

The Magnetic Dipole Moment

As we have just discussed, a torque acts to rotate a current-carrying coil placed in a magnetic field. In that sense, the coil behaves like a bar magnet placed in the magnetic field. Thus, like a bar magnet, a current-carrying coil is said to be a *magnetic dipole*. Moreover, to account for the torque on the coil due to the magnetic field, we assign a **magnetic dipole moment** $\vec{\mu}$ to the coil. The direction of $\vec{\mu}$ is that of the normal vector \vec{n} to the plane of the coil and thus is given by the same right-hand rule shown in Fig. 28-19. That is, grasp the coil with the fingers of your right hand in the direction of current i ; the outstretched thumb of that hand gives the direction of $\vec{\mu}$. The magnitude of $\vec{\mu}$ is given by

$$\mu = NiA \quad (\text{magnetic moment}), \quad (28-35)$$

in which N is the number of turns in the coil, i is the current through the coil, and A is the area enclosed by each turn of the coil. From this equation, with i in amperes and A in square meters, we see that the unit of $\vec{\mu}$ is the ampere-square meter ($\text{A} \cdot \text{m}^2$).

Torque. Using $\vec{\mu}$, we can rewrite Eq. 28-33 for the torque on the coil due to a magnetic field as

$$\tau = \mu B \sin \theta, \quad (28-36)$$

in which θ is the angle between the vectors $\vec{\mu}$ and \vec{B} .

We can generalize this to the vector relation

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad (28-37)$$

which reminds us very much of the corresponding equation for the torque exerted by an *electric* field on an *electric* dipole—namely, Eq. 22-34:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

In each case the torque due to the field—either magnetic or electric—is equal to the vector product of the corresponding dipole moment and the field vector.

Energy. A magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation in the field. For electric dipoles we have shown (Eq. 22-38) that

$$U(\theta) = -\vec{p} \cdot \vec{E}.$$

In strict analogy, we can write for the magnetic case

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \quad (28-38)$$

In each case the energy due to the field is equal to the negative of the scalar product of the corresponding dipole moment and the field vector.

A magnetic dipole has its lowest energy ($= -\mu B \cos 0 = -\mu B$) when its dipole moment $\vec{\mu}$ is lined up with the magnetic field (Fig. 28-20). It has its highest energy ($= -\mu B \cos 180^\circ = +\mu B$) when $\vec{\mu}$ is directed opposite the field. From Eq. 28-38, with U in joules and \vec{B} in teslas, we see that the unit of $\vec{\mu}$ can be the joule per tesla (J/T) instead of the ampere-square meter as suggested by Eq. 28-35.

Work. If an applied torque (due to “an external agent”) rotates a magnetic dipole from an initial orientation θ_i to another orientation θ_f , then work W_a is done on the dipole by the applied torque. If the dipole is stationary before and after the change in its orientation, then work W_a is

$$W_a = U_f - U_i, \quad (28-39)$$

where U_f and U_i are calculated with Eq. 28-38.

The magnetic moment vector attempts to align with the magnetic field.

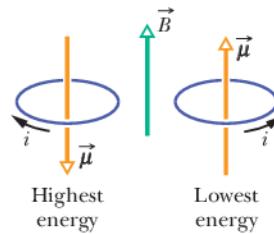


Figure 28-20 The orientations of highest and lowest energy of a magnetic dipole (here a coil carrying current) in an external magnetic field \vec{B} . The direction of the current i gives the direction of the magnetic dipole moment $\vec{\mu}$ via the right-hand rule shown for \vec{n} in Fig. 28-19b.

Table 28-2 Some Magnetic Dipole Moments

Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

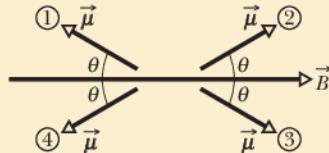
So far, we have identified only a current-carrying coil and a permanent magnet as a magnetic dipole. However, a rotating sphere of charge is also a magnetic dipole, as is Earth itself (approximately). Finally, most subatomic particles, including the electron, the proton, and the neutron, have magnetic dipole moments. As you will see in Chapter 32, all these quantities can be viewed as current loops. For comparison, some approximate magnetic dipole moments are shown in Table 28-2.

Language. Some instructors refer to U in Eq. 28-38 as a potential energy and relate it to work done by the magnetic field when the orientation of the dipole changes. Here we shall avoid the debate and say that U is an energy associated with the dipole orientation.



Checkpoint 5

The figure shows four orientations, at angle θ , of a magnetic dipole moment $\vec{\mu}$ in a magnetic field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the orientation energy of the dipole, greatest first.



Sample Problem 28.07 Rotating a magnetic dipole in a magnetic field

Figure 28-21 shows a circular coil with 250 turns, an area A of 2.52×10^{-4} m 2 , and a current of $100 \mu\text{A}$. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with \vec{B} .

- (a) In Fig. 28-21, what is the direction of the current in the coil?

Right-hand rule: Imagine cupping the coil with your right hand so that your right thumb is outstretched in the direction of $\vec{\mu}$. The direction in which your fingers curl around the coil is the direction of the current in the coil. Thus, in the wires on the near side of the coil—those we see in Fig. 28-21—the current is from top to bottom.

- (b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its ini-

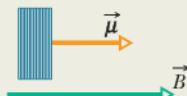


Figure 28-21 A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment is aligned with magnetic field \vec{B} .

tial orientation, so that $\vec{\mu}$ is perpendicular to \vec{B} and the coil is again at rest?

KEY IDEA

The work W_a done by the applied torque would be equal to the change in the coil's orientation energy due to its change in orientation.

Calculations: From Eq. 28-39 ($W_a = U_f - U_i$), we find

$$\begin{aligned} W_a &= U(90^\circ) - U(0^\circ) \\ &= -\mu B \cos 90^\circ - (-\mu B \cos 0^\circ) = 0 + \mu B \\ &= \mu B. \end{aligned}$$

Substituting for μ from Eq. 28-35 ($\mu = NiA$), we find that

$$\begin{aligned} W_a &= (NiA)B \\ &= (250)(100 \times 10^{-6} \text{ A})(2.52 \times 10^{-4} \text{ m}^2)(0.85 \text{ T}) \\ &= 5.355 \times 10^{-6} \text{ J} \approx 5.4 \mu\text{J}. \quad (\text{Answer}) \end{aligned}$$

Similarly, we can show that to change the orientation by another 90° , so that the dipole moment is opposite the field, another $5.4 \mu\text{J}$ is required.



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Review & Summary

Magnetic Field \vec{B} A magnetic field \vec{B} is defined in terms of the force \vec{F}_B acting on a test particle with charge q moving through the field with velocity \vec{v} :

$$\vec{F}_B = q\vec{v} \times \vec{B}. \quad (28-2)$$

The SI unit for \vec{B} is the **tesla** (T): $1 \text{ T} = 1 \text{ N}/(\text{A} \cdot \text{m}) = 10^4 \text{ gauss}$.

The Hall Effect When a conducting strip carrying a current i is placed in a uniform magnetic field \vec{B} , some charge carriers (with charge e) build up on one side of the conductor, creating a potential difference V across the strip. The polarities of the sides indicate the sign of the charge carriers.

A Charged Particle Circulating in a Magnetic Field A charged particle with mass m and charge magnitude $|q|$ moving with velocity \vec{v} perpendicular to a uniform magnetic field \vec{B} will travel in a circle. Applying Newton's second law to the circular motion yields

$$|q|\vec{v}B = \frac{mv^2}{r}, \quad (28-15)$$

from which we find the radius r of the circle to be

$$r = \frac{mv}{|q|B}. \quad (28-16)$$

The frequency of revolution f , the angular frequency ω , and the period of the motion T are given by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{|q|B}{2\pi m}. \quad (28-19, 28-18, 28-17)$$

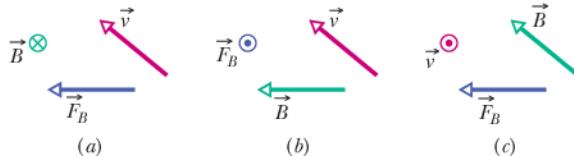


Figure 28-22 Question 1.

2 Figure 28-23 shows a wire that carries current to the right through a uniform magnetic field. It also shows four choices for the direction of that field. (a) Rank the choices according to the magnitude of the electric potential difference that would be set up across the width of the wire, greatest first. (b) For which choice is the top side of the wire at higher potential than the bottom side of the wire?

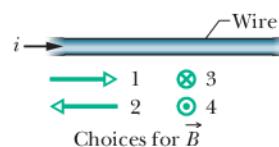


Figure 28-23 Question 2.

3 Figure 28-24 shows a metallic, rectangular solid that is to move at a certain speed v through the uniform magnetic field \vec{B} . The dimensions of the solid are multiples of d , as shown. You have six choices for the direction of the velocity: parallel to x , y , or z in ei-

Magnetic Force on a Current-Carrying Wire A straight wire carrying a current i in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}. \quad (28-26)$$

The force acting on a current element $i d\vec{L}$ in a magnetic field is

$$d\vec{F}_B = i d\vec{L} \times \vec{B}. \quad (28-28)$$

The direction of the length vector \vec{L} or $d\vec{L}$ is that of the current i .

Torque on a Current-Carrying Coil A coil (of area A and N turns, carrying current i) in a uniform magnetic field \vec{B} will experience a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \quad (28-37)$$

Here $\vec{\mu}$ is the **magnetic dipole moment** of the coil, with magnitude $\mu = NiA$ and direction given by the right-hand rule.

Orientation Energy of a Magnetic Dipole The orientation energy of a magnetic dipole in a magnetic field is

$$U(\theta) = -\vec{\mu} \cdot \vec{B}. \quad (28-38)$$

If an external agent rotates a magnetic dipole from an initial orientation θ_i to some other orientation θ_f and the dipole is stationary both initially and finally, the work W_a done on the dipole by the agent is

$$W_a = \Delta U = U_f - U_i. \quad (28-39)$$

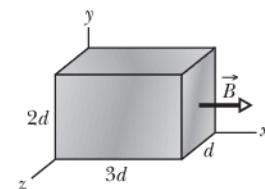


Figure 28-24 Question 3.

ther the positive or negative direction. (a) Rank the six choices according to the potential difference set up across the solid, greatest first. (b) For which choice is the front face at lower potential?

4 Figure 28-25 shows the path of a particle through six regions of uniform magnetic field, where the path is either a half-circle or a quarter-circle. Upon leaving the last region, the particle travels between two charged, parallel plates and is deflected toward the plate of higher potential. What is the direction of the magnetic field in each of the six regions?

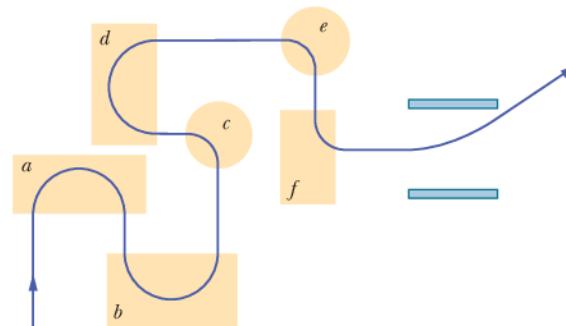


Figure 28-25 Question 4.

5 In Module 28-2, we discussed a charged particle moving through crossed fields with the forces \vec{F}_E and \vec{F}_B in opposition. We found that the particle moves in a straight line (that is, neither force dominates the motion) if its speed is given by Eq. 28-7 ($v = E/B$). Which of the two forces dominates if the speed of the particle is (a) $v < E/B$ and (b) $v > E/B$?

6 Figure 28-26 shows crossed uniform electric and magnetic fields \vec{E} and \vec{B} and, at a certain instant, the velocity vectors of the 10 charged particles listed in Table 28-3. (The vectors are not drawn to scale.) The speeds given in the table are either less than or greater than E/B (see Question 5). Which particles will move out of the page toward you after the instant shown in Fig. 28-26?

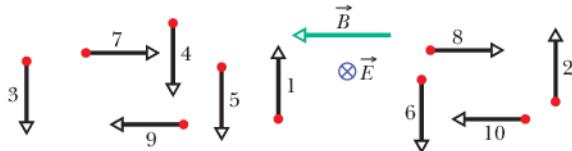


Figure 28-26 Question 6.

Table 28-3 Question 6

Particle	Charge	Speed	Particle	Charge	Speed
1	+	Less	6	-	Greater
2	+	Greater	7	+	Less
3	+	Less	8	+	Greater
4	+	Greater	9	-	Less
5	-	Less	10	-	Greater

7 Figure 28-27 shows the path of an electron that passes through two regions containing uniform magnetic fields of magnitudes B_1 and B_2 . Its path in each region is a half-circle. (a) Which field is stronger? (b) What is the direction of each field? (c) Is the time spent by the electron in the B_1 region greater than, less than, or the same as the time spent in the B_2 region?

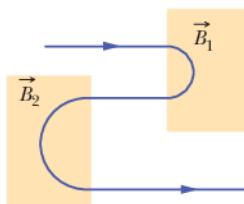


Figure 28-27 Question 7.

8 Figure 28-28 shows the path of an electron in a region of uniform magnetic field. The path consists of two straight sections, each between a pair of uniformly charged plates, and two half-circles. Which plate is at the higher electric potential in (a) the top pair of plates and (b) the bottom pair? (c) What is the direction of the magnetic field?

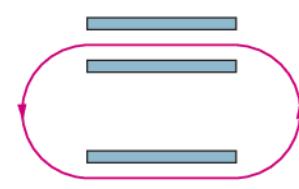


Figure 28-28 Question 8.

9 (a) In Checkpoint 5, if the dipole moment $\vec{\mu}$ is rotated from orientation 2 to orientation 1 by an external agent, is the work done on the dipole by the agent positive, negative, or zero? (b) Rank the work done on the dipole by the agent for these three rotations, greatest first: 2 → 1, 2 → 4, 2 → 3.

10 *Particle roundabout.* Figure 28-29 shows 11 paths through a region of uniform magnetic field. One path is a straight line; the rest are half-circles. Table 28-4 gives the masses, charges, and speeds of 11 particles that take these paths through the field in the directions shown. Which path in the figure corresponds to which

particle in the table? (The direction of the magnetic field can be determined by means of one of the paths, which is unique.)

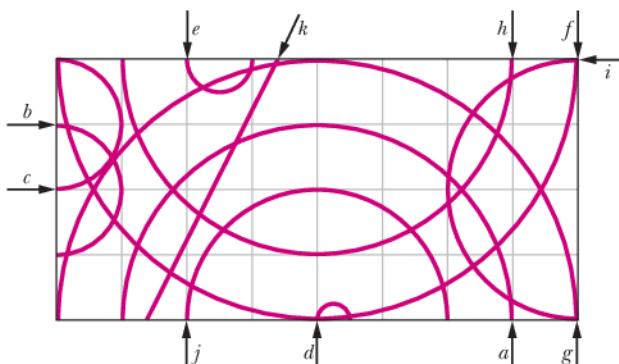


Figure 28-29 Question 10.

Table 28-4 Question 10

Particle	Mass	Charge	Speed
1	$2m$	q	v
2	m	$2q$	v
3	$m/2$	q	$2v$
4	$3m$	$3q$	$3v$
5	$2m$	q	$2v$
6	m	$-q$	$2v$
7	m	$-4q$	v
8	m	$-q$	v
9	$2m$	$-2q$	$3v$
10	m	$-2q$	$8v$
11	$3m$	0	$3v$

11 In Fig. 28-30, a charged particle enters a uniform magnetic field \vec{B} with speed v_0 , moves through a half-circle in time T_0 , and then leaves the field. (a) Is the charge positive or negative? (b) Is the final speed of the particle greater than, less than, or equal to v_0 ? (c) If the initial speed had been $0.5v_0$, would the time spent in field \vec{B} have been greater than, less than, or equal to T_0 ? (d) Would the path have been a half-circle, more than a half-circle, or less than a half-circle?

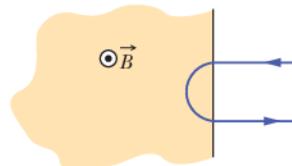


Figure 28-30 Question 11.

12 Figure 28-31 gives snapshots for three situations in which a positively charged particle passes through a uniform magnetic field \vec{B} . The velocities \vec{v} of the particle differ in orientation in the three snapshots but not in magnitude. Rank the situations according to (a) the period, (b) the frequency, and (c) the pitch of the particle's motion, greatest first.

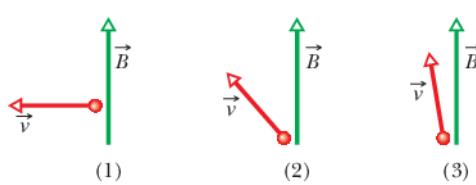


Figure 28-31 Question 12.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com



Worked-out solution is at



Interactive solution is at

<http://www.wiley.com/college/halliday>

Module 28-1 Magnetic Fields and the Definition of \vec{B}

- 1 **SSM ILW** A proton traveling at 23.0° with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of 6.50×10^{-17} N. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.

- 2 A particle of mass 10 g and charge $80 \mu\text{C}$ moves through a uniform magnetic field, in a region where the free-fall acceleration is $-9.8\hat{j}$ m/s². The velocity of the particle is a constant $20\hat{i}$ km/s, which is perpendicular to the magnetic field. What, then, is the magnetic field?

- 3 An electron that has an instantaneous velocity of

$$\vec{v} = (2.0 \times 10^6 \text{ m/s})\hat{i} + (3.0 \times 10^6 \text{ m/s})\hat{j}$$

- is moving through the uniform magnetic field $\vec{B} = (0.030 \text{ T})\hat{i} - (0.15 \text{ T})\hat{j}$. (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.

- 4 An alpha particle travels at a velocity \vec{v} of magnitude 550 m/s through a uniform magnetic field \vec{B} of magnitude 0.045 T. (An alpha particle has a charge of $+3.2 \times 10^{-19}$ C and a mass of 6.6×10^{-27} kg.) The angle between \vec{v} and \vec{B} is 52° . What is the magnitude of (a) the force \vec{F}_B acting on the particle due to the field and (b) the acceleration of the particle due to \vec{F}_B ? (c) Does the speed of the particle increase, decrease, or remain the same?

- 5 **GO** An electron moves through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (3.0B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19} \text{ N})\hat{k}$. Find B_x .

- 6 **GO** A proton moves through a uniform magnetic field given by $\vec{B} = (10\hat{i} - 20\hat{j} + 30\hat{k})$ mT. At time t_1 , the proton has a velocity given by $\vec{v} = v_x\hat{i} + v_y\hat{j} + (2.0 \text{ km/s})\hat{k}$ and the magnetic force on the proton is $\vec{F}_B = (4.0 \times 10^{-17} \text{ N})\hat{i} + (2.0 \times 10^{-17} \text{ N})\hat{j}$. At that instant, what are (a) v_x and (b) v_y ?

Module 28-2 Crossed Fields: Discovery of the Electron

- 7 An electron has an initial velocity of $(12.0\hat{j} + 15.0\hat{k})$ km/s and a constant acceleration of $(2.00 \times 10^{12} \text{ m/s}^2)\hat{i}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B} = (400 \mu\text{T})\hat{i}$, find the electric field \vec{E} .

- 8 An electric field of 1.50 kV/m and a perpendicular magnetic field of 0.400 T act on a moving electron to produce no net force. What is the electron's speed?

- 9 **ILW** In Fig. 28-32, an electron accelerated from rest through potential difference $V_1 = 1.00 \text{ kV}$ enters the gap between two parallel plates having separation $d = 20.0 \text{ mm}$ and potential difference

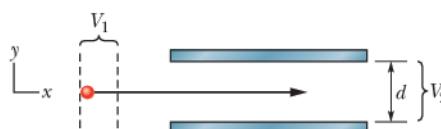


Figure 28-32 Problem 9.

$V_2 = 100 \text{ V}$. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

- 10 A proton travels through uniform magnetic and electric fields. The magnetic field is $\vec{B} = -2.50\hat{i}$ mT. At one instant the velocity of the proton is $\vec{v} = 2000\hat{j}$ m/s. At that instant and in unit-vector notation, what is the net force acting on the proton if the electric field is (a) $4.00\hat{k}$ V/m, (b) $-4.00\hat{k}$ V/m, and (c) $4.00\hat{i}$ V/m?

- 11 **GO** An ion source is producing ${}^6\text{Li}$ ions, which have charge $+e$ and mass 9.99×10^{-27} kg. The ions are accelerated by a potential difference of 10 kV and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude $B = 1.2 \text{ T}$. Calculate the strength of the smallest electric field, to be set up over the same region, that will allow the ${}^6\text{Li}$ ions to pass through undeflected.

- 12 **GO** At time t_1 , an electron is sent along the positive direction of an x axis, through both an electric field \vec{E} and a magnetic field \vec{B} , with \vec{E} directed parallel to the y axis. Figure 28-33 gives the y component $F_{\text{net},y}$ of the net force on the electron due to the two fields, as a function of the electron's speed v at time t_1 . The scale of the velocity axis is set by $v_s = 100.0 \text{ m/s}$. The x and z components of the net force are zero at t_1 . Assuming $B_x = 0$, find (a) the magnitude E and (b) \vec{B} in unit-vector notation.

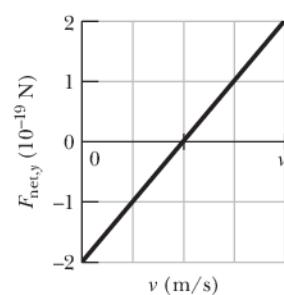
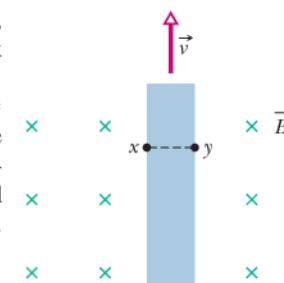


Figure 28-33 Problem 12.

Module 28-3 Crossed Fields: The Hall Effect

- 13 A strip of copper 150 μm thick and 4.5 mm wide is placed in a uniform magnetic field \vec{B} of magnitude 0.65 T, with \vec{B} perpendicular to the strip. A current $i = 23 \text{ A}$ is then sent through the strip such that a Hall potential difference V appears across the width of the strip. Calculate V . (The number of charge carriers per unit volume for copper is 8.47×10^{28} electrons/m³.)

- 14 A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity \vec{v} through a uniform magnetic field $B = 1.20 \text{ mT}$ directed perpendicular to the strip, as shown in Fig. 28-34. A potential difference of $3.90 \mu\text{V}$ is measured between points x and y across the strip. Calculate the speed v .



- 15 **GO** A conducting rectangular solid of dimensions $d_x = 5.00 \text{ m}$, $d_y = 3.00 \text{ m}$, and $d_z = 2.00 \text{ m}$ moves with a constant velocity $\vec{v} = (20.0 \text{ m/s})\hat{i}$ through a uniform magnetic field

Figure 28-34 Problem 14.

$\vec{B} = (30.0 \text{ mT})\hat{j}$ (Fig. 28-35). What are the resulting (a) electric field within the solid, in unit-vector notation, and (b) potential difference across the solid?

- 16 **GO** Figure 28-35 shows a metallic block, with its faces parallel to coordinate axes. The block is in a uniform magnetic field of magnitude 0.020 T. One edge length of the block is 25 cm; the block is *not* drawn to scale. The block is moved at 3.0 m/s parallel to each axis, in turn, and the resulting potential difference V that appears across the block is measured. With the motion parallel to the y axis, $V = 12 \text{ mV}$; with the motion parallel to the z axis, $V = 18 \text{ mV}$; with the motion parallel to the x axis, $V = 0$. What are the block lengths (a) d_x , (b) d_y , and (c) d_z ?

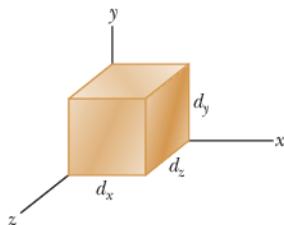


Figure 28-35 Problems 15 and 16.

Module 28-4 A Circulating Charged Particle

- 17 An alpha particle can be produced in certain radioactive decays of nuclei and consists of two protons and two neutrons. The particle has a charge of $q = +2e$ and a mass of 4.00 u, where u is the atomic mass unit, with $1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$. Suppose an alpha particle travels in a circular path of radius 4.50 cm in a uniform magnetic field with $B = 1.20 \text{ T}$. Calculate (a) its speed, (b) its period of revolution, (c) its kinetic energy, and (d) the potential difference through which it would have to be accelerated to achieve this energy.

- 18 **GO** In Fig. 28-36, a particle moves along a circle in a region of uniform magnetic field of magnitude $B = 4.00 \text{ mT}$. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude $3.20 \times 10^{-15} \text{ N}$. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of the motion?

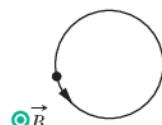


Figure 28-36 Problem 18.

- 19 A certain particle is sent into a uniform magnetic field, with the particle's velocity vector perpendicular to the direction of the field. Figure 28-37 gives the period T of the particle's motion versus the inverse of the field magnitude B . The vertical axis scale is set by $T_s = 40.0 \text{ ns}$, and the horizontal axis scale is set by $B_s^{-1} = 5.0 \text{ T}^{-1}$. What is the ratio m/q of the particle's mass to the magnitude of its charge?

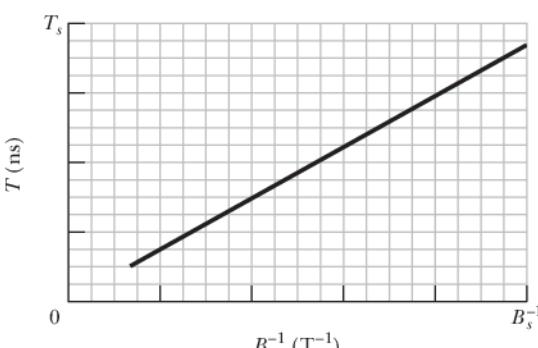


Figure 28-37 Problem 19.

