION OF LINEAR MOMENTUM

Learning Objectives

After reading this module, you should be able to . . .

9.26 For an isolated system of particles, apply the conservation of linear momenta to relate the initial momenta of the particles to their momenta at a later instant.

9.27 Identify that the conservation of linear momentum can be done along an individual axis by using components along that axis, *provided* that there is no net external force component along that axis.

Key Ideas

- If a system is closed and isolated so that no net *external* force acts on it, then the linear momentum \vec{P} must be constant even if there are internal changes:

$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}).$$

- This conservation of linear momentum can also be written in terms of the system's initial momentum and its momentum at some later instant:

$$\vec{P}_i = \vec{P}_f \quad (\text{closed, isolated system}),$$

Conservation of Linear Momentum

Suppose that the net external force \vec{F}_{net} (and thus the net impulse \vec{J}) acting on a system of particles is zero (the system is isolated) and that no particles leave or enter the system (the system is closed). Putting $\vec{F}_{\text{net}} = 0$ in Eq. 9-27 then yields $d\vec{P}/dt = 0$, which means that

$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}). \quad (9-42)$$

In words,



If no net external force acts on a system of particles, the total linear momentum \vec{P} of the system cannot change.

This result is called the **law of conservation of linear momentum** and is an extremely powerful tool in solving problems. In the homework we usually write the law as

$$\vec{P}_i = \vec{P}_f \quad (\text{closed, isolated system}). \quad (9-43)$$

In words, this equation says that, for a closed, isolated system,

$$\left(\begin{array}{c} \text{total linear momentum} \\ \text{at some initial time } t_i \end{array} \right) = \left(\begin{array}{c} \text{total linear momentum} \\ \text{at some later time } t_f \end{array} \right).$$

Caution: Momentum should not be confused with energy. In the sample problems of this module, momentum is conserved but energy is definitely not.

Equations 9-42 and 9-43 are vector equations and, as such, each is equivalent to three equations corresponding to the conservation of linear momentum in three mutually perpendicular directions as in, say, an *xyz* coordinate system. Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. However,



If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

In a homework problem, how can you know if linear momentum can be conserved along, say, an *x* axis? Check the force components along that axis. If the net of any such components is zero, then the conservation applies. As an example, suppose that you toss a grapefruit across a room. During its flight, the only external force acting on the grapefruit (which we take as the system) is the gravitational force \vec{F}_g , which is directed vertically downward. Thus, the vertical component of the linear

momentum of the grapefruit changes, but since no horizontal external force acts on the grapefruit, the horizontal component of the linear momentum cannot change.

Note that we focus on the external forces acting on a closed system. Although internal forces can change the linear momentum of portions of the system, they cannot change the total linear momentum of the entire system. For example, there are plenty of forces acting between the organs of your body, but they do not propel you across the room (thankfully).

The sample problems in this module involve explosions that are either one-dimensional (meaning that the motions before and after the explosion are along a single axis) or two-dimensional (meaning that they are in a plane containing two axes). In the following modules we consider collisions.



Checkpoint 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor, one of them in the positive x direction. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the x axis? (c) What is the direction of the momentum of the second piece?

Sample Problem 9.05 One-dimensional explosion, relative velocity, space hauler

One-dimensional explosion: Figure 9-12a shows a space hauler and cargo module, of total mass M , traveling along an x axis in deep space. They have an initial velocity \vec{v}_i of magnitude 2100 km/h relative to the Sun. With a small explosion, the hauler ejects the cargo module, of mass $0.20M$ (Fig. 9-12b). The hauler then travels 500 km/h faster than the module along the x axis; that is, the relative speed v_{rel} between the hauler and the module is 500 km/h. What then is the velocity \vec{v}_{HS} of the hauler relative to the Sun?

KEY IDEA

Because the hauler–module system is closed and isolated, its total linear momentum is conserved; that is,

$$\vec{P}_i = \vec{P}_f, \quad (9-44)$$

where the subscripts i and f refer to values before and after the ejection, respectively. (We need to be careful here: Although the momentum of the *system* does not change, the momenta of the hauler and module certainly do.)

Calculations: Because the motion is along a single axis, we can write momenta and velocities in terms of their x components, using a sign to indicate direction. Before the ejection, we have

$$P_i = Mv_i. \quad (9-45)$$

Let v_{MS} be the velocity of the ejected module relative to the Sun. The total linear momentum of the system after the ejection is then

$$P_f = (0.20M)v_{MS} + (0.80M)v_{HS}, \quad (9-46)$$

where the first term on the right is the linear momentum of the module and the second term is that of the hauler.

The explosive separation can change the momentum of the parts but not the momentum of the system.

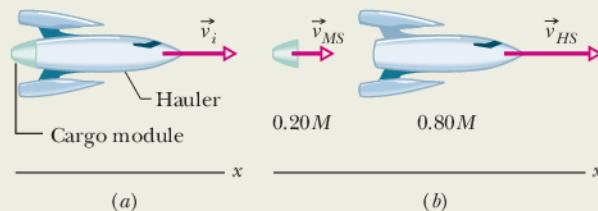


Figure 9-12 (a) A space hauler, with a cargo module, moving at initial velocity \vec{v}_i . (b) The hauler has ejected the cargo module. Now the velocities relative to the Sun are \vec{v}_{MS} for the module and \vec{v}_{HS} for the hauler.

We can relate the v_{MS} to the known velocities with

$$\begin{pmatrix} \text{velocity of} \\ \text{hauler relative} \\ \text{to Sun} \end{pmatrix} = \begin{pmatrix} \text{velocity of} \\ \text{hauler relative} \\ \text{to module} \end{pmatrix} + \begin{pmatrix} \text{velocity of} \\ \text{module relative} \\ \text{to Sun} \end{pmatrix}.$$

In symbols, this gives us

$$v_{HS} = v_{\text{rel}} + v_{MS} \quad (9-47)$$

or

$$v_{MS} = v_{HS} - v_{\text{rel}}.$$

Substituting this expression for v_{MS} into Eq. 9-46, and then substituting Eqs. 9-45 and 9-46 into Eq. 9-44, we find

$$Mv_i = 0.20M(v_{HS} - v_{\text{rel}}) + 0.80Mv_{HS},$$

which gives us

$$v_{HS} = v_i + 0.20v_{\text{rel}},$$

or

$$\begin{aligned} v_{HS} &= 2100 \text{ km/h} + (0.20)(500 \text{ km/h}) \\ &= 2200 \text{ km/h}. \end{aligned}$$

(Answer)



Additional examples, video, and practice available at WileyPLUS



Sample Problem 9.06 Two-dimensional explosion, momentum, coconut

Two-dimensional explosion: A firecracker placed inside a coconut of mass M , initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown in Fig. 9-13a. Piece C , with mass $0.30M$, has final speed $v_{fC} = 5.0 \text{ m/s}$.

(a) What is the speed of piece B , with mass $0.20M$?

KEY IDEA

First we need to see whether linear momentum is conserved. We note that (1) the coconut and its pieces form a closed system, (2) the explosion forces are internal to that system, and (3) no net external force acts on the system. Therefore, the linear momentum of the system is conserved. (We need to be careful here: Although the momentum of the system does not change, the momenta of the pieces certainly do.)

Calculations: To get started, we superimpose an xy coordinate system as shown in Fig. 9-13b, with the negative direction of the x axis coinciding with the direction of \vec{v}_{fA} . The x axis is at 80° with the direction of \vec{v}_{fC} and 50° with the direction of \vec{v}_{fB} .

Linear momentum is conserved separately along each axis. Let's use the y axis and write

$$P_{iy} = P_{fy}, \quad (9-48)$$

where subscript i refers to the initial value (before the explosion), and subscript y refers to the y component of \vec{P}_i or \vec{P}_f .

The component P_{iy} of the initial linear momentum is zero, because the coconut is initially at rest. To get an expression for P_{fy} , we find the y component of the final linear momentum of each piece, using the y -component version of Eq. 9-22 ($p_y = mv_y$):

$$\begin{aligned} p_{fA,y} &= 0, \\ p_{fB,y} &= -0.20Mv_{fB,y} = -0.20Mv_{fB} \sin 50^\circ, \\ p_{fC,y} &= 0.30Mv_{fC,y} = 0.30Mv_{fC} \sin 80^\circ. \end{aligned}$$

(Note that $p_{fA,y} = 0$ because of our nice choice of axes.) Equation 9-48 can now be written as

$$P_{iy} = P_{fy} = p_{fA,y} + p_{fB,y} + p_{fC,y}.$$

Then, with $v_{fC} = 5.0 \text{ m/s}$, we have

$$0 = 0 - 0.20Mv_{fB} \sin 50^\circ + (0.30M)(5.0 \text{ m/s}) \sin 80^\circ,$$

from which we find

$$v_{fB} = 9.64 \text{ m/s} \approx 9.6 \text{ m/s}. \quad (\text{Answer})$$

(b) What is the speed of piece A ?

Calculations: Linear momentum is also conserved along the x axis because there is no net external force acting on the coconut and pieces along that axis. Thus we have

$$P_{ix} = P_{fx}, \quad (9-49)$$

where $P_{ix} = 0$ because the coconut is initially at rest. To get P_{fx} , we find the x components of the final momenta, using the fact that piece A must have a mass of $0.50M$ ($= M - 0.20M - 0.30M$):

$$\begin{aligned} p_{fA,x} &= -0.50Mv_{fA}, \\ p_{fB,x} &= 0.20Mv_{fB,x} = 0.20Mv_{fB} \cos 50^\circ, \\ p_{fC,x} &= 0.30Mv_{fC,x} = 0.30Mv_{fC} \cos 80^\circ. \end{aligned}$$

Equation 9-49 for the conservation of momentum along the x axis can now be written as

$$P_{ix} = P_{fx} = p_{fA,x} + p_{fB,x} + p_{fC,x}.$$

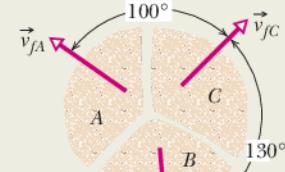
Then, with $v_{fC} = 5.0 \text{ m/s}$ and $v_{fB} = 9.64 \text{ m/s}$, we have

$$\begin{aligned} 0 &= -0.50Mv_{fA} + 0.20M(9.64 \text{ m/s}) \cos 50^\circ \\ &\quad + 0.30M(5.0 \text{ m/s}) \cos 80^\circ, \end{aligned}$$

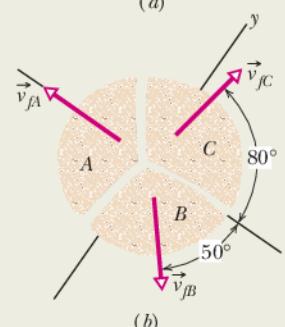
from which we find

$$v_{fA} = 3.0 \text{ m/s}. \quad (\text{Answer})$$

The explosive separation can change the momentum of the parts but not the momentum of the system.



(a)



(b)

Figure 9-13 Three pieces of an exploded coconut move off in three directions along a frictionless floor. (a) An overhead view of the event. (b) The same with a two-dimensional axis system imposed.



Additional examples, video, and practice available at WileyPLUS

9-6 MOMENTUM AND KINETIC ENERGY IN COLLISIONS

Learning Objectives

After reading this module, you should be able to . . .

- 9.28** Distinguish between elastic collisions, inelastic collisions, and completely inelastic collisions.
- 9.29** Identify a one-dimensional collision as one where the objects move along a single axis, both before and after the collision.

9.30 Apply the conservation of momentum for an isolated one-dimensional collision to relate the initial momenta of the objects to their momenta after the collision.

9.31 Identify that in an isolated system, the momentum and velocity of the center of mass are not changed even if the objects collide.

Key Ideas

- In an inelastic collision of two bodies, the kinetic energy of the two-body system is not conserved. If the system is closed and isolated, the total linear momentum of the system *must* be conserved, which we can write in vector form as

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f},$$

where subscripts *i* and *f* refer to values just before and just after the collision, respectively.

- If the motion of the bodies is along a single axis, the collision is one-dimensional and we can write the equation in terms of

velocity components along that axis:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

- If the bodies stick together, the collision is a completely inelastic collision and the bodies have the same final velocity *V* (because they are stuck together).
- The center of mass of a closed, isolated system of two colliding bodies is not affected by a collision. In particular, the velocity \vec{v}_{com} of the center of mass cannot be changed by the collision.

Momentum and Kinetic Energy in Collisions

In Module 9-4, we considered the collision of two particle-like bodies but focused on only one of the bodies at a time. For the next several modules we switch our focus to the system itself, with the assumption that the system is closed and isolated. In Module 9-5, we discussed a rule about such a system: The total linear momentum \vec{P} of the system cannot change because there is no net external force to change it. This is a very powerful rule because it can allow us to determine the results of a collision *without* knowing the details of the collision (such as how much damage is done).

We shall also be interested in the total kinetic energy of a system of two colliding bodies. If that total happens to be unchanged by the collision, then the kinetic energy of the system is *conserved* (it is the same before and after the collision). Such a collision is called an **elastic collision**. In everyday collisions of common bodies, such as two cars or a ball and a bat, some energy is always transferred from kinetic energy to other forms of energy, such as thermal energy or energy of sound. Thus, the kinetic energy of the system is *not* conserved. Such a collision is called an **inelastic collision**.

However, in some situations, we can *approximate* a collision of common bodies as elastic. Suppose that you drop a Superball onto a hard floor. If the collision between the ball and floor (or Earth) were elastic, the ball would lose no kinetic energy because of the collision and would rebound to its original height. However, the actual rebound height is somewhat short, showing that at least some kinetic energy is lost in the collision and thus that the collision is somewhat inelastic. Still, we might choose to neglect that small loss of kinetic energy to approximate the collision as elastic.

The inelastic collision of two bodies always involves a loss in the kinetic energy of the system. The greatest loss occurs if the bodies stick together, in which case the collision is called a **completely inelastic collision**. The collision of a baseball and a bat is inelastic. However, the collision of a wet putty ball and a bat is completely inelastic because the putty sticks to the bat.

Here is the generic setup for an inelastic collision.

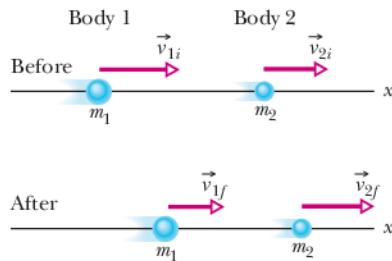


Figure 9-14 Bodies 1 and 2 move along an x axis, before and after they have an inelastic collision.

Inelastic Collisions in One Dimension

One-Dimensional Inelastic Collision

Figure 9-14 shows two bodies just before and just after they have a one-dimensional collision. The velocities before the collision (subscript i) and after the collision (subscript f) are indicated. The two bodies form our system, which is closed and isolated. We can write the law of conservation of linear momentum for this two-body system as

$$\left(\begin{array}{l} \text{total momentum } \vec{P}_i \\ \text{before the collision} \end{array} \right) = \left(\begin{array}{l} \text{total momentum } \vec{P}_f \\ \text{after the collision} \end{array} \right),$$

which we can symbolize as

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \quad (\text{conservation of linear momentum}). \quad (9-50)$$

Because the motion is one-dimensional, we can drop the overhead arrows for vectors and use only components along the axis, indicating direction with a sign. Thus, from $p = mv$, we can rewrite Eq. 9-50 as

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad (9-51)$$

If we know values for, say, the masses, the initial velocities, and one of the final velocities, we can find the other final velocity with Eq. 9-51.

One-Dimensional Completely Inelastic Collision

Figure 9-15 shows two bodies before and after they have a completely inelastic collision (meaning they stick together). The body with mass m_2 happens to be initially at rest ($v_{2i} = 0$). We can refer to that body as the *target* and to the incoming body as the *projectile*. After the collision, the stuck-together bodies move with velocity V . For this situation, we can rewrite Eq. 9-51 as

$$m_1 v_{1i} = (m_1 + m_2) V \quad (9-52)$$

$$\text{or} \quad V = \frac{m_1}{m_1 + m_2} v_{1i}. \quad (9-53)$$

If we know values for, say, the masses and the initial velocity v_{1i} of the projectile, we can find the final velocity V with Eq. 9-53. Note that V must be less than v_{1i} because the mass ratio $m_1/(m_1 + m_2)$ must be less than unity.

Velocity of the Center of Mass

In a closed, isolated system, the velocity \vec{v}_{com} of the center of mass of the system cannot be changed by a collision because, with the system isolated, there is no net external force to change it. To get an expression for \vec{v}_{com} , let us return to the

In a completely inelastic collision, the bodies stick together.

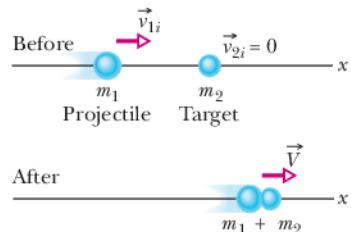


Figure 9-15 A completely inelastic collision between two bodies. Before the collision, the body with mass m_2 is at rest and the body with mass m_1 moves directly toward it. After the collision, the stuck-together bodies move with the same velocity \vec{V} .

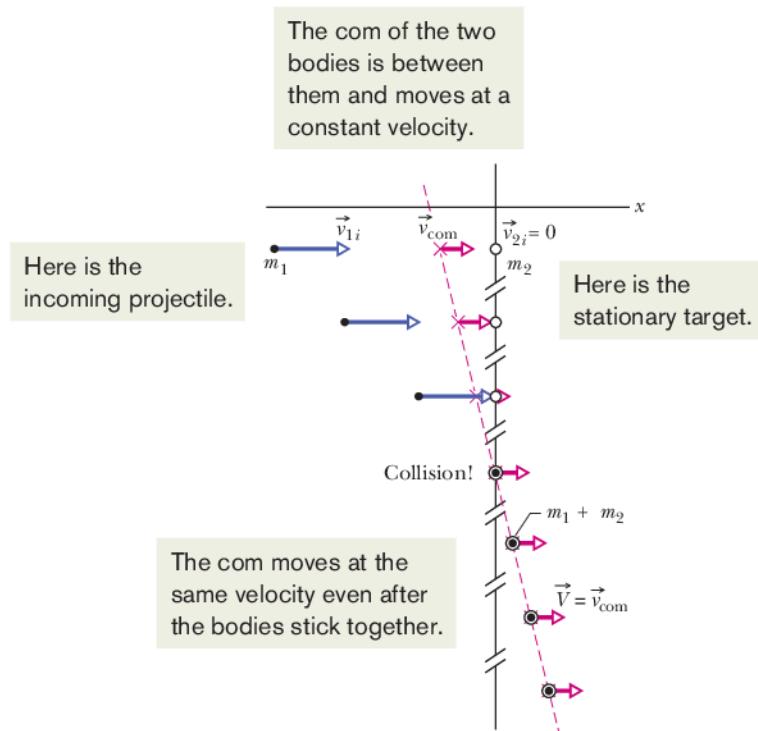


Figure 9-16 Some freeze-frames of the two-body system in Fig. 9-15, which undergoes a completely inelastic collision. The system's center of mass is shown in each freeze-frame. The velocity \vec{v}_{com} of the center of mass is unaffected by the collision. Because the bodies stick together after the collision, their common velocity \vec{V} must be equal to \vec{v}_{com} .

two-body system and one-dimensional collision of Fig. 9-14. From Eq. 9-25 ($\vec{P} = M\vec{v}_{\text{com}}$), we can relate \vec{v}_{com} to the total linear momentum \vec{P} of that two-body system by writing

$$\vec{P} = M\vec{v}_{\text{com}} = (m_1 + m_2)\vec{v}_{\text{com}}. \quad (9-54)$$

The total linear momentum \vec{P} is conserved during the collision; so it is given by either side of Eq. 9-50. Let us use the left side to write

$$\vec{P} = \vec{p}_{1i} + \vec{p}_{2i}. \quad (9-55)$$

Substituting this expression for \vec{P} in Eq. 9-54 and solving for \vec{v}_{com} give us

$$\vec{v}_{\text{com}} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}. \quad (9-56)$$

The right side of this equation is a constant, and \vec{v}_{com} has that same constant value before and after the collision.

For example, Fig. 9-16 shows, in a series of freeze-frames, the motion of the center of mass for the completely inelastic collision of Fig. 9-15. Body 2 is the target, and its initial linear momentum in Eq. 9-56 is $\vec{p}_{2i} = m_2\vec{v}_{2i} = 0$. Body 1 is the projectile, and its initial linear momentum in Eq. 9-56 is $\vec{p}_{1i} = m_1\vec{v}_{1i}$. Note that as the series of freeze-frames progresses to and then beyond the collision, the center of mass moves at a constant velocity to the right. After the collision, the common final speed V of the bodies is equal to \vec{v}_{com} because then the center of mass travels with the stuck-together bodies.



Checkpoint 7

- Body 1 and body 2 are in a completely inelastic one-dimensional collision. What is their final momentum if their initial momenta are, respectively, (a) 10 kg·m/s and 0; (b) 10 kg·m/s and 4 kg·m/s; (c) 10 kg·m/s and -4 kg·m/s?



Sample Problem 9.07 Conservation of momentum, ballistic pendulum

Here is an example of a common technique in physics. We have a demonstration that cannot be worked out as a whole (we don't have a workable equation for it). So, we break it up into steps that can be worked separately (we have equations for them).

The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown in Fig. 9-17 consists of a large block of wood of mass $M = 5.4 \text{ kg}$, hanging from two long cords. A bullet of mass $m = 9.5 \text{ g}$ is fired into the block, coming quickly to rest. The *block + bullet* then swing upward, their center of mass rising a vertical distance $h = 6.3 \text{ cm}$ before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?

KEY IDEAS

We can see that the bullet's speed v must determine the rise height h . However, we cannot use the conservation of mechanical energy to relate these two quantities because surely energy is transferred from mechanical energy to other forms (such as thermal energy and energy to break apart the wood) as the bullet penetrates the block. Nevertheless, we can split this complicated motion into two steps that we can separately analyze: (1) the bullet-block collision and (2) the bullet-block rise, during which mechanical energy *is* conserved.

Reasoning step 1: Because the collision within the bullet-block system is so brief, we can make two important assumptions: (1) During the collision, the gravitational force on the block and the force on the block from the cords are still balanced. Thus, during the collision, the net external impulse on the bullet-block system is zero. Therefore, the system is isolated and its total linear momentum is conserved:

$$\left(\begin{array}{l} \text{total momentum} \\ \text{before the collision} \end{array} \right) = \left(\begin{array}{l} \text{total momentum} \\ \text{after the collision} \end{array} \right). \quad (9-57)$$

(2) The collision is one-dimensional in the sense that the direction of the bullet and block *just after the collision* is in the bullet's original direction of motion.

Because the collision is one-dimensional, the block is initially at rest, and the bullet sticks in the block, we use Eq. 9-53 to express the conservation of linear momentum. Replacing the symbols there with the corresponding symbols here, we have

$$V = \frac{m}{m+M} v. \quad (9-58)$$

Reasoning step 2: As the bullet and block now swing up together, the mechanical energy of the bullet-block-Earth

system is conserved:

$$\left(\begin{array}{l} \text{mechanical energy} \\ \text{at bottom} \end{array} \right) = \left(\begin{array}{l} \text{mechanical energy} \\ \text{at top} \end{array} \right). \quad (9-59)$$

(This mechanical energy is not changed by the force of the cords on the block, because that force is always directed perpendicular to the block's direction of travel.) Let's take the block's initial level as our reference level of zero gravitational potential energy. Then conservation of mechanical energy means that the system's kinetic energy at the start of the swing must equal its gravitational potential energy at the highest point of the swing. Because the speed of the bullet and block at the start of the swing is the speed V immediately after the collision, we may write this conservation as

$$\frac{1}{2}(m+M)V^2 = (m+M)gh. \quad (9-60)$$

Combining steps: Substituting for V from Eq. 9-58 leads to

$$\begin{aligned} v &= \frac{m+M}{m} \sqrt{2gh} \\ &= \left(\frac{0.0095 \text{ kg} + 5.4 \text{ kg}}{0.0095 \text{ kg}} \right) \sqrt{(2)(9.8 \text{ m/s}^2)(0.063 \text{ m})} \\ &= 630 \text{ m/s}. \end{aligned} \quad (\text{Answer}) \quad (9-61)$$

The ballistic pendulum is a kind of "transformer," exchanging the high speed of a light object (the bullet) for the low—and thus more easily measurable—speed of a massive object (the block).

