

- 2** Identical isolated conducting spheres 1 and 2 have equal charges and are separated by a distance that is large compared with their diameters (Fig. 21-22a). The electrostatic force acting on sphere 2 due to sphere 1 is \vec{F} . Suppose now that a third identical sphere 3, having an insulating handle and initially neutral, is touched first to sphere 1 (Fig. 21-22b), then to sphere 2 (Fig. 21-22c), and finally removed (Fig. 21-22d). The electrostatic force that now acts on sphere 2 has magnitude F' . What is the ratio F'/F ?

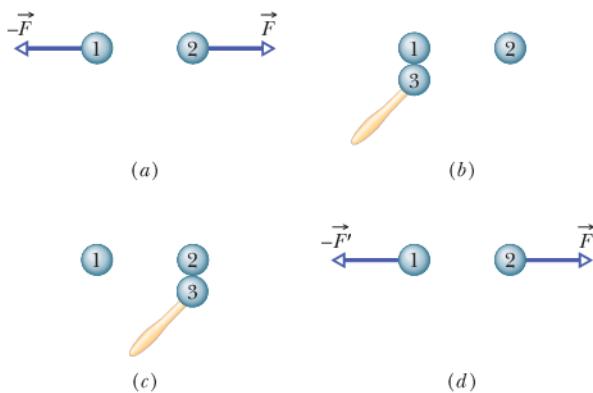


Figure 21-22 Problem 2.

- 3 SSM** What must be the distance between point charge $q_1 = 26.0 \mu\text{C}$ and point charge $q_2 = -47.0 \mu\text{C}$ for the electrostatic force between them to have a magnitude of 5.70 N?

- 4** In the return stroke of a typical lightning bolt, a current of $2.5 \times 10^4 \text{ A}$ exists for $20 \mu\text{s}$. How much charge is transferred in this event?

- 5** A particle of charge $+3.00 \times 10^{-6} \text{ C}$ is 12.0 cm distant from a second particle of charge $-1.50 \times 10^{-6} \text{ C}$. Calculate the magnitude of the electrostatic force between the particles.

- 6 ILW** Two equally charged particles are held $3.2 \times 10^{-3} \text{ m}$ apart and then released from rest. The initial acceleration of the first particle is observed to be 7.0 m/s^2 and that of the second to be 9.0 m/s^2 . If the mass of the first particle is $6.3 \times 10^{-7} \text{ kg}$, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle?

- 7** In Fig. 21-23, three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio q_1/q_2 ?

- 8** In Fig. 21-24, three identical conducting spheres initially have the following charges: sphere A, $4Q$; sphere B, $-6Q$; and sphere C, 0. Spheres A and B are fixed in place, with a center-to-center separation that is much larger than the spheres. Two experiments are conducted. In experiment 1, sphere C is touched to sphere A and then (separately) to sphere B, and then it is removed. In experiment 2, starting with the same initial states, the procedure is reversed: Sphere C is touched to sphere B and then (separately) to sphere A, and then it is removed. What is the ratio of the electro-

static force between A and B at the end of experiment 2 to that at the end of experiment 1?

- 9 SSM WWW** Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.0360 N. Of the initial charges on the spheres, with a positive net charge, what was (a) the negative charge on one of them and (b) the positive charge on the other?

- 10 GO** In Fig. 21-25, four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. (a)

What is Q/q if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of q that makes the net electrostatic force on each of the four particles zero? Explain.

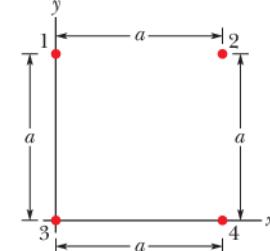


Figure 21-25

Problems 10, 11, and 70.

- 11 ILW** In Fig. 21-25, the particles have charges $q_1 = -q_2 = 100 \text{ nC}$ and $q_3 = -q_4 = 200 \text{ nC}$, and distance $a = 5.0 \text{ cm}$. What are the (a) x and (b) y components of the net electrostatic force on particle 3?

- 12** Two particles are fixed on an x axis. Particle 1 of charge $40 \mu\text{C}$ is located at $x = -2.0 \text{ cm}$; particle 2 of charge Q is located at $x = 3.0 \text{ cm}$. Particle 3 of charge magnitude $20 \mu\text{C}$ is released from rest on the y axis at $y = 2.0 \text{ cm}$. What is the value of Q if the initial acceleration of particle 3 is in the positive direction of (a) the x axis and (b) the y axis?

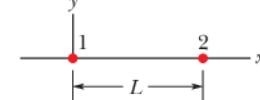


Figure 21-26 Problems 13, 19, 30, 58, and 67.

- 13 GO** In Fig. 21-26, particle 1 of charge $+1.0 \mu\text{C}$ and particle 2 of charge $-3.0 \mu\text{C}$ are held at separation $L = 10.0 \text{ cm}$ on an x axis. If particle 3 of unknown charge q_3 is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) y coordinates of particle 3?

- 14** Three particles are fixed on an x axis. Particle 1 of charge q_1 is at $x = -a$, and particle 2 of charge q_2 is at $x = +a$. If their net electrostatic force on particle 3 of charge $+Q$ is to be zero, what must be the ratio q_1/q_2 when particle 3 is at (a) $x = +0.500a$ and (b) $x = +1.50a$?

- 15 GO** The charges and coordinates of two charged particles held fixed in an xy plane are $q_1 = +3.0 \mu\text{C}$, $x_1 = 3.5 \text{ cm}$, $y_1 = 0.50 \text{ cm}$, and $q_2 = -4.0 \mu\text{C}$, $x_2 = -2.0 \text{ cm}$, $y_2 = 1.5 \text{ cm}$. Find the (a) magnitude and (b) direction of the electrostatic force on particle 2 due to particle 1. At what (c) x and (d) y coordinates should a third particle of charge $q_3 = +4.0 \mu\text{C}$ be placed such that the net electrostatic force on particle 2 due to particles 1 and 3 is zero?

- 16 GO** In Fig. 21-27a, particle 1 (of charge q_1) and particle 2 (of charge q_2) are fixed in place on an x axis, 8.00 cm apart. Particle 3 (of

Figure 21-23 Problems 7 and 40.

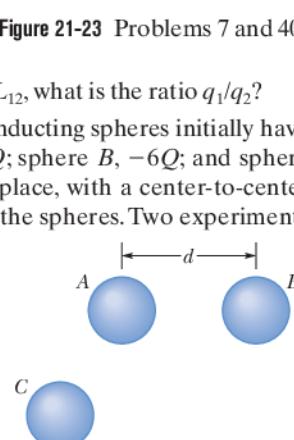


Figure 21-24
Problems 8 and 65.

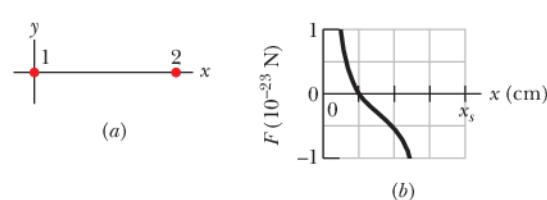


Figure 21-27 Problem 16.

charge $q_3 = +8.00 \times 10^{-19} \text{ C}$) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $\vec{F}_{3,\text{net}}$ on it. Figure 21-27b gives the x component of that force versus the coordinate x at which particle 3 is placed. The scale of the x axis is set by $x_s = 8.0 \text{ cm}$. What are (a) the sign of charge q_1 and (b) the ratio q_2/q_1 ?

- 17** In Fig. 21-28a, particles 1 and 2 have charge $20.0 \mu\text{C}$ each and are held at separation distance $d = 1.50 \text{ m}$. (a) What is the magnitude of the electrostatic force on particle 1 due to particle 2? In Fig. 21-28b, particle 3 of charge $20.0 \mu\text{C}$ is positioned so as to complete an equilateral triangle. (b) What is the magnitude of the net electrostatic force on particle 1 due to particles 2 and 3?

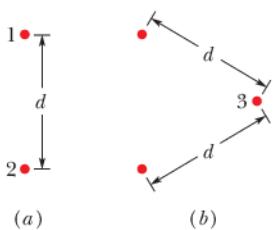


Figure 21-28 Problem 17.

- 18** In Fig. 21-29a, three positively charged particles are fixed on an x axis. Particles B and C are so close to each other that they can be considered to be at the same distance from particle A . The net force on particle A due to particles B and C is $2.014 \times 10^{-23} \text{ N}$ in the negative direction of the x axis. In Fig. 21-29b, particle B has been moved to the opposite side of A but is still at the same distance from it. The net force on A is now $2.877 \times 10^{-24} \text{ N}$ in the negative direction of the x axis. What is the ratio q_C/q_B ?

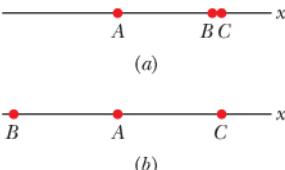


Figure 21-29 Problem 18.

- 19 SSM WWW** In Fig. 21-26, particle 1 of charge $+q$ and particle 2 of charge $+4.00q$ are held at separation $L = 9.00 \text{ cm}$ on an x axis. If particle 3 of charge q_3 is to be located such that the three particles remain in place when released, what must be the (a) x and (b) y coordinates of particle 3, and (c) the ratio q_3/q ?

- 20 GO** Figure 21-30a shows an arrangement of three charged particles separated by distance d . Particles A and C are fixed on the x axis, but particle B can be moved along a circle centered on particle A . During the movement, a radial line between A and B makes an angle θ relative to the positive direction of the x axis (Fig. 21-30b). The curves in Fig. 21-30c give, for two situations, the magnitude F_{net} of the net electrostatic force on particle A due to the other particles. That net force is given as a function of angle θ and as a multiple of a basic amount F_0 . For example on curve 1, at $\theta = 180^\circ$, we see that $F_{\text{net}} = 2F_0$. (a) For the situation corresponding to curve 1, what is the ratio of the charge of particle C to that of particle B (including sign)? (b) For the situation corresponding to curve 2, what is that ratio?

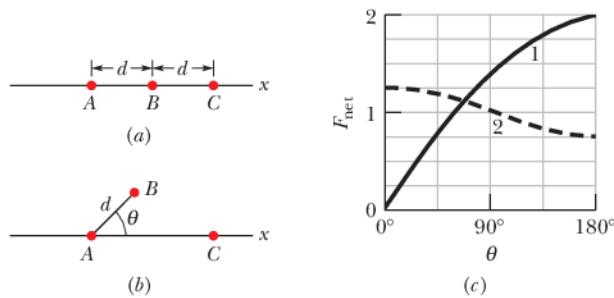


Figure 21-30 Problem 20.

- 21 GO** A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm , has charge spread nonuniformly through its volume between its inner and outer surfaces. The *volume charge density* ρ is the charge per unit volume, with the unit coulomb per cubic meter. For this shell $\rho = b/r$, where r is the distance in meters from the center of the shell and $b = 3.0 \mu\text{C}/\text{m}^3$. What is the net charge in the shell?

- 22 GO** Figure 21-31 shows an arrangement of four charged particles, with angle $\theta = 30.0^\circ$ and distance $d = 2.00 \text{ cm}$. Particle 2 has charge $q_2 = +8.00 \times 10^{-19} \text{ C}$; particles 3 and 4 have charges $q_3 = q_4 = -1.60 \times 10^{-19} \text{ C}$. (a) What is distance D between the origin and particle 2 if the net electrostatic force on particle 1 due to the other particles is zero? (b) If particles 3 and 4 were moved closer to the x axis but maintained their symmetry about that axis, would the required value of D be greater than, less than, or the same as in part (a)?

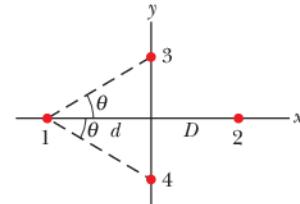


Figure 21-31 Problem 22.

- 23 GO** In Fig. 21-32, particles 1 and 2 of charge $q_1 = q_2 = +3.20 \times 10^{-19} \text{ C}$ are on a y axis at distance $d = 17.0 \text{ cm}$ from the origin. Particle 3 of charge $q_3 = +6.40 \times 10^{-19} \text{ C}$ is moved gradually along the x axis from $x = 0$ to $x = +5.0 \text{ m}$. At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?

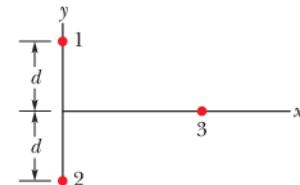


Figure 21-32 Problem 23.

Module 21-2 Charge Is Quantized

- 24** Two tiny, spherical water drops, with identical charges of $-1.00 \times 10^{-16} \text{ C}$, have a center-to-center separation of 1.00 cm . (a) What is the magnitude of the electrostatic force acting between them? (b) How many excess electrons are on each drop, giving it its charge imbalance?

- 25 ILW** How many electrons would have to be removed from a coin to leave it with a charge of $+1.0 \times 10^{-7} \text{ C}$?

- 26** What is the magnitude of the electrostatic force between a singly charged sodium ion (Na^+ , of charge $+e$) and an adjacent singly charged chlorine ion (Cl^- , of charge $-e$) in a salt crystal if their separation is $2.82 \times 10^{-10} \text{ m}$?

- 27 SSM** The magnitude of the electrostatic force between two identical ions that are separated by a distance of $5.0 \times 10^{-10} \text{ m}$ is $3.7 \times 10^{-9} \text{ N}$. (a) What is the charge of each ion? (b) How many electrons are “missing” from each ion (thus giving the ion its charge imbalance)?

- 28** A current of 0.300 A through your chest can send your heart into fibrillation, ruining the normal rhythm of heartbeat and disrupting the flow of blood (and thus oxygen) to your brain. If that current persists for 2.00 min , how many conduction electrons pass through your chest?

- 29 GO** In Fig. 21-33, particles 2 and 4, of charge $-e$, are fixed in place on a y axis, at $y_2 = -10.0 \text{ cm}$

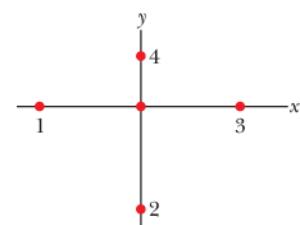


Figure 21-33 Problem 29.

and $y_4 = 5.00 \text{ cm}$. Particles 1 and 3, of charge $-e$, can be moved along the x axis. Particle 5, of charge $+e$, is fixed at the origin. Initially particle 1 is at $x_1 = -10.0 \text{ cm}$ and particle 3 is at $x_3 = 10.0 \text{ cm}$. (a) To what x value must particle 1 be moved to rotate the direction of the net electric force \vec{F}_{net} on particle 5 by 30° counterclockwise? (b) With particle 1 fixed at its new position, to what x value must you move particle 3 to rotate \vec{F}_{net} back to its original direction?

••30 In Fig. 21-26, particles 1 and 2 are fixed in place on an x axis, at a separation of $L = 8.00 \text{ cm}$. Their charges are $q_1 = +e$ and $q_2 = -27e$. Particle 3 with charge $q_3 = +4e$ is to be placed on the line between particles 1 and 2, so that they produce a net electrostatic force $\vec{F}_{3,\text{net}}$ on it. (a) At what coordinate should particle 3 be placed to minimize the magnitude of that force? (b) What is that minimum magnitude?

••31 **ILW** Earth's atmosphere is constantly bombarded by *cosmic ray protons* that originate somewhere in space. If the protons all passed through the atmosphere, each square meter of Earth's surface would intercept protons at the average rate of 1500 protons per second. What would be the electric current intercepted by the total surface area of the planet?

••32 **GO** Figure 21-34a shows charged particles 1 and 2 that are fixed in place on an x axis. Particle 1 has a charge with a magnitude of $|q_1| = 8.00e$. Particle 3 of charge $q_3 = +8.00e$ is initially on the x axis near particle 2. Then particle 3 is gradually moved in the positive direction of the x axis. As a result, the magnitude of the net electrostatic force $\vec{F}_{2,\text{net}}$ on particle 2 due to particles 1 and 3 changes. Figure 21-34b gives the x component of that net force as a function of the position x of particle 3. The scale of the x axis is set by $x_s = 0.80 \text{ m}$. The plot has an asymptote of $F_{2,\text{net}} = 1.5 \times 10^{-25} \text{ N}$ as $x \rightarrow \infty$. As a multiple of e and including the sign, what is the charge q_2 of particle 2?

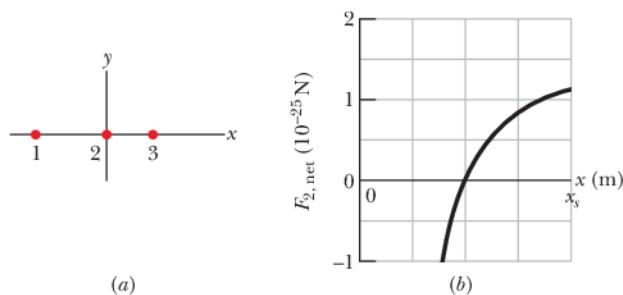


Figure 21-34 Problem 32.

••33 Calculate the number of coulombs of positive charge in 250 cm^3 of (neutral) water. (*Hint:* A hydrogen atom contains one proton; an oxygen atom contains eight protons.)

••34 **GO** Figure 21-35 shows electrons 1 and 2 on an x axis and charged ions 3 and 4 of identical charge $-q$ and at identical angles θ . Electron 2 is free to move; the other three particles are fixed in place at horizontal distances R from electron 2 and are intended to hold electron 2 in place. For physically possible values of $q \leq 5e$, what are the (a) smallest, (b) second smallest, and (c) third smallest values of θ for which electron 2 is held in place?

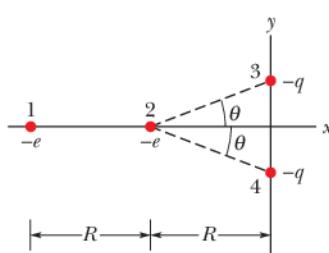


Figure 21-35 Problem 34.

••35 **SSM** In crystals of the salt cesium chloride, cesium ions Cs^+ form the eight corners of a cube and a chlorine ion Cl^- is at the cube's center (Fig. 21-36). The edge length of the cube is 0.40 nm . The Cs^+ ions are each deficient by one electron (and thus each has a charge of $+e$), and the Cl^- ion has one excess electron (and thus has a charge of $-e$). (a) What is the magnitude of the net electrostatic force exerted on the Cl^- ion by the eight Cs^+ ions at the corners of the cube? (b) If one of the Cs^+ ions is missing, the crystal is said to have a *defect*; what is the magnitude of the net electrostatic force exerted on the Cl^- ion by the seven remaining Cs^+ ions?

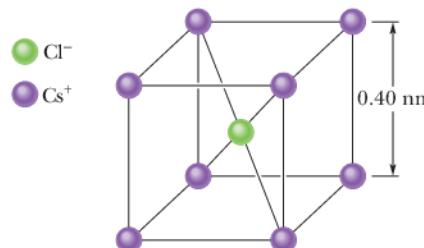


Figure 21-36 Problem 35.

Module 21-3 Charge Is Conserved

•36 Electrons and positrons are produced by the nuclear transformations of protons and neutrons known as *beta decay*. (a) If a proton transforms into a neutron, is an electron or a positron produced? (b) If a neutron transforms into a proton, is an electron or a positron produced?

•37 **SSM** Identify X in the following nuclear reactions: (a) ${}^1\text{H} + {}^9\text{Be} \rightarrow \text{X} + \text{n}$; (b) ${}^{12}\text{C} + {}^1\text{H} \rightarrow \text{X}$; (c) ${}^{15}\text{N} + {}^1\text{H} \rightarrow {}^4\text{He} + \text{X}$. Appendix F will help.

Additional Problems

38 **GO** Figure 21-37 shows four identical conducting spheres that are actually well separated from one another. Sphere W (with an initial charge of zero) is touched to sphere A and then they are separated. Next, sphere W is touched to sphere B (with an initial charge of $-32e$) and then they are separated. Finally, sphere W is touched to sphere C (with an initial charge of $+48e$), and then they are separated. The final charge on sphere W is $+18e$. What was the initial charge on sphere A?

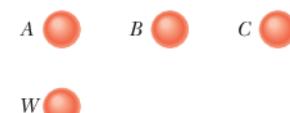


Figure 21-37 Problem 38.

39 **SSM** In Fig. 21-38, particle 1 of charge $+4e$ is above a floor by distance $d_1 = 2.00 \text{ mm}$ and particle 2 of charge $+6e$ is on the floor, at distance $d_2 = 6.00 \text{ mm}$ horizontally from particle 1. What is the x component of the electrostatic force on particle 2 due to particle 1?

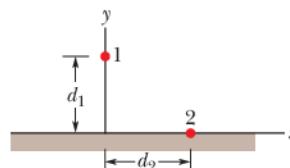


Figure 21-38 Problem 39.

40 In Fig. 21-23, particles 1 and 2 are fixed in place, but particle 3 is free to move. If the net electrostatic force on particle 3 due to particles 1 and 2 is zero and $L_{23} = 2.00L_{12}$, what is the ratio q_1/q_2 ?

41 (a) What equal positive charges would have to be placed on Earth and on the Moon to neutralize their gravitational attraction? (b) Why don't you need to know the lunar distance to solve this problem? (c) How many kilograms of hydrogen ions (that is, protons) would be needed to provide the positive charge calculated in (a)?

- 42** In Fig. 21-39, two tiny conducting balls of identical mass m and identical charge q hang from nonconducting threads of length L . Assume that θ is so small that $\tan \theta$ can be replaced by its approximate equal, $\sin \theta$. (a) Show that

$$x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3}$$

gives the equilibrium separation x of the balls. (b) If $L = 120$ cm, $m = 10$ g, and $x = 5.0$ cm, what is $|q|$?

- 43** (a) Explain what happens to the balls of Problem 42 if one of them is discharged (loses its charge q to, say, the ground). (b) Find the new equilibrium separation x , using the given values of L and m and the computed value of $|q|$.

- 44 SSM** How far apart must two protons be if the magnitude of the electrostatic force acting on either one due to the other is equal to the magnitude of the gravitational force on a proton at Earth's surface?

- 45** How many megacoulombs of positive charge are in 1.00 mol of neutral molecular-hydrogen gas (H_2)?

- 46** In Fig. 21-40, four particles are fixed along an x axis, separated by distances $d = 2.00$ cm. The charges are $q_1 = +2e$, $q_2 = -e$, $q_3 = +e$, and $q_4 = +4e$, with $e = 1.60 \times 10^{-19}$ C. In unit-vector notation, what is the net electrostatic force on (a) particle 1 and (b) particle 2 due to the other particles?

- 47 GO** Point charges of $+6.0 \mu\text{C}$ and $-4.0 \mu\text{C}$ are placed on an x axis, at $x = 8.0$ m and $x = 16$ m, respectively. What charge must be placed at $x = 24$ m so that any charge placed at the origin would experience no electrostatic force?

- 48** In Fig. 21-41, three identical conducting spheres form an equilateral triangle of side length $d = 20.0$ cm. The sphere radii are much smaller than d , and the sphere charges are $q_A = -2.00$ nC, $q_B = -4.00$ nC, and $q_C = +8.00$ nC. (a) What is the magnitude of the electrostatic force between spheres A and C ? The following steps are then taken: A and B are connected by a thin wire and then disconnected; B is grounded by the wire, and the wire is then removed; B and C are connected by the wire and then disconnected. What now are the magnitudes of the electrostatic force (b) between spheres A and C and (c) between spheres B and C ?

- 49** A neutron consists of one "up" quark of charge $+2e/3$ and two "down" quarks each having charge $-e/3$. If we assume that the down quarks are 2.6×10^{-15} m apart inside the neutron, what is the magnitude of the electrostatic force between them?

- 50** Figure 21-42 shows a long, nonconducting, massless rod of length L , pivoted at its center and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q . (a) Find the distance x when the rod is horizontal and balanced. (b)

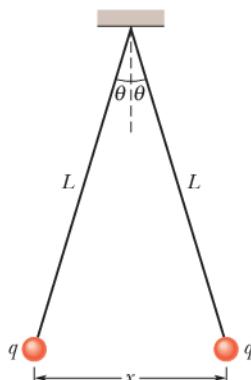


Figure 21-39
Problems 42 and 43.

What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?

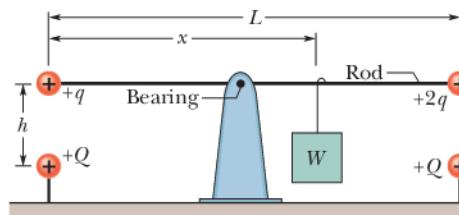


Figure 21-42 Problem 50.

- 51** A charged nonconducting rod, with a length of 2.00 m and a cross-sectional area of 4.00 cm^2 , lies along the positive side of an x axis with one end at the origin. The *volume charge density* ρ is charge per unit volume in coulombs per cubic meter. How many excess electrons are on the rod if ρ is (a) uniform, with a value of $-4.00 \mu\text{C}/\text{m}^3$, and (b) nonuniform, with a value given by $\rho = bx^2$, where $b = -2.00 \mu\text{C}/\text{m}^5$?

- 52** A particle of charge Q is fixed at the origin of an xy coordinate system. At $t = 0$ a particle ($m = 0.800$ g, $q = 4.00 \mu\text{C}$) is located on the x axis at $x = 20.0$ cm, moving with a speed of 50.0 m/s in the positive y direction. For what value of Q will the moving particle execute circular motion? (Neglect the gravitational force on the particle.)

- 53** What would be the magnitude of the electrostatic force between two 1.00 C point charges separated by a distance of (a) 1.00 m and (b) 1.00 km if such point charges existed (they do not) and this configuration could be set up?

- 54** A charge of $6.0 \mu\text{C}$ is to be split into two parts that are then separated by 3.0 mm. What is the maximum possible magnitude of the electrostatic force between those two parts?

- 55** Of the charge Q on a tiny sphere, a fraction α is to be transferred to a second, nearby sphere. The spheres can be treated as particles. (a) What value of α maximizes the magnitude F of the electrostatic force between the two spheres? What are the (b) smaller and (c) larger values of α that put F at half the maximum magnitude?

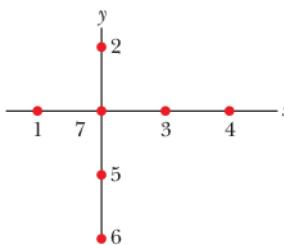
- 56** If a cat repeatedly rubs against your cotton slacks on a dry day, the charge transfer between the cat hair and the cotton can leave you with an excess charge of $-2.00 \mu\text{C}$. (a) How many electrons are transferred between the cat?