- 20 An electron is accelerated from rest through potential difference V and then enters a region of uniform magnetic field, where it

undergoes uniform circular motion. Figure 28-38 gives the radius r of that motion versus $V^{1/2}$. The vertical axis scale is set by $r_s = 3.0 \text{ mm}$, and the horizontal axis scale is set by $V_s^{1/2} = 40.0 \text{ V}^{1/2}$. What is the magnitude of the magnetic field?

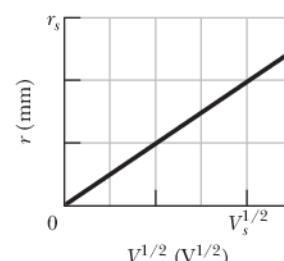


Figure 28-38 Problem 20.

- 21 **SSM** An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion.

- 22 In a nuclear experiment a proton with kinetic energy 1.0 MeV moves in a circular path in a uniform magnetic field. What energy must (a) an alpha particle ($q = +2e$, $m = 4.0 \text{ u}$) and (b) a deuteron ($q = +e$, $m = 2.0 \text{ u}$) have if they are to circulate in the same circular path?

- 23 What uniform magnetic field, applied perpendicular to a beam of electrons moving at $1.30 \times 10^6 \text{ m/s}$, is required to make the electrons travel in a circular arc of radius 0.350 m?

- 24 An electron is accelerated from rest by a potential difference of 350 V. It then enters a uniform magnetic field of magnitude 200 mT with its velocity perpendicular to the field. Calculate (a) the speed of the electron and (b) the radius of its path in the magnetic field.

- 25 (a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude $35.0 \mu\text{T}$. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field.

- 26 In Fig. 28-39, a charged particle moves into a region of uniform magnetic field \vec{B} , goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 130 ns in the region. (a) What is the magnitude of \vec{B} ? (b) If the particle is sent back through the magnetic field (along the same initial path) but with 2.00 times its previous kinetic energy, how much time does it spend in the field during this trip?

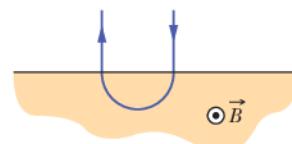


Figure 28-39 Problem 26.

- 27 A mass spectrometer (Fig. 28-12) is used to separate uranium ions of mass $3.92 \times 10^{-25} \text{ kg}$ and charge $3.20 \times 10^{-19} \text{ C}$ from related species. The ions are accelerated through a potential difference of 100 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 1.00 m. After traveling through 180° and passing through a slit of width 1.00 mm and height 1.00 cm, they are collected in a cup. (a) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is used to separate out 100 mg of material per hour, calculate (b) the current of the desired ions in the machine and (c) the thermal energy produced in the cup in 1.00 h.

- 28 A particle undergoes uniform circular motion of radius $26.1 \mu\text{m}$ in a uniform magnetic field. The magnetic force on the particle has a magnitude of $1.60 \times 10^{-17} \text{ N}$. What is the kinetic energy of the particle?

- 29 An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T. The pitch of the path is $6.00 \mu\text{m}$, and the

magnitude of the magnetic force on the electron is 2.00×10^{-15} N. What is the electron's speed?

- 30 GO** In Fig. 28-40, an electron with an initial kinetic energy of 4.0 keV enters region 1 at time $t = 0$. That region contains a uniform magnetic field directed into the page, with magnitude 0.010 T. The electron goes through a half-circle and then exits region 1, headed toward region 2 across a gap of 25.0 cm. There is an electric potential difference $\Delta V = 2000$ V across the gap, with a polarity such that the electron's speed increases uniformly as it traverses the gap. Region 2 contains a uniform magnetic field directed out of the page, with magnitude 0.020 T. The electron goes through a half-circle and then leaves region 2. At what time t does it leave?

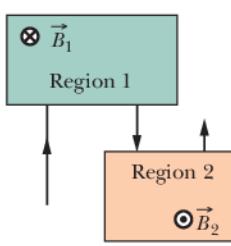


Figure 28-40
Problem 30.

- 31** A particular type of fundamental particle decays by transforming into an electron e^- and a positron e^+ . Suppose the decaying particle is at rest in a uniform magnetic field \vec{B} of magnitude 3.53 mT and the e^- and e^+ move away from the decay point in paths lying in a plane perpendicular to \vec{B} . How long after the decay do the e^- and e^+ collide?

- 32** A source injects an electron of speed $v = 1.5 \times 10^7$ m/s into a uniform magnetic field of magnitude $B = 1.0 \times 10^{-3}$ T. The velocity of the electron makes an angle $\theta = 10^\circ$ with the direction of the magnetic field. Find the distance d from the point of injection at which the electron next crosses the field line that passes through the injection point.

- 33 SSM WWW** A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field \vec{B} of magnitude 0.100 T, with its velocity vector making an angle of 89.0° with \vec{B} . Find (a) the period, (b) the pitch p , and (c) the radius r of its helical path.

- 34** An electron follows a helical path in a uniform magnetic field given by $\vec{B} = (20\hat{i} - 50\hat{j} - 30\hat{k})$ mT. At time $t = 0$, the electron's velocity is given by $\vec{v} = (20\hat{i} - 30\hat{j} + 50\hat{k})$ m/s. (a) What is the angle ϕ between \vec{v} and \vec{B} ? The electron's velocity changes with time. Do (b) its speed and (c) the angle ϕ change with time? (d) What is the radius of the helical path?

Module 28-5 Cyclotrons and Synchrotrons

- 35** A proton circulates in a cyclotron, beginning approximately at rest at the center. Whenever it passes through the gap between dees, the electric potential difference between the dees is 200 V. (a) By how much does its kinetic energy increase with each passage through the gap? (b) What is its kinetic energy as it completes 100 passes through the gap? Let r_{100} be the radius of the proton's circular path as it completes those 100 passes and enters a dee, and let r_{101} be its next radius, as it enters a dee the next time. (c) By what percentage does the radius increase when it changes from r_{100} to r_{101} ? That is, what is

$$\text{percentage increase} = \frac{r_{101} - r_{100}}{r_{100}} \cdot 100\%?$$

- 36** A cyclotron with dee radius 53.0 cm is operated at an oscillator frequency of 12.0 MHz to accelerate protons. (a) What magnitude B of magnetic field is required to achieve resonance? (b) At that field magnitude, what is the kinetic energy of a proton emerging from the cyclotron? Suppose, instead, that $B = 1.57$ T. (c) What oscillator frequency is required to achieve resonance now? (d) At that frequency, what is the kinetic energy of an emerging proton?

- 37** Estimate the total path length traveled by a deuteron in a cyclotron of radius 53 cm and operating frequency 12 MHz during the (entire) acceleration process. Assume that the accelerating potential between the dees is 80 kV.

- 38** In a certain cyclotron a proton moves in a circle of radius 0.500 m. The magnitude of the magnetic field is 1.20 T. (a) What is the oscillator frequency? (b) What is the kinetic energy of the proton, in electron-volts?

Module 28-6 Magnetic Force on a Current-Carrying Wire

- 39 SSM** A horizontal power line carries a current of 5000 A from south to north. Earth's magnetic field ($60.0 \mu\text{T}$) is directed toward the north and inclined downward at 70.0° to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earth's field.

- 40** A wire 1.80 m long carries a current of 13.0 A and makes an angle of 35.0° with a uniform magnetic field of magnitude $B = 1.50$ T. Calculate the magnetic force on the wire.

- 41 ILW** A 13.0 g wire of length $L = 62.0$ cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (Fig. 28-41). What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads?



Figure 28-41 Problem 41.

- 42** The bent wire shown in Fig. 28-42 lies in a uniform magnetic field. Each straight section is 2.0 m long and makes an angle of $\theta = 60^\circ$ with the x axis, and the wire carries a current of 2.0 A. What is the net magnetic force on the wire in unit-vector notation if the magnetic field is given by (a) $4.0\hat{k}$ T and (b) $4.0\hat{i}$ T?

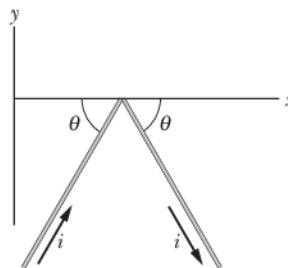


Figure 28-42 Problem 42.

- 43** A single-turn current loop, carrying a current of 4.00 A, is in the shape of a right triangle with sides 50.0, 120, and 130 cm. The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130 cm side of the loop. What is the magnitude of the magnetic force on (a) the 130 cm side, (b) the 50.0 cm side, and (c) the 120 cm side? (d) What is the magnitude of the net force on the loop?

- 44** Figure 28-43 shows a wire ring of radius $a = 1.8$ cm that is perpendicular to the general direction of a radially symmetric, diverging magnetic field. The magnetic field at the ring is everywhere of the same magnitude $B = 3.4$ mT, and its direction at the ring everywhere makes an angle $\theta = 20^\circ$ with a normal to the plane of the ring. The twisted lead wires have no effect on the problem. Find the magnitude of the force the field exerts on the ring if the ring carries a current $i = 4.6$ mA.

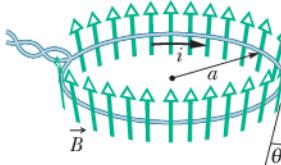


Figure 28-43 Problem 44.

- 45** A wire 50.0 cm long carries a 0.500 A current in the positive direction of an x axis through a magnetic field $\vec{B} = (3.00 \text{ mT})\hat{j} + (10.0 \text{ mT})\hat{k}$. In unit-vector notation, what is the magnetic force on the wire?

- 46** In Fig. 28-44, a metal wire of mass $m = 24.1\text{ mg}$ can slide with negligible friction on two horizontal parallel rails separated by distance $d = 2.56\text{ cm}$. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT . At time $t = 0$, device G is connected to the rails, producing a constant current $i = 9.13\text{ mA}$ in the wire and rails (even as the wire moves). At $t = 61.1\text{ ms}$, what are the wire's (a) speed and (b) direction of motion (left or right)?

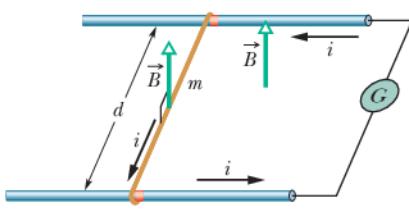


Figure 28-44 Problem 46.

- 47** A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60 . What are the (a) magnitude and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?

- 48** A long, rigid conductor, lying along an x axis, carries a current of 5.0 A in the negative x direction. A magnetic field \vec{B} is present, given by $\vec{B} = 3.0\hat{i} + 8.0x^2\hat{j}$, with x in meters and \vec{B} in milliteslas. Find, in unit-vector notation, the force on the 2.0 m segment of the conductor that lies between $x = 1.0\text{ m}$ and $x = 3.0\text{ m}$.

Module 28-7 Torque on a Current Loop

- 49 SSM** Figure 28-45 shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm . It carries a current of 0.10 A and is hinged along one long side. It is mounted in the xy plane, at angle $\theta = 30^\circ$ to the direction of a uniform magnetic field of magnitude 0.50 T . In unit-vector notation, what is the torque acting on the coil about the hinge line?

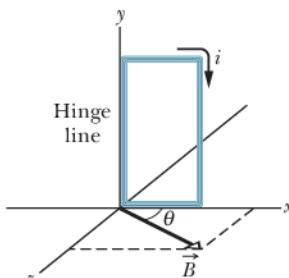


Figure 28-45 Problem 49.

- 50** An electron moves in a circle of radius $r = 5.29 \times 10^{-11}\text{ m}$ with speed $2.19 \times 10^6\text{ m/s}$. Treat the circular path as a current loop with a constant current equal to the ratio of the electron's charge magnitude to the period of the motion. If the circle lies in a uniform magnetic field of magnitude $B = 7.10\text{ mT}$, what is the maximum possible magnitude of the torque produced on the loop by the field?

- 51** Figure 28-46 shows a wood cylinder of mass $m = 0.250\text{ kg}$ and length $L = 0.100\text{ m}$, with $N = 10.0$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T , what is the least current i through the coil that keeps the cylinder from rolling down the plane?

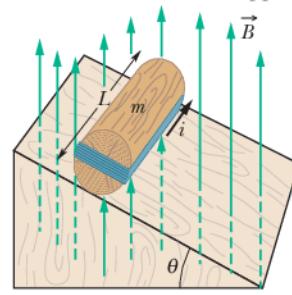
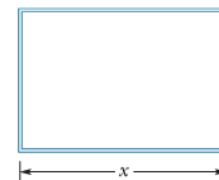


Figure 28-46 Problem 51.

- 52** In Fig. 28-47, a rectangular loop carrying current lies in the plane of a uniform magnetic field of magnitude 0.040 T . The loop consists of a single turn of flexible conducting wire that is wrapped around a flexible mount such that the dimensions of the rectangle can be changed. (The total length of the wire is not changed.) As edge length x is varied from approximately zero to its maximum value of approximately 4.0 cm , the magnitude τ of the torque on the loop changes. The maximum value of τ is $4.80 \times 10^{-8}\text{ N} \cdot \text{m}$. What is the current in the loop?

Figure 28-47
Problem 52.

- 53** Prove that the relation $\tau = NiAB \sin \theta$ holds not only for the rectangular loop of Fig. 28-19 but also for a closed loop of any shape. (Hint: Replace the loop of arbitrary shape with an assembly of adjacent long, thin, approximately rectangular loops that are nearly equivalent to the loop of arbitrary shape as far as the distribution of current is concerned.)

Module 28-8 The Magnetic Dipole Moment

- 54** A magnetic dipole with a dipole moment of magnitude 0.020 J/T is released from rest in a uniform magnetic field of magnitude 52 mT . The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientation where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.80 mJ . (a) What is the initial angle between the dipole moment and the magnetic field? (b) What is the angle when the dipole is next (momentarily) at rest?

- 55 SSM** Two concentric, circular wire loops, of radii $r_1 = 20.0\text{ cm}$ and $r_2 = 30.0\text{ cm}$, are located in an xy plane; each carries a clockwise current of 7.00 A (Fig. 28-48). (a) Find the magnitude of the net magnetic dipole moment of the system. (b) Repeat for reversed current in the inner loop.

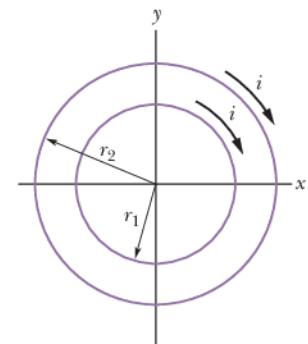


Figure 28-48 Problem 55.

- 56** A circular wire loop of radius 15.0 cm carries a current of 2.60 A . It is placed so that the normal to its plane makes an angle of 41.0° with a uniform magnetic field of magnitude 12.0 T . (a) Calculate the magnitude of the magnetic dipole moment of the loop. (b) What is the magnitude of the torque acting on the loop?

- 57 SSM** A circular coil of 160 turns has a radius of 1.90 cm . (a) Calculate the current that results in a magnetic dipole moment of magnitude $2.30\text{ A} \cdot \text{m}^2$. (b) Find the maximum magnitude of the torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.

- 58** The magnetic dipole moment of Earth has magnitude $8.00 \times 10^{22}\text{ J/T}$. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500 km , calculate the current they produce.

- 59** A current loop, carrying a current of 5.0 A , is in the shape of a right triangle with sides 30 , 40 , and 50 cm . The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Find the magnitude of (a) the magnetic dipole moment of the loop and (b) the torque on the loop.

- 60** Figure 28-49 shows a current loop $ABCDEF$ carrying a current $i = 5.00 \text{ A}$. The sides of the loop are parallel to the coordinate axes shown, with $AB = 20.0 \text{ cm}$, $BC = 30.0 \text{ cm}$, and $FA = 10.0 \text{ cm}$. In unit-vector notation, what is the magnetic dipole moment of this loop? (Hint: Imagine equal and opposite currents i in the line segment AD ; then treat the two rectangular loops $ABCDA$ and $ADEFA$.)

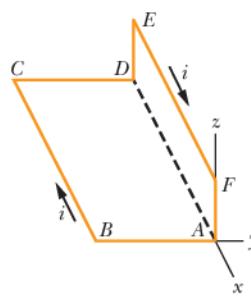


Figure 28-49 Problem 60.

- 61 SSM** The coil in Fig. 28-50 carries current $i = 2.00 \text{ A}$ in the direction indicated, is parallel to an xz plane, has 3.00 turns and an area of $4.00 \times 10^{-3} \text{ m}^2$, and lies in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 3.00\hat{j} - 4.00\hat{k}) \text{ mT}$. What are (a) the orientation energy of the coil in the magnetic field and (b) the torque (in unit-vector notation) on the coil due to the magnetic field?

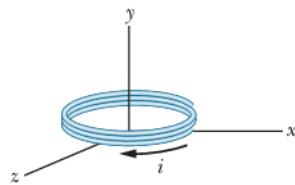


Figure 28-50 Problem 61.

- 62 GO** In Fig. 28-51a, two concentric coils, lying in the same plane, carry currents in opposite directions. The current in the larger coil 1 is fixed. Current i_2 in coil 2 can be varied. Figure 28-51b gives the net magnetic moment of the two-coil system as a function of i_2 . The vertical axis scale is set by $\mu_{\text{net},s} = 2.0 \times 10^{-5} \text{ A} \cdot \text{m}^2$, and the horizontal axis scale is set by $i_{2s} = 10.0 \text{ mA}$. If the current in coil 2 is then reversed, what is the magnitude of the net magnetic moment of the two-coil system when $i_2 = 7.0 \text{ mA}$?

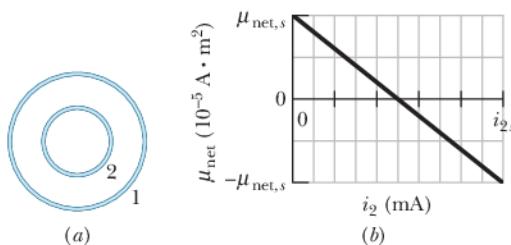


Figure 28-51 Problem 62.

- 63** A circular loop of wire having a radius of 8.0 cm carries a current of 0.20 A . A vector of unit length and parallel to the dipole moment $\vec{\mu}$ of the loop is given by $0.60\hat{i} - 0.80\hat{j}$. (This unit vector gives the orientation of the magnetic dipole moment vector.) If the loop is located in a uniform magnetic field given by $\vec{B} = (0.25 \text{ T})\hat{i} + (0.30 \text{ T})\hat{k}$, find (a) the torque on the loop (in unit-vector notation) and (b) the orientation energy of the loop.

- 64 GO** Figure 28-52 gives the orientation energy U of a magnetic dipole in an external magnetic field \vec{B} , as a function of angle ϕ between the directions of \vec{B} and the dipole moment. The vertical axis scale is set by $U_s = 2.0 \times 10^{-4} \text{ J}$. The dipole can be rotated about an axle with negligible friction in order to change ϕ . Counterclockwise rotation from $\phi = 0$ yields positive values of ϕ ,

and clockwise rotations yield negative values. The dipole is to be released at angle $\phi = 0$ with a rotational kinetic energy of $6.7 \times 10^{-4} \text{ J}$, so that it rotates counterclockwise. To what maximum value of ϕ will it rotate? (In the language of Module 8-3, what value ϕ is the turning point in the potential well of Fig. 28-52?)

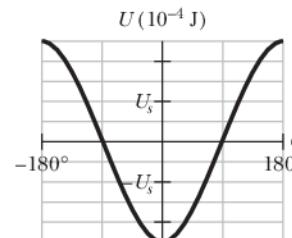


Figure 28-52 Problem 64.

- 65 SSM ILW** A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field \vec{B} of magnitude 5.71 mT . If the torque on the coil from the field is maximized, what are (a) the angle between \vec{B} and the coil's magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?

Additional Problems

- 66** A proton of charge $+e$ and mass m enters a uniform magnetic field $\vec{B} = B\hat{i}$ with an initial velocity $\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j}$. Find an expression in unit-vector notation for its velocity \vec{v} at any later time t .

- 67** A stationary circular wall clock has a face with a radius of 15 cm . Six turns of wire are wound around its perimeter; the wire carries a current of 2.0 A in the clockwise direction. The clock is located where there is a constant, uniform external magnetic field of magnitude 70 mT (but the clock still keeps perfect time). At exactly 1:00 P.M., the hour hand of the clock points in the direction of the external magnetic field. (a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field? (b) Find the torque magnitude.

- 68** A wire lying along a y axis from $y = 0$ to $y = 0.250 \text{ m}$ carries a current of 2.00 mA in the negative direction of the axis. The wire fully lies in a nonuniform magnetic field that is given by $\vec{B} = (0.300 \text{ T/m})y\hat{i} + (0.400 \text{ T/m})y\hat{j}$. In unit-vector notation, what is the magnetic force on the wire?

- 69** Atom 1 of mass 35 u and atom 2 of mass 37 u are both singly ionized with a charge of $+e$. After being introduced into a mass spectrometer (Fig. 28-12) and accelerated from rest through a potential difference $V = 7.3 \text{ kV}$, each ion follows a circular path in a uniform magnetic field of magnitude $B = 0.50 \text{ T}$. What is the distance Δx between the points where the ions strike the detector?

- 70** An electron with kinetic energy 2.5 keV moving along the positive direction of an x axis enters a region in which a uniform electric field of magnitude 10 kV/m is in the negative direction of the y axis. A uniform magnetic field \vec{B} is to be set up to keep the electron moving along the x axis, and the direction of \vec{B} is to be chosen to minimize the required magnitude of \vec{B} . In unit-vector notation, what \vec{B} should be set up?

- 71** Physicist S. A. Goudsmit devised a method for measuring the mass of heavy ions by timing their period of revolution in a known magnetic field. A singly charged ion of iodine makes 7.00 rev in a 45.0 mT field in 1.29 ms . Calculate its mass in atomic mass units.

72 A beam of electrons whose kinetic energy is K emerges from a thin-foil “window” at the end of an accelerator tube. A metal plate at distance d from this window is perpendicular to the direction of the emerging beam (Fig. 28-53). (a) Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field such that

$$B \geq \sqrt{\frac{2mK}{e^2d^2}},$$

in which m and e are the electron mass and charge. (b) How should \vec{B} be oriented?

73 SSM At time $t = 0$, an electron with kinetic energy 12 keV moves through $x = 0$ in the positive direction of an x axis that is parallel to the horizontal component of Earth’s magnetic field \vec{B} . The field’s vertical component is downward and has magnitude 55.0 μT . (a) What is the magnitude of the electron’s acceleration due to \vec{B} ? (b) What is the electron’s distance from the x axis when the electron reaches coordinate $x = 20$ cm?

74 SSM A particle with charge 2.0 C moves through a uniform magnetic field. At one instant the velocity of the particle is $(2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k})$ m/s and the magnetic force on the particle is $(4.0\hat{i} - 20\hat{j} + 12\hat{k})$ N. The x and y components of the magnetic field are equal. What is \vec{B} ?

75 A proton, a deuteron ($q = +e, m = 2.0$ u), and an alpha particle ($q = +2e, m = 4.0$ u) all having the same kinetic energy enter a region of uniform magnetic field \vec{B} , moving perpendicular to \vec{B} . What is the ratio of (a) the radius r_d of the deuteron path to the radius r_p of the proton path and (b) the radius r_α of the alpha particle path to r_p ?

76 Bainbridge’s mass spectrometer, shown in Fig. 28-54, separates ions having the same velocity. The ions, after entering through slits, S_1 and S_2 , pass through a velocity selector composed of an electric field produced by the charged plates P and P' , and a magnetic field \vec{B} perpendicular to the electric field and the ion path. The ions that then pass undeviated through the crossed \vec{E} and \vec{B} fields enter into a region where a second magnetic field \vec{B}' exists, where they are made to follow circular paths. A photographic plate (or a modern detector) registers their arrival. Show that, for the ions, $q/m = E/rBB'$, where r is the radius of the circular orbit.

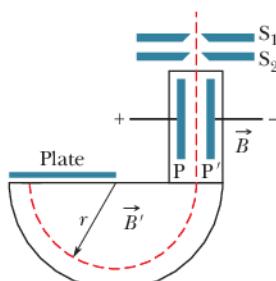


Figure 28-54 Problem 76.

77 SSM In Fig. 28-55, an electron moves at speed $v = 100$ m/s along an x axis through uniform electric and magnetic fields. The magnetic field \vec{B} is directed into the page and has magnitude 5.00 T. In unit-vector notation, what is the electric field?

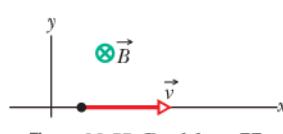


Figure 28-55 Problem 77.

78 (a) In Fig. 28-8, show that the ratio of the Hall electric field magnitude E to the magnitude E_C of the electric field responsible for moving charge (the current) along the length of

the strip is

$$\frac{E}{E_C} = \frac{B}{nep},$$

where ρ is the resistivity of the material and n is the number density of the charge carriers. (b) Compute this ratio numerically for Problem 13. (See Table 26-1.)

79 SSM A proton, a deuteron ($q = +e, m = 2.0$ u), and an alpha particle ($q = +2e, m = 4.0$ u) are accelerated through the same potential difference and then enter the same region of uniform magnetic field \vec{B} , moving perpendicular to \vec{B} . What is the ratio of (a) the proton’s kinetic energy K_p to the alpha particle’s kinetic energy K_α and (b) the deuteron’s kinetic energy K_d to K_α ? If the radius of the proton’s circular path is 10 cm, what is the radius of (c) the deuteron’s path and (d) the alpha particle’s path?

80 An electron is moving at 7.20×10^6 m/s in a magnetic field of strength 83.0 mT. What is the (a) maximum and (b) minimum magnitude of the force acting on the electron due to the field? (c) At one point the electron has an acceleration of magnitude 4.90×10^{14} m/s². What is the angle between the electron’s velocity and the magnetic field?