There are two events here. The bullet collides with the block. Then the bullet-block system swings upward by height h .

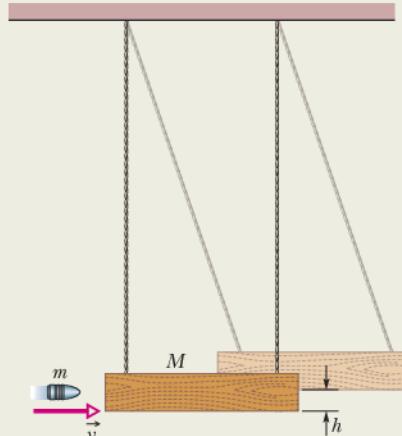


Figure 9-17 A ballistic pendulum, used to measure the speeds of bullets.



Additional examples, video, and practice available at WileyPLUS

9-7 ELASTIC COLLISIONS IN ONE DIMENSION

Learning Objectives

After reading this module, you should be able to . . .

9.32 For isolated elastic collisions in one dimension, apply the conservation laws for both the total energy and the net momentum of the colliding bodies to relate the initial values to the values after the collision.

Key Idea

- An elastic collision is a special type of collision in which the kinetic energy of a system of colliding bodies is conserved. If the system is closed and isolated, its linear momentum is also conserved. For a one-dimensional collision in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum

9.33 For a projectile hitting a stationary target, identify the resulting motion for the three general cases: equal masses, target more massive than projectile, projectile more massive than target.

yield the following expressions for the velocities immediately after the collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Elastic Collisions in One Dimension

As we discussed in Module 9-6, everyday collisions are inelastic but we can approximate some of them as being elastic; that is, we can approximate that the total kinetic energy of the colliding bodies is conserved and is not transferred to other forms of energy:

$$\left(\begin{array}{l} \text{total kinetic energy} \\ \text{before the collision} \end{array} \right) = \left(\begin{array}{l} \text{total kinetic energy} \\ \text{after the collision} \end{array} \right). \quad (9-62)$$

This means:



In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

For example, the collision of a cue ball with an object ball in a game of pool can be approximated as being an elastic collision. If the collision is head-on (the cue ball heads directly toward the object ball), the kinetic energy of the cue ball can be transferred almost entirely to the object ball. (Still, the collision transfers some of the energy to the sound you hear.)

Stationary Target

Figure 9-18 shows two bodies before and after they have a one-dimensional collision, like a head-on collision between pool balls. A projectile body of mass m_1 and initial velocity v_{1i} moves toward a target body of mass m_2 that is initially at rest ($v_{2i} = 0$). Let's assume that this two-body system is closed and isolated. Then the net linear momentum of the system is conserved, and from Eq. 9-51 we can write that conservation as

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad (\text{linear momentum}). \quad (9-63)$$

If the collision is also elastic, then the total kinetic energy is conserved and we can write that conservation as

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (\text{kinetic energy}). \quad (9-64)$$

In each of these equations, the subscript i identifies the initial velocities and the subscript f the final velocities of the bodies. If we know the masses of the bodies and if we also know v_{1i} , the initial velocity of body 1, the only unknown quantities are v_{1f} and v_{2f} , the final velocities of the two bodies. With two equations at our disposal, we should be able to find these two unknowns.

Here is the generic setup for an elastic collision with a stationary target.

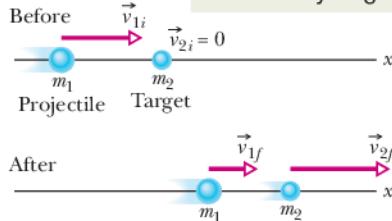


Figure 9-18 Body 1 moves along an x axis before having an elastic collision with body 2, which is initially at rest. Both bodies move along that axis after the collision.

To do so, we rewrite Eq. 9-63 as

$$m_1(v_{1i} - v_{1f}) = m_2v_{2f} \quad (9-65)$$

and Eq. 9-64 as*

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2v_{2f}^2. \quad (9-66)$$

After dividing Eq. 9-66 by Eq. 9-65 and doing some more algebra, we obtain

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad (9-67)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}. \quad (9-68)$$

Note that v_{2f} is always positive (the initially stationary target body with mass m_2 always moves forward). From Eq. 9-67 we see that v_{1f} may be of either sign (the projectile body with mass m_1 moves forward if $m_1 > m_2$ but rebounds if $m_1 < m_2$).

Let us look at a few special situations.

1. Equal masses If $m_1 = m_2$, Eqs. 9-67 and 9-68 reduce to

$$v_{1f} = 0 \quad \text{and} \quad v_{2f} = v_{1i},$$

which we might call a pool player's result. It predicts that after a head-on collision of bodies with equal masses, body 1 (initially moving) stops dead in its tracks and body 2 (initially at rest) takes off with the initial speed of body 1. In head-on collisions, bodies of equal mass simply exchange velocities. This is true even if body 2 is not initially at rest.

2. A massive target In Fig. 9-18, a massive target means that $m_2 \gg m_1$. For example, we might fire a golf ball at a stationary cannonball. Equations 9-67 and 9-68 then reduce to

$$v_{1f} \approx -v_{1i} \quad \text{and} \quad v_{2f} \approx \left(\frac{2m_1}{m_2}\right)v_{1i}. \quad (9-69)$$

This tells us that body 1 (the golf ball) simply bounces back along its incoming path, its speed essentially unchanged. Initially stationary body 2 (the cannonball) moves forward at a low speed, because the quantity in parentheses in Eq. 9-69 is much less than unity. All this is what we should expect.

3. A massive projectile This is the opposite case; that is, $m_1 \gg m_2$. This time, we fire a cannonball at a stationary golf ball. Equations 9-67 and 9-68 reduce to

$$v_{1f} \approx v_{1i} \quad \text{and} \quad v_{2f} \approx 2v_{1i}. \quad (9-70)$$

Equation 9-70 tells us that body 1 (the cannonball) simply keeps on going, scarcely slowed by the collision. Body 2 (the golf ball) charges ahead at twice the speed of the cannonball. Why twice the speed? Recall the collision described by Eq. 9-69, in which the velocity of the incident light body (the golf ball) changed from $+v$ to $-v$, a velocity *change* of $2v$. The same *change* in velocity (but now from zero to $2v$) occurs in this example also.

Moving Target

Now that we have examined the elastic collision of a projectile and a stationary target, let us examine the situation in which both bodies are moving before they undergo an elastic collision.

For the situation of Fig. 9-19, the conservation of linear momentum is written as

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}, \quad (9-71)$$

*In this step, we use the identity $a^2 - b^2 = (a - b)(a + b)$. It reduces the amount of algebra needed to solve the simultaneous equations Eqs. 9-65 and 9-66.

and the conservation of kinetic energy is written as

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2. \quad (9-72)$$

To solve these simultaneous equations for v_{1f} and v_{2f} , we first rewrite Eq. 9-71 as

$$m_1(v_{1i} - v_{1f}) = -m_2(v_{2i} - v_{2f}), \quad (9-73)$$

and Eq. 9-72 as

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f}). \quad (9-74)$$

After dividing Eq. 9-74 by Eq. 9-73 and doing some more algebra, we obtain

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \quad (9-75)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}. \quad (9-76)$$

Note that the assignment of subscripts 1 and 2 to the bodies is arbitrary. If we exchange those subscripts in Fig. 9-19 and in Eqs. 9-75 and 9-76, we end up with the same set of equations. Note also that if we set $v_{2i} = 0$, body 2 becomes a stationary target as in Fig. 9-18, and Eqs. 9-75 and 9-76 reduce to Eqs. 9-67 and 9-68, respectively.

Here is the generic setup for an elastic collision with a moving target.



Figure 9-19 Two bodies headed for a one-dimensional elastic collision.



Checkpoint 8

What is the final linear momentum of the target in Fig. 9-18 if the initial linear momentum of the projectile is 6 kg · m/s and the final linear momentum of the projectile is (a) 2 kg · m/s and (b) -2 kg · m/s? (c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are, respectively, 5 J and 2 J?

Sample Problem 9.08 Chain reaction of elastic collisions

In Fig. 9-20a, block 1 approaches a line of two stationary blocks with a velocity of $v_{1i} = 10 \text{ m/s}$. It collides with block 2, which then collides with block 3, which has mass $m_3 = 6.0 \text{ kg}$. After the second collision, block 2 is again stationary and block 3 has velocity $v_{3f} = 5.0 \text{ m/s}$ (Fig. 9-20b). Assume that the collisions are elastic. What are the masses of blocks 1 and 2? What is the final velocity v_{1f} of block 1?

KEY IDEAS

Because we assume that the collisions are elastic, we are to conserve mechanical energy (thus energy losses to sound, heating, and oscillations of the blocks are negligible). Because no external horizontal force acts on the blocks, we are to conserve linear momentum along the x axis. For these

two reasons, we can apply Eqs. 9-67 and 9-68 to each of the collisions.

Calculations: If we start with the first collision, we have too many unknowns to make any progress: we do not know the masses or the final velocities of the blocks. So, let's start with the second collision in which block 2 stops because of its collision with block 3. Applying Eq. 9-67 to this collision, with changes in notation, we have

$$v_{2f} = \frac{m_2 - m_3}{m_2 + m_3} v_{2i},$$

where v_{2i} is the velocity of block 2 just before the collision and v_{2f} is the velocity just afterward. Substituting $v_{2f} = 0$ (block 2 stops) and then $m_3 = 6.0 \text{ kg}$ gives us

$$m_2 = m_3 = 6.00 \text{ kg.} \quad (\text{Answer})$$

With similar notation changes, we can rewrite Eq. 9-68 for the second collision as

$$v_{3f} = \frac{2m_2}{m_2 + m_3} v_{2i},$$

where v_{3f} is the final velocity of block 3. Substituting $m_2 = m_3$ and the given $v_{3f} = 5.0 \text{ m/s}$, we find

$$v_{2i} = v_{3f} = 5.0 \text{ m/s.}$$

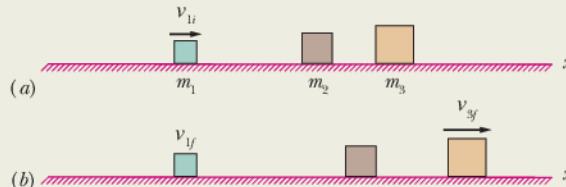


Figure 9-20 Block 1 collides with stationary block 2, which then collides with stationary block 3.

Next, let's reconsider the first collision, but we have to be careful with the notation for block 2: its velocity v_{2f} just after the first collision is the same as its velocity v_{2i} ($= 5.0 \text{ m/s}$) just before the second collision. Applying Eq. 9-68 to the first collision and using the given $v_{1i} = 10 \text{ m/s}$, we have

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i},$$

$$5.0 \text{ m/s} = \frac{2m_1}{m_1 + m_2} (10 \text{ m/s}),$$

which leads to

$$m_1 = \frac{1}{3}m_2 = \frac{1}{3}(6.0 \text{ kg}) = 2.0 \text{ kg}. \quad (\text{Answer})$$

Finally, applying Eq. 9-67 to the first collision with this result and the given v_{1i} , we write

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i},$$

$$= \frac{\frac{1}{3}m_2 - m_2}{\frac{1}{3}m_2 + m_2} (10 \text{ m/s}) = -5.0 \text{ m/s}. \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS

9-8 COLLISIONS IN TWO DIMENSIONS

Learning Objectives

After reading this module, you should be able to . . .

9.34 For an isolated system in which a two-dimensional collision occurs, apply the conservation of momentum along each axis of a coordinate system to relate the momentum components along an axis before the collision to the momentum components *along the same axis* after the collision.

9.35 For an isolated system in which a two-dimensional *elastic* collision occurs, (a) apply the conservation of momentum along each axis of a coordinate system to relate the momentum components along an axis before the collision to the momentum components *along the same axis* after the collision and (b) apply the conservation of total kinetic energy to relate the kinetic energies before and after the collision.

Key Idea

- If two bodies collide and their motion is not along a single axis (the collision is not head-on), the collision is two-dimensional. If the two-body system is closed and isolated, the law of conservation of momentum applies to the collision and can be written as

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}.$$

In component form, the law gives two equations that describe the collision (one equation for each of the two dimensions). If the collision is also elastic (a special case), the conservation of kinetic energy during the collision gives a third equation:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}.$$

Collisions in Two Dimensions

When two bodies collide, the impulse between them determines the directions in which they then travel. In particular, when the collision is not head-on, the bodies do not end up traveling along their initial axis. For such two-dimensional collisions in a closed, isolated system, the total linear momentum must still be conserved:

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}. \quad (9-77)$$

If the collision is also elastic (a special case), then the total kinetic energy is also conserved:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}. \quad (9-78)$$

Equation 9-77 is often more useful for analyzing a two-dimensional collision if we write it in terms of components on an xy coordinate system. For example, Fig. 9-21 shows a *glancing collision* (it is not head-on) between a projectile body and a target body initially at rest. The impulses between the bodies have sent the bodies off at angles θ_1 and θ_2 to the x axis, along which the projectile initially traveled. In this situ-

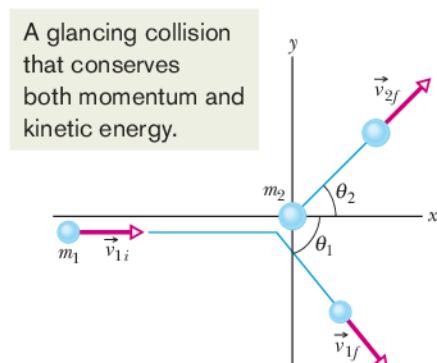


Figure 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

ation we would rewrite Eq. 9-77 for components along the x axis as

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2, \quad (9-79)$$

and along the y axis as

$$0 = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2. \quad (9-80)$$

We can also write Eq. 9-78 (for the special case of an elastic collision) in terms of speeds:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (\text{kinetic energy}). \quad (9-81)$$

Equations 9-79 to 9-81 contain seven variables: two masses, m_1 and m_2 ; three speeds, v_{1i} , v_{1f} , and v_{2f} ; and two angles, θ_1 and θ_2 . If we know any four of these quantities, we can solve the three equations for the remaining three quantities.



Checkpoint 9

In Fig. 9-21, suppose that the projectile has an initial momentum of $6 \text{ kg} \cdot \text{m/s}$, a final x component of momentum of $4 \text{ kg} \cdot \text{m/s}$, and a final y component of momentum of $-3 \text{ kg} \cdot \text{m/s}$. For the target, what then are (a) the final x component of momentum and (b) the final y component of momentum?

9-9 SYSTEMS WITH VARYING MASS: A ROCKET

Learning Objectives

After reading this module, you should be able to . . .

9.36 Apply the first rocket equation to relate the rate at which the rocket loses mass, the speed of the exhaust products relative to the rocket, the mass of the rocket, and the acceleration of the rocket.

9.37 Apply the second rocket equation to relate the change in the rocket's speed to the relative speed of the exhaust products and the initial and final mass of the rocket.

9.38 For a moving system undergoing a change in mass at a given rate, relate that rate to the change in momentum.

Key Ideas

● In the absence of external forces a rocket accelerates at an instantaneous rate given by

$$Rv_{\text{rel}} = Ma \quad (\text{first rocket equation}),$$

in which M is the rocket's instantaneous mass (including unexpended fuel), R is the fuel consumption rate, and v_{rel} is

the fuel's exhaust speed relative to the rocket. The term Rv_{rel} is the thrust of the rocket engine.

● For a rocket with constant R and v_{rel} , whose speed changes from v_i to v_f when its mass changes from M_i to M_f ,

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \quad (\text{second rocket equation}).$$

Systems with Varying Mass: A Rocket

So far, we have assumed that the total mass of the system remains constant. Sometimes, as in a rocket, it does not. Most of the mass of a rocket on its launching pad is fuel, all of which will eventually be burned and ejected from the nozzle of the rocket engine. We handle the variation of the mass of the rocket as the rocket accelerates by applying Newton's second law, not to the rocket alone but to the rocket and its ejected combustion products taken together. The mass of *this* system does *not* change as the rocket accelerates.

Finding the Acceleration

Assume that we are at rest relative to an inertial reference frame, watching a rocket accelerate through deep space with no gravitational or atmospheric drag

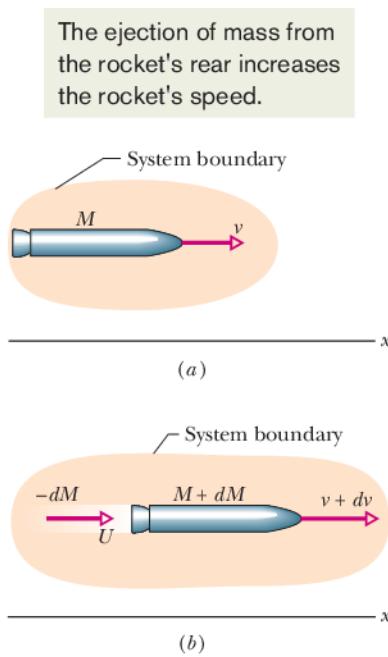


Figure 9-22 (a) An accelerating rocket of mass M at time t , as seen from an inertial reference frame. (b) The same but at time $t + dt$. The exhaust products released during interval dt are shown.

forces acting on it. For this one-dimensional motion, let M be the mass of the rocket and v its velocity at an arbitrary time t (see Fig. 9-22a).

Figure 9-22b shows how things stand a time interval dt later. The rocket now has velocity $v + dv$ and mass $M + dM$, where the change in mass dM is a *negative quantity*. The exhaust products released by the rocket during interval dt have mass $-dM$ and velocity U relative to our inertial reference frame.

Conserve Momentum. Our system consists of the rocket and the exhaust products released during interval dt . The system is closed and isolated, so the linear momentum of the system must be conserved during dt ; that is,

$$P_i = P_f, \quad (9-82)$$

where the subscripts i and f indicate the values at the beginning and end of time interval dt . We can rewrite Eq. 9-82 as

$$Mv = -dM U + (M + dM)(v + dv), \quad (9-83)$$

where the first term on the right is the linear momentum of the exhaust products released during interval dt and the second term is the linear momentum of the rocket at the end of interval dt .

Use Relative Speed. We can simplify Eq. 9-83 by using the relative speed v_{rel} between the rocket and the exhaust products, which is related to the velocities relative to the frame with

$$\left(\begin{array}{l} \text{velocity of rocket} \\ \text{relative to frame} \end{array} \right) = \left(\begin{array}{l} \text{velocity of rocket} \\ \text{relative to products} \end{array} \right) + \left(\begin{array}{l} \text{velocity of products} \\ \text{relative to frame} \end{array} \right).$$

In symbols, this means

$$(v + dv) = v_{\text{rel}} + U,$$

$$\text{or} \quad U = v + dv - v_{\text{rel}}. \quad (9-84)$$

Substituting this result for U into Eq. 9-83 yields, with a little algebra,

$$-dM v_{\text{rel}} = M dv. \quad (9-85)$$

Dividing each side by dt gives us

$$-\frac{dM}{dt} v_{\text{rel}} = M \frac{dv}{dt}. \quad (9-86)$$

We replace dM/dt (the rate at which the rocket loses mass) by $-R$, where R is the (positive) mass rate of fuel consumption, and we recognize that dv/dt is the acceleration of the rocket. With these changes, Eq. 9-86 becomes

$$Rv_{\text{rel}} = Ma \quad (\text{first rocket equation}). \quad (9-87)$$

Equation 9-87 holds for the values at any given instant.

Note the left side of Eq. 9-87 has the dimensions of force ($\text{kg}\cdot\text{s}\cdot\text{m}/\text{s} = \text{kg}\cdot\text{m}/\text{s}^2 = \text{N}$) and depends only on design characteristics of the rocket engine—namely, the rate R at which it consumes fuel mass and the speed v_{rel} with which that mass is ejected relative to the rocket. We call this term Rv_{rel} the **thrust** of the rocket engine and represent it with T . Newton's second law emerges if we write Eq. 9-87 as $T = Ma$, in which a is the acceleration of the rocket at the time that its mass is M .

Finding the Velocity

How will the velocity of a rocket change as it consumes its fuel? From Eq. 9-85 we have

$$dv = -v_{\text{rel}} \frac{dM}{M}.$$

Integrating leads to

$$\int_{v_i}^{v_f} dv = -v_{\text{rel}} \ln \frac{M_i}{M_f},$$

in which M_i is the initial mass of the rocket and M_f its final mass. Evaluating the integrals then gives

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \quad (\text{second rocket equation}) \quad (9-88)$$

for the increase in the speed of the rocket during the change in mass from M_i to M_f . (The symbol “ln” in Eq. 9-88 means the *natural logarithm*.) We see here the advantage of multistage rockets, in which M_f is reduced by discarding successive stages when their fuel is depleted. An ideal rocket would reach its destination with only its payload remaining.

Sample Problem 9.09 Rocket engine, thrust, acceleration

In all previous examples in this chapter, the mass of a system is constant (fixed as a certain number). Here is an example of a system (a rocket) that is losing mass. A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3 \text{ kg/s}$. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

KEY IDEA

Thrust T is equal to the product of the fuel consumption rate R and the relative speed v_{rel} at which exhaust gases are expelled, as given by Eq. 9-87.

Calculation: Here we find

$$\begin{aligned} T &= Rv_{\text{rel}} = (2.3 \text{ kg/s})(2800 \text{ m/s}) \\ &= 6440 \text{ N} \approx 6400 \text{ N}. \end{aligned} \quad (\text{Answer})$$

(b) What is the initial acceleration of the rocket?

KEY IDEA

We can relate the thrust T of a rocket to the magnitude a of the resulting acceleration with $T = Ma$, where M is the

rocket’s mass. However, M decreases and a increases as fuel is consumed. Because we want the initial value of a here, we must use the initial value M_i of the mass.

Calculation: We find

$$a = \frac{T}{M_i} = \frac{6440 \text{ N}}{850 \text{ kg}} = 7.6 \text{ m/s}^2. \quad (\text{Answer})$$

To be launched from Earth’s surface, a rocket must have an initial acceleration greater than $g = 9.8 \text{ m/s}^2$. That is, it must be greater than the gravitational acceleration at the surface. Put another way, the thrust T of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude $M_i g$, which gives us

$$(850 \text{ kg})(9.8 \text{ m/s}^2) = 8330 \text{ N}.$$

Because the acceleration or thrust requirement is not met (here $T = 6400 \text{ N}$), our rocket could not be launched from Earth’s surface by itself; it would require another, more powerful, rocket.



Additional examples, video, and practice available at WileyPLUS

Review & Summary

Center of Mass The **center of mass** of a system of n particles is defined to be the point whose coordinates are given by

$$x_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i x_i, \quad y_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i y_i, \quad z_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i z_i, \quad (9-5)$$

or $\vec{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$,

where M is the total mass of the system.

Newton’s Second Law for a System of Particles The motion of the center of mass of any system of particles is governed by **Newton’s second law for a system of particles**, which is

$$\vec{F}_{\text{net}} = M \vec{a}_{\text{com}}. \quad (9-14)$$

Here \vec{F}_{net} is the net force of all the *external* forces acting on the system, M is the total mass of the system, and \vec{a}_{com} is the acceleration of the system’s center of mass.