- You will gradually discharge via the floor, but if instead of waiting, you immediately reach toward a faucet, a painful spark can suddenly appear as your fingers near the faucet. (b) In that spark, do electrons flow from you to the faucet or vice versa? (c) Just before the spark appears, do you induce positive or negative charge in the faucet? (d) If, instead, the cat reaches a paw toward the faucet, which way do electrons flow in the resulting spark? (e) If you stroke a cat with a bare hand on a dry day, you should take care not to bring your fingers near the cat's nose or you will hurt it with a spark. Considering that cat hair is an insulator, explain how the spark can appear.

- 57** We know that the negative charge on the electron and the positive charge on the proton are equal. Suppose, however, that these magnitudes differ from each other by 0.00010%. With what force would two copper coins, placed 1.0 m apart, repel each other? Assume that each coin contains 3×10^{22} copper atoms. (Hint: A neutral copper atom contains 29 protons and 29 electrons.) What do you conclude?

58 In Fig. 21-26, particle 1 of charge $-80.0 \mu\text{C}$ and particle 2 of charge $+40.0 \mu\text{C}$ are held at separation $L = 20.0 \text{ cm}$ on an x axis. In unit-vector notation, what is the net electrostatic force on particle 3, of charge $q_3 = 20.0 \mu\text{C}$, if particle 3 is placed at (a) $x = 40.0 \text{ cm}$ and (b) $x = 80.0 \text{ cm}$? What should be the (c) x and (d) y coordinates of particle 3 if the net electrostatic force on it due to particles 1 and 2 is zero?

59 What is the total charge in coulombs of 75.0 kg of electrons?

60  In Fig. 21-43, six charged particles surround particle 7 at radial distances of either $d = 1.0 \text{ cm}$ or $2d$, as drawn. The charges are $q_1 = +2e, q_2 = +4e, q_3 = +e, q_4 = +4e, q_5 = +2e, q_6 = +8e, q_7 = +6e$, with $e = 1.60 \times 10^{-19} \text{ C}$. What is the magnitude of the net electrostatic force on particle 7?

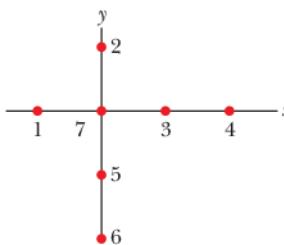
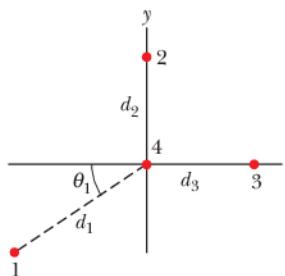


Figure 21-43 Problem 60.

61 Three charged particles form a triangle: particle 1 with charge $Q_1 = 80.0 \text{ nC}$ is at xy coordinates $(0, 3.00 \text{ mm})$, particle 2 with charge Q_2 is at $(0, -3.00 \text{ mm})$, and particle 3 with charge $q = 18.0 \text{ nC}$ is at $(4.00 \text{ mm}, 0)$. In unit-vector notation, what is the electrostatic force on particle 3 due to the other two particles if Q_2 is equal to (a) 80.0 nC and (b) -80.0 nC ?

62  In Fig. 21-44, what are the (a) magnitude and (b) direction of the net electrostatic force on particle 4 due to the other three particles? All four particles are fixed in the xy plane, and $q_1 = -3.20 \times 10^{-19} \text{ C}, q_2 = +3.20 \times 10^{-19} \text{ C}, q_3 = +6.40 \times 10^{-19} \text{ C}, q_4 = +3.20 \times 10^{-19} \text{ C}, \theta_1 = 35.0^\circ, d_1 = 3.00 \text{ cm}$, and $d_2 = d_3 = 2.00 \text{ cm}$.

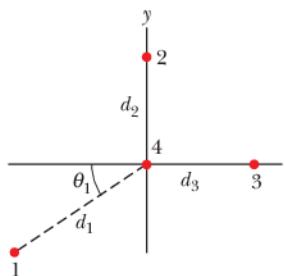


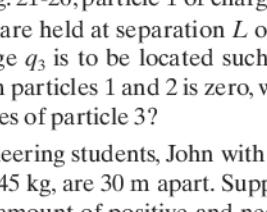
Figure 21-44 Problem 62.

63 Two point charges of 30 nC and -40 nC are held fixed on an x axis, at the origin and at $x = 72 \text{ cm}$, respectively. A particle with a charge of $42 \mu\text{C}$ is released from rest at $x = 28 \text{ cm}$. If the initial acceleration of the particle has a magnitude of 100 km/s^2 , what is the particle's mass?

64 Two small, positively charged spheres have a combined charge of $5.0 \times 10^{-5} \text{ C}$. If each sphere is repelled from the other by an electrostatic force of 1.0 N when the spheres are 2.0 m apart, what is the charge on the sphere with the smaller charge?

65 The initial charges on the three identical metal spheres in Fig. 21-24 are the following: sphere A, Q ; sphere $B, -Q/4$; and sphere $C, Q/2$, where $Q = 2.00 \times 10^{-14} \text{ C}$. Spheres A and B are fixed in place, with a center-to-center separation of $d = 1.20 \text{ m}$, which is much larger than the spheres. Sphere C is touched first to sphere A and then to sphere B and is then removed. What then is the magnitude of the electrostatic force between spheres A and B ?

66 An electron is in a vacuum near Earth's surface and located at $y = 0$ on a vertical y axis. At what value of y should a second electron be placed such that its electrostatic force on the first electron balances the gravitational force on the first electron?

67  In Fig. 21-26, particle 1 of charge $-5.00q$ and particle 2 of charge $+2.00q$ are held at separation L on an x axis. If particle 3 of unknown charge q_3 is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) y coordinates of particle 3?

68 Two engineering students, John with a mass of 90 kg and Mary with a mass of 45 kg , are 30 m apart. Suppose each has a 0.01% imbalance in the amount of positive and negative charge, one student being positive and the other negative. Find the order of magnitude of the electrostatic force of attraction between them by replacing each student with a sphere of water having the same mass as the student.

69 In the radioactive decay of Eq. 21-13, a ^{238}U nucleus transforms to ^{234}Th and an ejected ^4He . (These are nuclei, not atoms, and thus electrons are not involved.) When the separation between ^{234}Th and ^4He is $9.0 \times 10^{-15} \text{ m}$, what are the magnitudes of (a) the electrostatic force between them and (b) the acceleration of the ^4He particle?

70 In Fig. 21-25, four particles form a square. The charges are $q_1 = +Q, q_2 = q_3 = q$, and $q_4 = -2.00Q$. What is q/Q if the net electrostatic force on particle 1 is zero?

71 In a spherical metal shell of radius R , an electron is shot from the center directly toward a tiny hole in the shell, through which it escapes. The shell is negatively charged with a *surface charge density* (charge per unit area) of $6.90 \times 10^{-15} \text{ C/m}^2$. What is the magnitude of the electron's acceleration when it reaches radial distances (a) $r = 0.500R$ and (b) $2.00R$?

72 An electron is projected with an initial speed $v_i = 3.2 \times 10^5 \text{ m/s}$ directly toward a very distant proton that is at rest. Because the proton mass is large relative to the electron mass, assume that the proton remains at rest. By calculating the work done on the electron by the electrostatic force, determine the distance between the two particles when the electron instantaneously has speed $2v_i$.

73 In an early model of the hydrogen atom (the *Bohr model*), the electron orbits the proton in uniformly circular motion. The radius of the circle is restricted (*quantized*) to certain values given by

$$r = n^2 a_0, \quad \text{for } n = 1, 2, 3, \dots,$$

where $a_0 = 52.92 \text{ pm}$. What is the speed of the electron if it orbits in (a) the smallest allowed orbit and (b) the second smallest orbit? (c) If the electron moves to larger orbits, does its speed increase, decrease, or stay the same?

74 A 100 W lamp has a steady current of 0.83 A in its filament. How long is required for 1 mol of electrons to pass through the lamp?

75 The charges of an electron and a positron are $-e$ and $+e$. The mass of each is $9.11 \times 10^{-31} \text{ kg}$. What is the ratio of the electrical force to the gravitational force between an electron and a positron?

Electric Fields

22-1 THE ELECTRIC FIELD

Learning Objectives

After reading this module, you should be able to ...

- 22.01** Identify that at every point in the space surrounding a charged particle, the particle sets up an electric field \vec{E} , which is a vector quantity and thus has both magnitude and direction.
- 22.02** Identify how an electric field \vec{E} can be used to explain how a charged particle can exert an electrostatic force \vec{F}

on a second charged particle even though there is no contact between the particles.

- 22.03** Explain how a small positive test charge is used (in principle) to measure the electric field at any given point.
- 22.04** Explain electric field lines, including where they originate and terminate and what their spacing represents.

Key Ideas

- A charged particle sets up an electric field (a vector quantity) in the surrounding space. If a second charged particle is located in that space, an electrostatic force acts on it due to the magnitude and direction of the field at its location.
- The electric field \vec{E} at any point is defined in terms of the electrostatic force \vec{F} that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

- Electric field lines help us visualize the direction and magnitude of electric fields. The electric field vector at any point is tangent to the field line through that point. The density of field lines in that region is proportional to the magnitude of the electric field there. Thus, closer field lines represent a stronger field.
- Electric field lines originate on positive charges and terminate on negative charges. So, a field line extending from a positive charge must end on a negative charge.

What Is Physics?

Figure 22-1 shows two positively charged particles. From the preceding chapter we know that an electrostatic force acts on particle 1 due to the presence of particle 2. We also know the force direction and, given some data, we can calculate the force magnitude. However, here is a leftover nagging question. How does particle 1 “know” of the presence of particle 2? That is, since the particles do not touch, how can particle 2 push on particle 1—how can there be such an *action at a distance*?

One purpose of physics is to record observations about our world, such as the magnitude and direction of the push on particle 1. Another purpose is to provide an explanation of what is recorded. Our purpose in this chapter is to provide such an explanation to this nagging question about electric force at a distance.

The explanation that we shall examine here is this: Particle 2 sets up an **electric field** at all points in the surrounding space, even if the space is a vacuum. If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point. Thus, particle 2 pushes on particle 1 not by touching it as you would push on a coffee mug by making contact. Instead, particle 2 pushes by means of the electric field it has set up.



Figure 22-1 How does charged particle 2 push on charged particle 1 when they have no contact?

Our goals in this chapter are to (1) define electric field, (2) discuss how to calculate it for various arrangements of charged particles and objects, and (3) discuss how an electric field can affect a charged particle (as in making it move).

The Electric Field

A lot of different fields are used in science and engineering. For example, a *temperature field* for an auditorium is the distribution of temperatures we would find by measuring the temperature at many points within the auditorium. Similarly, we could define a *pressure field* in a swimming pool. Such fields are examples of *scalar fields* because temperature and pressure are scalar quantities, having only magnitudes and not directions.

In contrast, an electric field is a *vector field* because it is responsible for conveying the information for a force, which involves both magnitude and direction. This field consists of a distribution of electric field vectors \vec{E} , one for each point in the space around a charged object. In principle, we can define \vec{E} at some point near the charged object, such as point P in Fig. 22-2a, with this procedure: At P , we place a particle with a small positive charge q_0 , called a *test charge* because we use it to test the field. (We want the charge to be small so that it does not disturb the object's charge distribution.) We then measure the electrostatic force \vec{F} that acts on the test charge. The electric field at that point is then

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (\text{electric field}). \quad (22-1)$$

Because the test charge is positive, the two vectors in Eq. 22-1 are in the same direction, so the direction of \vec{E} is the direction we measure for \vec{F} . The magnitude of \vec{E} at point P is F/q_0 . As shown in Fig. 22-2b, we always represent an electric field with an arrow with its tail anchored on the point where the measurement is made. (This may sound trivial, but drawing the vectors any other way usually results in errors. Also, another common error is to mix up the terms *force* and *field* because they both start with the letter f. Electric force is a push or pull. Electric field is an abstract property set up by a charged object.) From Eq. 22-1, we see that the SI unit for the electric field is the newton per coulomb (N/C).

We can shift the test charge around to various other points, to measure the electric fields there, so that we can figure out the distribution of the electric field set up by the charged object. That field exists independent of the test charge. It is something that a charged object sets up in the surrounding space (even vacuum), independent of whether we happen to come along to measure it.

For the next several modules, we determine the field around charged particles and various charged objects. First, however, let's examine a way of visualizing electric fields.

Electric Field Lines

Look at the space in the room around you. Can you visualize a field of vectors throughout that space—vectors with different magnitudes and directions? As impossible as that seems, Michael Faraday, who introduced the idea of electric fields in the 19th century, found a way. He envisioned lines, now called **electric field lines**, in the space around any given charged particle or object.

Figure 22-3 gives an example in which a sphere is uniformly covered with negative charge. If we place a positive test charge at any point near the sphere (Fig. 22-3a), we find that an electrostatic force pulls on it toward the center of the sphere. Thus at every point around the sphere, an electric field vector points radially inward toward the sphere. We can represent this electric field with

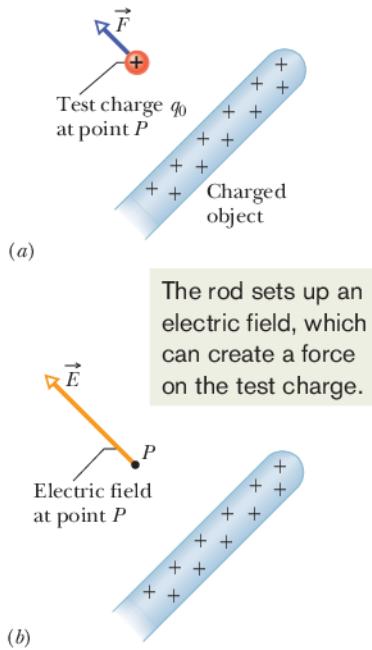


Figure 22-2 (a) A positive test charge q_0 placed at point P near a charged object. An electrostatic force \vec{F} acts on the test charge. (b) The electric field \vec{E} at point P produced by the charged object.

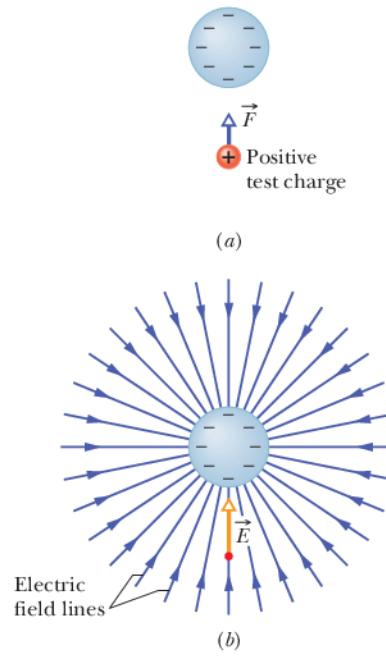


Figure 22-3 (a) The electrostatic force \vec{F} acting on a positive test charge near a sphere of uniform negative charge. (b) The electric field vector \vec{E} at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend toward the negatively charged sphere. (They originate on distant positive charges.)

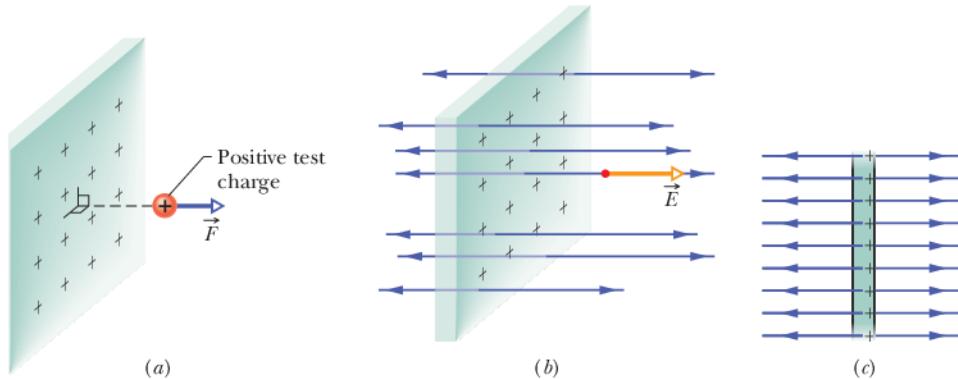


Figure 22-4 (a) The force on a positive test charge near a very large, nonconducting sheet with uniform positive charge on one side. (b) The electric field vector \vec{E} at the test charge's location, and the nearby electric field lines, extending away from the sheet. (c) Side view.

electric field lines as in Fig. 22-3b. At any point, such as the one shown, the direction of the field line through the point matches the direction of the electric vector at that point.

The rules for drawing electric fields lines are these: (1) At any point, the electric field vector must be tangent to the electric field line through that point and in the same direction. (This is easy to see in Fig. 22-3 where the lines are straight, but we'll see some curved lines soon.) (2) In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.

If the sphere in Fig. 22-3 were uniformly covered with positive charge, the electric field vectors at all points around it would be radially outward and thus so would the electric field lines. So, we have the following rule:



Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

In Fig. 22-3b, they originate on distant positive charges that are not shown.

For another example, Fig. 22-4a shows part of an infinitely large, nonconducting *sheet* (or plane) with a uniform distribution of positive charge on one side. If we place a positive test charge at any point near the sheet (on either side), we find that the electrostatic force on the particle is outward and perpendicular to the sheet. The perpendicular orientation is reasonable because any force component that is, say, upward is balanced out by an equal component that is downward. That leaves only outward, and thus the electric field vectors and the electric field lines must also be outward and perpendicular to the sheet, as shown in Figs. 22-4b and c.

Because the charge on the sheet is uniform, the field vectors and the field lines are also. Such a field is a *uniform electric field*, meaning that the electric field has the same magnitude and direction at every point within the field. (This is a lot easier to work with than a *nonuniform field*, where there is variation from point to point.) Of course, there is no such thing as an infinitely large sheet. That is just a way of saying that we are measuring the field at points close to the sheet relative to the size of the sheet and that we are not near an edge.

Figure 22-5 shows the field lines for two particles with equal positive charges. Now the field lines are curved, but the rules still hold: (1) the electric field vector at any given point must be tangent to the field line at that point and in the same direction, as shown for one vector, and (2) a closer spacing means a larger field magnitude. To imagine the full three-dimensional pattern of field lines around the particles, mentally rotate the pattern in Fig. 22-5 around the *axis of symmetry*, which is a vertical line through both particles.

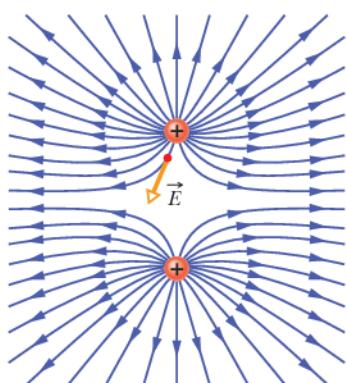


Figure 22-5 Field lines for two particles with equal positive charge. Doesn't the pattern itself suggest that the particles repel each other?

22-2 THE ELECTRIC FIELD DUE TO A CHARGED PARTICLE

Learning Objectives

After reading this module, you should be able to...

- 22.05** In a sketch, draw a charged particle, indicate its sign, pick a nearby point, and then draw the electric field vector \vec{E} at that point, with its tail anchored on the point.
- 22.06** For a given point in the electric field of a charged particle, identify the direction of the field vector \vec{E} when the particle is positively charged and when it is negatively charged.
- 22.07** For a given point in the electric field of a charged particle, apply the relationship between the field

magnitude E , the charge magnitude $|q|$, and the distance r between the point and the particle.

- 22.08** Identify that the equation given here for the magnitude of an electric field applies only to a particle, not an extended object.
- 22.09** If more than one electric field is set up at a point, draw each electric field vector and then find the net electric field by adding the individual electric fields as vectors (not as scalars).

Key Ideas

- The magnitude of the electric field \vec{E} set up by a particle with charge q at distance r from the particle is

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}.$$

- The electric field vectors set up by a positively charged particle all point directly away from the particle. Those set up

by a negatively charged particle all point directly toward the particle.

- If more than one charged particle sets up an electric field at a point, the net electric field is the *vector* sum of the individual electric fields—electric fields obey the superposition principle.

The Electric Field Due to a Point Charge

To find the electric field due to a charged particle (often called a *point charge*), we place a positive test charge at any point near the particle, at distance r . From Coulomb's law (Eq. 21-4), the force on the test charge due to the particle with charge q is

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \hat{r}.$$

As previously, the direction of \vec{F} is directly away from the particle if q is positive (because q_0 is positive) and directly toward it if q is negative. From Eq. 22-1, we can now write the electric field set up by the particle (at the location of the test charge) as

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (\text{charged particle}). \quad (22-2)$$

Let's think through the directions again. The direction of \vec{E} matches that of the force on the positive test charge: directly away from the point charge if q is positive and directly toward it if q is negative.

So, if given another charged particle, we can immediately determine the directions of the electric field vectors near it by just looking at the sign of the charge q . We can find the magnitude at any given distance r by converting Eq. 22-2 to a magnitude form:

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \quad (\text{charged particle}). \quad (22-3)$$

We write $|q|$ to avoid the danger of getting a negative E when q is negative, and then thinking the negative sign has something to do with direction. Equation 22-3 gives magnitude E only. We must think about the direction separately.

Figure 22-6 gives a number of electric field vectors at points around a positively charged particle, but be careful. Each vector represents the vector

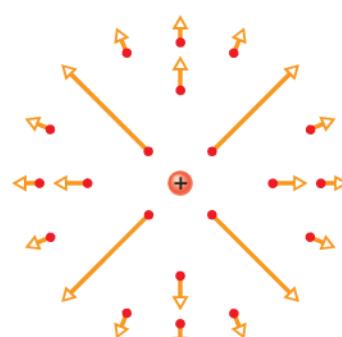


Figure 22-6 The electric field vectors at various points around a positive point charge.

quantity at the point where the tail of the arrow is anchored. The vector is not something that stretches from a “here” to a “there” as with a displacement vector.

In general, if several electric fields are set up at a given point by several charged particles, we can find the net field by placing a positive test particle at the point and then writing out the force acting on it due to each particle, such as \vec{F}_{01} due to particle 1. Forces obey the principle of superposition, so we just add the forces as vectors:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

To change over to electric field, we repeatedly use Eq. 22-1 for each of the individual forces:

$$\begin{aligned}\vec{E} &= \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.\end{aligned}\quad (22-4)$$

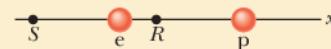
This tells us that electric fields also obey the principle of superposition. If you want the net electric field at a given point due to several particles, find the electric field due to each particle (such as \vec{E}_1 due to particle 1) and then sum the fields as vectors. (As with electrostatic forces, you cannot just willy-nilly add up the magnitudes.) This addition of fields is the subject of many of the homework problems.



Checkpoint 1

The figure here shows a proton p and an electron e on an x axis. What is the direction of the electric field due to the electron at (a) point S and (b) point R? What is the direction of the

net electric field at (c) point R and (d) point S?



Sample Problem 22.01 Net electric field due to three charged particles

Figure 22-7a shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field \vec{E} is produced at the origin?

KEY IDEA

Charges q_1 , q_2 , and q_3 produce electric field vectors \vec{E}_1 , \vec{E}_2 , and \vec{E}_3 , respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of \vec{E}_1 , which is due to q_1 , we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of \vec{E}_2 and \vec{E}_3 to be

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\epsilon_0} \frac{4Q}{d^2}.$$

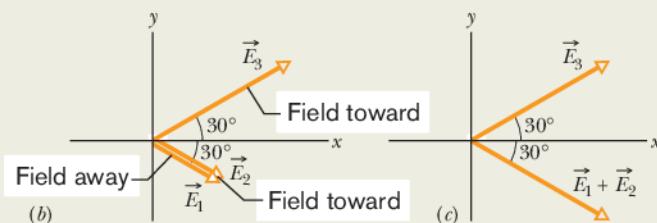
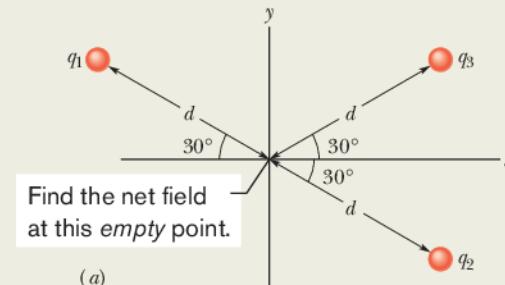


Figure 22-7 (a) Three particles with charges q_1 , q_2 , and q_3 are at the same distance d from the origin. (b) The electric field vectors \vec{E}_1 , \vec{E}_2 , and \vec{E}_3 , at the origin due to the three particles. (c) The electric field vector \vec{E}_3 and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7b. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7b, we see that electric fields \vec{E}_1 and \vec{E}_2 have the same direction. Hence, their vector sum has that direction and has the magnitude

$$\begin{aligned} E_1 + E_2 &= \frac{1}{4\pi\epsilon_0} \frac{2Q}{d^2} + \frac{1}{4\pi\epsilon_0} \frac{2Q}{d^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{4Q}{d^2}, \end{aligned}$$

which happens to equal the magnitude of field \vec{E}_3 .

We must now combine two vectors, \vec{E}_3 and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7c. From the symmetry of Fig. 22-7c, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field \vec{E} at the origin is in the positive direction of the x axis and has the magnitude

$$\begin{aligned} E &= 2E_{3x} = 2E_3 \cos 30^\circ \\ &= (2) \frac{1}{4\pi\epsilon_0} \frac{4Q}{d^2} (0.866) = \frac{6.93Q}{4\pi\epsilon_0 d^2}. \quad (\text{Answer}) \end{aligned}$$



Additional examples, video, and practice available at WileyPLUS



22-3 THE ELECTRIC FIELD DUE TO A DIPOLE

Learning Objectives

After reading this module, you should be able to . . .

- 22.10 Draw an electric dipole, identifying the charges (sizes and signs), dipole axis, and direction of the electric dipole moment.
- 22.11 Identify the direction of the electric field at any given point along the dipole axis, including between the charges.
- 22.12 Outline how the equation for the electric field due to an electric dipole is derived from the equations for the electric field due to the individual charged particles that form the dipole.
- 22.13 For a single charged particle and an electric dipole, compare the rate at which the electric field magnitude

decreases with increase in distance. That is, identify which drops off faster.

- 22.14 For an electric dipole, apply the relationship between the magnitude p of the dipole moment, the separation d between the charges, and the magnitude q of either of the charges.
- 22.15 For any distant point along a dipole axis, apply the relationship between the electric field magnitude E , the distance z from the center of the dipole, and either the dipole moment magnitude p or the product of charge magnitude q and charge separation d .