81 A 5.0 μC particle moves through a region containing the uniform magnetic field $-20\hat{i}$ mT and the uniform electric field $300\hat{j}$ V/m. At a certain instant the velocity of the particle is $(17\hat{i} - 11\hat{j} + 7.0\hat{k})$ km/s. At that instant and in unit-vector notation, what is the net electromagnetic force (the sum of the electric and magnetic forces) on the particle?

82 In a Hall-effect experiment, a current of 3.0 A sent lengthwise through a conductor 1.0 cm wide, 4.0 cm long, and 10 μm thick produces a transverse (across the width) Hall potential difference of 10 μV when a magnetic field of 1.5 T is passed perpendicularly through the thickness of the conductor. From these data, find (a) the drift velocity of the charge carriers and (b) the number density of charge carriers. (c) Show on a diagram the polarity of the Hall potential difference with assumed current and magnetic field directions, assuming also that the charge carriers are electrons.

83 SSM A particle of mass 6.0 g moves at 4.0 km/s in an xy plane, in a region with a uniform magnetic field given by $5.0\hat{i}$ mT. At one instant, when the particle’s velocity is directed 37° counterclockwise from the positive direction of the x axis, the magnetic force on the particle is $0.48\hat{k}$ N. What is the particle’s charge?

84 A wire lying along an x axis from $x = 0$ to $x = 1.00$ m carries a current of 3.00 A in the positive x direction. The wire is immersed in a nonuniform magnetic field that is given by $\vec{B} = (4.00 \text{ T/m}^2)x^2\hat{i} - (0.600 \text{ T/m}^2)x^2\hat{j}$. In unit-vector notation, what is the magnetic force on the wire?

85 At one instant, $\vec{v} = (-2.00\hat{i} + 4.00\hat{j} - 6.00\hat{k})$ m/s is the velocity of a proton in a uniform magnetic field $\vec{B} = (2.00\hat{i} - 4.00\hat{j} + 8.00\hat{k})$ mT. At that instant, what are (a) the magnetic force \vec{F} acting on the proton, in unit-vector notation, (b) the angle between \vec{v} and \vec{F} , and (c) the angle between \vec{v} and \vec{B} ?

86 An electron has velocity $\vec{v} = (32\hat{i} + 40\hat{j})$ km/s as it enters a uniform magnetic field $\vec{B} = 60\hat{i}$ μT . What are (a) the radius of the helical path taken by the electron and (b) the pitch of that path? (c) To an observer looking into the magnetic field region from the entrance point of the electron, does the electron spiral clockwise or counterclockwise as it moves?

87 Figure 28-56 shows a *homopolar generator*, which has a solid conducting disk as rotor and which is rotated by a motor (not shown). Conducting brushes connect this emf device to a circuit through which the device drives current. The device can produce a greater emf than wire loop rotors because they can spin at a much higher angular speed without rupturing. The disk has radius $R = 0.250\text{ m}$ and rotation frequency $f = 4000\text{ Hz}$, and the device is in a uniform magnetic field of magnitude $B = 60.0\text{ mT}$ that is perpendicular to the disk. As the disk is rotated, conduction electrons along the conducting path (dashed line) are forced to move through the magnetic field. (a) For the indicated rotation, is the magnetic force on those electrons up or down in the figure? (b) Is the magnitude of that force greater at the rim or near the center of the disk? (c) What is the work per unit charge done by that force in moving charge along the radial line, between the rim and the center? (d) What, then, is the emf of the device? (e) If the current is 50.0 A , what is the power at which electrical energy is being produced?

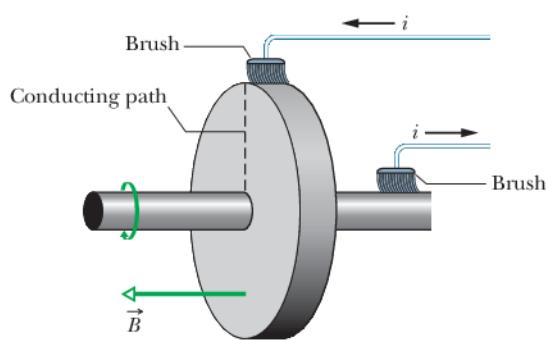


Figure 28-56 Problem 87.

88 In Fig. 28-57, the two ends of a U-shaped wire of mass $m = 10.0\text{ g}$ and length $L = 20.0\text{ cm}$ are immersed in mercury (which is a conductor). The wire is in a uniform field of magnitude $B = 0.100\text{ T}$. A switch (unshown) is rapidly closed and then reopened, sending a pulse of current through the wire, which causes the wire to jump upward. If jump height $h = 3.00\text{ m}$, how much charge was in the pulse? Assume that the duration of the pulse is much less than the time of flight. Consider the definition of impulse (Eq. 9-30) and its

relationship with momentum (Eq. 9-31). Also consider the relationship between charge and current (Eq. 26-2).

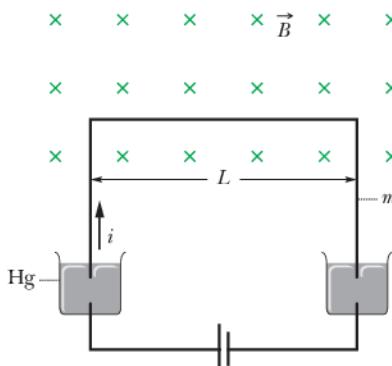


Figure 28-57 Problem 88.

89 In Fig. 28-58, an electron of mass m , charge $-e$, and low (negligible) speed enters the region between two plates of potential difference V and plate separation d , initially headed directly toward the top plate. A uniform magnetic field of magnitude B is normal to the plane of the figure. Find the minimum value of B such that the electron will not strike the top plate.

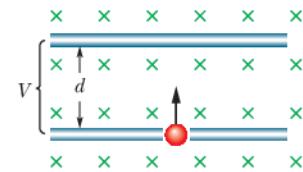


Figure 28-58 Problem 89.

90 A particle of charge q moves in a circle of radius r with speed v . Treating the circular path as a current loop with an average current, find the maximum torque exerted on the loop by a uniform field of magnitude B .

91 In a Hall-effect experiment, express the number density of charge carriers in terms of the Hall-effect electric field magnitude E , the current density magnitude J , and the magnetic field magnitude B .

92 An electron that is moving through a uniform magnetic field has velocity $\vec{v} = (40\text{ km/s})\hat{i} + (35\text{ km/s})\hat{j}$ when it experiences a force $\vec{F} = -(4.2\text{ fN})\hat{i} + (4.8\text{ fN})\hat{j}$ due to the magnetic field. If $B_x = 0$, calculate the magnetic field \vec{B} .

Magnetic Fields Due to Currents

29-1 MAGNETIC FIELD DUE TO A CURRENT

Learning Objectives

After reading this module, you should be able to ...

- 29.01** Sketch a current-length element in a wire and indicate the direction of the magnetic field that it sets up at a given point near the wire.
- 29.02** For a given point near a wire and a given current-element in the wire, determine the magnitude and direction of the magnetic field due to that element.
- 29.03** Identify the magnitude of the magnetic field set up by a current-length element at a point in line with the direction of that element.
- 29.04** For a point to one side of a long straight wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.
- 29.05** For a point to one side of a long straight wire carrying

current, use a right-hand rule to determine the direction of the field vector.

- 29.06** Identify that around a long straight wire carrying current, the magnetic field lines form circles.
- 29.07** For a point to one side of the end of a semi-infinite wire carrying current, apply the relationship between the magnetic field magnitude, the current, and the distance to the point.
- 29.08** For the center of curvature of a circular arc of wire carrying current, apply the relationship between the magnetic field magnitude, the current, the radius of curvature, and the angle subtended by the arc (in radians).
- 29.09** For a point to one side of a short straight wire carrying current, integrate the Biot–Savart law to find the magnetic field set up at the point by the current.

Key Ideas

- The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law. This law asserts that the contribution $d\vec{B}$ to the field produced by a current-length element $i d\vec{s}$ at a point P located a distance r from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \quad (\text{Biot–Savart law}).$$

Here \hat{r} is a unit vector that points from the element toward P . The quantity μ_0 , called the permeability constant, has the value

$$4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \approx 1.26 \times 10^{-6} \text{ T}\cdot\text{m/A}.$$

- For a long straight wire carrying a current i , the Biot–Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire,

$$B = \frac{\mu_0 i}{2\pi R} \quad (\text{long straight wire}).$$

- The magnitude of the magnetic field at the center of a circular arc, of radius R and central angle ϕ (in radians), carrying current i , is

$$B = \frac{\mu_0 i \phi}{4\pi R} \quad (\text{at center of circular arc}).$$

What Is Physics?

One basic observation of physics is that a moving charged particle produces a magnetic field around itself. Thus a current of moving charged particles produces a magnetic field around the current. This feature of *electromagnetism*, which is the combined study of electric and magnetic effects, came as a surprise to the people who discovered it. Surprise or not, this feature has become enormously important in everyday life because it is the basis of countless electromagnetic devices. For example, a magnetic field is produced in maglev trains and other devices used to lift heavy loads.

Our first step in this chapter is to find the magnetic field due to the current in a very small section of current-carrying wire. Then we shall find the magnetic field due to the entire wire for several different arrangements of the wire.

Calculating the Magnetic Field Due to a Current

Figure 29-1 shows a wire of arbitrary shape carrying a current i . We want to find the magnetic field \vec{B} at a nearby point P . We first mentally divide the wire into differential elements ds and then define for each element a length vector $d\vec{s}$ that has length ds and whose direction is the direction of the current in ds . We can then define a differential *current-length element* to be $i d\vec{s}$; we wish to calculate the field $d\vec{B}$ produced at P by a typical current-length element. From experiment we find that magnetic fields, like electric fields, can be superimposed to find a net field. Thus, we can calculate the net field \vec{B} at P by summing, via integration, the contributions $d\vec{B}$ from all the current-length elements. However, this summation is more challenging than the process associated with electric fields because of a complexity; whereas a charge element dq producing an electric field is a scalar, a current-length element $i d\vec{s}$ producing a magnetic field is a vector, being the product of a scalar and a vector.

Magnitude. The magnitude of the field $d\vec{B}$ produced at point P at distance r by a current-length element $i d\vec{s}$ turns out to be

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin \theta}{r^2}, \quad (29-1)$$

where θ is the angle between the directions of $d\vec{s}$ and \hat{r} , a unit vector that points from ds toward P . Symbol μ_0 is a constant, called the *permeability constant*, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \approx 1.26 \times 10^{-6} \text{ T}\cdot\text{m/A}. \quad (29-2)$$

Direction. The direction of $d\vec{B}$, shown as being into the page in Fig. 29-1, is that of the cross product $d\vec{s} \times \hat{r}$. We can therefore write Eq. 29-1 in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \quad (\text{Biot-Savart law}). \quad (29-3)$$

This vector equation and its scalar form, Eq. 29-1, are known as the **law of Biot and Savart** (rhymes with “Leo and bazaar”). The law, which is experimentally deduced, is an inverse-square law. We shall use this law to calculate the net magnetic field \vec{B} produced at a point by various distributions of current.

Here is one easy distribution: If current in a wire is either directly toward or directly away from a point P of measurement, can you see from Eq. 29-1 that the magnetic field at P from the current is simply zero (the angle θ is either 0° for *toward* or 180° for *away*, and both result in $\sin \theta = 0$)?

Magnetic Field Due to a Current in a Long Straight Wire

Shortly we shall use the law of Biot and Savart to prove that the magnitude of the magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by

$$B = \frac{\mu_0 i}{2\pi R} \quad (\text{long straight wire}). \quad (29-4)$$

The field magnitude B in Eq. 29-4 depends only on the current and the perpendicular distance R of the point from the wire. We shall show in our derivation that the field lines of \vec{B} form concentric circles around the wire, as Fig. 29-2 shows

This element of current creates a magnetic field at P , into the page.

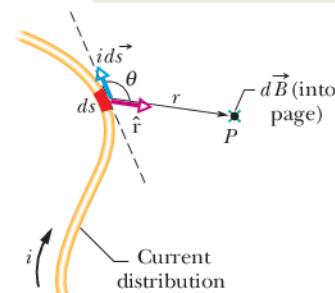


Figure 29-1 A current-length element $i d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P . The green \times (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

The magnetic field vector at any point is tangent to a circle.

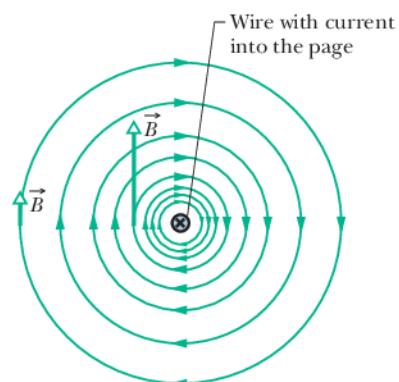
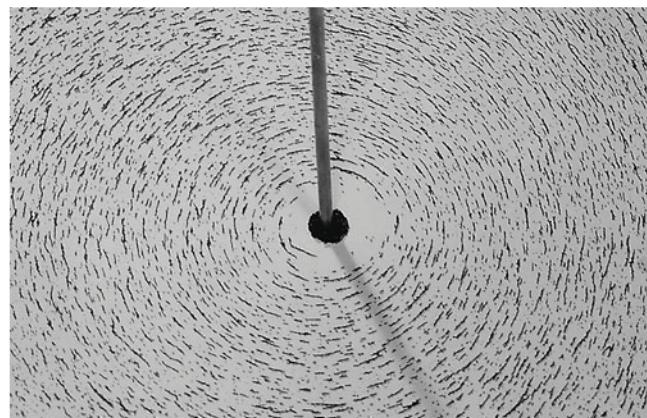


Figure 29-2 The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the \times .



Courtesy Education Development Center

Figure 29-3 Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current.

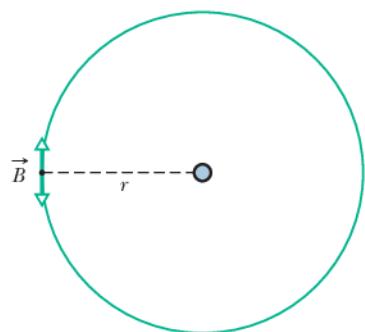


Figure 29-4 The magnetic field vector \vec{B} is perpendicular to the radial line extending from a long straight wire with current, but which of the two perpendicular vectors is it?

and as the iron filings in Fig. 29-3 suggest. The increase in the spacing of the lines in Fig. 29-2 with increasing distance from the wire represents the $1/R$ decrease in the magnitude of \vec{B} predicted by Eq. 29-4. The lengths of the two vectors \vec{B} in the figure also show the $1/R$ decrease.

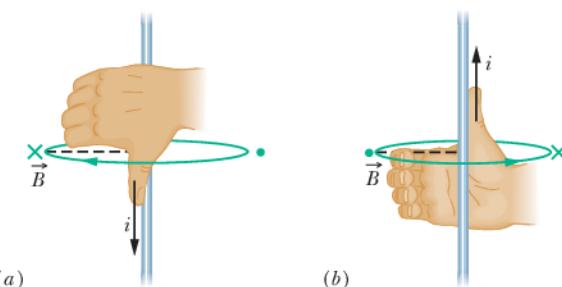
Directions. Plugging values into Eq. 29-4 to find the field magnitude B at a given radius is easy. What is difficult for many students is finding the direction of a field vector \vec{B} at a given point. The field lines form circles around a long straight wire, and the field vector at any point on a circle must be tangent to the circle. That means it must be perpendicular to a radial line extending to the point from the wire. But there are two possible directions for that perpendicular vector, as shown in Fig. 29-4. One is correct for current into the figure, and the other is correct for current out of the figure. How can you tell which is which? Here is a simple right-hand rule for telling which vector is correct:



Curved-straight right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

The result of applying this right-hand rule to the current in the straight wire of Fig. 29-2 is shown in a side view in Fig. 29-5a. To determine the direction of the magnetic field \vec{B} set up at any particular point by this current, mentally wrap your right hand around the wire with your thumb in the direction of the current. Let your fingertips pass through the point; their direction is then the direction of the magnetic field at that point. In the view of Fig. 29-2, \vec{B} at any point is *tangent to a magnetic field line*; in the view of Fig. 29-5, it is *perpendicular to a dashed radial line connecting the point and the current*.

Figure 29-5 A right-hand rule gives the direction of the magnetic field due to a current in a wire. (a) The situation of Fig. 29-2, seen from the side. The magnetic field \vec{B} at any point to the left of the wire is perpendicular to the dashed radial line and directed into the page, in the direction of the fingertips, as indicated by the \times . (b) If the current is reversed, \vec{B} at any point to the left is still perpendicular to the dashed radial line but now is directed out of the page, as indicated by the dot.



The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.

Proof of Equation 29-4

Figure 29-6, which is just like Fig. 29-1 except that now the wire is straight and of infinite length, illustrates the task at hand. We seek the field \vec{B} at point P , a perpendicular distance R from the wire. The magnitude of the differential magnetic field produced at P by the current-length element $i d\vec{s}$ located a distance r from P is given by Eq. 29-1:

$$dB = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \sin \theta}{r^2}.$$

The direction of $d\vec{B}$ in Fig. 29-6 is that of the vector $d\vec{s} \times \hat{r}$ —namely, directly into the page.

Note that $d\vec{B}$ at point P has this same direction for all the current-length elements into which the wire can be divided. Thus, we can find the magnitude of the magnetic field produced at P by the current-length elements in the upper half of the infinitely long wire by integrating dB in Eq. 29-1 from 0 to ∞ .

Now consider a current-length element in the lower half of the wire, one that is as far below P as $d\vec{s}$ is above P . By Eq. 29-3, the magnetic field produced at P by this current-length element has the same magnitude and direction as that from element $i d\vec{s}$ in Fig. 29-6. Further, the magnetic field produced by the lower half of the wire is exactly the same as that produced by the upper half. To find the magnitude of the *total* magnetic field \vec{B} at P , we need only multiply the result of our integration by 2. We get

$$B = 2 \int_0^\infty dB = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{\sin \theta ds}{r^2}. \quad (29-5)$$

The variables θ , s , and r in this equation are not independent; Fig. 29-6 shows that they are related by

$$r = \sqrt{s^2 + R^2}$$

and

$$\sin \theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

With these substitutions and integral 19 in Appendix E, Eq. 29-5 becomes

$$\begin{aligned} B &= \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{R ds}{(s^2 + R^2)^{3/2}} \\ &= \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2 + R^2)^{1/2}} \right]_0^\infty = \frac{\mu_0 i}{2\pi R}, \end{aligned} \quad (29-6)$$

as we wanted. Note that the magnetic field at P due to either the lower half or the upper half of the infinite wire in Fig. 29-6 is half this value; that is,

$$B = \frac{\mu_0 i}{4\pi R} \quad (\text{semi-infinite straight wire}). \quad (29-7)$$

Magnetic Field Due to a Current in a Circular Arc of Wire

To find the magnetic field produced at a point by a current in a curved wire, we would again use Eq. 29-1 to write the magnitude of the field produced by a single current-length element, and we would again integrate to find the net field produced by all the current-length elements. That integration can be difficult, depending on the shape of the wire; it is fairly straightforward, however, when the wire is a circular arc and the point is the center of curvature.

Figure 29-7a shows such an arc-shaped wire with central angle ϕ , radius R , and center C , carrying current i . At C , each current-length element $i d\vec{s}$ of the wire produces a magnetic field of magnitude dB given by Eq. 29-1. Moreover, as Fig. 29-7b shows, no matter where the element is located on the wire, the angle θ

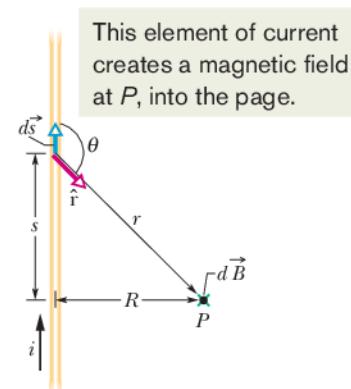
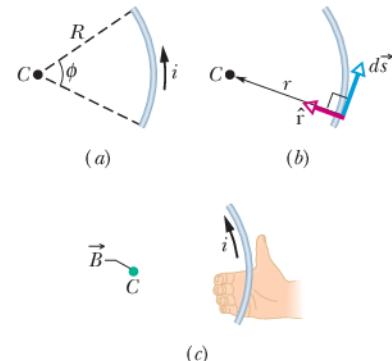


Figure 29-6 Calculating the magnetic field produced by a current i in a long straight wire. The field $d\vec{B}$ at P associated with the current-length element $i d\vec{s}$ is directed into the page, as shown.



The right-hand rule reveals the field's direction at the center.

Figure 29-7 (a) A wire in the shape of a circular arc with center C carries current i . (b) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and \hat{r} is 90° . (c) Determining the direction of the magnetic field at the center C due to the current in the wire; the field is out of the page, in the direction of the fingertips, as indicated by the colored dot at C .

between the vectors $d\vec{s}$ and \hat{f} is 90° ; also, $r = R$. Thus, by substituting R for r and 90° for θ in Eq. 29-1, we obtain

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{i ds}{R^2}. \quad (29-8)$$

The field at C due to each current-length element in the arc has this magnitude.

Directions. How about the direction of the differential field $d\vec{B}$ set up by an element? From above we know that the vector must be perpendicular to a radial line extending through point C from the element, either into the plane of Fig. 29-7a or out of it. To tell which direction is correct, we use the right-hand rule for any of the elements, as shown in Fig. 29-7c. Grasping the wire with the thumb in the direction of the current and bringing the fingers into the region near C , we see that the vector $d\vec{B}$ due to any of the differential elements is out of the plane of the figure, not into it.

Total Field. To find the total field at C due to all the elements on the arc, we need to add all the differential field vectors $d\vec{B}$. However, because the vectors are all in the same direction, we do not need to find components. We just sum the magnitudes dB as given by Eq. 29-8. Since we have a vast number of those magnitudes, we sum via integration. We want the result to indicate how the total field depends on the angle ϕ of the arc (rather than the arc length). So, in Eq. 29-8 we switch from ds to $d\phi$ by using the identity $ds = R d\phi$. The summation by integration then becomes

$$B = \int dB = \int_0^\phi \frac{\mu_0}{4\pi} \frac{iR d\phi}{R^2} = \frac{\mu_0 i}{4\pi R} \int_0^\phi d\phi.$$

Integrating, we find that

$$B = \frac{\mu_0 i \phi}{4\pi R} \quad (\text{at center of circular arc}). \quad (29-9)$$

Heads Up. Note that this equation gives us the magnetic field *only* at the center of curvature of a circular arc of current. When you insert data into the equation, you must be careful to express ϕ in radians rather than degrees. For example, to find the magnitude of the magnetic field at the center of a full circle of current, you would substitute 2π rad for ϕ in Eq. 29-9, finding

$$B = \frac{\mu_0 i (2\pi)}{4\pi R} = \frac{\mu_0 i}{2R} \quad (\text{at center of full circle}). \quad (29-10)$$



Sample Problem 29.01 Magnetic field at the center of a circular arc of current

The wire in Fig. 29-8a carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field \vec{B} (magnitude and direction) does the current produce at C ?

KEY IDEAS

We can find the magnetic field \vec{B} at point C by applying the Biot–Savart law of Eq. 29-3 to the wire, point by point along the full length of the wire. However, the application of Eq. 29-3 can be simplified by evaluating \vec{B} separately for the three distinguishable sections of the wire—namely, (1) the

straight section at the left, (2) the straight section at the right, and (3) the circular arc.

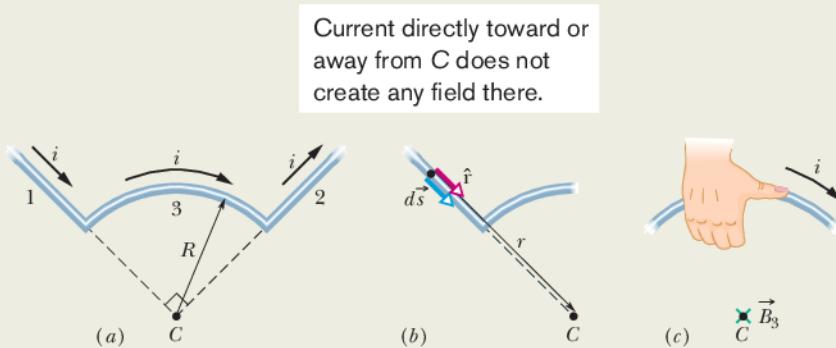
Straight sections: For any current-length element in section 1, the angle θ between $d\vec{s}$ and \hat{f} is zero (Fig. 29-8b); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi} \frac{i ds \sin \theta}{r^2} = \frac{\mu_0}{4\pi} \frac{i ds \sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C :

$$B_1 = 0.$$

Figure 29-8 (a) A wire consists of two straight sections (1 and 2) and a circular arc (3), and carries current i . (b) For a current-length element in section 1, the angle between $d\vec{s}$ and \hat{r} for any current-length element is zero. (c) Determining the direction of magnetic field \vec{B}_3 at C due to the current in the circular arc; the field is into the page there.



The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$ and \hat{r} for any current-length element is 180° . Thus,

$$B_2 = 0.$$

Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i \phi / 4\pi R$). Here the central angle ϕ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field \vec{B}_3 at the arc's center C is

$$B_3 = \frac{\mu_0 i (\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

To find the direction of \vec{B}_3 , we apply the right-hand rule displayed in Fig. 29-5. Mentally grasp the circular arc with your right hand as in Fig. 29-8c, with your thumb in the

direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point *into the plane* of the page. Thus, \vec{B}_3 is directed into that plane.

Net field: Generally, we combine multiple magnetic fields as vectors. Here, however, only the circular arc produces a magnetic field at point C . Thus, we can write the magnitude of the net field \vec{B} as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \quad (\text{Answer})$$

The direction of \vec{B} is the direction of \vec{B}_3 —namely, into the plane of Fig. 29-8.