Linear Momentum and Newton's Second Law For a single particle, we define a quantity \vec{p} called its **linear momentum** as

$$\vec{p} = m\vec{v}, \quad (9-22)$$

and can write Newton's second law in terms of this momentum:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}. \quad (9-23)$$

For a system of particles these relations become

$$\vec{P} = M\vec{v}_{\text{com}} \quad \text{and} \quad \vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}. \quad (9-25, 9-27)$$

Collision and Impulse Applying Newton's second law in momentum form to a particle-like body involved in a collision leads to the **impulse–linear momentum theorem**:

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J}, \quad (9-31, 9-32)$$

where $\vec{p}_f - \vec{p}_i = \Delta\vec{p}$ is the change in the body's linear momentum, and \vec{J} is the **impulse** due to the force $\vec{F}(t)$ exerted on the body by the other body in the collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t) dt. \quad (9-30)$$

If F_{avg} is the average magnitude of $\vec{F}(t)$ during the collision and Δt is the duration of the collision, then for one-dimensional motion

$$J = F_{\text{avg}} \Delta t. \quad (9-35)$$

When a steady stream of bodies, each with mass m and speed v , collides with a body whose position is fixed, the average force on the fixed body is

$$F_{\text{avg}} = -\frac{n}{\Delta t} \Delta p = -\frac{n}{\Delta t} m \Delta v, \quad (9-37)$$

where $n/\Delta t$ is the rate at which the bodies collide with the fixed body, and Δv is the change in velocity of each colliding body. This average force can also be written as

$$F_{\text{avg}} = -\frac{\Delta m}{\Delta t} \Delta v, \quad (9-40)$$

where $\Delta m/\Delta t$ is the rate at which mass collides with the fixed body. In Eqs. 9-37 and 9-40, $\Delta v = -v$ if the bodies stop upon impact and $\Delta v = -2v$ if they bounce directly backward with no change in their speed.

Conservation of Linear Momentum If a system is isolated so that no net external force acts on it, the linear momentum \vec{P} of the system remains constant:

$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}). \quad (9-42)$$

This can also be written as

$$\vec{P}_i = \vec{P}_f \quad (\text{closed, isolated system}), \quad (9-43)$$

where the subscripts refer to the values of \vec{P} at some initial time and at a later time. Equations 9-42 and 9-43 are equivalent statements of the **law of conservation of linear momentum**.

Inelastic Collision in One Dimension In an *inelastic collision* of two bodies, the kinetic energy of the two-body system is not conserved (it is not a constant). If the system is closed and isolated, the total linear momentum of the system

must be conserved (it is a constant), which we can write in vector form as

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}, \quad (9-50)$$

where subscripts i and f refer to values just before and just after the collision, respectively.

If the motion of the bodies is along a single axis, the collision is one-dimensional and we can write Eq. 9-50 in terms of velocity components along that axis:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}. \quad (9-51)$$

If the bodies stick together, the collision is a *completely inelastic collision* and the bodies have the same final velocity V (because they are stuck together).

Motion of the Center of Mass The center of mass of a closed, isolated system of two colliding bodies is not affected by a collision. In particular, the velocity \vec{v}_{com} of the center of mass cannot be changed by the collision.

Elastic Collisions in One Dimension An *elastic collision* is a special type of collision in which the kinetic energy of a system of colliding bodies is conserved. If the system is closed and isolated, its linear momentum is also conserved. For a one-dimensional collision in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum yield the following expressions for the velocities immediately after the collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad (9-67)$$

$$\text{and} \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}. \quad (9-68)$$

Collisions in Two Dimensions If two bodies collide and their motion is not along a single axis (the collision is not head-on), the collision is two-dimensional. If the two-body system is closed and isolated, the law of conservation of momentum applies to the collision and can be written as

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}. \quad (9-77)$$

In component form, the law gives two equations that describe the collision (one equation for each of the two dimensions). If the collision is also elastic (a special case), the conservation of kinetic energy during the collision gives a third equation:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}. \quad (9-78)$$

Variable-Mass Systems In the absence of external forces a rocket accelerates at an instantaneous rate given by

$$Rv_{\text{rel}} = Ma \quad (\text{first rocket equation}), \quad (9-87)$$

in which M is the rocket's instantaneous mass (including unexpended fuel), R is the fuel consumption rate, and v_{rel} is the fuel's exhaust speed relative to the rocket. The term Rv_{rel} is the **thrust** of the rocket engine. For a rocket with constant R and v_{rel} , whose speed changes from v_i to v_f when its mass changes from M_i to M_f ,

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \quad (\text{second rocket equation}). \quad (9-88)$$

Questions

- 1** Figure 9-23 shows an overhead view of three particles on which external forces act. The magnitudes and directions of the forces on two of the particles are indicated. What are the magnitude and direction of the force acting on the third particle if the center of mass of the three-particle system is (a) stationary, (b) moving at a constant velocity rightward, and (c) accelerating rightward?

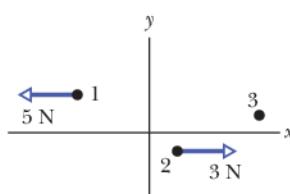


Figure 9-23 Question 1.

- 2** Figure 9-24 shows an overhead view of four particles of equal mass sliding over a frictionless surface at constant velocity. The directions of the velocities are indicated; their magnitudes are equal. Consider pairing the particles. Which pairs form a system with a center of mass that (a) is stationary, (b) is stationary and at the origin, and (c) passes through the origin?

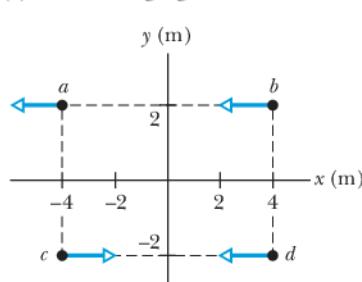


Figure 9-24 Question 2.

- 3** Consider a box that explodes into two pieces while moving with a constant positive velocity along an x axis. If one piece, with mass m_1 , ends up with positive velocity \vec{v}_1 , then the second piece, with mass m_2 , could end up with (a) a positive velocity \vec{v}_2 (Fig. 9-25a), (b) a negative velocity \vec{v}_2 (Fig. 9-25b), or (c) zero velocity (Fig. 9-25c). Rank those three possible results for the second piece according to the corresponding magnitude of \vec{v}_1 , greatest first.

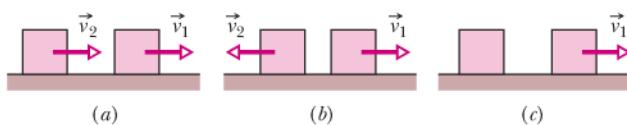


Figure 9-25 Question 3.

- 4** Figure 9-26 shows graphs of force magnitude versus time for a body involved in a collision. Rank the graphs according to the magnitude of the impulse on the body, greatest first.

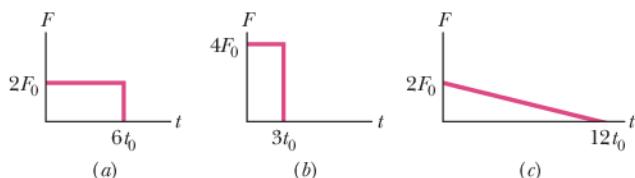


Figure 9-26 Question 4.

- 5** The free-body diagrams in Fig. 9-27 give, from overhead views, the horizontal forces acting on three boxes of chocolates as the

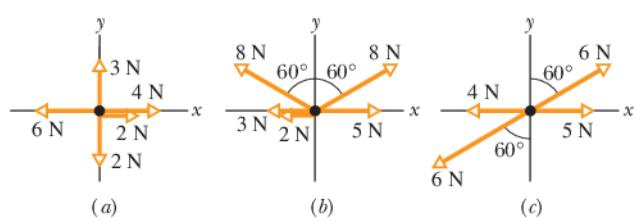


Figure 9-27 Question 5.

boxes move over a frictionless confectioner's counter. For each box, is its linear momentum conserved along the x axis and the y axis?

- 6** Figure 9-28 shows four groups of three or four identical particles that move parallel to either the x axis or the y axis, at identical speeds. Rank the groups according to center-of-mass speed, greatest first.

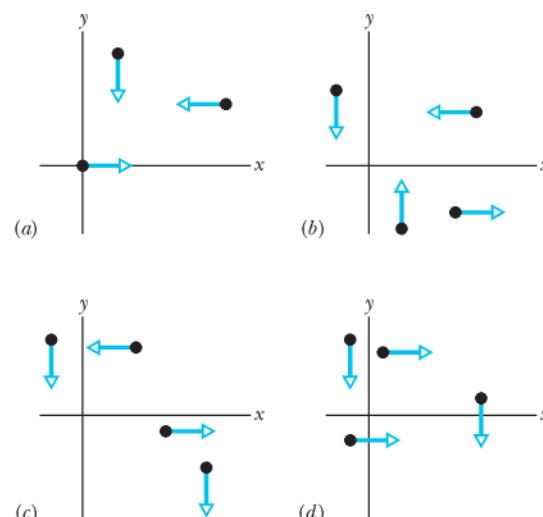


Figure 9-28 Question 6.

- 7** A block slides along a frictionless floor and into a stationary second block with the same mass. Figure 9-29 shows four choices for a graph of the kinetic energies K of the blocks. (a) Determine which represent physically impossible situations. Of the others, which best represents (b) an elastic collision and (c) an inelastic collision?

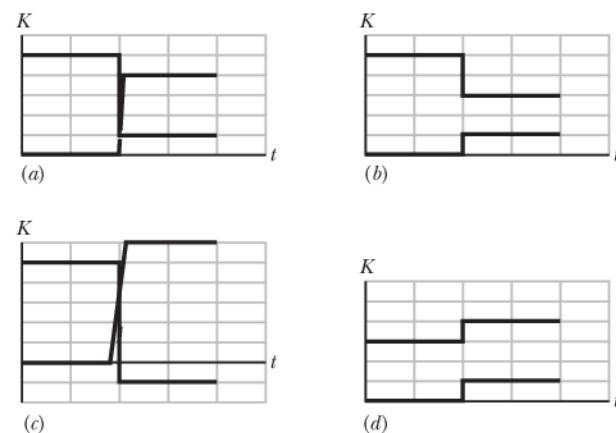


Figure 9-29 Question 7.

- 8** Figure 9-30 shows a snapshot of block 1 as it slides along an x axis on a frictionless floor, before it undergoes an elastic collision with stationary block 2. The figure also shows three possible positions of the center of mass (com) of the two-block system at the time of the snapshot. (Point B is halfway between the centers of the two blocks.) Is block 1 stationary, moving forward, or moving backward after the collision if the com is located in the snapshot at (a) A, (b) B, and (c) C?



Figure 9-30 Question 8.

- 9** Two bodies have undergone an elastic one-dimensional collision along an x axis. Figure 9-31 is a graph of position versus time for those bodies and for their center of mass. (a) Were both bodies initially moving, or was one initially stationary? Which line segment corresponds to the motion of the center of mass (b) before the collision and (c) after the collision? (d) Is the mass of the body that was moving faster before the collision greater than, less than, or equal to that of the other body?

- 10** Figure 9-32: A block on a horizontal floor is initially either stationary, sliding in the positive direction of an x axis, or sliding in

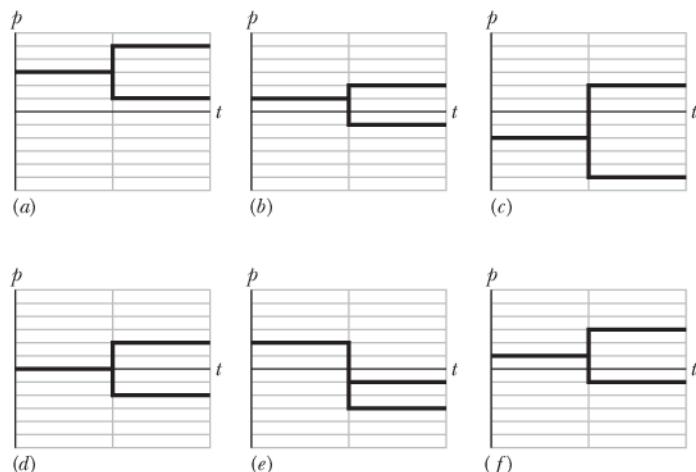


Figure 9-32 Question 10.

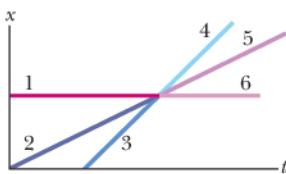


Figure 9-31 Question 9.

the negative direction of that axis. Then the block explodes into two pieces that slide along the x axis. Assume the block and the two pieces form a closed, isolated system. Six choices for a graph of the momenta of the block and the pieces are given, all versus time t . Determine which choices represent physically impossible situations and explain why.

- 11** Block 1 with mass m_1 slides along an x axis across a frictionless floor and then undergoes an elastic collision with a stationary block 2 with mass m_2 . Figure 9-33 shows a plot of position x versus time t of block 1 until the collision occurs at position x_c and time t_c . In which of the lettered regions on the graph will the plot be continued (after the collision) if (a) $m_1 < m_2$ and (b) $m_1 > m_2$? (c) Along which of the numbered dashed lines will the plot be continued if $m_1 = m_2$?

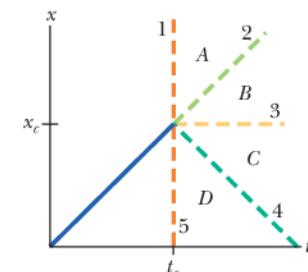


Figure 9-33 Question 11.

- 12** Figure 9-34 shows four graphs of position versus time for two bodies and their center of mass. The two bodies form a closed, isolated system and undergo a completely inelastic, one-dimensional collision on an x axis. In graph 1, are (a) the two bodies and (b) the center of mass moving in the positive or negative direction of the x axis? (c) Which of the graphs correspond to a physically impossible situation? Explain.

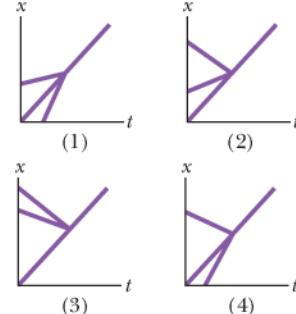


Figure 9-34 Question 12.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

WWW Worked-out solution is at

ILW Interactive solution is at

<http://www.wiley.com/college/halliday>

Module 9-1 Center of Mass

- 1** A 2.00 kg particle has the xy coordinates $(-1.20 \text{ m}, 0.500 \text{ m})$, and a 4.00 kg particle has the xy coordinates $(0.600 \text{ m}, -0.750 \text{ m})$. Both lie on a horizontal plane. At what (a) x and (b) y coordinates must you place a 3.00 kg particle such that the center of mass of the three-particle system has the coordinates $(-0.500 \text{ m}, -0.700 \text{ m})$?

- 2** Figure 9-35 shows a three-particle system, with masses $m_1 = 3.0 \text{ kg}$, $m_2 = 4.0 \text{ kg}$, and $m_3 = 8.0 \text{ kg}$. The scales on the axes are set by $x_s = 2.0 \text{ m}$ and $y_s = 2.0 \text{ m}$. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass? (c) If m_3 is gradually increased, does the center of mass of the system shift toward or away from that particle, or does it remain stationary?

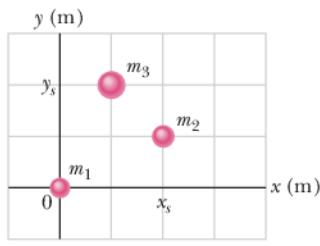


Figure 9-35 Problem 2.

- 3** Figure 9-36 shows a slab with dimensions $d_1 = 11.0 \text{ cm}$, $d_2 = 2.80 \text{ cm}$, and $d_3 = 13.0 \text{ cm}$. Half the slab consists of aluminum (den-

sity = 2.70 g/cm^3) and half consists of iron (density = 7.85 g/cm^3). What are (a) the x coordinate, (b) the y coordinate, and (c) the z coordinate of the slab's center of mass?

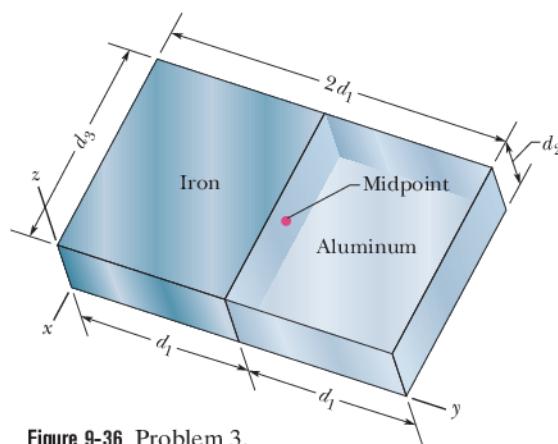


Figure 9-36 Problem 3.

- 4** In Fig. 9-37, three uniform thin rods, each of length $L = 22\text{ cm}$, form an inverted U. The vertical rods each have a mass of 14 g; the horizontal rod has a mass of 42 g. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass?

- 5 GO** What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in Fig. 9-38 if $L = 5.0\text{ cm}$?

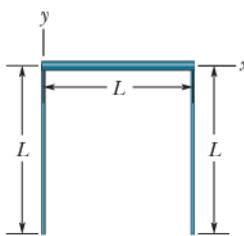


Figure 9-37 Problem 4.

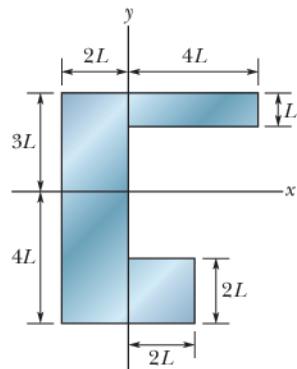


Figure 9-38 Problem 5.

- 6** Figure 9-39 shows a cubical box that has been constructed from uniform metal plate of negligible thickness. The box is open at the top and has edge length $L = 40\text{ cm}$. Find (a) the x coordinate, (b) the y coordinate, and (c) the z coordinate of the center of mass of the box.

- 7 ILW** In the ammonia (NH_3) molecule of Fig. 9-40, three hydrogen (H) atoms form an equilateral triangle, with the center of the triangle at distance $d = 9.40 \times 10^{-11}\text{ m}$ from each hydrogen atom. The nitrogen (N) atom is at the apex of a pyramid, with the three hydrogen atoms forming the base. The nitrogen-to-hydrogen atomic mass ratio is 13.9, and the nitrogen-to-hydrogen distance is $L = 10.14 \times 10^{-11}\text{ m}$. What are the (a) x and (b) y coordinates of the molecule's center of mass?

- 8 GO** A uniform soda can of mass 0.140 kg is 12.0 cm tall and filled with 0.354 kg of soda (Fig. 9-41). Then small holes are drilled in the top and bottom (with negligible loss of metal) to drain the soda. What is the height h of the com of the can and contents (a) initially and (b) after the can loses all the soda? (c) What happens to h as the soda drains out? (d) If x is the height of the remaining soda at any given instant, find x when the com reaches its lowest point.

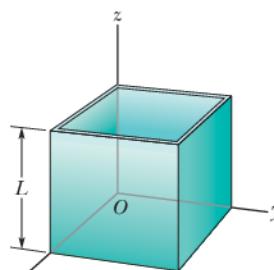


Figure 9-39 Problem 6.

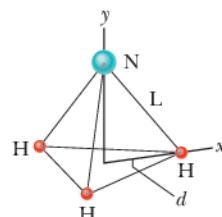


Figure 9-40 Problem 7.

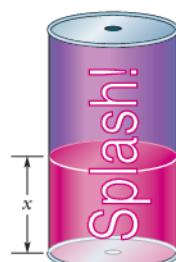


Figure 9-41 Problem 8.

Module 9-2 Newton's Second Law for a System of Particles

- 9 ILW** A stone is dropped at $t = 0$. A second stone, with twice the mass of the first, is dropped from the same point at $t = 100\text{ ms}$. (a) How far below the release point is the center of mass of the two stones at $t = 300\text{ ms}$? (Neither stone has yet reached the ground.) (b) How fast is the center of mass of the two-stone system moving at that time?

- 10 GO** A 1000 kg automobile is at rest at a traffic signal. At the instant the light turns green, the automobile starts to move with a constant acceleration of 4.0 m/s^2 . At the same instant a 2000 kg truck, traveling at a constant speed of 8.0 m/s , overtakes and passes the automobile. (a) How far is the com of the automobile–truck system from the traffic light at $t = 3.0\text{ s}$? (b) What is the speed of the com then?

- 11** A big olive ($m = 0.50\text{ kg}$) lies at the origin of an xy coordinate system, and a big Brazil nut ($M = 1.5\text{ kg}$) lies at the point $(1.0, 2.0)\text{ m}$. At $t = 0$, a force $\vec{F}_o = (2.0\hat{i} + 3.0\hat{j})\text{ N}$ begins to act on the olive, and a force $\vec{F}_n = (-3.0\hat{i} - 2.0\hat{j})\text{ N}$ begins to act on the nut. In unit-vector notation, what is the displacement of the center of mass of the olive–nut system at $t = 4.0\text{ s}$, with respect to its position at $t = 0$?

- 12** Two skaters, one with mass 65 kg and the other with mass 40 kg , stand on an ice rink holding a pole of length 10 m and negligible mass. Starting from the ends of the pole, the skaters pull themselves along the pole until they meet. How far does the 40 kg skater move?

- 13 SSM** A shell is shot with an initial velocity \vec{v}_0 of 20 m/s , at an angle of $\theta_0 = 60^\circ$ with the horizontal. At the top of the trajectory, the shell explodes into two fragments of equal mass (Fig. 9-42). One fragment, whose speed immediately after the explosion is zero, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?

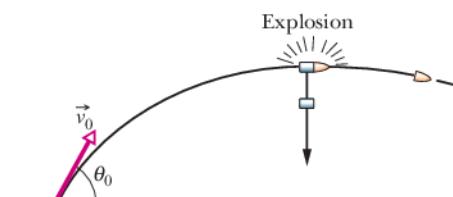


Figure 9-42 Problem 13.

- 14** In Figure 9-43, two particles are launched from the origin of the coordinate system at time $t = 0$. Particle 1 of mass $m_1 = 5.00\text{ g}$ is shot directly along the x axis on a frictionless floor, with constant speed 10.0 m/s . Particle 2 of mass $m_2 = 3.00\text{ g}$ is shot with a velocity of magnitude 20.0 m/s , at an upward angle such that it always stays directly above particle 1. (a) What is the maximum height H_{\max} reached by the com of the two-particle system? In unit-vector notation, what are the (b) velocity and (c) acceleration of the com when the com reaches H_{\max} ?

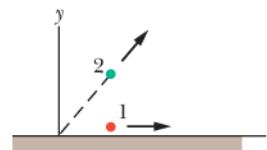


Figure 9-43 Problem 14.

••15 Figure 9-44 shows an arrangement with an air track, in which a cart is connected by a cord to a hanging block. The cart has mass $m_1 = 0.600 \text{ kg}$, and its center is initially at xy coordinates $(-0.500 \text{ m}, 0 \text{ m})$; the block has mass $m_2 = 0.400 \text{ kg}$, and its center is initially at xy coordinates $(0, -0.100 \text{ m})$. The mass of the cord and pulley are negligible. The cart is released from rest, and both cart and block move until the cart hits the pulley. The friction between the cart and the air track and between the pulley and its axle is negligible. (a) In unit-vector notation, what is the acceleration of the center of mass of the cart-block system? (b) What is the velocity of the com as a function of time t ? (c) Sketch the path taken by the com. (d) If the path is curved, determine whether it bulges upward to the right or downward to the left, and if it is straight, find the angle between it and the x axis.

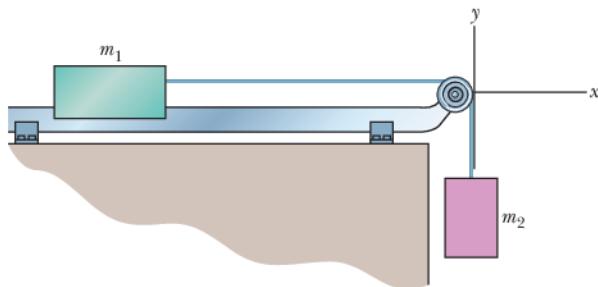


Figure 9-44 Problem 15.