Key Ideas

- An electric dipole consists of two particles with charges of equal magnitude q but opposite signs, separated by a small distance d .
- The electric dipole moment \vec{p} has magnitude qd and points from the negative charge to the positive charge.
- The magnitude of the electric field set up by an electric dipole at a distant point on the dipole axis (which runs through both particles) can be written in terms of either the product qd or the magnitude p of the dipole moment:

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3},$$

where z is the distance between the point and the center of the dipole.

- Because of the $1/z^3$ dependence, the field magnitude of an electric dipole decreases more rapidly with distance than the field magnitude of either of the individual charges forming the dipole, which depends on $1/r^2$.

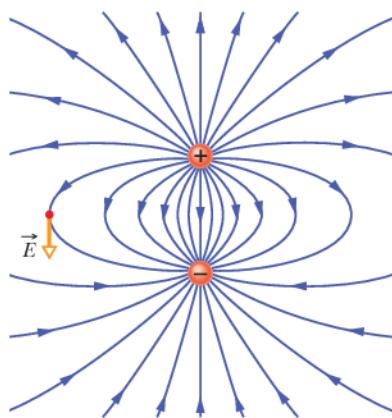


Figure 22-8 The pattern of electric field lines around an electric dipole, with an electric field vector \vec{E} shown at one point (tangent to the field line through that point).

The Electric Field Due to an Electric Dipole

Figure 22-8 shows the pattern of electric field lines for two particles that have the same charge magnitude q but opposite signs, a very common and important arrangement known as an **electric dipole**. The particles are separated by distance d and lie along the *dipole axis*, an axis of symmetry around which you can imagine rotating the pattern in Fig. 22-8. Let's label that axis as a z axis. Here we restrict our interest to the magnitude and direction of the electric field \vec{E} at an arbitrary point P along the dipole axis, at distance z from the dipole's midpoint.

Figure 22-9a shows the electric fields set up at P by each particle. The nearer particle with charge $+q$ sets up field $E_{(+)}$ in the positive direction of the z axis (directly away from the particle). The farther particle with charge $-q$ sets up a smaller field $E_{(-)}$ in the negative direction (directly toward the particle). We want the net field at P , as given by Eq. 22-4. However, because the field vectors are along the same axis, let's simply indicate the vector directions with plus and minus signs, as we commonly do with forces along a single axis. Then we can write the magnitude of the net field at P as

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\epsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\epsilon_0(z + \frac{1}{2}d)^2}. \end{aligned} \quad (22-5)$$

After a little algebra, we can rewrite this equation as

$$E = \frac{q}{4\pi\epsilon_0 z^2} \left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right). \quad (22-6)$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\epsilon_0 z^2} \frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}. \quad (22-7)$$

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$. At such large distances, we have $d/2z \ll 1$ in Eq. 22-7. Thus, in our approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3}. \quad (22-8)$$

The product qd , which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the **electric dipole moment** \vec{p} of the dipole. (The unit of \vec{p} is the coulomb-meter.) Thus, we can write Eq. 22-8 as

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3} \quad (\text{electric dipole}). \quad (22-9)$$

The direction of \vec{p} is taken to be from the negative to the positive end of the dipole, as indicated in Fig. 22-9b. We can use the direction of \vec{p} to specify the orientation of a dipole.

Equation 22-9 shows that, if we measure the electric field of a dipole only at distant points, we can never find q and d separately; instead, we can find only their product. The field at distant points would be unchanged if, for example, q

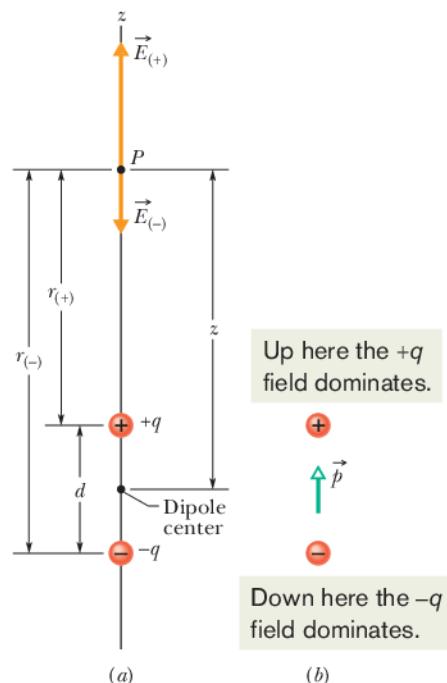


Figure 22-9 (a) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (b) The dipole moment \vec{p} of the dipole points from the negative charge to the positive charge.

were doubled and d simultaneously halved. Although Eq. 22-9 holds only for distant points along the dipole axis, it turns out that E for a dipole varies as $1/r^3$ for all distant points, regardless of whether they lie on the dipole axis; here r is the distance between the point in question and the dipole center.

Inspection of Fig. 22-9 and of the field lines in Fig. 22-8 shows that the direction of \vec{E} for distant points on the dipole axis is always the direction of the dipole moment vector \vec{p} . This is true whether point P in Fig. 22-9a is on the upper or the lower part of the dipole axis.

Inspection of Eq. 22-9 shows that if you double the distance of a point from a dipole, the electric field at the point drops by a factor of 8. If you double the distance from a single point charge, however (see Eq. 22-3), the electric field drops only by a factor of 4. Thus the electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge. The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two particles that almost—but not quite—coincide. Thus, because they have charges of equal magnitude but opposite signs, their electric fields at distant points almost—but not quite—cancel each other.

Sample Problem 22.02 Electric dipole and atmospheric sprites

Sprites (Fig. 22-10a) are huge flashes that occur far above a large thunderstorm. They were seen for decades by pilots flying at night, but they were so brief and dim that most pilots figured they were just illusions. Then in the 1990s sprites were captured on video. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge $-q$ from the ground to the base of the clouds (Fig. 22-10b).

Just after such a transfer, the ground has a complicated distribution of positive charge. However, we can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge $-q$ at cloud height h and charge $+q$ at below-ground depth h (Fig. 22-10c). If $q = 200 \text{ C}$ and $h = 6.0 \text{ km}$, what is the magnitude of the dipole's electric field at altitude $z_1 = 30 \text{ km}$ somewhat above the clouds and altitude $z_2 = 60 \text{ km}$ somewhat above the stratosphere?



(a) Courtesy NASA

KEY IDEA

We can approximate the magnitude E of an electric dipole's electric field on the dipole axis with Eq. 22-8.

Calculations: We write that equation as

$$E = \frac{1}{2\pi\epsilon_0} \frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-10c. For the electric field at altitude $z_1 = 30 \text{ km}$, we find

$$E = \frac{1}{2\pi\epsilon_0} \frac{(200 \text{ C})(2)(6.0 \times 10^3 \text{ m})}{(30 \times 10^3 \text{ m})^3} = 1.6 \times 10^3 \text{ N/C.} \quad (\text{Answer})$$

Similarly, for altitude $z_2 = 60 \text{ km}$, we find

$$E = 2.0 \times 10^2 \text{ N/C.} \quad (\text{Answer})$$

As we discuss in Module 22-6, when the magnitude of

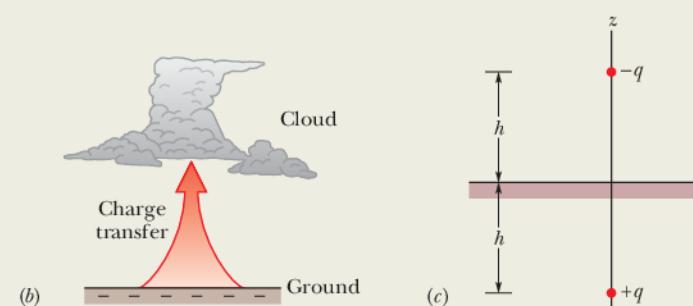


Figure 22-10 (a) Photograph of a sprite. (b) Lightning in which a large amount of negative charge is transferred from ground to cloud base. (c) The cloud-ground system modeled as a vertical electric dipole.

an electric field exceeds a certain critical value E_c , the field can pull electrons out of atoms (ionize the atoms), and then the freed electrons can run into other atoms, causing those atoms to emit light. The value of E_c depends on the density of the air in which the electric field exists. At altitude $z_2 = 60$ km the density of the air is so low that

$E = 2.0 \times 10^2$ N/C exceeds E_c , and thus light is emitted by the atoms in the air. That light forms sprites. Lower down, just above the clouds at $z_1 = 30$ km, the density of the air is much higher, $E = 1.6 \times 10^3$ N/C does not exceed E_c , and no light is emitted. Hence, sprites occur only far above storm clouds.



Additional examples, video, and practice available at WileyPLUS

22-4 THE ELECTRIC FIELD DUE TO A LINE OF CHARGE

Learning Objectives

After reading this module, you should be able to...

- 22.16** For a uniform distribution of charge, find the linear charge density λ for charge along a line, the surface charge density σ for charge on a surface, and the volume charge density ρ for charge in a volume.
- 22.17** For charge that is distributed uniformly along a line, find the net electric field at a given point near the line by

Key Ideas

- The equation for the electric field set up by a particle does not apply to an extended object with charge (said to have a continuous charge distribution).
- To find the electric field of an extended object at a point, we first consider the electric field set up by a charge element dq in the object, where the element is small enough for us to apply

splitting the distribution up into charge elements dq and then summing (by integration) the electric field vectors $d\vec{E}$ set up at the point by each element.

- 22.18** Explain how symmetry can be used to simplify the calculation of the electric field at a point near a line of uniformly distributed charge.

the equation for a particle. Then we sum, via integration, components of the electric fields $d\vec{E}$ from all the charge elements.

- Because the individual electric fields $d\vec{E}$ have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

The Electric Field Due to a Line of Charge

So far we have dealt with only charged particles, a single particle or a simple collection of them. We now turn to a much more challenging situation in which a thin (approximately one-dimensional) object such as a rod or ring is charged with a huge number of particles, more than we could ever even count. In the next module, we consider two-dimensional objects, such as a disk with charge spread over a surface. In the next chapter we tackle three-dimensional objects, such as a sphere with charge spread through a volume.

Heads Up. Many students consider this module to be the most difficult in the book for a variety of reasons. There are lots of steps to take, a lot of vector features to keep track of, and after all that, we set up and then solve an integral. The worst part, however, is that the procedure can be different for different arrangements of the charge. Here, as we focus on a particular arrangement (a charged ring), be aware of the general approach, so that you can tackle other arrangements in the homework (such as rods and partial circles).

Figure 22-11 shows a thin ring of radius R with a uniform distribution of positive charge along its circumference. It is made of plastic, which means that the charge is fixed in place. The ring is surrounded by a pattern of electric field lines, but here we restrict our interest to an arbitrary point P on the central axis (the axis through the ring's center and perpendicular to the plane of the ring), at distance z from the center point.

The charge of an extended object is often conveyed in terms of a charge density rather than the total charge. For a line of charge, we use the *linear charge*

density λ (the charge per unit length), with the SI unit of coulomb per meter. Table 22-1 shows the other charge densities that we shall be using for charged surfaces and volumes.

First Big Problem. So far, we have an equation for the electric field of a particle. (We can combine the field of several particles as we did for the electric dipole to generate a special equation, but we are still basically using Eq. 22-3). Now take a look at the ring in Fig. 22-11. That clearly is not a particle and so Eq. 22-3 does not apply. So what do we do?

The answer is to mentally divide the ring into differential elements of charge that are so small that we can treat them as though they *are* particles. Then we *can* apply Eq. 22-3.

Second Big Problem. We now know to apply Eq. 22-3 to each charge element dq (the front d emphasizes that the charge is very small) and can write an expression for its contribution of electric field $d\vec{E}$ (the front d emphasizes that the contribution is very small). However, each such contributed field vector at P is in its own direction. How can we add them to get the net field at P ?

The answer is to split the vectors into components and then separately sum one set of components and then the other set. However, first we check to see if one set simply all cancels out. (Canceling out components saves lots of work.)

Third Big Problem. There is a huge number of dq elements in the ring and thus a huge number of $d\vec{E}$ components to add up, even if we can cancel out one set of components. How can we add up more components than we could even count? The answer is to add them by means of integration.

Do It. Let's do all this (but again, be aware of the general procedure, not just the fine details). We arbitrarily pick the charge element shown in Fig. 22-11. Let ds be the arc length of that (or any other) dq element. Then in terms of the linear density λ (the charge per unit length), we have

$$dq = \lambda ds. \quad (22-10)$$

An Element's Field. This charge element sets up the differential electric field $d\vec{E}$ at P , at distance r from the element, as shown in Fig. 22-11. (Yes, we are introducing a new symbol that is not given in the problem statement, but soon we shall replace it with “legal symbols.”) Next we rewrite the field equation for a particle (Eq. 22-3) in terms of our new symbols dE and dq , but then we replace dq using Eq. 22-10. The field magnitude due to the charge element is

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2}. \quad (22-11)$$

Notice that the illegal symbol r is the hypotenuse of the right triangle displayed in Fig. 22-11. Thus, we can replace r by rewriting Eq. 22-11 as

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)}. \quad (22-12)$$

Because every charge element has the same charge and the same distance from point P , Eq. 22-12 gives the field magnitude contributed by each of them. Figure 22-11 also tells us that each contributed $d\vec{E}$ leans at angle θ to the central axis (the z axis) and thus has components perpendicular and parallel to that axis.

Cancelling Components. Now comes the neat part, where we eliminate one set of those components. In Fig. 22-11, consider the charge element on the opposite side of the ring. It too contributes the field magnitude dE but the field vector leans at angle θ in the opposite direction from the vector from our first charge

Table 22-1 Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m ²
Volume charge density	ρ	C/m ³

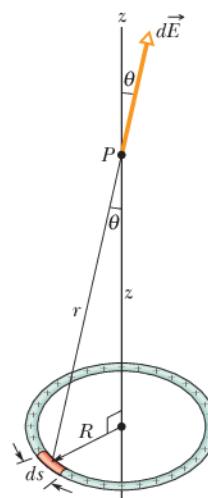


Figure 22-11 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P .

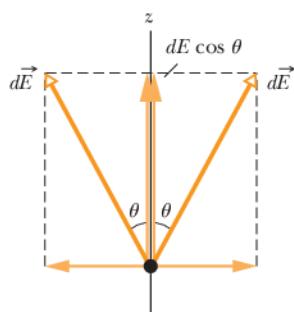


Figure 22-12 The electric fields set up at P by a charge element and its symmetric partner (on the opposite side of the ring). The components perpendicular to the z axis cancel; the parallel components add.

element, as indicated in the side view of Fig. 22-12. Thus the two perpendicular components cancel. All around the ring, this cancellation occurs for every charge element and its *symmetric partner* on the opposite side of the ring. So we can neglect all the perpendicular components.

Adding Components. We have another big win here. All the remaining components are in the positive direction of the z axis, so we can just add them up as scalars. Thus we can already tell the direction of the net electric field at P : directly away from the ring. From Fig. 22-12, we see that the parallel components each have magnitude $dE \cos \theta$, but θ is another illegal symbol. We can replace $\cos \theta$ with legal symbols by again using the right triangle in Fig. 22-11 to write

$$\cos \theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}. \quad (22-13)$$

Multiplying Eq. 22-12 by Eq. 22-13 gives us the parallel field component from each charge element:

$$dE \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{z\lambda}{(z^2 + R^2)^{3/2}} ds. \quad (22-14)$$

Integrating. Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it $s = 0$) through the full circumference ($s = 2\pi R$). Only the quantity s varies as we go through the elements; the other symbols in Eq. 22-14 remain the same, so we move them outside the integral. We find

$$\begin{aligned} E &= \int dE \cos \theta = \frac{z\lambda}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds \\ &= \frac{z\lambda(2\pi R)}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}. \end{aligned} \quad (22-15)$$

This is a fine answer, but we can also switch to the total charge by using $\lambda = q/(2\pi R)$:

$$E = \frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \quad (\text{charged ring}). \quad (22-16)$$

If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at P is still given by Eq. 22-16. However, the electric field vector then points toward the ring instead of away from it.

Let us check Eq. 22-16 for a point on the central axis that is so far away that $z \gg R$. For such a point, the expression $z^2 + R^2$ in Eq. 22-16 can be approximated as z^2 , and Eq. 22-16 becomes

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2} \quad (\text{charged ring at large distance}). \quad (22-17)$$

This is a reasonable result because from a large distance, the ring “looks like” a point charge. If we replace z with r in Eq. 22-17, we indeed do have the magnitude of the electric field due to a point charge, as given by Eq. 22-3.

Let us next check Eq. 22-16 for a point at the center of the ring—that is, for $z = 0$. At that point, Eq. 22-16 tells us that $E = 0$. This is a reasonable result because if we were to place a test charge at the center of the ring, there would be no net electrostatic force acting on it; the force due to any element of the ring would be canceled by the force due to the element on the opposite side of the ring. By Eq. 22-1, if the force at the center of the ring were zero, the electric field there would also have to be zero.



Sample Problem 22.03 Electric field of a charged circular rod

Figure 22-13a shows a plastic rod with a uniform charge $-Q$. It is bent in a 120° circular arc of radius r and symmetrically placed across an x axis with the origin at the center of curvature P of the rod. In terms of Q and r , what is the electric field \vec{E} due to the rod at point P ?

KEY IDEA

Because the rod has a continuous charge distribution, we must find an expression for the electric fields due to differential elements of the rod and then sum those fields via calculus.

An element: Consider a differential element having arc length ds and located at an angle θ above the x axis (Figs. 22-13b and c). If we let λ represent the linear charge density of the rod, our element ds has a differential charge of magnitude

$$dq = \lambda ds. \quad (22-18)$$

The element's field: Our element produces a differential electric field $d\vec{E}$ at point P , which is a distance r from the element. Treating the element as a point charge, we can

rewrite Eq. 22-3 to express the magnitude of $d\vec{E}$ as

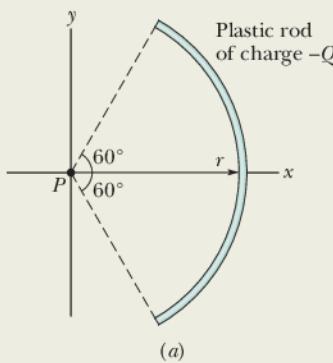
$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2}. \quad (22-19)$$

The direction of $d\vec{E}$ is toward ds because charge dq is negative.

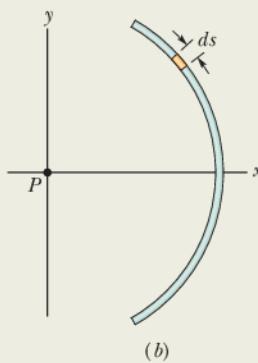
Symmetric partner: Our element has a symmetrically located (mirror image) element ds' in the bottom half of the rod. The electric field $d\vec{E}'$ set up at P by ds' also has the magnitude given by Eq. 22-19, but the field vector points toward ds' as shown in Fig. 22-13d. If we resolve the electric field vectors of ds and ds' into x and y components as shown in Figs. 22-13e and f, we see that their y components cancel (because they have equal magnitudes and are in opposite directions). We also see that their x components have equal magnitudes and are in the same direction.

Summing: Thus, to find the electric field set up by the rod, we need sum (via integration) only the x components of the differential electric fields set up by all the differential elements of the rod. From Fig. 22-13f and Eq. 22-19, we can write

This negatively charged rod is obviously not a particle.



But we can treat this element as a particle.



Here is the field the element creates.

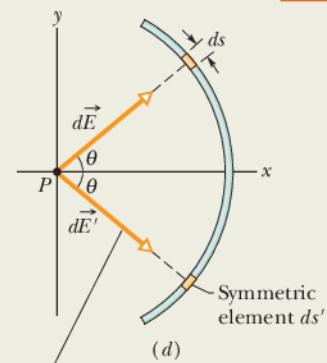
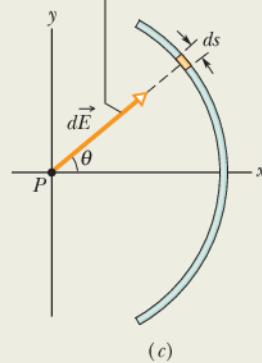
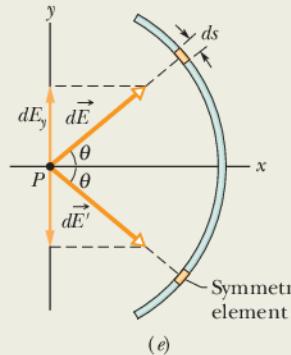
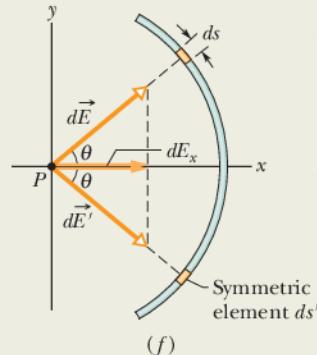


Figure 22-13 Available in WileyPLUS as an animation with voiceover. (a) A plastic rod of charge $-Q$ is a circular section of radius r and central angle 120° ; point P is the center of curvature of the rod. (b)-(c) A differential element in the top half of the rod, at an angle θ to the x axis and of arc length ds , sets up a differential electric field $d\vec{E}$ at P . (d) An element ds' , symmetric to ds about the x axis, sets up a field $d\vec{E}'$ at P with the same magnitude. (e)-(f) The field components. (g) Arc length ds makes an angle $d\theta$ about point P .

These y components just cancel, so neglect them.

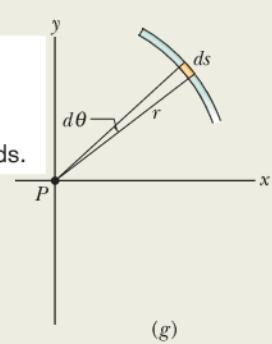


These x components add. Our job is to add all such components.



Here is the field created by the symmetric element, same size and angle.

We use this to relate the element's arc length to the angle that it subtends.



the component dE_x set up by ds as

$$dE_x = dE \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \theta ds. \quad (22-20)$$

Equation 22-20 has two variables, θ and s . Before we can integrate it, we must eliminate one variable. We do so by replacing ds , using the relation

$$ds = r d\theta,$$

in which $d\theta$ is the angle at P that includes arc length ds (Fig. 22-13g). With this replacement, we can integrate Eq. 22-20 over the angle made by the rod at P , from $\theta = -60^\circ$ to $\theta = 60^\circ$; that will give us the field magnitude at P :

$$\begin{aligned} E &= \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \theta r d\theta \\ &= \frac{\lambda}{4\pi\epsilon_0 r} \int_{-60^\circ}^{60^\circ} \cos \theta d\theta = \frac{\lambda}{4\pi\epsilon_0 r} \left[\sin \theta \right]_{-60^\circ}^{60^\circ} \\ &= \frac{\lambda}{4\pi\epsilon_0 r} [\sin 60^\circ - \sin(-60^\circ)] \\ &= \frac{1.73\lambda}{4\pi\epsilon_0 r}. \end{aligned} \quad (22-21)$$

(If we had reversed the limits on the integration, we would have gotten the same result but with a minus sign. Since the integration gives only the magnitude of \vec{E} , we would then have discarded the minus sign.)

Charge density: To evaluate λ , we note that the full rod subtends an angle of 120° and so is one-third of a full circle. Its arc length is then $2\pi r/3$, and its linear charge density must be

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

Substituting this into Eq. 22-21 and simplifying give us

$$\begin{aligned} E &= \frac{(1.73)(0.477Q)}{4\pi\epsilon_0 r^2} \\ &= \frac{0.83Q}{4\pi\epsilon_0 r^2}. \end{aligned} \quad (\text{Answer})$$

The direction of \vec{E} is toward the rod, along the axis of symmetry of the charge distribution. We can write \vec{E} in unit-vector notation as

$$\vec{E} = \frac{0.83Q}{4\pi\epsilon_0 r^2} \hat{i}.$$

Problem-Solving Tactics A Field Guide for Lines of Charge

Here is a generic guide for finding the electric field \vec{E} produced at a point P by a line of uniform charge, either circular or straight. The general strategy is to pick out an element dq of the charge, find $d\vec{E}$ due to that element, and integrate $d\vec{E}$ over the entire line of charge.

Step 1. If the line of charge is circular, let ds be the arc length of an element of the distribution. If the line is straight, run an x axis along it and let dx be the length of an element. Mark the element on a sketch.

Step 2. Relate the charge dq of the element to the length of the element with either $dq = \lambda ds$ or $dq = \lambda dx$. Consider dq and λ to be positive, even if the charge is actually negative. (The sign of the charge is used in the next step.)

Step 3. Express the field $d\vec{E}$ produced at P by dq with Eq. 22-3, replacing q in that equation with either λds or λdx . If the charge on the line is positive, then at P draw a vector $d\vec{E}$ that points directly away from dq . If the charge is negative, draw the vector pointing directly toward dq .

Step 4. Always look for any symmetry in the situation. If P is on an axis of symmetry of the charge distribution, resolve the field $d\vec{E}$ produced by dq into components that are perpendicular and parallel to the axis of symmetry. Then consider a second element dq' that is located symmetrically to dq about the line of symmetry. At P draw the vector $d\vec{E}'$ that this symmetrical element pro-

duces and resolve it into components. One of the components produced by dq is a *cancelling component*; it is canceled by the corresponding component produced by dq' and needs no further attention. The other component produced by dq is an *adding component*; it adds to the corresponding component produced by dq' . Add the adding components of all the elements via integration.

Step 5. Here are four general types of uniform charge distributions, with strategies for the integral of step 4.

Ring, with point P on (central) axis of symmetry, as in Fig. 22-11. In the expression for dE , replace r^2 with $z^2 + R^2$, as in Eq. 22-12. Express the adding component of $d\vec{E}$ in terms of θ . That introduces $\cos \theta$, but θ is identical for all elements and thus is not a variable. Replace $\cos \theta$ as in Eq. 22-13. Integrate over s , around the circumference of the ring.

Circular arc, with point P at the center of curvature, as in Fig. 22-13. Express the adding component of $d\vec{E}$ in terms of θ . That introduces either $\sin \theta$ or $\cos \theta$. Reduce the resulting two variables s and θ to one, θ , by replacing ds with $r d\theta$. Integrate over θ from one end of the arc to the other end.

Straight line, with point P on an extension of the line, as in Fig. 22-14a. In the expression for dE , replace r with x . Integrate over x , from end to end of the line of charge.