Sample Problem 29.02 Magnetic field off to the side of two long straight currents

Figure 29-9a shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P ? Assume the following values: $i_1 = 15 \text{ A}$, $i_2 = 32 \text{ A}$, and $d = 5.3 \text{ cm}$.

KEY IDEAS

(1) The net magnetic field \vec{B} at point P is the vector sum of the magnetic fields due to the currents in the two wires. (2) We can find the magnetic field due to any current by applying the Biot–Savart law to the current. For points near the current in a long straight wire, that law leads to Eq. 29-4.

Finding the vectors: In Fig. 29-9a, point P is distance R from both currents i_1 and i_2 . Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields \vec{B}_1 and \vec{B}_2 with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-9a, note that the base angles (between sides R and d) are both 45° . This allows us to write

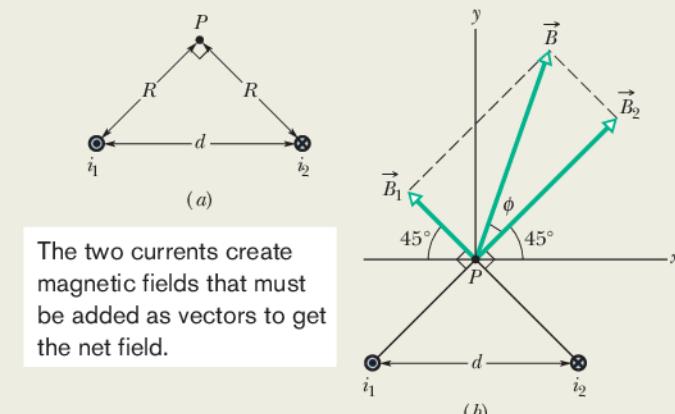


Figure 29-9 (a) Two wires carry currents i_1 and i_2 in opposite directions (out of and into the page). Note the right angle at P . (b) The separate fields \vec{B}_1 and \vec{B}_2 are combined vectorially to yield the net field \vec{B} .

$\cos 45^\circ = R/d$ and replace R with $d \cos 45^\circ$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45^\circ} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45^\circ}.$$

We want to combine \vec{B}_1 and \vec{B}_2 to find their vector sum, which is the net field \vec{B} at P . To find the directions of \vec{B}_1 and \vec{B}_2 , we apply the right-hand rule of Fig. 29-5 to each current in Fig. 29-9a. For wire 1, with current out of the page, we mentally grasp the wire with the right hand, with the thumb pointing out of the page. Then the curled fingers indicate that the field lines run counterclockwise. In particular, in the region of point P , they are directed upward to the left. Recall that the magnetic field at a point near a long, straight current-carrying wire must be directed perpendicular to a radial line between the point and the current. Thus, \vec{B}_1 must be directed upward to the left as drawn in Fig. 29-9b. (Note carefully the perpendicular symbol between vector \vec{B}_1 and the line connecting point P and wire 1.)

Repeating this analysis for the current in wire 2, we find that \vec{B}_2 is directed upward to the right as drawn in Fig. 29-9b.

Adding the vectors: We can now vectorially add \vec{B}_1 and \vec{B}_2 to find the net magnetic field \vec{B} at point P , either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of \vec{B} .



Additional examples, video, and practice available at WileyPLUS

29-2 FORCE BETWEEN TWO PARALLEL CURRENTS

Learning Objectives

After reading this module, you should be able to . . .

29.10 Given two parallel or antiparallel currents, find the magnetic field of the first current at the location of the second current and then find the force acting on that second current.

Key Ideas

- Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length L of either wire is

However, in Fig. 29-9b, there is a third method: Because \vec{B}_1 and \vec{B}_2 are perpendicular to each other, they form the legs of a right triangle, with \vec{B} as the hypotenuse. So,

$$\begin{aligned} B &= \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45^\circ)} \sqrt{i_1^2 + i_2^2} \\ &= \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A})\sqrt{(15 \text{ A})^2 + (32 \text{ A})^2}}{(2\pi)(5.3 \times 10^{-2} \text{ m})(\cos 45^\circ)} \\ &= 1.89 \times 10^{-4} \text{ T} \approx 190 \mu\text{T}. \end{aligned} \quad (\text{Answer})$$

The angle ϕ between the directions of \vec{B} and \vec{B}_2 in Fig. 29-9b follows from

$$\phi = \tan^{-1} \frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1} \frac{i_1}{i_2} = \tan^{-1} \frac{15 \text{ A}}{32 \text{ A}} = 25^\circ.$$

The angle between \vec{B} and the x axis shown in Fig. 29-9b is then

$$\phi + 45^\circ = 25^\circ + 45^\circ = 70^\circ. \quad (\text{Answer})$$

29.11 Identify that parallel currents attract each other, and antiparallel currents repel each other.

29.12 Describe how a rail gun works.

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d},$$

where d is the wire separation, and i_a and i_b are the currents in the wires.

Force Between Two Parallel Currents

Two long parallel wires carrying currents exert forces on each other. Figure 29-10 shows two such wires, separated by a distance d and carrying currents i_a and i_b . Let us analyze the forces on these wires due to each other.

We seek first the force on wire b in Fig. 29-10 due to the current in wire a . That current produces a magnetic field \vec{B}_a and it is this magnetic field that actually causes the force we seek. To find the force, then, we need the magnitude and direction of the field \vec{B}_a at the site of wire b . The magnitude of \vec{B}_a at every point of wire b is, from Eq. 29-4,

$$B_a = \frac{\mu_0 i_a}{2\pi d}. \quad (29-11)$$

The curled-straight right-hand rule tells us that the direction of \vec{B}_a at wire b is down, as Fig. 29-10 shows. Now that we have the field, we can find the force it produces on wire b . Equation 28-26 tells us that the force \vec{F}_{ba} on a length L of wire b due to the external magnetic field \vec{B}_a is

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a, \quad (29-12)$$

where \vec{L} is the length vector of the wire. In Fig. 29-10, vectors \vec{L} and \vec{B}_a are perpendicular to each other, and so with Eq. 29-11, we can write

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}. \quad (29-13)$$

The direction of \vec{F}_{ba} is the direction of the cross product $\vec{L} \times \vec{B}_a$. Applying the right-hand rule for cross products to \vec{L} and \vec{B}_a in Fig. 29-10, we see that \vec{F}_{ba} is directly toward wire a , as shown.

The general procedure for finding the force on a current-carrying wire is this:



To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

We could now use this procedure to compute the force on wire a due to the current in wire b . We would find that the force is directly toward wire b ; hence, the two wires with parallel currents attract each other. Similarly, if the two currents were antiparallel, we could show that the two wires repel each other. Thus,



Parallel currents attract each other, and antiparallel currents repel each other.

The force acting between currents in parallel wires is the basis for the definition of the ampere, which is one of the seven SI base units. The definition, adopted in 1946, is this: The ampere is that constant current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce on each of these conductors a force of magnitude 2×10^{-7} newton per meter of wire length.

Rail Gun

The basics of a rail gun are shown in Fig. 29-11a. A large current is sent out along one of two parallel conducting rails, across a conducting “fuse” (such as a narrow piece of copper) between the rails, and then back to the current source along the second rail. The projectile to be fired lies on the far side of the fuse and fits loosely between the rails. Immediately after the current begins, the fuse element melts and vaporizes, creating a conducting gas between the rails where the fuse had been.

The curled-straight right-hand rule of Fig. 29-5 reveals that the currents in the rails of Fig. 29-11a produce magnetic fields that are directed downward between the rails. The net magnetic field \vec{B} exerts a force \vec{F} on the gas due to the current i through the gas (Fig. 29-11b). With Eq. 29-12 and the right-hand rule for cross products, we find that \vec{F} points outward along the rails. As the gas is forced outward along the rails, it pushes the projectile, accelerating it by as much as 5×10^6 g, and then launches it with a speed of 10 km/s, all within 1 ms. Someday rail guns may be used to launch materials into space from mining operations on the Moon or an asteroid.



Checkpoint 1

The figure here shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. Rank the wires according to the magnitude of the force on each due to the currents in the other two wires, greatest first.

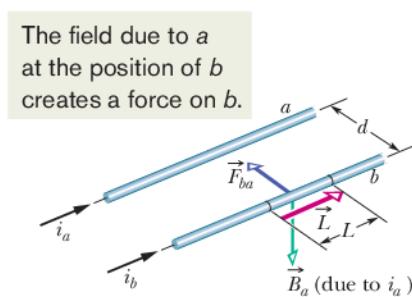
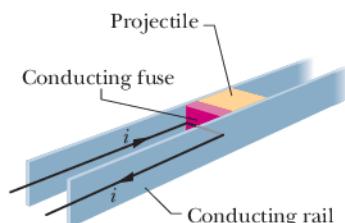
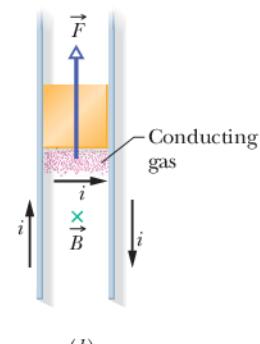


Figure 29-10 Two parallel wires carrying currents in the same direction attract each other. \vec{B}_a is the magnetic field at wire b produced by the current in wire a . \vec{F}_{ba} is the resulting force acting on wire b because it carries current in \vec{B}_a .



(a)



(b)

Figure 29-11 (a) A rail gun, as a current i is set up in it. The current rapidly causes the conducting fuse to vaporize. (b) The current produces a magnetic field \vec{B} between the rails, and the field causes a force \vec{F} to act on the conducting gas, which is part of the current path. The gas propels the projectile along the rails, launching it.



29-3 AMPERE'S LAW

Learning Objectives

After reading this module, you should be able to . . .

29.13 Apply Ampere's law to a loop that encircles current.

29.14 With Ampere's law, use a right-hand rule for determining the algebraic sign of an encircled current.

29.15 For more than one current within an Amperian loop, determine the net current to be used in Ampere's law.

29.16 Apply Ampere's law to a long straight wire with current, to find the magnetic field magnitude inside and outside the wire, identifying that only the current encircled by the Amperian loop matters.

Key Idea

- Ampere's law states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere's law}).$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current i on the right side is the *net* current encircled by the loop.

Ampere's Law

We can find the net electric field due to *any* distribution of charges by first writing the differential electric field $d\vec{E}$ due to a charge element and then summing the contributions of $d\vec{E}$ from all the elements. However, if the distribution is complicated, we may have to use a computer. Recall, however, that if the distribution has planar, cylindrical, or spherical symmetry, we can apply Gauss' law to find the net electric field with considerably less effort.

Similarly, we can find the net magnetic field due to *any* distribution of currents by first writing the differential magnetic field $d\vec{B}$ (Eq. 29-3) due to a current-length element and then summing the contributions of $d\vec{B}$ from all the elements. Again we may have to use a computer for a complicated distribution. However, if the distribution has some symmetry, we may be able to apply **Ampere's law** to find the magnetic field with considerably less effort. This law, which can be derived from the Biot–Savart law, has traditionally been credited to André-Marie Ampère (1775–1836), for whom the SI unit of current is named. However, the law actually was advanced by English physicist James Clerk Maxwell. Ampere's law is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere's law}). \quad (29-14)$$

The loop on the integral sign means that the scalar (dot) product $\vec{B} \cdot d\vec{s}$ is to be integrated around a *closed* loop, called an *Amperian loop*. The current i_{enc} is the *net* current encircled by that closed loop.

To see the meaning of the scalar product $\vec{B} \cdot d\vec{s}$ and its integral, let us first apply Ampere's law to the general situation of Fig. 29-12. The figure shows cross sections of three long straight wires that carry currents i_1 , i_2 , and i_3 either directly into or directly out of the page. An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third. The counterclockwise direction marked on the loop indicates the arbitrarily chosen direction of integration for Eq. 29-14.

To apply Ampere's law, we mentally divide the loop into differential vector elements $d\vec{s}$ that are everywhere directed along the tangent to the loop in the direction of integration. Assume that at the location of the element $d\vec{s}$ shown in Fig. 29-12, the net magnetic field due to the three currents is \vec{B} . Because the wires are perpendicular to the page, we know that the magnetic

field at $d\vec{s}$ due to each current is in the plane of Fig. 29-12; thus, their net magnetic field \vec{B} at $d\vec{s}$ must also be in that plane. However, we do not know the orientation of \vec{B} within the plane. In Fig. 29-12, \vec{B} is arbitrarily drawn at an angle θ to the direction of $d\vec{s}$. The scalar product $\vec{B} \cdot d\vec{s}$ on the left side of Eq. 29-14 is equal to $B \cos \theta ds$. Thus, Ampere's law can be written as

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \theta ds = \mu_0 i_{\text{enc}}. \quad (29-15)$$

We can now interpret the scalar product $\vec{B} \cdot d\vec{s}$ as being the product of a length ds of the Amperian loop and the field component $B \cos \theta$ tangent to the loop. Then we can interpret the integration as being the summation of all such products around the entire loop.

Signs. When we can actually perform this integration, we do not need to know the direction of \vec{B} before integrating. Instead, we arbitrarily assume \vec{B} to be generally in the direction of integration (as in Fig. 29-12). Then we use the following curled-straight right-hand rule to assign a plus sign or a minus sign to each of the currents that make up the net encircled current i_{enc} :



Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Finally, we solve Eq. 29-15 for the magnitude of \vec{B} . If B turns out positive, then the direction we assumed for \vec{B} is correct. If it turns out negative, we neglect the minus sign and redraw \vec{B} in the opposite direction.

Net Current. In Fig. 29-13 we apply the curled-straight right-hand rule for Ampere's law to the situation of Fig. 29-12. With the indicated counterclockwise direction of integration, the net current encircled by the loop is

$$i_{\text{enc}} = i_1 - i_2.$$

(Current i_3 is not encircled by the loop.) We can then rewrite Eq. 29-15 as

$$\oint B \cos \theta ds = \mu_0(i_1 - i_2). \quad (29-16)$$

You might wonder why, since current i_3 contributes to the magnetic-field magnitude B on the left side of Eq. 29-16, it is not needed on the right side. The answer is that the contributions of current i_3 to the magnetic field cancel out because the integration in Eq. 29-16 is made around the full loop. In contrast, the contributions of an encircled current to the magnetic field do not cancel out.

We cannot solve Eq. 29-16 for the magnitude B of the magnetic field because for the situation of Fig. 29-12 we do not have enough information to simplify and solve the integral. However, we do know the outcome of the integration; it must be equal to $\mu_0(i_1 - i_2)$, the value of which is set by the net current passing through the loop.

We shall now apply Ampere's law to two situations in which symmetry does allow us to simplify and solve the integral, hence to find the magnetic field.

Magnetic Field Outside a Long Straight Wire with Current

Figure 29-14 shows a long straight wire that carries current i directly out of the page. Equation 29-4 tells us that the magnetic field \vec{B} produced by the current has the same magnitude at all points that are the same distance r from the wire; that is, the field \vec{B} has cylindrical symmetry about the wire. We can take advantage of that symmetry to simplify the integral in Ampere's law (Eqs. 29-14 and 29-15) if we encircle the wire with a concentric circular Amperian loop of radius r , as in Fig. 29-14. The magnetic field then has the same magnitude B at every point on the loop. We shall integrate counterclockwise, so that $d\vec{s}$ has the direction shown in Fig. 29-14.

Only the currents encircled by the loop are used in Ampere's law.

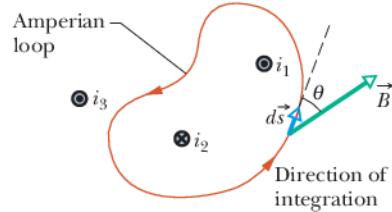


Figure 29-12 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

This is how to assign a sign to a current used in Ampere's law.

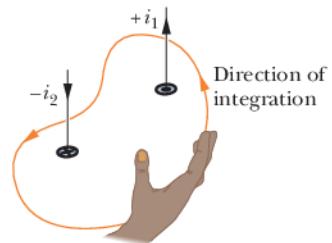


Figure 29-13 A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-12.

All of the current is encircled and thus all is used in Ampere's law.

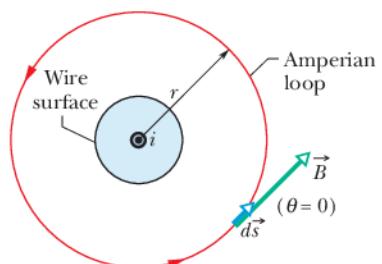


Figure 29-14 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

We can further simplify the quantity $B \cos \theta$ in Eq. 29-15 by noting that \vec{B} is tangent to the loop at every point along the loop, as is $d\vec{s}$. Thus, \vec{B} and $d\vec{s}$ are either parallel or antiparallel at each point of the loop, and we shall arbitrarily assume the former. Then at every point the angle θ between $d\vec{s}$ and \vec{B} is 0° , so $\cos \theta = \cos 0^\circ = 1$. The integral in Eq. 29-15 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \theta ds = B \oint ds = B(2\pi r).$$

Note that $\oint ds$ is the summation of all the line segment lengths ds around the circular loop; that is, it simply gives the circumference $2\pi r$ of the loop.

Our right-hand rule gives us a plus sign for the current of Fig. 29-14. The right side of Ampere's law becomes $+\mu_0 i$, and we then have

$$B(2\pi r) = \mu_0 i$$

or $B = \frac{\mu_0 i}{2\pi r}$ (outside straight wire). (29-17)

With a slight change in notation, this is Eq. 29-4, which we derived earlier—with considerably more effort—using the law of Biot and Savart. In addition, because the magnitude B turned out positive, we know that the correct direction of \vec{B} must be the one shown in Fig. 29-14.

Only the current encircled by the loop is used in Ampere's law.

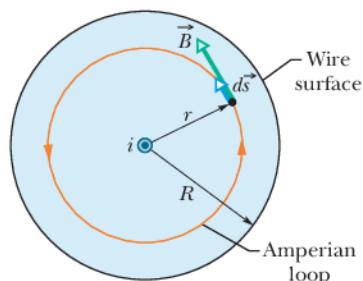


Figure 29-15 Using Ampere's law to find the magnetic field that a current i produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Magnetic Field Inside a Long Straight Wire with Current

Figure 29-15 shows the cross section of a long straight wire of radius R that carries a uniformly distributed current i directly out of the page. Because the current is uniformly distributed over a cross section of the wire, the magnetic field \vec{B} produced by the current must be cylindrically symmetrical. Thus, to find the magnetic field at points inside the wire, we can again use an Amperian loop of radius r , as shown in Fig. 29-15, where now $r < R$. Symmetry again suggests that \vec{B} is tangent to the loop, as shown; so the left side of Ampere's law again yields

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r). (29-18)$$

Because the current is uniformly distributed, the current i_{enc} encircled by the loop is proportional to the area encircled by the loop; that is,

$$i_{\text{enc}} = i \frac{\pi r^2}{\pi R^2}. (29-19)$$

Our right-hand rule tells us that i_{enc} gets a plus sign. Then Ampere's law gives us

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

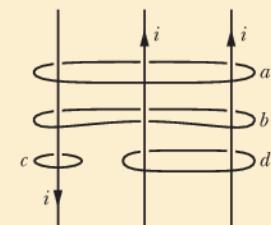
or $B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r$ (inside straight wire). (29-20)

Thus, inside the wire, the magnitude B of the magnetic field is proportional to r , is zero at the center, and is maximum at $r = R$ (the surface). Note that Eqs. 29-17 and 29-20 give the same value for B at the surface.



Checkpoint 2

The figure here shows three equal currents i (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ along each, greatest first.





Sample Problem 29.03 Ampere's law to find the field inside a long cylinder of current

Figure 29-16a shows the cross section of a long conducting cylinder with inner radius $a = 2.0\text{ cm}$ and outer radius $b = 4.0\text{ cm}$. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6 \text{ A/m}^4$ and r in meters. What is the magnetic field \vec{B} at the dot in Fig. 29-16a, which is at radius $r = 3.0\text{ cm}$ from the central axis of the cylinder?

KEY IDEAS

The point at which we want to evaluate \vec{B} is inside the material of the conducting cylinder, between its inner and outer radii. We note that the current distribution has cylindrical symmetry (it is the same all around the cross section for any given radius). Thus, the symmetry allows us to use Ampere's law to find \vec{B} at the point. We first draw the Amperian loop shown in Fig. 29-16b. The loop is concentric with the cylinder and has radius $r = 3.0\text{ cm}$ because we want to evaluate \vec{B} at that distance from the cylinder's central axis.

Next, we must compute the current i_{enc} that is encircled by the Amperian loop. However, we *cannot* set up a proportionality as in Eq. 29-19, because here the current is not uniformly distributed. Instead, we must integrate the current density magnitude from the cylinder's inner radius a to the loop radius r , using the steps shown in Figs. 29-16c through h.

Calculations: We write the integral as

$$\begin{aligned} i_{\text{enc}} &= \int J dA = \int_a^r cr^2(2\pi r dr) \\ &= 2\pi c \int_a^r r^3 dr = 2\pi c \left[\frac{r^4}{4} \right]_a^r \\ &= \frac{\pi c(r^4 - a^4)}{2}. \end{aligned}$$

Note that in these steps we took the differential area dA to be the area of the thin ring in Figs. 29-16d-f and then

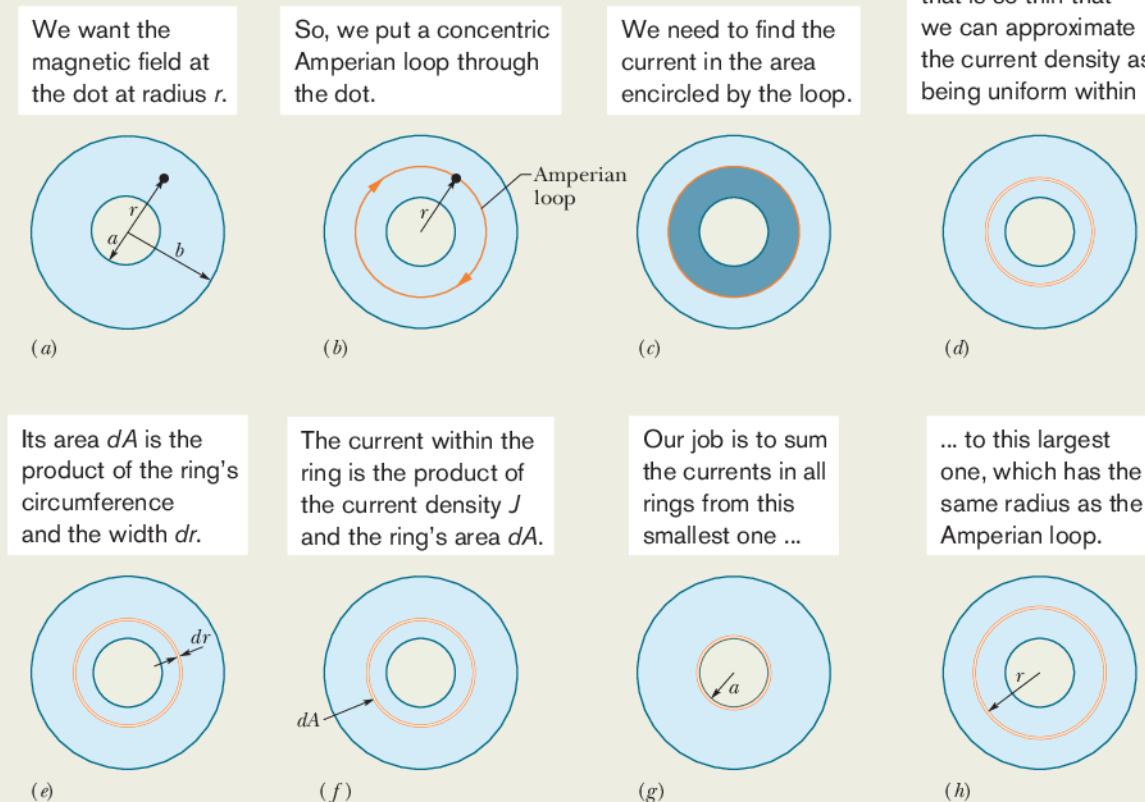


Figure 29-16 (a)-(b) To find the magnetic field at a point within this conducting cylinder, we use a concentric Amperian loop through the point. We then need the current encircled by the loop. (c)-(h) Because the current density is nonuniform, we start with a thin ring and then sum (via integration) the currents in all such rings in the encircled area.

replaced it with its equivalent, the product of the ring's circumference $2\pi r$ and its thickness dr .

For the Amperian loop, the direction of integration indicated in Fig. 29-16b is (arbitrarily) clockwise. Applying the right-hand rule for Ampere's law to that loop, we find that we should take i_{enc} as negative because the current is directed out of the page but our thumb is directed into the page.

We next evaluate the left side of Ampere's law as we did in Fig. 29-15, and we again obtain Eq. 29-18. Then Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 C}{2} (r^4 - a^4).$$

Solving for B and substituting known data yield

$$\begin{aligned} B &= -\frac{\mu_0 C}{4r} (r^4 - a^4) \\ &= -\frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A})(3.0 \times 10^6 \text{ A}/\text{m}^4)}{4(0.030 \text{ m})} \\ &\quad \times [(0.030 \text{ m})^4 - (0.020 \text{ m})^4] \\ &= -2.0 \times 10^{-5} \text{ T}. \end{aligned}$$

Thus, the magnetic field \vec{B} at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5} \text{ T} \quad (\text{Answer})$$

and forms magnetic field lines that are directed opposite our direction of integration, hence counterclockwise in Fig. 29-16b.



Additional examples, video, and practice available at WileyPLUS

29-4 SOLENOIDS AND TOROIDS

Learning Objectives

After reading this module, you should be able to . . .