••16 GO Ricardo, of mass 80 kg, and Carmelita, who is lighter, are enjoying Lake Merced at dusk in a 30 kg canoe. When the canoe is at rest in the placid water, they exchange seats, which are 3.0 m apart and symmetrically located with respect to the canoe's center. If the canoe moves 40 cm horizontally relative to a pier post, what is Carmelita's mass?

••17 GO In Fig. 9-45a, a 4.5 kg dog stands on an 18 kg flatboat at distance $D = 6.1 \text{ m}$ from the shore. It walks 2.4 m along the boat toward shore and then stops. Assuming no friction between the boat and the water, find how far the dog is then from the shore. (Hint: See Fig. 9-45b.)

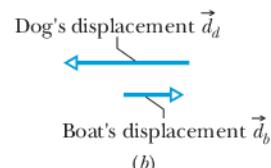
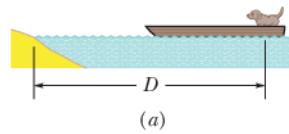


Figure 9-45 Problem 17.

Module 9-3 Linear Momentum

•18 A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?

•19 ILW A 2100 kg truck traveling north at 41 km/h turns east and accelerates to 51 km/h. (a) What is the change in the truck's kinetic energy? What are the (b) magnitude and (c) direction of the change in its momentum?

•20 GO At time $t = 0$, a ball is struck at ground level and sent over level ground. The momentum p versus t during the flight is given by Fig. 9-46 (with $p_0 = 6.0 \text{ kg} \cdot \text{m/s}$ and $p_1 = 4.0 \text{ kg} \cdot \text{m/s}$). At what initial angle is the ball launched? (Hint: Find a solution that does not require you to read the time of the low point of the plot.)

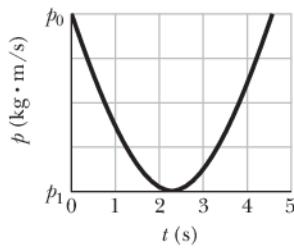


Figure 9-46 Problem 20.

•21 A 0.30 kg softball has a velocity of 15 m/s at an angle of 35° below the horizontal just before making contact with the bat. What is the magnitude of the change in momentum of the ball while in contact with the bat if the ball leaves with a velocity of (a) 20 m/s, vertically downward, and (b) 20 m/s, horizontally back toward the pitcher?

•22 Figure 9-47 gives an overhead view of the path taken by a 0.165 kg cue ball as it bounces from a rail of a pool table. The ball's initial speed is 2.00 m/s, and the angle θ_1 is 30.0° . The bounce reverses the y component of the ball's velocity but does not alter the x component. What are (a) angle θ_2 and (b) the change in the ball's linear momentum in unit-vector notation? (The fact that the ball rolls is irrelevant to the problem.)

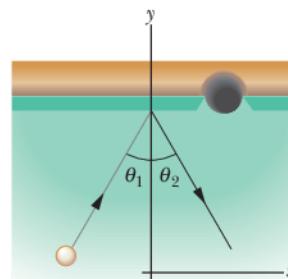


Figure 9-47 Problem 22.

Module 9-4 Collision and Impulse

•23 Until his seventies, Henri LaMothe (Fig. 9-48) excited audiences by belly-flopping from a height of 12 m into 30 cm of water. Assuming that he stops just as he reaches the bottom of the water and estimating his mass, find the magnitude of the impulse on him from the water.



George Long/Getty Images, Inc.

Figure 9-48 Problem 23. Belly-flopping into 30 cm of water.

•24 In February 1955, a paratrooper fell 370 m from an airplane without being able to open his chute but happened to land in snow, suffering only minor injuries. Assume that his speed at impact was 56 m/s (terminal speed), that his mass (including gear) was 85 kg, and that the magnitude of the force on him from the

snow was at the survivable limit of $1.2 \times 10^5 \text{ N}$. What are (a) the minimum depth of snow that would have stopped him safely and (b) the magnitude of the impulse on him from the snow?

•25 A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?

•26 In a common but dangerous prank, a chair is pulled away as a person is moving downward to sit on it, causing the victim to land hard on the floor. Suppose the victim falls by 0.50 m, the mass that moves downward is 70 kg, and the collision on the floor lasts 0.082 s. What are the magnitudes of the (a) impulse and (b) average force acting on the victim from the floor during the collision?

•27 SSM A force in the negative direction of an x axis is applied for 27 ms to a 0.40 kg ball initially moving at 14 m/s in the positive direction of the axis. The force varies in magnitude, and the impulse has magnitude $32.4 \text{ N} \cdot \text{s}$. What are the ball's (a) speed and (b) direction of travel just after the force is applied? What are (c) the average magnitude of the force and (d) the direction of the impulse on the ball?

•28 In tae-kwon-do, a hand is slammed down onto a target at a speed of 13 m/s and comes to a stop during the 5.0 ms collision. Assume that during the impact the hand is independent of the arm and has a mass of 0.70 kg. What are the magnitudes of the (a) impulse and (b) average force on the hand from the target?

•29 Suppose a gangster sprays Superman's chest with 3 g bullets at the rate of 100 bullets/min, and the speed of each bullet is 500 m/s. Suppose too that the bullets rebound straight back with no change in speed. What is the magnitude of the average force on Superman's chest?

•30 Two average forces. A steady stream of 0.250 kg snowballs is shot perpendicularly into a wall at a speed of 4.00 m/s. Each ball sticks to the wall. Figure 9-49 gives the magnitude F of the force on the wall as a function of time t for two of the snowball impacts. Impacts occur with a repetition time interval $\Delta t_r = 50.0 \text{ ms}$, last a duration time interval $\Delta t_d = 10 \text{ ms}$, and produce isosceles triangles on the graph, with each impact reaching a force maximum $F_{\max} = 200 \text{ N}$. During each impact, what are the magnitudes of (a) the impulse and (b) the average force on the wall? (c) During a time interval of many impacts, what is the magnitude of the average force on the wall?

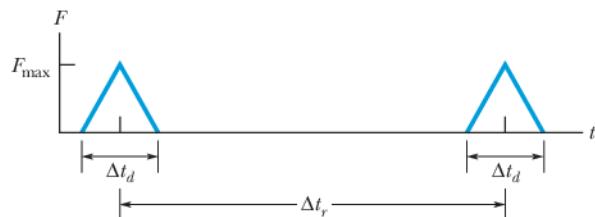


Figure 9-49 Problem 30.

•31 Jumping up before the elevator hits. After the cable snaps and the safety system fails, an elevator cab free-falls from a height of 36 m. During the collision at the bottom of the elevator shaft, a 90 kg passenger is stopped in 5.0 ms. (Assume that neither the passenger nor the cab rebounds.) What are the magnitudes of the (a) impulse and (b) average force on the passenger during the collision? If the passenger were to jump upward with a speed of 7.0 m/s relative to the cab floor just before the cab hits the bottom of the shaft, what

are the magnitudes of the (c) impulse and (d) average force (assuming the same stopping time)?

•32 A 5.0 kg toy car can move along an x axis; Fig. 9-50 gives F_x of the force acting on the car, which begins at rest at time $t = 0$. The scale on the F_x axis is set by $F_{xs} = 5.0 \text{ N}$. In unit-vector notation, what is \vec{p} at (a) $t = 4.0 \text{ s}$ and (b) $t = 7.0 \text{ s}$, and (c) what is \vec{v} at $t = 9.0 \text{ s}$?

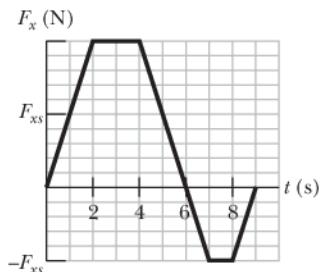


Figure 9-50 Problem 32.

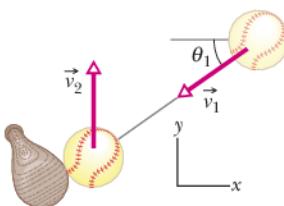


Figure 9-51 Problem 33.

•33 GO Figure 9-51 shows a 0.300 kg baseball just before and just after it collides with a bat. Just before, the ball has velocity \vec{v}_1 of magnitude 12.0 m/s and angle $\theta_1 = 35.0^\circ$. Just after, it is traveling directly upward with velocity \vec{v}_2 of magnitude 10.0 m/s. The duration of the collision is 2.00 ms. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the impulse on the ball from the bat? What are the (c) magnitude and (d) direction of the average force on the ball from the bat?

•34 Basilisk lizards can run across the top of a water surface (Fig. 9-52). With each step, a lizard first slaps its foot against the water and then pushes it down into the water rapidly enough to form an air cavity around the top of the foot. To avoid having to pull the foot back up against water drag in order to complete the step, the lizard withdraws the foot before water can flow into the air cavity. If the lizard is not to sink, the average upward impulse on the lizard during this full action of slap, downward push, and withdrawal must match the downward impulse due to the gravitational force. Suppose the mass of a basilisk lizard is 90.0 g, the mass of each foot is 3.00 g, the speed of a foot as it slaps the water is 1.50 m/s, and the time for a single step is 0.600 s. (a) What is the magnitude of the impulse on the lizard during the slap? (Assume this impulse is directly upward.) (b) During the 0.600 s duration of a step, what is the downward impulse on the lizard due to the gravitational force? (c) Which action, the slap or the push, provides the primary support for the lizard, or are they approximately equal in their support?



Stephen Dalton/Photo Researchers, Inc.

Figure 9-52 Problem 34. Lizard running across water.

- 35 GO** Figure 9-53 shows an approximate plot of force magnitude F versus time t during the collision of a 58 g Superball with a wall. The initial velocity of the ball is 34 m/s perpendicular to the wall; the ball rebounds directly back with approximately the same speed, also perpendicular to the wall. What is F_{\max} , the maximum magnitude of the force on the ball from the wall during the collision?

••36 A 0.25 kg puck is initially stationary on an ice surface with negligible friction. At time $t = 0$, a horizontal force begins to move the puck. The force is given by $\vec{F} = (12.0 - 3.00t^2)\hat{i}$, with \vec{F} in newtons and t in seconds, and it acts until its magnitude is zero. (a) What is the magnitude of the impulse on the puck from the force between $t = 0.500$ s and $t = 1.25$ s? (b) What is the change in momentum of the puck between $t = 0$ and the instant at which $F = 0$?

••37 SSM A soccer player kicks a soccer ball of mass 0.45 kg that is initially at rest. The foot of the player is in contact with the ball for 3.0×10^{-3} s, and the force of the kick is given by

$$F(t) = [(6.0 \times 10^6)t - (2.0 \times 10^9)t^2] \text{ N}$$

for $0 \leq t \leq 3.0 \times 10^{-3}$ s, where t is in seconds. Find the magnitudes of (a) the impulse on the ball due to the kick, (b) the average force on the ball from the player's foot during the period of contact, (c) the maximum force on the ball from the player's foot during the period of contact, and (d) the ball's velocity immediately after it loses contact with the player's foot.

- 38** In the overhead view of Fig. 9-54, a 300 g ball with a speed v of 6.0 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit-vector notation, what are (a) the impulse on the ball from the wall and (b) the average force on the wall from the ball?

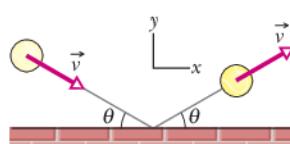


Figure 9-54 Problem 38.

Module 9-5 Conservation of Linear Momentum

- 39 SSM** A 91 kg man lying on a surface of negligible friction shoves a 68 g stone away from himself, giving it a speed of 4.0 m/s. What speed does the man acquire as a result?

•40 A space vehicle is traveling at 4300 km/h relative to Earth when the exhausted rocket motor (mass $4m$) is disengaged and sent backward with a speed of 82 km/h relative to the command module (mass m). What is the speed of the command module relative to Earth just after the separation?

- 41** Figure 9-55 shows a two-ended "rocket" that is initially stationary on a frictionless floor, with its center at the origin of an x axis. The rocket consists of a central block C (of mass $M = 6.00$ kg) and blocks L and R (each of mass $m = 2.00$ kg) on the left and right sides. Small explosions can shoot either of the side blocks away from block C and along the x axis. Here is the sequence: (1) At time $t = 0$, block L is shot to the left with a speed of 3.00 m/s relative to the ve-

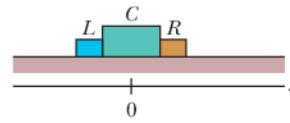


Figure 9-55 Problem 41.

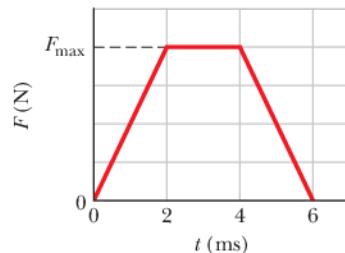


Figure 9-53 Problem 35.

locity that the explosion gives the rest of the rocket. (2) Next, at time $t = 0.80$ s, block R is shot to the right with a speed of 3.00 m/s relative to the velocity that block C then has. At $t = 2.80$ s, what are (a) the velocity of block C and (b) the position of its center?

- 42** An object, with mass m and speed v relative to an observer, explodes into two pieces, one three times as massive as the other; the explosion takes place in deep space. The less massive piece stops relative to the observer. How much kinetic energy is added to the system during the explosion, as measured in the observer's reference frame?

••43 In the Olympiad of 708 B.C., some athletes competing in the standing long jump used handheld weights called *halteres* to lengthen their jumps (Fig. 9-56). The weights were swung up in front just before liftoff and then swung down and thrown backward during the flight. Suppose a modern 78 kg long jumper similarly uses two 5.50 kg halteres, throwing them horizontally to the rear at his maximum height such that their horizontal velocity is zero relative to the ground. Let his liftoff velocity be $\vec{v} = (9.5\hat{i} + 4.0\hat{j})$ m/s with or without the halteres, and assume that he lands at the liftoff level. What distance would the use of the halteres add to his range?



Réunion des Musées Nationaux/
Art Resource

Figure 9-56 Problem 43.

- 44 GO** In Fig. 9-57, a stationary block explodes into two pieces L and R that slide across a frictionless floor and then into regions with friction, where they stop. Piece L , with a mass of 2.0 kg, encounters a coefficient of kinetic friction $\mu_L = 0.40$ and slides to a stop in distance $d_L = 0.15$ m. Piece R encounters a coefficient of kinetic friction $\mu_R = 0.50$ and slides to a stop in distance $d_R = 0.25$ m. What was the mass of the block?

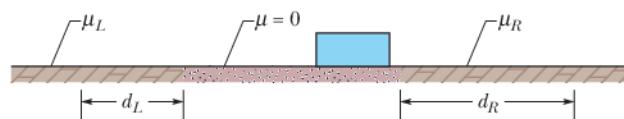


Figure 9-57 Problem 44.

- 45 SSM WWW** A 20.0 kg body is moving through space in the positive direction of an x axis with a speed of 200 m/s when, due to an internal explosion, it breaks into three parts. One part, with a mass of 10.0 kg, moves away from the point of explosion with a speed of 100 m/s in the positive y direction. A second part, with a mass of 4.00 kg, moves in the negative x direction with a speed of 500 m/s. (a) In unit-vector notation, what is the velocity of the third part? (b) How much energy is released in the explosion? Ignore effects due to the gravitational force.

••46 A 4.0 kg mess kit sliding on a frictionless surface explodes into two 2.0 kg parts: 3.0 m/s, due north, and 5.0 m/s, 30° north of east. What is the original speed of the mess kit?

••47 A vessel at rest at the origin of an xy coordinate system explodes into three pieces. Just after the explosion, one piece, of mass m , moves with velocity $(-30 \text{ m/s})\hat{i}$ and a second piece, also of mass m , moves with velocity $(-30 \text{ m/s})\hat{j}$. The third piece has mass $3m$. Just after the explosion, what are the (a) magnitude and (b) direction of the velocity of the third piece?

••48 GO Particle A and particle B are held together with a compressed spring between them. When they are released, the spring pushes them apart, and they then fly off in opposite directions, free of the spring. The mass of A is 2.00 times the mass of B , and the energy stored in the spring was 60 J. Assume that the spring has negligible mass and that all its stored energy is transferred to the particles. Once that transfer is complete, what are the kinetic energies of (a) particle A and (b) particle B ?

Module 9-6 Momentum and Kinetic Energy in Collisions

•49 A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

•50 A 5.20 g bullet moving at 672 m/s strikes a 700 g wooden block at rest on a frictionless surface. The bullet emerges, traveling in the same direction with its speed reduced to 428 m/s. (a) What is the resulting speed of the block? (b) What is the speed of the bullet-block center of mass?

•51 GO In Fig. 9-58a, a 3.50 g bullet is fired horizontally at two blocks at rest on a frictionless table. The bullet passes through block 1 (mass 1.20 kg) and embeds itself in block 2 (mass 1.80 kg). The blocks end up with speeds $v_1 = 0.630 \text{ m/s}$ and $v_2 = 1.40 \text{ m/s}$ (Fig. 9-58b). Neglecting the material removed from block 1 by the bullet, find the speed of the bullet as it (a) leaves and (b) enters block 1.

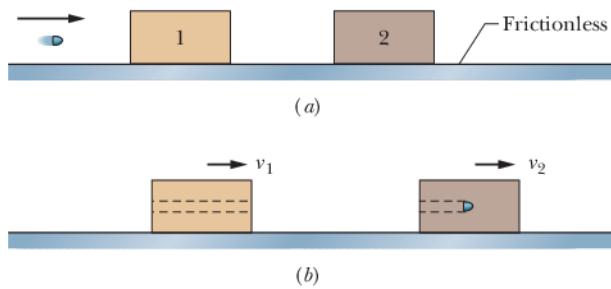


Figure 9-58 Problem 51.

•52 GO In Fig. 9-59, a 10 g bullet moving directly upward at 1000 m/s strikes and passes through the center of mass of a 5.0 kg block initially at rest. The bullet emerges from the block moving directly upward at 400 m/s. To what maximum height does the block then rise above its initial position?

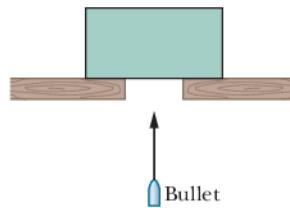


Figure 9-59 Problem 52.

•53 In Anchorage, collisions of a vehicle with a moose are so common that they are referred to with the abbreviation MVC. Suppose a 1000 kg car slides into a stationary 500 kg moose on a very slippery road, with the moose being thrown through the windshield (a common MVC result). (a) What percent of the original kinetic energy is lost in the collision to other forms of energy? A similar danger occurs in Saudi Arabia because of camel-vehicle

collisions (CVC). (b) What percent of the original kinetic energy is lost if the car hits a 300 kg camel? (c) Generally, does the percent loss increase or decrease if the animal mass decreases?

•54 A completely inelastic collision occurs between two balls of wet putty that move directly toward each other along a vertical axis. Just before the collision, one ball, of mass 3.0 kg, is moving upward at 20 m/s and the other ball, of mass 2.0 kg, is moving downward at 12 m/s. How high do the combined two balls of putty rise above the collision point? (Neglect air drag.)

•55 ILW A 5.0 kg block with a speed of 3.0 m/s collides with a 10 kg block that has a speed of 2.0 m/s in the same direction. After the collision, the 10 kg block travels in the original direction with a speed of 2.5 m/s. (a) What is the velocity of the 5.0 kg block immediately after the collision? (b) By how much does the total kinetic energy of the system of two blocks change because of the collision? (c) Suppose, instead, that the 10 kg block ends up with a speed of 4.0 m/s. What then is the change in the total kinetic energy? (d) Account for the result you obtained in (c).

•56 In the “before” part of Fig. 9-60, car A (mass 1100 kg) is stopped at a traffic light when it is rear-ended by car B (mass 1400 kg). Both cars then slide with locked wheels until the frictional force from the slick road (with a low μ_k of 0.13) stops them, at distances $d_A = 8.2 \text{ m}$ and $d_B = 6.1 \text{ m}$. What are the speeds of (a) car A and (b) car B at the start of the sliding, just after the collision? (c) Assuming that linear momentum is conserved during the collision, find the speed of car B just before the collision. (d) Explain why this assumption may be invalid.

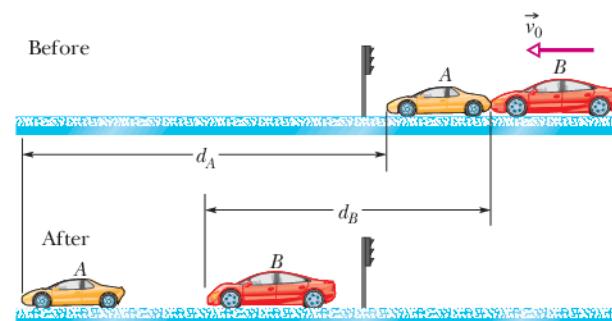


Figure 9-60 Problem 56.

•57 GO In Fig. 9-61, a ball of mass $m = 60 \text{ g}$ is shot with speed $v_i = 22 \text{ m/s}$ into the barrel of a spring gun of mass $M = 240 \text{ g}$ initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase in thermal energy due to friction between the ball and the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b) What fraction of the initial kinetic energy of the ball is stored in the spring?



Figure 9-61 Problem 57.

•58 In Fig. 9-62, block 2 (mass 1.0 kg) is at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. The other end of the spring is fixed to a wall. Block 1 (mass 2.0 kg), traveling at speed $v_1 = 4.0 \text{ m/s}$, collides with block 2, and the two blocks stick together. When the blocks momentarily stop, by what distance is the spring compressed?

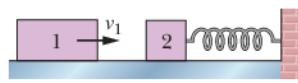


Figure 9-62 Problem 58.

- 59 ILW** In Fig. 9-63, block 1 (mass 2.0 kg) is moving rightward at 10 m/s and block 2 (mass 5.0 kg) is moving rightward at 3.0 m/s. The surface is frictionless, and a spring with a spring constant of 1120 N/m is fixed to block 2. When the blocks collide, the compression of the spring is maximum at the instant the blocks have the same velocity. Find the maximum compression.



Figure 9-63 Problem 59.

Module 9-7 Elastic Collisions in One Dimension

- 60** In Fig. 9-64, block *A* (mass 1.6 kg) slides into block *B* (mass 2.4 kg), along a frictionless surface. The directions of three velocities before (*i*) and after (*f*) the collision are indicated; the corresponding speeds are $v_{Ai} = 5.5$ m/s, $v_{Bi} = 2.5$ m/s, and $v_{Bf} = 4.9$ m/s. What are the (a) speed and (b) direction (left or right) of velocity \vec{v}_{Af} ? (c) Is the collision elastic?

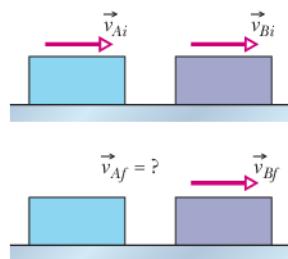


Figure 9-64 Problem 60.

- 61 SSM** A cart with mass 340 g moving on a frictionless linear air track at an initial speed of 1.2 m/s undergoes an elastic collision with an initially stationary cart of unknown mass. After the collision, the first cart continues in its original direction at 0.66 m/s. (a) What is the mass of the second cart? (b) What is its speed after impact? (c) What is the speed of the two-cart center of mass?

- 62** Two titanium spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is 300 g, remains at rest. (a) What is the mass of the other sphere? (b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2.00 m/s?

- 63** Block 1 of mass m_1 slides along a frictionless floor and into a one-dimensional elastic collision with stationary block 2 of mass $m_2 = 3m_1$. Prior to the collision, the center of mass of the two-block system had a speed of 3.00 m/s. Afterward, what are the speeds of (a) the center of mass and (b) block 2?

