

via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

**Calculations:** At any point, the electric field  $\vec{E}_{(+)}$  due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$\begin{aligned} E_{(+)} &= \frac{\sigma_{(+)}}{2\epsilon_0} = \frac{6.8 \times 10^{-6} \text{ C/m}^2}{(2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \\ &= 3.84 \times 10^5 \text{ N/C}. \end{aligned}$$

Similarly, at any point, the electric field  $\vec{E}_{(-)}$  due to the negative sheet is directed *toward* that sheet and has the magnitude

$$\begin{aligned} E_{(-)} &= \frac{\sigma_{(-)}}{2\epsilon_0} = \frac{4.3 \times 10^{-6} \text{ C/m}^2}{(2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \\ &= 2.43 \times 10^5 \text{ N/C}. \end{aligned}$$



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Figure 23-19b shows the fields set up by the sheets to the left of the sheets ( $L$ ), between them ( $B$ ), and to their right ( $R$ ).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$\begin{aligned} E_L &= E_{(+)} - E_{(-)} \\ &= 3.84 \times 10^5 \text{ N/C} - 2.43 \times 10^5 \text{ N/C} \\ &= 1.4 \times 10^5 \text{ N/C}. \end{aligned} \quad (\text{Answer})$$

Because  $E_{(+)}$  is larger than  $E_{(-)}$ , the net electric field  $\vec{E}_L$  in this region is directed to the left, as Fig. 23-19c shows. To the right of the sheets, the net electric field has the same magnitude but is directed to the right, as Fig. 23-19c shows.

Between the sheets, the two fields add and we have

$$\begin{aligned} E_B &= E_{(+)} + E_{(-)} \\ &= 3.84 \times 10^5 \text{ N/C} + 2.43 \times 10^5 \text{ N/C} \\ &= 6.3 \times 10^5 \text{ N/C}. \end{aligned} \quad (\text{Answer})$$

The electric field  $\vec{E}_B$  is directed to the right.

## 23-6 APPLYING GAUSS' LAW: SPHERICAL SYMMETRY

### Learning Objectives

After reading this module, you should be able to . . .

- 23.26 Identify that a shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge is concentrated at the center of the shell.
- 23.27 Identify that if a charged particle is enclosed by a shell of uniform charge, there is no electrostatic force on the particle from the shell.
- 23.28 For a point outside a spherical shell with uniform

charge, apply the relationship between the electric field magnitude  $E$ , the charge  $q$  on the shell, and the distance  $r$  from the shell's center.

- 23.29 Identify the magnitude of the electric field for points enclosed by a spherical shell with uniform charge.
- 23.30 For a uniform spherical charge distribution (a uniform ball of charge), determine the magnitude and direction of the electric field at interior and exterior points.

### Key Ideas

- Outside a spherical shell of uniform charge  $q$ , the electric field due to the shell is radial (inward or outward, depending on the sign of the charge) and has the magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{outside spherical shell}),$$

where  $r$  is the distance to the point of measurement from the center of the shell. The field is the same as though all of the charge is concentrated as a particle at the center of the shell.

- Inside the shell, the field due to the shell is zero.
- Inside a sphere with a uniform volume charge density, the field is radial and has the magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} r \quad (\text{inside sphere of charge}),$$

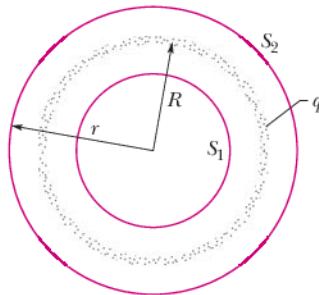
where  $q$  is the total charge,  $R$  is the sphere's radius, and  $r$  is the radial distance from the center of the sphere to the point of measurement.

### Applying Gauss' Law: Spherical Symmetry

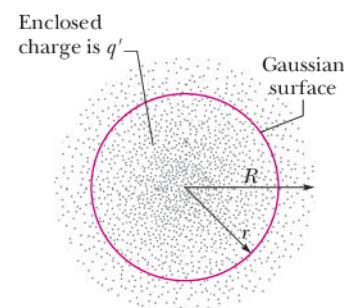
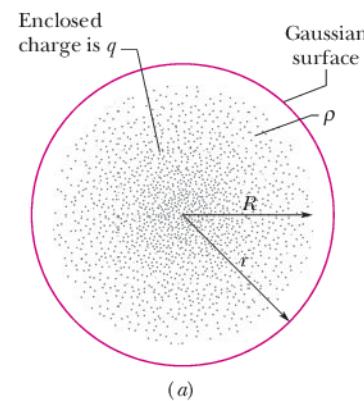
Here we use Gauss' law to prove the two shell theorems presented without proof in Module 21-1:



A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.



**Figure 23-20** A thin, uniformly charged, spherical shell with total charge  $q$ , in cross section. Two concentric spherical Gaussian surfaces  $S_1$  and  $S_2$  are also shown in cross section. Surface  $S_2$  encloses the shell, and  $S_1$  encloses only the empty interior of the shell.



**Figure 23-21** The dots represent a spherically symmetric distribution of charge of radius  $R$ , whose volume charge density  $\rho$  is a function only of distance from the center. The charged object is not a conductor, and therefore the charge is assumed to be fixed in position. A concentric spherical Gaussian surface with  $r > R$  is shown in (a). A similar Gaussian surface with  $r < R$  is shown in (b).

Figure 23-20 shows a charged spherical shell of total charge  $q$  and radius  $R$  and two concentric spherical Gaussian surfaces,  $S_1$  and  $S_2$ . If we followed the procedure of Module 23-2 as we applied Gauss' law to surface  $S_2$ , for which  $r \geq R$ , we would find that

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, field at } r \geq R). \quad (23-15)$$

This field is the same as one set up by a particle with charge  $q$  at the center of the shell of charge. Thus, the force produced by a shell of charge  $q$  on a charged particle placed outside the shell is the same as if all the shell's charge is concentrated as a particle at the shell's center. This proves the first shell theorem.

Applying Gauss' law to surface  $S_1$ , for which  $r < R$ , leads directly to

$$E = 0 \quad (\text{spherical shell, field at } r < R), \quad (23-16)$$

because this Gaussian surface encloses no charge. Thus, if a charged particle were enclosed by the shell, the shell would exert no net electrostatic force on the particle. This proves the second shell theorem.



If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

Any spherically symmetric charge distribution, such as that of Fig. 23-21, can be constructed with a nest of concentric spherical shells. For purposes of applying the two shell theorems, the volume charge density  $\rho$  should have a single value for each shell but need not be the same from shell to shell. Thus, for the charge distribution as a whole,  $\rho$  can vary, but only with  $r$ , the radial distance from the center. We can then examine the effect of the charge distribution "shell by shell."

In Fig. 23-21a, the entire charge lies within a Gaussian surface with  $r > R$ . The charge produces an electric field on the Gaussian surface as if the charge were that of a particle located at the center, and Eq. 23-15 holds.

Figure 23-21b shows a Gaussian surface with  $r < R$ . To find the electric field at points on this Gaussian surface, we separately consider the charge inside it and the charge outside it. From Eq. 23-16, the outside charge does not set up a field on the Gaussian surface. From Eq. 23-15, the inside charge sets up a field as though it is concentrated at the center. Letting  $q'$  represent that enclosed charge, we can then rewrite Eq. 23-15 as

$$E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2} \quad (\text{spherical distribution, field at } r \leq R). \quad (23-17)$$

If the full charge  $q$  enclosed within radius  $R$  is uniform, then  $q'$  enclosed within radius  $r$  in Fig. 23-21b is proportional to  $q$ :

$$\frac{\left(\begin{array}{l} \text{charge enclosed by} \\ \text{sphere of radius } r \end{array}\right)}{\left(\begin{array}{l} \text{volume enclosed by} \\ \text{sphere of radius } r \end{array}\right)} = \frac{\text{full charge}}{\text{full volume}}$$

or

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}. \quad (23-18)$$

This gives us

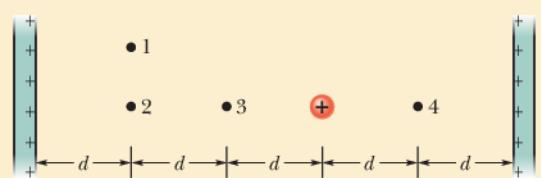
$$q' = q \frac{r^3}{R^3}. \quad (23-19)$$

Substituting this into Eq. 23-17 yields

$$E = \left( \frac{q}{4\pi\epsilon_0 R^3} \right) r \quad (\text{uniform charge, field at } r \leq R). \quad (23-20)$$

**Checkpoint 4**

The figure shows two large, parallel, nonconducting sheets with identical (positive) uniform surface charge densities, and a sphere with a uniform (positive) volume charge density. Rank the four numbered points according to the magnitude of the net electric field there, greatest first.

 **Review & Summary**

**Gauss' Law** Gauss' law and Coulomb's law are different ways of describing the relation between charge and electric field in static situations. Gauss' law is

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (\text{Gauss' law}), \quad (23-6)$$

in which  $q_{\text{enc}}$  is the net charge inside an imaginary closed surface (a *Gaussian surface*) and  $\Phi$  is the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{electric flux through a Gaussian surface}). \quad (23-4)$$

Coulomb's law can be derived from Gauss' law.

**Applications of Gauss' Law** Using Gauss' law and, in some cases, symmetry arguments, we can derive several important results in electrostatic situations. Among these are:

1. An excess charge on an isolated *conductor* is located entirely on the outer surface of the conductor.
2. The external electric field near the *surface of a charged conductor* is perpendicular to the surface and has a magnitude that depends on the surface charge density  $\sigma$ :

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{conducting surface}). \quad (23-11)$$

Within the conductor,  $E = 0$ .

3. The electric field at any point due to an infinite *line of charge*

with uniform linear charge density  $\lambda$  is perpendicular to the line of charge and has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge}), \quad (23-12)$$

where  $r$  is the perpendicular distance from the line of charge to the point.

4. The electric field due to an *infinite nonconducting sheet* with uniform surface charge density  $\sigma$  is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{sheet of charge}). \quad (23-13)$$

5. The electric field *outside a spherical shell of charge* with radius  $R$  and total charge  $q$  is directed radially and has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, for } r \geq R). \quad (23-15)$$

Here  $r$  is the distance from the center of the shell to the point at which  $E$  is measured. (The charge behaves, for external points, as if it were all located at the center of the sphere.) The field *inside* a uniform spherical shell of charge is exactly zero:

$$E = 0 \quad (\text{spherical shell, for } r < R). \quad (23-16)$$

6. The electric field *inside a uniform sphere of charge* is directed radially and has magnitude

$$E = \left( \frac{q}{4\pi\epsilon_0 R^3} \right) r. \quad (23-20)$$

**Questions**

- 1 A surface has the area vector  $\vec{A} = (2\hat{i} + 3\hat{j}) \text{ m}^2$ . What is the flux of a uniform electric field through the area if the field is (a)  $\vec{E} = 4\hat{i} \text{ N/C}$  and (b)  $\vec{E} = 4\hat{k} \text{ N/C}$ ?

- 2 Figure 23-22 shows, in cross section, three solid cylinders, each of length  $L$  and uniform charge  $Q$ . Concentric with each cylinder is a cylindrical Gaussian surface, with all three surfaces having the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface, greatest first.

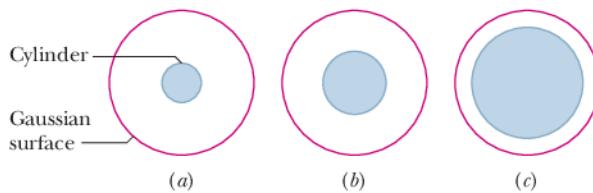


Figure 23-22 Question 2.

- 3 Figure 23-23 shows, in cross section, a central metal ball, two spherical metal shells, and three spherical Gaussian surfaces of radii  $R$ ,  $2R$ , and  $3R$ , all with the same center. The uniform charges on the three objects are: ball,  $Q$ ; smaller shell,  $3Q$ ; larger shell,  $5Q$ . Rank the Gaussian surfaces according to the magnitude of the electric field at any point on the surface, greatest first.

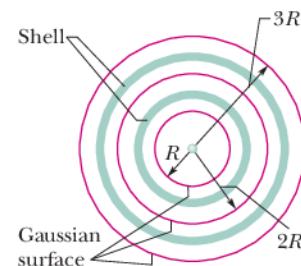


Figure 23-23 Question 3.

- 4** Figure 23-24 shows, in cross section, two Gaussian spheres and two Gaussian cubes that are centered on a positively charged particle. (a) Rank the net flux through the four Gaussian surfaces, greatest first. (b) Rank the magnitudes of the electric fields on the surfaces, greatest first, and indicate whether the magnitudes are uniform or variable along each surface.

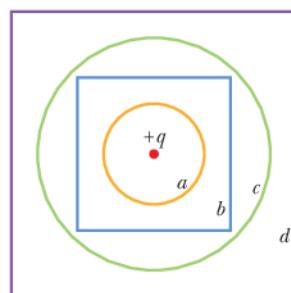


Figure 23-24 Question 4.

- 5** In Fig. 23-25, an electron is released between two infinite nonconducting sheets that are horizontal and have uniform surface charge densities  $\sigma_{(+)}$  and  $\sigma_{(-)}$ , as indicated. The electron is subjected to the following three situations involving surface charge densities and sheet separations. Rank the magnitudes of the electron's acceleration, greatest first.

Situation	$\sigma_{(+)}$	$\sigma_{(-)}$	Separation
1	$+4\sigma$	$-4\sigma$	$d$
2	$+7\sigma$	$-\sigma$	$4d$
3	$+3\sigma$	$-5\sigma$	$9d$

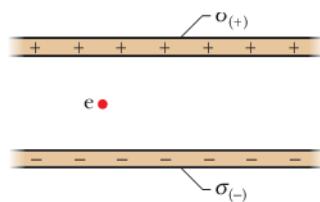


Figure 23-25 Question 5.

- 6** Three infinite nonconducting sheets, with uniform positive surface charge densities  $\sigma$ ,  $2\sigma$ , and  $3\sigma$ , are arranged to be parallel like the two sheets in Fig. 23-19a. What is their order, from left to right, if the electric field  $\vec{E}$  produced by the arrangement has magnitude  $E = 0$  in one region and  $E = 2\sigma/\epsilon_0$  in another region?

- 7** Figure 23-26 shows four situations in which four very long rods extend into and out of the page (we see only their cross sections). The value below each cross section gives that particular rod's uniform charge density in microcoulombs per meter. The rods are separated by either  $d$  or  $2d$  as drawn, and a central point is shown midway between the inner rods. Rank the situations according to the magnitude of the net electric field at that central point, greatest first.

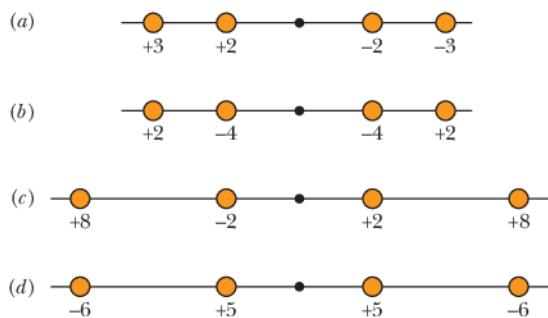


Figure 23-26 Question 7.

- 8** Figure 23-27 shows four solid spheres, each with charge  $Q$  uniformly distributed through its volume. (a) Rank the spheres according to their volume charge density, greatest first. The figure also shows a point  $P$  for each sphere, all at the same distance from the center of the sphere. (b) Rank the spheres according to the magnitude of the electric field they produce at point  $P$ , greatest first.

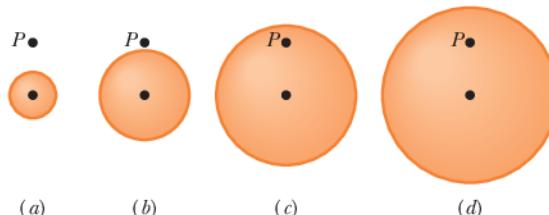


Figure 23-27 Question 8.

- 9** A small charged ball lies within the hollow of a metallic spherical shell of radius  $R$ . For three situations, the net charges on the ball and shell, respectively, are (1)  $+4q$ , 0; (2)  $-6q$ ,  $+10q$ ; (3)  $+16q$ ,  $-12q$ . Rank the situations according to the charge on (a) the inner surface of the shell and (b) the outer surface, most positive first.

- 10** Rank the situations of Question 9 according to the magnitude of the electric field (a) halfway through the shell and (b) at a point  $2R$  from the center of the shell, greatest first.

- 11** Figure 23-28 shows a section of three long charged cylinders centered on the same axis. Central cylinder  $A$  has a uniform charge  $q_A = +3q_0$ . What uniform charges  $q_B$  and  $q_C$  should be on cylinders  $B$  and  $C$  so that (if possible) the net electric field is zero at (a) point 1, (b) point 2, and (c) point 3?

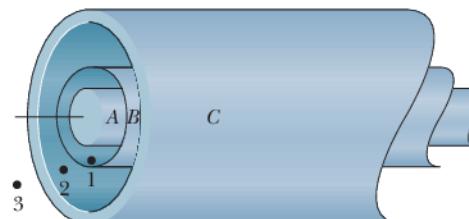


Figure 23-28 Question 11.

- 12** Figure 23-29 shows four Gaussian surfaces consisting of identical cylindrical midsections but different end caps. The surfaces are in a uniform electric field  $\vec{E}$  that is directed parallel to the central axis of each cylindrical midsection. The end caps have these shapes:  $S_1$ , convex hemispheres;  $S_2$ , concave hemispheres;  $S_3$ , cones;  $S_4$ , flat disks. Rank the surfaces according to (a) the net electric flux through them and (b) the electric flux through the top end caps, greatest first.

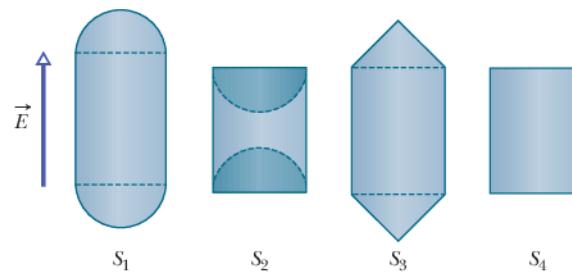


Figure 23-29 Question 12.

## Problems

**GO** Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign

**SSM** Worked-out solution available in Student Solutions Manual

• • Number of dots indicates level of problem difficulty

 Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://flyingcircusofphysics.com)

**WWW** Worked-out solution is at

**ILW** Interactive solution is at

<http://www.wiley.com/college/halliday>

### Module 23-1 Electric Flux

- 1 **SSM** The square surface shown in Fig. 23-30 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude  $E = 1800 \text{ N/C}$  and with field lines at an angle of  $\theta = 35^\circ$  with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.

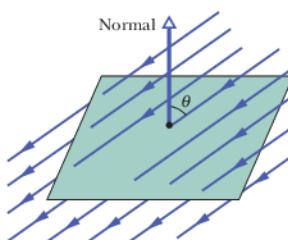


Figure 23-30 Problem 1.

- 2 An electric field given by  $\vec{E} = 4.0\hat{i} - 3.0(y^2 + 2.0)\hat{j}$  pierces a Gaussian cube of edge length 2.0 m and positioned as shown in Fig. 23-7. (The magnitude  $E$  is in newtons per coulomb and the position  $x$  is in meters.) What is the electric flux through the (a) top face, (b) bottom face, (c) left face, and (d) back face? (e) What is the net electric flux through the cube?

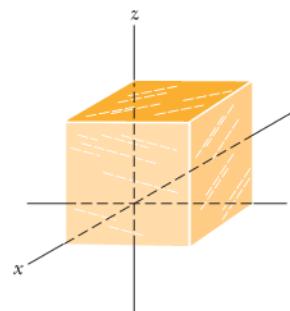


Figure 23-31 Problems 3, 6, and 9.

- 3 The cube in Fig. 23-31 has edge length 1.40 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by (a)  $6.00\hat{i}$ , (b)  $-2.00\hat{j}$ , and (c)  $-3.00\hat{i} + 4.00\hat{k}$ . (d) What is the total flux through the cube for each field?

### Module 23-2 Gauss' Law

- 4 In Fig. 23-32, a butterfly net is in a uniform electric field of magnitude  $E = 3.0 \text{ mN/C}$ . The rim, a circle of radius  $a = 11 \text{ cm}$ , is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the netting.

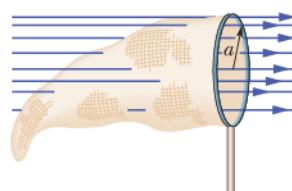


Figure 23-32 Problem 4.

- 5 In Fig. 23-33, a proton is a distance  $d/2$  directly above the center of a square of side  $d$ . What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge  $d$ .)

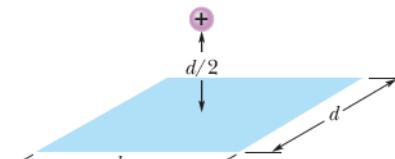


Figure 23-33 Problem 5.

- 6 At each point on the surface of the cube shown in Fig. 23-31, the electric field is parallel to the  $z$  axis. The length of each edge of the cube is 3.0 m. On the top face of the cube the field is

$\vec{E} = -34\hat{k} \text{ N/C}$ , and on the bottom face it is  $\vec{E} = +20\hat{k} \text{ N/C}$ . Determine the net charge contained within the cube.

- 7 A particle of charge  $1.8 \mu\text{C}$  is at the center of a Gaussian cube 55 cm on edge. What is the net electric flux through the surface?

- 8  When a shower is turned on in a closed bathroom, the splashing of the water on the bare tub can fill the room's air with negatively charged ions and produce an electric field in the air as great as  $1000 \text{ N/C}$ . Consider a bathroom with dimensions  $2.5 \text{ m} \times 3.0 \text{ m} \times 2.0 \text{ m}$ . Along the ceiling, floor, and four walls, approximate the electric field in the air as being directed perpendicular to the surface and as having a uniform magnitude of  $600 \text{ N/C}$ . Also, treat those surfaces as forming a closed Gaussian surface around the room's air. What are (a) the volume charge density  $\rho$  and (b) the number of excess elementary charges  $e$  per cubic meter in the room's air?

- 9 **ILW** Fig. 23-31 shows a Gaussian surface in the shape of a cube with edge length 1.40 m. What are (a) the net flux  $\Phi$  through the surface and (b) the net charge  $q_{\text{enc}}$  enclosed by the surface if  $\vec{E} = (3.00y\hat{j}) \text{ N/C}$ , with  $y$  in meters? What are (c)  $\Phi$  and (d)  $q_{\text{enc}}$  if  $\vec{E} = [-4.00\hat{i} + (6.00 + 3.00y)\hat{j}] \text{ N/C}$ ?

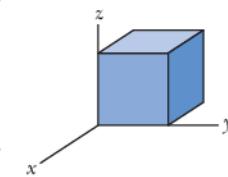


Figure 23-34 Problem 10.

- 10 Figure 23-34 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by  $\vec{E} = (3.00x + 4.00)\hat{i} + 6.00\hat{j} + 7.00\hat{k} \text{ N/C}$ , with  $x$  in meters. What is the net charge contained by the cube?

- 11 **GO** Figure 23-35 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at  $x_1 = 5.00 \text{ m}$ ,  $y_1 = 4.00 \text{ m}$ . The cube lies in a region where the electric field vector is given by  $\vec{E} = -3.00\hat{i} - 4.00y\hat{j} + 3.00\hat{k} \text{ N/C}$ , with  $y$  in meters. What is the net charge contained by the cube?

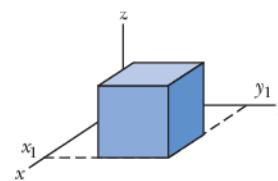


Figure 23-35 Problem 11.

- 12 Figure 23-36 shows two nonconducting spherical shells fixed in place. Shell 1 has uniform surface charge density  $+6.0 \mu\text{C/m}^2$  on its outer surface and radius 3.0 cm; shell 2 has uniform surface charge density  $+4.0 \mu\text{C/m}^2$  on its outer surface and radius 2.0 cm; the shell centers are separated by  $L = 10 \text{ cm}$ . In unit-vector notation, what is the net electric field at  $x = 2.0 \text{ cm}$ ?

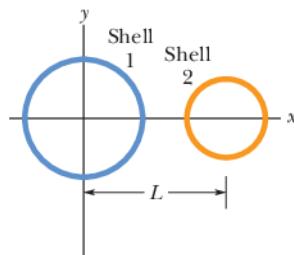


Figure 23-36 Problem 12.

**••13 SSM** The electric field in a certain region of Earth's atmosphere is directed vertically down. At an altitude of 300 m the field has magnitude 60.0 N/C; at an altitude of 200 m, the magnitude is 100 N/C. Find the net amount of charge contained in a cube 100 m on edge, with horizontal faces at altitudes of 200 and 300 m.

**••14 GO Flux and nonconducting shells.** A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure 23-37a shows a cross section. Figure 23-37b gives the net flux  $\Phi$  through a Gaussian sphere centered on the particle, as a function of the radius  $r$  of the sphere. The scale of the vertical axis is set by  $\Phi_s = 5.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$ . (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?

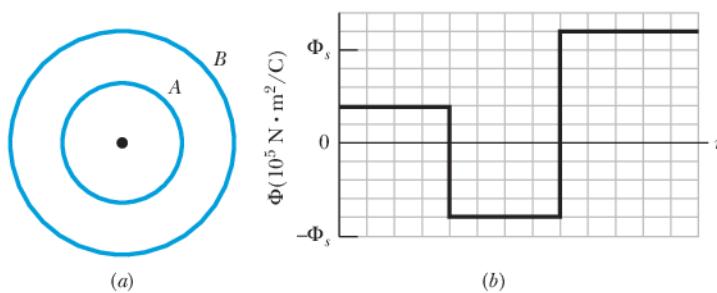


Figure 23-37 Problem 14.

**••15** A particle of charge  $+q$  is placed at one corner of a Gaussian cube. What multiple of  $q/\epsilon_0$  gives the flux through (a) each cube face forming that corner and (b) each of the other cube faces?

**••16 GO** The box-like Gaussian surface shown in Fig. 23-38 encloses a net charge of  $+24.0\epsilon_0 \text{ C}$  and lies in an electric field given by  $\vec{E} = [(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}] \text{ N/C}$ , with  $x$  and  $z$  in meters and  $b$  a constant. The bottom face is in the  $xz$  plane; the top face is in the horizontal plane passing through  $y_2 = 1.00 \text{ m}$ . For  $x_1 = 1.00 \text{ m}$ ,  $x_2 = 4.00 \text{ m}$ ,  $z_1 = 1.00 \text{ m}$ , and  $z_2 = 3.00 \text{ m}$ , what is  $b$ ?

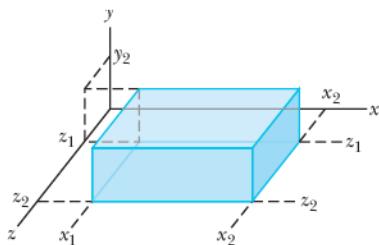


Figure 23-38 Problem 16.

### Module 23-3 A Charged Isolated Conductor

**•17 SSM** A uniformly charged conducting sphere of 1.2 m diameter has surface charge density  $8.1 \mu\text{C/m}^2$ . Find (a) the net charge on the sphere and (b) the total electric flux leaving the surface.

**•18** The electric field just above the surface of the charged conducting drum of a photocopying machine has a magnitude  $E$  of  $2.3 \times 10^5 \text{ N/C}$ . What is the surface charge density on the drum?

**•19** Space vehicles traveling through Earth's radiation belts can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metal satellite 1.3 m in diameter accumulates  $2.4 \mu\text{C}$  of charge in one orbital revolution. (a) Find the resulting surface charge density. (b) Calculate the magnitude of the electric field just outside the surface of the satellite, due to the surface charge.