- 29.17** Describe a solenoid and a toroid and sketch their magnetic field lines.
- 29.18** Explain how Ampere's law is used to find the magnetic field inside a solenoid.
- 29.19** Apply the relationship between a solenoid's internal magnetic field B , the current i , and the number of turns per

unit length n of the solenoid.

- 29.20** Explain how Ampere's law is used to find the magnetic field inside a toroid.
- 29.21** Apply the relationship between a toroid's internal magnetic field B , the current i , the radius r , and the total number of turns N .

Key Ideas

- Inside a long solenoid carrying current i , at points not near its ends, the magnitude B of the magnetic field is

$$B = \mu_0 i n \quad (\text{ideal solenoid}),$$

where n is the number of turns per unit length.

- At a point inside a toroid, the magnitude B of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \quad (\text{toroid}),$$

where r is the distance from the center of the toroid to the point.

Solenoids and Toroids

Magnetic Field of a Solenoid

We now turn our attention to another situation in which Ampere's law proves useful. It concerns the magnetic field produced by the current in a long, tightly wound helical coil of wire. Such a coil is called a **solenoid** (Fig. 29-17). We assume that the length of the solenoid is much greater than the diameter.

Figure 29-18 shows a section through a portion of a "stretched-out" solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (*windings*) that make up the solenoid. For points very



Figure 29-17 A solenoid carrying current i .

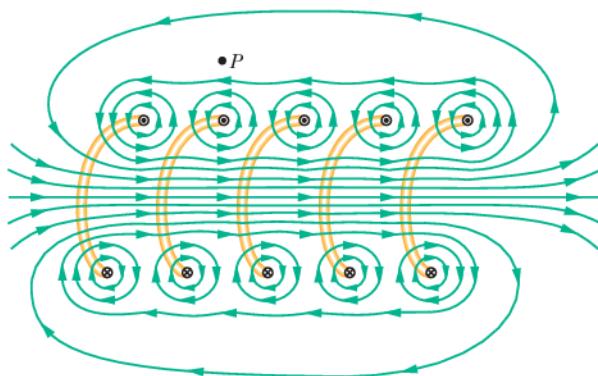


Figure 29-18 A vertical cross section through the central axis of a “stretched-out” solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid’s axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

close to a turn, the wire behaves magnetically almost like a long straight wire, and the lines of \vec{B} there are almost concentric circles. Figure 29-18 suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire, \vec{B} is approximately parallel to the (central) solenoid axis. In the limiting case of an *ideal solenoid*, which is infinitely long and consists of tightly packed (*close-packed*) turns of square wire, the field inside the coil is uniform and parallel to the solenoid axis.

At points above the solenoid, such as P in Fig. 29-18, the magnetic field set up by the upper parts of the solenoid turns (these upper turns are marked \odot) is directed to the left (as drawn near P) and tends to cancel the field set up at P by the lower parts of the turns (these lower turns are marked \otimes), which is directed to the right (not drawn). In the limiting case of an ideal solenoid, the magnetic field outside the solenoid is zero. Taking the external field to be zero is an excellent assumption for a real solenoid if its length is much greater than its diameter and if we consider external points such as point P that are not at either end of the solenoid. The direction of the magnetic field along the solenoid axis is given by a curled-straight right-hand rule: Grasp the solenoid with your right hand so that your fingers follow the direction of the current in the windings; your extended right thumb then points in the direction of the axial magnetic field.

Figure 29-19 shows the lines of \vec{B} for a real solenoid. The spacing of these lines in the central region shows that the field inside the coil is fairly strong and uniform over the cross section of the coil. The external field, however, is relatively weak.

Ampere’s Law. Let us now apply Ampere’s law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}, \quad (29-21)$$

to the ideal solenoid of Fig. 29-20, where \vec{B} is uniform within the solenoid and zero outside it, using the rectangular Amperian loop $abcd$. We write $\oint \vec{B} \cdot d\vec{s}$ as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}. \quad (29-22)$$

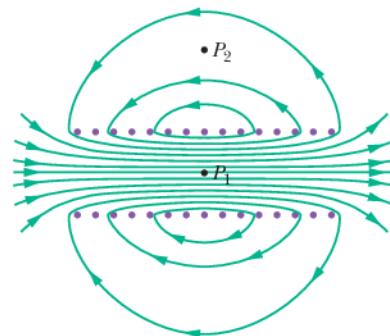


Figure 29-19 Magnetic field lines for a real solenoid of finite length. The field is strong and uniform at interior points such as P_1 but relatively weak at external points such as P_2 .

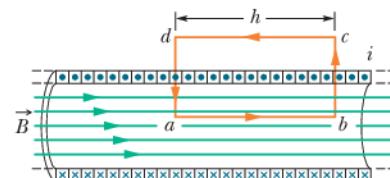


Figure 29-20 Application of Ampere’s law to a section of a long ideal solenoid carrying a current i . The Amperian loop is the rectangle $abcd$.

The first integral on the right of Eq. 29-22 is Bh , where B is the magnitude of the uniform field \vec{B} inside the solenoid and h is the (arbitrary) length of the segment from a to b . The second and fourth integrals are zero because for every element ds of these segments, \vec{B} either is perpendicular to ds or is zero, and thus the product $\vec{B} \cdot d\vec{s}$ is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because $B = 0$ at all external points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value Bh .

Net Current. The net current i_{enc} encircled by the rectangular Amperian loop in Fig. 29-20 is not the same as the current i in the solenoid windings because the windings pass more than once through this loop. Let n be the number of turns per unit length of the solenoid; then the loop encloses nh turns and

$$i_{\text{enc}} = i(nh).$$

Ampere's law then gives us

$$Bh = \mu_0 i_{\text{enc}}$$

or

$$B = \mu_0 i n \quad (\text{ideal solenoid}). \quad (29-23)$$

Although we derived Eq. 29-23 for an infinitely long ideal solenoid, it holds quite well for actual solenoids if we apply it only at interior points and well away from the solenoid ends. Equation 29-23 is consistent with the experimental fact that the magnetic field magnitude B within a solenoid does not depend on the diameter or the length of the solenoid and that B is uniform over the solenoidal cross section. A solenoid thus provides a practical way to set up a known uniform magnetic field for experimentation, just as a parallel-plate capacitor provides a practical way to set up a known uniform electric field.

Magnetic Field of a Toroid

Figure 29-21a shows a **toroid**, which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet. What magnetic field \vec{B} is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet.

From the symmetry, we see that the lines of \vec{B} form concentric circles inside the toroid, directed as shown in Fig. 29-21b. Let us choose a concentric circle of radius r as an Amperian loop and traverse it in the clockwise direction. Ampere's law (Eq. 29-14) yields

$$(B)(2\pi r) = \mu_0 i N,$$

where i is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and N is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi r} \quad (\text{toroid}). \quad (29-24)$$

In contrast to the situation for a solenoid, B is not constant over the cross section of a toroid.

It is easy to show, with Ampere's law, that $B = 0$ for points outside an ideal toroid (as if the toroid were made from an ideal solenoid). The direction of the magnetic field within a toroid follows from our curled-straight right-hand rule: Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.

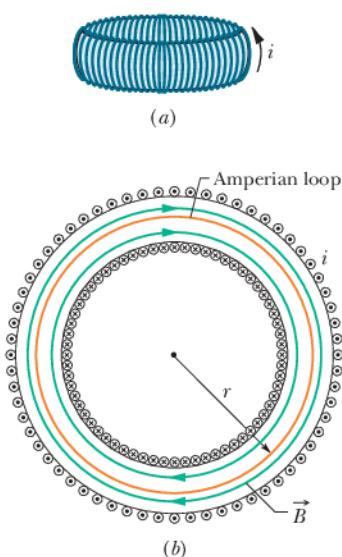


Figure 29-21 (a) A toroid carrying a current i . (b) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.



Sample Problem 29.04 The field inside a solenoid (a long coil of current)

A solenoid has length $L = 1.23\text{ m}$ and inner diameter $d = 3.55\text{ cm}$, and it carries a current $i = 5.57\text{ A}$. It consists of five close-packed layers, each with 850 turns along length L . What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).



Additional examples, video, and practice available at WileyPLUS

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in = (4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A})(5.57 \text{ A}) \frac{5 \times 850 \text{ turns}}{1.23 \text{ m}} \\ = 2.42 \times 10^{-2} \text{ T} = 24.2 \text{ mT.} \quad (\text{Answer})$$

To a good approximation, this is the field magnitude throughout most of the solenoid.



29-5 A CURRENT-CARRYING COIL AS A MAGNETIC DIPOLE

Learning Objectives

After reading this module, you should be able to . . .

29.22 Sketch the magnetic field lines of a flat coil that is carrying current.

29.23 For a current-carrying coil, apply the relationship between the dipole moment magnitude μ and the coil's

current i , number of turns N , and area per turn A .

29.24 For a point along the central axis, apply the relationship between the magnetic field magnitude B , the magnetic moment μ , and the distance z from the center of the coil.

Key Idea

- The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point P located a distance z along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3},$$

where $\vec{\mu}$ is the dipole moment of the coil. This equation applies only when z is much greater than the dimensions of the coil.

A Current-Carrying Coil as a Magnetic Dipole

So far we have examined the magnetic fields produced by current in a long straight wire, a solenoid, and a toroid. We turn our attention here to the field produced by a coil carrying a current. You saw in Module 28-8 that such a coil behaves as a magnetic dipole in that, if we place it in an external magnetic field \vec{B} , a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (29-25)$$

acts on it. Here $\vec{\mu}$ is the magnetic dipole moment of the coil and has the magnitude NiA , where N is the number of turns, i is the current in each turn, and A is the area enclosed by each turn. (Caution: Don't confuse the magnetic dipole moment $\vec{\mu}$ with the permeability constant μ_0 .)

Recall that the direction of $\vec{\mu}$ is given by a curled-straight right-hand rule: Grasp the coil so that the fingers of your right hand curl around it in the direction of the current; your extended thumb then points in the direction of the dipole moment $\vec{\mu}$.

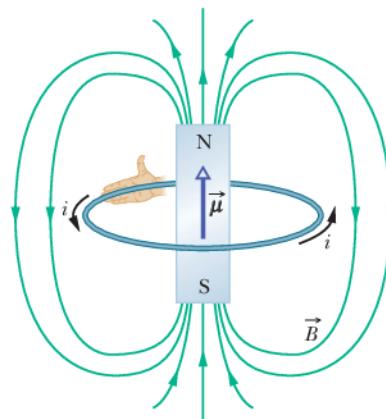


Figure 29-22 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled-straight right-hand rule, points from the south pole to the north pole, in the direction of the field \vec{B} within the loop.

Magnetic Field of a Coil

We turn now to the other aspect of a current-carrying coil as a magnetic dipole. What magnetic field does it produce at a point in the surrounding space? The problem does not have enough symmetry to make Ampere's law useful; so we must turn to the law of Biot and Savart. For simplicity, we first consider only a coil with a single circular loop and only points on its perpendicular central axis, which we take to be a z axis. We shall show that the magnitude of the magnetic field at such points is

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}, \quad (29-26)$$

in which R is the radius of the circular loop and z is the distance of the point in question from the center of the loop. Furthermore, the direction of the magnetic field \vec{B} is the same as the direction of the magnetic dipole moment $\vec{\mu}$ of the loop.

Large z . For axial points far from the loop, we have $z \gg R$ in Eq. 29-26. With that approximation, the equation reduces to

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3}.$$

Recalling that πR^2 is the area A of the loop and extending our result to include a coil of N turns, we can write this equation as

$$B(z) = \frac{\mu_0}{2\pi} \frac{NiA}{z^3}.$$

Further, because \vec{B} and $\vec{\mu}$ have the same direction, we can write the equation in vector form, substituting from the identity $\mu = NiA$:

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3} \quad (\text{current-carrying coil}). \quad (29-27)$$

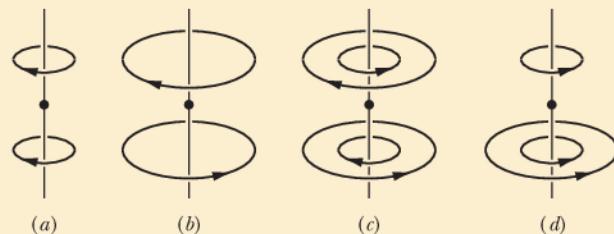
Thus, we have two ways in which we can regard a current-carrying coil as a magnetic dipole: (1) it experiences a torque when we place it in an external magnetic field; (2) it generates its own intrinsic magnetic field, given, for distant points along its axis, by Eq. 29-27. Figure 29-22 shows the magnetic field of a current loop; one side of the loop acts as a north pole (in the direction of $\vec{\mu}$)

and the other side as a south pole, as suggested by the lightly drawn magnet in the figure. If we were to place a current-carrying coil in an external magnetic field, it would tend to rotate just like a bar magnet would.



Checkpoint 3

The figure here shows four arrangements of circular loops of radius r or $2r$, centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.



Proof of Equation 29-26

Figure 29-23 shows the back half of a circular loop of radius R carrying a current i . Consider a point P on the central axis of the loop, a distance z from its plane. Let us apply the law of Biot and Savart to a differential element ds of the loop, located at the left side of the loop. The length vector $d\vec{s}$ for this element points perpendicularly out of the page. The angle θ between $d\vec{s}$ and \hat{r} in Fig. 29-23 is 90° ; the plane formed by these two vectors is perpendicular to the plane of the page and contains both \hat{r} and $d\vec{s}$. From the law of Biot and Savart (and the right-hand rule), the differential field $d\vec{B}$ produced at point P by the current in this element is perpendicular to this plane and thus is directed in the plane of the figure, perpendicular to \hat{r} , as indicated in Fig. 29-23.

Let us resolve $d\vec{B}$ into two components: dB_{\parallel} along the axis of the loop and dB_{\perp} perpendicular to this axis. From the symmetry, the vector sum of all the perpendicular components dB_{\perp} due to all the loop elements ds is zero. This leaves only the axial (parallel) components dB_{\parallel} and we have

$$B = \int dB_{\parallel}.$$

For the element $d\vec{s}$ in Fig. 29-23, the law of Biot and Savart (Eq. 29-1) tells us that the magnetic field at distance r is

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin 90^\circ}{r^2}.$$

We also have

$$dB_{\parallel} = dB \cos \alpha.$$

Combining these two relations, we obtain

$$dB_{\parallel} = \frac{\mu_0 i \cos \alpha ds}{4\pi r^2}. \quad (29-28)$$

Figure 29-23 shows that r and α are related to each other. Let us express each in terms of the variable z , the distance between point P and the center of the loop. The relations are

$$r = \sqrt{R^2 + z^2} \quad (29-29)$$

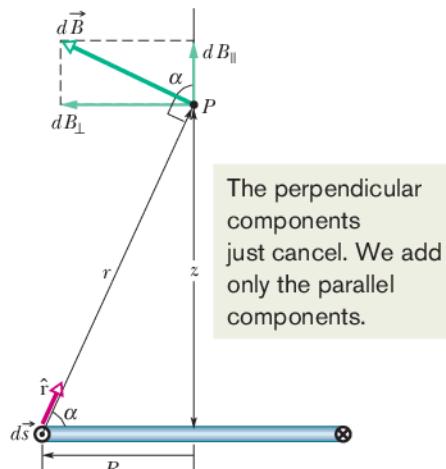


Figure 29-23 Cross section through a current loop of radius R . The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point P on the central perpendicular axis of the loop.

and

$$\cos \alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}. \quad (29-30)$$

Substituting Eqs. 29-29 and 29-30 into Eq. 29-28, we find

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}} ds.$$

Note that i , R , and z have the same values for all elements ds around the loop; so when we integrate this equation, we find that

$$\begin{aligned} B &= \int dB_{\parallel} \\ &= \frac{\mu_0 i R}{4\pi(R^2 + z^2)^{3/2}} \int ds \end{aligned}$$

or, because $\int ds$ is simply the circumference $2\pi R$ of the loop,

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

This is Eq. 29-26, the relation we sought to prove.

Review & Summary

The Biot-Savart Law The magnetic field set up by a current-carrying conductor can be found from the *Biot-Savart law*. This law asserts that the contribution $d\vec{B}$ to the field produced by a current-length element $i d\vec{s}$ at a point P located a distance r from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \quad (\text{Biot-Savart law}). \quad (29-3)$$

Here \hat{r} is a unit vector that points from the element toward P . The quantity μ_0 , called the permeability constant, has the value

$$4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \approx 1.26 \times 10^{-6} \text{ T}\cdot\text{m/A}.$$

Magnetic Field of a Long Straight Wire For a long straight wire carrying a current i , the Biot-Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire,

$$B = \frac{\mu_0 i}{2\pi R} \quad (\text{long straight wire}). \quad (29-4)$$

Magnetic Field of a Circular Arc The magnitude of the magnetic field at the center of a circular arc, of radius R and central angle ϕ (in radians), carrying current i , is

$$B = \frac{\mu_0 i \phi}{4\pi R} \quad (\text{at center of circular arc}). \quad (29-9)$$

Force Between Parallel Currents Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}, \quad (29-13)$$

where d is the wire separation, and i_a and i_b are the currents in the wires.

Ampere's Law Ampere's law states that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere's law}). \quad (29-14)$$

The line integral in this equation is evaluated around a closed loop called an *Amperian loop*. The current i on the right side is the *net* current encircled by the loop. For some current distributions, Eq. 29-14 is easier to use than Eq. 29-3 to calculate the magnetic field due to the currents.

Fields of a Solenoid and a Toroid Inside a *long solenoid* carrying current i , at points not near its ends, the magnitude B of the magnetic field is

$$B = \mu_0 i n \quad (\text{ideal solenoid}), \quad (29-23)$$

where n is the number of turns per unit length. Thus the internal magnetic field is uniform. Outside the solenoid, the magnetic field is approximately zero.

At a point inside a *toroid*, the magnitude B of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \quad (\text{toroid}), \quad (29-24)$$

where r is the distance from the center of the toroid to the point.

Field of a Magnetic Dipole The magnetic field produced by a current-carrying coil, which is a *magnetic dipole*, at a point P located a distance z along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}, \quad (29-27)$$

where $\vec{\mu}$ is the dipole moment of the coil. This equation applies only when z is much greater than the dimensions of the coil.

Questions

- 1** Figure 29-24 shows three circuits, each consisting of two radial lengths and two concentric circular arcs, one of radius r and the other of radius $R > r$. The circuits have the same current through them and the same angle between the two radial lengths. Rank the circuits according to the magnitude of the net magnetic field at the center, greatest first.

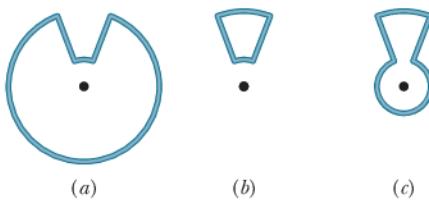


Figure 29-24 Question 1.

- 2** Figure 29-25 represents a snapshot of the velocity vectors of four electrons near a wire carrying current i . The four velocities have the same magnitude; velocity \vec{v}_2 is directed into the page. Electrons 1 and 2 are at the same distance from the wire, as are electrons 3 and 4. Rank the electrons according to the magnitudes of the magnetic forces on them due to current i , greatest first.

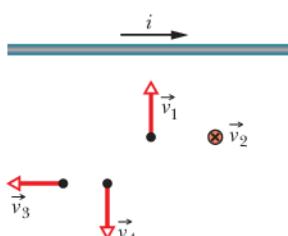


Figure 29-25 Question 2.

- 3** Figure 29-26 shows four arrangements in which long parallel wires carry equal currents directly into or out of the page at the corners of identical squares. Rank the arrangements according to the magnitude of the net magnetic field at the center of the square, greatest first.

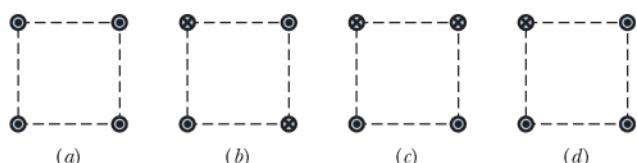


Figure 29-26 Question 3.

- 4** Figure 29-27 shows cross sections of two long straight wires; the left-hand wire carries current i_1 directly out of the page. If the net magnetic field due to the two currents is to be zero at point P , (a) should the direction of current i_2 in the right-hand wire be directly into or out of the page and (b) should i_2 be greater than, less than, or equal to i_1 ?

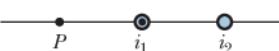


Figure 29-27 Question 4.

- 5** Figure 29-28 shows three circuits consisting of straight radial lengths and concentric circular arcs (either half- or quarter-circles

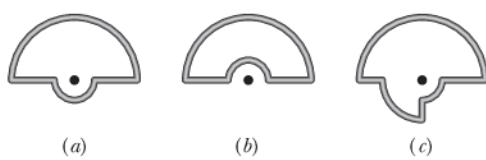


Figure 29-28 Question 5.

of radii r , $2r$, and $3r$). The circuits carry the same current. Rank them according to the magnitude of the magnetic field produced at the center of curvature (the dot), greatest first.

- 6** Figure 29-29 gives, as a function of radial distance r , the magnitude B of the magnetic field inside and outside four wires (a , b , c , and d), each of which carries a current that is uniformly distributed across the wire's cross section. Overlapping portions of the plots (drawn slightly separated) are indicated by double labels. Rank the wires according to (a) radius, (b) the magnitude of the magnetic field on the surface, and (c) the value of the current, greatest first. (d) Is the magnitude of the current density in wire a greater than, less than, or equal to that in wire c ?

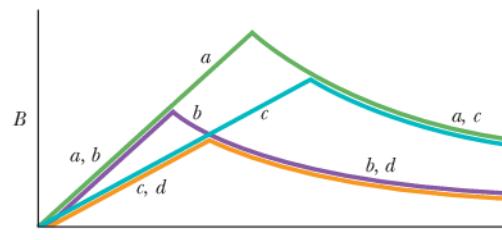


Figure 29-29 Question 6.

- 7** Figure 29-30 shows four circular Amperian loops (a , b , c , d) concentric with a wire whose current is directed out of the page. The current is uniform across the wire's circular cross section (the shaded region). Rank the loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ around each, greatest first.

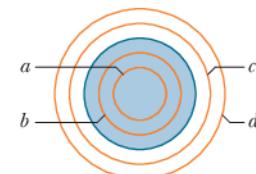


Figure 29-30 Question 7.

- 8** Figure 29-31 shows four arrangements in which long, parallel, equally spaced wires carry equal currents directly into or out of the page. Rank the arrangements according to the magnitude of the net force on the central wire due to the currents in the other wires, greatest first.

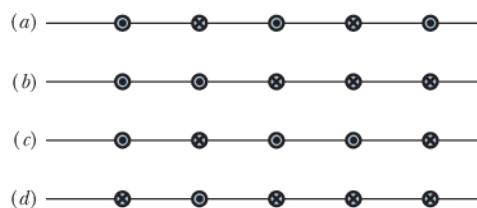


Figure 29-31 Question 8.

- 9** Figure 29-32 shows four circular Amperian loops (a , b , c , d) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of

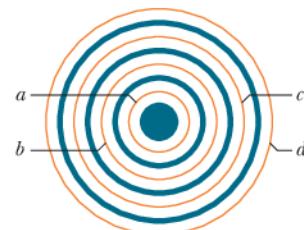


Figure 29-32 Question 9.

the page, 9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ around each, greatest first.

10 Figure 29-33 shows four identical currents i and five Amperian paths (*a* through *e*) encircling them. Rank the paths according to the value of $\oint \vec{B} \cdot d\vec{s}$ taken in the directions shown, most positive first.

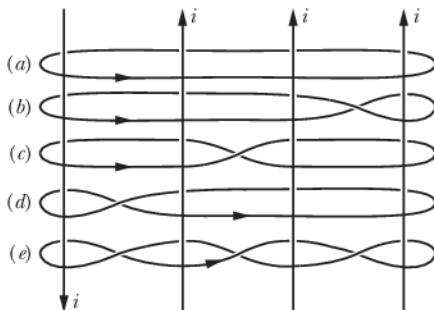


Figure 29-33 Question 10.

- 11** Figure 29-34 shows three arrangements of three long straight wires carrying equal currents directly into or out of the page. (a) Rank the arrangements according to the magnitude of the net force on wire *A* due to the currents in the other wires, greatest first. (b) In arrangement 3, is the angle between the net force on wire *A* and the dashed line equal to, less than, or more than 45° ?

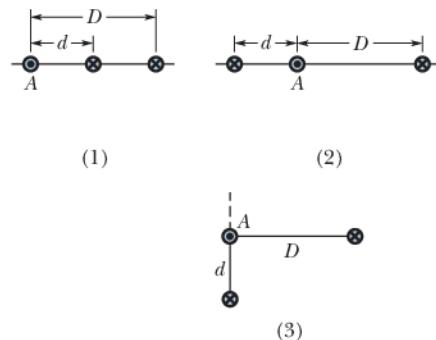


Figure 29-34 Question 11.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

WWW Worked-out solution is at

ILW Interactive solution is at

<http://www.wiley.com/college/halliday>

Module 29-1 Magnetic Field Due to a Current

- 1** A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is 20 μT .

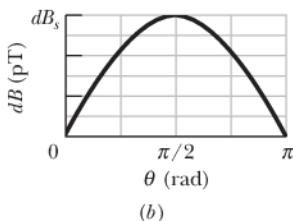
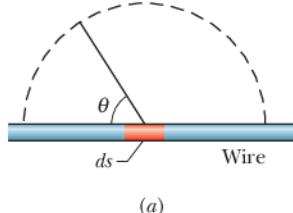


Figure 29-35 Problem 2.

- 2** Figure 29-35a shows an element of length $ds = 1.00 \mu\text{m}$ in a very long straight wire carrying current. The current in that element sets up a differential magnetic field $d\vec{B}$ at points in the surrounding space. Figure 29-35b gives the magnitude dB of the field for points 2.5 cm from the element, as a function of angle θ between the wire and a straight line to the point. The vertical scale is set by $dB_s = 60.0 \text{ pT}$. What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm from the wire?