Straight line, with point P at perpendicular distance y from the line of charge, as in Fig. 22-14b. In the expression for dE , replace r with an expression involving x and y . If P is on the perpendicular bisector of the line of charge, find an expression for the adding component of $d\vec{E}$. That will introduce either $\sin \theta$ or $\cos \theta$. Reduce the resulting two variables x and θ to one, x , by replacing the trigonometric function with an expression (its definition) involving x and y . Integrate over x from end to end of the line of charge. If P is not on a line of symmetry, as in Fig. 22-14c, set up an integral to sum the components dE_x , and integrate over x to find E_x . Also set up an integral to sum the components dE_y , and integrate over x again to find E_y . Use the components E_x and E_y in the usual way to find the magnitude E and the orientation of \vec{E} .

Step 6. One arrangement of the integration limits gives a positive result. The reverse gives the same result with a mi-

nus sign; discard the minus sign. If the result is to be stated in terms of the total charge Q of the distribution, replace λ with Q/L , in which L is the length of the distribution.

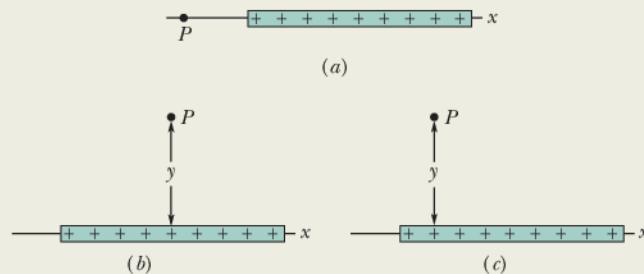


Figure 22-14 (a) Point P is on an extension of the line of charge. (b) P is on a line of symmetry of the line of charge, at perpendicular distance y from that line. (c) Same as (b) except that P is not on a line of symmetry.

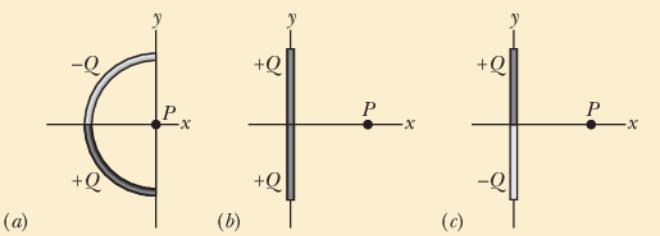


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Checkpoint 2

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude Q along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point P ?



22-5 THE ELECTRIC FIELD DUE TO A CHARGED DISK

Learning Objectives

After reading this module, you should be able to . . .

- 22.19** Sketch a disk with uniform charge and indicate the direction of the electric field at a point on the central axis if the charge is positive and if it is negative.
22.20 Explain how the equation for the electric field on the central axis of a uniformly charged ring can be used to find

Key Idea

- On the central axis through a uniformly charged disk,

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

the equation for the electric field on the central axis of a uniformly charged disk.

- 22.21** For a point on the central axis of a uniformly charged disk, apply the relationship between the surface charge density σ , the disk radius R , and the distance z to that point.

gives the electric field magnitude. Here z is the distance along the axis from the center of the disk, R is the radius of the disk, and σ is the surface charge density.

The Electric Field Due to a Charged Disk

Now we switch from a line of charge to a surface of charge by examining the electric field of a circular plastic disk, with a radius R and a uniform surface charge density σ (charge per unit area, Table 22-1) on its top surface. The disk sets up a

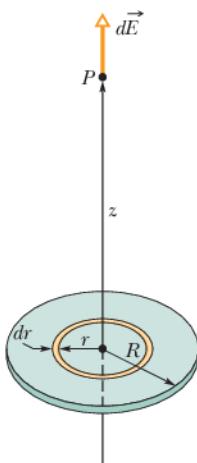


Figure 22-15 A disk of radius R and uniform positive charge. The ring shown has radius r and radial width dr . It sets up a differential electric field $d\vec{E}$ at point P on its central axis.

pattern of electric field lines around it, but here we restrict our attention to the electric field at an arbitrary point P on the central axis, at distance z from the center of the disk, as indicated in Fig. 22-15.

We could proceed as in the preceding module but set up a two-dimensional integral to include all of the field contributions from the two-dimensional distribution of charge on the top surface. However, we can save a lot of work with a neat shortcut using our earlier work with the field on the central axis of a thin ring.

We superimpose a ring on the disk as shown in Fig. 22-15, at an arbitrary radius $r \leq R$. The ring is so thin that we can treat the charge on it as a charge element dq . To find its small contribution dE to the electric field at point P , we rewrite Eq. 22-16 in terms of the ring's charge dq and radius r :

$$dE = \frac{dq z}{4\pi\epsilon_0(z^2 + r^2)^{3/2}}. \quad (22-22)$$

The ring's field points in the positive direction of the z axis.

To find the total field at P , we are going to integrate Eq. 22-22 from the center of the disk at $r = 0$ out to the rim at $r = R$ so that we sum all the dE contributions (by sweeping our arbitrary ring over the entire disk surface). However, that means we want to integrate with respect to a variable radius r of the ring.

We get dr into the expression by substituting for dq in Eq. 22-22. Because the ring is so thin, call its thickness dr . Then its surface area dA is the product of its circumference $2\pi r$ and thickness dr . So, in terms of the surface charge density σ , we have

$$dq = \sigma dA = \sigma(2\pi r dr). \quad (22-23)$$

After substituting this into Eq. 22-22 and simplifying slightly, we can sum all the dE contributions with

$$E = \int dE = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2 + r^2)^{-3/2} (2r) dr, \quad (22-24)$$

where we have pulled the constants (including z) out of the integral. To solve this integral, we cast it in the form $\int X^m dX$ by setting $X = (z^2 + r^2)$, $m = -\frac{3}{2}$, and $dX = (2r) dr$. For the recast integral we have

$$\int X^m dX = \frac{X^{m+1}}{m+1},$$

and so Eq. 22-24 becomes

$$E = \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R. \quad (22-25)$$

Taking the limits in Eq. 22-25 and rearranging, we find

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \quad (\text{charged disk}) \quad (22-26)$$

as the magnitude of the electric field produced by a flat, circular, charged disk at points on its central axis. (In carrying out the integration, we assumed that $z \geq 0$.)

If we let $R \rightarrow \infty$ while keeping z finite, the second term in the parentheses in Eq. 22-26 approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{infinite sheet}). \quad (22-27)$$

This is the electric field produced by an infinite sheet of uniform charge located on one side of a nonconductor such as plastic. The electric field lines for such a situation are shown in Fig. 22-4.

We also get Eq. 22-27 if we let $z \rightarrow 0$ in Eq. 22-26 while keeping R finite. This shows that at points very close to the disk, the electric field set up by the disk is the same as if the disk were infinite in extent.

22-6 A POINT CHARGE IN AN ELECTRIC FIELD

Learning Objectives

After reading this module, you should be able to ...

- 22.22** For a charged particle placed in an external electric field (a field due to other charged objects), apply the relationship between the electric field \vec{E} at that point, the particle's charge q , and the electrostatic force \vec{F} that acts on the particle, and identify the relative directions of the force

and the field when the particle is positively charged and negatively charged.

- 22.23** Explain Millikan's procedure of measuring the elementary charge.

- 22.24** Explain the general mechanism of ink-jet printing.

Key Ideas

- If a particle with charge q is placed in an external electric field \vec{E} , an electrostatic force \vec{F} acts on the particle:

$$\vec{F} = q\vec{E}.$$

- If charge q is positive, the force vector is in the same direction as the field vector. If charge q is negative, the force vector is in the opposite direction (the minus sign in the equation reverses the force vector from the field vector).

A Point Charge in an Electric Field

In the preceding four modules we worked at the first of our two tasks: given a charge distribution, to find the electric field it produces in the surrounding space. Here we begin the second task: to determine what happens to a charged particle when it is in an electric field set up by other stationary or slowly moving charges.

What happens is that an electrostatic force acts on the particle, as given by

$$\vec{F} = q\vec{E}, \quad (22-28)$$

in which q is the charge of the particle (including its sign) and \vec{E} is the electric field that other charges have produced at the location of the particle. (The field is *not* the field set up by the particle itself; to distinguish the two fields, the field acting on the particle in Eq. 22-28 is often called the *external field*. A charged particle or object is not affected by its own electric field.) Equation 22-28 tells us



The electrostatic force \vec{F} acting on a charged particle located in an external electric field \vec{E} has the direction of \vec{E} if the charge q of the particle is positive and has the opposite direction if q is negative.

Measuring the Elementary Charge

Equation 22-28 played a role in the measurement of the elementary charge e by American physicist Robert A. Millikan in 1910–1913. Figure 22-16 is a representation of his apparatus. When tiny oil drops are sprayed into chamber A, some of them become charged, either positively or negatively, in the process. Consider a drop that drifts downward through the small hole in plate P_1 and into chamber C. Let us assume that this drop has a negative charge q .

If switch S in Fig. 22-16 is open as shown, battery B has no electrical effect on chamber C. If the switch is closed (the connection between chamber C and the positive terminal of the battery is then complete), the battery causes an excess positive charge on conducting plate P_1 and an excess negative charge on conducting plate P_2 . The charged plates set up a downward-directed electric field \vec{E} in chamber C. According to Eq. 22-28, this field exerts an electrostatic force on any charged drop that happens to be in the chamber and affects its motion. In particular, our negatively charged drop will tend to drift upward.

By timing the motion of oil drops with the switch opened and with it closed and thus determining the effect of the charge q , Millikan discovered that the

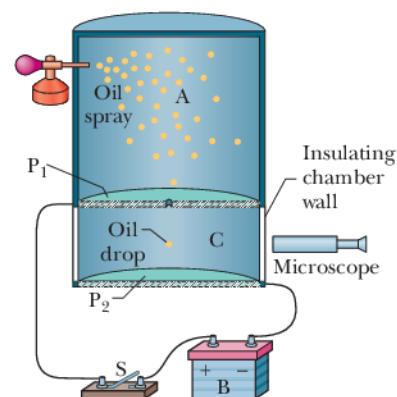


Figure 22-16 The Millikan oil-drop apparatus for measuring the elementary charge e . When a charged oil drop drifted into chamber C through the hole in plate P_1 , its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

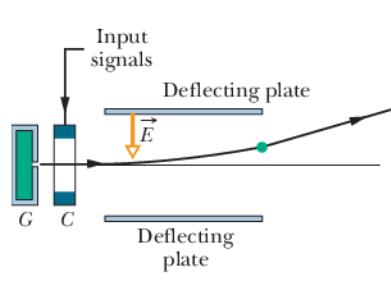


Figure 22-17 Ink-jet printer. Drops shot from generator G receive a charge in charging unit C . An input signal from a computer controls the charge and thus the effect of field \vec{E} on where the drop lands on the paper.

values of q were always given by

$$q = ne, \quad \text{for } n = 0, \pm 1, \pm 2, \pm 3, \dots, \quad (22-29)$$

in which e turned out to be the fundamental constant we call the *elementary charge*, 1.60×10^{-19} C. Millikan's experiment is convincing proof that charge is quantized, and he earned the 1923 Nobel Prize in physics in part for this work. Modern measurements of the elementary charge rely on a variety of interlocking experiments, all more precise than the pioneering experiment of Millikan.

Ink-Jet Printing

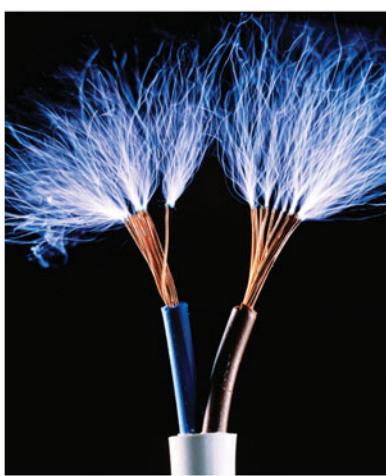
The need for high-quality, high-speed printing has caused a search for an alternative to impact printing, such as occurs in a standard typewriter. Building up letters by squirting tiny drops of ink at the paper is one such alternative.

Figure 22-17 shows a negatively charged drop moving between two conducting deflecting plates, between which a uniform, downward-directed electric field \vec{E} has been set up. The drop is deflected upward according to Eq. 22-28 and then strikes the paper at a position that is determined by the magnitudes of \vec{E} and the charge q of the drop.

In practice, E is held constant and the position of the drop is determined by the charge q delivered to the drop in the charging unit, through which the drop must pass before entering the deflecting system. The charging unit, in turn, is activated by electronic signals that encode the material to be printed.

Electrical Breakdown and Sparking

If the magnitude of an electric field in air exceeds a certain critical value E_c , the air undergoes *electrical breakdown*, a process whereby the field removes electrons from the atoms in the air. The air then begins to conduct electric current because the freed electrons are propelled into motion by the field. As they move, they collide with any atoms in their path, causing those atoms to emit light. We can see the paths, commonly called sparks, taken by the freed electrons because of that emitted light. Figure 22-18 shows sparks above charged metal wires where the electric fields due to the wires cause electrical breakdown of the air.



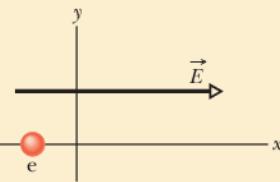
Adam Hart-Davis/Photo Researchers, Inc.

Figure 22-18 The metal wires are so charged that the electric fields they produce in the surrounding space cause the air there to undergo electrical breakdown.



Checkpoint 3

- (a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the y axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?





Sample Problem 22.04 Motion of a charged particle in an electric field

Figure 22-19 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field \vec{E} is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

KEY IDEA

The drop is negatively charged and the electric field is directed *downward*. From Eq. 22-28, a constant electrostatic force of

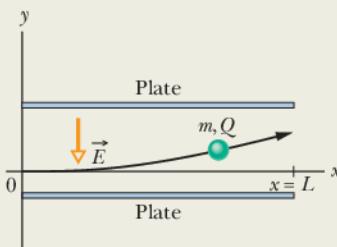


Figure 22-19 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

magnitude QE acts *upward* on the charged drop. Thus, as the drop travels parallel to the x axis at constant speed v_x , it accelerates upward with some constant acceleration a_y .

Calculations: Applying Newton's second law ($F = ma$) for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \quad (22-30)$$

Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \frac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t, \quad (22-31)$$

respectively. Eliminating t between these two equations and substituting Eq. 22-30 for a_y , we find

$$\begin{aligned} y &= \frac{QEL^2}{2mv_x^2} \\ &= \frac{(1.5 \times 10^{-13} \text{ C})(1.4 \times 10^6 \text{ N/C})(1.6 \times 10^{-2} \text{ m})^2}{(2)(1.3 \times 10^{-10} \text{ kg})(18 \text{ m/s})^2} \\ &= 6.4 \times 10^{-4} \text{ m} \\ &= 0.64 \text{ mm}. \end{aligned} \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS

22-7 A DIPOLE IN AN ELECTRIC FIELD

Learning Objectives

After reading this module, you should be able to . . .

22.25 On a sketch of an electric dipole in an external electric field, indicate the direction of the field, the direction of the dipole moment, the direction of the electrostatic forces on the two ends of the dipole, and the direction in which those forces tend to rotate the dipole, and identify the value of the net force on the dipole.

22.26 Calculate the torque on an electric dipole in an external electric field by evaluating a cross product of the dipole moment vector and the electric field vector, in magnitude-angle notation and unit-vector notation.

22.27 For an electric dipole in an external electric field, relate the potential energy of the dipole to the work done by a torque as the dipole rotates in the electric field.

22.28 For an electric dipole in an external electric field, calculate the potential energy by taking a dot product of the dipole moment vector and the electric field vector, in magnitude-angle notation and unit-vector notation.

22.29 For an electric dipole in an external electric field, identify the angles for the minimum and maximum potential energies and the angles for the minimum and maximum torque magnitudes.

Key Ideas

- The torque on an electric dipole of dipole moment \vec{p} when placed in an external electric field \vec{E} is given by a cross product:

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

- A potential energy U is associated with the orientation of the dipole moment in the field, as given by a dot product:

$$U = -\vec{p} \cdot \vec{E}.$$

- If the dipole orientation changes, the work done by the electric field is

$$W = -\Delta U.$$

If the change in orientation is due to an external agent, the work done by the agent is $W_a = -W$.

A Dipole in an Electric Field

We have defined the electric dipole moment \vec{p} of an electric dipole to be a vector that points from the negative to the positive end of the dipole. As you will see, the behavior of a dipole in a uniform external electric field \vec{E} can be described completely in terms of the two vectors \vec{E} and \vec{p} , with no need of any details about the dipole's structure.

A molecule of water (H_2O) is an electric dipole; Fig. 22-20 shows why. There the black dots represent the oxygen nucleus (having eight protons) and the two hydrogen nuclei (having one proton each). The colored enclosed areas represent the regions in which electrons can be located around the nuclei.

In a water molecule, the two hydrogen atoms and the oxygen atom do not lie on a straight line but form an angle of about 105° , as shown in Fig. 22-20. As a result, the molecule has a definite "oxygen side" and "hydrogen side." Moreover, the 10 electrons of the molecule tend to remain closer to the oxygen nucleus than to the hydrogen nuclei. This makes the oxygen side of the molecule slightly more negative than the hydrogen side and creates an electric dipole moment \vec{p} that points along the symmetry axis of the molecule as shown. If the water molecule is placed in an external electric field, it behaves as would be expected of the more abstract electric dipole of Fig. 22-9.

To examine this behavior, we now consider such an abstract dipole in a uniform external electric field \vec{E} , as shown in Fig. 22-21a. We assume that the dipole is a rigid structure that consists of two centers of opposite charge, each of magnitude q , separated by a distance d . The dipole moment \vec{p} makes an angle θ with field \vec{E} .

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig. 22-21a) and with the same magnitude $F = qE$. Thus, *because the field is uniform*, the net force on the dipole from the field is zero and the center of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance x from one end and thus a distance $d - x$ from the other end. From Eq. 10-39 ($\tau = rF \sin \phi$), we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta. \quad (22-32)$$

We can also write the magnitude of $\vec{\tau}$ in terms of the magnitudes of the electric field E and the dipole moment $p = qd$. To do so, we substitute qE for F and p/q for d in Eq. 22-32, finding that the magnitude of $\vec{\tau}$ is

$$\tau = pE \sin \theta. \quad (22-33)$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\text{torque on a dipole}). \quad (22-34)$$

Vectors \vec{p} and \vec{E} are shown in Fig. 22-21b. The torque acting on a dipole tends to rotate \vec{p} (hence the dipole) into the direction of field \vec{E} , thereby reducing θ . In Fig. 22-21, such rotation is clockwise. As we discussed in Chapter 10, we can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque. With that notation, the torque of Fig. 22-21 is

$$\tau = -pE \sin \theta. \quad (22-35)$$

Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment \vec{p} is lined up with the field \vec{E} (then $\vec{\tau} = \vec{p} \times \vec{E} = 0$). It has greater potential energy in all other orientations. Thus the dipole is like a pendulum, which has *its* least gravitational potential

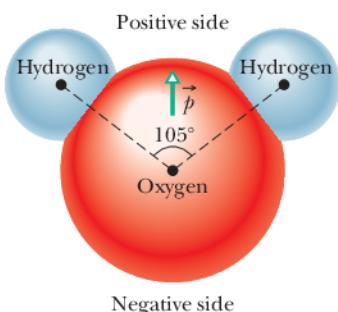


Figure 22-20 A molecule of H_2O , showing the three nuclei (represented by dots) and the regions in which the electrons can be located. The electric dipole moment \vec{p} points from the (negative) oxygen side to the (positive) hydrogen side of the molecule.

energy in *its* equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

In any situation involving potential energy, we are free to define the zero-potential-energy configuration in an arbitrary way because only differences in potential energy have physical meaning. The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ in Fig. 22-21 is 90° . We then can find the potential energy U of the dipole at any other value of θ with Eq. 8-1 ($\Delta U = -W$) by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90° . With the aid of Eq. 10-53 ($W = \int \tau d\theta$) and Eq. 22-35, we find that the potential energy U at any angle θ is

$$U = -W = - \int_{90^\circ}^{\theta} \tau d\theta = \int_{90^\circ}^{\theta} pE \sin \theta d\theta. \quad (22-36)$$

Evaluating the integral leads to

$$U = -pE \cos \theta. \quad (22-37)$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E} \quad (\text{potential energy of a dipole}). \quad (22-38)$$

Equations 22-37 and 22-38 show us that the potential energy of the dipole is least ($U = -pE$) when $\theta = 0$ (\vec{p} and \vec{E} are in the same direction); the potential energy is greatest ($U = pE$) when $\theta = 180^\circ$ (\vec{p} and \vec{E} are in opposite directions).

When a dipole rotates from an initial orientation θ_i to another orientation θ_f , the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i), \quad (22-39)$$

where U_f and U_i are calculated with Eq. 22-38. If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$W_a = -W = (U_f - U_i). \quad (22-40)$$

Microwave Cooking

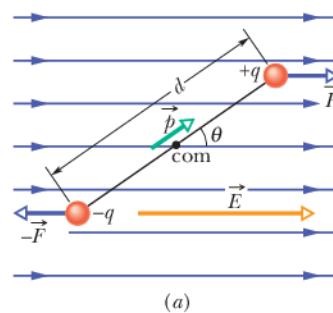
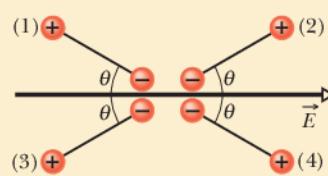
Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field \vec{E} within the oven and thus also within the food. From Eq. 22-34, we see that any electric field \vec{E} produces a torque on an electric dipole moment \vec{p} to align \vec{p} with \vec{E} . Because the oven's \vec{E} oscillates, the water molecules continuously flip-flop in a frustrated attempt to align with \vec{E} .

Energy is transferred from the electric field to the thermal energy of the water (and thus of the food) where three water molecules happened to have bonded together to form a group. The flip-flop breaks some of the bonds. When the molecules reform the bonds, energy is transferred to the random motion of the group and then to the surrounding molecules. Soon, the thermal energy of the water is enough to cook the food.



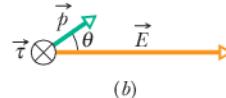
Checkpoint 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



(a)

The dipole is being torqued into alignment.



(b)

Figure 22-21 (a) An electric dipole in a uniform external electric field \vec{E} . Two centers of equal but opposite charge are separated by distance d . The line between them represents their rigid connection. (b) Field \vec{E} causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol \otimes .



Sample Problem 22.05 Torque and energy of an electric dipole in an electric field

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude $6.2 \times 10^{-30} \text{ C} \cdot \text{m}$.

- (a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d .

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$\begin{aligned} d &= \frac{p}{10e} = \frac{6.2 \times 10^{-30} \text{ C} \cdot \text{m}}{(10)(1.60 \times 10^{-19} \text{ C})} \\ &= 3.9 \times 10^{-12} \text{ m} = 3.9 \text{ pm}. \end{aligned} \quad (\text{Answer})$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

- (b) If the molecule is placed in an electric field of $1.5 \times 10^4 \text{ N/C}$, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)



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Review & Summary

Electric Field To explain the electrostatic force between two charges, we assume that each charge sets up an electric field in the space around it. The force acting on each charge is then due to the electric field set up at its location by the other charge.

Definition of Electric Field The *electric field* \vec{E} at any point is defined in terms of the electrostatic force \vec{F} that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}. \quad (22-1)$$

Electric Field Lines *Electric field lines* provide a means for visualizing the direction and magnitude of electric fields. The electric field vector at any point is tangent to a field line through that point. The density of field lines in any region is proportional to the magnitude of the electric field in that region. Field lines originate on positive charges and terminate on negative charges.

Field Due to a Point Charge The magnitude of the electric field \vec{E} set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}. \quad (22-3)$$

KEY IDEA

The torque on a dipole is maximum when the angle θ between \vec{p} and \vec{E} is 90° .

Calculation: Substituting $\theta = 90^\circ$ in Eq. 22-33 yields

$$\begin{aligned} \tau &= pE \sin \theta \\ &= (6.2 \times 10^{-30} \text{ C} \cdot \text{m})(1.5 \times 10^4 \text{ N/C})(\sin 90^\circ) \\ &= 9.3 \times 10^{-26} \text{ N} \cdot \text{m}. \end{aligned} \quad (\text{Answer})$$

- (c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0^\circ$?

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$\begin{aligned} W_a &= U_{180^\circ} - U_0 \\ &= (-pE \cos 180^\circ) - (-pE \cos 0^\circ) \\ &= 2pE = (2)(6.2 \times 10^{-30} \text{ C} \cdot \text{m})(1.5 \times 10^4 \text{ N/C}) \\ &= 1.9 \times 10^{-25} \text{ J}. \end{aligned} \quad (\text{Answer})$$

The direction of \vec{E} is away from the point charge if the charge is positive and toward it if the charge is negative.

Field Due to an Electric Dipole An *electric dipole* consists of two particles with charges of equal magnitude q but opposite sign, separated by a small distance d . Their **electric dipole moment** \vec{p} has magnitude qd and points from the negative charge to the positive charge. The magnitude of the electric field set up by the dipole at a distant point on the dipole axis (which runs through both charges) is

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}, \quad (22-9)$$

where z is the distance between the point and the center of the dipole.

Field Due to a Continuous Charge Distribution The electric field due to a *continuous charge distribution* is found by treating charge elements as point charges and then summing, via integration, the electric field vectors produced by all the charge elements to find the net vector.

Field Due to a Charged Disk The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right), \quad (22-26)$$

where z is the distance along the axis from the center of the disk, R is the radius of the disk, and σ is the surface charge density.

Force on a Point Charge in an Electric Field When a point charge q is placed in an external electric field \vec{E} , the electrostatic force \vec{F} that acts on the point charge is

$$\vec{F} = q\vec{E}. \quad (22-28)$$

Questions

- 1 Figure 22-22 shows three arrangements of electric field lines. In each arrangement, a proton is released from rest at point A and is then accelerated through point B by the electric field. Points A and B have equal separations in the three arrangements. Rank the arrangements according to the linear momentum of the proton at point B , greatest first.

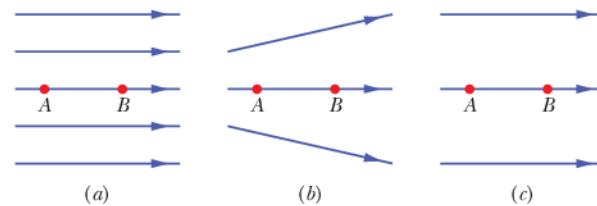


Figure 22-22 Question 1.

- 2 Figure 22-23 shows two square arrays of charged particles. The squares, which are centered on point P , are misaligned. The particles are separated by either d or $d/2$ along the perimeters of the squares. What are the magnitude and direction of the net electric field at P ?