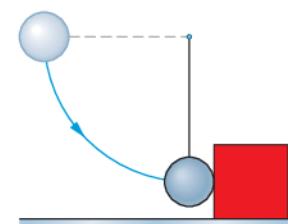


Figure 9-65 Problem 64.

- 64 GO** A steel ball of mass 0.500 kg is fastened to a cord that is 70.0 cm long and fixed at the far end. The ball is then released when the cord is horizontal (Fig. 9-65). At the bottom of its path, the ball strikes a 2.50 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.

- 65 SSM** A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its original speed. (a) What is the mass of the other body? (b) What is the speed of the two-body center of mass if the initial speed of the 2.0 kg body was 4.0 m/s?

- 66** Block 1, with mass m_1 and speed 4.0 m/s, slides along an *x* axis on a frictionless floor and then undergoes a one-dimensional elastic collision with stationary block 2, with mass $m_2 = 0.40m_1$. The two blocks then slide into a region where the coefficient of kinetic

friction is 0.50; there they stop. How far into that region do (a) block 1 and (b) block 2 slide?

- 67** In Fig. 9-66, particle 1 of mass $m_1 = 0.30$ kg slides rightward along an *x* axis on a frictionless floor with a speed of 2.0 m/s. When it reaches $x = 0$, it undergoes a one-dimensional elastic collision with stationary particle 2 of mass $m_2 = 0.40$ kg. When particle 2 then reaches a wall at $x_w = 70$ cm, it bounces from the wall with no loss of speed. At what position on the *x* axis does particle 2 then collide with particle 1?

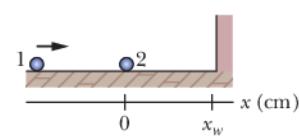


Figure 9-66 Problem 67.

- 68 GO** In Fig. 9-67, block 1 of mass m_1 slides from rest along a frictionless ramp from height $h = 2.50$ m and then collides with stationary block 2, which has mass $m_2 = 2.00m_1$. After the collision, block 2 slides into a region where the coefficient of kinetic friction μ_k is 0.500 and comes to a stop in distance d within that region. What is the value of distance d if the collision is (a) elastic and (b) completely inelastic?

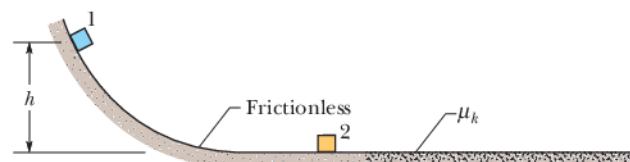


Figure 9-67 Problem 68.

- 69 GO** A small ball of mass m is aligned above a larger ball of mass $M = 0.63$ kg (with a slight separation, as with the baseball and basketball of Fig. 9-68a), and the two are dropped simultaneously from a height of $h = 1.8$ m. (Assume the radius of each ball is negligible relative to h .) (a) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of m results in the larger ball stopping when it collides with the small ball? (b) What height does the small ball then reach (Fig. 9-68b)?

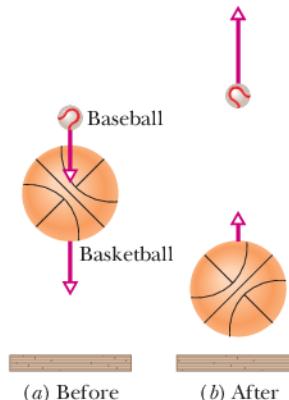


Figure 9-68 Problem 69.

- 70 GO** In Fig. 9-69, puck 1 of mass $m_1 = 0.20$ kg is sent sliding across a frictionless lab bench, to undergo a one-dimensional elastic collision with stationary puck 2. Puck 2 then slides off the bench and lands a distance d from the base of the bench. Puck 1 rebounds from the collision and slides off the opposite edge of the bench, landing a distance $2d$ from the base of the bench. What is the mass of puck 2? (Hint: Be careful with signs.)

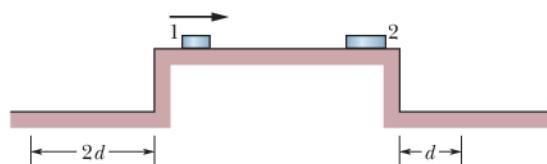


Figure 9-69 Problem 70.

Module 9-8 Collisions in Two Dimensions

•71 ILW In Fig. 9-21, projectile particle 1 is an alpha particle and target particle 2 is an oxygen nucleus. The alpha particle is scattered at angle $\theta_1 = 64.0^\circ$ and the oxygen nucleus recoils with speed 1.20×10^5 m/s and at angle $\theta_2 = 51.0^\circ$. In atomic mass units, the mass of the alpha particle is 4.00 u and the mass of the oxygen nucleus is 16.0 u. What are the (a) final and (b) initial speeds of the alpha particle?

•72 Ball *B*, moving in the positive direction of an *x* axis at speed *v*, collides with stationary ball *A* at the origin. *A* and *B* have different masses. After the collision, *B* moves in the negative direction of the *y* axis at speed *v*/2. (a) In what direction does *A* move? (b) Show that the speed of *A* cannot be determined from the given information.

•73 After a completely inelastic collision, two objects of the same mass and same initial speed move away together at half their initial speed. Find the angle between the initial velocities of the objects.

•74 Two 2.0 kg bodies, *A* and *B*, collide. The velocities before the collision are $\vec{v}_A = (15\hat{i} + 30\hat{j})$ m/s and $\vec{v}_B = (-10\hat{i} + 5.0\hat{j})$ m/s. After the collision, $\vec{v}'_A = (-5.0\hat{i} + 20\hat{j})$ m/s. What are (a) the final velocity of *B* and (b) the change in the total kinetic energy (including sign)?

•75 GO A projectile proton with a speed of 500 m/s collides elastically with a target proton initially at rest. The two protons then move along perpendicular paths, with the projectile path at 60° from the original direction. After the collision, what are the speeds of (a) the target proton and (b) the projectile proton?

Module 9-9 Systems with Varying Mass: A Rocket

•76 A 6090 kg space probe moving nose-first toward Jupiter at 105 m/s relative to the Sun fires its rocket engine, ejecting 80.0 kg of exhaust at a speed of 253 m/s relative to the space probe. What is the final velocity of the probe?

•77 SSM In Fig. 9-70, two long barges are moving in the same direction in still water, one with a speed of 10 km/h and the other with a speed of 20 km/h. While they are passing each other, coal is shoveled from the slower to the faster one at a rate of 1000 kg/min. How much additional force must be provided by the driving engines of (a) the faster barge and (b) the slower barge if neither is to change speed? Assume that the shoveling is always perfectly sideways and that the frictional forces between the barges and the water do not depend on the mass of the barges.

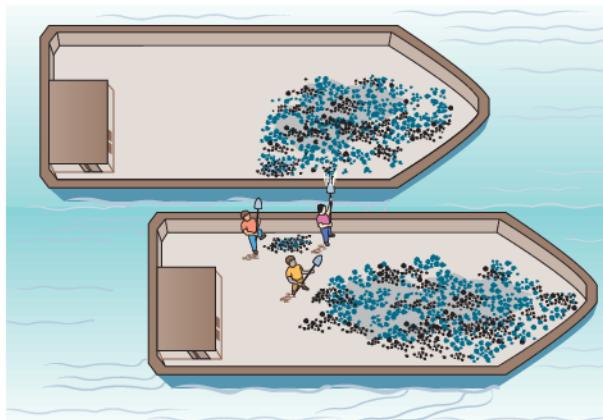


Figure 9-70 Problem 77.

•78 Consider a rocket that is in deep space and at rest relative to an inertial reference frame. The rocket's engine is to be fired for a

certain interval. What must be the rocket's *mass ratio* (ratio of initial to final mass) over that interval if the rocket's original speed relative to the inertial frame is to be equal to (a) the exhaust speed (speed of the exhaust products relative to the rocket) and (b) 2.0 times the exhaust speed?

•79 SSM ILW A rocket that is in deep space and initially at rest relative to an inertial reference frame has a mass of 2.55×10^5 kg, of which 1.81×10^5 kg is fuel. The rocket engine is then fired for 250 s while fuel is consumed at the rate of 480 kg/s. The speed of the exhaust products relative to the rocket is 3.27 km/s. (a) What is the rocket's thrust? After the 250 s firing, what are (b) the mass and (c) the speed of the rocket?

Additional Problems

80 An object is tracked by a radar station and determined to have a position vector given by $\vec{r} = (3500 - 160t)\hat{i} + 2700\hat{j} + 300\hat{k}$, with \vec{r} in meters and *t* in seconds. The radar station's *x* axis points east, its *y* axis north, and its *z* axis vertically up. If the object is a 250 kg meteorological missile, what are (a) its linear momentum, (b) its direction of motion, and (c) the net force on it?

81 The last stage of a rocket, which is traveling at a speed of 7600 m/s, consists of two parts that are clamped together: a rocket case with a mass of 290.0 kg and a payload capsule with a mass of 150.0 kg. When the clamp is released, a compressed spring causes the two parts to separate with a relative speed of 910.0 m/s. What are the speeds of (a) the rocket case and (b) the payload after they have separated? Assume that all velocities are along the same line. Find the total kinetic energy of the two parts (c) before and (d) after they separate. (e) Account for the difference.

82 *Pancake collapse of a tall building.* In the section of a tall building shown in Fig. 9-71a, the infrastructure of any given floor *K* must support the weight *W* of all higher floors. Normally the infrastructure is constructed with a safety factor *s* so that it can withstand an even greater downward force of *sW*. If, however, the support columns between *K* and *L* suddenly

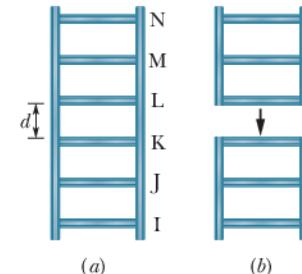


Figure 9-71 Problem 82.

collapse and allow the higher floors to free-fall together onto floor *K* (Fig. 9-71b), the force in the collision can exceed *sW* and, after a brief pause, cause *K* to collapse onto floor *J*, which collapses on floor *I*, and so on until the ground is reached. Assume that the floors are separated by $d = 4.0$ m and have the same mass. Also assume that when the floors above *K* free-fall onto *K*, the collision lasts 1.5 ms. Under these simplified conditions, what value must the safety factor *s* exceed to prevent pancake collapse of the building?

83 “*Relative*” is an important word. In Fig. 9-72, block *L* of mass $m_L = 1.00$ kg and block *R* of mass $m_R = 0.500$ kg are held in place with a compressed spring between them.

When the blocks are released, the spring sends them sliding across a frictionless floor. (The spring has negligible mass and falls to the floor after the blocks leave it.) (a) If the spring gives block *L* a release speed of 1.20 m/s relative to the floor, how far does block *R* travel in the next 0.800 s? (b) If, instead, the spring gives block *L* a release speed of 1.20 m/s relative to the velocity that the spring gives block *R*, how far does block *R* travel in the next 0.800 s?

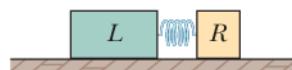


Figure 9-72 Problem 83.

84 Figure 9-73 shows an overhead view of two particles sliding at constant velocity over a frictionless surface. The particles have the same mass and the same initial speed $v = 4.00 \text{ m/s}$, and they collide where their paths intersect. An x axis is arranged to bisect the angle between their incoming paths, such that $\theta = 40.0^\circ$. The region to the right of the collision is divided into four lettered sections by the x axis and four numbered dashed lines. In what region or along what line do the particles travel if the collision is (a) completely inelastic, (b) elastic, and (c) inelastic? What are their final speeds if the collision is (d) completely inelastic and (e) elastic?

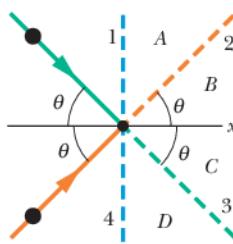


Figure 9-73 Problem 84.

85 **Speed deamplifier.** In Fig. 9-74, block 1 of mass m_1 slides along an x axis on a frictionless floor at speed 4.00 m/s . Then it undergoes a one-dimensional elastic collision with stationary block 2 of mass $m_2 = 2.00m_1$. Next, block 2 undergoes a one-dimensional elastic collision with stationary block 3 of mass $m_3 = 2.00m_2$. (a) What then is the speed of block 3? Are (b) the speed, (c) the kinetic energy, and (d) the momentum of block 3 greater than, less than, or the same as the initial values for block 1?

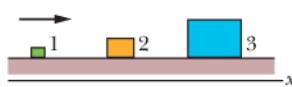


Figure 9-74 Problem 85.

86 **Speed amplifier.** In Fig. 9-75, block 1 of mass m_1 slides along an x axis on a frictionless floor with a speed of $v_{1i} = 4.00 \text{ m/s}$. Then it undergoes a one-dimensional elastic collision with stationary block 2 of mass $m_2 = 0.500m_1$. Next, block 2 undergoes a one-dimensional elastic collision with stationary block 3 of mass $m_3 = 0.500m_2$. (a) What then is the speed of block 3? Are (b) the speed, (c) the kinetic energy, and (d) the momentum of block 3 greater than, less than, or the same as the initial values for block 1?

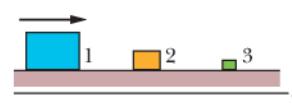


Figure 9-75 Problem 86.

87 A ball having a mass of 150 g strikes a wall with a speed of 5.2 m/s and rebounds with only 50% of its initial kinetic energy. (a) What is the speed of the ball immediately after rebounding? (b) What is the magnitude of the impulse on the wall from the ball? (c) If the ball is in contact with the wall for 7.6 ms , what is the magnitude of the average force on the ball from the wall during this time interval?

88 A spacecraft is separated into two parts by detonating the explosive bolts that hold them together. The masses of the parts are 1200 kg and 1800 kg ; the magnitude of the impulse on each part from the bolts is $300 \text{ N}\cdot\text{s}$. With what relative speed do the two parts separate because of the detonation?

89 A 1400 kg car moving at 5.3 m/s is initially traveling north along the positive direction of a y axis. After completing a 90° right-hand turn in 4.6 s , the inattentive operator drives into a tree, which stops the car in 350 ms . In unit-vector notation, what is the impulse on the car (a) due to the turn and (b) due to the collision? What is the magnitude of the average force that acts on the car (c) during the turn and (d) during the collision? (e) What is the direction of the average force during the turn?

90 A certain radioactive (parent) nucleus transforms to a different (daughter) nucleus by emitting an electron and a neutrino. The parent nucleus was at rest at the origin of an xy coordinate system. The electron moves away from the origin with linear momentum $(-1.2 \times 10^{-22} \text{ kg}\cdot\text{m/s})\hat{i}$; the neutrino moves away from the

origin with linear momentum $(-6.4 \times 10^{-23} \text{ kg}\cdot\text{m/s})\hat{j}$. What are the (a) magnitude and (b) direction of the linear momentum of the daughter nucleus? (c) If the daughter nucleus has a mass of $5.8 \times 10^{-26} \text{ kg}$, what is its kinetic energy?

91 A 75 kg man rides on a 39 kg cart moving at a velocity of 2.3 m/s . He jumps off with zero horizontal velocity relative to the ground. What is the resulting change in the cart's velocity, including sign?

92 Two blocks of masses 1.0 kg and 3.0 kg are connected by a spring and rest on a frictionless surface. They are given velocities toward each other such that the 1.0 kg block travels initially at 1.7 m/s toward the center of mass, which remains at rest. What is the initial speed of the other block?

93 A railroad freight car of mass $3.18 \times 10^4 \text{ kg}$ collides with a stationary caboose car. They couple together, and 27.0% of the initial kinetic energy is transferred to thermal energy, sound, vibrations, and so on. Find the mass of the caboose.

94 An old Chrysler with mass 2400 kg is moving along a straight stretch of road at 80 km/h . It is followed by a Ford with mass 1600 kg moving at 60 km/h . How fast is the center of mass of the two cars moving?

95 In the arrangement of Fig. 9-21, billiard ball 1 moving at a speed of 2.2 m/s undergoes a glancing collision with identical billiard ball 2 that is at rest. After the collision, ball 2 moves at speed 1.1 m/s , at an angle of $\theta_2 = 60^\circ$. What are (a) the magnitude and (b) the direction of the velocity of ball 1 after the collision? (c) Do the given data suggest the collision is elastic or inelastic?

96 A rocket is moving away from the solar system at a speed of $6.0 \times 10^3 \text{ m/s}$. It fires its engine, which ejects exhaust with a speed of $3.0 \times 10^3 \text{ m/s}$ relative to the rocket. The mass of the rocket at this time is $4.0 \times 10^4 \text{ kg}$, and its acceleration is 2.0 m/s^2 . (a) What is the thrust of the engine? (b) At what rate, in kilograms per second, is exhaust ejected during the firing?

97 The three balls in the overhead view of Fig. 9-76 are identical. Balls 2 and 3 touch each other and are aligned perpendicular to the path of ball 1. The velocity of ball 1 has magnitude $v_0 = 10 \text{ m/s}$ and is directed at the contact point of balls 1 and 2. After the collision, what are the (a) speed and (b) direction of the velocity of ball 2, the (c) speed and (d) direction of the velocity of ball 3, and the (e) speed and (f) direction of the velocity of ball 1? (Hint: With friction absent, each impulse is directed along the line connecting the centers of the colliding balls, normal to the path of the ball.)

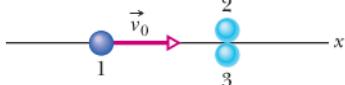
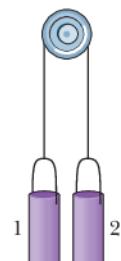


Figure 9-76 Problem 97.

The velocity of ball 1 has magnitude $v_0 = 10 \text{ m/s}$ and is directed at the contact point of balls 1 and 2. After the collision, what are the (a) speed and (b) direction of the velocity of ball 2, the (c) speed and (d) direction of the velocity of ball 3, and the (e) speed and (f) direction of the velocity of ball 1? (Hint: With friction absent, each impulse is directed along the line connecting the centers of the colliding balls, normal to the path of the ball.)

98 A 0.15 kg ball hits a wall with a velocity of $(5.00 \text{ m/s})\hat{i} + (6.50 \text{ m/s})\hat{j} + (4.00 \text{ m/s})\hat{k}$. It rebounds from the wall with a velocity of $(2.00 \text{ m/s})\hat{i} + (3.50 \text{ m/s})\hat{j} + (-3.20 \text{ m/s})\hat{k}$. What are (a) the change in the ball's momentum, (b) the impulse on the ball, and (c) the impulse on the wall?

99 In Fig. 9-77, two identical containers of sugar are connected by a cord that passes over a frictionless pulley. The cord and pulley have negligible mass, each container and its sugar together have a mass of 500 g , the centers of the containers are separated by 50 mm , and the containers are held fixed at the same height. What is the horizontal distance between the center of container 1 and the center of mass of the two-container system (a) initially and

Figure 9-77
Problem 99.

(b) after 20 g of sugar is transferred from container 1 to container 2? After the transfer and after the containers are released, (c) in what direction and (d) at what acceleration magnitude does the center of mass move?

100 In a game of pool, the cue ball strikes another ball of the same mass and initially at rest. After the collision, the cue ball moves at 3.50 m/s along a line making an angle of 22.0° with the cue ball's original direction of motion, and the second ball has a speed of 2.00 m/s. Find (a) the angle between the direction of motion of the second ball and the original direction of motion of the cue ball and (b) the original speed of the cue ball. (c) Is kinetic energy (of the centers of mass, don't consider the rotation) conserved?

101 In Fig. 9-78, a 3.2 kg box of running shoes slides on a horizontal frictionless table and collides with a 2.0 kg box of ballet slippers initially at rest on the edge of the table, at height $h = 0.40$ m. The speed of the 3.2 kg box is 3.0 m/s just before the collision. If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

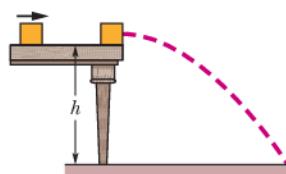


Figure 9-78 Problem 101.

102 In Fig. 9-79, an 80 kg man is on a ladder hanging from a balloon that has a total mass of 320 kg (including the basket passenger). The balloon is initially stationary relative to the ground. If the man on the ladder begins to climb at 2.5 m/s relative to the ladder, (a) in what direction and (b) at what speed does the balloon move? (c) If the man then stops climbing, what is the speed of the balloon?



Figure 9-79
Problem 102.

103 In Fig. 9-80, block 1 of mass $m_1 = 6.6$ kg is at rest on a long frictionless table that is up against a wall. Block 2 of mass m_2 is placed between block 1 and the wall and sent sliding to the left, toward block 1, with constant speed v_{2i} . Find the value of m_2 for which both blocks move with the same velocity after block 2 has collided once with block 1 and once with the wall. Assume all collisions are elastic (the collision with the wall does not change the speed of block 2).

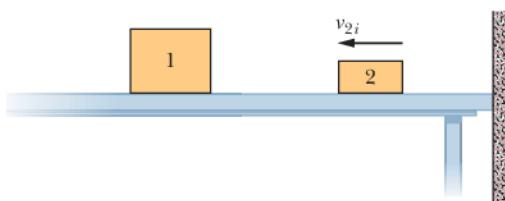


Figure 9-80 Problem 103.

104 The script for an action movie calls for a small race car (of mass 1500 kg and length 3.0 m) to accelerate along a flattop boat (of mass 4000 kg and length 14 m), from one end of the boat to the other, where the car will then jump the gap between the boat and a somewhat lower dock. You are the technical advisor for the movie. The

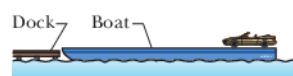


Figure 9-81 Problem 104.

boat will initially touch the dock, as in Fig. 9-81; the boat can slide through the water without significant resistance; both the car and the boat can be approximated as uniform in their mass distribution. Determine what the width of the gap will be just as the car is about to make the jump.

105 SSM A 3.0 kg object moving at 8.0 m/s in the positive direction of an x axis has a one-dimensional elastic collision with an object of mass M , initially at rest. After the collision the object of mass M has a velocity of 6.0 m/s in the positive direction of the axis. What is mass M ?

106 A 2140 kg railroad flatcar, which can move with negligible friction, is motionless next to a platform. A 242 kg sumo wrestler runs at 5.3 m/s along the platform (parallel to the track) and then jumps onto the flatcar. What is the speed of the flatcar if he then (a) stands on it, (b) runs at 5.3 m/s relative to it in his original direction, and (c) turns and runs at 5.3 m/s relative to the flatcar opposite his original direction?

107 SSM A 6100 kg rocket is set for vertical firing from the ground. If the exhaust speed is 1200 m/s, how much gas must be ejected each second if the thrust (a) is to equal the magnitude of the gravitational force on the rocket and (b) is to give the rocket an initial upward acceleration of 21 m/s^2 ?

108 A 500.0 kg module is attached to a 400.0 kg shuttle craft, which moves at 1000 m/s relative to the stationary main spaceship. Then a small explosion sends the module backward with speed 100.0 m/s relative to the new speed of the shuttle craft. As measured by someone on the main spaceship, by what fraction did the kinetic energy of the module and shuttle craft increase because of the explosion?

109 SSM (a) How far is the center of mass of the Earth–Moon system from the center of Earth? (Appendix C gives the masses of Earth and the Moon and the distance between the two.) (b) What percentage of Earth's radius is that distance?

110 A 140 g ball with speed 7.8 m/s strikes a wall perpendicularly and rebounds in the opposite direction with the same speed. The collision lasts 3.80 ms. What are the magnitudes of the (a) impulse and (b) average force on the wall from the ball during the elastic collision?

111 SSM A rocket sled with a mass of 2900 kg moves at 250 m/s on a set of rails. At a certain point, a scoop on the sled dips into a trough of water located between the tracks and scoops water into an empty tank on the sled. By applying the principle of conservation of linear momentum, determine the speed of the sled after 920 kg of water has been scooped up. Ignore any retarding force on the scoop.