- 3 SSM** At a certain location in the Philippines, Earth's magnetic field of 39 μT is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?

- 4** A straight conductor carrying current $i = 5.0 \text{ A}$ splits into identical semicircular arcs as shown in Fig. 29-36. What is the magnetic field at the center *C* of the resulting circular loop?

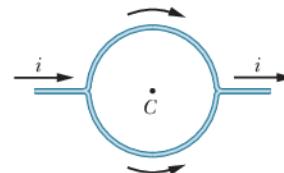


Figure 29-36 Problem 4.

- 5** In Fig. 29-37, a current $i = 10 \text{ A}$ is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R = 5.0 \text{ mm}$. Point *b* is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of \vec{B} at *a* and the (c) magnitude and (d) direction of \vec{B} at *b*?



Figure 29-37 Problem 5.

- 6** In Fig. 29-38, point *P* is at perpendicular distance $R = 2.00 \text{ cm}$ from a very long straight wire carrying a current. The magnetic field \vec{B} set up at point *P* is due to contributions from all the identical current-length elements $i ds$ along the wire. What is the distance s to the

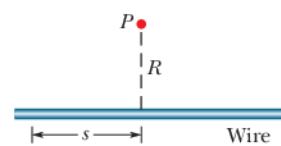


Figure 29-38 Problem 6.

element making (a) the greatest contribution to field \vec{B} and (b) 10.0% of the greatest contribution?

- 7 GO In Fig. 29-39, two circular arcs have radii $a = 13.5$ cm and $b = 10.7$ cm, subtend angle $\theta = 74.0^\circ$, carry current $i = 0.411$ A, and share the same center of curvature P . What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at P ?

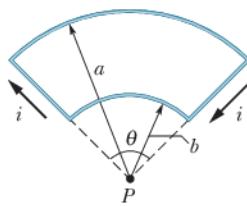


Figure 29-39 Problem 7.

- 8 In Fig. 29-40, two semicircular arcs have radii $R_2 = 7.80$ cm and $R_1 = 3.15$ cm, carry current $i = 0.281$ A, and have the same center of curvature C . What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C ?

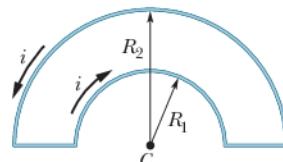


Figure 29-40 Problem 8.

- 9 SSM Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude $300 \mu\text{T}$. (a) Should the currents be in the same or opposite directions? (b) How much current is needed?

- 10 In Fig. 29-41, a wire forms a semicircle of radius $R = 9.26$ cm and two (radial) straight segments each of length $L = 13.1$ cm. The wire carries current $i = 34.8$ mA. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature C ?

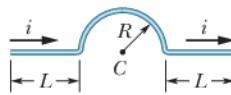


Figure 29-41 Problem 10.

- 11 In Fig. 29-42, two long straight wires are perpendicular to the page and separated by distance $d_1 = 0.75$ cm. Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point P located at distance $d_2 = 1.50$ cm from wire 2?

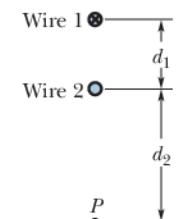


Figure 29-42 Problem 11.

- 12 In Fig. 29-43, two long straight wires at separation $d = 16.0$ cm carry currents $i_1 = 3.61$ mA and $i_2 = 3.00i_1$ out of the page. (a) Where on the x axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1, shifted toward wire 2, or unchanged?

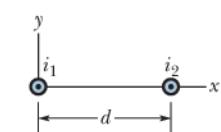


Figure 29-43 Problem 12.

- 13 In Fig. 29-44, point P_1 is at distance $R = 13.1$ cm on the perpendicular bisector of a straight wire of length $L = 18.0$ cm carrying

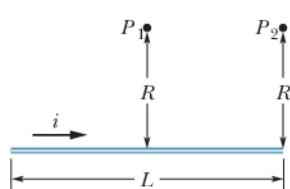


Figure 29-44 Problems 13 and 17.

current $i = 58.2$ mA. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at P_1 due to i ?

- 14 Equation 29-4 gives the magnitude B of the magnetic field set up by a current in an *infinitely long* straight wire, at a point P at perpendicular distance R from the wire. Suppose that point P is actually at perpendicular distance R from the midpoint of a wire with a *finite* length L . Using Eq. 29-4 to calculate B then results in a certain percentage error. What value must the ratio L/R exceed if the percentage error is to be less than 1.00%? That is, what L/R gives

$$\frac{(B \text{ from Eq. 29-4}) - (B \text{ actual})}{(B \text{ actual})} (100\%) = 1.00\%$$

- 15 Figure 29-45 shows two current segments. The lower segment carries a current of $i_1 = 0.40$ A and includes a semicircular arc with radius 5.0 cm, angle 180° , and center point P . The upper segment carries current $i_2 = 2i_1$ and includes a circular arc with radius 4.0 cm, angle 120° , and the same center point P . What are the (a) magnitude and (b) direction of the net magnetic field \vec{B} at P for the indicated current directions? What are the (c) magnitude and (d) direction of \vec{B} if i_1 is reversed?

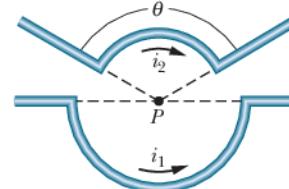


Figure 29-45 Problem 15.

- 16 GO In Fig. 29-46, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has radius 1.50 cm and carries 4.00 mA. Loop 2 has radius 2.50 cm and carries 6.00 mA. Loop 2 is to be rotated about a diameter while the net magnetic field \vec{B} set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT?

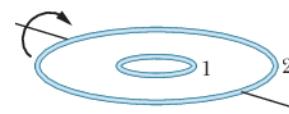


Figure 29-46 Problem 16.

- 17 SSM In Fig. 29-44, point P_2 is at perpendicular distance $R = 25.1$ cm from one end of a straight wire of length $L = 13.6$ cm carrying current $i = 0.693$ A. (Note that the wire is *not* long.) What is the magnitude of the magnetic field at P_2 ?

- 18 A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-47a shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is $47.25 \mu\text{T}$. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-47b). The magnetic field produced at the (same) center of curvature now has magnitude $15.75 \mu\text{T}$, and its direction is reversed from the initial magnetic field. What is the radius of the smaller semicircle?

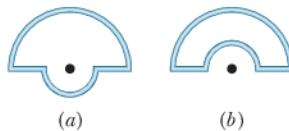


Figure 29-47 Problem 18.

- 19 One long wire lies along an x axis and carries a current of 30 A in the positive x direction. A second long wire is perpendicular to the xy plane, passes through the point $(0, 4.0 \text{ m}, 0)$, and carries a current of 40 A in the positive z direction. What is the magnitude of the resulting magnetic field at the point $(0, 2.0 \text{ m}, 0)$?

••20 In Fig. 29-48, part of a long insulated wire carrying current $i = 5.78 \text{ mA}$ is bent into a circular section of radius $R = 1.89 \text{ cm}$. In unit-vector notation, what is the magnetic field at the center of curvature C if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated 90° counterclockwise as indicated?

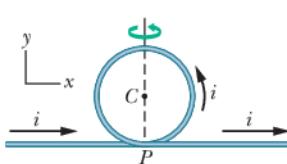


Figure 29-48 Problem 20.

••21 GO Figure 29-49 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance $d_1 = 6.00 \text{ m}$ and distance $d_2 = 4.00 \text{ m}$. What is the magnitude of the net magnetic field at point P , which lies on a perpendicular bisector to the wires?

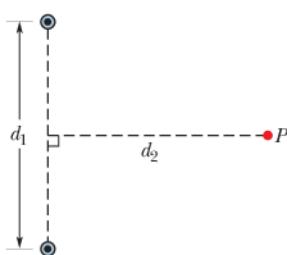


Figure 29-49 Problem 21.

••22 GO Figure 29-50a shows, in cross section, two long, parallel wires carrying current and separated by distance L . The ratio i_1/i_2 of their currents is 4.00; the directions of the currents are not indicated. Figure 29-50b shows the y component B_y of their net magnetic field along the x axis to the right of wire 2. The vertical scale is set by $B_{ys} = 4.0 \text{ nT}$, and the horizontal scale is set by $x_s = 20.0 \text{ cm}$. (a) At what value of $x > 0$ is B_y maximum? (b) If $i_2 = 3 \text{ mA}$, what is the value of that maximum? What is the direction (into or out of the page) of (c) i_1 and (d) i_2 ?

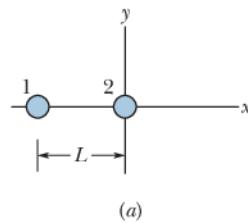
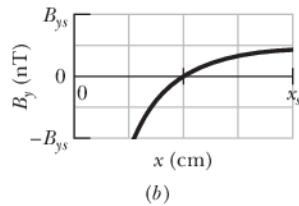


Figure 29-50 Problem 22.



••23 ILW Figure 29-51 shows a snapshot of a proton moving at velocity $\vec{v} = (-200 \text{ m/s})\hat{j}$ toward a long straight wire with current $i = 350 \text{ mA}$. At the instant shown, the proton's distance from the wire is $d = 2.89 \text{ cm}$. In unit-vector notation, what is the magnetic force on the proton due to the current?

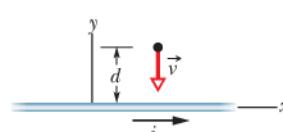
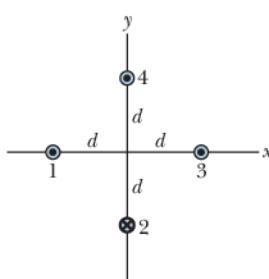


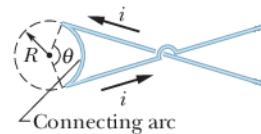
Figure 29-51 Problem 23.

••24 GO Figure 29-52 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance $d = 15.0 \text{ cm}$ from the origin of the coordinate system, where they create a net magnetic field \vec{B} . (a) To what value of x must you move wire 1 along the x axis in order to rotate \vec{B} counterclockwise by 30° ? (b) With wire 1 in that new position, to what value of x

Figure 29-52
Problem 24.

must you move wire 3 along the x axis to rotate \vec{B} by 30° back to its initial orientation?

••25 SSM A wire with current $i = 3.00 \text{ A}$ is shown in Fig. 29-53. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle θ and runs along the circumference of the circle. The arc and the two straight sections all lie in the same plane. If $B = 0$ at the circle's center, what is θ ?

Figure 29-53
Problem 25.

••26 GO In Fig. 29-54a, wire 1 consists of a circular arc and two radial lengths; it carries current $i_1 = 0.50 \text{ A}$ in the direction indicated. Wire 2, shown in cross section, is long, straight, and perpendicular to the plane of the figure. Its distance from the center of the arc is equal to the radius R of the arc, and it carries a current i_2 that can be varied. The two currents set up a net magnetic field \vec{B} at the center of the arc. Figure 29-54b gives the square of the field's magnitude B^2 plotted versus the square of the current i_2^2 . The vertical scale is set by $B_s^2 = 10.0 \times 10^{-10} \text{ T}^2$. What angle is subtended by the arc?

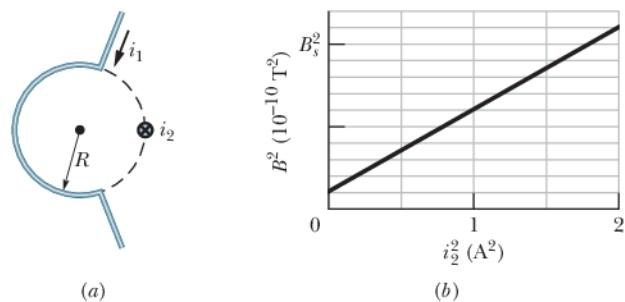


Figure 29-54 Problem 26.

••27 In Fig. 29-55, two long straight wires (shown in cross section) carry the currents $i_1 = 30.0 \text{ mA}$ and $i_2 = 40.0 \text{ mA}$ directly out of the page. They are equal distances from the origin, where they set up a magnetic field \vec{B} . To what value must current i_1 be changed in order to rotate \vec{B} 20.0° clockwise?

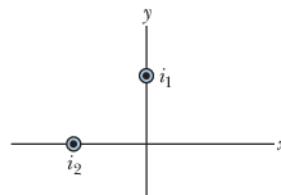


Figure 29-55 Problem 27.

••28 GO Figure 29-56a shows two wires, each carrying a current. Wire 1 consists of a circular arc of

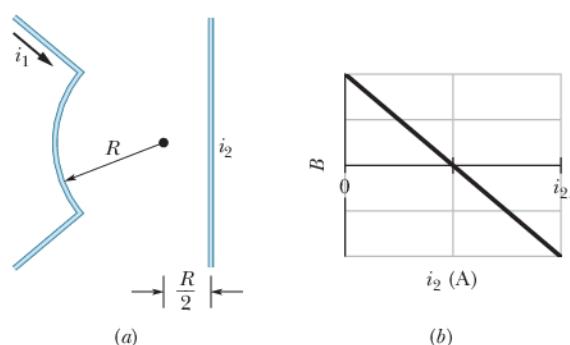


Figure 29-56 Problem 28.

radius R and two radial lengths; it carries current $i_1 = 2.0 \text{ A}$ in the direction indicated. Wire 2 is long and straight; it carries a current i_2 that can be varied; and it is at distance $R/2$ from the center of the arc. The net magnetic field \vec{B} due to the two currents is measured at the center of curvature of the arc. Figure 29-56b is a plot of the component of \vec{B} in the direction perpendicular to the figure as a function of current i_2 . The horizontal scale is set by $i_{2s} = 1.00 \text{ A}$. What is the angle subtended by the arc?

••29 SSM In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 20 \text{ cm}$. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A. In unit-vector notation, what is the net magnetic field at the square's center?

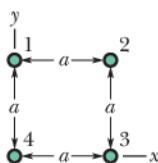


Figure 29-57 Problems 29, 37, and 40.

••30 GO Two long straight thin wires with current lie against an equally long plastic cylinder, at radius $R = 20.0 \text{ cm}$ from the cylinder's central axis. Figure 29-58a shows, in cross section, the cylinder and wire 1 but not wire 2. With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle $\theta_1 = 0^\circ$ to angle $\theta_1 = 180^\circ$, through the first and second quadrants of the xy coordinate system. The net magnetic field \vec{B} at the center of the cylinder is measured as a function of θ_1 . Figure 29-58b gives the x component B_x of that field as a function of θ_1 (the vertical scale is set by $B_{xs} = 6.0 \mu\text{T}$), and Fig. 29-58c gives the y component B_y (the vertical scale is set by $B_{ys} = 4.0 \mu\text{T}$). (a) At what angle θ_2 is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current in wire 1 and the (d) size and (e) direction of the current in wire 2?

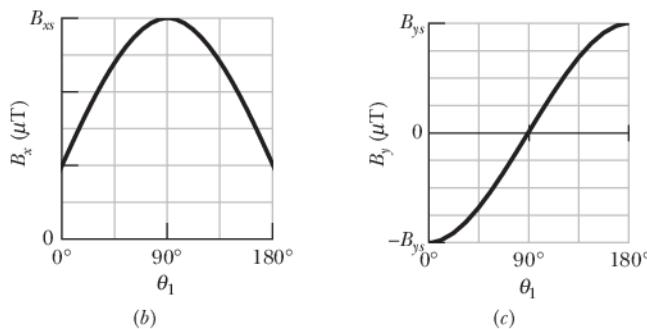
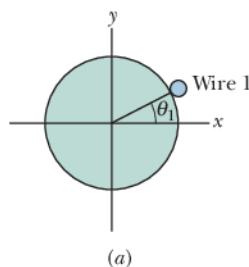


Figure 29-58 Problem 30.

••31 In Fig. 29-59, length a is 4.7 cm (short) and current i is 13 A. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at point P ?

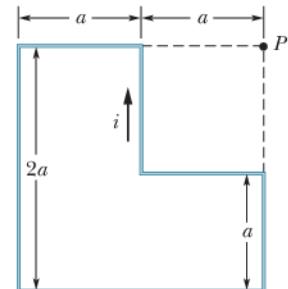


Figure 29-59 Problem 31.

••32 GO The current-carrying wire loop in Fig. 29-60a lies all in one plane and consists of a semicircle of radius 10.0 cm, a smaller semicircle with the same center, and two radial lengths. The smaller semicircle is rotated out of that plane by angle θ , until it is perpendicular to the plane (Fig. 29-60b). Figure 29-60c gives the magnitude of the net magnetic field at the center of curvature versus angle θ . The vertical scale is set by $B_a = 10.0 \mu\text{T}$ and $B_b = 12.0 \mu\text{T}$. What is the radius of the smaller semicircle?

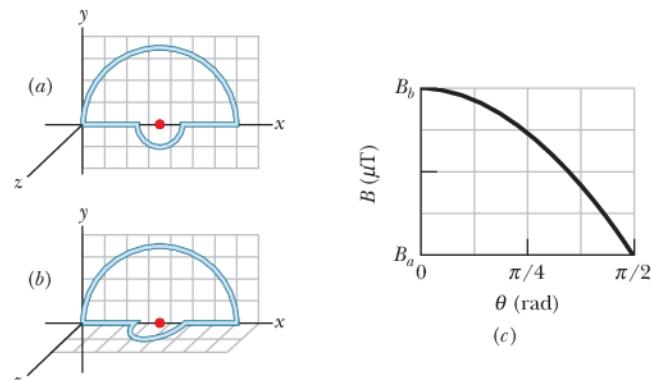


Figure 29-60 Problem 32.

••33 SSM ILW Figure 29-61 shows a cross section of a long thin ribbon of width $w = 4.91 \text{ cm}$ that is carrying a uniformly distributed total current $i = 4.61 \mu\text{A}$ into the page. In unit-vector notation, what is the magnetic field \vec{B} at a point P in the plane of the ribbon at a distance $d = 2.16 \text{ cm}$ from its edge? (Hint: Imagine the ribbon as being constructed from many long, thin, parallel wires.)

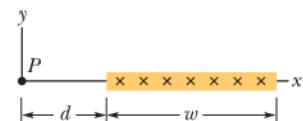


Figure 29-61 Problem 33.

••34 GO Figure 29-62 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm. Wire 1 carries current $i_1 = 60.0 \text{ mA}$ out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current $i_2 = 40.0 \text{ mA}$ out of the page and can be moved around the cylinder. At what (positive) angle θ_2 should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude 80.0 nT?

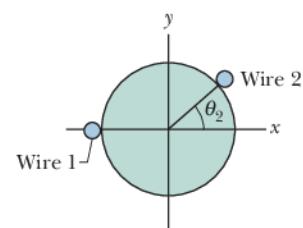


Figure 29-62 Problem 34.

Module 29-2 Force Between Two Parallel Currents

••35 SSM Figure 29-63 shows wire 1 in cross section; the wire is long

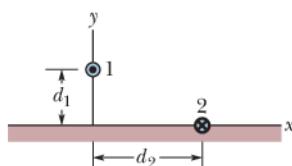


Figure 29-63 Problem 35.

and straight, carries a current of 4.00 mA out of the page, and is at distance $d_1 = 2.40$ cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance $d_2 = 5.00$ cm from wire 1 and carries a current of 6.80 mA into the page. What is the x component of the magnetic force *per unit length* on wire 2 due to wire 1?

- 36** In Fig. 29-64, five long parallel wires in an xy plane are separated by distance $d = 8.00$ cm, have lengths of 10.0 m, and carry identical currents of 3.00 A out of the page. Each wire experiences a magnetic force due to the currents in the other wires. In unit-vector notation, what is the net magnetic force on (a) wire 1, (b) wire 2, (c) wire 3, (d) wire 4, and (e) wire 5?

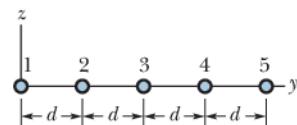


Figure 29-64 Problems 36 and 39.

- 37** **GO** In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 13.5$ cm. Each wire carries 7.50 A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 4?

- 38** **GO** Figure 29-65a shows, in cross section, three current-carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an x axis, with separation d . Wire 1 has a current of 0.750 A, but the direction of the current is not given. Wire 3, with a current of 0.250 A out of the page, can be moved along the x axis to the right of wire 2. As wire 3 is moved, the magnitude of the net magnetic force \vec{F}_2 on wire 2 due to the currents in wires 1 and 3 changes. The x component of that force is F_{2x} and the value per unit length of wire 2 is F_{2x}/L_2 . Figure 29-65b gives F_{2x}/L_2 versus the position x of wire 3. The plot has an asymptote $F_{2x}/L_2 = -0.627 \mu\text{N/m}$ as $x \rightarrow \infty$. The horizontal scale is set by $x_s = 12.0$ cm. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2?

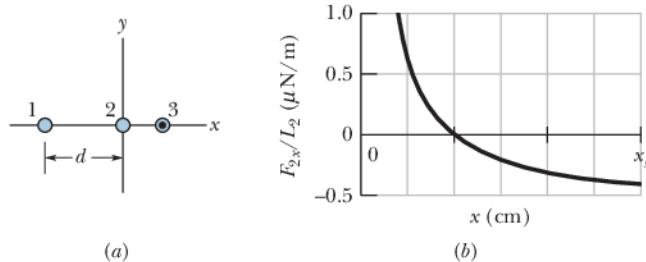


Figure 29-65 Problem 38.

- 39** **GO** In Fig. 29-64, five long parallel wires in an xy plane are separated by distance $d = 50.0$ cm. The currents into the page are $i_1 = 2.00$ A, $i_3 = 0.250$ A, $i_4 = 4.00$ A, and $i_5 = 2.00$ A; the current out of the page is $i_2 = 4.00$ A. What is the magnitude of the net force *per unit length* acting on wire 3 due to the currents in the other wires?

- 40** In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 8.50$ cm. Each wire carries 15.0 A, and all the currents are out of the page. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 1?

- 41** **ILW** In Fig. 29-66, a long straight wire carries a current $i_1 = 30.0$ A and a rectangular loop carries current $i_2 = 20.0$ A. Take the dimensions to be $a = 1.00$ cm, $b = 8.00$ cm, and $L = 30.0$ cm. In unit-vector notation, what is the net force on the loop due to i_1 ?

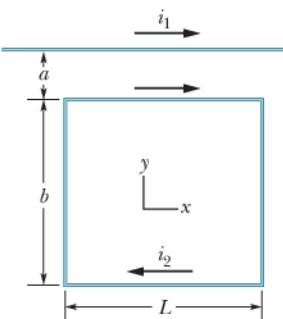


Figure 29-66 Problem 41.

Module 29-3 Ampere's Law

- 42** In a particular region there is a uniform current density of 15 A/m² in the positive z direction. What is the value of $\oint \vec{B} \cdot d\vec{s}$ when that line integral is calculated along a closed path consisting of the three straight-line segments from (x, y, z) coordinates $(4d, 0, 0)$ to $(4d, 3d, 0)$ to $(0, 0, 0)$ to $(4d, 0, 0)$, where $d = 20$ cm?

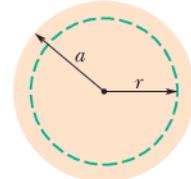


Figure 29-67 Problem 43.

- 43** Figure 29-67 shows a cross section across a diameter of a long cylindrical conductor of radius $a = 2.00$ cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm?

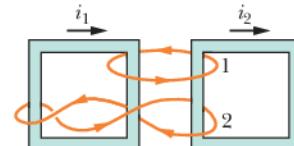


Figure 29-68 Problem 44.

- 44** Figure 29-68 shows two closed paths wrapped around two conducting loops carrying currents $i_1 = 5.0$ A and $i_2 = 3.0$ A. What is the value of the integral $\oint \vec{B} \cdot d\vec{s}$ for (a) path 1 and (b) path 2?

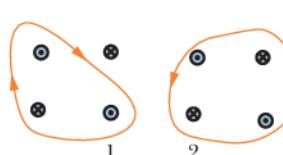


Figure 29-69 Problem 45.

- 46** Eight wires cut the page perpendicularly at the points shown in Fig. 29-70. A wire labeled with the integer k ($k = 1, 2, \dots, 8$) carries the current ki , where $i = 4.50$ mA. For those wires with odd k , the current is out of the page; for those with even k , it is into the page. Evaluate $\oint \vec{B} \cdot d\vec{s}$ along the closed path indicated and in the direction shown.

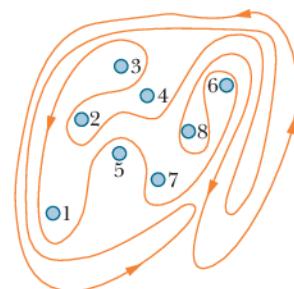


Figure 29-70 Problem 46.

- 47** **ILW** The current density \vec{J} inside a long, solid, cylindrical wire of radius $a = 3.1$ mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = J_0 r/a$, where $J_0 =$

310 A/m^2 . Find the magnitude of the magnetic field at (a) $r = 0$, (b) $r = a/2$, and (c) $r = a$.

- 48 In Fig. 29-71, a long circular pipe with outside radius $R = 2.6 \text{ cm}$ carries a (uniformly distributed) current $i = 8.00 \text{ mA}$ into the page. A wire runs parallel to the pipe at a distance of $3.00R$ from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

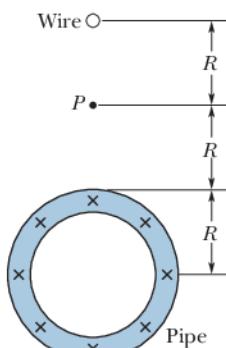


Figure 29-71
Problem 48.

Module 29-4 Solenoids and Toroids

- 49 A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A . (It is made up of a square solenoid—instead of a round one as in Fig. 29-17—bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?