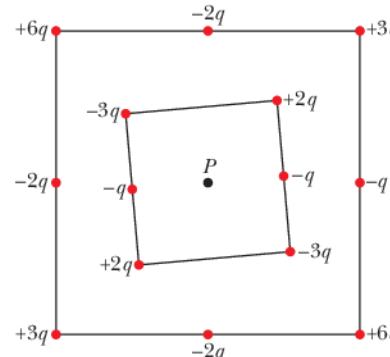


Figure 22-23 Question 2.

- 3 In Fig. 22-24, two particles of charge $-q$ are arranged symmetrically about the y axis; each produces an electric field at point P on that axis. (a) Are the magnitudes of the fields at P equal? (b) Is each electric field directed toward or away from the charge producing it? (c) Is the magnitude of the net electric field at P equal to the sum of the magnitudes E of the two field vectors (is it equal to $2E$)? (d) Do the x components of those two field vectors add or cancel? (e) Do their y components add or cancel? (f) Is the direction of the net field at P that of the canceling components or the adding components? (g) What is the direction of the net field?

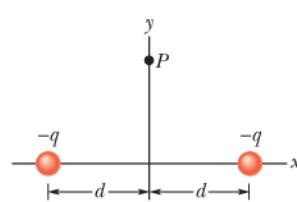


Figure 22-24 Question 3.

Force \vec{F} has the same direction as \vec{E} if q is positive and the opposite direction if q is negative.

Dipole in an Electric Field When an electric dipole of dipole moment \vec{p} is placed in an electric field \vec{E} , the field exerts a torque $\vec{\tau}$ on the dipole:

$$\vec{\tau} = \vec{p} \times \vec{E}. \quad (22-34)$$

The dipole has a potential energy U associated with its orientation in the field:

$$U = -\vec{p} \cdot \vec{E}. \quad (22-38)$$

This potential energy is defined to be zero when \vec{p} is perpendicular to \vec{E} ; it is least ($U = -pE$) when \vec{p} is aligned with \vec{E} and greatest ($U = pE$) when \vec{p} is directed opposite \vec{E} .

- 4 Figure 22-25 shows four situations in which four charged particles are evenly spaced to the left and right of a central point. The charge values are indicated. Rank the situations according to the magnitude of the net electric field at the central point, greatest first.

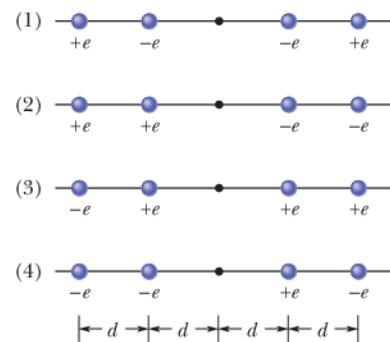


Figure 22-25 Question 4.

- 5 Figure 22-26 shows two charged particles fixed in place on an axis. (a) Where on the axis (other than at an infinite distance) is there a point at which their net electric field is zero: between the charges, to their left, or to their right? (b) Is there a point of zero net electric field anywhere off the axis (other than at an infinite distance)?



Figure 22-26 Question 5.

- 6 In Fig. 22-27, two identical circular nonconducting rings are centered on the same line with their planes perpendicular to the line. Each ring has charge that is uniformly distributed along its circumference. The rings each produce electric fields at points along the line. For three situations, the charges on rings A and B are, respectively, (1) q_0 and q_0 , (2) $-q_0$ and $-q_0$, and (3) $-q_0$ and q_0 . Rank the situations according to the magnitude of the net electric field at (a) point P_1 midway between the rings, (b) point P_2 at the center of ring B , and (c) point P_3 to the right of ring B , greatest first.

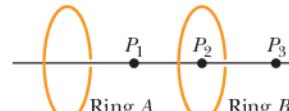


Figure 22-27 Question 6.

- 7 The potential energies associated with four orientations of an electric dipole in an electric field are (1) $-5U_0$, (2) $-7U_0$, (3) $3U_0$, and (4) $5U_0$, where U_0 is positive. Rank the orientations according to (a) the angle between the electric dipole moment \vec{p} and the electric field \vec{E} and (b) the magnitude of the torque on the electric dipole, greatest first.

- 8 (a) In Checkpoint 4, if the dipole rotates from orientation 1 to orientation 2, is the work done on the dipole by the field positive, negative, or zero? (b) If, instead, the dipole rotates from orientation 1 to orientation 4, is the work done by the field more than, less than, or the same as in (a)?

- 9** Figure 22-28 shows two disks and a flat ring, each with the same uniform charge Q . Rank the objects according to the magnitude of the electric field they create at points P (which are at the same vertical heights), greatest first.

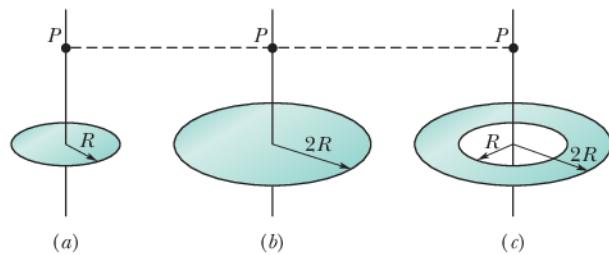


Figure 22-28 Question 9.

- 10** In Fig. 22-29, an electron e travels through a small hole in plate A and then toward plate B . A uniform electric field in the region between the plates then slows the electron without deflecting it. (a) What is the direction of the field? (b) Four other particles similarly travel through small holes in either plate A or plate B and then into the region between the plates. Three have charges $+q_1$, $+q_2$, and $-q_3$. The fourth (labeled n) is a neutron, which is electrically neutral. Does the speed of each of those four other particles increase, decrease, or remain the same in the region between the plates?

- 11** In Fig. 22-30a, a circular plastic rod with uniform charge $+Q$ produces an electric field of magnitude E at the center of

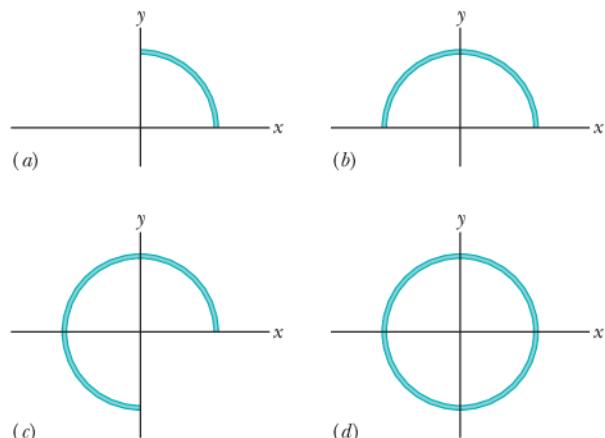


Figure 22-30 Question 11.

curvature (at the origin). In Figs. 22-30b, c, and d, more circular rods, each with identical uniform charges $+Q$, are added until the circle is complete. A fifth arrangement (which would be labeled e) is like that in d except the rod in the fourth quadrant has charge $-Q$. Rank the five arrangements according to the magnitude of the electric field at the center of curvature, greatest first.

- 12** When three electric dipoles are near each other, they each experience the electric field of the other two, and the three-dipole system has a certain potential energy. Figure 22-31 shows two arrangements in which three electric dipoles are side by side. Each dipole has the same magnitude of electric dipole moment, and the spacings between adjacent dipoles are identical. In which arrangement is the potential energy of the three-dipole system greater?

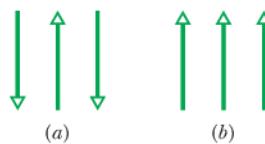


Figure 22-31 Question 12.

- 13** Figure 22-32 shows three rods, each with the same charge Q spread uniformly along its length. Rods a (of length L) and b (of length $L/2$) are straight, and points P are aligned with their midpoints. Rod c (of length $L/2$) forms a complete circle about point P . Rank the rods according to the magnitude of the electric field they create at points P , greatest first.

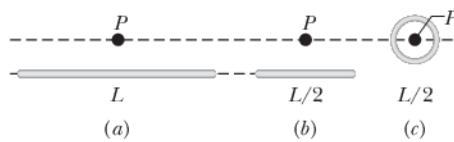


Figure 22-32 Question 13.

- 14** Figure 22-33 shows five protons that are launched in a uniform electric field \vec{E} ; the magnitude and direction of the launch velocities are indicated. Rank the protons according to the magnitude of their accelerations due to the field, greatest first.

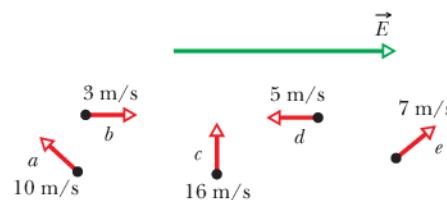


Figure 22-33 Question 14.

Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Worked-out solution is at

<http://www.wiley.com/college/halliday>



Number of dots indicates level of problem difficulty



Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Module 22-1 The Electric Field

- Sketch qualitatively the electric field lines both between and outside two concentric conducting spherical shells when a uniform

positive charge q_1 is on the inner shell and a uniform negative charge $-q_2$ is on the outer. Consider the cases $q_1 > q_2$, $q_1 = q_2$, and $q_1 < q_2$.

- 2** In Fig. 22-34 the electric field lines on the left have twice the separation of those on the right. (a) If the magnitude of the field at A is 40 N/C, what is the magnitude of the force on a proton at A ? (b) What is the magnitude of the field at B ?

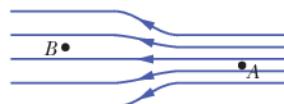


Figure 22-34 Problem 2.

Module 22-2 The Electric Field Due to a Charged Particle

- 3 SSM** The nucleus of a plutonium-239 atom contains 94 protons. Assume that the nucleus is a sphere with radius 6.64 fm and with the charge of the protons uniformly spread through the sphere. At the surface of the nucleus, what are the (a) magnitude and (b) direction (radially inward or outward) of the electric field produced by the protons?

- 4** Two charged particles are attached to an x axis: Particle 1 of charge -2.00×10^{-7} C is at position $x = 6.00$ cm and particle 2 of charge $+2.00 \times 10^{-7}$ C is at position $x = 21.0$ cm. Midway between the particles, what is their net electric field in unit-vector notation?

- 5 SSM** A charged particle produces an electric field with a magnitude of 2.0 N/C at a point that is 50 cm away from the particle. What is the magnitude of the particle's charge?

- 6** What is the magnitude of a point charge that would create an electric field of 1.00 N/C at points 1.00 m away?

- 7 SSM ILW WWW** In Fig. 22-35, the four particles form a square of edge length $a = 5.00$ cm and have charges $q_1 = +10.0$ nC, $q_2 = -20.0$ nC, $q_3 = +20.0$ nC, and $q_4 = -10.0$ nC. In unit-vector notation, what net electric field do the particles produce at the square's center?

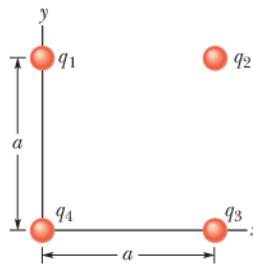


Figure 22-35 Problem 7.

- 8 GO** In Fig. 22-36, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5.0 \mu\text{m}$. What is the magnitude of the net electric field at point P due to the particles?

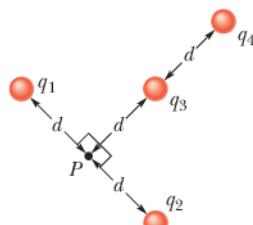


Figure 22-36 Problem 8.

- 9 GO** Figure 22-37 shows two charged particles on an x axis: $-q = -3.20 \times 10^{-19}$ C at $x = -3.00$ m and $q = 3.20 \times 10^{-19}$ C at $x = +3.00$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at point P at $y = 4.00$ m?

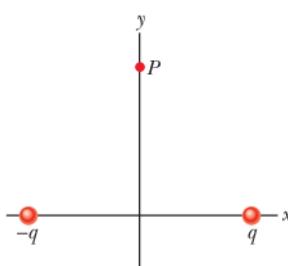
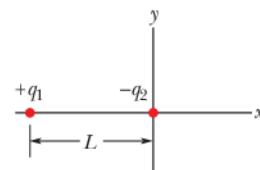
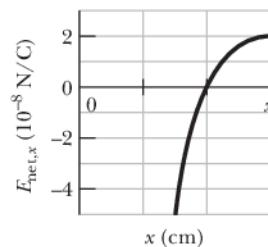


Figure 22-37 Problem 9.

- 10 GO** Figure 22-38a shows two charged particles fixed in place on an x axis with separation L . The ratio q_1/q_2 of their charge magnitudes is 4.00. Figure 22-38b shows the x component $E_{\text{net},x}$ of their net electric field along the x axis just to the right of particle 2. The x axis scale is set by $x_s = 30.0$ cm. (a) At what value of $x > 0$ is $E_{\text{net},x}$ maximum? (b) If particle 2 has charge $-q_2 = -3e$, what is the value of that maximum?



(a)



(b)

Figure 22-38 Problem 10.

- 11 SSM** Two charged particles are fixed to an x axis: Particle 1 of charge $q_1 = 2.1 \times 10^{-8}$ C is at position $x = 20$ cm and particle 2 of charge $q_2 = -4.00q_1$ is at position $x = 70$ cm. At what coordinate on the axis (other than at infinity) is the net electric field produced by the two particles equal to zero?

- 12 GO** Figure 22-39 shows an uneven arrangement of electrons (e) and protons (p) on a circular arc of radius $r = 2.00$ cm, with angles $\theta_1 = 30.0^\circ$, $\theta_2 = 50.0^\circ$, $\theta_3 = 30.0^\circ$, and $\theta_4 = 20.0^\circ$. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the net electric field produced at the center of the arc?

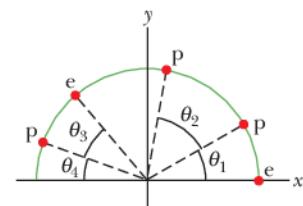


Figure 22-39 Problem 12.

- 13 GO** Figure 22-40 shows a proton (p) on the central axis through a disk with a uniform charge density due to excess electrons. The disk is seen from an edge-on view. Three of those electrons are shown: electron e_c at the disk center and electrons e_s at opposite sides of the disk, at radius R from the center. The proton is initially at distance $z = R = 2.00$ cm from the disk. At that location, what are the magnitudes of (a) the electric field \vec{E}_c due to electron e_c and (b) the net electric field $\vec{E}_{s,\text{net}}$ due to electrons e_s ? The proton is then moved to $z = R/10.0$. What then are the magnitudes of (c) \vec{E}_c and (d) $\vec{E}_{s,\text{net}}$ at the proton's location? (e) From (a) and (c) we see that as the proton gets nearer to the disk, the magnitude of \vec{E}_c increases, as expected. Why does the magnitude of $\vec{E}_{s,\text{net}}$ from the two side electrons decrease, as we see from (b) and (d)?

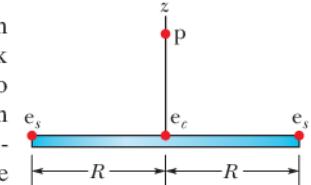


Figure 22-40 Problem 13.

- 14** In Fig. 22-41, particle 1 of charge $q_1 = -5.00q$ and particle 2 of charge $q_2 = +2.00q$ are fixed to an x axis. (a) As a multiple of distance L , at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines between and around the particles.

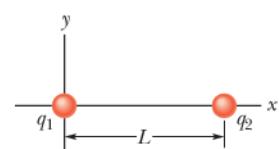


Figure 22-41 Problem 14.

••15 In Fig. 22-42, the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00 \mu\text{m}$. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?

••16 Figure 22-43 shows a plastic ring of radius $R = 50.0 \text{ cm}$. Two small charged beads are on the ring: Bead 1 of charge $+2.00 \mu\text{C}$ is fixed in place at the left side; bead 2 of charge $+6.00 \mu\text{C}$ can be moved along the ring. The two beads produce a net electric field of magnitude E at the center of the ring. At what (a) positive and (b) negative value of angle θ should bead 2 be positioned such that $E = 2.00 \times 10^5 \text{ N/C}$?

••17 Two charged beads are on the plastic ring in Fig. 22-44a. Bead 2, which is not shown, is fixed in place on the ring, which has radius $R = 60.0 \text{ cm}$. Bead 1, which is not fixed in place, is initially on the x axis at angle $\theta = 0^\circ$. It is then moved to the opposite side, at angle $\theta = 180^\circ$, through the first and second quadrants of the xy coordinate system. Figure 22-44b gives the x component of the net electric field produced at the origin by the two beads as a function of θ , and Fig. 22-44c gives the y component of that net electric field. The vertical axis scales are set by $E_{xs} = 5.0 \times 10^4 \text{ N/C}$ and $E_{ys} = -9.0 \times 10^4 \text{ N/C}$. (a) At what angle θ is bead 2 located? What are the charges of (b) bead 1 and (c) bead 2?

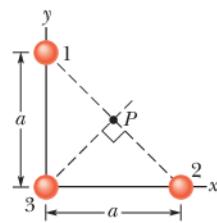


Figure 22-42
Problem 15.

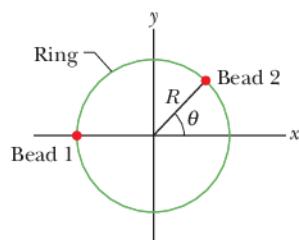


Figure 22-43 Problem 16.

••18 Two charged beads are on the plastic ring in Fig. 22-44a. Bead 2, which is not shown, is fixed in place on the ring, which has radius $R = 60.0 \text{ cm}$. Bead 1, which is not fixed in place, is initially on the x axis at angle $\theta = 0^\circ$. It is then moved to the opposite side, at angle $\theta = 180^\circ$, through the first and second quadrants of the xy coordinate system. Figure 22-44b gives the x component of the net electric field produced at the origin by the two beads as a function of θ , and Fig. 22-44c gives the y component of that net electric field. The vertical axis scales are set by $E_{xs} = 5.0 \times 10^4 \text{ N/C}$ and $E_{ys} = -9.0 \times 10^4 \text{ N/C}$. (a) At what angle θ is bead 2 located? What are the charges of (b) bead 1 and (c) bead 2?

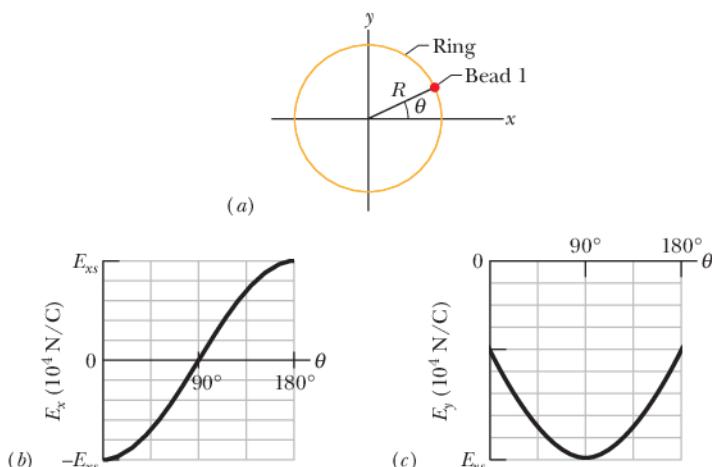


Figure 22-44 Problem 17.

Module 22-3 The Electric Field Due to a Dipole

••18 The electric field of an electric dipole along the dipole axis is approximated by Eqs. 22-8 and 22-9. If a binomial expansion is made of Eq. 22-7, what is the next term in the expression for the dipole's electric field along the dipole axis? That is, what is E_{next} in the expression

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} + E_{\text{next}}?$$

••19 Figure 22-45 shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P , located at distance $r \gg d$?

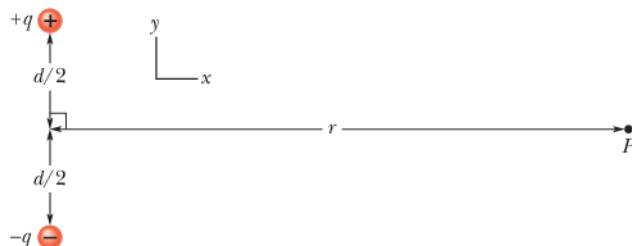


Figure 22-45 Problem 19.

••20 Equations 22-8 and 22-9 are approximations of the magnitude of the electric field of an electric dipole, at points along the dipole axis. Consider a point P on that axis at distance $z = 5.00d$ from the dipole center (d is the separation distance between the particles of the dipole). Let E_{appr} be the magnitude of the field at point P as approximated by Eqs. 22-8 and 22-9. Let E_{act} be the actual magnitude. What is the ratio $E_{\text{appr}}/E_{\text{act}}$?

••21 SSM *Electric quadrupole.* Figure 22-46 shows a generic electric quadrupole. It consists of two dipoles with dipole moments that are equal in magnitude but opposite in direction. Show that the value of E on the axis of the quadrupole for a point P a distance z from its center (assume $z \gg d$) is given by

$$E = \frac{3Q}{4\pi\epsilon_0 z^4},$$

in which $Q (= 2qd^2)$ is known as the *quadrupole moment* of the charge distribution.

Module 22-4 The Electric Field Due to a Line of Charge

•22 *Density, density, density.* (a) A charge $-300e$ is uniformly distributed along a circular arc of radius 4.00 cm , which subtends an angle of 40° . What is the linear charge density along the arc? (b) A charge $-300e$ is uniformly distributed over one face of a circular disk of radius 2.00 cm . What is the surface charge density over that face? (c) A charge $-300e$ is uniformly distributed over the surface of a sphere of radius 2.00 cm . What is the surface charge density over that surface? (d) A charge $-300e$ is uniformly spread through the volume of a sphere of radius 2.00 cm . What is the volume charge density in that sphere?

•23 Figure 22-47 shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge q_1 and radius R ; ring 2 has uniform charge q_2 and the same radius R . The rings are separated by distance $d = 3.00R$. The net electric field at point P on the common line, at distance R from ring 1, is zero. What is the ratio q_1/q_2 ?

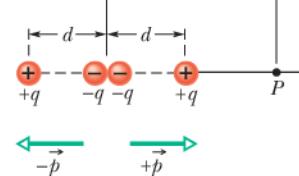


Figure 22-46 Problem 21.

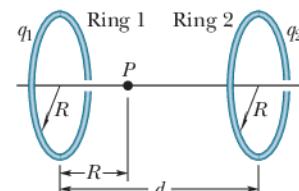


Figure 22-47 Problem 23.

•24 A thin nonconducting rod with a uniform distribution of positive charge Q is bent into a complete circle of radius R

(Fig. 22-48). The central perpendicular axis through the ring is a z axis, with the origin at the center of the ring. What is the magnitude of the electric field due to the rod at (a) $z = 0$ and (b) $z = \infty$? (c) In terms of R , at what positive value of z is that magnitude maximum? (d) If $R = 2.00\text{ cm}$ and $Q = 4.00\text{ }\mu\text{C}$, what is the maximum magnitude?

••25 Figure 22-49 shows three circular arcs centered on the origin of a coordinate system. On each arc, the uniformly distributed charge is given in terms of $Q = 2.00\text{ }\mu\text{C}$. The radii are given in terms of $R = 10.0\text{ cm}$. What are the (a) magnitude and (b) direction (relative to the positive x direction) of the net electric field at the origin due to the arcs?

••26 **GO ILW** In Fig. 22-50, a thin glass rod forms a semicircle of radius $r = 5.00\text{ cm}$. Charge is uniformly distributed along the rod, with $+q = 4.50\text{ pC}$ in the upper half and $-q = -4.50\text{ pC}$ in the lower half. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field \vec{E} at P , the center of the semicircle?

••27 **GO** In Fig. 22-51, two curved plastic rods, one of charge $+q$ and the other of charge $-q$, form a circle of radius $R = 8.50\text{ cm}$ in an xy plane. The x axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If $q = 15.0\text{ pC}$, what are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field \vec{E} produced at P , the center of the circle?

••28 Charge is uniformly distributed around a ring of radius $R = 2.40\text{ cm}$, and the resulting electric field magnitude E is measured along the ring's central axis (perpendicular to the plane of the ring). At what distance from the ring's center is E maximum?

••29 **GO** Figure 22-52a shows a nonconducting rod with a uniformly distributed charge $+Q$. The rod forms a half-circle with radius R and produces an electric field of magnitude E_{arc} at its center of curvature P . If the arc is collapsed to a point at distance R from P (Fig. 22-52b), by what factor is the magnitude of the electric field at P multiplied?

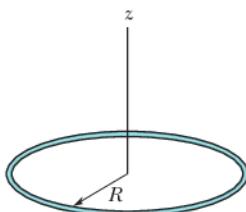


Figure 22-48 Problem 24.

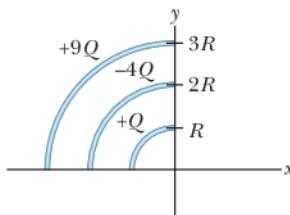


Figure 22-49 Problem 25.

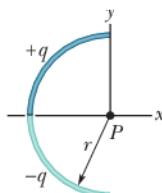


Figure 22-50
Problem 26.

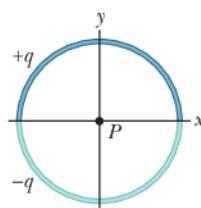


Figure 22-51
Problem 27.

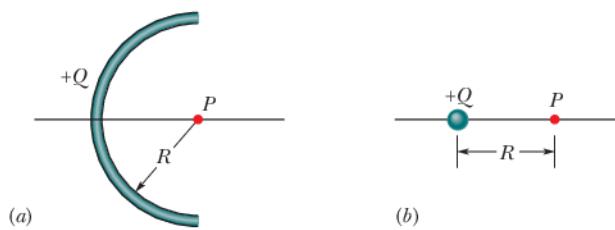


Figure 22-52 Problem 29.

••30 **GO** Figure 22-53 shows two concentric rings, of radii R and $R' = 3.00R$, that lie on the same plane. Point P lies on the central z axis, at distance $D = 2.00R$ from the center of the rings. The smaller ring has uniformly distributed charge $+Q$. In terms of Q , what is the uniformly distributed charge on the larger ring if the net electric field at P is zero?

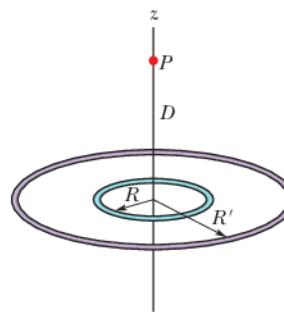


Figure 22-53 Problem 30.