**•20 GO Flux and conducting shells.** A charged particle is held at the center of two concentric conducting spherical shells. Figure 23-39a shows a cross section. Figure 23-39b gives the net flux  $\Phi$  through a Gaussian sphere centered on the particle, as a function of the radius  $r$  of the sphere. The scale of the vertical axis is set by  $\Phi_s = 5.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$ . What are (a) the charge of the central particle and the net charges of (b) shell A and (c) shell B?

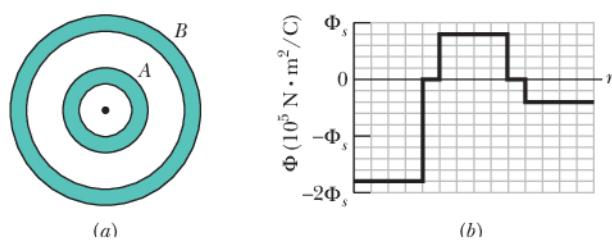


Figure 23-39 Problem 20.

**•21** An isolated conductor has net charge  $+10 \times 10^{-6} \text{ C}$  and a cavity with a particle of charge  $q = +3.0 \times 10^{-6} \text{ C}$ . What is the charge on (a) the cavity wall and (b) the outer surface?

### Module 23-4 Applying Gauss' Law: Cylindrical Symmetry

**•22** An electron is released 9.0 cm from a very long nonconducting rod with a uniform  $6.0 \mu\text{C/m}$ . What is the magnitude of the electron's initial acceleration?

**•23** (a) The drum of a photocopying machine has a length of 42 cm and a diameter of 12 cm. The electric field just above the drum's surface is  $2.3 \times 10^5 \text{ N/C}$ . What is the total charge on the drum? (b) The manufacturer wishes to produce a desktop version of the machine. This requires reducing the drum length to 28 cm and the diameter to 8.0 cm. The electric field at the drum surface must not change. What must be the charge on this new drum?

**•24** Figure 23-40 shows a section of a long, thin-walled metal tube of radius  $R = 3.00 \text{ cm}$ , with a charge per unit length of  $\lambda = 2.00 \times 10^{-8} \text{ C/m}$ . What is the magnitude  $E$  of the electric field at radial distance (a)  $r = R/2.00$  and (b)  $r = 2.00R$ ? (c) Graph  $E$  versus  $r$  for the range  $r = 0$  to  $2.00R$ .

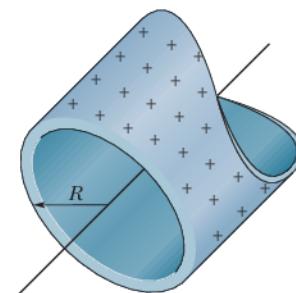


Figure 23-40 Problem 24.

**•25 SSM** An infinite line of charge produces a field of magnitude  $4.5 \times 10^4 \text{ N/C}$  at distance 2.0 m. Find the linear charge density.

**•26** Figure 23-41a shows a narrow charged solid cylinder that is coaxial with a larger charged cylindrical shell. Both are noncon-

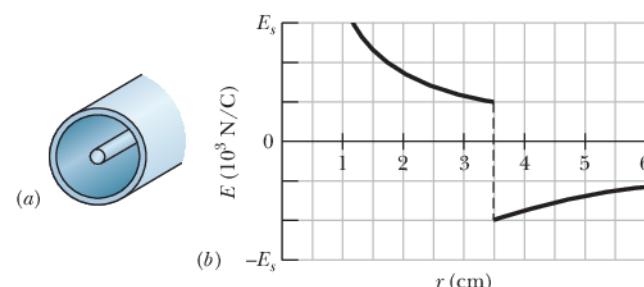


Figure 23-41 Problem 26.

ducting and thin and have uniform surface charge densities on their outer surfaces. Figure 23-41b gives the radial component  $E$  of the electric field versus radial distance  $r$  from the common axis, and  $E_s = 3.0 \times 10^3 \text{ N/C}$ . What is the shell's linear charge density?

**••27 GO** A long, straight wire has fixed negative charge with a linear charge density of magnitude  $3.6 \text{ nC/m}$ . The wire is to be enclosed by a coaxial, thin-walled nonconducting cylindrical shell of radius  $1.5 \text{ cm}$ . The shell is to have positive charge on its outside surface with a surface charge density  $\sigma$  that makes the net external electric field zero. Calculate  $\sigma$ .

**••28 GO** A charge of uniform linear density  $2.0 \text{ nC/m}$  is distributed along a long, thin, nonconducting rod. The rod is coaxial with a long conducting cylindrical shell (inner radius =  $5.0 \text{ cm}$ , outer radius =  $10 \text{ cm}$ ). The net charge on the shell is zero. (a) What is the magnitude of the electric field  $15 \text{ cm}$  from the axis of the shell? What is the surface charge density on the (b) inner and (c) outer surface of the shell?

**••29 SSM WWW** Figure 23-42 is a section of a conducting rod of radius  $R_1 = 1.30 \text{ mm}$  and length  $L = 11.00 \text{ m}$  inside a thin-walled coaxial conducting cylindrical shell of radius  $R_2 = 10.0R_1$  and the (same) length  $L$ . The net charge on the rod is  $Q_1 = +3.40 \times 10^{-12} \text{ C}$ ; that on the shell is  $Q_2 = -2.00Q_1$ . What are the (a) magnitude  $E$  and (b) direction (radially inward or outward) of the electric field at radial distance  $r = 2.00R_2$ ? What are (c)  $E$  and (d) the direction at  $r = 5.00R_1$ ? What is the charge on the (e) interior and (f) exterior surface of the shell?

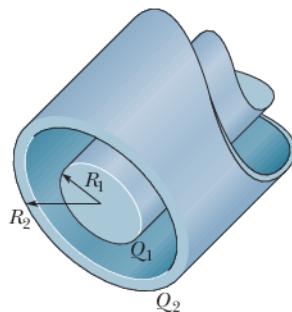


Figure 23-42 Problem 29.

**••30** In Fig. 23-43, short sections of two very long parallel lines of charge are shown, fixed in place, separated by  $L = 8.0 \text{ cm}$ . The uniform linear charge densities are  $+6.0 \mu\text{C/m}$  for line 1 and  $-2.0 \mu\text{C/m}$  for line 2. Where along the  $x$  axis shown is the net electric field from the two lines zero?

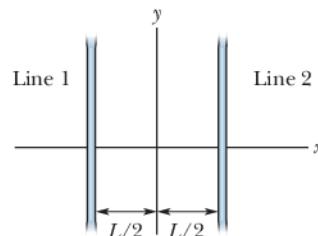
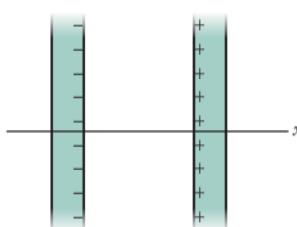


Figure 23-43 Problem 30.

**••31 ILW** Two long, charged, thin-walled, concentric cylindrical shells have radii of  $3.0$  and  $6.0 \text{ cm}$ . The charge per unit length is  $5.0 \times 10^{-6} \text{ C/m}$  on the inner shell and  $-7.0 \times 10^{-6} \text{ C/m}$  on the outer shell. What are the (a) magnitude  $E$  and (b) direction (radially inward or outward) of the electric field at radial distance  $r = 4.0 \text{ cm}$ ? What are (c)  $E$  and (d) the direction at  $r = 8.0 \text{ cm}$ ?

**••32 GO** A long, nonconducting, solid cylinder of radius  $4.0 \text{ cm}$  has a nonuniform volume charge density  $\rho$  that is a function of radial distance  $r$  from the cylinder axis:  $\rho = Ar^2$ . For  $A = 2.5 \mu\text{C/m}^3$ , what is the magnitude of the electric field at (a)  $r = 3.0 \text{ cm}$  and (b)  $r = 5.0 \text{ cm}$ ?



#### Module 23-5 Applying Gauss' Law: Planar Symmetry

**•33** In Fig. 23-44, two large, thin metal plates are parallel and close to each other. On their inner faces,

the plates have excess surface charge densities of opposite signs and magnitude  $7.00 \times 10^{-22} \text{ C/m}^2$ . In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them?

**•34** In Fig. 23-45, a small circular hole of radius  $R = 1.80 \text{ cm}$  has been cut in the middle of an infinite, flat, nonconducting surface that has uniform charge density  $\sigma = 4.50 \text{ pC/m}^2$ . A  $z$  axis, with its origin at the hole's center, is perpendicular to the surface. In unit-vector notation, what is the electric field at point  $P$  at  $z = 2.56 \text{ cm}$ ? (Hint: See Eq. 22-26 and use superposition.)

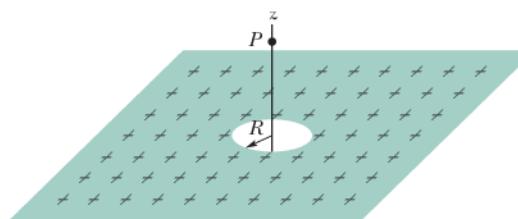


Figure 23-45 Problem 34.

**•35 GO** Figure 23-46a shows three plastic sheets that are large, parallel, and uniformly charged. Figure 23-46b gives the component of the net electric field along an  $x$  axis through the sheets. The scale of the vertical axis is set by  $E_s = 6.0 \times 10^5 \text{ N/C}$ . What is the ratio of the charge density on sheet 3 to that on sheet 2?

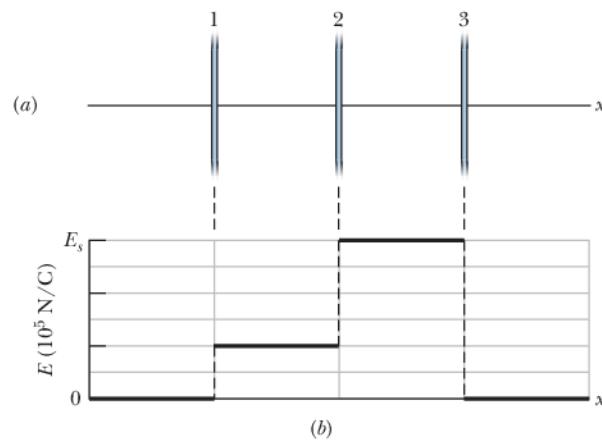


Figure 23-46 Problem 35.

**•36** Figure 23-47 shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density  $\sigma = 1.77 \times 10^{-22} \text{ C/m}^2$ . In unit-vector notation, what is  $\vec{E}$  at points (a) above the sheets, (b) between them, and (c) below them?

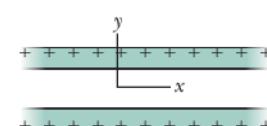


Figure 23-47  
Problem 36.

**•37 SSM WWW** A square metal plate of edge length  $8.0 \text{ cm}$  and negligible thickness has a total charge of  $6.0 \times 10^{-6} \text{ C}$ . (a) Estimate the magnitude  $E$  of the electric field just off the center of the plate (at, say, a distance of  $0.50 \text{ mm}$  from the center) by assuming that the charge is spread uniformly over the two faces of the plate. (b) Estimate  $E$  at a distance of  $30 \text{ m}$  (large relative to the plate size) by assuming that the plate is a charged particle.

- 38 GO** In Fig. 23-48a, an electron is shot directly away from a uniformly charged plastic sheet, at speed  $v_s = 2.0 \times 10^5 \text{ m/s}$ . The sheet is nonconducting, flat, and very large. Figure 23-48b gives the electron's vertical velocity component  $v$  versus time  $t$  until the return to the launch point. What is the sheet's surface charge density?

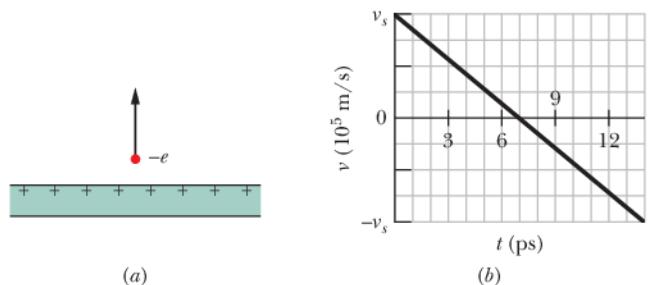


Figure 23-48 Problem 38.

- 39 SSM** In Fig. 23-49, a small, nonconducting ball of mass  $m = 1.0 \text{ mg}$  and charge  $q = 2.0 \times 10^{-8} \text{ C}$  (distributed uniformly through its volume) hangs from an insulating thread that makes an angle  $\theta = 30^\circ$  with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density  $\sigma$  of the sheet.

- 40** Figure 23-50 shows a very large nonconducting sheet that has a uniform surface charge density of  $\sigma = -2.00 \mu\text{C/m}^2$ ; it also shows a particle of charge  $Q = 6.00 \mu\text{C}$ , at distance  $d$  from the sheet.

Both are fixed in place. If  $d = 0.200 \text{ m}$ , at what (a) positive and (b) negative coordinate on the  $x$  axis (other than infinity) is the net electric field  $\vec{E}_{\text{net}}$  of the sheet and particle zero? (c) If  $d = 0.800 \text{ m}$ , at what coordinate on the  $x$  axis is  $\vec{E}_{\text{net}} = 0$ ?

- 41 GO** An electron is shot directly toward the center of a large metal plate that has surface charge density  $-2.0 \times 10^{-6} \text{ C/m}^2$ . If the initial kinetic energy of the electron is  $1.60 \times 10^{-17} \text{ J}$  and if the electron is to stop (due to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must the launch point be?

- 42** Two large metal plates of area  $1.0 \text{ m}^2$  face each other,  $5.0 \text{ cm}$  apart, with equal charge magnitudes  $|q|$  but opposite signs. The field magnitude  $E$  between them (neglect fringing) is  $55 \text{ N/C}$ . Find  $|q|$ .

- 43 GO** Figure 23-51 shows a cross section through a very large nonconducting slab of thickness  $d = 9.40 \text{ mm}$  and uniform volume charge density  $\rho = 5.80 \text{ fC/m}^3$ . The origin of an  $x$  axis is at the slab's center. What is the magnitude of the slab's electric field at an  $x$  coordinate of (a) 0, (b)  $2.00 \text{ mm}$ , (c)  $4.70 \text{ mm}$ , and (d)  $26.0 \text{ mm}$ ?

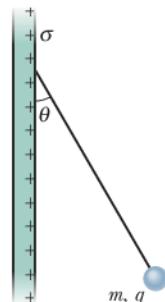
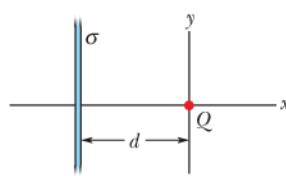
Figure 23-49  
Problem 39.

Figure 23-50 Problem 40.

### Module 23-6 Applying Gauss' Law: Spherical Symmetry

- 44** Figure 23-52 gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. The scale of the vertical axis is set by  $E_s = 5.0 \times 10^7 \text{ N/C}$ . What is the charge on the sphere?

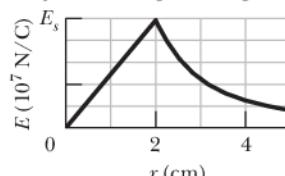


Figure 23-52 Problem 44.

- 45** Two charged concentric spherical shells have radii  $10.0 \text{ cm}$  and  $15.0 \text{ cm}$ . The charge on the inner shell is  $4.00 \times 10^{-8} \text{ C}$ , and that on the outer shell is  $2.00 \times 10^{-8} \text{ C}$ . Find the electric field (a) at  $r = 12.0 \text{ cm}$  and (b) at  $r = 20.0 \text{ cm}$ .

- 46** Assume that a ball of charged particles has a uniformly distributed negative charge density except for a narrow radial tunnel through its center, from the surface on one side to the surface on the opposite side. Also assume that we can position a proton anywhere along the tunnel or outside the ball. Let  $F_R$  be the magnitude of the electrostatic force on the proton when it is located at the ball's surface, at radius  $R$ . As a multiple of  $R$ , how far from the surface is there a point where the force magnitude is  $0.50F_R$  if we move the proton (a) away from the ball and (b) into the tunnel?

- 47 SSM** An unknown charge sits on a conducting solid sphere of radius  $10 \text{ cm}$ . If the electric field  $15 \text{ cm}$  from the center of the sphere has the magnitude  $3.0 \times 10^3 \text{ N/C}$  and is directed radially inward, what is the net charge on the sphere?

- 48 GO** A charged particle is held at the center of a spherical shell. Figure 23-53 gives the magnitude  $E$  of the electric field versus radial distance  $r$ . The scale of the vertical axis is set by  $E_s = 10.0 \times 10^7 \text{ N/C}$ . Approximately, what is the net charge on the shell?

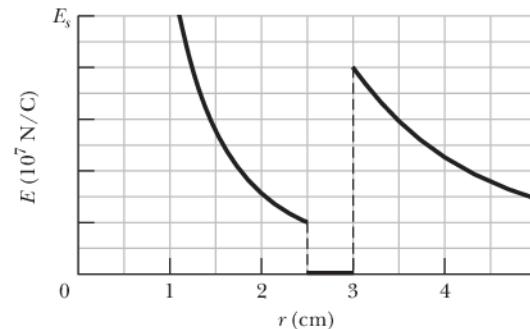


Figure 23-53 Problem 48.

- 49** In Fig. 23-54, a solid sphere of radius  $a = 2.00 \text{ cm}$  is concentric with a spherical conducting shell of inner radius  $b = 2.00a$  and outer radius  $c = 2.40a$ . The sphere has a net uniform charge  $q_1 = +5.00 \text{ fC}$ ; the shell has a net charge  $q_2 = -q_1$ . What is the magnitude of the electric field at radial distances (a)  $r = 0$ , (b)  $r = a/2.00$ , (c)  $r = a$ , (d)  $r = 1.50a$ , (e)  $r = 2.30a$ , and (f)  $r = 3.50a$ ? What is the net charge on the (g) inner and (h) outer surface of the shell?

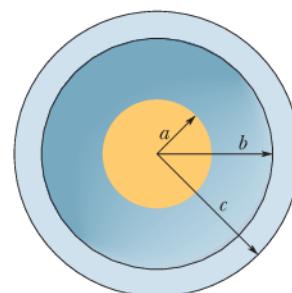


Figure 23-54 Problem 49.

- 50 GO** Figure 23-55 shows two nonconducting spherical shells fixed in place on an  $x$  axis. Shell 1 has uniform surface charge density  $+4.0 \mu\text{C}/\text{m}^2$  on its outer surface and radius 0.50 cm, and shell 2 has uniform surface charge density  $-2.0 \mu\text{C}/\text{m}^2$  on its outer surface and radius 2.0 cm; the centers are separated by  $L = 6.0$  cm. Other than at  $x = \infty$ , where on the  $x$  axis is the net electric field equal to zero?

- 51 SSM WWW** In Fig. 23-56, a nonconducting spherical shell of inner radius  $a = 2.00$  cm and outer radius  $b = 2.40$  cm has (within its thickness) a positive volume charge density  $\rho = A/r$ , where  $A$  is a constant and  $r$  is the distance from the center of the shell. In addition, a small ball of charge  $q = 45.0$  fC is located at that center. What value should  $A$  have if the electric field in the shell ( $a \leq r \leq b$ ) is to be uniform?

- 52 GO** Figure 23-57 shows a spherical shell with uniform volume charge density  $\rho = 1.84 \text{nC}/\text{m}^3$ , inner radius  $a = 10.0$  cm, and outer radius  $b = 2.00a$ . What is the magnitude of the electric field at radial distances (a)  $r = 0$ ; (b)  $r = a/2.00$ , (c)  $r = a$ , (d)  $r = 1.50a$ , (e)  $r = b$ , and (f)  $r = 3.00b$ ?

- 53 ILW** The volume charge density of a solid nonconducting sphere of radius  $R = 5.60$  cm varies with radial distance  $r$  as given by  $\rho = (14.1 \text{ pC}/\text{m}^3)r/R$ . (a) What is the sphere's total charge? What is the field magnitude  $E$  at (b)  $r = 0$ , (c)  $r = R/2.00$ , and (d)  $r = R$ ? (e) Graph  $E$  versus  $r$ .

- 54** Figure 23-58 shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius  $R$ . Point  $P$  lies on a line connecting the centers of the spheres, at radial distance  $R/2.00$  from the center of sphere 1. If the net electric field at point  $P$  is zero, what is the ratio  $q_2/q_1$  of the total charges?

- 55** A charge distribution that is spherically symmetric but not uniform radially produces an electric field of magnitude  $E = Kr^4$ , directed radially outward from the center of the sphere. Here  $r$  is the radial distance from that center, and  $K$  is a constant. What is the volume density  $\rho$  of the charge distribution?

#### Additional Problems

- 56** The electric field in a particular space is  $\vec{E} = (x + 2)\hat{i}$  N/C, with  $x$  in meters. Consider a cylindrical Gaussian surface of radius 20 cm that is coaxial with the  $x$  axis. One end of the cylinder is at  $x = 0$ . (a) What is the magnitude of the electric flux through the other end of the cylinder at  $x = 2.0$  m? (b) What net charge is enclosed within the cylinder?

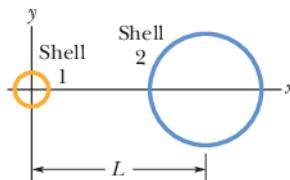


Figure 23-55 Problem 50.

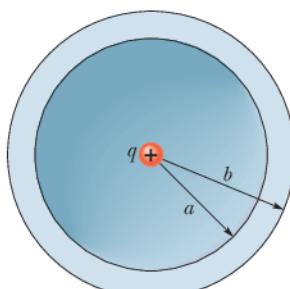


Figure 23-56 Problem 51.

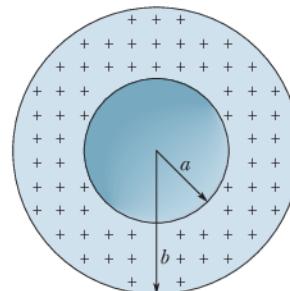


Figure 23-57 Problem 52.

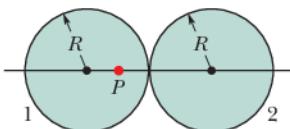


Figure 23-58 Problem 54.

- 57** A thin-walled metal spherical shell has radius 25.0 cm and charge  $2.00 \times 10^{-7}$  C. Find  $E$  for a point (a) inside the shell, (b) just outside it, and (c) 3.00 m from the center.

- 58** A uniform surface charge of density  $8.0 \text{nC}/\text{m}^2$  is distributed over the entire  $xy$  plane. What is the electric flux through a spherical Gaussian surface centered on the origin and having a radius of 5.0 cm?

- 59** Charge of uniform volume density  $\rho = 1.2 \text{nC}/\text{m}^3$  fills an infinite slab between  $x = -5.0$  cm and  $x = +5.0$  cm. What is the magnitude of the electric field at any point with the coordinate (a)  $x = 4.0$  cm and (b)  $x = 6.0$  cm?

- 60** *The chocolate crumb mystery.* Explosions ignited by electrostatic discharges (sparks) constitute a serious danger in facilities handling grain or powder. Such an explosion occurred in chocolate crumb powder at a biscuit factory in the 1970s. Workers usually emptied newly delivered sacks of the powder into a loading bin, from which it was blown through electrically grounded plastic pipes to a silo for storage. Somewhere along this route, two conditions for an explosion were met: (1) The magnitude of an electric field became  $3.0 \times 10^6$  N/C or greater, so that electrical breakdown and thus sparking could occur. (2) The energy of a spark was 150 mJ or greater so that it could ignite the powder explosively. Let us check for the first condition in the powder flow through the plastic pipes.

Suppose a stream of *negatively* charged powder was blown through a cylindrical pipe of radius  $R = 5.0$  cm. Assume that the powder and its charge were spread uniformly through the pipe with a volume charge density  $\rho$ . (a) Using Gauss' law, find an expression for the magnitude of the electric field  $\vec{E}$  in the pipe as a function of radial distance  $r$  from the pipe center. (b) Does  $E$  increase or decrease with increasing  $r$ ? (c) Is  $\vec{E}$  directed radially inward or outward? (d) For  $\rho = 1.1 \times 10^{-3} \text{ C}/\text{m}^3$  (a typical value at the factory), find the maximum  $E$  and determine where that maximum field occurs. (e) Could sparking occur, and if so, where? (The story continues with Problem 70 in Chapter 24.)

- 61 SSM** A thin-walled metal spherical shell of radius  $a$  has a charge  $q_a$ . Concentric with it is a thin-walled metal spherical shell of radius  $b > a$  and charge  $q_b$ . Find the electric field at points a distance  $r$  from the common center, where (a)  $r < a$ , (b)  $a < r < b$ , and (c)  $r > b$ . (d) Discuss the criterion you would use to determine how the charges are distributed on the inner and outer surfaces of the shells.

- 62** A particle of charge  $q = 1.0 \times 10^{-7}$  C is at the center of a spherical cavity of radius 3.0 cm in a chunk of metal. Find the electric field (a) 1.5 cm from the cavity center and (b) anywhere in the metal.

- 63** A proton at speed  $v = 3.00 \times 10^5$  m/s orbits at radius  $r = 1.00$  cm outside a charged sphere. Find the sphere's charge.

- 64** Equation 23-11 ( $E = \sigma/\epsilon_0$ ) gives the electric field at points near a charged conducting surface. Apply this equation to a conducting sphere of radius  $r$  and charge  $q$ , and show that the electric field outside the sphere is the same as the field of a charged particle located at the center of the sphere.

- 65** Charge  $Q$  is uniformly distributed in a sphere of radius  $R$ . (a) What fraction of the charge is contained within the radius  $r = R/2.00$ ? (b) What is the ratio of the electric field magnitude at  $r = R/2.00$  to that on the surface of the sphere?

- 66** A charged particle causes an electric flux of  $-750 \text{ N} \cdot \text{m}^2/\text{C}$  to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were

doubled, how much flux would pass through the surface? (b) What is the charge of the particle?

**67 SSM** The electric field at point  $P$  just outside the outer surface of a hollow spherical conductor of inner radius 10 cm and outer radius 20 cm has magnitude 450 N/C and is directed outward. When a particle of unknown charge  $Q$  is introduced into the center of the sphere, the electric field at  $P$  is still directed outward but is now 180 N/C. (a) What was the net charge enclosed by the outer surface before  $Q$  was introduced? (b) What is charge  $Q$ ? After  $Q$  is introduced, what is the charge on the (c) inner and (d) outer surface of the conductor?

**68** The net electric flux through each face of a die (singular of dice) has a magnitude in units of  $10^3 \text{ N} \cdot \text{m}^2/\text{C}$  that is exactly equal to the number of spots  $N$  on the face (1 through 6). The flux is inward for  $N$  odd and outward for  $N$  even. What is the net charge inside the die?

**69** Figure 23-59 shows, in cross section, three infinitely large nonconducting sheets on which charge is uniformly spread. The surface charge densities are  $\sigma_1 = +2.00 \mu\text{C}/\text{m}^2$ ,  $\sigma_2 = +4.00 \mu\text{C}/\text{m}^2$ , and  $\sigma_3 = -5.00 \mu\text{C}/\text{m}^2$ , and distance  $L = 1.50 \text{ cm}$ . In unit-vector notation, what is the net electric field at point  $P$ ?

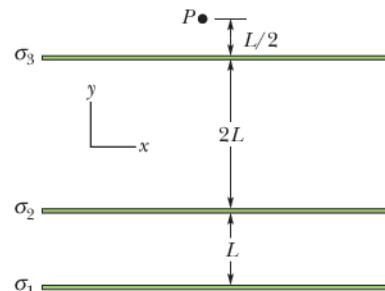


Figure 23-59 Problem 69.

**70** Charge of uniform volume density  $\rho = 3.2 \mu\text{C}/\text{m}^3$  fills a nonconducting solid sphere of radius 5.0 cm. What is the magnitude of the electric field (a) 3.5 cm and (b) 8.0 cm from the sphere's center?