112 SSM A pellet gun fires ten 2.0 g pellets per second with a speed of 500 m/s. The pellets are stopped by a rigid wall. What are (a) the magnitude of the momentum of each pellet, (b) the kinetic energy of each pellet, and (c) the magnitude of the average force on the wall from the stream of pellets? (d) If each pellet is in contact with the wall for 0.60 ms, what is the magnitude of the average force on the wall from each pellet during contact? (e) Why is this average force so different from the average force calculated in (c)?

113 A railroad car moves under a grain elevator at a constant speed of 3.20 m/s. Grain drops into the car at the rate of 540 kg/min. What is the magnitude of the force needed to keep the car moving at constant speed if friction is negligible?

- 114** Figure 9-82 shows a uniform square plate of edge length $6d = 6.0\text{ m}$ from which a square piece of edge length $2d$ has been removed. What are (a) the x coordinate and (b) the y coordinate of the center of mass of the remaining piece?

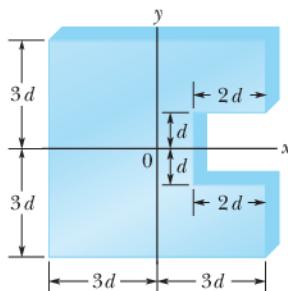


Figure 9-82 Problem 114.

- 115 [SSM]** At time $t = 0$, force $\vec{F}_1 = (-4.00\hat{i} + 5.00\hat{j})\text{ N}$ acts on an initially stationary particle of mass $2.00 \times 10^{-3}\text{ kg}$ and force $\vec{F}_2 = (2.00\hat{i} - 4.00\hat{j})\text{ N}$ acts on an initially stationary particle of mass $4.00 \times 10^{-3}\text{ kg}$. From time $t = 0$ to $t = 2.00\text{ ms}$, what are the (a) magnitude and (b) angle (relative to the positive direction of the x axis) of the displacement of the center of mass of the two-particle system? (c) What is the kinetic energy of the center of mass at $t = 2.00\text{ ms}$?

- 116** Two particles P and Q are released from rest 1.0 m apart. P has a mass of 0.10 kg , and Q a mass of 0.30 kg . P and Q attract each other with a constant force of $1.0 \times 10^{-2}\text{ N}$. No external forces act on the system. (a) What is the speed of the center of mass of P and Q when the separation is 0.50 m ? (b) At what distance from P 's original position do the particles collide?

- 117** A collision occurs between a 2.00 kg particle traveling with velocity $\vec{v}_1 = (-4.00\text{ m/s})\hat{i} + (-5.00\text{ m/s})\hat{j}$ and a 4.00 kg particle traveling with velocity $\vec{v}_2 = (6.00\text{ m/s})\hat{i} + (-2.00\text{ m/s})\hat{j}$. The collision connects the two particles. What then is their velocity in (a) unit-vector notation and as a (b) magnitude and (c) angle?

- 118** In the two-sphere arrangement of Fig. 9-20, assume that sphere 1 has a mass of 50 g and an initial height of $h_1 = 9.0\text{ cm}$, and that sphere 2 has a mass of 85 g . After sphere 1 is released and collides elastically with sphere 2, what height is reached by (a) sphere 1 and (b) sphere 2? After the next (elastic) collision, what height is reached by (c) sphere 1 and (d) sphere 2? (*Hint:* Do not use rounded-off values.)

- 119** In Fig. 9-83, block 1 slides along an x axis on a frictionless floor with a speed of 0.75 m/s . When it reaches stationary block 2, the two blocks undergo an elastic collision. The following table gives the mass and length of the (uniform) blocks and also the locations of their centers at time $t = 0$. Where is the center of mass of the two-block system located (a) at $t = 0$, (b) when the two blocks first touch, and (c) at $t = 4.0\text{ s}$?



Figure 9-83 Problem 119.

Block	Mass (kg)	Length (cm)	Center at $t = 0$
1	0.25	5.0	$x = -1.50\text{ m}$
2	0.50	6.0	$x = 0$

- 120** A body is traveling at 2.0 m/s along the positive direction of an x axis; no net force acts on the body. An internal explosion sepa-

rates the body into two parts, each of 4.0 kg , and increases the total kinetic energy by 16 J . The forward part continues to move in the original direction of motion. What are the speeds of (a) the rear part and (b) the forward part?

- 121** An electron undergoes a one-dimensional elastic collision with an initially stationary hydrogen atom. What percentage of the electron's initial kinetic energy is transferred to kinetic energy of the hydrogen atom? (The mass of the hydrogen atom is 1840 times the mass of the electron.)

- 122** A man (weighing 915 N) stands on a long railroad flatcar (weighing 2415 N) as it rolls at 18.2 m/s in the positive direction of an x axis, with negligible friction. Then the man runs along the flatcar in the negative x direction at 4.00 m/s relative to the flatcar. What is the resulting increase in the speed of the flatcar?

- 123** An unmanned space probe (of mass m and speed v relative to the Sun) approaches the planet Jupiter (of mass M and speed V_J relative to the Sun) as shown in Fig. 9-84. The spacecraft rounds the planet and departs in the opposite direction. What is its speed (in kilometers per second), relative to the Sun, after this slingshot encounter, which can be analyzed as a collision? Assume $v = 10.5\text{ km/s}$ and $V_J = 13.0\text{ km/s}$ (the orbital speed of Jupiter). The mass of Jupiter is very much greater than the mass of the spacecraft ($M \gg m$).

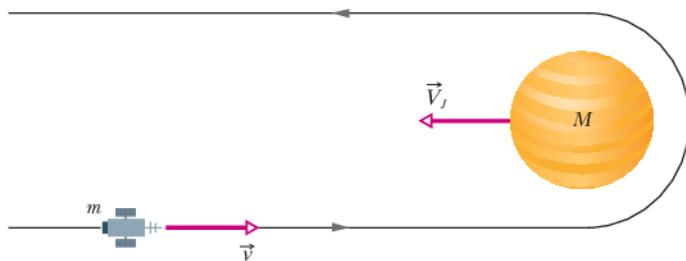


Figure 9-84 Problem 123.

- 124** A 0.550 kg ball falls directly down onto concrete, hitting it with a speed of 12.0 m/s and rebounding directly upward with a speed of 3.00 m/s . Extend a y axis upward. In unit-vector notation, what are (a) the change in the ball's momentum, (b) the impulse on the ball, and (c) the impulse on the concrete?

- 125** An atomic nucleus at rest at the origin of an xy coordinate system transforms into three particles. Particle 1, mass $16.7 \times 10^{-27}\text{ kg}$, moves away from the origin at velocity $(6.00 \times 10^6\text{ m/s})\hat{i}$; particle 2, mass $8.35 \times 10^{-27}\text{ kg}$, moves away at velocity $(-8.00 \times 10^6\text{ m/s})\hat{j}$. (a) In unit-vector notation, what is the linear momentum of the third particle, mass $11.7 \times 10^{-27}\text{ kg}$? (b) How much kinetic energy appears in this transformation?

- 126** Particle 1 of mass 200 g and speed 3.00 m/s undergoes a one-dimensional collision with stationary particle 2 of mass 400 g . What is the magnitude of the impulse on particle 1 if the collision is (a) elastic and (b) completely inelastic?

- 127** During a lunar mission, it is necessary to increase the speed of a spacecraft by 2.2 m/s when it is moving at 400 m/s relative to the Moon. The speed of the exhaust products from the rocket engine is 1000 m/s relative to the spacecraft. What fraction of the initial mass of the spacecraft must be burned and ejected to accomplish the speed increase?

- 128** A cue stick strikes a stationary pool ball, with an average force of 32 N over a time of 14 ms . If the ball has mass 0.20 kg , what speed does it have just after impact?

Rotation

10-1 ROTATIONAL VARIABLES

Learning Objectives

After reading this module, you should be able to . . .

- 10.01** Identify that if all parts of a body rotate around a fixed axis locked together, the body is a rigid body. (This chapter is about the motion of such bodies.)
- 10.02** Identify that the angular position of a rotating rigid body is the angle that an internal reference line makes with a fixed, external reference line.
- 10.03** Apply the relationship between angular displacement and the initial and final angular positions.
- 10.04** Apply the relationship between average angular velocity, angular displacement, and the time interval for that displacement.
- 10.05** Apply the relationship between average angular acceleration, change in angular velocity, and the time interval for that change.
- 10.06** Identify that counterclockwise motion is in the positive direction and clockwise motion is in the negative direction.
- 10.07** Given angular position as a *function of time*, calculate the instantaneous angular velocity at any particular time and the average angular velocity between any two particular times.

Key Ideas

- To describe the rotation of a rigid body about a fixed axis, called the rotation axis, we assume a reference line is fixed in the body, perpendicular to that axis and rotating with the body. We measure the angular position θ of this line relative to a fixed direction. When θ is measured in radians,

$$\theta = \frac{s}{r} \quad (\text{radian measure}),$$

where s is the arc length of a circular path of radius r and angle θ .

- Radian measure is related to angle measure in revolutions and degrees by

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad.}$$

- A body that rotates about a rotation axis, changing its angular position from θ_1 to θ_2 , undergoes an angular displacement

$$\Delta\theta = \theta_2 - \theta_1,$$

where $\Delta\theta$ is positive for counterclockwise rotation and negative for clockwise rotation.

- If a body rotates through an angular displacement $\Delta\theta$ in a time interval Δt , its average angular velocity ω_{avg} is

- 10.08** Given a *graph* of angular position versus time, determine the instantaneous angular velocity at a particular time and the average angular velocity between any two particular times.
- 10.09** Identify instantaneous angular speed as the magnitude of the instantaneous angular velocity.
- 10.10** Given angular velocity as a *function of time*, calculate the instantaneous angular acceleration at any particular time and the average angular acceleration between any two particular times.
- 10.11** Given a *graph* of angular velocity versus time, determine the instantaneous angular acceleration at any particular time and the average angular acceleration between any two particular times.
- 10.12** Calculate a body's change in angular velocity by integrating its angular acceleration function with respect to time.
- 10.13** Calculate a body's change in angular position by integrating its angular velocity function with respect to time.

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}.$$

The (instantaneous) angular velocity ω of the body is

$$\omega = \frac{d\theta}{dt}.$$

Both ω_{avg} and ω are vectors, with directions given by a right-hand rule. They are positive for counterclockwise rotation and negative for clockwise rotation. The magnitude of the body's angular velocity is the angular speed.

- If the angular velocity of a body changes from ω_1 to ω_2 in a time interval $\Delta t = t_2 - t_1$, the average angular acceleration α_{avg} of the body is

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}.$$

The (instantaneous) angular acceleration α of the body is

$$\alpha = \frac{d\omega}{dt}.$$

Both α_{avg} and α are vectors.

What Is Physics?

As we have discussed, one focus of physics is motion. However, so far we have examined only the motion of **translation**, in which an object moves along a straight or curved line, as in Fig. 10-1a. We now turn to the motion of **rotation**, in which an object turns about an axis, as in Fig. 10-1b.

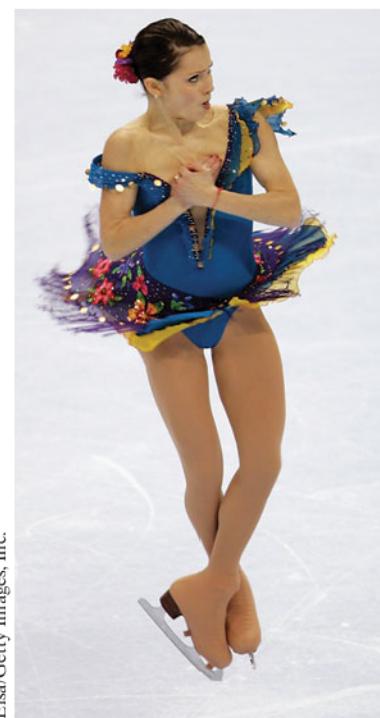
You see rotation in nearly every machine, you use it every time you open a beverage can with a pull tab, and you pay to experience it every time you go to an amusement park. Rotation is the key to many fun activities, such as hitting a long drive in golf (the ball needs to rotate in order for the air to keep it aloft longer) and throwing a curveball in baseball (the ball needs to rotate in order for the air to push it left or right). Rotation is also the key to more serious matters, such as metal failure in aging airplanes.

We begin our discussion of rotation by defining the variables for the motion, just as we did for translation in Chapter 2. As we shall see, the variables for rotation are analogous to those for one-dimensional motion and, as in Chapter 2, an important special situation is where the acceleration (here the rotational acceleration) is constant. We shall also see that Newton's second law can be written for rotational motion, but we must use a new quantity called *torque* instead of just force. Work and the work–kinetic energy theorem can also be applied to rotational motion, but we must use a new quantity called *rotational inertia* instead of just mass. In short, much of what we have discussed so far can be applied to rotational motion with, perhaps, a few changes.

Caution: In spite of this repetition of physics ideas, many students find this and the next chapter very challenging. Instructors have a variety of reasons as to why, but two reasons stand out: (1) There are a lot of symbols (with Greek



(a)



(b)

Figure 10-1 Figure skater Sasha Cohen in motion of (a) pure translation in a fixed direction and (b) pure rotation about a vertical axis.

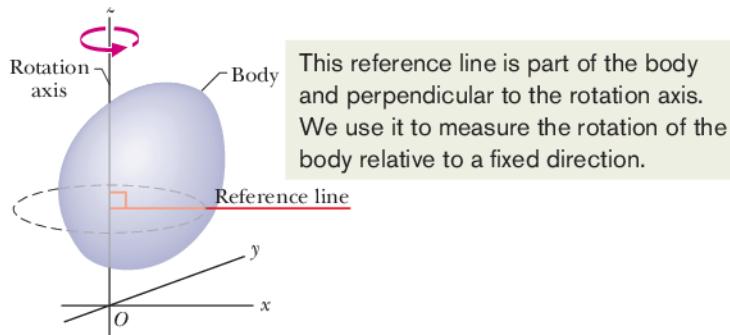


Figure 10-2 A rigid body of arbitrary shape in pure rotation about the z axis of a coordinate system. The position of the *reference line* with respect to the rigid body is arbitrary, but it is perpendicular to the rotation axis. It is fixed in the body and rotates with the body.

letters) to sort out. (2) Although you are very familiar with linear motion (you can get across the room and down the road just fine), you are probably very unfamiliar with rotation (and that is one reason why you are willing to pay so much for amusement park rides). If a homework problem looks like a foreign language to you, see if translating it into the one-dimensional linear motion of Chapter 2 helps. For example, if you are to find, say, an *angular* distance, temporarily delete the word *angular* and see if you can work the problem with the Chapter 2 notation and ideas.

Rotational Variables

We wish to examine the rotation of a rigid body about a fixed axis. A **rigid body** is a body that can rotate with all its parts locked together and without any change in its shape. A **fixed axis** means that the rotation occurs about an axis that does not move. Thus, we shall not examine an object like the Sun, because the parts of the Sun (a ball of gas) are not locked together. We also shall not examine an object like a bowling ball rolling along a lane, because the ball rotates about a moving axis (the ball's motion is a mixture of rotation and translation).

Figure 10-2 shows a rigid body of arbitrary shape in rotation about a fixed axis, called the **axis of rotation** or the **rotation axis**. In pure rotation (*angular motion*), every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval. In pure translation (*linear motion*), every point of the body moves in a straight line, and every point moves through the same *linear distance* during a particular time interval.

We deal now—one at a time—with the angular equivalents of the linear quantities position, displacement, velocity, and acceleration.

Angular Position

Figure 10-2 shows a *reference line*, fixed in the body, perpendicular to the rotation axis and rotating with the body. The **angular position** of this line is the angle of the line relative to a fixed direction, which we take as the **zero angular position**. In Fig. 10-3, the angular position θ is measured relative to the positive direction of the x axis. From geometry, we know that θ is given by

$$\theta = \frac{s}{r} \quad (\text{radian measure}). \quad (10-1)$$

Here s is the length of a circular arc that extends from the x axis (the zero angular position) to the reference line, and r is the radius of the circle.

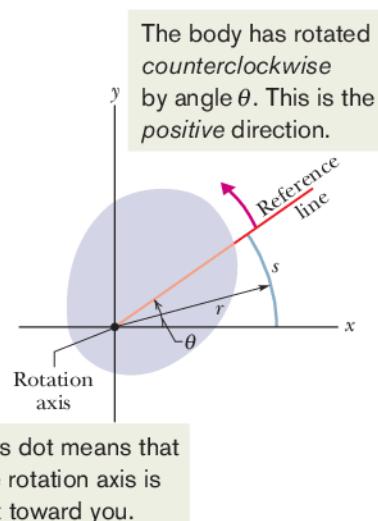


Figure 10-3 The rotating rigid body of Fig. 10-2 in cross section, viewed from above. The plane of the cross section is perpendicular to the rotation axis, which now extends out of the page, toward you. In this position of the body, the reference line makes an angle θ with the x axis.

An angle defined in this way is measured in **radians** (rad) rather than in revolutions (rev) or degrees. The radian, being the ratio of two lengths, is a pure number and thus has no dimension. Because the circumference of a circle of radius r is $2\pi r$, there are 2π radians in a complete circle:

$$1 \text{ rev} = 360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad}, \quad (10-2)$$

and thus

$$1 \text{ rad} = 57.3^\circ = 0.159 \text{ rev}. \quad (10-3)$$

We do *not* reset θ to zero with each complete rotation of the reference line about the rotation axis. If the reference line completes two revolutions from the zero angular position, then the angular position θ of the line is $\theta = 4\pi$ rad.

For pure translation along an x axis, we can know all there is to know about a moving body if we know $x(t)$, its position as a function of time. Similarly, for pure rotation, we can know all there is to know about a rotating body if we know $\theta(t)$, the angular position of the body's reference line as a function of time.

Angular Displacement

If the body of Fig. 10-3 rotates about the rotation axis as in Fig. 10-4, changing the angular position of the reference line from θ_1 to θ_2 , the body undergoes an **angular displacement** $\Delta\theta$ given by

$$\Delta\theta = \theta_2 - \theta_1. \quad (10-4)$$

This definition of angular displacement holds not only for the rigid body as a whole but also for *every particle within that body*.

Clocks Are Negative. If a body is in translational motion along an x axis, its displacement Δx is either positive or negative, depending on whether the body is moving in the positive or negative direction of the axis. Similarly, the angular displacement $\Delta\theta$ of a rotating body is either positive or negative, according to the following rule:



An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.

The phrase “*clocks are negative*” can help you remember this rule (they certainly are negative when their alarms sound off early in the morning).



Checkpoint 1

A disk can rotate about its central axis like a merry-go-round. Which of the following pairs of values for its initial and final angular positions, respectively, give a negative angular displacement: (a) -3 rad, $+5$ rad, (b) -3 rad, -7 rad, (c) 7 rad, -3 rad?

Angular Velocity

Suppose that our rotating body is at angular position θ_1 at time t_1 and at angular position θ_2 at time t_2 as in Fig. 10-4. We define the **average angular velocity** of the body in the time interval Δt from t_1 to t_2 to be

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}, \quad (10-5)$$

where $\Delta\theta$ is the angular displacement during Δt (ω is the lowercase omega).

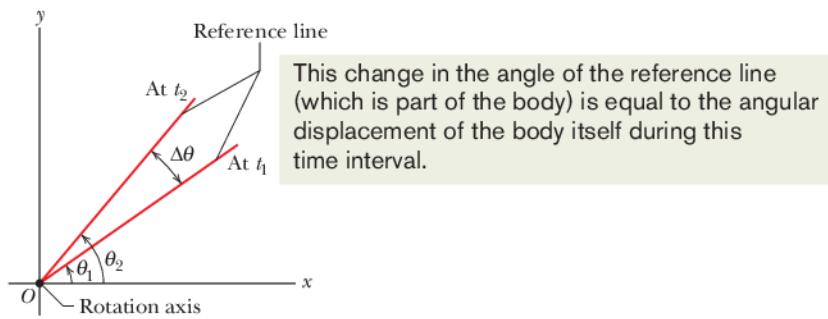


Figure 10-4 The reference line of the rigid body of Figs. 10-2 and 10-3 is at angular position θ_1 at time t_1 and at angular position θ_2 at a later time t_2 . The quantity $\Delta\theta (= \theta_2 - \theta_1)$ is the angular displacement that occurs during the interval $\Delta t (= t_2 - t_1)$. The body itself is not shown.

The (**instantaneous**) **angular velocity** ω , with which we shall be most concerned, is the limit of the ratio in Eq. 10-5 as Δt approaches zero. Thus,

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \quad (10-6)$$

If we know $\theta(t)$, we can find the angular velocity ω by differentiation.

Equations 10-5 and 10-6 hold not only for the rotating rigid body as a whole but also for *every particle of that body* because the particles are all locked together. The unit of angular velocity is commonly the radian per second (rad/s) or the revolution per second (rev/s). Another measure of angular velocity was used during at least the first three decades of rock: Music was produced by vinyl (phonograph) records that were played on turntables at “33 $\frac{1}{3}$ rpm” or “45 rpm,” meaning at 33 $\frac{1}{3}$ rev/min or 45 rev/min.

If a particle moves in translation along an x axis, its linear velocity v is either positive or negative, depending on its direction along the axis. Similarly, the angular velocity ω of a rotating rigid body is either positive or negative, depending on whether the body is rotating counterclockwise (positive) or clockwise (negative). (“Clocks are negative” still works.) The magnitude of an angular velocity is called the **angular speed**, which is also represented with ω .

Angular Acceleration

If the angular velocity of a rotating body is not constant, then the body has an angular acceleration. Let ω_2 and ω_1 be its angular velocities at times t_2 and t_1 , respectively. The **average angular acceleration** of the rotating body in the interval from t_1 to t_2 is defined as

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}, \quad (10-7)$$

in which $\Delta\omega$ is the change in the angular velocity that occurs during the time interval Δt . The (**instantaneous**) **angular acceleration** α , with which we shall be most concerned, is the limit of this quantity as Δt approaches zero. Thus,

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}. \quad (10-8)$$

As the name suggests, this is the angular acceleration of the body at a given instant. Equations 10-7 and 10-8 also hold for *every particle of that body*. The unit of angular acceleration is commonly the radian per second-squared (rad/s²) or the revolution per second-squared (rev/s²).



Sample Problem 10.01 Angular velocity derived from angular position

The disk in Fig. 10-5a is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by

$$\theta = -1.00 - 0.600t + 0.250t^2, \quad (10-9)$$

with t in seconds, θ in radians, and the zero angular position as indicated in the figure. (If you like, you can translate all this into Chapter 2 notation by momentarily dropping the word “angular” from “angular position” and replacing the symbol θ with the symbol x . What you then have is an equation that gives the position as a function of time, for the one-dimensional motion of Chapter 2.)

- (a) Graph the angular position of the disk versus time from $t = -3.0$ s to $t = 5.4$ s. Sketch the disk and its angular position reference line at $t = -2.0$ s, 0 s, and 4.0 s, and when the curve crosses the t axis.

KEY IDEA

The angular position of the disk is the angular position $\theta(t)$ of its reference line, which is given by Eq. 10-9 as a function of time t . So we graph Eq. 10-9; the result is shown in Fig. 10-5b.