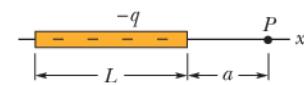


Figure 22-54 Problem 31.

••31 **SSM ILW WWW** In Fig. 22-54, a nonconducting rod of length $L = 8.15\text{ cm}$ has a charge $-q = -4.23\text{ fC}$ uniformly distributed along its length. (a) What is the linear charge density of the rod? What are the (b) magnitude and (c) direction (relative to the positive direction of the x axis) of the electric field produced at point P , at distance $a = 12.0\text{ cm}$ from the rod? What is the electric field magnitude produced at distance $a = 50\text{ m}$ by (d) the rod and (e) a particle of charge $-q = -4.23\text{ fC}$ that we use to replace the rod? (At that distance, the rod "looks" like a particle.)

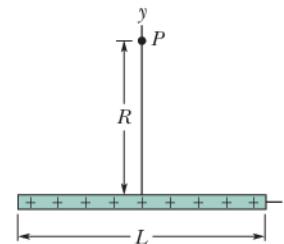


Figure 22-55 Problem 32.

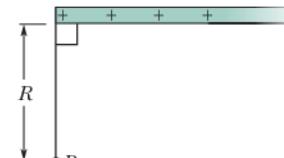


Figure 22-56 Problem 33.

••32 **GO** In Fig. 22-55, positive charge $q = 7.81\text{ pC}$ is spread uniformly along a thin nonconducting rod of length $L = 14.5\text{ cm}$. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field produced at point P , at distance $R = 6.00\text{ cm}$ from the rod along its perpendicular bisector?

••33 **GO** In Fig. 22-56, a "semi-infinite" nonconducting rod (that is, infinite in one direction only) has uniform linear charge density λ . Show that the electric field \vec{E}_p at point P makes an angle of 45° with the rod and that this result is independent of the distance R . (Hint: Separately find the component of \vec{E}_p parallel to the rod and the component perpendicular to the rod.)

Module 22-5 The Electric Field Due to a Charged Disk

•34 A disk of radius 2.5 cm has a surface charge density of $5.3\text{ }\mu\text{C/m}^2$ on its upper face. What is the magnitude of the electric field produced by the disk at a point on its central axis at distance $z = 12\text{ cm}$ from the disk?

•35 **SSM WWW** At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.600 m is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk?

•36 A circular plastic disk with radius $R = 2.00\text{ cm}$ has a uniformly distributed charge $Q = +(2.00 \times 10^6)e$ on one face. A circular ring of width $30\text{ }\mu\text{m}$ is centered on that face, with the center of that width at radius $r = 0.50\text{ cm}$. In coulombs, what charge is contained within the width of the ring?

••37 Suppose you design an apparatus in which a uniformly charged disk of radius R is to produce an electric field. The field magnitude is most important along the central perpendicular axis of the disk, at a point P at distance $2.00R$ from the disk (Fig. 22-57a). Cost analysis suggests that you switch to a ring of the same outer radius R but with inner radius $R/2.00$ (Fig. 22-57b). Assume that the ring will have the same surface charge density as the original disk. If you switch to the ring, by what percentage will you decrease the electric field magnitude at P ?

••38 Figure 22-58a shows a circular disk that is uniformly charged. The central z axis is perpendicular to the disk face, with the origin at the disk. Figure 22-58b gives the magnitude of the electric field along that axis in terms of the maximum magnitude E_m at the disk surface. The z axis scale is set by $z_s = 8.0$ cm. What is the radius of the disk?

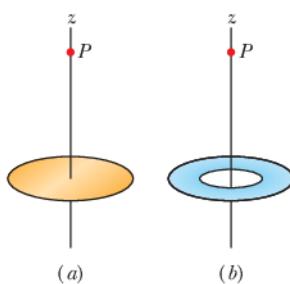


Figure 22-57 Problem 37.

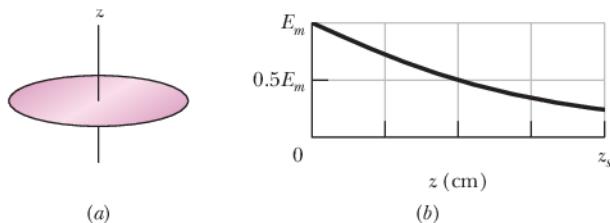


Figure 22-58 Problem 38.

Module 22-6 A Point Charge in an Electric Field

•39 In Millikan's experiment, an oil drop of radius $1.64 \mu\text{m}$ and density 0.851 g/cm^3 is suspended in chamber C (Fig. 22-16) when a downward electric field of $1.92 \times 10^5 \text{ N/C}$ is applied. Find the charge on the drop, in terms of e .

•40 GO An electron with a speed of $5.00 \times 10^8 \text{ cm/s}$ enters an electric field of magnitude $1.00 \times 10^3 \text{ N/C}$, traveling along a field line in the direction that retards its motion. (a) How far will the electron travel in the field before stopping momentarily, and (b) how much time will have elapsed? (c) If the region containing the electric field is 8.00 mm long (too short for the electron to stop within it), what fraction of the electron's initial kinetic energy will be lost in that region?

•41 SSM A charged cloud system produces an electric field in the air near Earth's surface. A particle of charge $-2.0 \times 10^{-9} \text{ C}$ is acted on by a downward electrostatic force of $3.0 \times 10^{-6} \text{ N}$ when placed in this field. (a) What is the magnitude of the electric field? What are the (b) magnitude and (c) direction of the electrostatic force \vec{F}_{el} on the proton placed in this field? (d) What is the magnitude of the gravitational force \vec{F}_g on the proton? (e) What is the ratio F_{el}/F_g in this case?

•42 Humid air breaks down (its molecules become ionized) in an electric field of $3.0 \times 10^6 \text{ N/C}$. In that field, what is the magnitude of the electrostatic force on (a) an electron and (b) an ion with a single electron missing?

•43 SSM An electron is released from rest in a uniform electric field of magnitude $2.00 \times 10^4 \text{ N/C}$. Calculate the acceleration of the electron. (Ignore gravitation.)

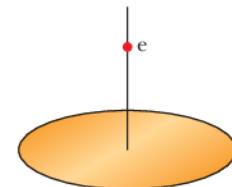
•44 An alpha particle (the nucleus of a helium atom) has a mass of $6.64 \times 10^{-27} \text{ kg}$ and a charge of $+2e$. What are the (a) magnitude and (b) direction of the electric field that will balance the gravitational force on the particle?

•45 ILW An electron on the axis of an electric dipole is 25 nm from the center of the dipole. What is the magnitude of the electrostatic force on the electron if the dipole moment is $3.6 \times 10^{-29} \text{ C}\cdot\text{m}$? Assume that 25 nm is much larger than the separation of the charged particles that form the dipole.

•46 An electron is accelerated eastward at $1.80 \times 10^9 \text{ m/s}^2$ by an electric field. Determine the field (a) magnitude and (b) direction.

•47 SSM Beams of high-speed protons can be produced in "guns" using electric fields to accelerate the protons. (a) What acceleration would a proton experience if the gun's electric field were $2.00 \times 10^4 \text{ N/C}$? (b) What speed would the proton attain if the field accelerated the proton through a distance of 1.00 cm ?

•48 In Fig. 22-59, an electron (e) is to be released from rest on the central axis of a uniformly charged disk of radius R . The surface charge density on the disk is $+4.00 \mu\text{C/m}^2$. What is the magnitude of the electron's initial acceleration if it is released at a distance (a) R , (b) $R/100$, and (c) $R/1000$ from the center of the disk? (d) Why does the acceleration magnitude increase only slightly as the release point is moved closer to the disk?

Figure 22-59
Problem 48.

•49 A 10.0 g block with a charge of $+8.00 \times 10^{-5} \text{ C}$ is placed in an electric field $\vec{E} = (3000\hat{i} - 600\hat{j}) \text{ N/C}$. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electrostatic force on the block? If the block is released from rest at the origin at time $t = 0$, what are its (c) x and (d) y coordinates at $t = 3.00 \text{ s}$?

•50 At some instant the velocity components of an electron moving between two charged parallel plates are $v_x = 1.5 \times 10^5 \text{ m/s}$ and $v_y = 3.0 \times 10^3 \text{ m/s}$. Suppose the electric field between the plates is uniform and given by $\vec{E} = (120 \text{ N/C})\hat{j}$. In unit-vector notation, what are (a) the electron's acceleration in that field and (b) the electron's velocity when its x coordinate has changed by 2.0 cm ?

•51 Assume that a honeybee is a sphere of diameter 1.00 cm with a charge of $+45.0 \text{ pC}$ uniformly spread over its surface. Assume also that a spherical pollen grain of diameter $40.0 \mu\text{m}$ is electrically held on the surface of the bee because the bee's charge induces a charge of -1.00 pC on the near side of the grain and a charge of $+1.00 \text{ pC}$ on the far side. (a) What is the magnitude of the net electrostatic force on the grain due to the bee? Next, assume that the bee brings the grain to a distance of 1.000 mm from the tip of a flower's stigma and that the tip is a particle of charge -45.0 pC . (b) What is the magnitude of the net electrostatic force on the grain due to the stigma? (c) Does the grain remain on the bee or does it move to the stigma?

•52 An electron enters a region of uniform electric field with an initial velocity of 40 km/s in the same direction as the electric field, which has magnitude $E = 50 \text{ N/C}$. (a) What is the speed of the electron 1.5 ns after entering this region? (b) How far does the electron travel during the 1.5 ns interval?

- 53 GO** Two large parallel copper plates are 5.0 cm apart and have a uniform electric field between them as depicted in Fig. 22-60. An electron is released from the negative plate at the same time that a proton is released from the positive plate. Neglect the force of the particles on each other and find their distance from the positive plate when they pass each other. (Does it surprise you that you need not know the electric field to solve this problem?)

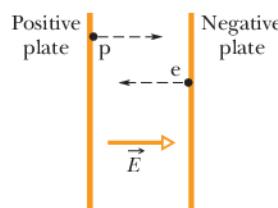


Figure 22-60 Problem 53.

- 54 GO** In Fig. 22-61, an electron is shot at an initial speed of $v_0 = 2.00 \times 10^6 \text{ m/s}$, at angle $\theta_0 = 40.0^\circ$ from an x axis. It moves through a uniform electric field $\vec{E} = (5.00 \text{ N/C})\hat{j}$. A screen for detecting electrons is positioned parallel to the y axis, at distance $x = 3.00 \text{ m}$. In unit-vector notation, what is the velocity of the electron when it hits the screen?

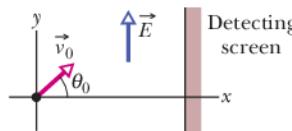


Figure 22-61 Problem 54.

- 55 ILW** A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away, in a time $1.5 \times 10^{-8} \text{ s}$. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field \vec{E} ?

Module 22-7 A Dipole in an Electric Field

- 56** An electric dipole consists of charges $+2e$ and $-2e$ separated by 0.78 nm. It is in an electric field of strength $3.4 \times 10^6 \text{ N/C}$. Calculate the magnitude of the torque on the dipole when the dipole moment is (a) parallel to, (b) perpendicular to, and (c) antiparallel to the electric field.

- 57 SSM** An electric dipole consisting of charges of magnitude 1.50 nC separated by $6.20 \mu\text{m}$ is in an electric field of strength 1100 N/C . What are (a) the magnitude of the electric dipole moment and (b) the difference between the potential energies for dipole orientations parallel and antiparallel to \vec{E} ?

- 58** A certain electric dipole is placed in a uniform electric field \vec{E} of magnitude 20 N/C . Figure 22-62 gives the potential energy U of the dipole versus the angle θ between \vec{E} and the dipole moment \vec{p} . The vertical axis scale is set by $U_s = 100 \times 10^{-28} \text{ J}$. What is the magnitude of \vec{p} ?

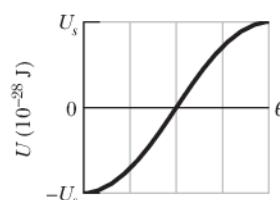


Figure 22-62 Problem 58.

- 59** How much work is required to turn an electric dipole 180° in a uniform electric field of magnitude $E = 46.0 \text{ N/C}$ if the dipole moment has a magnitude of $p = 3.02 \times 10^{-25} \text{ C}\cdot\text{m}$ and the initial angle is 64° ?

- 60** A certain electric dipole is placed in a uniform electric field \vec{E} of magnitude 40 N/C . Figure 22-63 gives the magnitude τ of the torque on the dipole versus the angle θ between field \vec{E} and the dipole moment \vec{p} . The vertical axis scale is set by $\tau_s = 100 \times 10^{-28} \text{ N}\cdot\text{m}$. What is the magnitude of \vec{p} ?

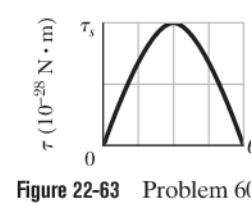


Figure 22-63 Problem 60.

- 61** Find an expression for the oscillation frequency of an electric dipole of dipole moment \vec{p} and rotational inertia I for small amplitudes of oscillation about its equilibrium position in a uniform electric field of magnitude E .

Additional Problems

- 62** (a) What is the magnitude of an electron's acceleration in a uniform electric field of magnitude $1.40 \times 10^6 \text{ N/C}$? (b) How long would the electron take, starting from rest, to attain one-tenth the speed of light? (c) How far would it travel in that time?

- 63** A spherical water drop $1.20 \mu\text{m}$ in diameter is suspended in calm air due to a downward-directed atmospheric electric field of magnitude $E = 462 \text{ N/C}$. (a) What is the magnitude of the gravitational force on the drop? (b) How many excess electrons does it have?

- 64** Three particles, each with positive charge Q , form an equilateral triangle, with each side of length d . What is the magnitude of the electric field produced by the particles at the midpoint of any side?

- 65** In Fig. 22-64a, a particle of charge $+Q$ produces an electric field of magnitude E_{part} at point P , at distance R from the particle. In Fig. 22-64b, that same amount of charge is spread uniformly along a circular arc that has radius R and subtends an angle θ . The charge on the arc produces an electric field of magnitude E_{arc} at its center of curvature P . For what value of θ does $E_{\text{arc}} = 0.500E_{\text{part}}$? (Hint: You will probably resort to a graphical solution.)

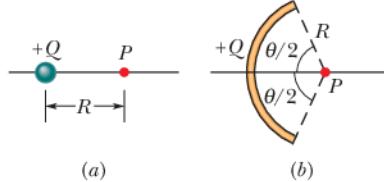
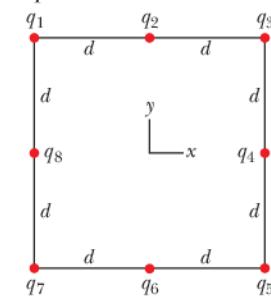


Figure 22-64 Problem 65.

- 66** A proton and an electron form two corners of an equilateral triangle of side length $2.0 \times 10^{-6} \text{ m}$. What is the magnitude of the net electric field these two particles produce at the third corner?

- 67** A charge (uniform linear density = 9.0 nC/m) lies on a string that is stretched along an x axis from $x = 0$ to $x = 3.0 \text{ m}$. Determine the magnitude of the electric field at $x = 4.0 \text{ m}$ on the x axis.

- 68** In Fig. 22-65, eight particles form a square in which distance $d = 2.0 \text{ cm}$. The charges are $q_1 = +3e$, $q_2 = +e$, $q_3 = -5e$, $q_4 = -2e$, $q_5 = +3e$, $q_6 = +e$, $q_7 = -5e$, and $q_8 = +e$. In unit-vector notation, what is the net electric field at the square's center?

Figure 22-65
Problem 68.

- 69** Two particles, each with a charge of magnitude 12 nC , are at two of the vertices of an equilateral triangle with edge length 2.0 m . What is the magnitude of the electric field at the third vertex if (a) both charges are positive and (b) one charge is positive and the other is negative?

- 70** The following table gives the charge seen by Millikan at different times on a single drop in his experiment. From the data, calculate the elementary charge e .

$6.563 \times 10^{-19} \text{ C}$	$13.13 \times 10^{-19} \text{ C}$	$19.71 \times 10^{-19} \text{ C}$
$8.204 \times 10^{-19} \text{ C}$	$16.48 \times 10^{-19} \text{ C}$	$22.89 \times 10^{-19} \text{ C}$
$11.50 \times 10^{-19} \text{ C}$	$18.08 \times 10^{-19} \text{ C}$	$26.13 \times 10^{-19} \text{ C}$

71 A charge of 20 nC is uniformly distributed along a straight rod of length 4.0 m that is bent into a circular arc with a radius of 2.0 m . What is the magnitude of the electric field at the center of curvature of the arc?

72 An electron is constrained to the central axis of the ring of charge of radius R in Fig. 22-11, with $z \ll R$. Show that the electrostatic force on the electron can cause it to oscillate through the ring center with an angular frequency

$$\omega = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}},$$

where q is the ring's charge and m is the electron's mass.

73 SSM The electric field in an xy plane produced by a positively charged particle is $7.2(4.0\hat{i} + 3.0\hat{j}) \text{ N/C}$ at the point $(3.0, 3.0) \text{ cm}$ and $100\hat{i} \text{ N/C}$ at the point $(2.0, 0) \text{ cm}$. What are the (a) x and (b) y coordinates of the particle? (c) What is the charge of the particle?

74 (a) What total (excess) charge q must the disk in Fig. 22-15 have for the electric field on the surface of the disk at its center to have magnitude $3.0 \times 10^6 \text{ N/C}$, the E value at which air breaks down electrically, producing sparks? Take the disk radius as 2.5 cm . (b) Suppose each surface atom has an effective cross-sectional area of 0.015 nm^2 . How many atoms are needed to make up the disk surface? (c) The charge calculated in (a) results from some of the surface atoms having one excess electron. What fraction of these atoms must be so charged?

75 In Fig. 22-66, particle 1 (of charge $+1.00 \mu\text{C}$), particle 2 (of charge $+1.00 \mu\text{C}$), and particle 3 (of charge Q) form an equilateral triangle of edge length a . For what value of Q (both sign and magnitude) does the net electric field produced by the particles at the center of the triangle vanish?

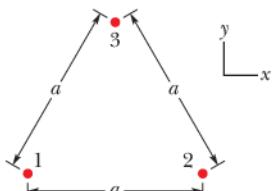


Figure 22-66 Problems 75 and 86.

76 In Fig. 22-67, an electric dipole swings from an initial orientation i ($\theta_i = 20.0^\circ$) to a final orientation f ($\theta_f = 20.0^\circ$) in a uniform external electric field \vec{E} . The electric dipole moment is $1.60 \times 10^{-27} \text{ C}\cdot\text{m}$; the field magnitude is $3.00 \times 10^6 \text{ N/C}$. What is the change in the dipole's potential energy?

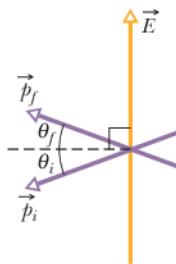


Figure 22-67 Problem 76.

77 A particle of charge $-q_1$ is at the origin of an x axis. (a) At what location on the axis should a particle of charge $-4q_1$ be placed so that the net electric field is zero at $x = 2.0 \text{ mm}$ on the axis? (b) If, instead, a particle of charge $+4q_1$ is placed at that location, what is the direction (relative to the positive direction of the x axis) of the net electric field at $x = 2.0 \text{ mm}$?

78 Two particles, each of positive charge q , are fixed in place on a y axis, one at $y = d$ and the other at $y = -d$. (a) Write an expression that gives the magnitude E of the net electric field at points on the x axis given by $x = ad$. (b) Graph E versus α for the range $0 < \alpha < 4$. From the graph, determine the values of α that give (c) the maximum value of E and (d) half the maximum value of E .

79 A clock face has negative point charges $-q, -2q, -3q, \dots, -12q$ fixed at the positions of the corresponding numerals. The clock hands do not perturb the net field due to the point charges. At

what time does the hour hand point in the same direction as the electric field vector at the center of the dial? (Hint: Use symmetry.)

80 Calculate the electric dipole moment of an electron and a proton 4.30 nm apart.

81 An electric field \vec{E} with an average magnitude of about 150 N/C points downward in the atmosphere near Earth's surface. We wish to "float" a sulfur sphere weighing 4.4 N in this field by charging the sphere. (a) What charge (both sign and magnitude) must be used? (b) Why is the experiment impractical?

82 A circular rod has a radius of curvature $R = 9.00 \text{ cm}$ and a uniformly distributed positive charge $Q = 6.25 \text{ pC}$ and subtends an angle $\theta = 2.40 \text{ rad}$. What is the magnitude of the electric field that Q produces at the center of curvature?

83 SSM An electric dipole with dipole moment

$$\vec{p} = (3.00\hat{i} + 4.00\hat{j})(1.24 \times 10^{-30} \text{ C}\cdot\text{m})$$

is in an electric field $\vec{E} = (4000 \text{ N/C})\hat{i}$. (a) What is the potential energy of the electric dipole? (b) What is the torque acting on it? (c) If an external agent turns the dipole until its electric dipole moment is

$$\vec{p} = (-4.00\hat{i} + 3.00\hat{j})(1.24 \times 10^{-30} \text{ C}\cdot\text{m}),$$

how much work is done by the agent?

84 In Fig. 22-68, a uniform, upward electric field \vec{E} of magnitude $2.00 \times 10^3 \text{ N/C}$ has been set up between two horizontal plates by charging the lower plate positively and the upper plate negatively. The plates have length $L = 10.0 \text{ cm}$ and separation $d = 2.00 \text{ cm}$. An electron is then shot

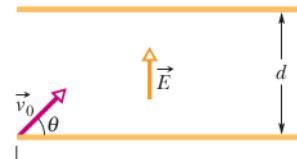


Figure 22-68 Problem 84.

between the plates from the left edge of the lower plate. The initial velocity \vec{v}_0 of the electron makes an angle $\theta = 45.0^\circ$ with the lower plate and has a magnitude of $6.00 \times 10^6 \text{ m/s}$. (a) Will the electron strike one of the plates? (b) If so, which plate and how far horizontally from the left edge will the electron strike?

85 For the data of Problem 70, assume that the charge q on the drop is given by $q = ne$, where n is an integer and e is the elementary charge. (a) Find n for each given value of q . (b) Do a linear regression fit of the values of q versus the values of n and then use that fit to find e .

86 In Fig. 22-66, particle 1 (of charge $+2.00 \text{ pC}$), particle 2 (of charge -2.00 pC), and particle 3 (of charge $+5.00 \text{ pC}$) form an equilateral triangle of edge length $a = 9.50 \text{ cm}$. (a) Relative to the positive direction of the x axis, determine the direction of the force \vec{F}_3 on particle 3 due to the other particles by sketching electric field lines of the other particles. (b) Calculate the magnitude of \vec{F}_3 .

87 In Fig. 22-69, particle 1 of charge $q_1 = 1.00 \text{ pC}$ and particle 2 of charge $q_2 = -2.00 \text{ pC}$ are fixed at a distance $d = 5.00 \text{ cm}$ apart. In unit-vector notation, what is the net electric field at points (a) A , (b) B , and (c) C ? (d) Sketch the electric field lines.

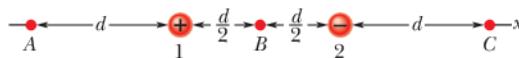


Figure 22-69 Problem 87.

Gauss' Law

23-1 ELECTRIC FLUX

Learning Objectives

After reading this module, you should be able to ...

- 23.01** Identify that Gauss' law relates the electric field at points on a closed surface (real or imaginary, said to be a Gaussian surface) to the net charge enclosed by that surface.
- 23.02** Identify that the amount of electric field piercing a surface (not skimming along the surface) is the electric flux Φ through the surface.
- 23.03** Identify that an area vector for a flat surface is a vector that is perpendicular to the surface and that has a magnitude equal to the area of the surface.
- 23.04** Identify that any surface can be divided into area elements (patch elements) that are each small enough and flat enough for an area vector $d\vec{A}$ to be assigned to it, with the vector perpendicular to the element and having a magnitude equal to the area of the element.

Key Ideas

- The electric flux Φ through a surface is the amount of electric field that pierces the surface.
- The area vector $d\vec{A}$ for an area element (patch element) on a surface is a vector that is perpendicular to the element and has a magnitude equal to the area dA of the element.
- The electric flux $d\Phi$ through a patch element with area vector $d\vec{A}$ is given by a dot product:

$$d\Phi = \vec{E} \cdot d\vec{A}.$$

- The total flux through a surface is given by

- 23.05** Calculate the flux Φ through a surface by integrating the dot product of the electric field vector \vec{E} and the area vector $d\vec{A}$ (for patch elements) over the surface, in magnitude-angle notation and unit-vector notation.
- 23.06** For a closed surface, explain the algebraic signs associated with inward flux and outward flux.
- 23.07** Calculate the *net* flux Φ through a *closed* surface, algebraic sign included, by integrating the dot product of the electric field vector \vec{E} and the area vector $d\vec{A}$ (for patch elements) over the full surface.
- 23.08** Determine whether a closed surface can be broken up into parts (such as the sides of a cube) to simplify the integration that yields the net flux through the surface.

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad (\text{total flux}),$$

where the integration is carried out over the surface.

- The net flux through a closed surface (which is used in Gauss' law) is given by

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{net flux}),$$

where the integration is carried out over the entire surface.

What Is Physics?

In the preceding chapter we found the electric field at points near extended charged objects, such as rods. Our technique was labor-intensive: We split the charge distribution up into charge elements dq , found the field $d\vec{E}$ due to an element, and resolved the vector into components. Then we determined whether the components from all the elements would end up canceling or adding. Finally we summed the adding components by integrating over all the elements, with several changes in notation along the way.

One of the primary goals of physics is to find simple ways of solving such labor-intensive problems. One of the main tools in reaching this goal is the use of symmetry. In this chapter we discuss a beautiful relationship between charge and

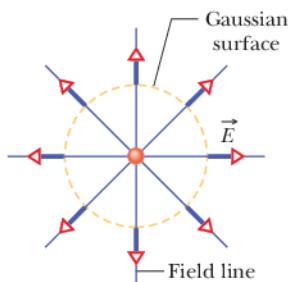


Figure 23-1 Electric field vectors and field lines pierce an imaginary, spherical Gaussian surface that encloses a particle with charge $+Q$.

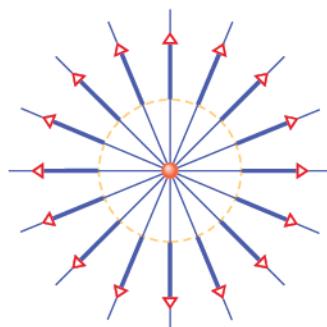


Figure 23-2 Now the enclosed particle has charge $+2Q$.