**71** A Gaussian surface in the form of a hemisphere of radius  $R = 5.68 \text{ cm}$  lies in a uniform electric field of magnitude  $E = 2.50 \text{ N/C}$ . The surface encloses no net charge. At the (flat) base of the surface, the field is perpendicular to the surface and directed into the surface. What is the flux through (a) the base and (b) the curved portion of the surface?

**72** What net charge is enclosed by the Gaussian cube of Problem 2?

**73** A nonconducting solid sphere has a uniform volume charge density  $\rho$ . Let  $\vec{r}$  be the vector from the center of the sphere to a general point  $P$  within the sphere. (a) Show that the electric field at  $P$  is given by  $\vec{E} = \rho \vec{r} / 3\epsilon_0$ . (Note that the result is independent of the radius of the sphere.) (b) A spherical cavity is hollowed out of the sphere, as shown in Fig. 23-60. Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to  $\vec{E} = \rho \vec{a} / 3\epsilon_0$ , where  $\vec{a}$  is the position vector from the center of the sphere to the center of the cavity.

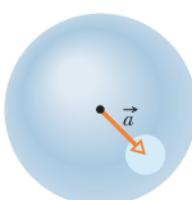


Figure 23-60  
Problem 73.

**74** A uniform charge density of  $500 \text{ nC/m}^3$  is distributed throughout a spherical volume of radius 6.00 cm. Consider a cubical Gaussian surface with its center at the center of the sphere. What is the electric flux through this cubical surface if its edge length is (a) 4.00 cm and (b) 14.0 cm?

**75** Figure 23-61 shows a Geiger counter, a device used to detect ionizing radiation, which causes ionization of atoms. A thin, posi-

tively charged central wire is surrounded by a concentric, circular, conducting cylindrical shell with an equal negative charge, creating a strong radial electric field. The shell contains a low-pressure inert gas. A particle of radiation entering the device through the shell wall ionizes a few of the gas atoms. The resulting free electrons ( $e^-$ ) are drawn to the positive wire. However, the electric field is so intense that, between collisions with gas atoms, the free electrons gain energy sufficient to ionize these atoms also. More free electrons are thereby created, and the process is repeated until the electrons reach the wire. The resulting "avalanche" of electrons is collected by the wire, generating a signal that is used to record the passage of the original particle of radiation. Suppose that the radius of the central wire is  $25 \mu\text{m}$ , the inner radius of the shell  $1.4 \text{ cm}$ , and the length of the shell  $16 \text{ cm}$ . If the electric field at the shell's inner wall is  $2.9 \times 10^4 \text{ N/C}$ , what is the total positive charge on the central wire?

**76** Charge is distributed uniformly throughout the volume of an infinitely long solid cylinder of radius  $R$ . (a) Show that, at a distance  $r < R$  from the cylinder axis,

$$E = \frac{\rho r}{2\epsilon_0},$$

where  $\rho$  is the volume charge density. (b) Write an expression for  $E$  when  $r > R$ .

**77 SSM** A spherical conducting shell has a charge of  $-14 \mu\text{C}$  on its outer surface and a charged particle in its hollow. If the net charge on the shell is  $-10 \mu\text{C}$ , what is the charge (a) on the inner surface of the shell and (b) of the particle?

**78** A charge of  $6.00 \text{ pC}$  is spread uniformly throughout the volume of a sphere of radius  $r = 4.00 \text{ cm}$ . What is the magnitude of the electric field at a radial distance of (a) 6.00 cm and (b) 3.00 cm?

**79** Water in an irrigation ditch of width  $w = 3.22 \text{ m}$  and depth  $d = 1.04 \text{ m}$  flows with a speed of  $0.207 \text{ m/s}$ . The mass flux of the flowing water through an imaginary surface is the product of the water's density ( $1000 \text{ kg/m}^3$ ) and its volume flux through that surface. Find the mass flux through the following imaginary surfaces: (a) a surface of area  $wd$ , entirely in the water, perpendicular to the flow; (b) a surface with area  $3wd/2$ , of which  $wd$  is in the water, perpendicular to the flow; (c) a surface of area  $wd/2$ , entirely in the water, perpendicular to the flow; (d) a surface of area  $wd$ , half in the water and half out, perpendicular to the flow; (e) a surface of area  $wd$ , entirely in the water, with its normal  $34.0^\circ$  from the direction of flow.

**80** Charge of uniform surface density  $8.00 \text{ nC/m}^2$  is distributed over an entire  $xy$  plane; charge of uniform surface density  $3.00 \text{ nC/m}^2$  is distributed over the parallel plane defined by  $z = 2.00 \text{ m}$ . Determine the magnitude of the electric field at any point having a  $z$  coordinate of (a) 1.00 m and (b) 3.00 m.

**81** A spherical ball of charged particles has a uniform charge density. In terms of the ball's radius  $R$ , at what radial distances (a) inside and (b) outside the ball is the magnitude of the ball's electric field equal to  $\frac{1}{4}$  of the maximum magnitude of that field?

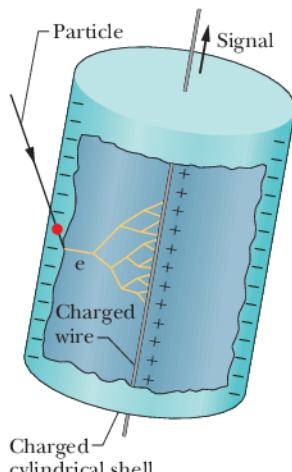


Figure 23-61 Problem 75.

# Electric Potential

## 24-1 ELECTRIC POTENTIAL

### Learning Objectives

After reading this module, you should be able to ...

- 24.01** Identify that the electric force is conservative and thus has an associated potential energy.
- 24.02** Identify that at every point in a charged object's electric field, the object sets up an electric potential  $V$ , which is a scalar quantity that can be positive or negative depending on the sign of the object's charge.
- 24.03** For a charged particle placed at a point in an object's electric field, apply the relationship between the object's electric potential  $V$  at that point, the particle's charge  $q$ , and the potential energy  $U$  of the particle-object system.
- 24.04** Convert energies between units of joules and electron-volts.
- 24.05** If a charged particle moves from an initial point to a final point in an electric field, apply the relationships

### Key Ideas

- The electric potential  $V$  at a point  $P$  in the electric field of a charged object is

$$V = \frac{-W_{\infty}}{q_0} = \frac{U}{q_0},$$

where  $W_{\infty}$  is the work that would be done by the electric force on a positive test charge  $q_0$  were it brought from an infinite distance to  $P$ , and  $U$  is the electric potential energy that would then be stored in the test charge-object system.

- If a particle with charge  $q$  is placed at a point where the electric potential of a charged object is  $V$ , the electric potential energy  $U$  of the particle-object system is

$$U = qV.$$

- If the particle moves through a potential difference  $\Delta V$ , the change in the electric potential energy is

between the change  $\Delta V$  in the potential, the particle's charge  $q$ , the change  $\Delta U$  in the potential energy, and the work  $W$  done by the electric force.

- 24.06** If a charged particle moves between two given points in the electric field of a charged object, identify that the amount of work done by the electric force is path independent.
- 24.07** If a charged particle moves through a change  $\Delta V$  in electric potential without an applied force acting on it, relate  $\Delta V$  and the change  $\Delta K$  in the particle's kinetic energy.
- 24.08** If a charged particle moves through a change  $\Delta V$  in electric potential while an applied force acts on it, relate  $\Delta V$ , the change  $\Delta K$  in the particle's kinetic energy, and the work  $W_{\text{app}}$  done by the applied force.

$$\Delta U = q \Delta V = q(V_f - V_i).$$

- If a particle moves through a change  $\Delta V$  in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

$$\Delta K = -q \Delta V.$$

- If, instead, an applied force acts on the particle, doing work  $W_{\text{app}}$ , the change in kinetic energy is

$$\Delta K = -q \Delta V + W_{\text{app}}.$$

- In the special case when  $\Delta K = 0$ , the work of an applied force involves only the motion of the particle through a potential difference:

$$W_{\text{app}} = q \Delta V.$$

### What Is Physics?

One goal of physics is to identify basic forces in our world, such as the electric force we discussed in Chapter 21. A related goal is to determine whether a force is conservative—that is, whether a potential energy can be associated with it. The motivation for associating a potential energy with a force is that we can then



**Figure 24-1** Particle 1 is located at point  $P$  in the electric field of particle 2.

apply the principle of the conservation of mechanical energy to closed systems involving the force. This extremely powerful principle allows us to calculate the results of experiments for which force calculations alone would be very difficult. Experimentally, physicists and engineers discovered that the electric force is conservative and thus has an associated electric potential energy. In this chapter we first define this type of potential energy and then put it to use.

For a quick taste, let's return to the situation we considered in Chapter 22: In Figure 24-1, particle 1 with positive charge  $q_1$  is located at point  $P$  near particle 2 with positive charge  $q_2$ . In Chapter 22 we explained how particle 2 is able to push on particle 1 without any contact. To account for the force  $\vec{F}$  (which is a vector quantity), we defined an electric field  $\vec{E}$  (also a vector quantity) that is set up at  $P$  by particle 2. That field exists regardless of whether particle 1 is at  $P$ . If we choose to place particle 1 there, the push on it is due to charge  $q_1$  and that pre-existing field  $\vec{E}$ .

Here is a related problem. If we release particle 1 at  $P$ , it begins to move and thus has kinetic energy. Energy cannot appear by magic, so from where does it come? It comes from the electric potential energy  $U$  associated with the force between the two particles in the arrangement of Fig. 24-1. To account for the potential energy  $U$  (which is a scalar quantity), we define an **electric potential**  $V$  (also a scalar quantity) that is set up at  $P$  by particle 2. The electric potential exists regardless of whether particle 1 is at  $P$ . If we choose to place particle 1 there, the potential energy of the two-particle system is then due to charge  $q_1$  and that pre-existing electric potential  $V$ .

Our goals in this chapter are to (1) define electric potential, (2) discuss how to calculate it for various arrangements of charged particles and objects, and (3) discuss how electric potential  $V$  is related to electric potential energy  $U$ .

## Electric Potential and Electric Potential Energy

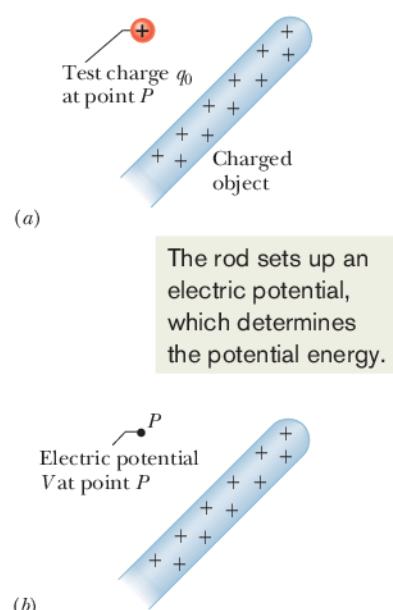
We are going to define the electric potential (or *potential* for short) in terms of electric potential energy, so our first job is to figure out how to measure that potential energy. Back in Chapter 8, we measured gravitational potential energy  $U$  of an object by (1) assigning  $U = 0$  for a reference configuration (such as the object at table level) and (2) then calculating the work  $W$  the gravitational force does if the object is moved up or down from that level. We then defined the potential energy as being

$$U = -W \quad (\text{potential energy}). \quad (24-1)$$

Let's follow the same procedure with our new conservative force, the electric force. In Fig. 24-2a, we want to find the potential energy  $U$  associated with a positive test charge  $q_0$  located at point  $P$  in the electric field of a charged rod. First, we need a reference configuration for which  $U = 0$ . A reasonable choice is for the test charge to be infinitely far from the rod, because then there is no interaction with the rod. Next, we bring the test charge in from infinity to point  $P$  to form the configuration of Fig. 24-2a. Along the way, we calculate the work done by the electric force on the test charge. The potential energy of the final configuration is then given by Eq. 24-1, where  $W$  is now the work done by the electric force. Let's use the notation  $W_\infty$  to emphasize that the test charge is brought in from infinity. The work and thus the potential energy can be positive or negative depending on the sign of the rod's charge.

Next, we define the electric potential  $V$  at  $P$  in terms of the work done by the electric force and the resulting potential energy:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0} \quad (\text{electric potential}). \quad (24-2)$$



**Figure 24-2** (a) A test charge has been brought in from infinity to point  $P$  in the electric field of the rod. (b) We define an electric potential  $V$  at  $P$  based on the potential energy of the configuration in (a).

That is, the electric potential is the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity. The rod sets up this potential  $V$  at  $P$  regardless of whether the test charge (or anything else) happens to be there (Fig. 24-2b). From Eq. 24-2 we see that  $V$  is a scalar quantity (because there is no direction associated with potential energy or charge) and can be positive or negative (because potential energy and charge have signs).

Repeating this procedure we find that an electric potential is set up at every point in the rod's electric field. In fact, every charged object sets up electric potential  $V$  at points throughout its electric field. If we happen to place a particle with, say, charge  $q$  at a point where we know the pre-existing  $V$ , we can immediately find the potential energy of the configuration:

$$\text{(electric potential energy)} = \text{(particle's charge)} \left( \frac{\text{electric potential energy}}{\text{unit charge}} \right),$$

or

$$U = qV, \quad (24-3)$$

where  $q$  can be positive or negative.

**Two Cautions.** (1) The (now very old) decision to call  $V$  a *potential* was unfortunate because the term is easily confused with *potential energy*. Yes, the two quantities are related (that is the point here) but they are very different and not interchangeable. (2) Electric potential is a scalar, not a vector. (When you come to the homework problems, you will rejoice on this point.)

**Language.** A potential energy is a property of a system (or configuration) of objects, but sometimes we can get away with assigning it to a single object. For example, the gravitational potential energy of a baseball hit to outfield is actually a potential energy of the baseball-Earth system (because it is associated with the force between the baseball and Earth). However, because only the baseball noticeably moves (its motion does not noticeably affect Earth), we might assign the gravitational potential energy to it alone. In a similar way, if a charged particle is placed in an electric field and has no noticeable effect on the field (or the charged object that sets up the field), we usually assign the electric potential energy to the particle alone.

**Units.** The SI unit for potential that follows from Eq. 24-2 is the joule per coulomb. This combination occurs so often that a special unit, the *volt* (abbreviated V), is used to represent it. Thus,

$$1 \text{ volt} = 1 \text{ joule per coulomb}.$$

With two unit conversions, we can now switch the unit for electric field from newtons per coulomb to a more conventional unit:

$$\begin{aligned} 1 \text{ N/C} &= \left( 1 \frac{\text{N}}{\text{C}} \right) \left( \frac{1 \text{ V}}{1 \text{ J/C}} \right) \left( \frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}} \right) \\ &= 1 \text{ V/m}. \end{aligned}$$

The conversion factor in the second set of parentheses comes from our definition of volt given above; that in the third set of parentheses is derived from the definition of the joule. From now on, we shall express values of the electric field in volts per meter rather than in newtons per coulomb.

### Motion Through an Electric Field

**Change in Electric Potential.** If we move from an initial point  $i$  to a second point  $f$  in the electric field of a charged object, the electric potential changes by

$$\Delta V = V_f - V_i$$

If we move a particle with charge  $q$  from  $i$  to  $f$ , then, from Eq. 24-3, the potential energy of the system changes by

$$\Delta U = q \Delta V = q(V_f - V_i). \quad (24-4)$$

The change can be positive or negative, depending on the signs of  $q$  and  $\Delta V$ . It can also be zero, if there is no change in potential from  $i$  to  $f$  (the points have the same value of potential). Because the electric force is conservative, the change in potential energy  $\Delta U$  between  $i$  and  $f$  is the same for all paths between those points (it is *path independent*).

**Work by the Field.** We can relate the potential energy change  $\Delta U$  to the work  $W$  done by the electric force as the particle moves from  $i$  to  $f$  by applying the general relation for a conservative force (Eq. 8-1):

$$W = -\Delta U \quad (\text{work, conservative force}). \quad (24-5)$$

Next, we can relate that work to the change in the potential by substituting from Eq. 24-4:

$$W = -\Delta U = -q \Delta V = -q(V_f - V_i). \quad (24-6)$$

Up until now, we have always attributed work to a force but here can also say that  $W$  is the work done on the particle by the electric field (because it, of course, produces the force). The work can be positive, negative, or zero. Because  $\Delta U$  between any two points is path independent, so is the work  $W$  done by the field. (If you need to calculate work for a difficult path, switch to an easier path—you get the same result.)

**Conservation of Energy.** If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved. Let's assume that we can assign the electric potential energy to the particle alone. Then we can write the conservation of mechanical energy of the particle that moves from point  $i$  to point  $f$  as

$$U_i + K_i = U_f + K_f, \quad (24-7)$$

$$\text{or} \quad \Delta K = -\Delta U. \quad (24-8)$$

Substituting Eq. 24-4, we find a very useful equation for the change in the particle's kinetic energy as a result of the particle moving through a potential difference:

$$\Delta K = -q \Delta V = -q(V_f - V_i). \quad (24-9)$$

**Work by an Applied Force.** If some force in addition to the electric force acts on the particle, we say that the additional force is an *applied force* or *external force*, which is often attributed to an *external agent*. Such an applied force can do work on the particle, but the force may not be conservative and thus, in general, we cannot associate a potential energy with it. We account for that work  $W_{\text{app}}$  by modifying Eq. 24-7:

$$\begin{aligned} &(\text{initial energy}) + (\text{work by applied force}) = (\text{final energy}) \\ \text{or} \quad &U_i + K_i + W_{\text{app}} = U_f + K_f. \end{aligned} \quad (24-10)$$

Rearranging and substituting from Eq. 24-4, we can also write this as

$$\Delta K = -\Delta U + W_{\text{app}} = -q \Delta V + W_{\text{app}}. \quad (24-11)$$

The work by the applied force can be positive, negative, or zero, and thus the energy of the system can increase, decrease, or remain the same.

In the special case where the particle is stationary before and after the move, the kinetic energy terms in Eqs. 24-10 and 24-11 are zero and we have

$$W_{\text{app}} = q \Delta V \quad (\text{for } K_i = K_f). \quad (24-12)$$

In this special case, the work  $W_{\text{app}}$  involves the motion of the particle through the potential difference  $\Delta V$  and not a change in the particle's kinetic energy.

By comparing Eqs. 24-6 and 24-12, we see that in this special case, the work by the applied force is the negative of the work by the field:

$$W_{\text{app}} = -W \quad (\text{for } K_i = K_f). \quad (24-13)$$

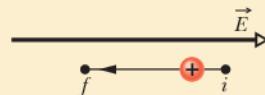
**Electron-volts.** In atomic and subatomic physics, energy measures in the SI unit of joules often require awkward powers of ten. A more convenient (but non-SI unit) is the *electron-volt* (eV), which is defined to be equal to the work required to move a single elementary charge  $e$  (such as that of an electron or proton) through a potential difference  $\Delta V$  of exactly one volt. From Eq. 24-6, we see that the magnitude of this work is  $q \Delta V$ . Thus,

$$\begin{aligned} 1 \text{ eV} &= e(1 \text{ V}) \\ &= (1.602 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.602 \times 10^{-19} \text{ J}. \end{aligned} \quad (24-14)$$



### Checkpoint 1

In the figure, we move a proton from point  $i$  to point  $f$  in a uniform electric field. Is positive or negative work done by (a) the electric field and (b) our force? (c) Does the electric potential energy increase or decrease? (d) Does the proton move to a point of higher or lower electric potential?



### Sample Problem 24.01 Work and potential energy in an electric field

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electric force  $\vec{F}$  due to the electric field  $\vec{E}$  that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude  $E = 150 \text{ N/C}$  and is directed downward. What is the change  $\Delta U$  in the electric potential energy of a released electron when the electric force causes it to move vertically upward through a distance  $d = 520 \text{ m}$  (Fig. 24-3)? Through what potential change does the electron move?

#### KEY IDEAS

- (1) The change  $\Delta U$  in the electric potential energy of the electron is related to the work  $W$  done on the electron by the electric field. Equation 24-5 ( $W = -\Delta U$ ) gives the relation.
- (2) The work done by a constant force  $\vec{F}$  on a particle undergoing a displacement  $\vec{d}$  is

$$W = \vec{F} \cdot \vec{d}.$$

- (3) The electric force and the electric field are related by the force equation  $\vec{F} = q\vec{E}$ , where here  $q$  is the charge of an electron ( $= -1.6 \times 10^{-19} \text{ C}$ ).

**Calculations:** Substituting the force equation into the work equation and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta,$$

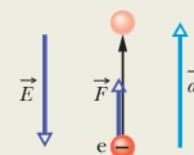


Figure 24-3 An electron in the atmosphere is moved upward through displacement  $\vec{d}$  by an electric force  $\vec{F}$  due to an electric field  $\vec{E}$ .

where  $\theta$  is the angle between the directions of  $\vec{E}$  and  $\vec{d}$ . The field  $\vec{E}$  is directed downward and the displacement  $\vec{d}$  is directed upward; so  $\theta = 180^\circ$ . We can now evaluate the work as

$$\begin{aligned} W &= (-1.6 \times 10^{-19} \text{ C})(150 \text{ N/C})(520 \text{ m}) \cos 180^\circ \\ &= 1.2 \times 10^{-14} \text{ J}. \end{aligned}$$

Equation 24-5 then yields

$$\Delta U = -W = -1.2 \times 10^{-14} \text{ J}. \quad (\text{Answer})$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by  $1.2 \times 10^{-14} \text{ J}$ . To find the change in electric potential, we apply Eq. 24-4:

$$\begin{aligned} \Delta V &= \frac{\Delta U}{-q} = \frac{-1.2 \times 10^{-14} \text{ J}}{-1.6 \times 10^{-19} \text{ C}} \\ &= 4.5 \times 10^4 \text{ V} = 45 \text{ kV}. \quad (\text{Answer}) \end{aligned}$$

This tells us that the electric force does work to move the electron to a *higher* potential.



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## 24-2 EQUIPOTENTIAL SURFACES AND THE ELECTRIC FIELD

### Learning Objectives

After reading this module, you should be able to...

- 24.09** Identify an equipotential surface and describe how it is related to the direction of the associated electric field.
- 24.10** Given an electric field as a function of position, calculate the change in potential  $\Delta V$  from an initial point to a final point by choosing a path between the points and integrating the dot product of the field  $\vec{E}$  and a length element  $d\vec{s}$  along the path.

### Key Ideas

- The points on an equipotential surface all have the same electric potential. The work done on a test charge in moving it from one such surface to another is independent of the locations of the initial and final points on these surfaces and of the path that joins the points. The electric field  $\vec{E}$  is always directed perpendicularly to corresponding equipotential surfaces.
- The electric potential difference between two points  $i$  and  $f$  is

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s},$$

where the integral is taken over any path connecting the points. If the integration is difficult along any particular path,

- 24.11** For a uniform electric field, relate the field magnitude  $E$  and the separation  $\Delta x$  and potential difference  $\Delta V$  between adjacent equipotential lines.
- 24.12** Given a graph of electric field  $E$  versus position along an axis, calculate the change in potential  $\Delta V$  from an initial point to a final point by graphical integration.
- 24.13** Explain the use of a zero-potential location.

we can choose a different path along which the integration might be easier.

- If we choose  $V_i = 0$ , we have, for the potential at a particular point,

$$V = - \int_i^f \vec{E} \cdot d\vec{s}.$$

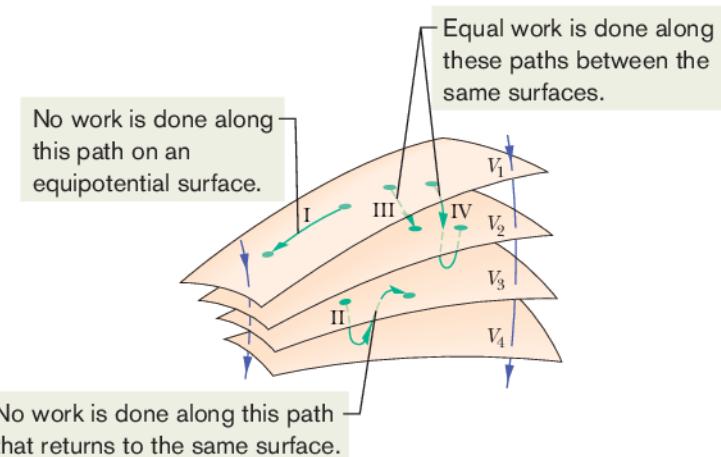
- In a uniform field of magnitude  $E$ , the change in potential from a higher equipotential surface to a lower one, separated by distance  $\Delta x$ , is

$$\Delta V = -E \Delta x.$$

### Equipotential Surfaces

Adjacent points that have the same electric potential form an **equipotential surface**, which can be either an imaginary surface or a real, physical surface. No net work  $W$  is done on a charged particle by an electric field when the particle moves between two points  $i$  and  $f$  on the same equipotential surface. This follows from Eq. 24-6, which tells us that  $W$  must be zero if  $V_f = V_i$ . Because of the path independence of work (and thus of potential energy and potential),  $W = 0$  for any path connecting points  $i$  and  $f$  on a given equipotential surface regardless of whether that path lies entirely on that surface.

Figure 24-4 shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. The work done by the electric field on a charged particle as the particle moves from one end to the other of paths



**Figure 24-4** Portions of four equipotential surfaces at electric potentials  $V_1 = 100$  V,  $V_2 = 80$  V,  $V_3 = 60$  V, and  $V_4 = 40$  V. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

I and II is zero because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential. The work done as the charged particle moves from one end to the other of paths III and IV is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths III and IV connect the same pair of equipotential surfaces.

From symmetry, the equipotential surfaces produced by a charged particle or a spherically symmetrical charge distribution are a family of concentric spheres. For a uniform electric field, the surfaces are a family of planes perpendicular to the field lines. In fact, equipotential surfaces are always perpendicular to electric field lines and thus to  $\vec{E}$ , which is always tangent to these lines. If  $\vec{E}$  were not perpendicular to an equipotential surface, it would have a component lying along that surface. This component would then do work on a charged particle as it moved along the surface. However, by Eq. 24-6 work cannot be done if the surface is truly an equipotential surface; the only possible conclusion is that  $\vec{E}$  must be everywhere perpendicular to the surface. Figure 24-5 shows electric field lines and cross sections of the equipotential surfaces for a uniform electric field and for the field associated with a charged particle and with an electric dipole.

## Calculating the Potential from the Field

We can calculate the potential difference between any two points  $i$  and  $f$  in an electric field if we know the electric field vector  $\vec{E}$  all along any path connecting those points. To make the calculation, we find the work done on a positive test charge by the field as the charge moves from  $i$  to  $f$ , and then use Eq. 24-6.