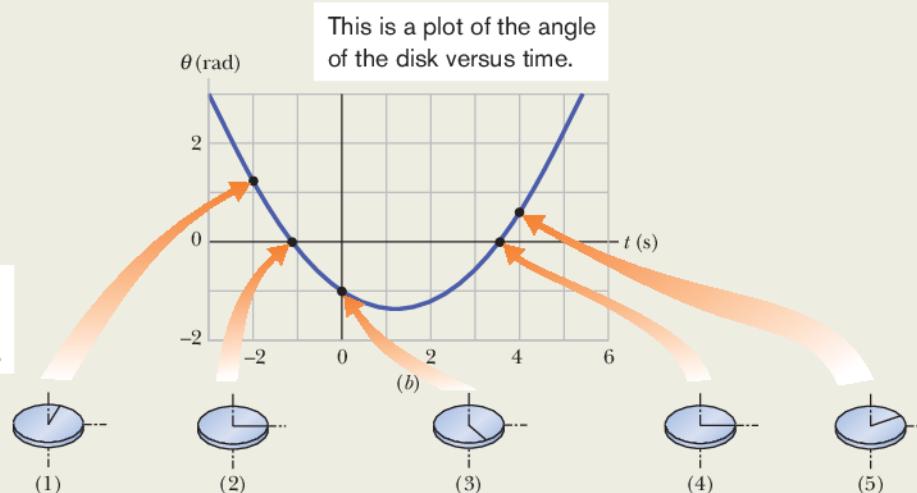
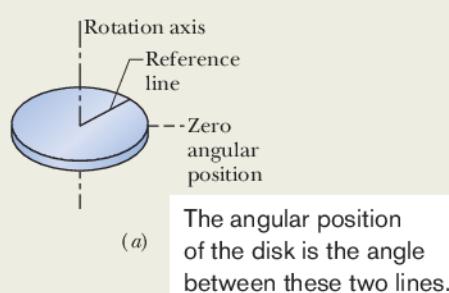
Calculations: To sketch the disk and its reference line at a particular time, we need to determine θ for that time. To do so, we substitute the time into Eq. 10-9. For $t = -2.0$ s, we get

$$\begin{aligned}\theta &= -1.00 - (0.600)(-2.0) + (0.250)(-2.0)^2 \\ &= 1.2 \text{ rad} = 1.2 \text{ rad} \frac{360^\circ}{2\pi \text{ rad}} = 69^\circ.\end{aligned}$$

This means that at $t = -2.0$ s the reference line on the disk is rotated counterclockwise from the zero position by angle $1.2 \text{ rad} = 69^\circ$ (counterclockwise because θ is positive). Sketch 1 in Fig. 10-5b shows this position of the reference line.

Similarly, for $t = 0$, we find $\theta = -1.00 \text{ rad} = -57^\circ$, which means that the reference line is rotated clockwise from the zero angular position by 1.0 rad , or 57° , as shown in sketch 3. For $t = 4.0$ s, we find $\theta = 0.60 \text{ rad} = 34^\circ$ (sketch 5). Drawing sketches for when the curve crosses the t axis is easy, because then $\theta = 0$ and the reference line is momentarily aligned with the zero angular position (sketches 2 and 4).

- (b) At what time t_{\min} does $\theta(t)$ reach the minimum value shown in Fig. 10-5b? What is that minimum value?



At $t = -2$ s, the disk is at a positive (counterclockwise) angle. So, a positive θ value is plotted.

Now, the disk is at a zero angle.

Now, it is at a negative (clockwise) angle. So, a negative θ value is plotted.

It has reversed its rotation and is again at a zero angle.

Now, it is back at a positive angle.

Figure 10-5 (a) A rotating disk. (b) A plot of the disk's angular position $\theta(t)$. Five sketches indicate the angular position of the reference line on the disk for five points on the curve. (c) A plot of the disk's angular velocity $\omega(t)$. Positive values of ω correspond to counterclockwise rotation, and negative values to clockwise rotation.

KEY IDEA

To find the extreme value (here the minimum) of a function, we take the first derivative of the function and set the result to zero.

Calculations: The first derivative of $\theta(t)$ is

$$\frac{d\theta}{dt} = -0.600 + 0.500t. \quad (10-10)$$

Setting this to zero and solving for t give us the time at which $\theta(t)$ is minimum:

$$t_{\min} = 1.20 \text{ s.} \quad (\text{Answer})$$

To get the minimum value of θ , we next substitute t_{\min} into Eq. 10-9, finding

$$\theta = -1.36 \text{ rad} \approx -77.9^\circ. \quad (\text{Answer})$$

This *minimum* of $\theta(t)$ (the bottom of the curve in Fig. 10-5b) corresponds to the *maximum clockwise* rotation of the disk from the zero angular position, somewhat more than is shown in sketch 3.

(c) Graph the angular velocity ω of the disk versus time from

$t = -3.0 \text{ s}$ to $t = 6.0 \text{ s}$. Sketch the disk and indicate the direction of turning and the sign of ω at $t = -2.0 \text{ s}$, 4.0 s , and t_{\min} .

KEY IDEA

From Eq. 10-6, the angular velocity ω is equal to $d\theta/dt$ as given in Eq. 10-10. So, we have

$$\omega = -0.600 + 0.500t. \quad (10-11)$$

The graph of this function $\omega(t)$ is shown in Fig. 10-5c. Because the function is linear, the plot is a straight line. The slope is 0.500 rad/s^2 and the intercept with the vertical axis (not shown) is -0.600 rad/s .

Calculations: To sketch the disk at $t = -2.0 \text{ s}$, we substitute that value into Eq. 10-11, obtaining

$$\omega = -1.6 \text{ rad/s.} \quad (\text{Answer})$$

The minus sign here tells us that at $t = -2.0 \text{ s}$, the disk is turning clockwise (as indicated by the left-hand sketch in Fig. 10-5c).

Substituting $t = 4.0 \text{ s}$ into Eq. 10-11 gives us

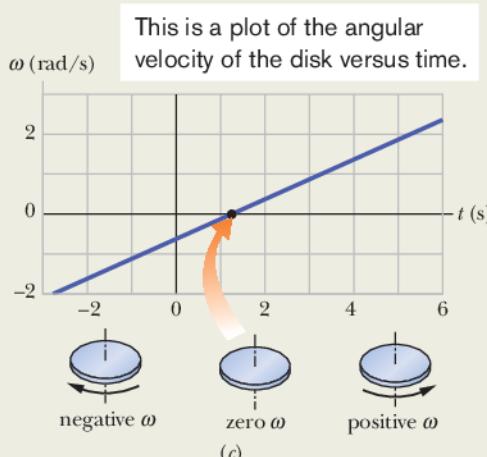
$$\omega = 1.4 \text{ rad/s.} \quad (\text{Answer})$$

The implied plus sign tells us that now the disk is turning counterclockwise (the right-hand sketch in Fig. 10-5c).

For t_{\min} , we already know that $d\theta/dt = 0$. So, we must also have $\omega = 0$. That is, the disk momentarily stops when the reference line reaches the minimum value of θ in Fig. 10-5b, as suggested by the center sketch in Fig. 10-5c. On the graph of ω versus t in Fig. 10-5c, this momentary stop is the zero point where the plot changes from the negative clockwise motion to the positive counterclockwise motion.

(d) Use the results in parts (a) through (c) to describe the motion of the disk from $t = -3.0 \text{ s}$ to $t = 6.0 \text{ s}$.

Description: When we first observe the disk at $t = -3.0 \text{ s}$, it has a positive angular position and is turning clockwise but slowing. It stops at angular position $\theta = -1.36 \text{ rad}$ and then begins to turn counterclockwise, with its angular position eventually becoming positive again.



The angular velocity is initially negative and slowing, then momentarily zero during reversal, and then positive and increasing.

Additional examples, video, and practice available at WileyPLUS





Sample Problem 10.02 Angular velocity derived from angular acceleration

A child's top is spun with angular acceleration

$$\alpha = 5t^3 - 4t,$$

with t in seconds and α in radians per second-squared. At $t = 0$, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad.

(a) Obtain an expression for the angular velocity $\omega(t)$ of the top. That is, find an expression that explicitly indicates how the angular velocity depends on time. (We can tell that there *is* such a dependence because the top is undergoing an angular acceleration, which means that its angular velocity *is* changing.)

KEY IDEA

By definition, $\alpha(t)$ is the derivative of $\omega(t)$ with respect to time. Thus, we can find $\omega(t)$ by integrating $\alpha(t)$ with respect to time.

Calculations: Equation 10-8 tells us

$$d\omega = \alpha dt,$$

so

$$\int d\omega = \int \alpha dt.$$

From this we find

$$\omega = \int (5t^3 - 4t) dt = \frac{5}{4}t^4 - \frac{4}{2}t^2 + C.$$

 Additional examples, video, and practice available at WileyPLUS

To evaluate the constant of integration C , we note that $\omega = 5$ rad/s at $t = 0$. Substituting these values in our expression for ω yields

$$5 \text{ rad/s} = 0 - 0 + C,$$

so $C = 5$ rad/s. Then

$$\omega = \frac{5}{4}t^4 - 2t^2 + 5. \quad (\text{Answer})$$

(b) Obtain an expression for the angular position $\theta(t)$ of the top.

KEY IDEA

By definition, $\omega(t)$ is the derivative of $\theta(t)$ with respect to time. Therefore, we can find $\theta(t)$ by integrating $\omega(t)$ with respect to time.

Calculations: Since Eq. 10-6 tells us that

$$d\theta = \omega dt,$$

we can write

$$\begin{aligned} \theta &= \int \omega dt = \int (\frac{5}{4}t^4 - 2t^2 + 5) dt \\ &= \frac{1}{4}t^5 - \frac{2}{3}t^3 + 5t + C' \\ &= \frac{1}{4}t^5 - \frac{2}{3}t^3 + 5t + 2, \end{aligned} \quad (\text{Answer})$$

where C' has been evaluated by noting that $\theta = 2$ rad at $t = 0$.

Are Angular Quantities Vectors?

We can describe the position, velocity, and acceleration of a single particle by means of vectors. If the particle is confined to a straight line, however, we do not really need vector notation. Such a particle has only two directions available to it, and we can indicate these directions with plus and minus signs.

In the same way, a rigid body rotating about a fixed axis can rotate only clockwise or counterclockwise as seen along the axis, and again we can select between the two directions by means of plus and minus signs. The question arises: “Can we treat the angular displacement, velocity, and acceleration of a rotating body as vectors?” The answer is a qualified “yes” (see the caution below, in connection with angular displacements).

Angular Velocities. Consider the angular velocity. Figure 10-6a shows a vinyl record rotating on a turntable. The record has a constant angular speed ω ($= 33\frac{1}{3}$ rev/min) in the clockwise direction. We can represent its angular velocity as a vector $\vec{\omega}$ pointing along the axis of rotation, as in Fig. 10-6b. Here’s how: We choose the length of this vector according to some convenient scale, for example, with 1 cm corresponding to 10 rev/min. Then we establish a direction for the vector $\vec{\omega}$ by using a **right-hand rule**, as Fig. 10-6c shows: Curl your right hand about the rotating record, your fingers pointing *in the direction of rotation*. Your extended thumb will then point in the direction of the angular velocity vector. If the record were to rotate in the opposite sense, the right-

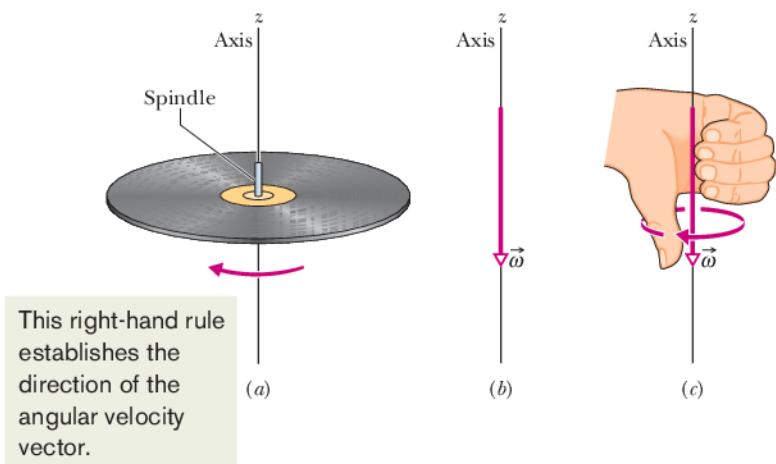


Figure 10-6 (a) A record rotating about a vertical axis that coincides with the axis of the spindle. (b) The angular velocity of the rotating record can be represented by the vector $\vec{\omega}$, lying along the axis and pointing down, as shown. (c) We establish the direction of the angular velocity vector as downward by using a right-hand rule. When the fingers of the right hand curl around the record and point the way it is moving, the extended thumb points in the direction of $\vec{\omega}$.

hand rule would tell you that the angular velocity vector then points in the opposite direction.

It is not easy to get used to representing angular quantities as vectors. We instinctively expect that something should be moving *along* the direction of a vector. That is not the case here. Instead, something (the rigid body) is rotating *around* the direction of the vector. In the world of pure rotation, a vector defines an axis of rotation, not a direction in which something moves. Nonetheless, the vector also defines the motion. Furthermore, it obeys all the rules for vector manipulation discussed in Chapter 3. The angular acceleration $\vec{\alpha}$ is another vector, and it too obeys those rules.

In this chapter we consider only rotations that are about a fixed axis. For such situations, we need not consider vectors—we can represent angular velocity with ω and angular acceleration with α , and we can indicate direction with an implied plus sign for counterclockwise or an explicit minus sign for clockwise.

Angular Displacements. Now for the caution: Angular *displacements* (unless they are very small) *cannot* be treated as vectors. Why not? We can certainly give them both magnitude and direction, as we did for the angular velocity vector in Fig. 10-6. However, to be represented as a vector, a quantity must *also* obey the rules of vector addition, one of which says that if you add two vectors, the order in which you add them does not matter. Angular displacements fail this test.

Figure 10-7 gives an example. An initially horizontal book is given two 90° angular displacements, first in the order of Fig. 10-7a and then in the order of Fig. 10-7b. Although the two angular displacements are identical, their order is not, and the book ends up with different orientations. Here's another example. Hold your right arm downward, palm toward your thigh. Keeping your wrist rigid, (1) lift the arm forward until it is horizontal, (2) move it horizontally until it points toward the right, and (3) then bring it down to your side. Your palm faces forward. If you start over, but reverse the steps, which way does your palm end up facing? From either example, we must conclude that the addition of two angular displacements depends on their order and they cannot be vectors.

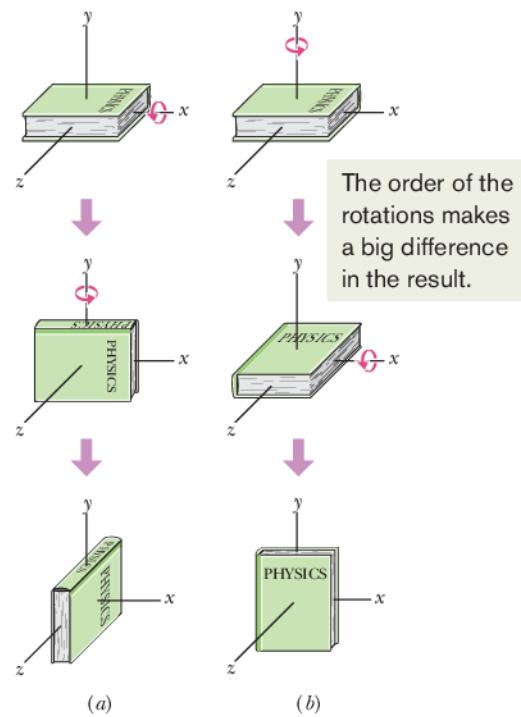


Figure 10-7 (a) From its initial position, at the top, the book is given two successive 90° rotations, first about the (horizontal) x axis and then about the (vertical) y axis. (b) The book is given the same rotations, but in the reverse order.

10-2 ROTATION WITH CONSTANT ANGULAR ACCELERATION

Learning Objective

After reading this module, you should be able to . . .

10.14 For constant angular acceleration, apply the relationships between angular position, angular displacement,

angular velocity, angular acceleration, and elapsed time (Table 10-1).

Key Idea

- Constant angular acceleration ($\alpha = \text{constant}$) is an important special case of rotational motion. The appropriate kinematic equations are

$$\begin{aligned}\omega &= \omega_0 + \alpha t, \\ \theta - \theta_0 &= \omega_0 t + \frac{1}{2} \alpha t^2, \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0), \\ \theta - \theta_0 &= \frac{1}{2}(\omega_0 + \omega)t, \\ \theta - \theta_0 &= \omega t - \frac{1}{2} \alpha t^2.\end{aligned}$$

Rotation with Constant Angular Acceleration

In pure translation, motion with a *constant linear acceleration* (for example, that of a falling body) is an important special case. In Table 2-1, we displayed a series of equations that hold for such motion.

In pure rotation, the case of *constant angular acceleration* is also important, and a parallel set of equations holds for this case also. We shall not derive them here, but simply write them from the corresponding linear equations, substituting equivalent angular quantities for the linear ones. This is done in Table 10-1, which lists both sets of equations (Eqs. 2-11 and 2-15 to 2-18; 10-12 to 10-16).

Recall that Eqs. 2-11 and 2-15 are basic equations for constant linear acceleration—the other equations in the Linear list can be derived from them. Similarly, Eqs. 10-12 and 10-13 are the basic equations for constant angular acceleration, and the other equations in the Angular list can be derived from them. To solve a simple problem involving constant angular acceleration, you can usually use an equation from the Angular list (*if* you have the list). Choose an equation for which the only unknown variable will be the variable requested in the problem. A better plan is to remember only Eqs. 10-12 and 10-13, and then solve them as simultaneous equations whenever needed.



Checkpoint 2

In four situations, a rotating body has angular position $\theta(t)$ given by (a) $\theta = 3t - 4$, (b) $\theta = -5t^3 + 4t^2 + 6$, (c) $\theta = 2/t^2 - 4/t$, and (d) $\theta = 5t^2 - 3$. To which situations do the angular equations of Table 10-1 apply?

Table 10-1 Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable	Angular Equation	Equation Number	
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	(10-16)



Sample Problem 10.03 Constant angular acceleration, grindstone

A grindstone (Fig. 10-8) rotates at constant angular acceleration $\alpha = 0.35 \text{ rad/s}^2$. At time $t = 0$, it has an angular velocity of $\omega_0 = -4.6 \text{ rad/s}$ and a reference line on it is horizontal, at the angular position $\theta_0 = 0$.

- (a) At what time after $t = 0$ is the reference line at the angular position $\theta = 5.0 \text{ rev}$?

KEY IDEA

The angular acceleration is constant, so we can use the rotation equations of Table 10-1. We choose Eq. 10-13,

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2,$$

because the only unknown variable it contains is the desired time t .

Calculations: Substituting known values and setting $\theta_0 = 0$ and $\theta = 5.0 \text{ rev} = 10\pi \text{ rad}$ give us

$$10\pi \text{ rad} = (-4.6 \text{ rad/s})t + \frac{1}{2}(0.35 \text{ rad/s}^2)t^2.$$

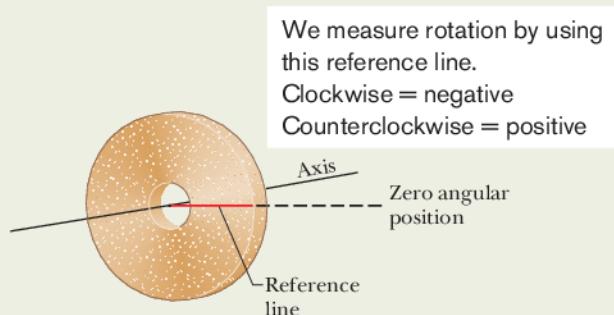


Figure 10-8 A grindstone. At $t = 0$ the reference line (which we imagine to be marked on the stone) is horizontal.

(We converted 5.0 rev to $10\pi \text{ rad}$ to keep the units consistent.) Solving this quadratic equation for t , we find

$$t = 32 \text{ s.} \quad (\text{Answer})$$

Now notice something a bit strange. We first see the wheel when it is rotating in the negative direction and through the $\theta = 0$ orientation. Yet, we just found out that 32 s later it is at the positive orientation of $\theta = 5.0 \text{ rev}$. What happened in that time interval so that it could be at a positive orientation?

- (b) Describe the grindstone's rotation between $t = 0$ and $t = 32 \text{ s}$.

Description: The wheel is initially rotating in the negative (clockwise) direction with angular velocity $\omega_0 = -4.6 \text{ rad/s}$, but its angular acceleration α is positive. This initial opposition of the signs of angular velocity and angular acceleration means that the wheel slows in its rotation in the negative direction, stops, and then reverses to rotate in the positive direction. After the reference line comes back through its initial orientation of $\theta = 0$, the wheel turns an additional 5.0 rev by time $t = 32 \text{ s}$.

- (c) At what time t does the grindstone momentarily stop?

Calculation: We again go to the table of equations for constant angular acceleration, and again we need an equation that contains only the desired unknown variable t . However, now the equation must also contain the variable ω , so that we can set it to 0 and then solve for the corresponding time t . We choose Eq. 10-12, which yields

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (-4.6 \text{ rad/s})}{0.35 \text{ rad/s}^2} = 13 \text{ s.} \quad (\text{Answer})$$

Sample Problem 10.04 Constant angular acceleration, riding a Rotor

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev , at constant angular acceleration. (The passenger is obviously more of a “translation person” than a “rotation person.”)

- (a) What is the constant angular acceleration during this decrease in angular speed?

KEY IDEA

Because the cylinder's angular acceleration is constant, we can relate it to the angular velocity and angular displacement via the basic equations for constant angular acceleration (Eqs. 10-12 and 10-13).

Calculations: Let's first do a quick check to see if we can solve the basic equations. The initial angular velocity is $\omega_0 = 3.40$

rad/s, the angular displacement is $\theta - \theta_0 = 20.0 \text{ rev}$, and the angular velocity at the end of that displacement is $\omega = 2.00 \text{ rad/s}$. In addition to the angular acceleration α that we want, both basic equations also contain time t , which we do not necessarily want.

To eliminate the unknown t , we use Eq. 10-12 to write

$$t = \frac{\omega - \omega_0}{\alpha},$$

which we then substitute into Eq. 10-13 to write

$$\theta - \theta_0 = \omega_0 \left(\frac{\omega - \omega_0}{\alpha} \right) + \frac{1}{2} \alpha \left(\frac{\omega - \omega_0}{\alpha} \right)^2.$$

Solving for α , substituting known data, and converting 20 rev to 125.7 rad , we find

$$\begin{aligned} \alpha &= \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{(2.00 \text{ rad/s})^2 - (3.40 \text{ rad/s})^2}{2(125.7 \text{ rad})} \\ &= -0.0301 \text{ rad/s}^2. \end{aligned} \quad (\text{Answer})$$

(b) How much time did the speed decrease take?

Calculation: Now that we know α , we can use Eq. 10-12 to solve for t :

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{2.00 \text{ rad/s} - 3.40 \text{ rad/s}}{-0.0301 \text{ rad/s}^2}$$

$$= 46.5 \text{ s.}$$

(Answer)



Additional examples, video, and practice available at WileyPLUS

10-3 RELATING THE LINEAR AND ANGULAR VARIABLES

Learning Objectives

After reading this module, you should be able to . . .

10.15 For a rigid body rotating about a fixed axis, relate the angular variables of the body (angular position, angular velocity, and angular acceleration) and the linear variables of a particle on the body (position, velocity, and acceleration) at any given radius.

Key Ideas

- A point in a rigid rotating body, at a perpendicular distance r from the rotation axis, moves in a circle with radius r . If the body rotates through an angle θ , the point moves along an arc with length s given by

$$s = \theta r \quad (\text{radian measure}),$$

where θ is in radians.

- The linear velocity \vec{v} of the point is tangent to the circle; the point's linear speed v is given by

$$v = \omega r \quad (\text{radian measure}),$$

where ω is the angular speed (in radians per second) of the body, and thus also the point.

10.16 Distinguish between tangential acceleration and radial acceleration, and draw a vector for each in a sketch of a particle on a body rotating about an axis, for both an increase in angular speed and a decrease.