- 50 A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1200 turns; it carries a current of 3.60 A . Calculate the magnitude of the magnetic field inside the solenoid.

- 51 A 200-turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A . Calculate the magnitude of the magnetic field \vec{B} inside the solenoid.

- 52 A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A . The magnetic field inside the solenoid is 23.0 mT . Find the length of the wire forming the solenoid.

- 53 A long solenoid has 100 turns/cm and carries current i . An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is $0.0460c$ (c = speed of light). Find the current i in the solenoid.

- 54 An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 m/s and its velocity vector makes an angle of 30° with the central axis of the solenoid. The solenoid carries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)

- 55 SSM ILW WWW A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA . A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at 45.0° to the axial direction? (b) What is the magnitude of the magnetic field there?

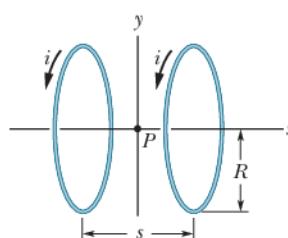


Figure 29-72
Problem 56.

Module 29-5 A Current-Carrying Coil as a Magnetic Dipole

- 56 Figure 29-72 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius $R = 25.0 \text{ cm}$, separated by a distance

$s = R$. The two coils carry equal currents $i = 12.2 \text{ mA}$ in the same direction. Find the magnitude of the net magnetic field at P , midway between the coils.

- 57 SSM A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter $d = 5.0 \text{ cm}$. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude $5.0 \mu\text{T}$ (approximately one-tenth that of Earth's magnetic field)?

- 58 Figure 29-73a shows a length of wire carrying a current i and bent into a circular coil of one turn. In Fig. 29-73b the same length of wire has been bent to give a coil of two turns, each of half the original radius. (a) If B_a and B_b are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio B_b/B_a ? (b) What is the ratio μ_b/μ_a of the dipole moment magnitudes of the coils?

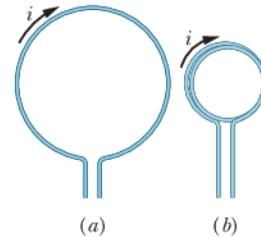


Figure 29-73 Problem 58.

- 59 SSM What is the magnitude of the magnetic dipole moment $\vec{\mu}$ of the solenoid described in Problem 51?

- 60 GO In Fig. 29-74a, two circular loops, with different currents but the same radius of 4.0 cm , are centered on a y axis. They are initially separated by distance $L = 3.0 \text{ cm}$, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with y component B_y . That component is to be measured as loop 2 is gradually moved in the positive direction of the y axis. Figure 29-74b gives B_y as a function of the position y of loop 2. The curve approaches an asymptote of $B_y = 7.20 \mu\text{T}$ as $y \rightarrow \infty$. The horizontal scale is set by $y_s = 10.0 \text{ cm}$. What are (a) current i_1 in loop 1 and (b) current i_2 in loop 2?

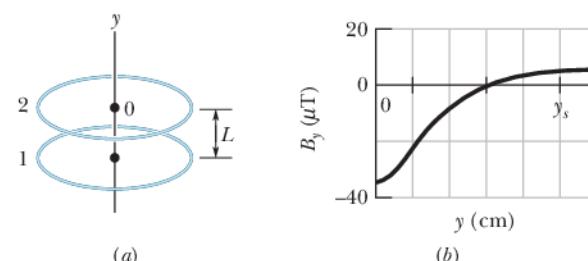


Figure 29-74 Problem 60.

- 61 A circular loop of radius 12 cm carries a current of 15 A . A flat coil of radius 0.82 cm , having 50 turns and a current of 1.3 A , is concentric with the loop. The plane of the loop is perpendicular to the plane of the coil. Assume the loop's magnetic field is uniform across the coil. What is the magnitude of (a) the magnetic field produced by the loop at its center and (b) the torque on the coil due to the loop?

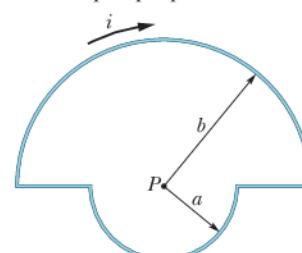


Figure 29-75 Problem 62.

- 62 In Fig. 29-75, current $i = 56.2 \text{ mA}$ is set up in a loop having two radial lengths and two semicir-

cles of radii $a = 5.72$ cm and $b = 9.36$ cm with a common center P . What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at P and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?

- 63** In Fig. 29-76, a conductor carries 6.0 A along the closed path $abcdefgha$ running along 8 of the 12 edges of a cube of edge length 10 cm . (a) Taking the path to be a combination of three square current loops ($bcfgb$, $abgha$, and $cdefc$), find the net magnetic moment of the path in unit-vector notation. (b) What is the magnitude of the net magnetic field at the xyz coordinates of $(0, 5.0\text{ m}, 0)$?

Additional Problems

- 64** In Fig. 29-77, a closed loop carries current $i = 200\text{ mA}$. The loop consists of two radial straight wires and two concentric circular arcs of radii 2.00 m and 4.00 m . The angle θ is $\pi/4\text{ rad}$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the center of curvature P ?

- 65** A cylindrical cable of radius 8.00 mm carries a current of 25.0 A , uniformly spread over its cross-sectional area. At what distance from the center of the wire is there a point within the wire where the magnetic field magnitude is 0.100 mT ?

- 66** Two long wires lie in an xy plane, and each carries a current in the positive direction of the x axis. Wire 1 is at $y = 10.0\text{ cm}$ and carries 6.00 A ; wire 2 is at $y = 5.00\text{ cm}$ and carries 10.0 A . (a) In unit-vector notation, what is the net magnetic field \vec{B} at the origin? (b) At what value of y does $\vec{B} = 0$? (c) If the current in wire 1 is reversed, at what value of y does $\vec{B} = 0$?

- 67** Two wires, both of length L , are formed into a circle and a square, and each carries current i . Show that the square produces a greater magnetic field at its center than the circle produces at its center.

- 68** A long straight wire carries a current of 50 A . An electron, traveling at $1.0 \times 10^7\text{ m/s}$, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?

- 69** Three long wires are parallel to a z axis, and each carries a current of 10 A in the positive z direction. Their points of intersection with the xy plane form an equilateral triangle with sides of 50 cm , as shown in Fig. 29-78. A fourth wire (wire b) passes through the midpoint of the base of the triangle and is parallel to the other three wires. If the net magnetic force on wire a is zero, what are the (a) size and (b) direction ($+z$ or $-z$) of the current in wire b ?

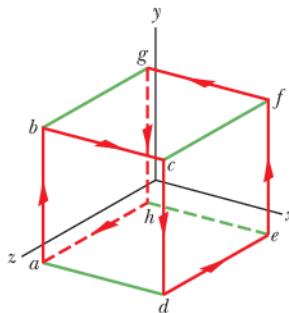


Figure 29-76 Problem 63.

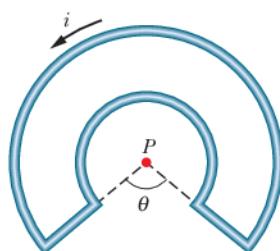


Figure 29-77 Problem 64.

- 70** Figure 29-79 shows a closed loop with current $i = 2.00\text{ A}$. The loop consists of a half-circle of radius 4.00 m , two quarter-circles each of radius 2.00 m , and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?

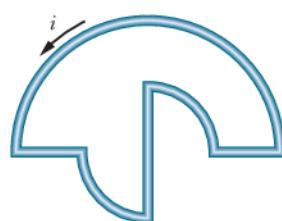


Figure 29-79 Problem 70.

- 71** A 10-gauge bare copper wire (2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

- 72** A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm . If the magnitude of the magnetic field at a point 5.0 mm from the wire is $1.0\text{ }\mu\text{T}$, what are the (a) size and (b) direction of the current in the wire?

- 73** Figure 29-80 shows a cross section of a long cylindrical conductor of radius $a = 4.00\text{ cm}$ containing a long cylindrical hole of radius $b = 1.50\text{ cm}$. The central axes of the cylinder and hole are parallel and are distance $d = 2.00\text{ cm}$ apart; current $i = 5.25\text{ A}$ is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases $b = 0$ and $d = 0$.

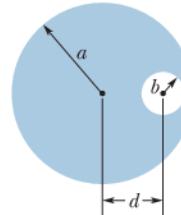


Figure 29-80 Problem 73.

- 74** The magnitude of the magnetic field at a point 88.0 cm from the central axis of a long straight wire is $7.30\text{ }\mu\text{T}$. What is the current in the wire?

- 75 SSM** Figure 29-81 shows a wire segment of length $\Delta s = 3.0\text{ cm}$, centered at the origin, carrying current $i = 2.0\text{ A}$ in the positive y direction (as part of some complete circuit). To calculate the magnitude of the magnetic field \vec{B} produced by the segment at a point several meters from the origin, we can use $B = (\mu_0/4\pi)i \Delta s (\sin \theta)/r^2$ as the Biot–Savart law. This is because r and θ are essentially constant over the segment. Calculate \vec{B} (in unit-vector notation) at the (x, y, z) coordinates (a) $(0, 0, 5.0\text{ m})$, (b) $(0, 6.0\text{ m}, 0)$, (c) $(7.0\text{ m}, 7.0\text{ m}, 0)$, and (d) $(-3.0\text{ m}, -4.0\text{ m}, 0)$.

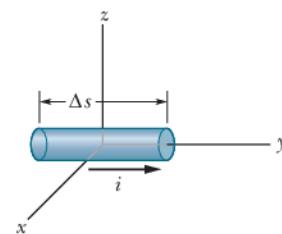


Figure 29-81 Problem 75.

- 76 GO** Figure 29-82 shows, in cross section, two long parallel wires spaced by distance $d = 10.0\text{ cm}$; each carries 100 A , out of the page in wire 1. Point P is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at P if the current in wire 2 is (a) out of the page and (b) into the page?

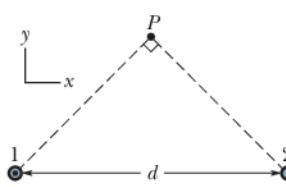


Figure 29-82 Problem 76.

77 In Fig. 29-83, two infinitely long wires carry equal currents i . Each follows a 90° arc on the circumference of the same circle of radius R . Show that the magnetic field \vec{B} at the center of the circle is the same as the field \vec{B} a distance R below an infinite straight wire carrying a current i to the left.

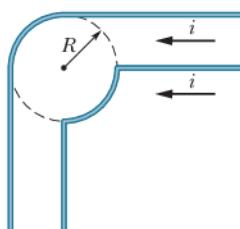


Figure 29-83
Problem 77.

78 A long wire carrying 100 A is perpendicular to the magnetic field lines of a uniform magnetic field of magnitude 5.0 mT. At what distance from the wire is the net magnetic field equal to zero?

79 A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm, (b) 3.0 mm, and (c) 5.0 mm from the central axis of the wire and cylinder?

80 A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?

81 SSM Figure 29-84 shows a cross section of an infinite conducting sheet carrying a current per unit x -length of λ ; the current emerges perpendicularly out of the page. (a) Use the Biot-Savart law and symmetry to show that for all points P above the sheet and all points P' below it, the magnetic field \vec{B} is parallel to the sheet and directed as shown. (b) Use Ampere's law to prove that $B = \frac{1}{2}\mu_0\lambda$ at all points P and P' .

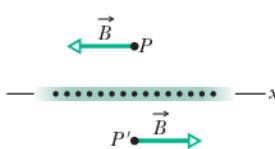


Figure 29-84 Problem 81.

82 Figure 29-85 shows, in cross section, two long parallel wires that are separated by distance $d = 18.6$ cm. Each carries 4.23 A, out of the page in wire 1 and into the page in wire 2. In unit-vector notation, what is the net magnetic field at point P at distance $R = 34.2$ cm, due to the two currents?

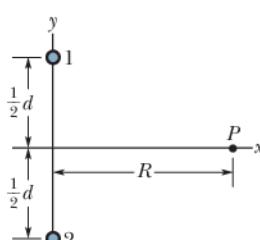


Figure 29-85 Problem 82.

83 SSM In unit-vector notation, what is the magnetic field at point P in Fig. 29-86 if $i = 10$ A and $a = 8.0$ cm? (Note that the wires are not long.)

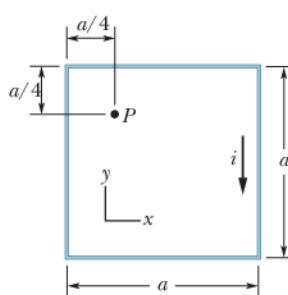


Figure 29-86 Problem 83.

84 Three long wires all lie in an xy plane parallel to the x axis. They are spaced equally, 10 cm apart. The two outer wires each carry a current of 5.0 A in the positive x direction. What is the magnitude of the force on a 3.0 m section of either of the outer wires if the current in the cen-

ter wire is 3.2 A (a) in the positive x direction and (b) in the negative x direction?

85 SSM Figure 29-87 shows a cross section of a hollow cylindrical conductor of radii a and b , carrying a uniformly distributed current i . (a) Show that the magnetic field magnitude $B(r)$ for the radial distance r in the range $b < r < a$ is given by

$$B = \frac{\mu_0 i}{2\pi(a^2 - b^2)} \frac{r^2 - b^2}{r}.$$

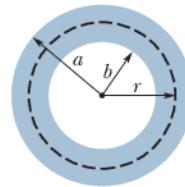


Figure 29-87
Problem 85.

(b) Show that when $r = a$, this equation gives the magnetic field magnitude B at the surface of a long straight wire carrying current i ; when $r = b$, it gives zero magnetic field; and when $b = 0$, it gives the magnetic field inside a solid conductor of radius a carrying current i . (c) Assume that $a = 2.0$ cm, $b = 1.8$ cm, and $i = 100$ A, and then plot $B(r)$ for the range $0 < r < 6$ cm.

86 Show that the magnitude of the magnetic field produced at the center of a rectangular loop of wire of length L and width W , carrying a current i , is

$$B = \frac{2\mu_0 i}{\pi} \frac{(L^2 + W^2)^{1/2}}{LW}.$$

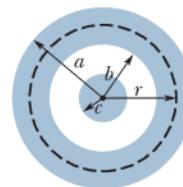


Figure 29-88
Problem 87.

87 Figure 29-88 shows a cross section of a long conducting coaxial cable and gives its radii (a, b, c). Equal but opposite currents i are uniformly distributed in the two conductors. Derive expressions for $B(r)$ with radial distance r in the ranges (a) $r < c$, (b) $c < r < b$, (c) $b < r < a$, and (d) $r > a$. (e) Test these expressions for all the special cases that occur to you.

(f) Assume that $a = 2.0$ cm, $b = 1.8$ cm, $c = 0.40$ cm, and $i = 120$ A and plot the function $B(r)$ over the range $0 < r < 3$ cm.

88 Figure 29-89 is an idealized schematic drawing of a rail gun. Projectile P sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let w be the distance between the rails, R the radius of each rail, and i the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$F = \frac{i^2\mu_0}{2\pi} \ln \frac{w+R}{R}.$$

(b) If the projectile starts from the left end of the rails at rest, find the speed v at which it is expelled at the right. Assume that $i = 450$ kA, $w = 12$ mm, $R = 6.7$ cm, $L = 4.0$ m, and the projectile mass is 10 g.

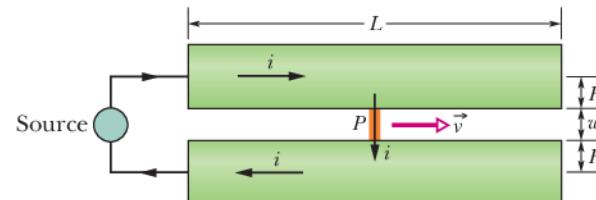


Figure 29-89 Problem 88.

Induction and Inductance

30-1 FARADAY'S LAW AND LENZ'S LAW

Learning Objectives

After reading this module, you should be able to . . .

- 30.01** Identify that the amount of magnetic field piercing a surface (not skimming along the surface) is the magnetic flux Φ through the surface.
- 30.02** Identify that an area vector for a flat surface is a vector that is perpendicular to the surface and that has a magnitude equal to the area of the surface.
- 30.03** Identify that any surface can be divided into area elements (patch elements) that are each small enough and flat enough for an area vector $d\vec{A}$ to be assigned to it, with the vector perpendicular to the element and having a magnitude equal to the area of the element.
- 30.04** Calculate the magnetic flux Φ through a surface by integrating the dot product of the magnetic field vector \vec{B} and the area vector $d\vec{A}$ (for patch elements) over the surface, in magnitude-angle notation and unit-vector notation.
- 30.05** Identify that a current is induced in a conducting loop while the number of magnetic field lines intercepted by the loop is changing.

Key Ideas

- The magnetic flux Φ_B through an area A in a magnetic field \vec{B} is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A},$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.

- If \vec{B} is perpendicular to the area and uniform over it, the flux is

$$\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}).$$

- If the magnetic flux Φ_B through an area bounded by a closed conducting loop changes with time, a current and

- 30.06** Identify that an induced current in a conducting loop is driven by an induced emf.
- 30.07** Apply Faraday's law, which is the relationship between an induced emf in a conducting loop and the rate at which magnetic flux through the loop changes.
- 30.08** Extend Faraday's law from a loop to a coil with multiple loops.
- 30.09** Identify the three general ways in which the magnetic flux through a coil can change.
- 30.10** Use a right-hand rule for Lenz's law to determine the direction of induced emf and induced current in a conducting loop.
- 30.11** Identify that when a magnetic flux through a loop changes, the induced current in the loop sets up a magnetic field to oppose that change.
- 30.12** If an emf is induced in a conducting loop containing a battery, determine the net emf and calculate the corresponding current in the loop.

an emf are produced in the loop; this process is called induction. The induced emf is

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}).$$

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}.$$

- An induced current has a direction such that the magnetic field *due to the current* opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.

What Is Physics?

In Chapter 29 we discussed the fact that a current produces a magnetic field. That fact came as a surprise to the scientists who discovered the effect. Perhaps even more surprising was the discovery of the reverse effect: A magnetic field can produce an electric field that can drive a current. This link between a magnetic field and the electric field it produces (*induces*) is now called *Faraday's law of induction*.

The observations by Michael Faraday and other scientists that led to this law were at first just basic science. Today, however, applications of that basic science are almost everywhere. For example, induction is the basis of the electric guitars that revolutionized early rock and still drive heavy metal and punk today. It is also the basis of the electric generators that power cities and transportation lines and of the huge induction furnaces that are commonplace in foundries where large amounts of metal must be melted rapidly.

Before we get to applications like the electric guitar, we must examine two simple experiments about Faraday's law of induction.

Two Experiments

Let us examine two simple experiments to prepare for our discussion of Faraday's law of induction.

First Experiment. Figure 30-1 shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit. The current disappears when the magnet stops. If we then move the magnet away, a current again suddenly appears, but now in the opposite direction. If we experimented for a while, we would discover the following:

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.

The current produced in the loop is called an **induced current**; the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf**; and the process of producing the current and emf is called **induction**.

Second Experiment. For this experiment we use the apparatus of Fig. 30-2, with the two conducting loops close to each other but not touching. If we close switch S, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction. We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).

The induced emf and induced current in these experiments are apparently caused when something changes—but what is that “something”? Faraday knew.

Faraday's Law of Induction

Faraday realized that an emf and a current can be induced in a loop, as in our two experiments, by changing the *amount of magnetic field* passing through the loop. He further realized that the “amount of magnetic field” can be visualized in terms of the magnetic field lines passing through the loop. **Faraday's law of induction**, stated in terms of our experiments, is this:



An emf is induced in the loop at the left in Figs. 30-1 and 30-2 when the number of magnetic field lines that pass through the loop is changing.

The magnet's motion creates a current in the loop.

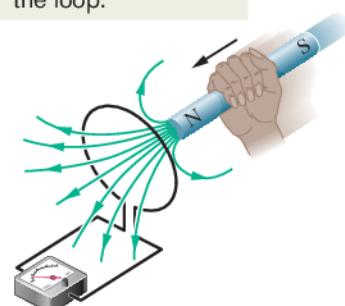
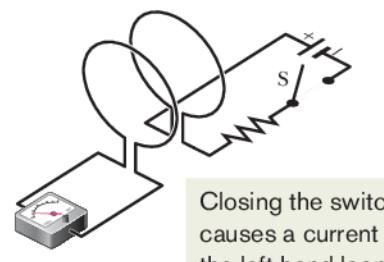


Figure 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.



Closing the switch causes a current in the left-hand loop.

Figure 30-2 An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

The actual number of field lines passing through the loop does not matter; the values of the induced emf and induced current are determined by the *rate* at which that number changes.

In our first experiment (Fig. 30-1), the magnetic field lines spread out from the north pole of the magnet. Thus, as we move the north pole closer to the loop, the number of field lines passing through the loop increases. That increase apparently causes conduction electrons in the loop to move (the induced current) and provides energy (the induced emf) for their motion. When the magnet stops moving, the number of field lines through the loop no longer changes and the induced current and induced emf disappear.

In our second experiment (Fig. 30-2), when the switch is open (no current), there are no field lines. However, when we turn on the current in the right-hand loop, the increasing current builds up a magnetic field around that loop and at the left-hand loop. While the field builds, the number of magnetic field lines through the left-hand loop increases. As in the first experiment, the increase in field lines through that loop apparently induces a current and an emf there. When the current in the right-hand loop reaches a final, steady value, the number of field lines through the left-hand loop no longer changes, and the induced current and induced emf disappear.

A Quantitative Treatment

To put Faraday's law to work, we need a way to calculate the *amount of magnetic field* that passes through a loop. In Chapter 23, in a similar situation, we needed to calculate the amount of electric field that passes through a surface. There we defined an electric flux $\Phi_E = \int \vec{E} \cdot d\vec{A}$. Here we define a *magnetic flux*: Suppose a loop enclosing an area A is placed in a magnetic field \vec{B} . Then the **magnetic flux** through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad (\text{magnetic flux through area } A). \quad (30-1)$$

As in Chapter 23, $d\vec{A}$ is a vector of magnitude dA that is perpendicular to a differential area dA . As with electric flux, we want the component of the field that *pierces* the surface (not skims along it). The dot product of the field and the area vector automatically gives us that piercing component.

Special Case. As a special case of Eq. 30-1, suppose that the loop lies in a plane and that the magnetic field is perpendicular to the plane of the loop. Then we can write the dot product in Eq. 30-1 as $B dA \cos 0^\circ = B dA$. If the magnetic field is also uniform, then B can be brought out in front of the integral sign. The remaining $\int dA$ then gives just the area A of the loop. Thus, Eq. 30-1 reduces to

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}). \quad (30-2)$$

Unit. From Eqs. 30-1 and 30-2, we see that the SI unit for magnetic flux is the tesla–square meter, which is called the *weber* (abbreviated Wb):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2. \quad (30-3)$$

Faraday's Law. With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:



The magnitude of the emf \mathcal{E} induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

As you will see below, the induced emf \mathcal{E} tends to oppose the flux change, so

Faraday's law is formally written as

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law}), \quad (30-4)$$

with the minus sign indicating that opposition. We often neglect the minus sign in Eq. 30-4, seeking only the magnitude of the induced emf.

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (*closely packed*), so that the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} \quad (\text{coil of } N \text{ turns}). \quad (30-5)$$

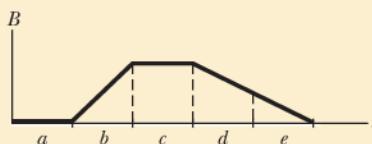
Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.
2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
3. Change the angle between the direction of the magnetic field \vec{B} and the plane of the coil (for example, by rotating the coil so that field \vec{B} is first perpendicular to the plane of the coil and then is along that plane).



Checkpoint 1

The graph gives the magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, with the direction of the field perpendicular to the plane of the loop. Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



Sample Problem 30.01 Induced emf in coil due to a solenoid

The long solenoid S shown (in cross section) in Fig. 30-3 has 220 turns/cm and carries a current $i = 1.5 \text{ A}$; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1 \text{ cm}$. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?

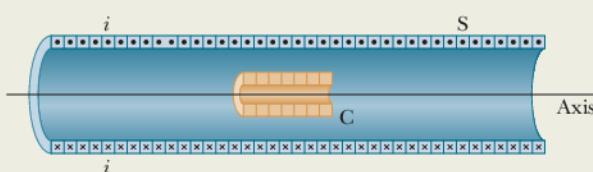


Figure 30-3 A coil C is located inside a solenoid S, which carries current i .

KEY IDEAS

1. Because it is located in the interior of the solenoid, coil C lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ_B through coil C.
2. Because current i decreases, flux Φ_B also decreases.
3. As Φ_B decreases, emf \mathcal{E} is induced in coil C.
4. The flux through each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field \vec{B} . Because \vec{B} is uniform and directed perpendicular to area A , the flux is given by Eq. 30-2 ($\Phi_B = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 29-23 ($B = \mu_0 in$).

Calculations: Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 30-5 ($\mathcal{E} = -N d\Phi_B/dt$), where the number of turns N is 130 and $d\Phi_B/dt$ is the rate at which the flux changes.