Consider an arbitrary electric field, represented by the field lines in Fig. 24-6, and a positive test charge  $q_0$  that moves along the path shown from point  $i$  to point  $f$ . At any point on the path, an electric force  $q_0\vec{E}$  acts on the charge as it moves through a differential displacement  $d\vec{s}$ . From Chapter 7, we know that the differential work  $dW$  done on a particle by a force  $\vec{F}$  during a displacement  $d\vec{s}$  is given by the dot product of the force and the displacement:

$$dW = \vec{F} \cdot d\vec{s}. \quad (24-15)$$

For the situation of Fig. 24-6,  $\vec{F} = q_0\vec{E}$  and Eq. 24-15 becomes

$$dW = q_0\vec{E} \cdot d\vec{s}. \quad (24-16)$$

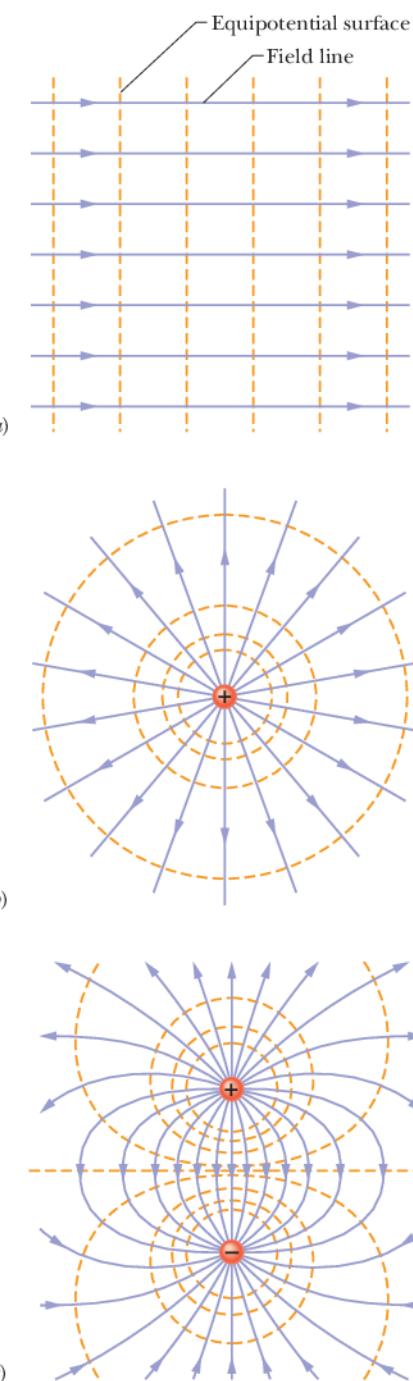
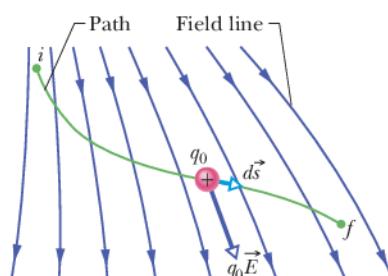
To find the total work  $W$  done on the particle by the field as the particle moves from point  $i$  to point  $f$ , we sum—via integration—the differential works done on the charge as it moves through all the displacements  $d\vec{s}$  along the path:

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-17)$$

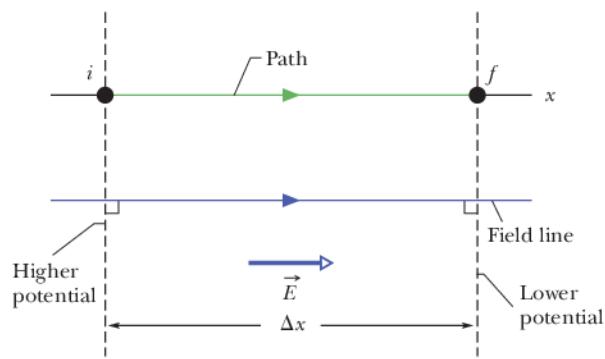
If we substitute the total work  $W$  from Eq. 24-17 into Eq. 24-6, we find

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-18)$$

**Figure 24-6** A test charge  $q_0$  moves from point  $i$  to point  $f$  along the path shown in a nonuniform electric field. During a displacement  $d\vec{s}$ , an electric force  $q_0\vec{E}$  acts on the test charge. This force points in the direction of the field line at the location of the test charge.



**Figure 24-5** Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (a) a uniform electric field, (b) the field due to a charged particle, and (c) the field due to an electric dipole.



**Figure 24-7** We move between points  $i$  and  $f$ , between adjacent equipotential lines in a uniform electric field  $\vec{E}$ , parallel to a field line.

Thus, the potential difference  $V_f - V_i$  between any two points  $i$  and  $f$  in an electric field is equal to the negative of the *line integral* (meaning the integral along a particular path) of  $\vec{E} \cdot d\vec{s}$  from  $i$  to  $f$ . However, because the electric force is conservative, all paths (whether easy or difficult to use) yield the same result.

Equation 24-18 allows us to calculate the difference in potential between any two points in the field. If we set potential  $V_i = 0$ , then Eq. 24-18 becomes

$$V = - \int_i^f \vec{E} \cdot d\vec{s}, \quad (24-19)$$

in which we have dropped the subscript  $f$  on  $V_f$ . Equation 24-19 gives us the potential  $V$  at any point  $f$  in the electric field *relative to the zero potential* at point  $i$ . If we let point  $i$  be at infinity, then Eq. 24-19 gives us the potential  $V$  at any point  $f$  relative to the zero potential at infinity.

**Uniform Field.** Let's apply Eq. 24-18 for a uniform field as shown in Fig. 24-7. We start at point  $i$  on an equipotential line with potential  $V_i$  and move to point  $f$  on an equipotential line with a lower potential  $V_f$ . The separation between the two equipotential lines is  $\Delta x$ . Let's also move along a path that is parallel to the electric field  $\vec{E}$  (and thus perpendicular to the equipotential lines). The angle between  $\vec{E}$  and  $d\vec{s}$  in Eq. 24-18 is zero, and the dot product gives us

$$\vec{E} \cdot d\vec{s} = E ds \cos 0 = E ds.$$

Because  $E$  is constant for a uniform field, Eq. 24-18 becomes

$$V_f - V_i = -E \int_i^f ds. \quad (24-20)$$

The integral is simply an instruction for us to add all the displacement elements  $ds$  from  $i$  to  $f$ , but we already know that the sum is length  $\Delta x$ . Thus we can write the change in potential  $V_f - V_i$  in this uniform field as

$$\Delta V = -E \Delta x \quad (\text{uniform field}). \quad (24-21)$$

This is the change in voltage  $\Delta V$  between two equipotential lines in a uniform field of magnitude  $E$ , separated by distance  $\Delta x$ . If we move in the direction of the field by distance  $\Delta x$ , the potential decreases. In the opposite direction, it increases.

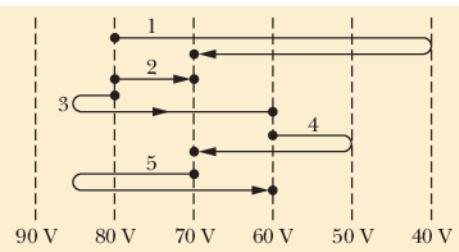


The electric field vector points from higher potential toward lower potential.



### Checkpoint 2

The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.





### Sample Problem 24.02 Finding the potential change from the electric field

(a) Figure 24-8a shows two points  $i$  and  $f$  in a uniform electric field  $\vec{E}$ . The points lie on the same electric field line (not shown) and are separated by a distance  $d$ . Find the potential difference  $V_f - V_i$  by moving a positive test charge  $q_0$  from  $i$  to  $f$  along the path shown, which is parallel to the field direction.

#### KEY IDEA

We can find the potential difference between any two points in an electric field by integrating  $\vec{E} \cdot d\vec{s}$  along a path connecting those two points according to Eq. 24-18.

**Calculations:** We have actually already done the calculation for such a path in the direction of an electric field line in a uniform field when we derived Eq. 24-21. With slight changes in notation, Eq. 24-21 gives us

$$V_f - V_i = -Ed. \quad (\text{Answer})$$

(b) Now find the potential difference  $V_f - V_i$  by moving the positive test charge  $q_0$  from  $i$  to  $f$  along the path  $icf$  shown in Fig. 24-8b.

**Calculations:** The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines:  $ic$  and  $cf$ . At all points along line  $ic$ , the displace-

ment  $d\vec{s}$  of the test charge is perpendicular to  $\vec{E}$ . Thus, the angle  $\theta$  between  $\vec{E}$  and  $d\vec{s}$  is  $90^\circ$ , and the dot product  $\vec{E} \cdot d\vec{s}$  is 0. Equation 24-18 then tells us that points  $i$  and  $c$  are at the same potential:  $V_c - V_i = 0$ . Ah, we should have seen this coming. The points are on the same equipotential surface, which is perpendicular to the electric field lines.

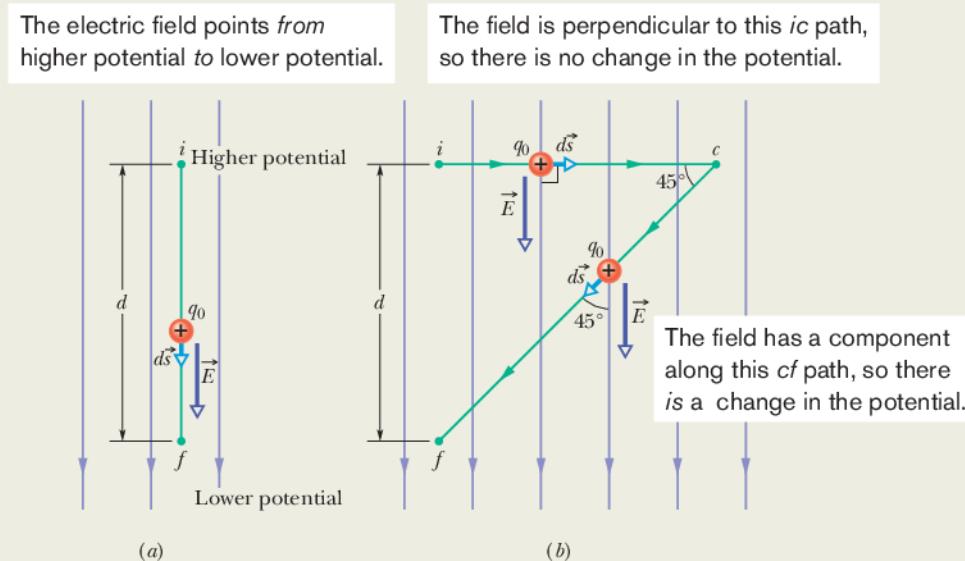
For line  $cf$  we have  $\theta = 45^\circ$  and, from Eq. 24-18,

$$\begin{aligned} V_f - V_i &= - \int_c^f \vec{E} \cdot d\vec{s} = - \int_c^f E(\cos 45^\circ) ds \\ &= -E(\cos 45^\circ) \int_c^f ds. \end{aligned}$$

The integral in this equation is just the length of line  $cf$ ; from Fig. 24-8b, that length is  $d/\cos 45^\circ$ . Thus,

$$V_f - V_i = -E(\cos 45^\circ) \frac{d}{\cos 45^\circ} = -Ed. \quad (\text{Answer})$$

This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them. Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a path that simplifies the use of Eq. 24-18.



**Figure 24-8** (a) A test charge  $q_0$  moves in a straight line from point  $i$  to point  $f$ , along the direction of a uniform external electric field. (b) Charge  $q_0$  moves along path  $icf$  in the same electric field.



## 24-3 POTENTIAL DUE TO A CHARGED PARTICLE

### Learning Objectives

After reading this module, you should be able to...

- 24.14** For a given point in the electric field of a charged particle, apply the relationship between the electric potential  $V$ , the charge of the particle  $q$ , and the distance  $r$  from the particle.
- 24.15** Identify the correlation between the algebraic signs of the potential set up by a particle and the charge of the particle.
- 24.16** For points outside or on the surface of a spherically

symmetric charge distribution, calculate the electric potential as if all the charge is concentrated as a particle at the center of the sphere.

- 24.17** Calculate the net potential at any given point due to several charged particles, identifying that algebraic addition is used, not vector addition.

- 24.18** Draw equipotential lines for a charged particle.

### Key Ideas

- The electric potential due to a single charged particle at a distance  $r$  from that charged particle is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r},$$

where  $V$  has the same sign as  $q$ .

- The potential due to a collection of charged particles is

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}.$$

Thus, the potential is the algebraic sum of the individual potentials, with no consideration of directions.

### Potential Due to a Charged Particle

We now use Eq. 24-18 to derive, for the space around a charged particle, an expression for the electric potential  $V$  relative to the zero potential at infinity. Consider a point  $P$  at distance  $R$  from a fixed particle of positive charge  $q$  (Fig. 24-9). To use Eq. 24-18, we imagine that we move a positive test charge  $q_0$  from point  $P$  to infinity. Because the path we take does not matter, let us choose the simplest one—a line that extends radially from the fixed particle through  $P$  to infinity.

To use Eq. 24-18, we must evaluate the dot product

$$\vec{E} \cdot d\vec{s} = E \cos \theta ds. \quad (24-22)$$

The electric field  $\vec{E}$  in Fig. 24-9 is directed radially outward from the fixed particle. Thus, the differential displacement  $d\vec{s}$  of the test particle along its path has the same direction as  $\vec{E}$ . That means that in Eq. 24-22, angle  $\theta = 0$  and  $\cos \theta = 1$ . Because the path is radial, let us write  $ds$  as  $dr$ . Then, substituting the limits  $R$  and  $\infty$ , we can write Eq. 24-18 as

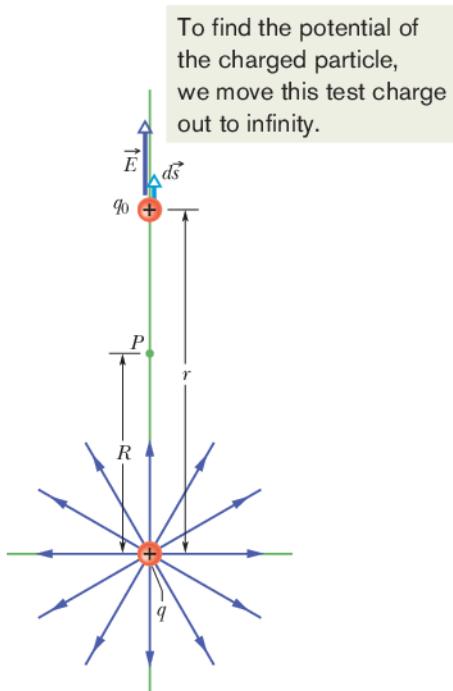
$$V_f - V_i = - \int_R^\infty E dr. \quad (24-23)$$

Next, we set  $V_f = 0$  (at  $\infty$ ) and  $V_i = V$  (at  $R$ ). Then, for the magnitude of the electric field at the site of the test charge, we substitute from Eq. 22-3:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}. \quad (24-24)$$

With these changes, Eq. 24-23 then gives us

$$\begin{aligned} 0 - V &= - \frac{q}{4\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} dr = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r} \right]_R^\infty \\ &= - \frac{1}{4\pi\epsilon_0} \frac{q}{R}. \end{aligned} \quad (24-25)$$



**Figure 24-9** The particle with positive charge  $q$  produces an electric field  $\vec{E}$  and an electric potential  $V$  at point  $P$ . We find the potential by moving a test charge  $q_0$  from  $P$  to infinity. The test charge is shown at distance  $r$  from the particle, during differential displacement  $d\vec{s}$ .

Solving for  $V$  and switching  $R$  to  $r$ , we then have

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (24-26)$$

as the electric potential  $V$  due to a particle of charge  $q$  at any radial distance  $r$  from the particle.

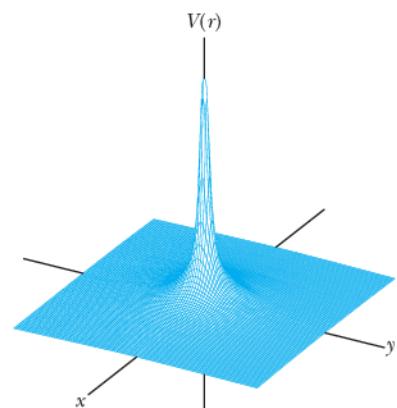
Although we have derived Eq. 24-26 for a positively charged particle, the derivation holds also for a negatively charged particle, in which case,  $q$  is a negative quantity. Note that the sign of  $V$  is the same as the sign of  $q$ :



A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Figure 24-10 shows a computer-generated plot of Eq. 24-26 for a positively charged particle; the magnitude of  $V$  is plotted vertically. Note that the magnitude increases as  $r \rightarrow 0$ . In fact, according to Eq. 24-26,  $V$  is infinite at  $r = 0$ , although Fig. 24-10 shows a finite, smoothed-off value there.

Equation 24-26 also gives the electric potential either *outside or on the external surface of* a spherically symmetric charge distribution. We can prove this by using one of the shell theorems of Modules 21-1 and 23-6 to replace the actual spherical charge distribution with an equal charge concentrated at its center. Then the derivation leading to Eq. 24-26 follows, provided we do not consider a point within the actual distribution.



**Figure 24-10** A computer-generated plot of the electric potential  $V(r)$  due to a positively charged particle located at the origin of an  $xy$  plane. The potentials at points in the  $xy$  plane are plotted vertically. (Curved lines have been added to help you visualize the plot.) The infinite value of  $V$  predicted by Eq. 24-26 for  $r = 0$  is not plotted.

## Potential Due to a Group of Charged Particles

We can find the net electric potential at a point due to a group of charged particles with the help of the superposition principle. Using Eq. 24-26 with the plus or minus sign of the charge included, we calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. Thus, for  $n$  charges, the net potential is

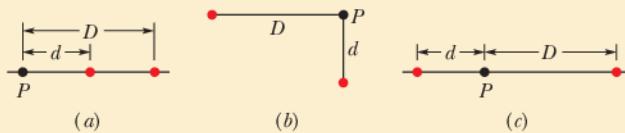
$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i} \quad (n \text{ charged particles}). \quad (24-27)$$

Here  $q_i$  is the value of the  $i$ th charge and  $r_i$  is the radial distance of the given point from the  $i$ th charge. The sum in Eq. 24-27 is an *algebraic sum*, not a vector sum like the sum that would be used to calculate the electric field resulting from a group of charged particles. Herein lies an important computational advantage of potential over electric field: It is a lot easier to sum several scalar quantities than to sum several vector quantities whose directions and components must be considered.



### Checkpoint 3

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point  $P$  by the protons, greatest first.





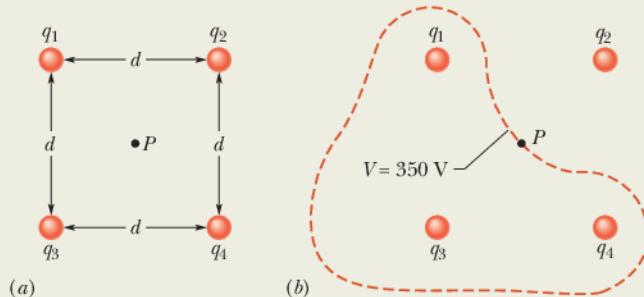
### Sample Problem 24.03 Net potential of several charged particles

What is the electric potential at point  $P$ , located at the center of the square of charged particles shown in Fig. 24-11a? The distance  $d$  is 1.3 m, and the charges are

$$\begin{aligned} q_1 &= +12 \text{ nC}, & q_3 &= +31 \text{ nC}, \\ q_2 &= -24 \text{ nC}, & q_4 &= +17 \text{ nC}. \end{aligned}$$

#### KEY IDEA

The electric potential  $V$  at point  $P$  is the algebraic sum of the electric potentials contributed by the four particles.



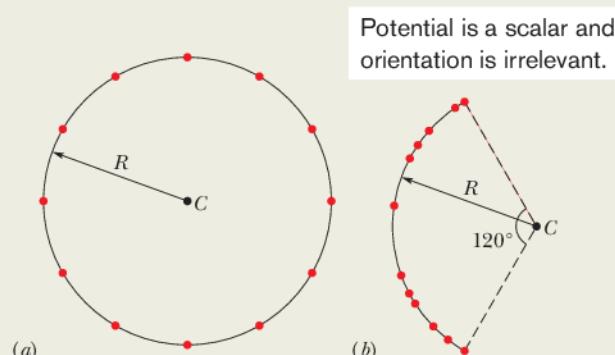
**Figure 24-11** (a) Four charged particles. (b) The closed curve is a (roughly drawn) cross section of the equipotential surface that contains point  $P$ .

### Sample Problem 24.04 Potential is not a vector, orientation is irrelevant

(a) In Fig. 24-12a, 12 electrons (of charge  $-e$ ) are equally spaced and fixed around a circle of radius  $R$ . Relative to  $V = 0$  at infinity, what are the electric potential and electric field at the center  $C$  of the circle due to these electrons?

#### KEY IDEAS

(1) The electric potential  $V$  at  $C$  is the algebraic sum of the electric potentials contributed by all the electrons. Because



**Figure 24-12** (a) Twelve electrons uniformly spaced around a circle. (b) The electrons nonuniformly spaced along an arc of the original circle.

(Because electric potential is a scalar, the orientations of the particles do not matter.)

**Calculations:** From Eq. 24-27, we have

$$V = \sum_{i=1}^4 V_i = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r} \right).$$

The distance  $r$  is  $d/\sqrt{2}$ , which is 0.919 m, and the sum of the charges is

$$\begin{aligned} q_1 + q_2 + q_3 + q_4 &= (12 - 24 + 31 + 17) \times 10^{-9} \text{ C} \\ &= 36 \times 10^{-9} \text{ C}. \end{aligned}$$

$$\begin{aligned} \text{Thus, } V &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}} \\ &\approx 350 \text{ V}. \end{aligned} \quad (\text{Answer})$$

Close to any of the three positively charged particles in Fig. 24-11a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point  $P$ . The curve in Fig. 24-11b shows the intersection of the plane of the figure with the equipotential surface that contains point  $P$ .

electric potential is a scalar, the orientations of the electrons do not matter. (2) The electric field at  $C$  is a vector quantity and thus the orientation of the electrons is important.

**Calculations:** Because the electrons all have the same negative charge  $-e$  and are all the same distance  $R$  from  $C$ , Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\epsilon_0} \frac{e}{R}. \quad (\text{Answer}) \quad (24-28)$$

Because of the symmetry of the arrangement in Fig. 24-12a, the electric field vector at  $C$  due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at  $C$ ,

$$\vec{E} = 0. \quad (\text{Answer})$$

(b) The electrons are moved along the circle until they are nonuniformly spaced over a  $120^\circ$  arc (Fig. 24-12b). At  $C$ , find the electric potential and describe the electric field.

**Reasoning:** The potential is still given by Eq. 24-28, because the distance between  $C$  and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.



Additional examples, video, and practice available at WileyPLUS

## 24-4 POTENTIAL DUE TO AN ELECTRIC DIPOLE

### Learning Objectives

After reading this module, you should be able to . . .

**24.19** Calculate the potential  $V$  at any given point due to an electric dipole, in terms of the magnitude  $p$  of the dipole moment or the product of the charge separation  $d$  and the magnitude  $q$  of either charge.

**24.20** For an electric dipole, identify the locations of positive potential, negative potential, and zero potential.

**24.21** Compare the decrease in potential with increasing distance for a single charged particle and an electric dipole.

### Key Idea

- At a distance  $r$  from an electric dipole with dipole moment magnitude  $p = qd$ , the electric potential of the dipole is

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

for  $r \gg d$ ; the angle  $\theta$  lies between the dipole moment vector and a line extending from the dipole midpoint to the point of measurement.

### Potential Due to an Electric Dipole

Now let us apply Eq. 24-27 to an electric dipole to find the potential at an arbitrary point  $P$  in Fig. 24-13a. At  $P$ , the positively charged particle (at distance  $r_{(+)}$ ) sets up potential  $V_{(+)}$  and the negatively charged particle (at distance  $r_{(-)}$ ) sets up potential  $V_{(-)}$ . Then the net potential at  $P$  is given by Eq. 24-27 as

$$\begin{aligned} V &= \sum_{i=1}^2 V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\epsilon_0} \left( \frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}} \right) \\ &= \frac{q}{4\pi\epsilon_0} \frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}. \end{aligned} \quad (24-29)$$

Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that  $r \gg d$ , where  $d$  is the distance between the charges and  $r$  is the distance from the dipole's midpoint to  $P$ . In that case, we can approximate the two lines to  $P$  as being parallel and their length difference as being the leg of a right triangle with hypotenuse  $d$  (Fig. 24-13b). Also, that difference is so small that the product of the lengths is approximately  $r^2$ . Thus,

$$r_{(-)} - r_{(+)} \approx d \cos \theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

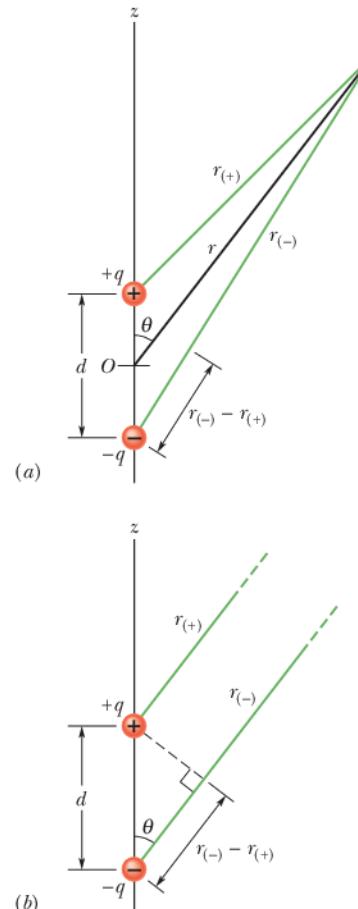
If we substitute these quantities into Eq. 24-29, we can approximate  $V$  to be

$$V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2},$$

where  $\theta$  is measured from the dipole axis as shown in Fig. 24-13a. We can now write  $V$  as

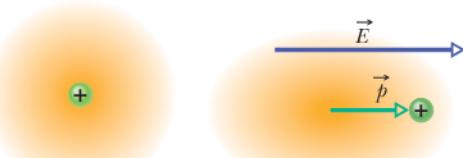
$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} \quad (\text{electric dipole}), \quad (24-30)$$

in which  $p (= qd)$  is the magnitude of the electric dipole moment  $\vec{p}$  defined in Module 22-3. The vector  $\vec{p}$  is directed along the dipole axis, from the negative to the positive charge. (Thus,  $\theta$  is measured from the direction of  $\vec{p}$ .) We use this vector to report the orientation of an electric dipole.



**Figure 24-13** (a) Point  $P$  is a distance  $r$  from the midpoint  $O$  of a dipole. The line  $OP$  makes an angle  $\theta$  with the dipole axis. (b) If  $P$  is far from the dipole, the lines of lengths  $r_{(+)}$  and  $r_{(-)}$  are approximately parallel to the line of length  $r$ , and the dashed black line is approximately perpendicular to the line of length  $r_{(-)}$ .

The electric field shifts the positive and negative charges, creating a dipole.



**Figure 24-14** (a) An atom, showing the positively charged nucleus (green) and the negatively charged electrons (gold shading). The centers of positive and negative charge coincide. (b) If the atom is placed in an external electric field  $\vec{E}$ , the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole moment  $\vec{p}$  appears. The distortion is greatly exaggerated here.

### Checkpoint 4

Suppose that three points are set at equal (large) distances  $r$  from the center of the dipole in Fig. 24-13: Point  $a$  is on the dipole axis above the positive charge, point  $b$  is on the axis below the negative charge, and point  $c$  is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.

### Induced Dipole Moment

Many molecules, such as water, have *permanent* electric dipole moments. In other molecules (called *nonpolar molecules*) and in every isolated atom, the centers of the positive and negative charges coincide (Fig. 24-14a) and thus no dipole moment is set up. However, if we place an atom or a nonpolar molecule in an external electric field, the field distorts the electron orbits and separates the centers of positive and negative charge (Fig. 24-14b). Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment  $\vec{p}$  that points in the direction of the field. This dipole moment is said to be *induced* by the field, and the atom or molecule is then said to be *polarized* by the field (that is, it has a positive side and a negative side). When the field is removed, the induced dipole moment and the polarization disappear.

## 24-5 POTENTIAL DUE TO A CONTINUOUS CHARGE DISTRIBUTION

### Learning Objective

After reading this module, you should be able to ...

**24.22** For charge that is distributed uniformly along a line or over a surface, find the net potential at a given point by splitting the distribution up into charge elements and summing (by integration) the potential due to each one.

### Key Ideas

- For a continuous distribution of charge (over an extended object), the potential is found by (1) dividing the distribution into charge elements  $dq$  that can be treated as particles and then (2) summing the potential due to each element by integrating over the full distribution:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}.$$

- In order to carry out the integration,  $dq$  is replaced with the product of either a linear charge density  $\lambda$  and a length element (such as  $dx$ ), or a surface charge density  $\sigma$  and area element (such as  $dx dy$ ).
- In some cases where the charge is symmetrically distributed, a two-dimensional integration can be reduced to a one-dimensional integration.