- The linear acceleration \vec{a} of the point has both tangential and radial components. The tangential component is

$$a_t = \alpha r \quad (\text{radian measure}),$$

where α is the magnitude of the angular acceleration (in radians per second-squared) of the body. The radial component of \vec{a} is

$$a_r = \frac{v^2}{r} = \omega^2 r \quad (\text{radian measure}).$$

- If the point moves in uniform circular motion, the period T of the motion for the point and the body is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \quad (\text{radian measure}).$$

Relating the Linear and Angular Variables

In Module 4-5, we discussed uniform circular motion, in which a particle travels at constant linear speed v along a circle and around an axis of rotation. When a rigid body, such as a merry-go-round, rotates around an axis, each particle in the body moves in its own circle around that axis. Since the body is rigid, all the particles make one revolution in the same amount of time; that is, they all have the same angular speed ω .

However, the farther a particle is from the axis, the greater the circumference of its circle is, and so the faster its linear speed v must be. You can notice this on a merry-go-round. You turn with the same angular speed ω regardless of your distance from the center, but your linear speed v increases noticeably if you move to the outside edge of the merry-go-round.

We often need to relate the linear variables s , v , and a for a particular point in a rotating body to the angular variables θ , ω , and α for that body. The two sets of variables are related by r , the *perpendicular distance* of the point from the rotation axis. This perpendicular distance is the distance between the point and the rotation axis, measured along a perpendicular to the axis. It is also the radius r of the circle traveled by the point around the axis of rotation.

The Position

If a reference line on a rigid body rotates through an angle θ , a point within the body at a position r from the rotation axis moves a distance s along a circular arc, where s is given by Eq. 10-1:

$$s = \theta r \quad (\text{radian measure}). \quad (10-17)$$

This is the first of our linear–angular relations. *Caution:* The angle θ here must be measured in radians because Eq. 10-17 is itself the definition of angular measure in radians.

The Speed

Differentiating Eq. 10-17 with respect to time—with r held constant—leads to

$$\frac{ds}{dt} = \frac{d\theta}{dt} r.$$

However, ds/dt is the linear speed (the magnitude of the linear velocity) of the point in question, and $d\theta/dt$ is the angular speed ω of the rotating body. So

$$v = \omega r \quad (\text{radian measure}). \quad (10-18)$$

Caution: The angular speed ω must be expressed in radian measure.

Equation 10-18 tells us that since all points within the rigid body have the same angular speed ω , points with greater radius r have greater linear speed v . Figure 10-9a reminds us that the linear velocity is always tangent to the circular path of the point in question.

If the angular speed ω of the rigid body is constant, then Eq. 10-18 tells us that the linear speed v of any point within it is also constant. Thus, each point within the body undergoes uniform circular motion. The period of revolution T for the motion of each point and for the rigid body itself is given by Eq. 4-35:

$$T = \frac{2\pi r}{v}. \quad (10-19)$$

This equation tells us that the time for one revolution is the distance $2\pi r$ traveled in one revolution divided by the speed at which that distance is traveled. Substituting for v from Eq. 10-18 and canceling r , we find also that

$$T = \frac{2\pi}{\omega} \quad (\text{radian measure}). \quad (10-20)$$

This equivalent equation says that the time for one revolution is the angular distance 2π rad traveled in one revolution divided by the angular speed (or rate) at which that angle is traveled.

The Acceleration

Differentiating Eq. 10-18 with respect to time—again with r held constant—leads to

$$\frac{dv}{dt} = \frac{d\omega}{dt} r. \quad (10-21)$$

Here we run up against a complication. In Eq. 10-21, dv/dt represents only the part of the linear acceleration that is responsible for changes in the *magnitude* v of the linear velocity \vec{v} . Like \vec{v} , that part of the linear acceleration is tangent to the path of the point in question. We call it the *tangential component* a_t of the linear acceleration of the point, and we write

$$a_t = \alpha r \quad (\text{radian measure}), \quad (10-22)$$

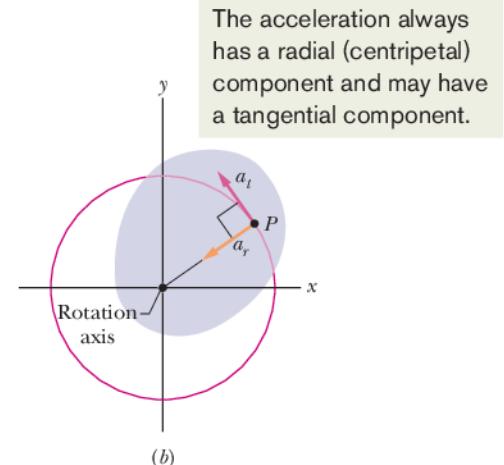
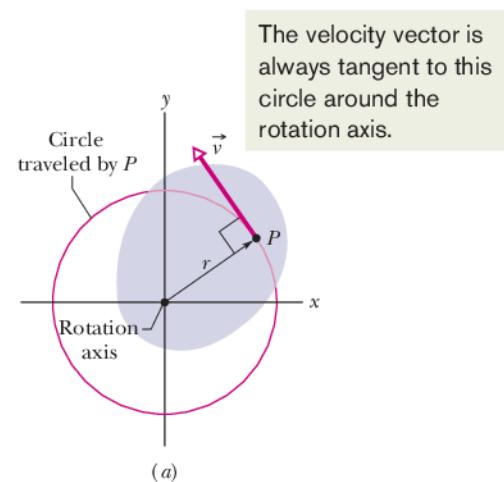


Figure 10-9 The rotating rigid body of Fig. 10-2, shown in cross section viewed from above. Every point of the body (such as P) moves in a circle around the rotation axis. (a) The linear velocity \vec{v} of every point is tangent to the circle in which the point moves. (b) The linear acceleration \vec{a} of the point has (in general) two components: tangential a_t and radial a_r .

where $\alpha = d\omega/dt$. Caution: The angular acceleration α in Eq. 10-22 must be expressed in radian measure.

In addition, as Eq. 4-34 tells us, a particle (or point) moving in a circular path has a *radial component* of linear acceleration, $a_r = v^2/r$ (directed radially inward), that is responsible for changes in the *direction* of the linear velocity \vec{v} . By substituting for v from Eq. 10-18, we can write this component as

$$a_r = \frac{v^2}{r} = \omega^2 r \quad (\text{radian measure}). \quad (10-23)$$

Thus, as Fig. 10-9b shows, the linear acceleration of a point on a rotating rigid body has, in general, two components. The radially inward component a_r (given by Eq. 10-23) is present whenever the angular velocity of the body is not zero. The tangential component a_t (given by Eq. 10-22) is present whenever the angular acceleration is not zero.



Checkpoint 3

A cockroach rides the rim of a rotating merry-go-round. If the angular speed of this system (*merry-go-round + cockroach*) is constant, does the cockroach have (a) radial acceleration and (b) tangential acceleration? If ω is decreasing, does the cockroach have (c) radial acceleration and (d) tangential acceleration?



Sample Problem 10.05 Designing The Giant Ring, a large-scale amusement park ride

We are given the job of designing a large horizontal ring that will rotate around a vertical axis and that will have a radius of $r = 33.1$ m (matching that of Beijing's The Great Observation Wheel, the largest Ferris wheel in the world). Passengers will enter through a door in the outer wall of the ring and then stand next to that wall (Fig. 10-10a). We decide that for the time interval $t = 0$ to $t = 2.30$ s, the angular position $\theta(t)$ of a reference line on the ring will be given by

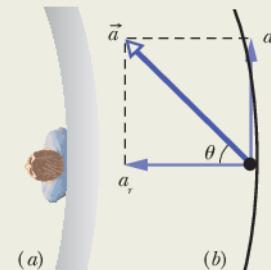
$$\theta = ct^3, \quad (10-24)$$

with $c = 6.39 \times 10^{-2}$ rad/s³. After $t = 2.30$ s, the angular speed will be held constant until the end of the ride. Once the ring begins to rotate, the floor of the ring will drop away from the riders but the riders will not fall—indeed, they feel as though they are pinned to the wall. For the time $t = 2.20$ s, let's determine a rider's angular speed ω , linear speed v , angular acceleration α , tangential acceleration a_t , radial acceleration a_r , and acceleration \vec{a} .

KEY IDEAS

- (1) The angular speed ω is given by Eq. 10-6 ($\omega = d\theta/dt$).
- (2) The linear speed v (along the circular path) is related to the angular speed (around the rotation axis) by Eq. 10-18 ($v = \omega r$).
- (3) The angular acceleration α is given by Eq. 10-8 ($\alpha = d\omega/dt$).
- (4) The tangential acceleration a_t (along the circular path) is related to the angular acceleration (around the rotation axis) by Eq. 10-22 ($a_t = \alpha r$).
- (5) The radial acceleration a_r is given Eq. 10-23 ($a_r = \omega^2 r$).
- (6) The tangential

Figure 10-10 (a) Overhead view of a passenger ready to ride The Giant Ring. (b) The radial and tangential acceleration components of the (full) acceleration.



and radial accelerations are the (perpendicular) components of the (full) acceleration \vec{a} .

Calculations: Let's go through the steps. We first find the angular velocity by taking the time derivative of the given angular position function and then substituting the given time of $t = 2.20$ s:

$$\begin{aligned} \omega &= \frac{d\theta}{dt} = \frac{d}{dt}(ct^3) = 3ct^2 & (10-25) \\ &= 3(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})^2 \\ &= 0.928 \text{ rad/s}. \end{aligned}$$

(Answer)

From Eq. 10-18, the linear speed just then is

$$\begin{aligned} v &= \omega r = 3ct^2 r \\ &= 3(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})^2(33.1 \text{ m}) \\ &= 30.7 \text{ m/s}. \end{aligned}$$

(Answer)

Although this is fast (111 km/h or 68.7 mi/h), such speeds are common in amusement parks and not alarming because (as mentioned in Chapter 2) your body reacts to accelerations but not to velocities. (It is an accelerometer, not a speedometer.) From Eq. 10-26 we see that the linear speed is increasing as the square of the time (but this increase will cut off at $t = 2.30\text{ s}$).

Next, let's tackle the angular acceleration by taking the time derivative of Eq. 10-25:

$$\begin{aligned}\alpha &= \frac{d\omega}{dt} = \frac{d}{dt}(3ct^2) = 6ct \\ &= 6(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s}) = 0.843 \text{ rad/s}^2. \quad (\text{Answer}) \\ a_t &= \alpha r = 6ctr \quad (10-27) \\ &= 6(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})(33.1 \text{ m}) \\ &= 27.91 \text{ m/s}^2 \approx 27.9 \text{ m/s}^2, \quad (\text{Answer})\end{aligned}$$

or $2.8g$ (which is reasonable and a bit exciting). Equation 10-27 tells us that the tangential acceleration is increasing with time (but it will cut off at $t = 2.30\text{ s}$). From Eq. 10-23, we write the radial acceleration as

$$a_r = \omega^2 r.$$

Substituting from Eq. 10-25 leads us to

$$\begin{aligned}a_r &= (3ct^2)^2 r = 9c^2 t^4 r \quad (10-28) \\ &= 9(6.39 \times 10^{-2} \text{ rad/s}^3)^2 (2.20 \text{ s})^4 (33.1 \text{ m}) \\ &= 28.49 \text{ m/s}^2 \approx 28.5 \text{ m/s}^2, \quad (\text{Answer})\end{aligned}$$

or $2.9g$ (which is also reasonable and a bit exciting).



Additional examples, video, and practice available at WileyPLUS



10-4 KINETIC ENERGY OF ROTATION

Learning Objectives

After reading this module, you should be able to . . .

- 10.17** Find the rotational inertia of a particle about a point.
- 10.18** Find the total rotational inertia of many particles moving around the same fixed axis.

Key Idea

- The kinetic energy K of a rigid body rotating about a fixed axis is given by

$$K = \frac{1}{2} I \omega^2 \quad (\text{radian measure}),$$

The radial and tangential accelerations are perpendicular to each other and form the components of the rider's acceleration \vec{a} (Fig. 10-10b). The magnitude of \vec{a} is given by

$$\begin{aligned}a &= \sqrt{a_r^2 + a_t^2} \quad (10-29) \\ &= \sqrt{(28.49 \text{ m/s}^2)^2 + (27.91 \text{ m/s}^2)^2} \\ &\approx 39.9 \text{ m/s}^2, \quad (\text{Answer})\end{aligned}$$

or $4.1g$ (which is really exciting!). All these values are acceptable.

To find the orientation of \vec{a} , we can calculate the angle θ shown in Fig. 10-10b:

$$\tan \theta = \frac{a_t}{a_r}.$$

However, instead of substituting our numerical results, let's use the algebraic results from Eqs. 10-27 and 10-28:

$$\theta = \tan^{-1} \left(\frac{6ctr}{9c^2 t^4 r} \right) = \tan^{-1} \left(\frac{2}{3ct^3} \right). \quad (10-30)$$

The big advantage of solving for the angle algebraically is that we can then see that the angle (1) does not depend on the ring's radius and (2) decreases as t goes from 0 to 2.20 s. That is, the acceleration vector \vec{a} swings toward being radially inward because the radial acceleration (which depends on t^4) quickly dominates over the tangential acceleration (which depends on only t). At our given time $t = 2.20\text{ s}$, we have

$$\theta = \tan^{-1} \frac{2}{3(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})^3} = 44.4^\circ. \quad (\text{Answer})$$

- 10.19** Calculate the rotational kinetic energy of a body in terms of its rotational inertia and its angular speed.

in which I is the rotational inertia of the body, defined as

$$I = \sum m_i r_i^2$$

for a system of discrete particles.

Kinetic Energy of Rotation

The rapidly rotating blade of a table saw certainly has kinetic energy due to that rotation. How can we express the energy? We cannot apply the familiar formula $K = \frac{1}{2}mv^2$ to the saw as a whole because that would give us the kinetic energy only of the saw's center of mass, which is zero.

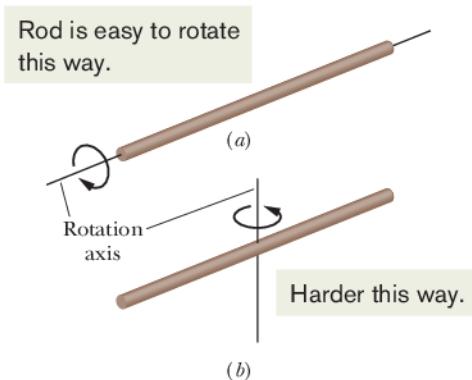


Figure 10-11 A long rod is much easier to rotate about (a) its central (longitudinal) axis than about (b) an axis through its center and perpendicular to its length. The reason for the difference is that the mass is distributed closer to the rotation axis in (a) than in (b).

Instead, we shall treat the table saw (and any other rotating rigid body) as a collection of particles with different speeds. We can then add up the kinetic energies of all the particles to find the kinetic energy of the body as a whole. In this way we obtain, for the kinetic energy of a rotating body,

$$\begin{aligned} K &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \dots \\ &= \sum \frac{1}{2}m_i v_i^2, \end{aligned} \quad (10-31)$$

in which m_i is the mass of the i th particle and v_i is its speed. The sum is taken over all the particles in the body.

The problem with Eq. 10-31 is that v_i is not the same for all particles. We solve this problem by substituting for v from Eq. 10-18 ($v = \omega r$), so that we have

$$K = \sum \frac{1}{2}m_i(\omega r_i)^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2, \quad (10-32)$$

in which ω is the same for all particles.

The quantity in parentheses on the right side of Eq. 10-32 tells us how the mass of the rotating body is distributed about its axis of rotation. We call that quantity the **rotational inertia** (or **moment of inertia**) I of the body with respect to the axis of rotation. It is a constant for a particular rigid body and a particular rotation axis. (Caution: That axis must always be specified if the value of I is to be meaningful.)

We may now write

$$I = \sum m_i r_i^2 \quad (\text{rotational inertia}) \quad (10-33)$$

and substitute into Eq. 10-32, obtaining

$$K = \frac{1}{2}I\omega^2 \quad (\text{radian measure}) \quad (10-34)$$

as the expression we seek. Because we have used the relation $v = \omega r$ in deriving Eq. 10-34, ω must be expressed in radian measure. The SI unit for I is the kilogram-square meter ($\text{kg} \cdot \text{m}^2$).

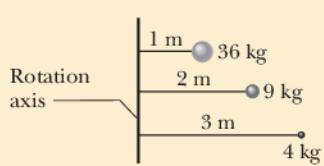
The Plan. If we have a few particles and a specified rotation axis, we find mr^2 for each particle and then add the results as in Eq. 10-33 to get the total rotational inertia I . If we want the total rotational kinetic energy, we can then substitute that I into Eq. 10-34. That is the plan for a few particles, but suppose we have a huge number of particles such as in a rod. In the next module we shall see how to handle such *continuous bodies* and do the calculation in only a few minutes.

Equation 10-34, which gives the kinetic energy of a rigid body in pure rotation, is the angular equivalent of the formula $K = \frac{1}{2}Mv_{\text{com}}^2$, which gives the kinetic energy of a rigid body in pure translation. In both formulas there is a factor of $\frac{1}{2}$. Where mass M appears in one equation, I (which involves both mass and its distribution) appears in the other. Finally, each equation contains as a factor the square of a speed—translational or rotational as appropriate. The kinetic energies of translation and of rotation are not different kinds of energy. They are both kinetic energy, expressed in ways that are appropriate to the motion at hand.

We noted previously that the rotational inertia of a rotating body involves not only its mass but also how that mass is distributed. Here is an example that you can literally feel. Rotate a long, fairly heavy rod (a pole, a length of lumber, or something similar), first around its central (longitudinal) axis (Fig. 10-11a) and then around an axis perpendicular to the rod and through the center (Fig. 10-11b). Both rotations involve the very same mass, but the first rotation is much easier than the second. The reason is that the mass is distributed much closer to the rotation axis in the first rotation. As a result, the rotational inertia of the rod is much smaller in Fig. 10-11a than in Fig. 10-11b. In general, smaller rotational inertia means easier rotation.

**Checkpoint 4**

The figure shows three small spheres that rotate about a vertical axis. The perpendicular distance between the axis and the center of each sphere is given. Rank the three spheres according to their rotational inertia about that axis, greatest first.



10-5 CALCULATING THE ROTATIONAL INERTIA

Learning Objectives

After reading this module, you should be able to . . .

10.20 Determine the rotational inertia of a body if it is given in Table 10-2.

10.21 Calculate the rotational inertia of a body by integration over the mass elements of the body.

10.22 Apply the parallel-axis theorem for a rotation axis that is displaced from a parallel axis through the center of mass of a body.

Key Ideas

- I is the rotational inertia of the body, defined as

$$I = \sum m_i r_i^2$$

for a system of discrete particles and defined as

$$I = \int r^2 dm$$

for a body with continuously distributed mass. The r and r_i in these expressions represent the perpendicular distance from the axis of rotation to each mass element in the body, and the integration is carried out over the entire body so as to include every mass element.

- The parallel-axis theorem relates the rotational inertia I of a body about any axis to that of the same body about a parallel axis through the center of mass:

$$I = I_{\text{com}} + Mh^2.$$

Here h is the perpendicular distance between the two axes, and I_{com} is the rotational inertia of the body about the axis through the com. We can describe h as being the distance the actual rotation axis has been shifted from the rotation axis through the com.

Calculating the Rotational Inertia

If a rigid body consists of a few particles, we can calculate its rotational inertia about a given rotation axis with Eq. 10-33 ($I = \sum m_i r_i^2$); that is, we can find the product mr^2 for each particle and then sum the products. (Recall that r is the perpendicular distance a particle is from the given rotation axis.)

If a rigid body consists of a great many adjacent particles (it is *continuous*, like a Frisbee), using Eq. 10-33 would require a computer. Thus, instead, we replace the sum in Eq. 10-33 with an integral and define the rotational inertia of the body as

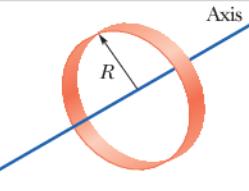
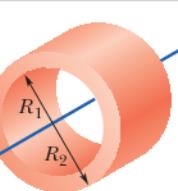
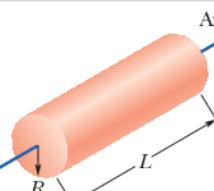
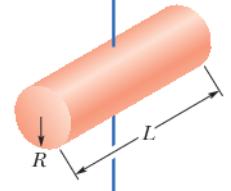
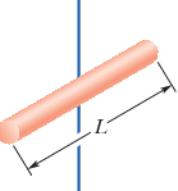
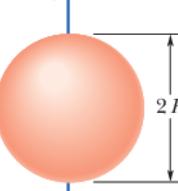
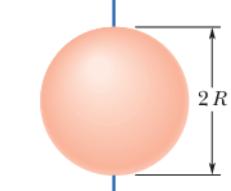
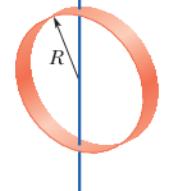
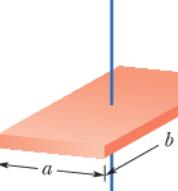
$$I = \int r^2 dm \quad (\text{rotational inertia, continuous body}). \quad (10-35)$$

Table 10-2 gives the results of such integration for nine common body shapes and the indicated axes of rotation.

Parallel-Axis Theorem

Suppose we want to find the rotational inertia I of a body of mass M about a given axis. In principle, we can always find I with the integration of Eq. 10-35. However, there is a neat shortcut if we happen to already know the rotational inertia I_{com} of the body about a *parallel* axis that extends through the body's center of mass. Let h be the perpendicular distance between the given axis and the axis

Table 10-2 Some Rotational Inertias

 $I = MR^2$	 $I = \frac{1}{2}M(R_1^2 + R_2^2)$	 $I = \frac{1}{2}MR^2$
 $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$	 $I = \frac{1}{12}ML^2$	 $I = \frac{2}{5}MR^2$
 $I = \frac{2}{3}MR^2$	 $I = \frac{1}{2}MR^2$	 $I = \frac{1}{12}M(a^2 + b^2)$

We need to relate the rotational inertia around the axis at P to that around the axis at the com.

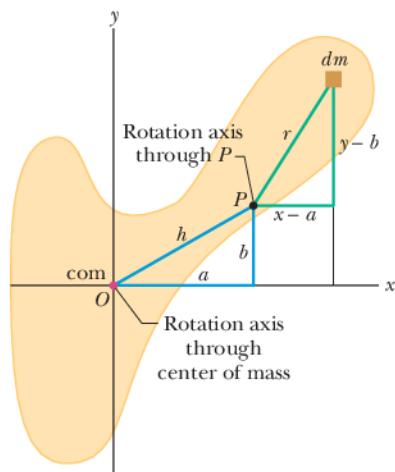


Figure 10-12 A rigid body in cross section, with its center of mass at O . The parallel-axis theorem (Eq. 10-36) relates the rotational inertia of the body about an axis through O to that about a parallel axis through a point such as P , a distance h from the body's center of mass.

through the center of mass (remember these two axes must be parallel). Then the rotational inertia I about the given axis is

$$I = I_{\text{com}} + Mh^2 \quad (\text{parallel-axis theorem}). \quad (10-36)$$

Think of the distance h as being the distance we have shifted the rotation axis from being through the com. This equation is known as the **parallel-axis theorem**. We shall now prove it.

Proof of the Parallel-Axis Theorem

Let O be the center of mass of the arbitrarily shaped body shown in cross section in Fig. 10-12. Place the origin of the coordinates at O . Consider an axis through O perpendicular to the plane of the figure, and another axis through point P parallel to the first axis. Let the x and y coordinates of P be a and b .

Let dm be a mass element with the general coordinates x and y . The rotational inertia of the body about the axis through P is then, from Eq. 10-35,

$$I = \int r^2 dm = \int [(x - a)^2 + (y - b)^2] dm,$$

which we can rearrange as

$$I = \int (x^2 + y^2) dm - 2a \int x dm - 2b \int y dm + \int (a^2 + b^2) dm. \quad (10-37)$$

From the definition of the center of mass (Eq. 9-9), the middle two integrals of Eq. 10-37 give the coordinates of the center of mass (multiplied by a constant)