Because the current in the solenoid decreases at a steady rate, flux Φ_B also decreases at a steady rate, and so we can write $d\Phi_B/dt$ as $\Delta\Phi_B/\Delta t$. Then, to evaluate $\Delta\Phi_B$, we need the final and initial flux values. The final flux $\Phi_{B,f}$ is zero because the final current in the solenoid is zero. To find the initial flux $\Phi_{B,i}$, we note that area A is $\frac{1}{4}\pi d^2$ ($= 3.464 \times 10^{-4} \text{ m}^2$) and the number n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 29-23 into Eq. 30-2 then leads to

$$\begin{aligned}\Phi_{B,i} &= BA = (\mu_0 in)A \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})(1.5 \text{ A})(22 000 \text{ turns/m}) \\ &\quad \times (3.464 \times 10^{-4} \text{ m}^2) \\ &= 1.44 \times 10^{-5} \text{ Wb}.\end{aligned}$$

Now we can write

$$\begin{aligned}\frac{d\Phi_B}{dt} &= \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \\ &= \frac{(0 - 1.44 \times 10^{-5} \text{ Wb})}{25 \times 10^{-3} \text{ s}} \\ &= -5.76 \times 10^{-4} \text{ Wb/s} \\ &= -5.76 \times 10^{-4} \text{ V}.\end{aligned}$$

We are interested only in magnitudes; so we ignore the minus signs here and in Eq. 30-5, writing

$$\begin{aligned}\mathcal{E} &= N \frac{d\Phi_B}{dt} = (130 \text{ turns})(5.76 \times 10^{-4} \text{ V}) \\ &= 7.5 \times 10^{-2} \text{ V} \\ &= 75 \text{ mV}. \quad (\text{Answer})\end{aligned}$$



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Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule for determining the direction of an induced current in a loop:



An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

Furthermore, the direction of an induced emf is that of the induced current. The key word in Lenz's law is "opposition." Let's apply the law to the motion of the north pole toward the conducting loop in Fig. 30-4.

1. Opposition to Pole Movement. The approach of the magnet's north pole in Fig. 30-4 increases the magnetic flux through the loop and thereby induces a current in the loop. From Fig. 29-22, we know that the loop then acts as a magnetic dipole with a south pole and a north pole, and that its magnetic dipole moment $\vec{\mu}$ is directed from south to north. To oppose the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and thus $\vec{\mu}$) must face toward the approaching north pole so as to repel it (Fig. 30-4). Then the curled-straight right-hand rule for $\vec{\mu}$ (Fig. 29-22) tells us that the current induced in the loop must be counterclockwise in Fig. 30-4.

If we next pull the magnet away from the loop, a current will again be induced in the loop. Now, however, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

2. Opposition to Flux Change. In Fig. 30-4, with the magnet initially distant, no magnetic flux passes through the loop. As the north pole of the magnet then nears the loop with its magnetic field \vec{B} directed downward, the flux through the loop increases. To oppose this increase in flux, the induced current i must set up its own field \vec{B}_{ind} directed upward inside the loop, as shown in Fig. 30-5a; then the upward flux of field \vec{B}_{ind} opposes the increasing downward flux of field \vec{B} . The curled-straight right-hand rule of Fig. 29-22 then tells us that i must be counterclockwise in Fig. 30-5a.

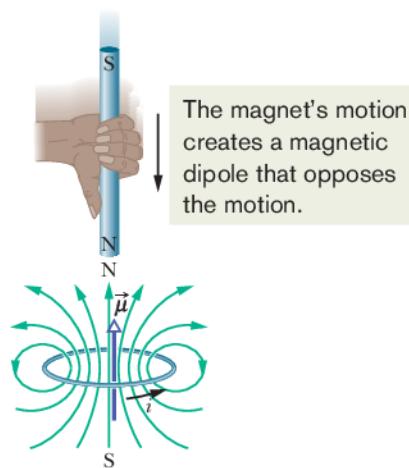


Figure 30-4 Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

Heads Up. The flux of \vec{B}_{ind} always opposes the *change* in the flux of \vec{B} , but \vec{B}_{ind} is not always opposite \vec{B} . For example, if we next pull the magnet away from the loop in Fig. 30-4, the magnet's flux Φ_B is still downward through the loop, but it is now decreasing. The flux of \vec{B}_{ind} must now be downward inside the loop, to oppose that *decrease* (Fig. 30-5b). Thus, \vec{B}_{ind} and \vec{B} are now in the same direction. In Figs. 30-5c and d, the south pole of the magnet approaches and retreats from the loop, again with opposition to change.

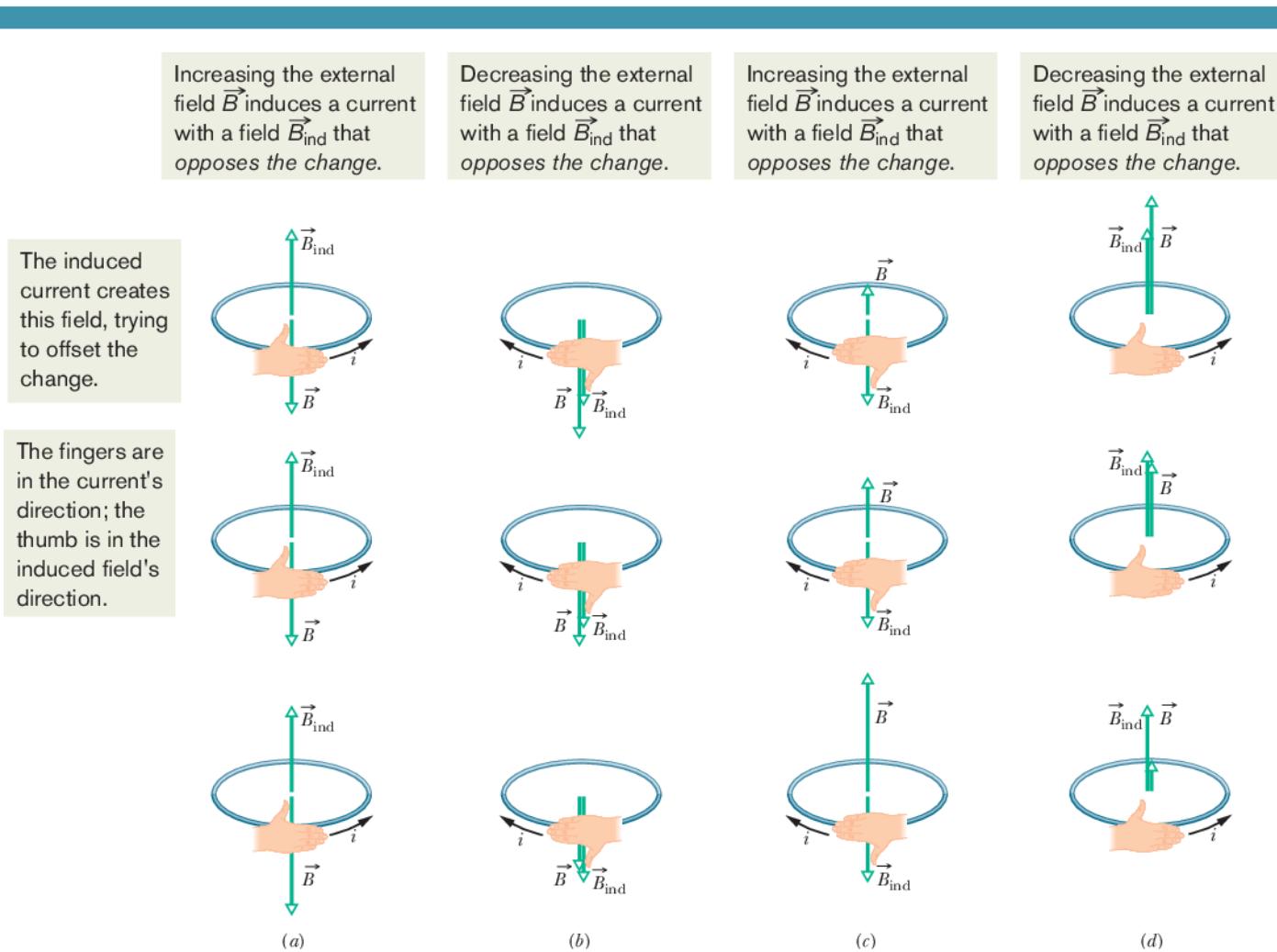
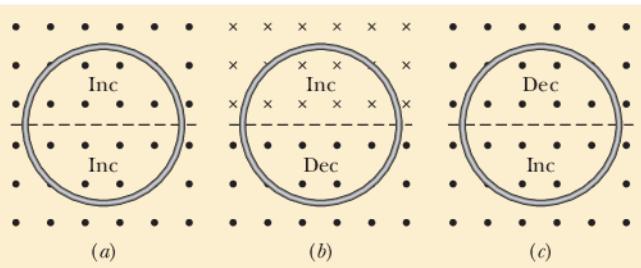


Figure 30-5 The direction of the current i induced in a loop is such that the current's magnetic field \vec{B}_{ind} opposes the *change* in the magnetic field \vec{B} inducing i . The field \vec{B}_{ind} is always directed opposite an increasing field \vec{B} (a, c) and in the same direction as a decreasing field \vec{B} (b, d). The curled-straight right-hand rule gives the direction of the induced current based on the direction of the induced field.



Checkpoint 2

The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.





Sample Problem 30.02 Induced emf and current due to a changing uniform B field

Figure 30-6 shows a conducting loop consisting of a half-circle of radius $r = 0.20\text{ m}$ and three straight sections. The half-circle lies in a uniform magnetic field \vec{B} that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathcal{E}_{\text{bat}} = 2.0\text{ V}$ is connected to the loop. The resistance of the loop is 2.0Ω .

- (a) What are the magnitude and direction of the emf \mathcal{E}_{ind} induced around the loop by field \vec{B} at $t = 10\text{ s}$?

KEY IDEAS

- According to Faraday's law, the magnitude of \mathcal{E}_{ind} is equal to the rate $d\Phi_B/dt$ at which the magnetic flux through the loop changes.
- The flux through the loop depends on how much of the loop's area lies within the flux and how the area is oriented in the magnetic field \vec{B} .
- Because \vec{B} is uniform and is perpendicular to the plane of the loop, the flux is given by Eq. 30-2 ($\Phi_B = BA$). (We don't need to integrate B over the area to get the flux.)
- The induced field B_{ind} (due to the induced current) must always oppose the *change* in the magnetic flux.

Magnitude: Using Eq. 30-2 and realizing that only the field magnitude B changes in time (not the area A), we rewrite Faraday's law, Eq. 30-4, as

$$\mathcal{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A \frac{dB}{dt}.$$

Because the flux penetrates the loop only within the half-circle, the area A in this equation is $\frac{1}{2}\pi r^2$. Substituting this and the given expression for B yields

$$\begin{aligned}\mathcal{E}_{\text{ind}} &= A \frac{dB}{dt} = \frac{\pi r^2}{2} \frac{d}{dt} (4.0t^2 + 2.0t + 3.0) \\ &= \frac{\pi r^2}{2} (8.0t + 2.0).\end{aligned}$$

Sample Problem 30.03 Induced emf due to a changing nonuniform B field

Figure 30-7 shows a rectangular loop of wire immersed in a nonuniform and varying magnetic field \vec{B} that is perpendicular to and directed into the page. The field's magnitude is given by $B = 4t^2x^2$, with B in teslas, t in seconds, and x in meters. (Note that the function depends on *both* time and position.) The loop has width $W = 3.0\text{ m}$ and height $H = 2.0\text{ m}$. What are the magnitude and direction of the induced emf \mathcal{E} around the loop at $t = 0.10\text{ s}$?

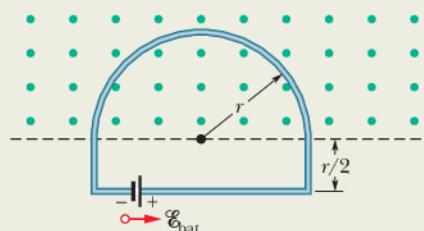


Figure 30-6 A battery is connected to a conducting loop that includes a half-circle of radius r lying in a uniform magnetic field. The field is directed out of the page; its magnitude is changing.

At $t = 10\text{ s}$, then,

$$\begin{aligned}\mathcal{E}_{\text{ind}} &= \frac{\pi(0.20\text{ m})^2}{2} [8.0(10) + 2.0] \\ &= 5.152\text{ V} \approx 5.2\text{ V}. \quad (\text{Answer})\end{aligned}$$

Direction: To find the direction of \mathcal{E}_{ind} , we first note that in Fig. 30-6 the flux through the loop is out of the page and increasing. Because the induced field B_{ind} (due to the induced current) must oppose that increase, it must be *into* the page. Using the curled-straight right-hand rule (Fig. 30-5c), we find that the induced current is clockwise around the loop, and thus so is the induced emf \mathcal{E}_{ind} .

- (b) What is the current in the loop at $t = 10\text{ s}$?

KEY IDEA

The point here is that *two* emfs tend to move charges around the loop.

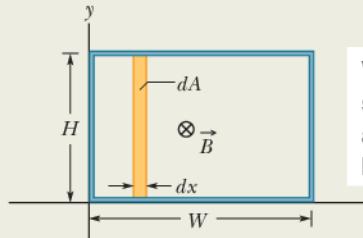
Calculation: The induced emf \mathcal{E}_{ind} tends to drive a current clockwise around the loop; the battery's emf \mathcal{E}_{bat} tends to drive a current counterclockwise. Because \mathcal{E}_{ind} is greater than \mathcal{E}_{bat} , the net emf \mathcal{E}_{net} is clockwise, and thus so is the current. To find the current at $t = 10\text{ s}$, we use Eq. 27-2 ($i = \mathcal{E}/R$):

$$\begin{aligned}i &= \frac{\mathcal{E}_{\text{net}}}{R} = \frac{\mathcal{E}_{\text{ind}} - \mathcal{E}_{\text{bat}}}{R} \\ &= \frac{5.152\text{ V} - 2.0\text{ V}}{2.0\Omega} = 1.58\text{ A} \approx 1.6\text{ A}. \quad (\text{Answer})\end{aligned}$$

KEY IDEAS

- Because the magnitude of the magnetic field \vec{B} is changing with time, the magnetic flux Φ_B through the loop is also changing.
- The changing flux induces an emf \mathcal{E} in the loop according to Faraday's law, which we can write as $\mathcal{E} = d\Phi_B/dt$.
- To use that law, we need an expression for the flux Φ_B at

If the field varies with position, we must integrate to get the flux through the loop.



We start with a strip so thin that we can approximate the field as being uniform within it.

Figure 30-7 A closed conducting loop, of width W and height H , lies in a nonuniform, varying magnetic field that points directly into the page. To apply Faraday's law, we use the vertical strip of height H , width dx , and area dA .

any time t . However, because B is *not* uniform over the area enclosed by the loop, we *cannot* use Eq. 30-2 ($\Phi_B = BA$) to find that expression; instead we must use Eq. 30-1 ($\Phi_B = \int \vec{B} \cdot d\vec{A}$).

Calculations: In Fig. 30-7, \vec{B} is perpendicular to the plane of the loop (and hence parallel to the differential area vector $d\vec{A}$); so the dot product in Eq. 30-1 gives $B dA$. Because the magnetic field varies with the coordinate x but not with the coordinate y , we can take the differential area

dA to be the area of a vertical strip of height H and width dx (as shown in Fig. 30-7). Then $dA = H dx$, and the flux through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA = \int BH dx = \int 4t^2x^2H dx.$$

Treating t as a constant for this integration and inserting the integration limits $x = 0$ and $x = 3.0$ m, we obtain

$$\Phi_B = 4t^2H \int_0^{3.0} x^2 dx = 4t^2H \left[\frac{x^3}{3} \right]_0^{3.0} = 72t^2,$$

where we have substituted $H = 2.0$ m and Φ_B is in webers. Now we can use Faraday's law to find the magnitude of \mathcal{E} at any time t :

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t,$$

in which \mathcal{E} is in volts. At $t = 0.10$ s,

$$\mathcal{E} = (144 \text{ V/s})(0.10 \text{ s}) \approx 14 \text{ V.} \quad (\text{Answer})$$

The flux of \vec{B} through the loop is into the page in Fig. 30-7 and is increasing in magnitude because B is increasing in magnitude with time. By Lenz's law, the field B_{ind} of the induced current opposes this increase and so is directed out of the page. The curled-straight right-hand rule in Fig. 30-5a then tells us that the induced current is counterclockwise around the loop, and thus so is the induced emf \mathcal{E} .



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30-2 INDUCTION AND ENERGY TRANSFERS

Learning Objectives

After reading this module, you should be able to . . .

30.13 For a conducting loop pulled into or out of a magnetic field, calculate the rate at which energy is transferred to thermal energy.

30.14 Apply the relationship between an induced current and the rate at which it produces thermal energy.

30.15 Describe eddy currents.

Key Idea

- The induction of a current by a changing flux means that energy is being transferred to that current. The energy can then be transferred to other forms, such as thermal energy.

Induction and Energy Transfers

By Lenz's law, whether you move the magnet toward or away from the loop in Fig. 30-1, a magnetic force resists the motion, requiring your applied force to do positive work. At the same time, thermal energy is produced in the material of the loop because of the material's electrical resistance to the current that is induced by the motion. The energy you transfer to the closed *loop + magnet* system via your applied force ends up in this thermal energy. (For now, we neglect energy that is radiated away from the loop as electromagnetic waves during the

induction.) The faster you move the magnet, the more rapidly your applied force does work and the greater the rate at which your energy is transferred to thermal energy in the loop; that is, the power of the transfer is greater.

Regardless of how current is induced in a loop, energy is always transferred to thermal energy during the process because of the electrical resistance of the loop (unless the loop is superconducting). For example, in Fig. 30-2, when switch S is closed and a current is briefly induced in the left-hand loop, energy is transferred from the battery to thermal energy in that loop.

Figure 30-8 shows another situation involving induced current. A rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in Fig. 30-8 show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity \vec{v} .

Flux Change. The situation of Fig. 30-8 does not differ in any essential way from that of Fig. 30-1. In each case a magnetic field and a conducting loop are in relative motion; in each case the flux of the field through the loop is changing with time. It is true that in Fig. 30-1 the flux is changing because \vec{B} is changing and in Fig. 30-8 the flux is changing because the area of the loop still in the magnetic field is changing, but that difference is not important. The important difference between the two arrangements is that the arrangement of Fig. 30-8 makes calculations easier. Let us now calculate the rate at which you do mechanical work as you pull steadily on the loop in Fig. 30-8.

Rate of Work. As you will see, to pull the loop at a constant velocity \vec{v} , you must apply a constant force \vec{F} to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you. From Eq. 7-48, the rate at which you do work—that is, the power—is then

$$P = Fv, \quad (30-6)$$

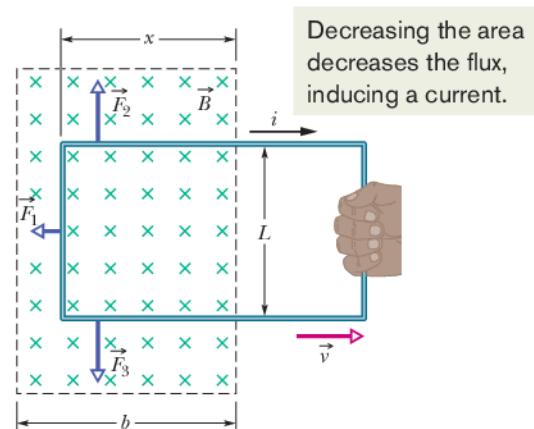
where F is the magnitude of your force. We wish to find an expression for P in terms of the magnitude B of the magnetic field and the characteristics of the loop—namely, its resistance R to current and its dimension L .

As you move the loop to the right in Fig. 30-8, the portion of its area within the magnetic field decreases. Thus, the flux through the loop also decreases and, according to Faraday's law, a current is produced in the loop. It is the presence of this current that causes the force that opposes your pull.

Induced emf. To find the current, we first apply Faraday's law. When x is the length of the loop still in the magnetic field, the area of the loop still in the field is Lx . Then from Eq. 30-2, the magnitude of the flux through the loop is

$$\Phi_B = BA = BLx. \quad (30-7)$$

Figure 30-8 You pull a closed conducting loop out of a magnetic field at constant velocity \vec{v} . While the loop is moving, a clockwise current i is induced in the loop, and the loop segments still within the magnetic field experience forces \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 .



As x decreases, the flux decreases. Faraday's law tells us that with this flux decrease, an emf is induced in the loop. Dropping the minus sign in Eq. 30-4 and using Eq. 30-7, we can write the magnitude of this emf as

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv, \quad (30-8)$$

in which we have replaced dx/dt with v , the speed at which the loop moves.

Figure 30-9 shows the loop as a circuit: induced emf \mathcal{E} is represented on the left, and the collective resistance R of the loop is represented on the right. The direction of the induced current i is obtained with a right-hand rule as in Fig. 30-5b for decreasing flux; applying the rule tells us that the current must be clockwise, and \mathcal{E} must have the same direction.

Induced Current. To find the magnitude of the induced current, we cannot apply the loop rule for potential differences in a circuit because, as you will see in Module 30-3, we cannot define a potential difference for an induced emf. However, we can apply the equation $i = \mathcal{E}/R$. With Eq. 30-8, this becomes

$$i = \frac{BLv}{R}. \quad (30-9)$$

Because three segments of the loop in Fig. 30-8 carry this current through the magnetic field, sideways deflecting forces act on those segments. From Eq. 28-26 we know that such a deflecting force is, in general notation,

$$\vec{F}_d = i\vec{L} \times \vec{B}. \quad (30-10)$$

In Fig. 30-8, the deflecting forces acting on the three segments of the loop are marked \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 . Note, however, that from the symmetry, forces \vec{F}_2 and \vec{F}_3 are equal in magnitude and cancel. This leaves only force \vec{F}_1 , which is directed opposite your force \vec{F} on the loop and thus is the force opposing you. So, $\vec{F} = -\vec{F}_1$.

Using Eq. 30-10 to obtain the magnitude of \vec{F}_1 and noting that the angle between \vec{B} and the length vector \vec{L} for the left segment is 90° , we write

$$F = F_1 = iLB \sin 90^\circ = iLB. \quad (30-11)$$

Substituting Eq. 30-9 for i in Eq. 30-11 then gives us

$$F = \frac{B^2 L^2 v}{R}. \quad (30-12)$$

Because B , L , and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

Rate of Work. By substituting Eq. 30-12 into Eq. 30-6, we find the rate at which you do work on the loop as you pull it from the magnetic field:

$$P = Fv = \frac{B^2 L^2 v^2}{R} \quad (\text{rate of doing work}). \quad (30-13)$$

Thermal Energy. To complete our analysis, let us find the rate at which thermal energy appears in the loop as you pull it along at constant speed. We calculate it from Eq. 26-27,

$$P = i^2 R. \quad (30-14)$$

Substituting for i from Eq. 30-9, we find

$$P = \left(\frac{BLv}{R} \right)^2 R = \frac{B^2 L^2 v^2}{R} \quad (\text{thermal energy rate}), \quad (30-15)$$

which is exactly equal to the rate at which you are doing work on the loop (Eq. 30-13). Thus, the work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop.

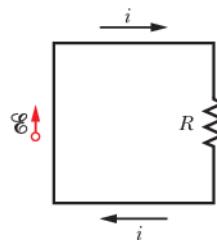


Figure 30-9 A circuit diagram for the loop of Fig. 30-8 while the loop is moving.

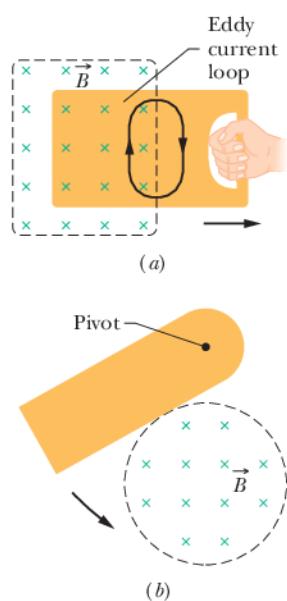


Figure 30-10 (a) As you pull a solid conducting plate out of a magnetic field, *eddy currents* are induced in the plate. A typical loop of eddy current is shown. (b) A conducting plate is allowed to swing like a pendulum about a pivot and into a region of magnetic field. As it enters and leaves the field, eddy currents are induced in the plate.

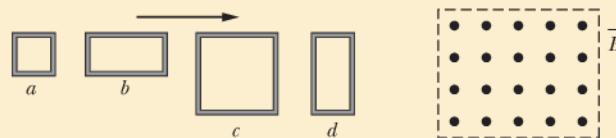
Eddy Currents

Suppose we replace the conducting loop of Fig. 30-8 with a solid conducting plate. If we then move the plate out of the magnetic field as we did the loop (Fig. 30-10a), the relative motion of the field and the conductor again induces a current in the conductor. Thus, we again encounter an opposing force and must do work because of the induced current. With the plate, however, the conduction electrons making up the induced current do not follow one path as they do with the loop. Instead, the electrons swirl about within the plate as if they were caught in an eddy (whirlpool) of water. Such a current is called an *eddy current* and can be represented, as it is in Fig. 30-10a, as if it followed a single path.

As with the conducting loop of Fig. 30-8, the current induced in the plate results in mechanical energy being dissipated as thermal energy. The dissipation is more apparent in the arrangement of Fig. 30-10b; a conducting plate, free to rotate about a pivot, is allowed to swing down through a magnetic field like a pendulum. Each time the plate enters and leaves the field, a portion of its mechanical energy is transferred to its thermal energy. After several swings, no mechanical energy remains and the warmed-up plate just hangs from its pivot.

Checkpoint 3

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field \vec{B} (directed out of the page) at the same constant velocity. Rank the four loops according to the maximum magnitude of the emf induced as they move through the field, greatest first.



30-3 INDUCED ELECTRIC FIELDS

Learning Objectives

After reading this module, you should be able to . . .

- 30.16** Identify that a changing magnetic field induces an electric field, regardless of whether there is a conducting loop.
30.17 Apply Faraday's law to relate the electric field \vec{E} induced along a closed path (whether it has conducting

material or not) to the rate of change $d\Phi/dt$ of the magnetic flux encircled by the path.

- 30.18** Identify that an electric potential cannot be associated with an induced electric field.

Key Ideas

- An emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field \vec{E} at every point of such a loop; the induced emf is related to \vec{E} by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$$

- Using the induced electric field, we can write Faraday's law in its most general form as

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}).$$

A changing magnetic field induces an electric field \vec{E} .