### Potential Due to a Continuous Charge Distribution

When a charge distribution  $q$  is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation of Eq. 24-27 to find the potential  $V$  at a point  $P$ . Instead, we must choose a differential element of charge  $dq$ , determine the potential  $dV$  at  $P$  due to  $dq$ , and then integrate over the entire charge distribution.

Let us again take the zero of potential to be at infinity. If we treat the element of charge  $dq$  as a particle, then we can use Eq. 24-26 to express the potential  $dV$  at point  $P$  due to  $dq$ :

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \quad (\text{positive or negative } dq). \quad (24-31)$$

Here  $r$  is the distance between  $P$  and  $dq$ . To find the total potential  $V$  at  $P$ , we

integrate to sum the potentials due to all the charge elements:

$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}. \quad (24-32)$$

The integral must be taken over the entire charge distribution. Note that because the electric potential is a scalar, there are *no vector components* to consider in Eq. 24-32.

We now examine two continuous charge distributions, a line and a disk.

### Line of Charge

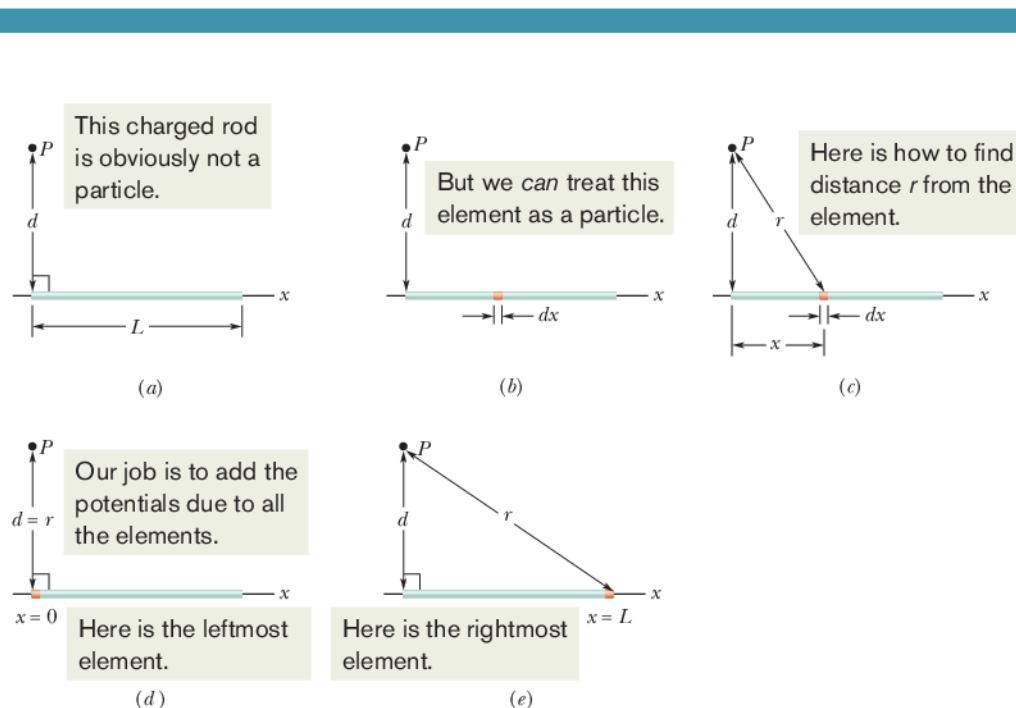
In Fig. 24-15a, a thin nonconducting rod of length  $L$  has a positive charge of uniform linear density  $\lambda$ . Let us determine the electric potential  $V$  due to the rod at point  $P$ , a perpendicular distance  $d$  from the left end of the rod.

We consider a differential element  $dx$  of the rod as shown in Fig. 24-15b. This (or any other) element of the rod has a differential charge of

$$dq = \lambda dx. \quad (24-33)$$

This element produces an electric potential  $dV$  at point  $P$ , which is a distance  $r = (x^2 + d^2)^{1/2}$  from the element (Fig. 24-15c). Treating the element as a point charge, we can use Eq. 24-31 to write the potential  $dV$  as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}}. \quad (24-34)$$



**Figure 24-15** (a) A thin, uniformly charged rod produces an electric potential  $V$  at point  $P$ . (b) An element can be treated as a particle. (c) The potential at  $P$  due to the element depends on the distance  $r$ . We need to sum the potentials due to all the elements, from the left side (d) to the right side (e).

Since the charge on the rod is positive and we have taken  $V = 0$  at infinity, we know from Module 24-3 that  $dV$  in Eq. 24-34 must be positive.

We now find the total potential  $V$  produced by the rod at point  $P$  by integrating Eq. 24-34 along the length of the rod, from  $x = 0$  to  $x = L$  (Figs. 24-15d and e), using integral 17 in Appendix E. We find

$$\begin{aligned} V &= \int dV = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx \\ &= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[ \ln(x + (x^2 + d^2)^{1/2}) \right]_0^L \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[ \ln(L + (L^2 + d^2)^{1/2}) - \ln d \right]. \end{aligned}$$

We can simplify this result by using the general relation  $\ln A - \ln B = \ln(A/B)$ . We then find

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[ \frac{L + (L^2 + d^2)^{1/2}}{d} \right]. \quad (24-35)$$

Because  $V$  is the sum of positive values of  $dV$ , it too is positive, consistent with the logarithm being positive for an argument greater than 1.

### Charged Disk

In Module 22-5, we calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius  $R$  that has a uniform charge density  $\sigma$  on one surface. Here we derive an expression for  $V(z)$ , the electric potential at any point on the central axis. Because we have a circular distribution of charge on the disk, we could start with a differential element that occupies angle  $d\theta$  and radial distance  $dr$ . We would then need to set up a two-dimensional integration. However, let's do something easier.

In Fig. 24-16, consider a differential element consisting of a flat ring of radius  $R'$  and radial width  $dR'$ . Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

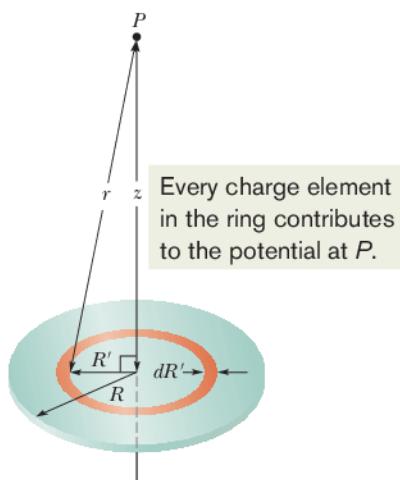
in which  $(2\pi R')(dR')$  is the upper surface area of the ring. All parts of this charged element are the same distance  $r$  from point  $P$  on the disk's axis. With the aid of Fig. 24-16, we can use Eq. 24-31 to write the contribution of this ring to the electric potential at  $P$  as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}. \quad (24-36)$$

We find the net potential at  $P$  by adding (via integration) the contributions of all the rings from  $R' = 0$  to  $R' = R$ :

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z). \quad (24-37)$$

Note that the variable in the second integral of Eq. 24-37 is  $R'$  and not  $z$ , which remains constant while the integration over the surface of the disk is carried out. (Note also that, in evaluating the integral, we have assumed that  $z \geq 0$ .)



**Figure 24-16** A plastic disk of radius  $R$ , charged on its top surface to a uniform surface charge density  $\sigma$ . We wish to find the potential  $V$  at point  $P$  on the central axis of the disk.

# 24-6 CALCULATING THE FIELD FROM THE POTENTIAL

## Learning Objectives

After reading this module, you should be able to ...

**24.23** Given an electric potential as a function of position along an axis, find the electric field along that axis.

**24.24** Given a graph of electric potential versus position along an axis, determine the electric field along the axis.

**24.25** For a uniform electric field, relate the field magnitude  $E$

and the separation  $\Delta x$  and potential difference  $\Delta V$  between adjacent equipotential lines.

**24.26** Relate the direction of the electric field and the directions in which the potential decreases and increases.

## Key Ideas

- The component of  $\vec{E}$  in any direction is the negative of the rate at which the potential changes with distance in that direction:

$$E_s = -\frac{\partial V}{\partial s}.$$

- The  $x$ ,  $y$ , and  $z$  components of  $\vec{E}$  may be found from

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}.$$

When  $\vec{E}$  is uniform, all this reduces to

$$E = -\frac{\Delta V}{\Delta s},$$

where  $s$  is perpendicular to the equipotential surfaces.

- The electric field is zero parallel to an equipotential surface.

## Calculating the Field from the Potential

In Module 24-2, you saw how to find the potential at a point  $f$  if you know the electric field along a path from a reference point to point  $f$ . In this module, we propose to go the other way—that is, to find the electric field when we know the potential. As Fig. 24-5 shows, solving this problem graphically is easy: If we know the potential  $V$  at all points near an assembly of charges, we can draw in a family of equipotential surfaces. The electric field lines, sketched perpendicular to those surfaces, reveal the variation of  $\vec{E}$ . What we are seeking here is the mathematical equivalent of this graphical procedure.

Figure 24-17 shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being  $dV$ . As the figure suggests, the field  $\vec{E}$  at any point  $P$  is perpendicular to the equipotential surface through  $P$ .

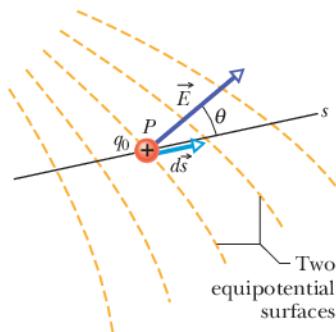
Suppose that a positive test charge  $q_0$  moves through a displacement  $d\vec{s}$  from one equipotential surface to the adjacent surface. From Eq. 24-6, we see that the work the electric field does on the test charge during the move is  $-q_0 dV$ . From Eq. 24-16 and Fig. 24-17, we see that the work done by the electric field may also be written as the scalar product  $(q_0 \vec{E}) \cdot d\vec{s}$ , or  $q_0 E(\cos \theta) ds$ . Equating these two expressions for the work yields

$$-q_0 dV = q_0 E(\cos \theta) ds, \quad (24-38)$$

or  $E \cos \theta = -\frac{dV}{ds}. \quad (24-39)$

Since  $E \cos \theta$  is the component of  $\vec{E}$  in the direction of  $d\vec{s}$ , Eq. 24-39 becomes

$$E_s = -\frac{\partial V}{\partial s}. \quad (24-40)$$



**Figure 24-17** A test charge  $q_0$  moves a distance  $d\vec{s}$  from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement  $d\vec{s}$  makes an angle  $\theta$  with the direction of the electric field  $\vec{E}$ .

We have added a subscript to  $E$  and switched to the partial derivative symbols to emphasize that Eq. 24-40 involves only the variation of  $V$  along a specified axis (here called the  $s$  axis) and only the component of  $\vec{E}$  along that axis. In words, Eq. 24-40 (which is essentially the reverse operation of Eq. 24-18) states:



The component of  $\vec{E}$  in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

If we take the  $s$  axis to be, in turn, the  $x$ ,  $y$ , and  $z$  axes, we find that the  $x$ ,  $y$ , and  $z$  components of  $\vec{E}$  at any point are

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}. \quad (24-41)$$

Thus, if we know  $V$  for all points in the region around a charge distribution—that is, if we know the function  $V(x, y, z)$ —we can find the components of  $\vec{E}$ , and thus  $\vec{E}$  itself, at any point by taking partial derivatives.

For the simple situation in which the electric field  $\vec{E}$  is uniform, Eq. 24-40 becomes

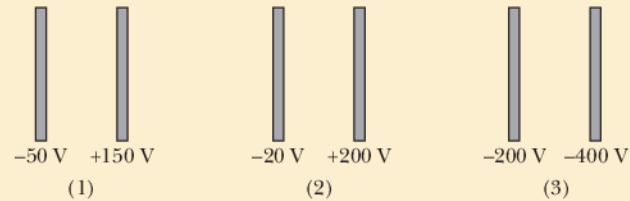
$$E = -\frac{\Delta V}{\Delta s}, \quad (24-42)$$

where  $s$  is perpendicular to the equipotential surfaces. The component of the electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along the surfaces.



### Checkpoint 5

The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and



perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?



### Sample Problem 24.05 Finding the field from the potential

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

#### KEY IDEAS

We want the electric field  $\vec{E}$  as a function of distance  $z$  along the axis of the disk. For any value of  $z$ , the direction of  $\vec{E}$  must be along that axis because the disk has circular symmetry

about that axis. Thus, we want the component  $E_z$  of  $\vec{E}$  in the direction of  $z$ . This component is the negative of the rate at which the electric potential changes with distance  $z$ .

**Calculation:** Thus, from the last of Eqs. 24-41, we can write

$$\begin{aligned} E_z &= -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\epsilon_0} \frac{d}{dz} (\sqrt{z^2 + R^2} - z) \\ &= \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right). \end{aligned} \quad (\text{Answer})$$

This is the same expression that we derived in Module 22-5 by integration, using Coulomb's law.



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## 24-7 ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF CHARGED PARTICLES

### Learning Objectives

After reading this module, you should be able to . . .

- 24.27** Identify that the total potential energy of a system of charged particles is equal to the work an applied force must do to assemble the system, starting with the particles infinitely far apart.
- 24.28** Calculate the potential energy of a pair of charged particles.
- 24.29** Identify that if a system has more than two charged parti-

cles, then the system's total potential energy is equal to the sum of the potential energies of every pair of the particles.

- 24.30** Apply the principle of the conservation of mechanical energy to a system of charged particles.
- 24.31** Calculate the escape speed of a charged particle from a system of charged particles (the minimum initial speed required to move infinitely far from the system).

### Key Idea

- The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other. For two particles at separation  $r$ ,

$$U = W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}.$$

### Electric Potential Energy of a System of Charged Particles

In this module we are going to calculate the potential energy of a system of two charged particles and then briefly discuss how to expand the result to a system of more than two particles. Our starting point is to examine the work we must do (as an external agent) to bring together two charged particles that are initially infinitely far apart and that end up near each other and stationary. If the two particles have the same sign of charge, we must fight against their mutual repulsion. Our work is then positive and results in a positive potential energy for the final two-particle system. If, instead, the two particles have opposite signs of charge, our job is easy because of the mutual attraction of the particles. Our work is then negative and results in a negative potential energy for the system.

Let's follow this procedure to build the two-particle system in Fig. 24-18, where particle 1 (with positive charge  $q_1$ ) and particle 2 (with positive charge  $q_2$ ) have separation  $r$ . Although both particles are positively charged, our result will apply also to situations where they are both negatively charged or have different signs.

We start with particle 2 fixed in place and particle 1 infinitely far away, with an initial potential energy  $U_i$  for the two-particle system. Next we bring particle 1 to its final position, and then the system's potential energy is  $U_f$ . Our work changes the system's potential energy by  $\Delta U = U_f - U_i$ .

With Eq. 24-4 ( $\Delta U = q(V_f - V_i)$ ), we can relate  $\Delta U$  to the change in potential through which we move particle 1:

$$U_f - U_i = q_1(V_f - V_i). \quad (24-43)$$

Let's evaluate these terms. The initial potential energy is  $U_i = 0$  because the particles are in the reference configuration (as discussed in Module 24-1). The two potentials in Eq. 24-43 are due to particle 2 and are given by Eq. 24-26:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r}. \quad (24-44)$$

This tells us that when particle 1 is initially at distance  $r = \infty$ , the potential at its location is  $V_i = 0$ . When we move it to the final position at distance  $r$ , the potential at its location is

$$V_f = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r}. \quad (24-45)$$

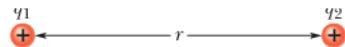


Figure 24-18 Two charges held a fixed distance  $r$  apart.

Substituting these results into Eq. 24-43 and dropping the subscript  $f$ , we find that the final configuration has a potential energy of

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (\text{two-particle system}). \quad (24-46)$$

Equation 24-46 includes the signs of the two charges. If the two charges have the same sign,  $U$  is positive. If they have opposite signs,  $U$  is negative.

If we next bring in a third particle, with charge  $q_3$ , we repeat our calculation, starting with particle 3 at an infinite distance and then bringing it to a final position at distance  $r_{31}$  from particle 1 and distance  $r_{32}$  from particle 2. At the final position, the potential  $V_f$  at the location of particle 3 is the algebraic sum of the potential  $V_1$  due to particle 1 and the potential  $V_2$  of particle 2. When we work out the algebra, we find that



The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

This result applies to a system for any given number of particles.

Now that we have an expression for the potential energy of a system of particles, we can apply the principle of the conservation of energy to the system as expressed in Eq. 24-10. For example, if the system consists of many particles, we might consider the kinetic energy (and the associated *escape speed*) required of one of the particles to escape from the rest of the particles.



### Sample Problem 24.06 Potential energy of a system of three charged particles

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy  $U$  of this system of charges? Assume that  $d = 12 \text{ cm}$  and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which  $q = 150 \text{ nC}$ .

#### KEY IDEA

The potential energy  $U$  of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

**Calculations:** Let's mentally build the system of Fig. 24-19, starting with one of the charges, say  $q_1$ , in place and the others at infinity. Then we bring another one, say  $q_2$ , in from infinity and put it in place. From Eq. 24-46 with  $d$  substituted for  $r$ , the potential energy  $U_{12}$  associated with the pair of charges  $q_1$  and  $q_2$  is

$$U_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d}.$$

We then bring the last charge  $q_3$  in from infinity and put it in

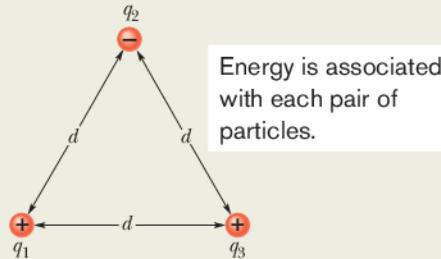


Figure 24-19 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

place. The work that we must do in this last step is equal to the sum of the work we must do to bring  $q_3$  near  $q_1$  and the work we must do to bring it near  $q_2$ . From Eq. 24-46, with  $d$  substituted for  $r$ , that sum is

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{d} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{d}.$$

The total potential energy  $U$  of the three-charge system is the sum of the potential energies associated with the three pairs of charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$\begin{aligned}
 U &= U_{12} + U_{13} + U_{23} \\
 &= \frac{1}{4\pi\epsilon_0} \left( \frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d} \right) \\
 &= -\frac{10q^2}{4\pi\epsilon_0 d} \\
 &= -\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9} \text{ C})^2}{0.12 \text{ m}} \\
 &= -1.7 \times 10^{-2} \text{ J} = -17 \text{ mJ.} \quad (\text{Answer})
 \end{aligned}$$

The negative potential energy means that negative work would have to be done to assemble this structure, starting with the three charges infinitely separated and at rest. Put another way, an external agent would have to do 17 mJ of positive work to disassemble the structure completely, ending with the three charges infinitely far apart.

The lesson here is this: If you are given an assembly of charged particles, you can find the potential energy of the assembly by finding the potential of every possible pair of the particles and then summing the results.

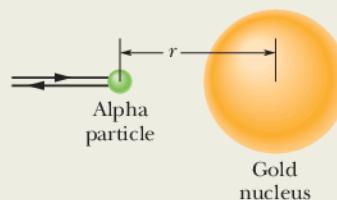
### Sample Problem 24.07 Conservation of mechanical energy with electric potential energy

An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus (Fig. 24-20). The alpha particle slows until it momentarily stops when its center is at radial distance  $r = 9.23 \text{ fm}$  from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy  $K_i$  of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force and treat each as a single charged particle.

#### KEY IDEA

During the entire process, the mechanical energy of the *alpha particle + gold atom* system is conserved.

**Reasoning:** When the alpha particle is outside the atom, the system's initial electric potential energy  $U_i$  is zero because the atom has an equal number of electrons and protons, which produce a *net* electric field of zero. However, once the alpha particle passes through the electron region surrounding the nucleus on its way to the nucleus, the electric field due to the electrons goes to zero. The reason is that the electrons act like a closed spherical shell of uniform negative charge and, as discussed in Module 23-6, such a shell produces zero electric field in the space it encloses. The alpha particle still experiences the electric field of the protons in the nucleus, which produces a repulsive force on the protons within the alpha particle.



**Figure 24-20** An alpha particle, traveling head-on toward the center of a gold nucleus, comes to a momentary stop (at which time all its kinetic energy has been transferred to electric potential energy) and then reverses its path.

As the incoming alpha particle is slowed by this repulsive force, its kinetic energy is transferred to electric potential energy of the system. The transfer is complete when the alpha particle momentarily stops and the kinetic energy is  $K_f = 0$ .

**Calculations:** The principle of conservation of mechanical energy tells us that

$$K_i + U_i = K_f + U_f. \quad (24-47)$$

We know two values:  $U_i = 0$  and  $K_f = 0$ . We also know that the potential energy  $U_f$  at the stopping point is given by the right side of Eq. 24-46, with  $q_1 = 2e$ ,  $q_2 = 79e$  (in which  $e$  is the elementary charge,  $1.60 \times 10^{-19} \text{ C}$ ), and  $r = 9.23 \text{ fm}$ . Thus, we can rewrite Eq. 24-47 as

$$\begin{aligned}
 K_i &= \frac{1}{4\pi\epsilon_0} \frac{(2e)(79e)}{9.23 \text{ fm}} \\
 &= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(158)(1.60 \times 10^{-19} \text{ C})^2}{9.23 \times 10^{-15} \text{ m}} \\
 &= 3.94 \times 10^{-12} \text{ J} = 24.6 \text{ MeV.} \quad (\text{Answer})
 \end{aligned}$$



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## 24-8 POTENTIAL OF A CHARGED ISOLATED CONDUCTOR

### Learning Objectives

After reading this module, you should be able to . . .

- 24.32 Identify that an excess charge placed on an isolated conductor (or connected isolated conductors) will distribute itself on the surface of the conductor so that all points of the conductor come to the same potential.
- 24.33 For an isolated spherical conducting shell, sketch graphs of the potential and the electric field magnitude versus distance from the center, both inside and outside the shell.
- 24.34 For an isolated spherical conducting shell, identify that internally the electric field is zero and the electric potential

has the same value as the surface and that externally the electric field and the electric potential have values as though all of the shell's charge is concentrated as a particle at its center.

- 24.35 For an isolated cylindrical conducting shell, identify that internally the electric field is zero and the electric potential has the same value as the surface and that externally the electric field and the electric potential have values as though all of the cylinder's charge is concentrated as a line of charge on the central axis.

### Key Ideas

- An excess charge placed on a conductor will, in the equilibrium state, be located entirely on the outer surface of the conductor.
- The entire conductor, including interior points, is at a uniform potential.
- If an isolated charged conductor is placed in an external

electric field, then at every internal point, the electric field due to the charge cancels the external electric field that otherwise would have been there.

- Also, the net electric field at every point on the surface is perpendicular to the surface.

### Potential of a Charged Isolated Conductor

In Module 23-3, we concluded that  $\vec{E} = 0$  for all points inside an isolated conductor. We then used Gauss' law to prove that an excess charge placed on an isolated conductor lies entirely on its surface. (This is true even if the conductor has an empty internal cavity.) Here we use the first of these facts to prove an extension of the second:



An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

Our proof follows directly from Eq. 24-18, which is

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}$$

Since  $\vec{E} = 0$  for all points within a conductor, it follows directly that  $V_f = V_i$  for all possible pairs of points  $i$  and  $f$  in the conductor.

Figure 24-21a is a plot of potential against radial distance  $r$  from the center for an isolated spherical conducting shell of 1.0 m radius, having a charge of 1.0  $\mu\text{C}$ . For points outside the shell, we can calculate  $V(r)$  from Eq. 24-26 because the charge  $q$  behaves for such external points as if it were concentrated at the center of the shell. That equation holds right up to the surface of the shell. Now let us push a small test charge through the shell—assuming a small hole exists—to its center. No extra work is needed to do this because no net electric force acts on the test charge once it is inside the shell. Thus, the potential at all points inside the shell has the same value as that on the surface, as Fig. 24-21a shows.

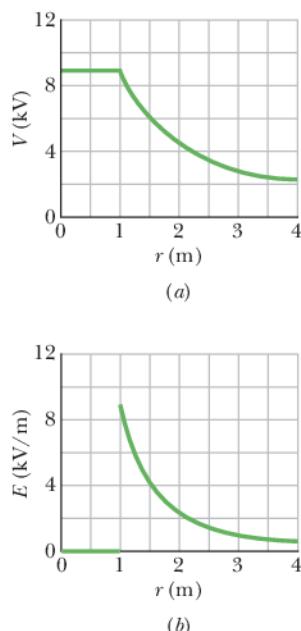


Figure 24-21 (a) A plot of  $V(r)$  both inside and outside a charged spherical shell of radius 1.0 m. (b) A plot of  $E(r)$  for the same shell.

Figure 24-21b shows the variation of electric field with radial distance for the same shell. Note that  $E = 0$  everywhere inside the shell. The curves of Fig. 24-21b can be derived from the curve of Fig. 24-21a by differentiating with respect to  $r$ , using Eq. 24-40 (recall that the derivative of any constant is zero). The curve of Fig. 24-21a can be derived from the curves of Fig. 24-21b by integrating with respect to  $r$ , using Eq. 24-19.

### Spark Discharge from a Charged Conductor

On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or sharp edges, the surface charge density—and thus the external electric field, which is proportional to it—may reach very high values. The air around such sharp points or edges may become ionized, producing the corona discharge that golfers and mountaineers see on the tips of bushes, golf clubs, and rock hammers when thunderstorms threaten. Such corona discharges, like hair that stands on end, are often the precursors of lightning strikes. In such circumstances, it is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal (Fig. 24-22).



### Isolated Conductor in an External Electric Field

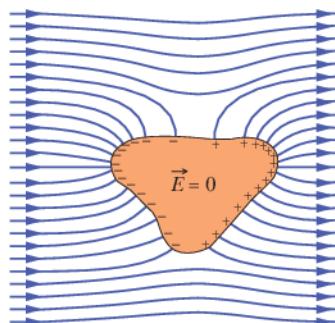
If an isolated conductor is placed in an *external electric field*, as in Fig. 24-23, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge. The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there. Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface. If the conductor in Fig. 24-23 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.

**Figure 24-23** An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field at the surface perpendicular to the surface.



Courtesy Westinghouse Electric Corporation

**Figure 24-22** A large spark jumps to a car's body and then exits by moving across the insulating left front tire (note the flash there), leaving the person inside unharmed.



## Review & Summary

**Electric Potential** The electric potential  $V$  at a point  $P$  in the electric field of a charged object is

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}, \quad (24-2)$$

where  $W_\infty$  is the work that would be done by the electric force on a positive test charge were it brought from an infinite distance to  $P$ , and  $U$  is the potential energy that would then be stored in the test charge-object system.

**Electric Potential Energy** If a particle with charge  $q$  is placed at a point where the electric potential of a charged object is  $V$ , the electric potential energy  $U$  of the particle-object system is

$$U = qV. \quad (24-3)$$

If the particle moves through a potential difference  $\Delta V$ , the change in the electric potential energy is

$$\Delta U = q \Delta V = q(V_f - V_i). \quad (24-4)$$

**Mechanical Energy** If a particle moves through a change  $\Delta V$  in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

$$\Delta K = -q \Delta V. \quad (24-9)$$

If, instead, an applied force acts on the particle, doing work  $W_{\text{app}}$ , the change in kinetic energy is

$$\Delta K = -q \Delta V + W_{\text{app}}. \quad (24-11)$$

In the special case when  $\Delta K = 0$ , the work of an applied force



- 3** Figure 24-26 shows four pairs of charged particles. For each pair, let  $V = 0$  at infinity and consider  $V_{\text{net}}$  at points on the  $x$  axis. For which pairs is there a point at which  $V_{\text{net}} = 0$  (a) between the particles and (b) to the right of the particles? (c) At such a point is  $\vec{E}_{\text{net}}$  due to the particles equal to zero? (d) For each pair, are there off-axis points (other than at infinity) where  $V_{\text{net}} = 0$ ?