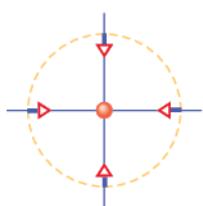


Figure 23-3 Can you tell what the enclosed charge is now?

electric field that allows us, in certain symmetric situations, to find the electric field of an extended charged object with a few lines of algebra. The relationship is called **Gauss' law**, which was developed by German mathematician and physicist Carl Friedrich Gauss (1777–1855).

Let's first take a quick look at some simple examples that give the spirit of Gauss' law. Figure 23-1 shows a particle with charge $+Q$ that is surrounded by an imaginary concentric sphere. At points on the sphere (said to be a *Gaussian surface*), the electric field vectors have a moderate magnitude (given by $E = kQ/r^2$) and point radially away from the particle (because it is positively charged). The electric field lines are also outward and have a moderate density (which, recall, is related to the field magnitude). We say that the field vectors and the field lines *pierce* the surface.

Figure 23-2 is similar except that the enclosed particle has charge $+2Q$. Because the enclosed charge is now twice as much, the magnitude of the field vectors piercing outward through the (same) Gaussian surface is twice as much as in Fig. 23-1, and the density of the field lines is also twice as much. That sentence, in a nutshell, is Gauss' law.



Guass' law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

Let's check this with a third example with a particle that is also enclosed by the same spherical Gaussian surface (a *Gaussian sphere*, if you like, or even the catchy *G-sphere*) as shown in Fig. 23-3. What is the amount and sign of the enclosed charge? Well, from the inward piercing we see immediately that the charge must be negative. From the fact that the density of field lines is half that of Fig. 23-1, we also see that the charge must be $-0.5Q$. (Using Gauss' law is like being able to tell what is inside a gift box by looking at the wrapping paper on the box.)

The problems in this chapter are of two types. Sometimes we know the charge and we use Gauss' law to find the field at some point. Sometimes we know the field on a Gaussian surface and we use Gauss' law to find the charge enclosed by the surface. However, we cannot do all this by simply comparing the density of field lines in a drawing as we just did. We need a quantitative way of determining how much electric field pierces a surface. That measure is called the electric flux.

Electric Flux

Flat Surface, Uniform Field. We begin with a flat surface with area A in a uniform electric field \vec{E} . Figure 23-4a shows one of the electric field vectors \vec{E} piercing a small square patch with area ΔA (where Δ indicates “small”). Actually, only the x component (with magnitude $E_x = E \cos \theta$ in Fig. 23-4b) pierces the patch. The y component merely skims along the surface (no piercing in that) and does not come into play in Gauss' law. The *amount* of electric field piercing the patch is defined to be the **electric flux** $\Delta\Phi$ through it:

$$\Delta\Phi = (E \cos \theta) \Delta A.$$

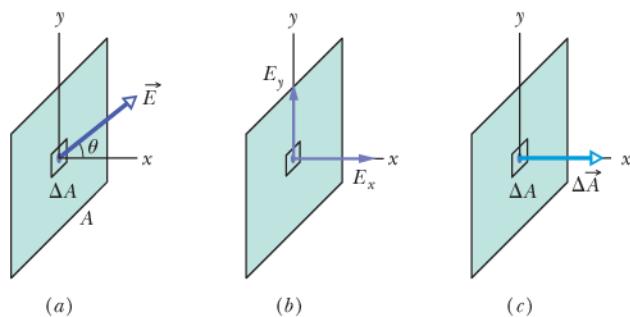


Figure 23-4 (a) An electric field vector pierces a small square patch on a flat surface. (b) Only the x component actually pierces the patch; the y component skims across it. (c) The area vector of the patch is perpendicular to the patch, with a magnitude equal to the patch's area.

There is another way to write the right side of this statement so that we have only the piercing component of \vec{E} . We define an area vector $\Delta\vec{A}$ that is perpendicular to the patch and that has a magnitude equal to the area ΔA of the patch (Fig. 23-4c). Then we can write

$$\Delta\Phi = \vec{E} \cdot \Delta\vec{A},$$

and the dot product automatically gives us the component of \vec{E} that is parallel to $\Delta\vec{A}$ and thus piercing the patch.

To find the total flux Φ through the surface in Fig. 23-4, we sum the flux through every patch on the surface:

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}. \quad (23-1)$$

However, because we do not want to sum hundreds (or more) flux values, we transform the summation into an integral by shrinking the patches from small squares with area ΔA to *patch elements* (or *area elements*) with area dA . The total flux is then

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad (\text{total flux}). \quad (23-2)$$

Now we can find the total flux by integrating the dot product over the full surface.

Dot Product. We can evaluate the dot product inside the integral by writing the two vectors in unit-vector notation. For example, in Fig. 23-4, $d\vec{A} = dA\hat{i}$ and \vec{E} might be, say, $(4\hat{i} + 4\hat{j})$ N/C. Instead, we can evaluate the dot product in magnitude-angle notation: $E \cos \theta dA$. When the electric field is uniform and the surface is flat, the product $E \cos \theta$ is a constant and comes outside the integral. The remaining $\int dA$ is just an instruction to sum the areas of all the patch elements to get the total area, but we already know that the total area is A . So the total flux in this simple situation is

$$\Phi = (E \cos \theta)A \quad (\text{uniform field, flat surface}). \quad (23-3)$$

Closed Surface. To use Gauss' law to relate flux and charge, we need a closed surface. Let's use the closed surface in Fig. 23-5 that sits in a nonuniform electric field. (Don't worry. The homework problems involve less complex surfaces.) As before, we first consider the flux through small square patches. However, now we are interested in not only the piercing components of the field but also on whether the piercing is inward or outward (just as we did with Figs. 23-1 through 23-3).

Directions. To keep track of the piercing direction, we again use an area vector $\Delta\vec{A}$ that is perpendicular to a patch, but now we always draw it pointing outward from the surface (*away from the interior*). Then if a field vector pierces outward, it and the area vector are in the same direction, the angle is $\theta = 0$, and $\cos \theta = 1$. Thus, the dot product $\vec{E} \cdot \Delta\vec{A}$ is positive and so is the flux. Conversely, if a field vector pierces inward, the angle is $\theta = 180^\circ$ and $\cos \theta = -1$. Thus, the dot product is negative and so is the flux. If a field vector skims the surface (no piercing), the dot product is zero (because $\cos 90^\circ = 0$) and so is the flux. Figure 23-5 gives some general examples and here is a summary:



An inward piercing field is negative flux. An outward piercing field is positive flux. A skimming field is zero flux.

Net Flux. In principle, to find the **net flux** through the surface in Fig. 23-5, we find the flux at every patch and then sum the results (with the algebraic signs included). However, we are not about to do that much work. Instead, we shrink the squares to patch elements with area vectors $d\vec{A}$ and then integrate:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{net flux}). \quad (23-4)$$

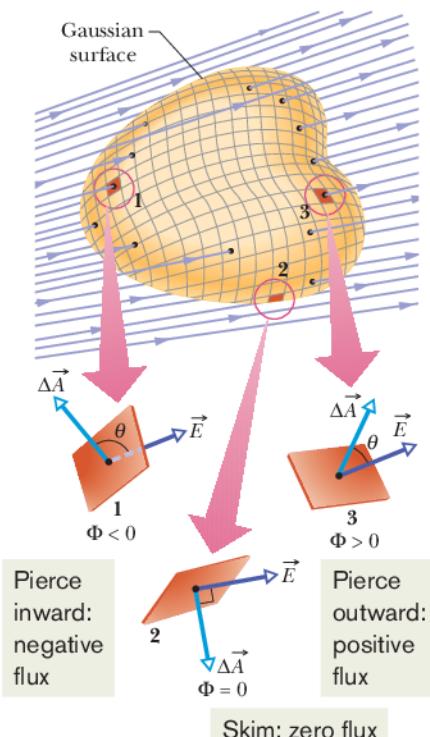


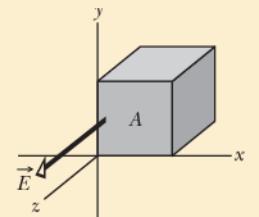
Figure 23-5 A Gaussian surface of arbitrary shape immersed in an electric field. The surface is divided into small squares of area ΔA . The electric field vectors \vec{E} and the area vectors $\Delta\vec{A}$ for three representative squares, marked 1, 2, and 3, are shown.

The loop on the integral sign indicates that we must integrate over the entire closed surface, to get the *net* flux through the surface (as in Fig. 23-5, flux might enter on one side and leave on another side). Keep in mind that we want to determine the net flux through a surface because that is what Gauss' law relates to the charge enclosed by the surface. (The law is coming up next.) Note that flux is a scalar (yes, we talk about field vectors but flux is the *amount* of piercing field, not a vector itself). The SI unit of flux is the newton-square-meter per coulomb ($N \cdot m^2/C$).



Checkpoint 1

The figure here shows a Gaussian cube of face area A immersed in a uniform electric field \vec{E} that has the positive direction of the z axis. In terms of E and A , what is the flux through (a) the front face (which is in the xy plane), (b) the rear face, (c) the top face, and (d) the whole cube?



Sample Problem 23.01 Flux through a closed cylinder, uniform field

Figure 23-6 shows a Gaussian surface in the form of a closed cylinder (a Gaussian cylinder or G-cylinder) of radius R . It lies in a uniform electric field \vec{E} with the cylinder's central axis (along the length of the cylinder) parallel to the field. What is the net flux Φ of the electric field through the cylinder?

KEY IDEAS

We can find the net flux Φ with Eq. 23-4 by integrating the dot product $\vec{E} \cdot d\vec{A}$ over the cylinder's surface. However, we cannot write out functions so that we can do that with one integral. Instead, we need to be a bit clever: We break up the surface into sections with which we can actually evaluate an integral.

Calculations: We break the integral of Eq. 23-4 into three terms: integrals over the left cylinder cap a , the curved cylindrical surface b , and the right cap c :

$$\begin{aligned}\Phi &= \oint \vec{E} \cdot d\vec{A} \\ &= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}. \quad (23-5)\end{aligned}$$

Pick a patch element on the left cap. Its area vector $d\vec{A}$ must be perpendicular to the patch and pointing away from the interior of the cylinder. In Fig. 23-6, that means the angle between it and the field piercing the patch is 180° . Also, note that the electric field through the end cap is uniform and thus E can be pulled out of the integration. So, we can write the flux through the left cap as

$$\int_a \vec{E} \cdot d\vec{A} = \int_a E(\cos 180^\circ) dA = -E \int_a dA = -EA,$$

where $\int dA$ gives the cap's area A ($= \pi R^2$). Similarly, for the right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int_c E(\cos 0) dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int_b E(\cos 90^\circ) dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \quad (\text{Answer})$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

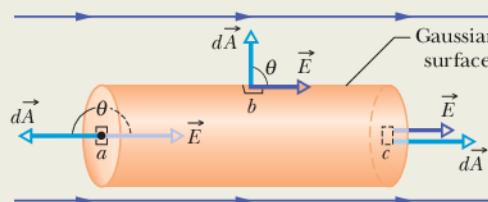


Figure 23-6 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.



Additional examples, video, and practice available at WileyPLUS



Sample Problem 23.02 Flux through a closed cube, nonuniform field

A *nonuniform* electric field given by $\vec{E} = 3.0\hat{x} + 4.0\hat{y}$ pierces the Gaussian cube shown in Fig. 23-7a. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

KEY IDEA

We can find the flux Φ through the surface by integrating the scalar product $\vec{E} \cdot d\vec{A}$ over each face.

Right face: An area vector \vec{A} is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for any patch element (small section) on the right face of the cube must point in the positive direction of the x axis. An example of such an element is shown in Figs. 23-7b and c, but we would have an identical vector for any other choice of a patch element on that face. The most convenient way to express the vector is in unit-vector notation,

$$d\vec{A} = dA\hat{i}.$$

From Eq. 23-4, the flux Φ_r through the right face is then

$$\begin{aligned}\Phi_r &= \int \vec{E} \cdot d\vec{A} = \int (3.0\hat{x} + 4.0\hat{y}) \cdot (dA\hat{i}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}] \\ &= \int (3.0x dA + 0) = 3.0 \int x dA.\end{aligned}$$

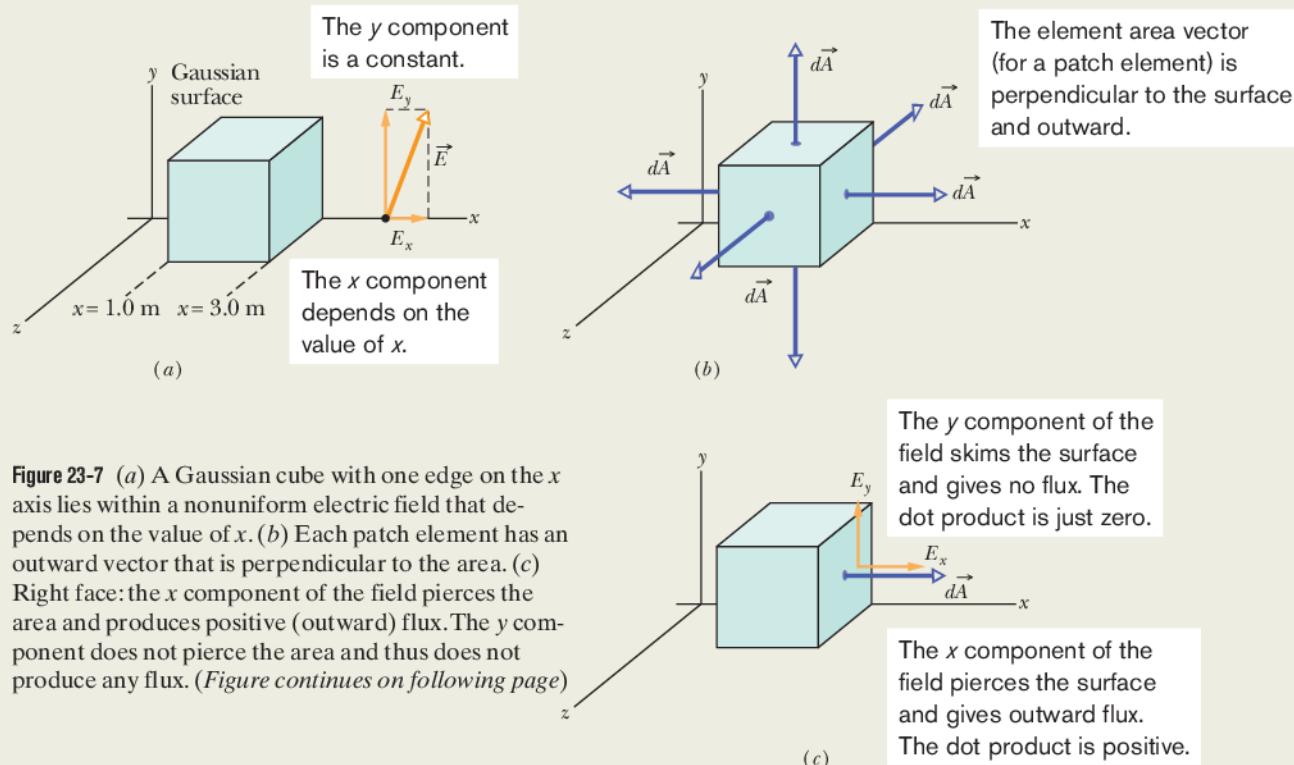
We are about to integrate over the right face, but we note that x has the same value everywhere on that face—namely, $x = 3.0$ m. This means we can substitute that constant value for x . This can be a confusing argument. Although x is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the x axis, every point on the face has the same x coordinate. (The y and z coordinates do not matter in our integral.) Thus, we have

$$\Phi_r = 3.0 \int (3.0) dA = 9.0 \int dA.$$

The integral $\int dA$ merely gives us the area $A = 4.0$ m² of the right face, so

$$\Phi_r = (9.0 \text{ N/C})(4.0 \text{ m}^2) = 36 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})$$

Left face: We repeat this procedure for the left face. However,



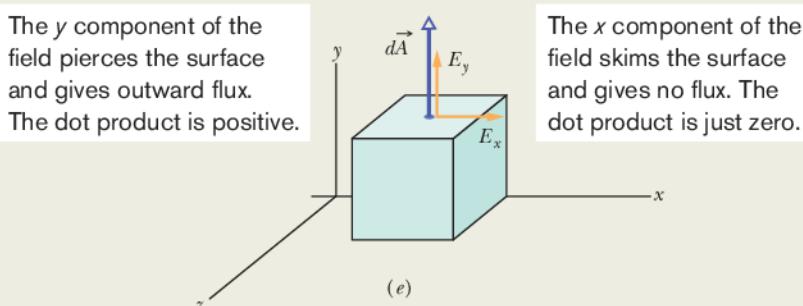
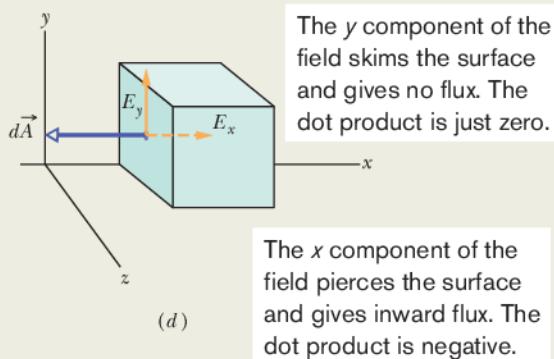


Figure 23-7 (Continued from previous page) (d) Left face: the x component of the field produces negative (inward) flux. (e) Top face: the y component of the field produces positive (outward) flux.

two factors change. (1) The element area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-7d). (2) On the left face, $x = 1.0 \text{ m}$. With these changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})$$

Top face: Now $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-7e). The flux Φ_t is

$$\begin{aligned}\Phi_t &= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)\hat{j} \cdot \hat{j}] \\ &= \int (0 + 4.0 dA) = 4.0 \int dA \\ &= 16 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})\end{aligned}$$



Additional examples, video, and practice available at WileyPLUS

23-2 GAUSS' LAW

Learning Objectives

After reading this module, you should be able to . . .

- 23.09 Apply Gauss' law to relate the net flux Φ through a closed surface to the net enclosed charge q_{enc} .
- 23.10 Identify how the algebraic sign of the net enclosed charge corresponds to the direction (inward or outward) of the net flux through a Gaussian surface.
- 23.11 Identify that charge outside a Gaussian surface makes

no contribution to the net flux through the closed surface.

- 23.12 Derive the expression for the magnitude of the electric field of a charged particle by using Gauss' law.
- 23.13 Identify that for a charged particle or uniformly charged sphere, Gauss' law is applied with a Gaussian surface that is a concentric sphere.

Key Ideas

- Gauss' law relates the net flux Φ penetrating a closed surface to the net charge q_{enc} enclosed by the surface:

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (\text{Gauss' law}).$$

- Gauss' law can also be written in terms of the electric field piercing the enclosing Gaussian surface:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad (\text{Gauss' law}).$$

Gauss' Law

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the net charge q_{enc} that is enclosed by that surface. It tells us that

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (\text{Gauss' law}).$$

(23-6)

By substituting Eq. 23-4, the definition of flux, we can also write Gauss' law as

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad (\text{Gauss' law}). \quad (23-7)$$

Equations 23-6 and 23-7 hold only when the net charge is located in a vacuum or (what is the same for most practical purposes) in air. In Chapter 25, we modify Gauss' law to include situations in which a material such as mica, oil, or glass is present.

In Eqs. 23-6 and 23-7, the net charge q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero. We include the sign, rather than just use the magnitude of the enclosed charge, because the sign tells us something about the net flux through the Gaussian surface: If q_{enc} is positive, the net flux is *outward*; if q_{enc} is negative, the net flux is *inward*.

Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc} in Gauss' law. The exact form and location of the charges inside the Gaussian surface are also of no concern; the only things that matter on the right side of Eqs. 23-6 and 23-7 are the magnitude and sign of the net enclosed charge. The quantity \vec{E} on the left side of Eq. 23-7, however, is the electric field resulting from *all* charges, both those inside and those outside the Gaussian surface. This statement may seem to be inconsistent, but keep this in mind: The electric field due to a charge outside the Gaussian surface contributes zero net flux *through* the surface, because as many field lines due to that charge enter the surface as leave it.

Let us apply these ideas to Fig. 23-8, which shows two particles, with charges equal in magnitude but opposite in sign, and the field lines describing the electric fields the particles set up in the surrounding space. Four Gaussian surfaces are also shown, in cross section. Let us consider each in turn.

Surface S_1 . The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires. (That is, in Eq. 23-6, if Φ is positive, q_{enc} must be also.)

Surface S_2 . The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

Surface S_3 . This surface encloses no charge, and thus $q_{\text{enc}} = 0$. Gauss' law (Eq. 23-6) requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

Surface S_4 . This surface encloses no *net* charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S_4 as entering it.

What would happen if we were to bring an enormous charge Q up close to surface S_4 in Fig. 23-8? The pattern of the field lines would certainly change, but the net flux for each of the four Gaussian surfaces would not change. Thus, the value of Q would not enter Gauss' law in any way, because Q lies outside all four of the Gaussian surfaces that we are considering.



Checkpoint 2

The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $\text{N} \cdot \text{m}^2/\text{C}$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?

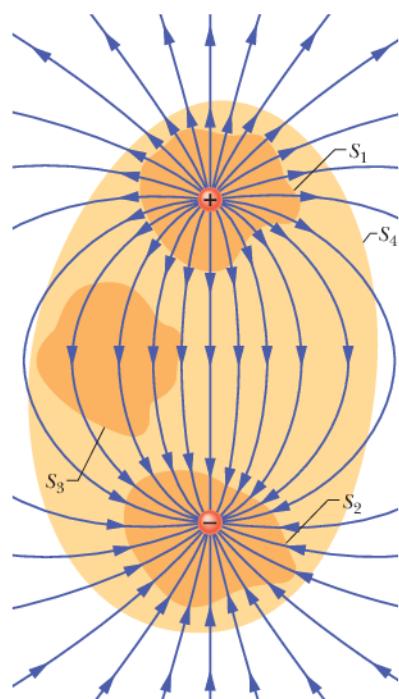
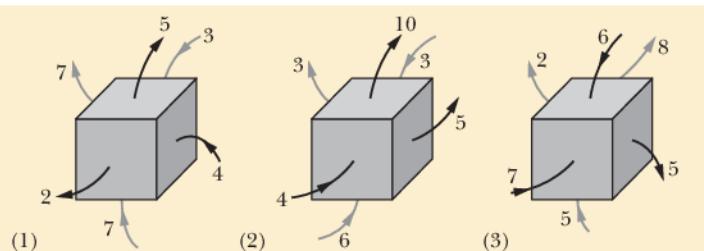


Figure 23-8 Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.



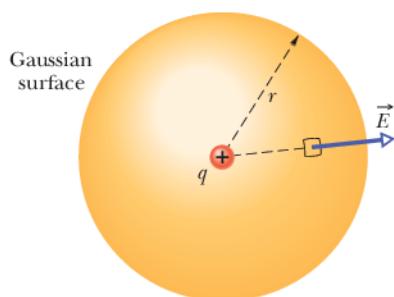


Figure 23-9 A spherical Gaussian surface centered on a particle with charge q .

Gauss' Law and Coulomb's Law

One of the situations in which we can apply Gauss' law is in finding the electric field of a charged particle. That field has spherical symmetry (the field depends on the distance r from the particle but not the direction). So, to make use of that symmetry, we enclose the particle in a Gaussian sphere that is centered on the particle, as shown in Fig. 23-9 for a particle with positive charge q . Then the electric field has the same magnitude E at any point on the sphere (all points are at the same distance r). That feature will simplify the integration.

The drill here is the same as previously. Pick a patch element on the surface and draw its area vector $d\vec{A}$ perpendicular to the patch and directed outward. From the symmetry of the situation, we know that the electric field \vec{E} at the patch is also radially outward and thus at angle $\theta = 0$ with $d\vec{A}$. So, we rewrite Gauss' law as

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 \oint E dA = q_{\text{enc}}. \quad (23-8)$$

Here $q_{\text{enc}} = q$. Because the field magnitude E is the same at every patch element, E can be pulled outside the integral:

$$\epsilon_0 E \oint dA = q. \quad (23-9)$$

The remaining integral is just an instruction to sum all the areas of the patch elements on the sphere, but we already know that the total area is $4\pi r^2$. Substituting this, we have

$$\begin{aligned} \epsilon_0 E (4\pi r^2) &= q \\ \text{or} \quad E &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}. \end{aligned} \quad (23-10)$$

This is exactly Eq. 22-3, which we found using Coulomb's law.



Checkpoint 3

There is a certain net flux Φ_i through a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r , and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to Φ_i ?



Sample Problem 23.03 Using Gauss' law to find the electric field

Figure 23-10a shows, in cross section, a plastic, spherical shell with uniform charge $Q = -16e$ and radius $R = 10$ cm. A particle with charge $q = +5e$ is at the center. What is the electric field (magnitude and direction) at (a) point P_1 at radial distance $r_1 = 6.00$ cm and (b) point P_2 at radial distance $r_2 = 12.0$ cm?

KEY IDEAS

- Because the situation in Fig. 23-10a has spherical symmetry, we can apply Gauss' law (Eq. 23-7) to find the electric field at a point if we use a Gaussian surface in the form of a sphere concentric with the particle and shell.
- To find the electric field at a point, we put that point on a Gaussian surface (so that the \vec{E} we want is the \vec{E} in the dot product inside the integral in Gauss' law).
- Gauss' law relates the net electric flux through a closed surface to the net enclosed charge. Any external charge is not included.

Calculations: To find the field at point P_1 , we construct a Gaussian sphere with P_1 on its surface and thus with a radius of r_1 . Because the charge enclosed by the Gaussian sphere is positive, the electric flux through the surface must be positive and thus outward. So, the electric field \vec{E} pierces the surface outward and, because of the spherical symmetry, must be *radially* outward, as drawn in Fig. 23-10b. That figure does not include the plastic shell because the shell is not enclosed by the Gaussian sphere.