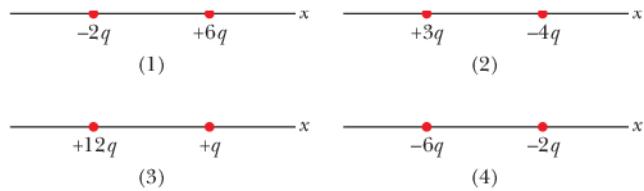


Figure 24-26 Questions 3 and 9.

- 4** Figure 24-27 gives the electric potential  $V$  as a function of  $x$ . (a) Rank the five regions according to the magnitude of the  $x$  component of the electric field within them, greatest first. What is the direction of the field along the  $x$  axis in (b) region 2 and (c) region 4?

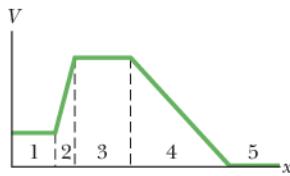


Figure 24-27 Question 4.

- 5** Figure 24-28 shows three paths along which we can move the positively charged sphere  $A$  closer to positively charged sphere  $B$ , which is held fixed in place. (a) Would sphere  $A$  be moved to a higher or lower electric potential? Is the work done (b) by our force and (c) by the electric field due to  $B$  positive, negative, or zero? (d) Rank the paths according to the work our force does, greatest first.

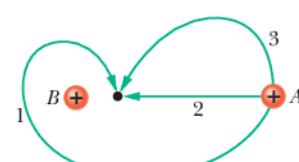


Figure 24-28 Question 5.

- 6** Figure 24-29 shows four arrangements of charged particles, all the same distance from the origin. Rank the situations according to the net electric potential at the origin, most positive first. Take the potential to be zero at infinity.

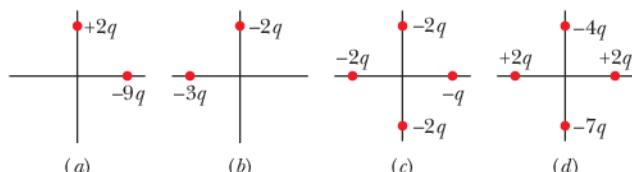


Figure 24-29 Question 6.

- 7** Figure 24-30 shows a system of three charged particles. If you move the particle of charge  $+q$  from point  $A$  to point  $D$ , are the following quantities positive, negative, or zero: (a) the change in the electric potential energy of the three-particle system, (b) the work done by the net electric force on the particle you moved (that is, the net force due to the other two particles), and (c) the work done by your force? (d) What are the answers to (a) through (c) if, instead, the particle is moved from  $B$  to  $C$ ?

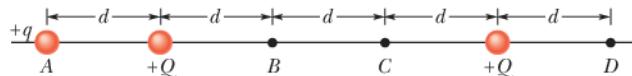


Figure 24-30 Questions 7 and 8.

- 8** In the situation of Question 7, is the work done by your force positive, negative, or zero if the particle is moved (a) from  $A$  to  $B$ , (b) from  $A$  to  $C$ , and (c) from  $B$  to  $D$ ? (d) Rank those moves according to the magnitude of the work done by your force, greatest first.

- 9** Figure 24-26 shows four pairs of charged particles with identical separations. (a) Rank the pairs according to their electric potential energy (that is, the energy of the two-particle system), greatest (most positive) first. (b) For each pair, if the separation between the particles is increased, does the potential energy of the pair increase or decrease?

- 10** (a) In Fig. 24-31a, what is the potential at point  $P$  due to charge  $Q$  at distance  $R$  from  $P$ ? Set  $V = 0$  at infinity. (b) In Fig. 24-31b, the same charge  $Q$  has been spread uniformly over a circular arc of radius  $R$  and central angle  $40^\circ$ . What is the potential at point  $P$ , the center of curvature of the arc? (c) In Fig. 24-31c, the same charge  $Q$  has been spread uniformly over a circle of radius  $R$ . What is the potential at point  $P$ , the center of the circle? (d) Rank the three situations according to the magnitude of the electric field that is set up at  $P$ , greatest first.

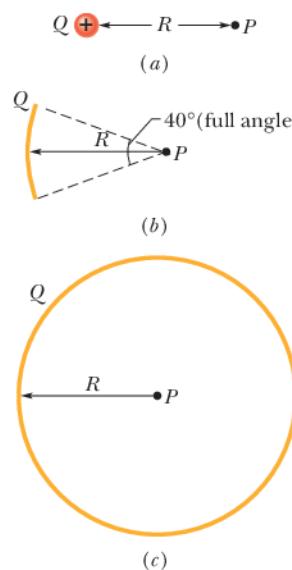


Figure 24-31 Question 10.

- 11** Figure 24-32 shows a thin, uniformly charged rod and three points at the same distance  $d$  from the rod. Rank the magnitude of the electric potential the rod produces at those three points, greatest first.

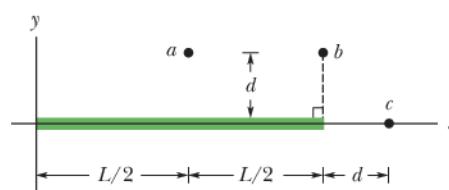


Figure 24-32 Question 11.

- 12** In Fig. 24-33, a particle is to be released at rest at point  $A$  and then is to be accelerated directly through point  $B$  by an electric field. The potential difference between points  $A$  and  $B$  is 100 V. Which point should be at higher electric potential if the particle is (a) an electron, (b) a proton, and (c) an alpha particle (a nucleus of two protons and two neutrons)? (d) Rank the kinetic energies of the particles at point  $B$ , greatest first.

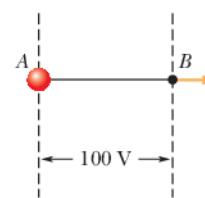


Figure 24-33  
Question 12.

## Problems

**GO** Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign

**SSM** Worked-out solution available in Student Solutions Manual

••• Number of dots indicates level of problem difficulty

**FC** Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

**WWW** Worked-out solution is at

**ILW** Interactive solution is at

<http://www.wiley.com/college/halliday>

### Module 24-1 Electric Potential

- 1 **SSM** A particular 12 V car battery can send a total charge of  $84 \text{ A} \cdot \text{h}$  (ampere-hours) through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (*Hint:* See Eq. 21-3.) (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

- 2 The electric potential difference between the ground and a cloud in a particular thunderstorm is  $1.2 \times 10^9 \text{ V}$ . In the unit electron-volts, what is the magnitude of the change in the electric potential energy of an electron that moves between the ground and the cloud?

- 3 Suppose that in a lightning flash the potential difference between a cloud and the ground is  $1.0 \times 10^9 \text{ V}$  and the quantity of charge transferred is 30 C. (a) What is the change in energy of that transferred charge? (b) If all the energy released could be used to accelerate a 1000 kg car from rest, what would be its final speed?

### Module 24-2 Equipotential Surfaces and the Electric Field

- 4 Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electric force of  $3.9 \times 10^{-15} \text{ N}$  acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?

- 5 **SSM** An infinite nonconducting sheet has a surface charge density  $\sigma = 0.10 \mu\text{C}/\text{m}^2$  on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

- 6 When an electron moves from A to B along an electric field line in Fig. 24-34, the electric field does  $3.94 \times 10^{-19} \text{ J}$  of work on it. What are the electric potential differences (a)  $V_B - V_A$ , (b)  $V_C - V_A$ , and (c)  $V_C - V_B$ ?

- 7 The electric field in a region of space has the components  $E_y = E_z = 0$  and  $E_x = (4.00 \text{ N/C})x$ . Point A is on the y axis at  $y = 3.00 \text{ m}$ , and point B is on the x axis at  $x = 4.00 \text{ m}$ . What is the potential difference  $V_B - V_A$ ?

- 8 A graph of the x component of the electric field as a function of x in a region of space is shown in Fig. 24-35. The scale of the vertical axis is set by  $E_{xs} = 20.0 \text{ N/C}$ . The y and z components of the electric field are zero in this region. If the electric potential at the origin is 10 V, (a) what is the electric potential at  $x = 2.0 \text{ m}$ , (b) what is the greatest positive value of the electric potential for points on the x axis for which  $0 \leq x \leq 6.0 \text{ m}$ , and (c) for what value of x is the electric potential zero?

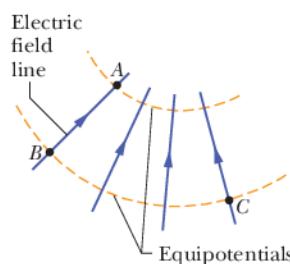


Figure 24-34 Problem 6.

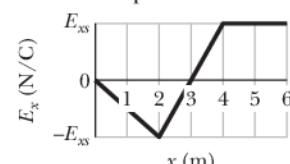


Figure 24-35 Problem 8.

- 9 An infinite nonconducting sheet has a surface charge density  $\sigma = +5.80 \text{ pC/m}^2$ . (a) How much work is done by the electric field due to the sheet if a particle of charge  $q = +1.60 \times 10^{-19} \text{ C}$  is moved from the sheet to a point P at distance  $d = 3.56 \text{ cm}$  from the sheet? (b) If the electric potential  $V$  is defined to be zero on the sheet, what is  $V$  at P?

- 10 **GO** Two uniformly charged, infinite, nonconducting planes are parallel to a  $yz$  plane and positioned at  $x = -50 \text{ cm}$  and  $x = +50 \text{ cm}$ . The charge densities on the planes are  $-50 \text{ nC/m}^2$  and  $+25 \text{ nC/m}^2$ , respectively. What is the magnitude of the potential difference between the origin and the point on the  $x$  axis at  $x = +80 \text{ cm}$ ? (*Hint:* Use Gauss' law.)

- 11 A nonconducting sphere has radius  $R = 2.31 \text{ cm}$  and uniformly distributed charge  $q = +3.50 \text{ fC}$ . Take the electric potential at the sphere's center to be  $V_0 = 0$ . What is  $V$  at radial distance (a)  $r = 1.45 \text{ cm}$  and (b)  $r = R$ . (*Hint:* See Module 23-6.)

### Module 24-3 Potential Due to a Charged Particle

- 12 As a space shuttle moves through the dilute ionized gas of Earth's ionosphere, the shuttle's potential is typically changed by  $-1.0 \text{ V}$  during one revolution. Assuming the shuttle is a sphere of radius 10 m, estimate the amount of charge it collects.

- 13 What are (a) the charge and (b) the charge density on the surface of a conducting sphere of radius 0.15 m whose potential is 200 V (with  $V = 0$  at infinity)?

- 14 Consider a particle with charge  $q = 1.0 \mu\text{C}$ , point A at distance  $d_1 = 2.0 \text{ m}$  from  $q$ , and point B at distance  $d_2 = 1.0 \text{ m}$ . (a) If A and B are diametrically opposite each other, as in Fig. 24-36a, what is the electric potential difference  $V_A - V_B$ ? (b) What is that electric potential difference if A and B are located as in Fig. 24-36b?

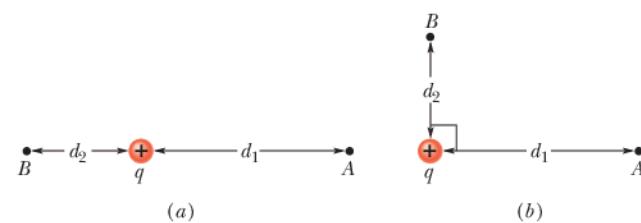


Figure 24-36 Problem 14.

- 15 **SSM ILW** A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with  $V = 0$  at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

- 16 **GO** Figure 24-37 shows a rectangular array of charged particles fixed in place, with distance  $a = 39.0 \text{ cm}$  and the charges shown as integer multiples of  $q_1 = 3.40 \text{ pC}$  and  $q_2 = 6.00 \text{ pC}$ . With  $V = 0$  at infinity, what

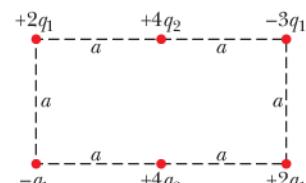


Figure 24-37 Problem 16.

is the net electric potential at the rectangle's center? (Hint: Thoughtful examination of the arrangement can reduce the calculation.)

- 17 GO In Fig. 24-38, what is the net electric potential at point  $P$  due to the four particles if  $V = 0$  at infinity,  $q = 5.00 \text{ fC}$ , and  $d = 4.00 \text{ cm}$ ?

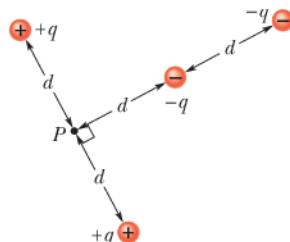


Figure 24-38 Problem 17.

••18 GO Two charged particles are shown in Fig. 24-39a. Particle 1, with charge  $q_1$ , is fixed in place at distance  $d$ . Particle 2, with charge  $q_2$ , can be moved along the  $x$  axis. Figure 24-39b gives the net electric potential  $V$  at the origin due to the two particles as a function of the  $x$  coordinate of particle 2. The scale of the  $x$  axis is set by  $x_s = 16.0 \text{ cm}$ . The plot has an asymptote of  $V = 5.76 \times 10^{-7} \text{ V}$  as  $x \rightarrow \infty$ . What is  $q_2$  in terms of  $e$ ?

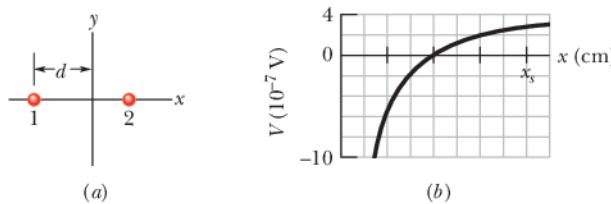


Figure 24-39 Problem 18.

- 19 In Fig. 24-40, particles with the charges  $q_1 = +5e$  and  $q_2 = -15e$  are fixed in place with a separation of  $d = 24.0 \text{ cm}$ . With electric potential defined to be  $V = 0$  at infinity, what are the finite (a) positive and (b) negative values of  $x$  at which the net electric potential on the  $x$  axis is zero?

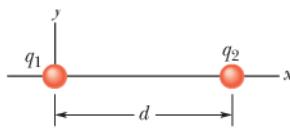


Figure 24-40 Problems 19 and 20.

- 20 Two particles, of charges  $q_1$  and  $q_2$ , are separated by distance  $d$  in Fig. 24-40. The net electric field due to the particles is zero at  $x = d/4$ . With  $V = 0$  at infinity, locate (in terms of  $d$ ) any point on the  $x$  axis (other than at infinity) at which the electric potential due to the two particles is zero.

#### Module 24-4 Potential Due to an Electric Dipole

- 21 ILW The ammonia molecule  $\text{NH}_3$  has a permanent electric dipole moment equal to  $1.47 \text{ D}$ , where  $1 \text{ D} = 1 \text{ debye unit} = 3.34 \times 10^{-30} \text{ C} \cdot \text{m}$ . Calculate the electric potential due to an ammonia molecule at a point  $52.0 \text{ nm}$  away along the axis of the dipole. (Set  $V = 0$  at infinity.)

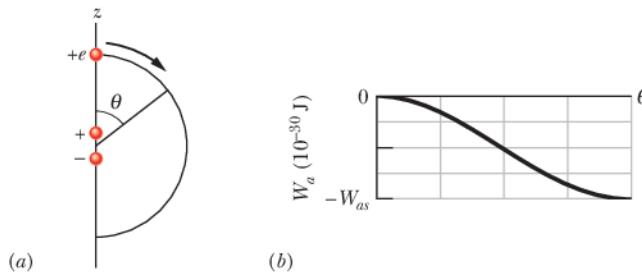


Figure 24-41 Problem 22.

- 22 In Fig. 24-41a, a particle of elementary charge  $+e$  is initially at coordinate  $z = 20 \text{ nm}$  on the dipole axis (here a  $z$  axis) through

an electric dipole, on the positive side of the dipole. (The origin of  $z$  is at the center of the dipole.) The particle is then moved along a circular path around the dipole center until it is at coordinate  $z = -20 \text{ nm}$ , on the negative side of the dipole axis. Figure 24-41b gives the work  $W_a$  done by the force moving the particle versus the angle  $\theta$  that locates the particle relative to the positive direction of the  $z$  axis. The scale of the vertical axis is set by  $W_{as} = 4.0 \times 10^{-30} \text{ J}$ . What is the magnitude of the dipole moment?

#### Module 24-5 Potential Due to a Continuous Charge Distribution

- 23 (a) Figure 24-42a shows a nonconducting rod of length  $L = 6.00 \text{ cm}$  and uniform linear charge density  $\lambda = +3.68 \text{ pC/m}$ . Assume that the electric potential is defined to be  $V = 0$  at infinity. What is  $V$  at point  $P$  at distance  $d = 8.00 \text{ cm}$  along the rod's perpendicular bisector? (b) Figure 24-42b shows an identical rod except that one half is now negatively charged. Both halves have a linear charge density of magnitude  $3.68 \text{ pC/m}$ . With  $V = 0$  at infinity, what is  $V$  at  $P$ ?

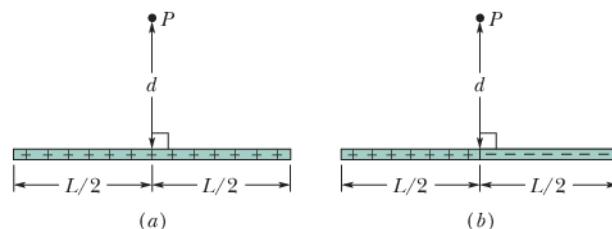


Figure 24-42 Problem 23.

- 24 In Fig. 24-43, a plastic rod having a uniformly distributed charge  $Q = -25.6 \text{ pC}$  has been bent into a circular arc of radius  $R = 3.71 \text{ cm}$  and central angle  $\phi = 120^\circ$ . With  $V = 0$  at infinity, what is the electric potential at  $P$ , the center of curvature of the rod?

- 25 A plastic rod has been bent into a circle of radius  $R = 8.20 \text{ cm}$ . It has a charge  $Q_1 = +4.20 \text{ pC}$  uniformly distributed along one-quarter of its circumference and a charge  $Q_2 = -6Q_1$  uniformly distributed along the rest of the circumference (Fig. 24-44). With  $V = 0$  at infinity, what is the electric potential at (a) the center  $C$  of the circle and (b) point  $P$ , on the central axis of the circle at distance  $D = 6.71 \text{ cm}$  from the center?

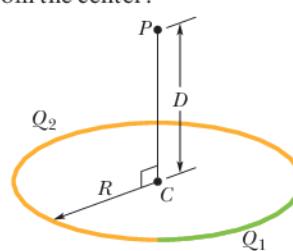


Figure 24-44 Problem 25.

- 26 GO Figure 24-45 shows a thin rod with a uniform charge density of  $2.00 \mu\text{C/m}$ . Evaluate the electric potential at point  $P$  if  $d = D = L/4.00$ . Assume that the potential is zero at infinity.

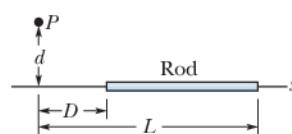


Figure 24-45 Problem 26.

- 27** In Fig. 24-46, three thin plastic rods form quarter-circles with a common center of curvature at the origin. The uniform charges on the three rods are  $Q_1 = +30 \text{ nC}$ ,  $Q_2 = +3.0Q_1$ , and  $Q_3 = -8.0Q_1$ . What is the net electric potential at the origin due to the rods?

**••28 GO** Figure 24-47 shows a thin plastic rod of length  $L = 12.0 \text{ cm}$  and uniform positive charge  $Q = 56.1 \text{ fC}$  lying on an  $x$  axis. With  $V = 0$  at infinity, find the electric potential at point  $P_1$  on the axis, at distance  $d = 2.50 \text{ cm}$  from the rod.

- 29** In Fig. 24-48, what is the net electric potential at the origin due to the circular arc of charge  $Q_1 = +7.21 \text{ pC}$  and the two particles of charges  $Q_2 = 4.00Q_1$  and  $Q_3 = -2.00Q_1$ ? The arc's center of curvature is at the origin and its radius is  $R = 2.00 \text{ m}$ ; the angle indicated is  $\theta = 20.0^\circ$ .

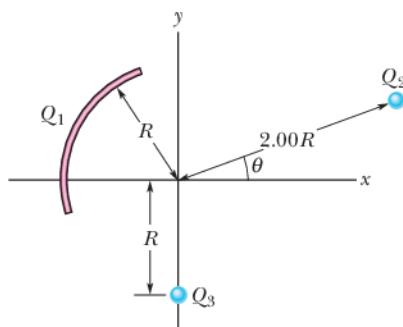


Figure 24-48 Problem 29.

- 30 GO** The smiling face of Fig. 24-49 consists of three items:

1. a thin rod of charge  $-3.0 \mu\text{C}$  that forms a full circle of radius  $6.0 \text{ cm}$ ;
2. a second thin rod of charge  $2.0 \mu\text{C}$  that forms a circular arc of radius  $4.0 \text{ cm}$ , subtending an angle of  $90^\circ$  about the center of the full circle;
3. an electric dipole with a dipole moment that is perpendicular to a radial line and has a magnitude of  $1.28 \times 10^{-21} \text{ C}\cdot\text{m}$ .

What is the net electric potential at the center?

- 31 SSM WWW** A plastic disk of radius  $R = 64.0 \text{ cm}$  is charged on one side with a uniform surface charge density  $\sigma = 7.73 \text{ fC/m}^2$ , and then three quadrants of the disk are removed. The remaining quadrant is shown in Fig. 24-50. With  $V = 0$  at infinity, what is the potential due to the remaining quadrant at point  $P$ , which is on the central axis of the original disk at distance  $D = 25.9 \text{ cm}$  from the original center?

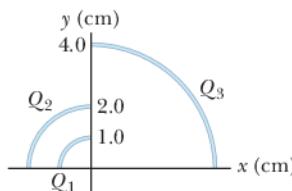


Figure 24-46 Problem 27.

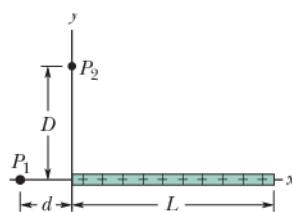


Figure 24-47 Problems 28, 33, 38, and 40.

- 32 GO** A nonuniform linear charge distribution given by  $\lambda = bx$ , where  $b$  is a constant, is located along an  $x$  axis from  $x = 0$  to  $x = 0.20 \text{ m}$ . If  $b = 20 \text{ nC/m}^2$  and  $V = 0$  at infinity, what is the electric potential at (a) the origin and (b) the point  $y = 0.15 \text{ m}$  on the  $y$  axis?

- 33 GO** The thin plastic rod shown in Fig. 24-47 has length  $L = 12.0 \text{ cm}$  and a nonuniform linear charge density  $\lambda = cx$ , where  $c = 28.9 \text{ pC/m}^2$ . With  $V = 0$  at infinity, find the electric potential at point  $P_1$  on the axis, at distance  $d = 3.00 \text{ cm}$  from one end.

#### Module 24-6 Calculating the Field from the Potential

- 34** Two large parallel metal plates are  $1.5 \text{ cm}$  apart and have charges of equal magnitudes but opposite signs on their facing surfaces. Take the potential of the negative plate to be zero. If the potential halfway between the plates is then  $+5.0 \text{ V}$ , what is the electric field in the region between the plates?

- 35** The electric potential at points in an  $xy$  plane is given by  $V = (2.0 \text{ V/m}^2)x^2 - (3.0 \text{ V/m}^2)y^2$ . In unit-vector notation, what is the electric field at the point  $(3.0 \text{ m}, 2.0 \text{ m})$ ?

- 36** The electric potential  $V$  in the space between two flat parallel plates 1 and 2 is given (in volts) by  $V = 1500x^2$ , where  $x$  (in meters) is the perpendicular distance from plate 1. At  $x = 1.3 \text{ cm}$ , (a) what is the magnitude of the electric field and (b) is the field directed toward or away from plate 1?

- 37 SSM** What is the magnitude of the electric field at the point  $(3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k}) \text{ m}$  if the electric potential in the region is given by  $V = 2.00xyz^2$ , where  $V$  is in volts and coordinates  $x$ ,  $y$ , and  $z$  are in meters?

- 38** Figure 24-47 shows a thin plastic rod of length  $L = 13.5 \text{ cm}$  and uniform charge  $43.6 \text{ fC}$ . (a) In terms of distance  $d$ , find an expression for the electric potential at point  $P_1$ . (b) Next, substitute variable  $x$  for  $d$  and find an expression for the magnitude of the component  $E_x$  of the electric field at  $P_1$ . (c) What is the direction of  $E_x$  relative to the positive direction of the  $x$  axis? (d) What is the value of  $E_x$  at  $P_1$  for  $x = d = 6.20 \text{ cm}$ ? (e) From the symmetry in Fig. 24-47, determine  $E_y$  at  $P_1$ .

- 39** An electron is placed in an  $xy$  plane where the electric potential depends on  $x$  and  $y$  as shown, for the coordinate axes, in Fig. 24-51 (the potential does not depend on  $z$ ). The scale of the vertical axis is set by  $V_s = 500 \text{ V}$ . In unit-vector notation, what is the electric force on the electron?

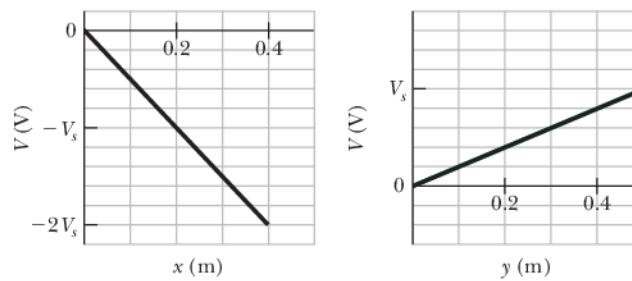


Figure 24-51 Problem 39.

- 40 GO** The thin plastic rod of length  $L = 10.0 \text{ cm}$  in Fig. 24-47 has a nonuniform linear charge density  $\lambda = cx$ , where  $c = 49.9 \text{ pC/m}^2$ . (a) With  $V = 0$  at infinity, find the electric potential at point  $P_2$  on the  $y$  axis at  $y = D = 3.56 \text{ cm}$ . (b) Find the electric field component  $E_y$  at  $P_2$ . (c) Why cannot the field component  $E_x$  at  $P_2$  be found using the result of (a)?

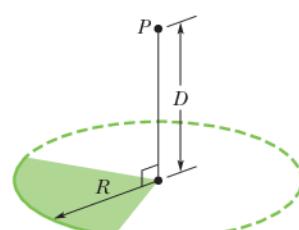


Figure 24-50 Problem 31.

**Module 24-7 Electric Potential Energy of a System of Charged Particles**

**•41** A particle of charge  $+7.5 \mu\text{C}$  is released from rest at the point  $x = 60 \text{ cm}$  on an  $x$  axis. The particle begins to move due to the presence of a charge  $Q$  that remains fixed at the origin. What is the kinetic energy of the particle at the instant it has moved 40 cm if (a)  $Q = +20 \mu\text{C}$  and (b)  $Q = -20 \mu\text{C}$ ?

**•42** (a) What is the electric potential energy of two electrons separated by 2.00 nm? (b) If the separation increases, does the potential energy increase or decrease?

**•43 SSM ILW WWW** How much work is required to set up the arrangement of Fig. 24-52 if  $q = 2.30 \text{ pC}$ ,  $a = 64.0 \text{ cm}$ , and the particles are initially infinitely far apart and at rest?

**•44** In Fig. 24-53, seven charged particles are fixed in place to form a square with an edge length of 4.0 cm. How much work must we do to bring a particle of charge  $+6e$  initially at rest from an infinite distance to the center of the square?

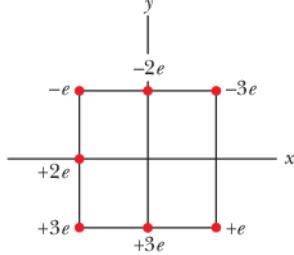


Figure 24-53 Problem 44.

**•45 ILW** A particle of charge  $q$  is fixed at point  $P$ , and a second particle of mass  $m$  and the same charge  $q$  is initially held a distance  $r_1$  from  $P$ . The second particle is then released. Determine its speed when it is a distance  $r_2$  from  $P$ . Let  $q = 3.1 \mu\text{C}$ ,  $m = 20 \text{ mg}$ ,  $r_1 = 0.90 \text{ mm}$ , and  $r_2 = 2.5 \text{ mm}$ .

**•46** A charge of  $-9.0 \text{ nC}$  is uniformly distributed around a thin plastic ring lying in a  $yz$  plane with the ring center at the origin. A  $-6.0 \text{ pC}$  particle is located on the  $x$  axis at  $x = 3.0 \text{ m}$ . For a ring radius of 1.5 m, how much work must an external force do on the particle to move it to the origin?

**•47 GO** What is the *escape speed* for an electron initially at rest on the surface of a sphere with a radius of 1.0 cm and a uniformly distributed charge of  $1.6 \times 10^{-15} \text{ C}$ ? That is, what initial speed must the electron have in order to reach an infinite distance from the sphere and have zero kinetic energy when it gets there?

**•48** A thin, spherical, conducting shell of radius  $R$  is mounted on an isolating support and charged to a potential of  $-125 \text{ V}$ . An electron is then fired directly toward the center of the shell, from point  $P$  at distance  $r$  from the center of the shell ( $r \gg R$ ). What initial speed  $v_0$  is needed for the electron to just reach the shell before reversing direction?

**•49 GO** Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity and stops midway between the two. What is its initial speed?

**•50** In Fig. 24-54, how much work must we do to bring a particle, of charge  $Q = +16e$  and initially at rest, along the dashed line from

infinity to the indicated point near two fixed particles of charges  $q_1 = +4e$  and  $q_2 = -q_1/2$ ? Distance  $d = 1.40 \text{ cm}$ ,  $\theta_1 = 43^\circ$ , and  $\theta_2 = 60^\circ$ .

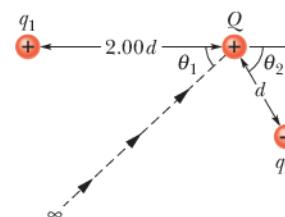


Figure 24-54 Problem 50.



Figure 24-55 Problem 51.

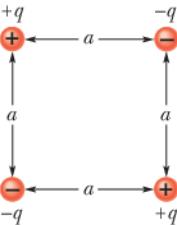


Figure 24-52  
Problem 43.

**•51 GO** In the rectangle of Fig. 24-55, the sides have lengths 5.0 cm and 15 cm,  $q_1 = -5.0 \mu\text{C}$ , and  $q_2 = +2.0 \mu\text{C}$ . With  $V = 0$  at infinity, what is the electric potential at (a) corner  $A$  and (b) corner  $B$ ? (c) How much work is required to move a charge  $q_3 = +3.0 \mu\text{C}$  from  $B$  to  $A$  along a diagonal of the rectangle? (d) Does this work increase or decrease the electric potential energy of the three-charge system? Is more, less, or the same work required if  $q_3$  is moved along a path that is (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?

**•52** Figure 24-56a shows an electron moving along an electric dipole axis toward the negative side of the dipole. The dipole is fixed in place. The electron was initially very far from the dipole, with kinetic energy 100 eV. Figure 24-56b gives the kinetic energy  $K$  of the electron versus its distance  $r$  from the dipole center. The scale of the horizontal axis is set by  $r_s = 0.10 \text{ m}$ . What is the magnitude of the dipole moment?

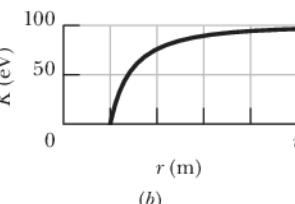
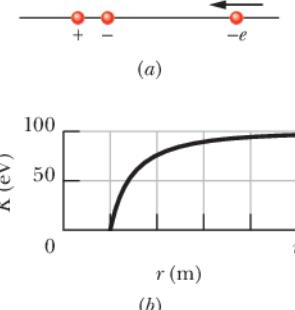


Figure 24-56 Problem 52.

**•53** Two tiny metal spheres  $A$  and  $B$ , mass  $m_A = 5.00 \text{ g}$  and  $m_B = 10.0 \text{ g}$ , have equal positive charge  $q = 5.00 \mu\text{C}$ . The spheres are connected by a massless nonconducting string of length  $d = 1.00 \text{ m}$ , which is much greater than the radii of the spheres. (a) What is the electric potential energy of the system? (b) Suppose you cut the string. At that instant, what is the acceleration of each sphere? (c) A long time after you cut the string, what is the speed of each sphere?

**•54 GO** A positron (charge  $+e$ , mass equal to the electron mass) is moving at  $1.0 \times 10^7 \text{ m/s}$  in the positive direction of an  $x$  axis when, at  $x = 0$ , it encounters an electric field directed along the  $x$  axis. The electric potential  $V$  associated with the field is given in Fig. 24-57. The scale of the vertical axis is set by  $V_s = 500.0 \text{ V}$ . (a) Does the positron emerge from the field at  $x = 0$  (which means its motion is reversed) or at  $x = 0.50 \text{ m}$  (which means its motion is not reversed)? (b) What is its speed when it emerges?

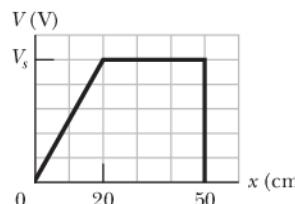


Figure 24-57 Problem 54.

**•55** An electron is projected with an initial speed of  $3.2 \times 10^5 \text{ m/s}$  directly toward a proton that is fixed in place. If the electron is initially a great distance from the proton, at what distance from the proton is the speed of the electron instantaneously equal to twice the initial value?

**•56** Particle 1 (with a charge of  $+5.0 \mu\text{C}$ ) and particle 2 (with a charge of  $+3.0 \mu\text{C}$ ) are fixed in place with separation  $d = 4.0 \text{ cm}$

on the  $x$  axis shown in Fig. 24-58a. Particle 3 can be moved along the  $x$  axis to the right of particle 2. Figure 24-58b gives the electric potential energy  $U$  of the three-particle system as a function of the  $x$  coordinate of particle 3. The scale of the vertical axis is set by  $U_s = 5.0 \text{ J}$ . What is the charge of particle 3?

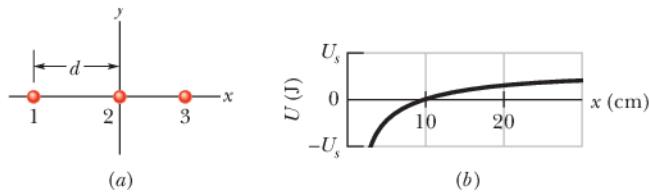


Figure 24-58 Problem 56.

- 57 SSM** Identical  $50 \mu\text{C}$  charges are fixed on an  $x$  axis at  $x = \pm 3.0 \text{ m}$ . A particle of charge  $q = -15 \mu\text{C}$  is then released from rest at a point on the positive part of the  $y$  axis. Due to the symmetry of the situation, the particle moves along the  $y$  axis and has kinetic energy  $1.2 \text{ J}$  as it passes through the point  $x = 0, y = 4.0 \text{ m}$ . (a) What is the kinetic energy of the particle as it passes through the origin? (b) At what negative value of  $y$  will the particle momentarily stop?

- 58 GO Proton in a well.** Figure 24-59 shows electric potential  $V$  along an  $x$  axis. The scale of the vertical axis is set by  $V_s = 10.0 \text{ V}$ . A proton is to be released at  $x = 3.5 \text{ cm}$  with initial kinetic energy  $4.00 \text{ eV}$ . (a) If it is initially moving in the negative direction of the axis, does it reach a turning point (if so, what is the  $x$  coordinate of that point) or does it escape from the plotted region (if so, what is its speed at  $x = 0$ )? (b) If it is initially moving in the positive direction of the axis, does it reach a turning point (if so, what is the  $x$  coordinate of that point) or does it escape from the plotted region (if so, what is its speed at  $x = 6.0 \text{ cm}$ )? What are the (c) magnitude  $F$  and (d) direction (positive or negative direction of the  $x$  axis) of the electric force on the proton if the proton moves just to the left of  $x = 3.0 \text{ cm}$ ? What are (e)  $F$  and (f) the direction if the proton moves just to the right of  $x = 5.0 \text{ cm}$ ?

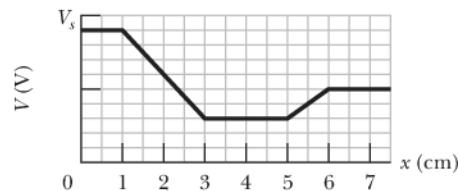
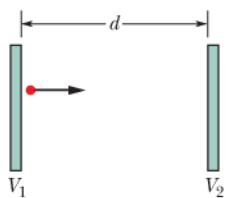


Figure 24-59 Problem 58.

- 59** In Fig. 24-60, a charged particle (either an electron or a proton) is moving rightward between two parallel charged plates separated by distance  $d = 2.00 \text{ mm}$ . The plate potentials are  $V_1 = -70.0 \text{ V}$  and  $V_2 = -50.0 \text{ V}$ . The particle is slowing from an initial speed of  $90.0 \text{ km/s}$  at the left plate. (a) Is the particle an electron or a proton? (b) What is its speed just as it reaches plate 2?

Figure 24-60  
Problem 59.

- 60** In Fig. 24-61a, we move an electron from an infinite distance to a point at distance  $R = 8.00 \text{ cm}$  from a tiny charged ball. The move requires work  $W = 2.16 \times 10^{-13} \text{ J}$  by us. (a) What is the charge  $Q$  on the ball? In Fig. 24-61b, the ball has been sliced up and the slices spread out so that an equal amount of charge is at the hour positions on a circular clock face of radius  $R = 8.00 \text{ cm}$ . Now the electron is brought from an infinite distance to the center of

the circle. (b) With that addition of the electron to the system of 12 charged particles, what is the change in the electric potential energy of the system?

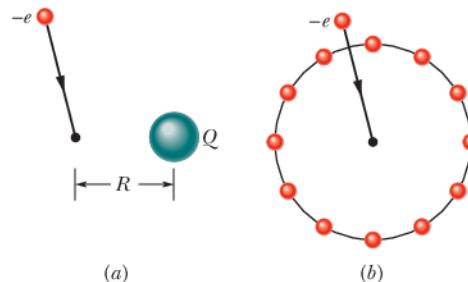


Figure 24-61 Problem 60.

- 61** Suppose  $N$  electrons can be placed in either of two configurations. In configuration 1, they are all placed on the circumference of a narrow ring of radius  $R$  and are uniformly distributed so that the distance between adjacent electrons is the same everywhere. In configuration 2,  $N - 1$  electrons are uniformly distributed on the ring and one electron is placed in the center of the ring. (a) What is the smallest value of  $N$  for which the second configuration is less energetic than the first? (b) For that value of  $N$ , consider any one circumference electron—call it  $e_0$ . How many other circumference electrons are closer to  $e_0$  than the central electron is?

#### Module 24-8 Potential of a Charged Isolated Conductor

- 62** Sphere 1 with radius  $R_1$  has positive charge  $q$ . Sphere 2 with radius  $2.00R_1$  is far from sphere 1 and initially uncharged. After the separated spheres are connected with a wire thin enough to retain only negligible charge, (a) is potential  $V_1$  of sphere 1 greater than, less than, or equal to potential  $V_2$  of sphere 2? What fraction of  $q$  ends up on (b) sphere 1 and (c) sphere 2? (d) What is the ratio  $\sigma_1/\sigma_2$  of the surface charge densities of the spheres?

- 63 SSM WWW** Two metal spheres, each of radius  $3.0 \text{ cm}$ , have a center-to-center separation of  $2.0 \text{ m}$ . Sphere 1 has charge  $+1.0 \times 10^{-8} \text{ C}$ ; sphere 2 has charge  $-3.0 \times 10^{-8} \text{ C}$ . Assume that the separation is large enough for us to say that the charge on each sphere is uniformly distributed (the spheres do not affect each other). With  $V = 0$  at infinity, calculate (a) the potential at the point halfway between the centers and the potential on the surface of (b) sphere 1 and (c) sphere 2.

- 64** A hollow metal sphere has a potential of  $+400 \text{ V}$  with respect to ground (defined to be at  $V = 0$ ) and a charge of  $5.0 \times 10^{-9} \text{ C}$ . Find the electric potential at the center of the sphere.

- 65 SSM** What is the excess charge on a conducting sphere of radius  $r = 0.15 \text{ m}$  if the potential of the sphere is  $1500 \text{ V}$  and  $V = 0$  at infinity?

- 66** Two isolated, concentric, conducting spherical shells have radii  $R_1 = 0.500 \text{ m}$  and  $R_2 = 1.00 \text{ m}$ , uniform charges  $q_1 = +2.00 \mu\text{C}$  and  $q_2 = +1.00 \mu\text{C}$ , and negligible thicknesses. What is the magnitude of the electric field  $E$  at radial distance (a)  $r = 4.00 \text{ m}$ , (b)  $r = 0.700 \text{ m}$ , and (c)  $r = 0.200 \text{ m}$ ? With  $V = 0$  at infinity, what is  $V$  at (d)  $r = 4.00 \text{ m}$ , (e)  $r = 1.00 \text{ m}$ , (f)  $r = 0.700 \text{ m}$ , (g)  $r = 0.500 \text{ m}$ , (h)  $r = 0.200 \text{ m}$ , and (i)  $r = 0$ ? (j) Sketch  $E(r)$  and  $V(r)$ .

- 67** A metal sphere of radius  $15 \text{ cm}$  has a net charge of  $3.0 \times 10^{-8} \text{ C}$ . (a) What is the electric field at the sphere's surface? (b) If  $V = 0$  at infinity, what is the electric potential at the sphere's surface? (c) At what distance from the sphere's surface has the electric potential decreased by  $500 \text{ V}$ ?

### Additional Problems

**68** Here are the charges and coordinates of two charged particles located in an  $xy$  plane:  $q_1 = +3.00 \times 10^{-6} \text{ C}$ ,  $x = +3.50 \text{ cm}$ ,  $y = +0.500 \text{ cm}$  and  $q_2 = -4.00 \times 10^{-6} \text{ C}$ ,  $x = -2.00 \text{ cm}$ ,  $y = +1.50 \text{ cm}$ . How much work must be done to locate these charges at their given positions, starting from infinite separation?

**69 SSM** A long, solid, conducting cylinder has a radius of 2.0 cm. The electric field at the surface of the cylinder is 160 N/C, directed radially outward. Let  $A$ ,  $B$ , and  $C$  be points that are 1.0 cm, 2.0 cm, and 5.0 cm, respectively, from the central axis of the cylinder. What are (a) the magnitude of the electric field at  $C$  and the electric potential differences (b)  $V_B - V_C$  and (c)  $V_A - V_B$ ?

**70** ~~SSM~~ *The chocolate crumb mystery.* This story begins with Problem 60 in Chapter 23. (a) From the answer to part (a) of that problem, find an expression for the electric potential as a function of the radial distance  $r$  from the center of the pipe. (The electric potential is zero on the grounded pipe wall.) (b) For the typical volume charge density  $\rho = -1.1 \times 10^{-3} \text{ C/m}^3$ , what is the difference in the electric potential between the pipe's center and its inside wall? (The story continues with Problem 60 in Chapter 25.)

**71 SSM** Starting from Eq. 24-30, derive an expression for the electric field due to a dipole at a point on the dipole axis.

**72** The magnitude  $E$  of an electric field depends on the radial distance  $r$  according to  $E = A/r^4$ , where  $A$  is a constant with the unit volt-cubic meter. As a multiple of  $A$ , what is the magnitude of the electric potential difference between  $r = 2.00 \text{ m}$  and  $r = 3.00 \text{ m}$ ?

**73** (a) If an isolated conducting sphere 10 cm in radius has a net charge of  $4.0 \mu\text{C}$  and if  $V = 0$  at infinity, what is the potential on the surface of the sphere? (b) Can this situation actually occur, given that the air around the sphere undergoes electrical breakdown when the field exceeds 3.0 MV/m?

**74** Three particles, charge  $q_1 = +10 \mu\text{C}$ ,  $q_2 = -20 \mu\text{C}$ , and  $q_3 = +30 \mu\text{C}$ , are positioned at the vertices of an isosceles triangle as shown in Fig. 24-62. If  $a = 10 \text{ cm}$  and  $b = 6.0 \text{ cm}$ , how much work must an external agent do to exchange the positions of (a)  $q_1$  and  $q_3$  and, instead, (b)  $q_1$  and  $q_2$ ?

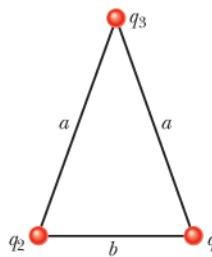


Figure 24-62  
Problem 74.

**75** An electric field of approximately 100 V/m is often observed near the surface of Earth. If this were the field over the entire surface, what would be the electric potential of a point on the surface? (Set  $V = 0$  at infinity.)

**76** A Gaussian sphere of radius 4.00 cm is centered on a ball that has a radius of 1.00 cm and a uniform charge distribution. The total (net) electric flux through the surface of the Gaussian sphere is  $+5.60 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C}$ . What is the electric potential 12.0 cm from the center of the ball?

**77** In a Millikan oil-drop experiment (Module 22-6), a uniform electric field of  $1.92 \times 10^5 \text{ N/C}$  is maintained in the region between two plates separated by 1.50 cm. Find the potential difference between the plates.

**78** Figure 24-63 shows three circular, nonconducting arcs of radius  $R = 8.50 \text{ cm}$ . The charges on the arcs are  $q_1 = 4.52 \text{ pC}$ ,

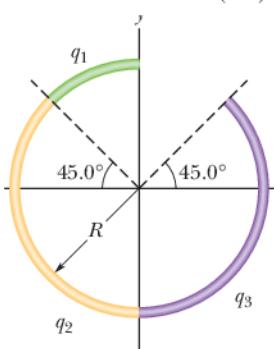


Figure 24-63 Problem 78.

$\text{pC}$ ,  $q_2 = -2.00q_1$ ,  $q_3 = +3.00q_1$ . With  $V = 0$  at infinity, what is the net electric potential of the arcs at the common center of curvature?

**79** An electron is released from rest on the axis of an electric dipole that has charge  $e$  and charge separation  $d = 20 \text{ pm}$  and that is fixed in place. The release point is on the positive side of the dipole, at distance  $7.0d$  from the dipole center. What is the electron's speed when it reaches a point  $5.0d$  from the dipole center?

**80** Figure 24-64 shows a ring of outer radius  $R = 13.0 \text{ cm}$ , inner radius  $r = 0.200R$ , and uniform surface charge density  $\sigma = 6.20 \text{ pC/m}^2$ . With  $V = 0$  at infinity, find the electric potential at point  $P$  on the central axis of the ring, at distance  $z = 2.00R$  from the center of the ring.

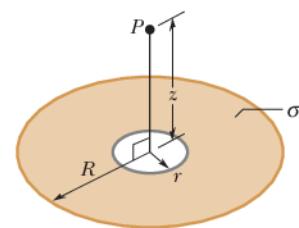


Figure 24-64 Problem 80.

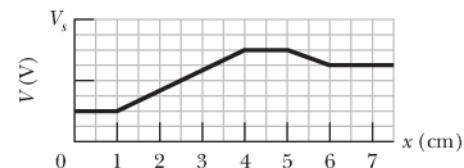


Figure 24-65 Problem 81.

**81 GO** *Electron in a well.* Figure 24-65 shows electric potential  $V$  along an  $x$  axis. The scale of the vertical axis is set by  $V_s = 8.0 \text{ V}$ . An electron is to be released at  $x = 4.5 \text{ cm}$  with initial kinetic energy 3.00 eV. (a) If it is initially moving in the negative direction of the axis, does it reach a turning point (if so, what is the  $x$  coordinate of that point) or does it escape from the plotted region (if so, what is its speed at  $x = 0$ )? (b) If it is initially moving in the positive direction of the axis, does it reach a turning point (if so, what is the  $x$  coordinate of that point) or does it escape from the plotted region (if so, what is its speed at  $x = 7.0 \text{ cm}$ )? What are the (c) magnitude  $F$  and (d) direction (positive or negative direction of the  $x$  axis) of the electric force on the electron if the electron moves just to the left of  $x = 4.0 \text{ cm}$ ? What are (e)  $F$  and (f) the direction if it moves just to the right of  $x = 5.0 \text{ cm}$ ?

**82** (a) If Earth had a uniform surface charge density of  $1.0 \text{ electron/m}^2$  (a very artificial assumption), what would its potential be? (Set  $V = 0$  at infinity.) What would be the (b) magnitude and (c) direction (radially inward or outward) of the electric field due to Earth just outside its surface?

**83** In Fig. 24-66, point  $P$  is at distance  $d_1 = 4.00 \text{ m}$  from particle 1 ( $q_1 = -2e$ ) and distance  $d_2 = 2.00 \text{ m}$  from particle 2 ( $q_2 = +2e$ ), with both particles fixed in place. (a) With  $V = 0$  at infinity, what is  $V$  at  $P$ ? If we bring a particle of charge  $q_3 = +2e$  from infinity to  $P$ , (b) how much work do we do and (c) what is the potential energy of the three-particle system?

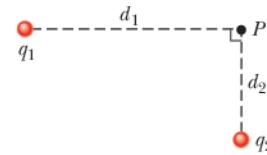


Figure 24-66 Problem 83.

**84** A solid conducting sphere of radius 3.0 cm has a charge of  $30 \text{ nC}$  distributed uniformly over its surface. Let  $A$  be a point 1.0 cm from the center of the sphere,  $S$  be a point on the surface of the sphere, and  $B$  be a point 5.0 cm from the center of the sphere. What are the electric potential differences (a)  $V_S - V_B$  and (b)  $V_A - V_B$ ?

**85** In Fig. 24-67, we move a particle of charge  $+2e$  in from infinity to the  $x$  axis. How much work do we do? Distance  $D$  is 4.00 m.

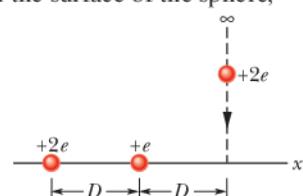


Figure 24-67 Problem 85.

**86** Figure 24-68 shows a hemisphere with a charge of  $4.00 \mu\text{C}$  distributed uniformly through its volume. The hemisphere lies on an  $xy$  plane the way half a grapefruit might lie face down on a kitchen table. Point  $P$  is located on the plane, along a radial line from the hemisphere's center of curvature, at radial distance 15 cm. What is the electric potential at point  $P$  due to the hemisphere?

**87 SSM** Three  $+0.12 \text{ C}$  charges form an equilateral triangle 1.7 m on a side. Using energy supplied at the rate of  $0.83 \text{ kW}$ , how many days would be required to move one of the charges to the midpoint of the line joining the other two charges?

**88** Two charges  $q = +2.0 \mu\text{C}$  are fixed a distance  $d = 2.0 \text{ cm}$  apart (Fig. 24-69). (a) With  $V = 0$  at infinity, what is the electric potential at point  $C$ ? (b) You bring a third charge  $q = +2.0 \mu\text{C}$  from infinity to  $C$ . How much work must you do? (c) What is the potential energy  $U$  of the three-charge configuration when the third charge is in place?

**89** Initially two electrons are fixed in place with a separation of  $2.00 \mu\text{m}$ . How much work must we do to bring a third electron in from infinity to complete an equilateral triangle?

**90** A particle of positive charge  $Q$  is fixed at point  $P$ . A second particle of mass  $m$  and negative charge  $-q$  moves at constant speed in a circle of radius  $r_1$ , centered at  $P$ . Derive an expression for the work  $W$  that must be done by an external agent on the second particle to increase the radius of the circle of motion to  $r_2$ .

**91** Two charged, parallel, flat conducting surfaces are spaced  $d = 1.00 \text{ cm}$  apart and produce a potential difference  $\Delta V = 625 \text{ V}$  between them. An electron is projected from one surface directly toward the second. What is the initial speed of the electron if it stops just at the second surface?

**92** In Fig. 24-70, point  $P$  is at the center of the rectangle. With  $V = 0$  at infinity,  $q_1 = 5.00 \text{ fC}$ ,  $q_2 = 2.00 \text{ fC}$ ,  $q_3 = 3.00 \text{ fC}$ , and  $d = 2.54 \text{ cm}$ , what is the net electric potential at  $P$  due to the six charged particles?

**93 SSM** A uniform charge of  $+16.0 \mu\text{C}$  is on a thin circular ring lying in an  $xy$  plane and centered on the origin. The ring's radius is 3.00 cm. If point  $A$  is at the origin and point  $B$  is on the  $z$  axis at  $z = 4.00 \text{ cm}$ , what is  $V_B - V_A$ ?

**94** Consider a particle with charge  $q = 1.50 \times 10^{-8} \text{ C}$ , and take  $V = 0$  at infinity. (a) What are the shape and dimensions of an equipotential surface having a potential of 30.0 V due to  $q$  alone? (b) Are surfaces whose potentials differ by a constant amount (1.0 V, say) evenly spaced?

**95 SSM** A thick spherical shell of charge  $Q$  and uniform volume charge density  $\rho$  is bounded by radii  $r_1$  and  $r_2 > r_1$ . With  $V = 0$  at infinity, find the electric potential  $V$  as a function of distance  $r$  from the center of the distribution, considering regions (a)  $r > r_2$ , (b)  $r_2 > r > r_1$ , and (c)  $r < r_1$ . (d) Do these solutions agree with each other at  $r = r_2$  and  $r = r_1$ ? (Hint: See Module 23-6.)

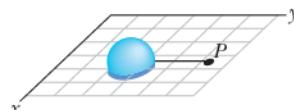


Figure 24-68 Problem 86.

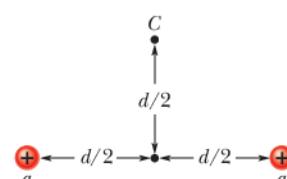


Figure 24-69 Problem 88.

**96** A charge  $q$  is distributed uniformly throughout a spherical volume of radius  $R$ . Let  $V = 0$  at infinity. What are (a)  $V$  at radial distance  $r < R$  and (b) the potential difference between points at  $r = R$  and the point at  $r = 0$ ?

**97 SSM** A solid copper sphere whose radius is 1.0 cm has a very thin surface coating of nickel. Some of the nickel atoms are radioactive, each atom emitting an electron as it decays. Half of these electrons enter the copper sphere, each depositing 100 keV of energy there. The other half of the electrons escape, each carrying away a charge  $-e$ . The nickel coating has an activity of  $3.70 \times 10^8$  radioactive decays per second. The sphere is hung from a long, nonconducting string and isolated from its surroundings. (a) How long will it take for the potential of the sphere to increase by 1000 V? (b) How long will it take for the temperature of the sphere to increase by 5.0 K due to the energy deposited by the electrons? The heat capacity of the sphere is 14 J/K.

**98** In Fig. 24-71, a metal sphere with charge  $q = 5.00 \mu\text{C}$  and radius  $r = 3.00 \text{ cm}$  is concentric with a larger metal sphere with charge  $Q = 15.0 \mu\text{C}$  and radius  $R = 6.00 \text{ cm}$ . (a) What is the potential difference between the spheres? If we connect the spheres with a wire, what then is the charge on (b) the smaller sphere and (c) the larger sphere?

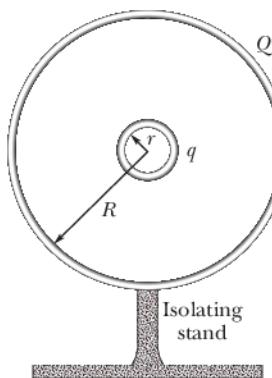


Figure 24-71 Problem 98.

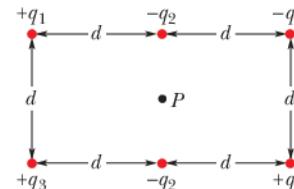


Figure 24-70 Problem 92.

**99** (a) Using Eq. 24-32, show that the electric potential at a point on the central axis of a thin ring (of charge  $q$  and radius  $R$ ) and at distance  $z$  from the ring is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{z^2 + R^2}}.$$

(b) From this result, derive an expression for the electric field magnitude  $E$  at points on the ring's axis; compare your result with the calculation of  $E$  in Module 22-4.

**100** An alpha particle (which has two protons) is sent directly toward a target nucleus containing 92 protons. The alpha particle has an initial kinetic energy of 0.48 pJ. What is the least center-to-center distance the alpha particle will be from the target nucleus, assuming the nucleus does not move?

**101** In the quark model of fundamental particles, a proton is composed of three quarks: two "up" quarks, each having charge  $+2e/3$ , and one "down" quark, having charge  $-e/3$ . Suppose that the three quarks are equidistant from one another. Take that separation distance to be  $1.32 \times 10^{-15} \text{ m}$  and calculate the electric potential energy of the system of (a) only the two up quarks and (b) all three quarks.

**102** A charge of  $1.50 \times 10^{-8} \text{ C}$  lies on an isolated metal sphere of radius 16.0 cm. With  $V = 0$  at infinity, what is the electric potential at points on the sphere's surface?

**103** In Fig. 24-72, two particles of charges  $q_1$  and  $q_2$  are fixed to an  $x$  axis. If a third particle, of charge  $+6.0 \mu\text{C}$ , is brought from an infinite distance to point  $P$ , the three-particle system has the same electric potential energy as the original two-particle system. What is the charge ratio  $q_1/q_2$ ?

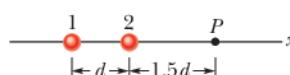


Figure 24-72 Problem 103.

# Capacitance

## 25-1 CAPACITANCE

### Learning Objectives

After reading this module, you should be able to . . .

**25.01** Sketch a schematic diagram of a circuit with a parallel-plate capacitor, a battery, and an open or closed switch.

**25.02** In a circuit with a battery, an open switch, and an uncharged capacitor, explain what happens to the conduction electrons when the switch is closed.

**25.03** For a capacitor, apply the relationship between the magnitude of charge  $q$  on either plate ("the charge on the capacitor"), the potential difference  $V$  between the plates ("the potential across the capacitor"), and the capacitance  $C$  of the capacitor.

### Key Ideas

- A capacitor consists of two isolated conductors (the plates) with charges  $+q$  and  $-q$ . Its capacitance  $C$  is defined from

$$q = CV,$$

where  $V$  is the potential difference between the plates.

- When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.

### What Is Physics?

One goal of physics is to provide the basic science for practical devices designed by engineers. The focus of this chapter is on one extremely common example—the capacitor, a device in which electrical energy can be stored. For example, the batteries in a camera store energy in the photoflash unit by charging a capacitor. The batteries can supply energy at only a modest rate, too slowly for the photoflash unit to emit a flash of light. However, once the capacitor is charged, it can supply energy at a much greater rate when the photoflash unit is triggered—enough energy to allow the unit to emit a burst of bright light.

The physics of capacitors can be generalized to other devices and to any situation involving electric fields. For example, Earth's atmospheric electric field is modeled by meteorologists as being produced by a huge spherical capacitor that partially discharges via lightning. The charge that skis collect as they slide along snow can be modeled as being stored in a capacitor that frequently discharges as sparks (which can be seen by nighttime skiers on dry snow).

The first step in our discussion of capacitors is to determine how much charge can be stored. This "how much" is called capacitance.

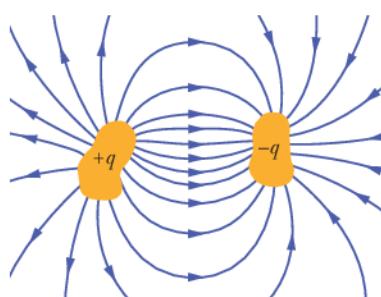
### Capacitance

Figure 25-1 shows some of the many sizes and shapes of capacitors. Figure 25-2 shows the basic elements of *any* capacitor—two isolated conductors of any



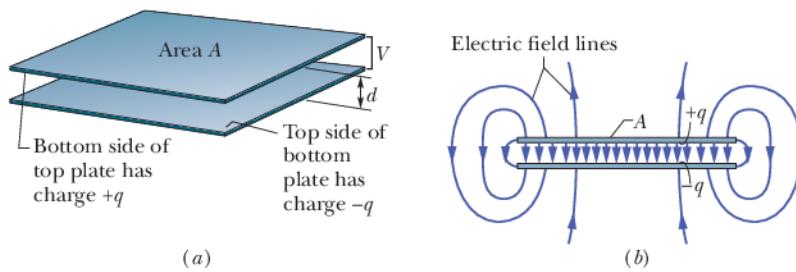
Paul Silvermann/Fundamental Photographs

**Figure 25-1** An assortment of capacitors.



**Figure 25-2** Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor*. When the capacitor is charged, the charges on the conductors, or *plates* as they are called, have the same magnitude  $q$  but opposite signs.

**Figure 25-3** (a) A parallel-plate capacitor, made up of two plates of area  $A$  separated by a distance  $d$ . The charges on the facing plate surfaces have the same magnitude  $q$  but opposite signs. (b) As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the “fringing” of the field lines there.



shape. No matter what their geometry, flat or not, we call these conductors *plates*.

Figure 25-3a shows a less general but more conventional arrangement, called a *parallel-plate capacitor*, consisting of two parallel conducting plates of area  $A$  separated by a distance  $d$ . The symbol we use to represent a capacitor ( $\text{+} \text{-}$ ) is based on the structure of a parallel-plate capacitor but is used for capacitors of all geometries. We assume for the time being that no material medium (such as glass or plastic) is present in the region between the plates. In Module 25-5, we shall remove this restriction.

When a capacitor is *charged*, its plates have charges of equal magnitudes but opposite signs:  $+q$  and  $-q$ . However, we refer to the *charge of a capacitor* as being  $q$ , the absolute value of these charges on the plates. (Note that  $q$  is not the net charge on the capacitor, which is zero.)

Because the plates are conductors, they are equipotential surfaces; all points on a plate are at the same electric potential. Moreover, there is a potential difference between the two plates. For historical reasons, we represent the absolute value of this potential difference with  $V$  rather than with the  $\Delta V$  we used in previous notation.

The charge  $q$  and the potential difference  $V$  for a capacitor are proportional to each other; that is,

$$q = CV. \quad (25-1)$$

The proportionality constant  $C$  is called the **capacitance** of the capacitor. Its value depends only on the geometry of the plates and *not* on their charge or potential difference. The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: The *greater the capacitance, the more charge is required*.

The SI unit of capacitance that follows from Eq. 25-1 is the coulomb per volt. This unit occurs so often that it is given a special name, the **farad** (F):

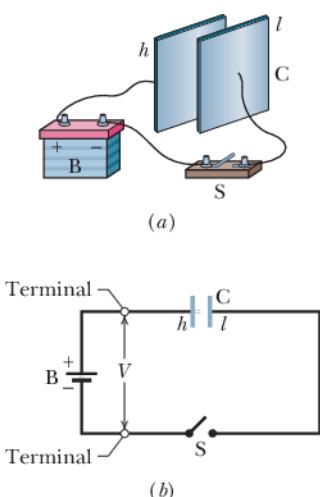
$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}. \quad (25-2)$$

As you will see, the farad is a very large unit. Submultiples of the farad, such as the microfarad ( $1 \mu\text{F} = 10^{-6} \text{ F}$ ) and the picofarad ( $1 \text{ pF} = 10^{-12} \text{ F}$ ), are more convenient units in practice.

### Charging a Capacitor

One way to charge a capacitor is to place it in an electric circuit with a battery. An *electric circuit* is a path through which charge can flow. A *battery* is a device that maintains a certain potential difference between its *terminals* (points at which charge can enter or leave the battery) by means of internal electrochemical reactions in which electric forces can move internal charge.

In Fig. 25-4a, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit. The same circuit is shown in the *schematic diagram* of Fig. 25-4b, in which the symbols for a battery, a switch, and a capacitor represent those devices. The battery maintains potential difference  $V$  between its terminals. The terminal of higher potential is labeled + and is often called the *positive terminal*; the terminal of lower potential is labeled - and is often called the *negative terminal*.



**Figure 25-4** (a) Battery B, switch S, and plates  $h$  and  $l$  of capacitor C, connected in a circuit. (b) A schematic diagram with the circuit elements represented by their symbols.

The circuit shown in Figs. 25-4a and b is said to be *incomplete* because switch S is *open*; that is, the switch does not electrically connect the wires attached to it. When the switch is *closed*, electrically connecting those wires, the circuit is complete and charge can then flow through the switch and the wires. As we discussed in Chapter 21, the charge that can flow through a conductor, such as a wire, is that of electrons. When the circuit of Fig. 25-4 is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires. The field drives electrons from capacitor plate *h* to the positive terminal of the battery; thus, plate *h*, losing electrons, becomes positively charged. The field drives just as many electrons from the negative terminal of the battery to capacitor plate *l*; thus, plate *l*, gaining electrons, becomes negatively charged *just as much* as plate *h*, losing electrons, becomes positively charged.

Initially, when the plates are uncharged, the potential difference between them is zero. As the plates become oppositely charged, that potential difference increases until it equals the potential difference *V* between the terminals of the battery. Then plate *h* and the positive terminal of the battery are at the same potential, and there is no longer an electric field in the wire between them. Similarly, plate *l* and the negative terminal reach the same potential, and there is then no electric field in the wire between them. Thus, with the field zero, there is no further drive of electrons. The capacitor is then said to be *fully charged*, with a potential difference *V* and charge *q* that are related by Eq. 25-1.

In this book we assume that during the charging of a capacitor and afterward, charge cannot pass from one plate to the other across the gap separating them. Also, we assume that a capacitor can retain (or *store*) charge indefinitely, until it is put into a circuit where it can be *discharged*.



### Checkpoint 1

Does the capacitance *C* of a capacitor increase, decrease, or remain the same (a) when the charge *q* on it is doubled and (b) when the potential difference *V* across it is tripled?

## 25-2 CALCULATING THE CAPACITANCE

### Learning Objectives

After reading this module, you should be able to . . .

**25.04** Explain how Gauss' law is used to find the capacitance of a parallel-plate capacitor.

**25.05** For a parallel-plate capacitor, a cylindrical capacitor, a spherical capacitor, and an isolated sphere, calculate the capacitance.

### Key Ideas

- We generally determine the capacitance of a particular capacitor configuration by (1) assuming a charge *q* to have been placed on the plates, (2) finding the electric field  $\vec{E}$  due to this charge, (3) evaluating the potential difference *V* between the plates, and (4) calculating *C* from  $q = CV$ . Some results are the following:

- A parallel-plate capacitor with flat parallel plates of area *A* and spacing *d* has capacitance

$$C = \frac{\epsilon_0 A}{d}.$$

- A cylindrical capacitor (two long coaxial cylinders) of length

*L* and radii *a* and *b* has capacitance

$$C = 2\pi\epsilon_0 \frac{L}{\ln(\frac{b}{a})}.$$

A spherical capacitor with concentric spherical plates of radii *a* and *b* has capacitance

$$C = 4\pi\epsilon_0 \frac{ab}{b - a}.$$

An isolated sphere of radius *R* has capacitance

$$C = 4\pi\epsilon_0 R.$$

## Calculating the Capacitance

Our goal here is to calculate the capacitance of a capacitor once we know its geometry. Because we shall consider a number of different geometries, it seems wise to develop a general plan to simplify the work. In brief our plan is as follows: (1) Assume a charge  $q$  on the plates; (2) calculate the electric field  $\vec{E}$  between the plates in terms of this charge, using Gauss' law; (3) knowing  $\vec{E}$ , calculate the potential difference  $V$  between the plates from Eq. 24-18; (4) calculate  $C$  from Eq. 25-1.

Before we start, we can simplify the calculation of both the electric field and the potential difference by making certain assumptions. We discuss each in turn.

### Calculating the Electric Field

To relate the electric field  $\vec{E}$  between the plates of a capacitor to the charge  $q$  on either plate, we shall use Gauss' law:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \quad (25-3)$$

Here  $q$  is the charge enclosed by a Gaussian surface and  $\oint \vec{E} \cdot d\vec{A}$  is the net electric flux through that surface. In all cases that we shall consider, the Gaussian surface will be such that whenever there is an electric flux through it,  $\vec{E}$  will have a uniform magnitude  $E$  and the vectors  $\vec{E}$  and  $d\vec{A}$  will be parallel. Equation 25-3 then reduces to

$$q = \epsilon_0 E A \quad (\text{special case of Eq. 25-3}), \quad (25-4)$$

in which  $A$  is the area of that part of the Gaussian surface through which there is a flux. For convenience, we shall always draw the Gaussian surface in such a way that it completely encloses the charge on the positive plate; see Fig. 25-5 for an example.

### Calculating the Potential Difference

In the notation of Chapter 24 (Eq. 24-18), the potential difference between the plates of a capacitor is related to the field  $\vec{E}$  by

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}, \quad (25-5)$$

in which the integral is to be evaluated along any path that starts on one plate and ends on the other. We shall always choose a path that follows an electric field line, from the negative plate to the positive plate. For this path, the vectors  $\vec{E}$  and  $d\vec{s}$  will have opposite directions; so the dot product  $\vec{E} \cdot d\vec{s}$  will be equal to  $-E ds$ . Thus, the right side of Eq. 25-5 will then be positive. Letting  $V$  represent the difference  $V_f - V_i$ , we can then recast Eq. 25-5 as

$$V = \int_{-}^{+} E ds \quad (\text{special case of Eq. 25-5}), \quad (25-6)$$

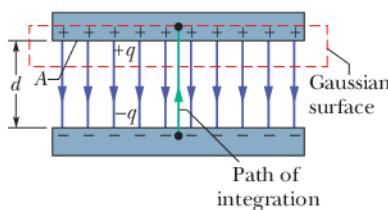
in which the  $-$  and  $+$  remind us that our path of integration starts on the negative plate and ends on the positive plate.

We are now ready to apply Eqs. 25-4 and 25-6 to some particular cases.

### A Parallel-Plate Capacitor

We assume, as Fig. 25-5 suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric field

We use Gauss' law to relate  $q$  and  $E$ . Then we integrate the  $E$  to get the potential difference.



**Figure 25-5** A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration of Eq. 25-6 is taken along a path extending directly from the negative plate to the positive plate.

at the edges of the plates, taking  $\vec{E}$  to be constant throughout the region between the plates.

We draw a Gaussian surface that encloses just the charge  $q$  on the positive plate, as in Fig. 25-5. From Eq. 25-4 we can then write

$$q = \epsilon_0 E A, \quad (25-7)$$

where  $A$  is the area of the plate.

Equation 25-6 yields

$$V = \int_{-}^{+} E ds = E \int_0^d ds = Ed. \quad (25-8)$$

In Eq. 25-8,  $E$  can be placed outside the integral because it is a constant; the second integral then is simply the plate separation  $d$ .

If we now substitute  $q$  from Eq. 25-7 and  $V$  from Eq. 25-8 into the relation  $q = CV$  (Eq. 25-1), we find

$$C = \frac{\epsilon_0 A}{d} \quad (\text{parallel-plate capacitor}). \quad (25-9)$$

Thus, the capacitance does indeed depend only on geometrical factors—namely, the plate area  $A$  and the plate separation  $d$ . Note that  $C$  increases as we increase area  $A$  or decrease separation  $d$ .

As an aside, we point out that Eq. 25-9 suggests one of our reasons for writing the electrostatic constant in Coulomb's law in the form  $1/4\pi\epsilon_0$ . If we had not done so, Eq. 25-9—which is used more often in engineering practice than Coulomb's law—would have been less simple in form. We note further that Eq. 25-9 permits us to express the permittivity constant  $\epsilon_0$  in a unit more appropriate for use in problems involving capacitors; namely,

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m} = 8.85 \text{ pF/m}. \quad (25-10)$$

We have previously expressed this constant as

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2. \quad (25-11)$$

### A Cylindrical Capacitor

Figure 25-6 shows, in cross section, a cylindrical capacitor of length  $L$  formed by two coaxial cylinders of radii  $a$  and  $b$ . We assume that  $L \gg b$  so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude  $q$ .

As a Gaussian surface, we choose a cylinder of length  $L$  and radius  $r$ , closed by end caps and placed as is shown in Fig. 25-6. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge  $q$  on that cylinder. Equation 25-4 then relates that charge and the field magnitude  $E$  as

$$q = \epsilon_0 E A = \epsilon_0 E (2\pi r L),$$

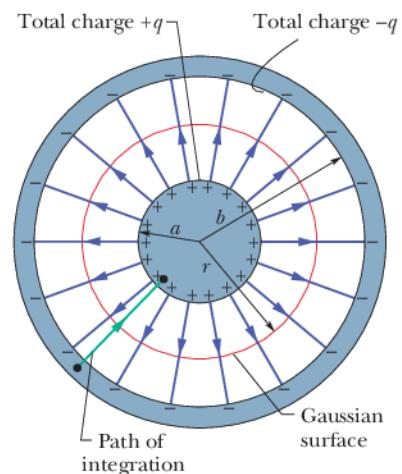
in which  $2\pi r L$  is the area of the curved part of the Gaussian surface. There is no flux through the end caps. Solving for  $E$  yields

$$E = \frac{q}{2\pi\epsilon_0 r L}. \quad (25-12)$$

Substitution of this result into Eq. 25-6 yields

$$V = \int_{-}^{+} E ds = -\frac{q}{2\pi\epsilon_0 L} \int_b^a \frac{dr}{r} = \frac{q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right), \quad (25-13)$$

where we have used the fact that here  $ds = -dr$  (we integrated radially inward).



**Figure 25-6** A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius  $r$  (that encloses the positive plate) and the radial path of integration along which Eq. 25-6 is to be applied. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

From the relation  $C = q/V$ , we then have

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)} \quad (\text{cylindrical capacitor}). \quad (25-14)$$

We see that the capacitance of a cylindrical capacitor, like that of a parallel-plate capacitor, depends only on geometrical factors, in this case the length  $L$  and the two radii  $b$  and  $a$ .

### A Spherical Capacitor

Figure 25-6 can also serve as a central cross section of a capacitor that consists of two concentric spherical shells, of radii  $a$  and  $b$ . As a Gaussian surface we draw a sphere of radius  $r$  concentric with the two shells; then Eq. 25-4 yields

$$q = \epsilon_0 EA = \epsilon_0 E(4\pi r^2),$$

in which  $4\pi r^2$  is the area of the spherical Gaussian surface. We solve this equation for  $E$ , obtaining

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}, \quad (25-15)$$

which we recognize as the expression for the electric field due to a uniform spherical charge distribution (Eq. 23-15).

If we substitute this expression into Eq. 25-6, we find

$$V = \int_{-}^{+} E ds = -\frac{q}{4\pi\epsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{b} \right) = \frac{q}{4\pi\epsilon_0} \frac{b-a}{ab}, \quad (25-16)$$

where again we have substituted  $-dr$  for  $ds$ . If we now substitute Eq. 25-16 into Eq. 25-1 and solve for  $C$ , we find

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} \quad (\text{spherical capacitor}). \quad (25-17)$$

### An Isolated Sphere

We can assign a capacitance to a *single* isolated spherical conductor of radius  $R$  by assuming that the “missing plate” is a conducting sphere of infinite radius. After all, the field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

To find the capacitance of the conductor, we first rewrite Eq. 25-17 as

$$C = 4\pi\epsilon_0 \frac{a}{1-a/b}.$$

If we then let  $b \rightarrow \infty$  and substitute  $R$  for  $a$ , we find

$$C = 4\pi\epsilon_0 R \quad (\text{isolated sphere}). \quad (25-18)$$

Note that this formula and the others we have derived for capacitance (Eqs. 25-9, 25-14, and 25-17) involve the constant  $\epsilon_0$  multiplied by a quantity that has the dimensions of a length.



### Checkpoint 2

For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.



### Sample Problem 25.01 Charging the plates in a parallel-plate capacitor

In Fig. 25-7a, switch S is closed to connect the uncharged capacitor of capacitance  $C = 0.25 \mu\text{F}$  to the battery of potential difference  $V = 12 \text{ V}$ . The lower capacitor plate has thickness  $L = 0.50 \text{ cm}$  and face area  $A = 2.0 \times 10^{-4} \text{ m}^2$ , and it consists of copper, in which the density of conduction electrons is  $n = 8.49 \times 10^{28} \text{ electrons/m}^3$ . From what depth  $d$  within the plate (Fig. 25-7b) must electrons move to the plate face as the capacitor becomes charged?

#### KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ( $q = CV$ ).

**Calculations:** Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge

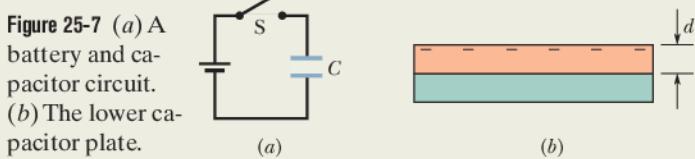
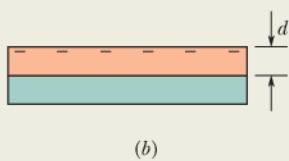
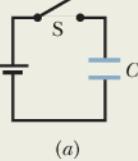


Figure 25-7 (a) A

battery and ca-  
pacitor circuit.  
(b) The lower ca-  
pacitor plate.



(b)



Additional examples, video, and practice available at WileyPLUS

## 25-3 CAPACITORS IN PARALLEL AND IN SERIES

#### Learning Objectives

After reading this module, you should be able to . . .

**25.06** Sketch schematic diagrams for a battery and (a) three capacitors in parallel and (b) three capacitors in series.

**25.07** Identify that capacitors in parallel have the same potential difference, which is the same value that their equivalent capacitor has.

**25.08** Calculate the equivalent of parallel capacitors.

**25.09** Identify that the total charge stored on parallel capacitors is the sum of the charges stored on the individual capacitors.

**25.10** Identify that capacitors in series have the same charge, which is the same value that their equivalent capacitor has.

**25.11** Calculate the equivalent of series capacitors.

**25.12** Identify that the potential applied to capacitors in series is equal to the sum of the potentials across the individual capacitors.

#### Key Idea

- The equivalent capacitances  $C_{\text{eq}}$  of combinations of individual capacitors connected in parallel and in series can be found from

$$C_{\text{eq}} = \sum_{j=1}^n C_j \quad (n \text{ capacitors in parallel})$$

magnitude that collects there is

$$\begin{aligned} q &= CV = (0.25 \times 10^{-6} \text{ F})(12 \text{ V}) \\ &= 3.0 \times 10^{-6} \text{ C}. \end{aligned}$$

Dividing this result by  $e$  gives us the number  $N$  of conduction electrons that come up to the face:

$$\begin{aligned} N &= \frac{q}{e} = \frac{3.0 \times 10^{-6} \text{ C}}{1.602 \times 10^{-19} \text{ C}} \\ &= 1.873 \times 10^{13} \text{ electrons}. \end{aligned}$$

These electrons come from a volume that is the product of the face area  $A$  and the depth  $d$  we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$\begin{aligned} d &= \frac{N}{An} = \frac{1.873 \times 10^{13} \text{ electrons}}{(2.0 \times 10^{-4} \text{ m}^2)(8.49 \times 10^{28} \text{ electrons/m}^3)} \\ &= 1.1 \times 10^{-12} \text{ m} = 1.1 \text{ pm}. \end{aligned} \quad (\text{Answer})$$

We commonly say that electrons move from the battery to the negative face but, actually, the battery sets up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

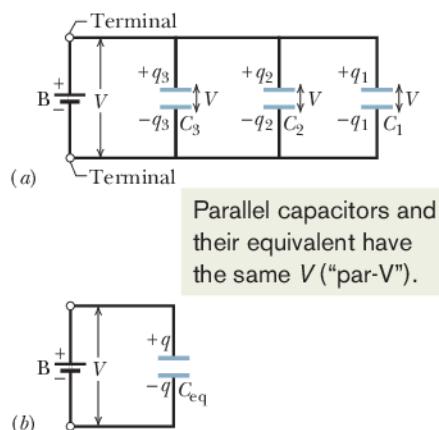
**25.13** For a circuit with a battery and some capacitors in parallel and some in series, simplify the circuit in steps by finding equivalent capacitors, until the charge and potential on the final equivalent capacitor can be determined, and then reverse the steps to find the charge and potential on the individual capacitors.

**25.14** For a circuit with a battery, an open switch, and one or more uncharged capacitors, determine the amount of charge that moves through a point in the circuit when the switch is closed.

**25.15** When a charged capacitor is connected in parallel to one or more uncharged capacitors, determine the charge and potential difference on each capacitor when equilibrium is reached.

and  $\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j} \quad (n \text{ capacitors in series}).$

Equivalent capacitances can be used to calculate the capacitances of more complicated series–parallel combinations.



**Figure 25-8** (a) Three capacitors connected in parallel to battery B. The battery maintains potential difference  $V$  across its terminals and thus across *each* capacitor. (b) The equivalent capacitor, with capacitance  $C_{eq}$ , replaces the parallel combination.

## Capacitors in Parallel and in Series

When there is a combination of capacitors in a circuit, we can sometimes replace that combination with an **equivalent capacitor**—that is, a single capacitor that has the same capacitance as the actual combination of capacitors. With such a replacement, we can simplify the circuit, affording easier solutions for unknown quantities of the circuit. Here we discuss two basic combinations of capacitors that allow such a replacement.

### Capacitors in Parallel

Figure 25-8a shows an electric circuit in which three capacitors are connected *in parallel* to battery B. This description has little to do with how the capacitor plates are drawn. Rather, “in parallel” means that the capacitors are directly wired together at one plate and directly wired together at the other plate, and that the same potential difference  $V$  is applied across the two groups of wired-together plates. Thus, each capacitor has the same potential difference  $V$ , which produces charge on the capacitor. (In Fig. 25-8a, the applied potential  $V$  is maintained by the battery.) In general:



When a potential difference  $V$  is applied across several capacitors connected in parallel, that potential difference  $V$  is applied across each capacitor. The total charge  $q$  stored on the capacitors is the sum of the charges stored on all the capacitors.

When we analyze a circuit of capacitors in parallel, we can simplify it with this mental replacement:



Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge  $q$  and the same potential difference  $V$  as the actual capacitors.

(You might remember this result with the nonsense word “par-V,” which is close to “party,” to mean “capacitors in parallel have the same  $V$ .”) Figure 25-8b shows the equivalent capacitor (with equivalent capacitance  $C_{eq}$ ) that has replaced the three capacitors (with actual capacitances  $C_1$ ,  $C_2$ , and  $C_3$ ) of Fig. 25-8a.

To derive an expression for  $C_{eq}$  in Fig. 25-8b, we first use Eq. 25-1 to find the charge on each actual capacitor:

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad \text{and} \quad q_3 = C_3 V.$$

The total charge on the parallel combination of Fig. 25-8a is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

The equivalent capacitance, with the same total charge  $q$  and applied potential difference  $V$  as the combination, is then

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number  $n$  of capacitors, as

$$C_{eq} = \sum_{j=1}^n C_j \quad (\text{n capacitors in parallel}). \quad (25-19)$$

Thus, to find the equivalent capacitance of a parallel combination, we simply add the individual capacitances.

### Capacitors in Series

Figure 25-9a shows three capacitors connected *in series* to battery B. This description has little to do with how the capacitors are drawn. Rather, “in series” means that the capacitors are wired serially, one after the other, and that a potential difference  $V$  is