Consider a patch element on the sphere at P_1 . Its area vector $d\vec{A}$ is radially outward (it must always be outward from a Gaussian surface). Thus the angle θ between \vec{E} and $d\vec{A}$ is zero. We can now rewrite the left side of Eq. 23-7 (Gauss' law) as

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 \oint E \cos 0 dA = \epsilon_0 \oint E dA = \epsilon_0 E \oint dA,$$

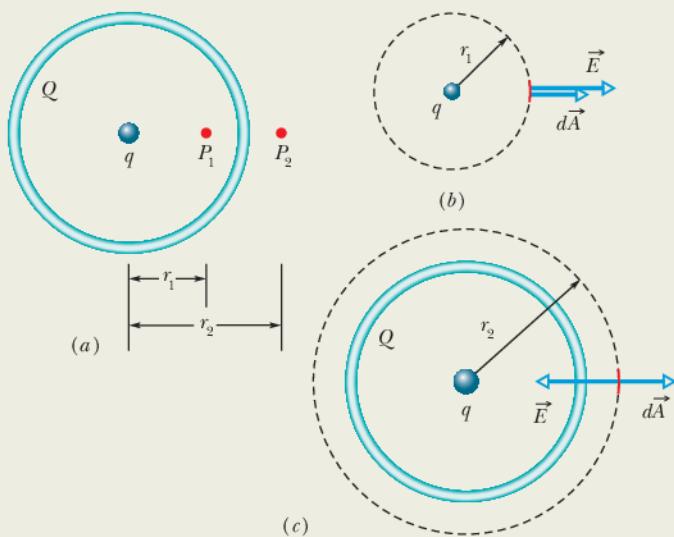


Figure 23-10 (a) A charged plastic spherical shell encloses a charged particle. (b) To find the electric field at P_1 , arrange for the point to be on a Gaussian sphere. The electric field pierces outward. The area vector for the patch element is outward. (c) P_2 is on a Gaussian sphere, \vec{E} is inward, and $d\vec{A}$ is still outward.

where in the last step we pull the field magnitude E out of the integral because it is the same at all points on the Gaussian sphere and thus is a constant. The remaining integral is simply an instruction for us to sum the areas of all the patch elements on the sphere, but we already know that the surface area of a sphere is $4\pi r^2$. Substituting these results, Eq. 23-7 for Gauss' law gives us

$$\epsilon_0 E 4\pi r^2 = q_{\text{enc}}$$

Sample Problem 23.04 Using Gauss' law to find the enclosed charge

What is the net charge enclosed by the Gaussian cube of Sample Problem 23.02?

KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\epsilon_0 \Phi = q_{\text{enc}}$).

Flux: To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36 \text{ N} \cdot \text{m}^2/\text{C}$), the left face ($\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C}$), and the top face ($\Phi_t = 16 \text{ N} \cdot \text{m}^2/\text{C}$).

For the bottom face, our calculation is just like that for the top face *except* that the element area vector $d\vec{A}$ is now directed downward along the y axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have

The only charge enclosed by the Gaussian surface through P_1 is that of the particle. Solving for E and substituting $q_{\text{enc}} = 5e$ and $r = r_1 = 6.00 \times 10^{-2} \text{ m}$, we find that the magnitude of the electric field at P_1 is

$$\begin{aligned} E &= \frac{q_{\text{enc}}}{4\pi\epsilon_0 r^2} \\ &= \frac{5(1.60 \times 10^{-19} \text{ C})}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0600 \text{ m})^2} \\ &= 2.00 \times 10^{-6} \text{ N/C.} \end{aligned} \quad (\text{Answer})$$

To find the electric field at P_2 , we follow the same procedure by constructing a Gaussian sphere with P_2 on its surface. This time, however, the net charge enclosed by the sphere is $q_{\text{enc}} = q + Q = 5e + (-16e) = -11e$. Because the net charge is negative, the electric field vectors on the sphere's surface pierce inward (Fig. 23-10c), the angle θ between \vec{E} and $d\vec{A}$ is 180° , and the dot product is $E(\cos 180^\circ) dA = -E dA$. Now solving Gauss' law for E and substituting $r = r_2 = 12.00 \times 10^{-2} \text{ m}$ and the new q_{enc} , we find

$$\begin{aligned} E &= \frac{-q_{\text{enc}}}{4\pi\epsilon_0 r^2} \\ &= \frac{-[-11(1.60 \times 10^{-19} \text{ C})]}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.120 \text{ m})^2} \\ &= 1.10 \times 10^{-6} \text{ N/C.} \end{aligned} \quad (\text{Answer})$$

Note how different the calculations would have been if we had put P_1 or P_2 on the surface of a Gaussian cube instead of mimicking the spherical symmetry with a Gaussian sphere. Then angle θ and magnitude E would have varied considerably over the surface of the cube and evaluation of the integral in Gauss' law would have been difficult.

$$d\vec{A} = -dA\hat{j}, \text{ and we find}$$

$$\Phi_b = -16 \text{ N} \cdot \text{m}^2/\text{C}.$$

For the front face we have $d\vec{A} = dA\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0\hat{x} + 4.0\hat{y}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\begin{aligned} \Phi &= (36 - 12 + 16 - 16 + 0 + 0) \text{ N} \cdot \text{m}^2/\text{C} \\ &= 24 \text{ N} \cdot \text{m}^2/\text{C}. \end{aligned}$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$\begin{aligned} q_{\text{enc}} &= \epsilon_0 \Phi = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(24 \text{ N} \cdot \text{m}^2/\text{C}) \\ &= 2.1 \times 10^{-10} \text{ C.} \end{aligned} \quad (\text{Answer})$$

Thus, the cube encloses a *net* positive charge.



Additional examples, video, and practice available at WileyPLUS

23-3 A CHARGED ISOLATED CONDUCTOR

Learning Objectives

After reading this module, you should be able to . . .

- 23.14 Apply the relationship between surface charge density σ and the area over which the charge is uniformly spread.
- 23.15 Identify that if excess charge (positive or negative) is placed on an isolated conductor, that charge moves to the surface and none is in the interior.
- 23.16 Identify the value of the electric field inside an isolated conductor.
- 23.17 For a conductor with a cavity that contains a charged

object, determine the charge on the cavity wall and on the external surface.

- 23.18 Explain how Gauss' law is used to find the electric field magnitude E near an isolated conducting surface with a uniform surface charge density σ .
- 23.19 For a uniformly charged conducting surface, apply the relationship between the charge density σ and the electric field magnitude E at points near the conductor, and identify the direction of the field vectors.

Key Ideas

- An excess charge on an isolated conductor is located entirely on the outer surface of the conductor.
- The internal electric field of a charged, isolated conductor is zero, and the external field (at nearby points) is perpendicular

to the surface and has a magnitude that depends on the surface charge density σ :

$$E = \frac{\sigma}{\epsilon_0}$$

A Charged Isolated Conductor

Gauss' law permits us to prove an important theorem about conductors:



If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

This might seem reasonable, considering that charges with the same sign repel one another. You might imagine that, by moving to the surface, the added charges are getting as far away from one another as they can. We turn to Gauss' law for verification of this speculation.

Figure 23-11a shows, in cross section, an isolated lump of copper hanging from an insulating thread and having an excess charge q . We place a Gaussian surface just inside the actual surface of the conductor.

The electric field inside this conductor must be zero. If this were not so, the field would exert forces on the conduction (free) electrons, which are always present in a conductor, and thus current would always exist within a conductor. (That is, charge would flow from place to place within the conductor.) Of course, there is no such perpetual current in an isolated conductor, and so the internal electric field is zero.

(An internal electric field *does* appear as a conductor is being charged. However, the added charge quickly distributes itself in such a way that the net internal electric field—the vector sum of the electric fields due to all the charges, both inside and outside—is zero. The movement of charge then ceases, because the net force on each charge is zero; the charges are then in *electrostatic equilibrium*.)

If \vec{E} is zero everywhere inside our copper conductor, it must be zero for all points on the Gaussian surface because that surface, though close to the surface of the conductor, is definitely inside the conductor. This means that the flux through the Gaussian surface must be zero. Gauss' law then tells us that the net charge inside the Gaussian surface must also be zero. Then because the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

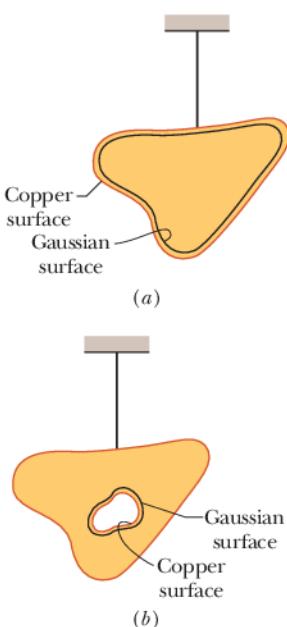


Figure 23-11 (a) A lump of copper with a charge q hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (b) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

An Isolated Conductor with a Cavity

Figure 23-11b shows the same hanging conductor, but now with a cavity that is totally within the conductor. It is perhaps reasonable to suppose that when we scoop out the electrically neutral material to form the cavity, we do not change the distribution of charge or the pattern of the electric field that exists in Fig. 23-11a. Again, we must turn to Gauss' law for a quantitative proof.

We draw a Gaussian surface surrounding the cavity, close to its surface but inside the conducting body. Because $\vec{E} = 0$ inside the conductor, there can be no flux through this new Gaussian surface. Therefore, from Gauss' law, that surface can enclose no net charge. We conclude that there is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor, as in Fig. 23-11a.

The Conductor Removed

Suppose that, by some magic, the excess charges could be “frozen” into position on the conductor's surface, perhaps by embedding them in a thin plastic coating, and suppose that then the conductor could be removed completely. This is equivalent to enlarging the cavity of Fig. 23-11b until it consumes the entire conductor, leaving only the charges. The electric field would not change at all; it would remain zero inside the thin shell of charge and would remain unchanged for all external points. This shows us that the electric field is set up by the charges and not by the conductor. The conductor simply provides an initial pathway for the charges to take up their positions.

The External Electric Field

You have seen that the excess charge on an isolated conductor moves entirely to the conductor's surface. However, unless the conductor is spherical, the charge does not distribute itself uniformly. Put another way, the surface charge density σ (charge per unit area) varies over the surface of any nonspherical conductor. Generally, this variation makes the determination of the electric field set up by the surface charges very difficult.

However, the electric field just outside the surface of a conductor is easy to determine using Gauss' law. To do this, we consider a section of the surface that is small enough to permit us to neglect any curvature and thus to take the section to be flat. We then imagine a tiny cylindrical Gaussian surface to be partially embedded in the section as shown in Fig. 23-12: One end cap is fully inside the conductor, the other is fully outside, and the cylinder is perpendicular to the conductor's surface.

The electric field \vec{E} at and just outside the conductor's surface must also be perpendicular to that surface. If it were not, then it would have a component along the conductor's surface that would exert forces on the surface charges, causing them to move. However, such motion would violate our implicit assumption that we are dealing with electrostatic equilibrium. Therefore, \vec{E} is perpendicular to the conductor's surface.

We now sum the flux through the Gaussian surface. There is no flux through the internal end cap, because the electric field within the conductor is zero. There is no flux through the curved surface of the cylinder, because internally (in the conductor) there is no electric field and externally the electric field is parallel to the curved portion of the Gaussian surface. The only flux through the Gaussian surface is that through the external end cap, where \vec{E} is perpendicular to the plane of the cap. We assume that the cap area A is small enough that the field magnitude E is constant over the cap. Then the flux through the cap is EA , and that is the net flux Φ through the Gaussian surface.

The charge q_{enc} enclosed by the Gaussian surface lies on the conductor's surface in an area A . (Think of the cylinder as a cookie cutter.) If σ is the charge per unit area, then q_{enc} is equal to σA . When we substitute σA for q_{enc} and EA for Φ ,

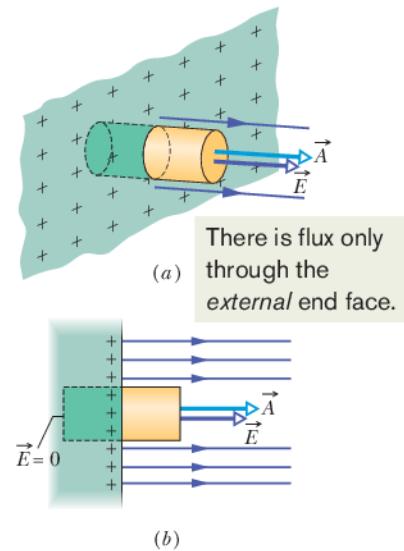


Figure 23-12 (a) Perspective view and (b) side view of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area A and area vector \vec{A} .

Gauss' law (Eq. 23-6) becomes

$$\epsilon_0 EA = \sigma A,$$

from which we find

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{conducting surface}). \quad (23-11)$$

Thus, the magnitude of the electric field just outside a conductor is proportional to the surface charge density on the conductor. The sign of the charge gives us the direction of the field. If the charge on the conductor is positive, the electric field is directed away from the conductor as in Fig. 23-12. It is directed toward the conductor if the charge is negative.

The field lines in Fig. 23-12 must terminate on negative charges somewhere in the environment. If we bring those charges near the conductor, the charge density at any given location on the conductor's surface changes, and so does the magnitude of the electric field. However, the relation between σ and E is still given by Eq. 23-11.



Sample Problem 23.05 Spherical metal shell, electric field and enclosed charge

Figure 23-13a shows a cross section of a spherical metal shell of inner radius R . A particle with a charge of $-5.0 \mu\text{C}$ is located at a distance $R/2$ from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its inner and outer surfaces? Are those charges uniformly distributed? What is the field pattern inside and outside the shell?

KEY IDEAS

Figure 23-13b shows a cross section of a spherical Gaussian surface within the metal, just outside the inner wall of the shell. The electric field must be zero inside the metal (and thus on the Gaussian surface inside the metal). This means that the electric flux through the Gaussian surface must also be zero. Gauss' law then tells us that the *net* charge enclosed by the Gaussian surface must be zero.

Reasoning: With a particle of charge $-5.0 \mu\text{C}$ within the shell, a charge of $+5.0 \mu\text{C}$ must lie on the inner wall of the shell in order that the net enclosed charge be zero. If the particle were centered, this positive charge would be uniformly distributed along the inner wall. However, since the particle is off-center, the distribution of positive charge is skewed, as suggested by Fig. 23-13b, because the positive charge tends to collect on the section of the inner wall nearest the (negative) particle.

Because the shell is electrically neutral, its inner wall can have a charge of $+5.0 \mu\text{C}$ only if electrons, with a total charge of $-5.0 \mu\text{C}$, leave the inner wall and move to the outer wall. There they spread out uniformly, as is also suggested by Fig. 23-13b. This distribution of negative charge is

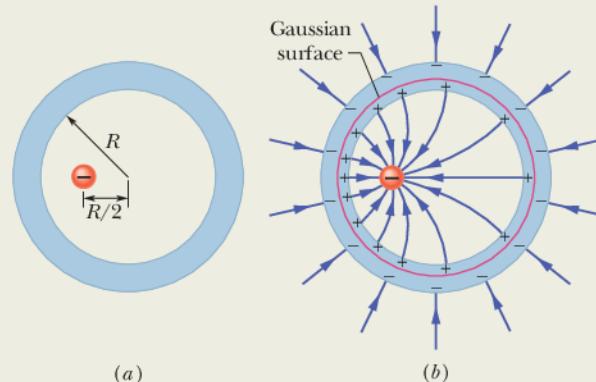


Figure 23-13 (a) A negatively charged particle is located within a spherical metal shell that is electrically neutral. (b) As a result, positive charge is nonuniformly distributed on the inner wall of the shell, and an equal amount of negative charge is uniformly distributed on the outer wall.

uniform because the shell is spherical and because the skewed distribution of positive charge on the inner wall cannot produce an electric field in the shell to affect the distribution of charge on the outer wall. Furthermore, these negative charges repel one another.

The field lines inside and outside the shell are shown approximately in Fig. 23-13b. All the field lines intersect the shell and the particle perpendicularly. Inside the shell the pattern of field lines is skewed because of the skew of the positive charge distribution. Outside the shell the pattern is the same as if the particle were centered and the shell were missing. In fact, this would be true no matter where inside the shell the particle happened to be located.



Additional examples, video, and practice available at WileyPLUS

23-4 APPLYING GAUSS' LAW: CYLINDRICAL SYMMETRY

Learning Objectives

After reading this module, you should be able to . . .

- 23.20** Explain how Gauss' law is used to derive the electric field magnitude outside a line of charge or a cylindrical surface (such as a plastic rod) with a uniform linear charge density λ .
- 23.21** Apply the relationship between linear charge density λ

on a cylindrical surface and the electric field magnitude E at radial distance r from the central axis.

- 23.22** Explain how Gauss' law can be used to find the electric field magnitude *inside* a cylindrical nonconducting surface (such as a plastic rod) with a uniform volume charge density ρ .

Key Idea

- The electric field at a point near an infinite line of charge (or charged rod) with uniform linear charge density λ is perpendicular to the line and has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge}),$$

where r is the perpendicular distance from the line to the point.

Applying Gauss' Law: Cylindrical Symmetry

Figure 23-14 shows a section of an infinitely long cylindrical plastic rod with a uniform charge density λ . We want to find an expression for the electric field magnitude E at radius r from the central axis of the rod, outside the rod. We could do that using the approach of Chapter 22 (charge element dq , field vector $d\vec{E}$, etc.). However, Gauss' law gives a much faster and easier (and prettier) approach.

The charge distribution and the field have cylindrical symmetry. To find the field at radius r , we enclose a section of the rod with a concentric Gaussian cylinder of radius r and height h . (If you want the field at a certain point, put a Gaussian surface through that point.) We can now apply Gauss' law to relate the charge enclosed by the cylinder and the net flux through the cylinder's surface.

First note that because of the symmetry, the electric field at any point must be radially outward (the charge is positive). That means that at any point on the end caps, the field only skims the surface and does not pierce it. So, the flux through each end cap is zero.

To find the flux through the cylinder's curved surface, first note that for any patch element on the surface, the area vector $d\vec{A}$ is radially outward (away from the interior of the Gaussian surface) and thus in the same direction as the field piercing the patch. The dot product in Gauss' law is then simply $E dA \cos 0 = E dA$, and we can pull E out of the integral. The remaining integral is just the instruction to sum the areas of all patch elements on the cylinder's curved surface, but we already know that the total area is the product of the cylinder's height h and circumference $2\pi r$. The net flux through the cylinder is then

$$\Phi = EA \cos \theta = E(2\pi rh)\cos 0 = E(2\pi rh).$$

On the other side of Gauss's law we have the charge q_{enc} enclosed by the cylinder. Because the linear charge density (charge per unit length, remember) is uniform, the enclosed charge is λh . Thus, Gauss' law,

$$\epsilon_0 \Phi = q_{\text{enc}},$$

reduces to

$$\epsilon_0 E(2\pi rh) = \lambda h,$$

yielding

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge}). \quad (23-12)$$

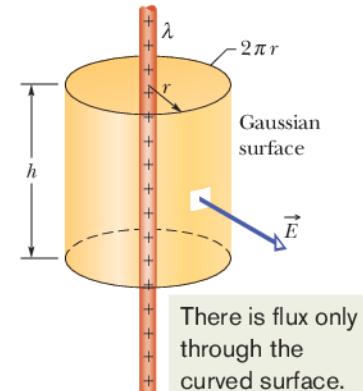


Figure 23-14 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

This is the electric field due to an infinitely long, straight line of charge, at a point that is a radial distance r from the line. The direction of \vec{E} is radially outward

from the line of charge if the charge is positive, and radially inward if it is negative. Equation 23-12 also approximates the field of a *finite* line of charge at points that are not too near the ends (compared with the distance from the line).

If the rod has a uniform volume charge density ρ , we could use a similar procedure to find the electric field magnitude *inside* the rod. We would just shrink the Gaussian cylinder shown in Fig. 23-14 until it is inside the rod. The charge q_{enc} enclosed by the cylinder would then be proportional to the volume of the rod enclosed by the cylinder because the charge density is uniform.



Sample Problem 23.06 Gauss' law and an upward streamer in a lightning storm

Upward streamer in a lightning storm. The woman in Fig. 23-15 was standing on a lookout platform high in the Sequoia National Park when a large storm cloud moved overhead. Some of the conduction electrons in her body were driven into the ground by the cloud's negatively charged base (Fig. 23-16a), leaving her positively charged. You can tell she was highly charged because her hair strands repelled one another and extended away from her along the electric field lines produced by the charge on her.

Lightning did not strike the woman, but she was in extreme danger because that electric field was on the verge of causing electrical breakdown in the surrounding air. Such a breakdown would have occurred along a path extending away from her in what is called an *upward streamer*. An upward streamer is dangerous because the resulting ionization of molecules in the air suddenly frees a tremendous number of electrons from those molecules. Had the woman in Fig. 23-15 developed an upward streamer, the free electrons in the air would have moved to neutralize her (Fig. 23-16b), producing a large, perhaps fatal, charge flow through her body. That charge flow is dangerous because it could have interfered with or even stopped her breathing (which is obviously necessary for oxygen) and the steady beat of her heart (which is obviously necessary for the blood flow that carries the oxygen). The charge flow could also have caused burns.

Let's model her body as a narrow vertical cylinder of height $L = 1.8 \text{ m}$ and radius $R = 0.10 \text{ m}$ (Fig. 23-16c). Assume that charge Q was uniformly distributed along the cylinder and that electrical breakdown would have occurred if the electric



Courtesy NOAA

Figure 23-15 This woman has become positively charged by an overhead storm cloud.

field magnitude along her body had exceeded the critical value $E_c = 2.4 \text{ MN/C}$. What value of Q would have put the air along her body on the verge of breakdown?

KEY IDEA

Because $R \ll L$, we can approximate the charge distribution as a long line of charge. Further, because we assume that the charge is uniformly distributed along this line, we can approximate the magnitude of the electric field along the side of her body with Eq. 23-12 ($E = \lambda/2\pi\epsilon_0 r$).

Calculations: Substituting the critical value E_c for E , the cylinder radius R for radial distance r , and the ratio Q/L for linear charge density λ , we have

$$E_c = \frac{Q/L}{2\pi\epsilon_0 R},$$

or

$$Q = 2\pi\epsilon_0 RLE_c.$$

Substituting given data then gives us

$$\begin{aligned} Q &= (2\pi)(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.10 \text{ m}) \\ &\quad \times (1.8 \text{ m})(2.4 \times 10^6 \text{ N/C}) \\ &= 2.402 \times 10^{-5} \text{ C} \approx 24 \mu\text{C}. \end{aligned} \quad (\text{Answer})$$

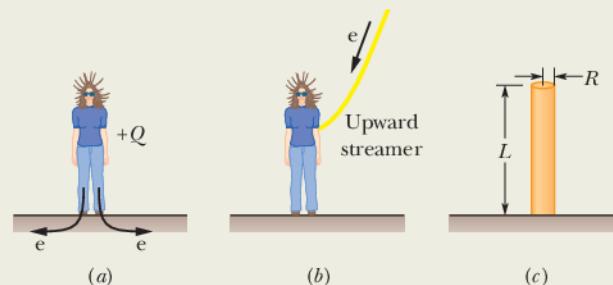


Figure 23-16 (a) Some of the conduction electrons in the woman's body are driven into the ground, leaving her positively charged. (b) An upward streamer develops if the air undergoes electrical breakdown, which provides a path for electrons freed from molecules in the air to move to the woman. (c) A cylinder represents the woman.



Additional examples, video, and practice available at WileyPLUS

23-5 APPLYING GAUSS' LAW: PLANAR SYMMETRY

Learning Objectives

After reading this module, you should be able to . . .

23.23 Apply Gauss' law to derive the electric field magnitude E near a large, flat, nonconducting surface with a uniform surface charge density σ .

23.24 For points near a large, flat *nonconducting* surface with a uniform charge density σ , apply the relationship be-

tween the charge density and the electric field magnitude E and also specify the direction of the field.

23.25 For points near two large, flat, parallel, *conducting* surfaces with a uniform charge density σ , apply the relationship between the charge density and the electric field magnitude E and also specify the direction of the field.

Key Ideas

- The electric field due to an infinite nonconducting sheet with uniform surface charge density σ is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{nonconducting sheet of charge}).$$

- The external electric field just outside the surface of an isolated charged conductor with surface charge density σ is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{external, charged conductor}).$$

Inside the conductor, the electric field is zero.

Applying Gauss' Law: Planar Symmetry

Nonconducting Sheet

Figure 23-17 shows a portion of a thin, infinite, nonconducting sheet with a uniform (positive) surface charge density σ . A sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple model. Let us find the electric field \vec{E} a distance r in front of the sheet.

A useful Gaussian surface is a closed cylinder with end caps of area A , arranged to pierce the sheet perpendicularly as shown. From symmetry, \vec{E} must be perpendicular to the sheet and hence to the end caps. Furthermore, since the charge is positive, \vec{E} is directed *away* from the sheet, and thus the electric field lines pierce the two Gaussian end caps in an outward direction. Because the field lines do not pierce the curved surface, there is no flux through this portion of the Gaussian surface. Thus $\vec{E} \cdot d\vec{A}$ is simply $E dA$; then Gauss' law,

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}},$$

becomes

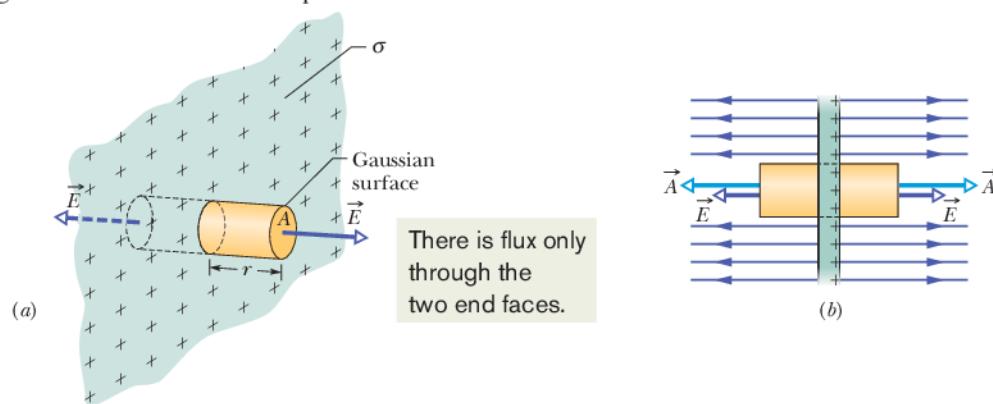
$$\epsilon_0(EA + EA) = \sigma A,$$

where σA is the charge enclosed by the Gaussian surface. This gives

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{sheet of charge}). \quad (23-13)$$

Since we are considering an infinite sheet with uniform charge density, this result holds for any point at a finite distance from the sheet. Equation 23-13 agrees with Eq. 22-27, which we found by integration of electric field components.

Figure 23-17 (a) Perspective view and (b) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ . A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.



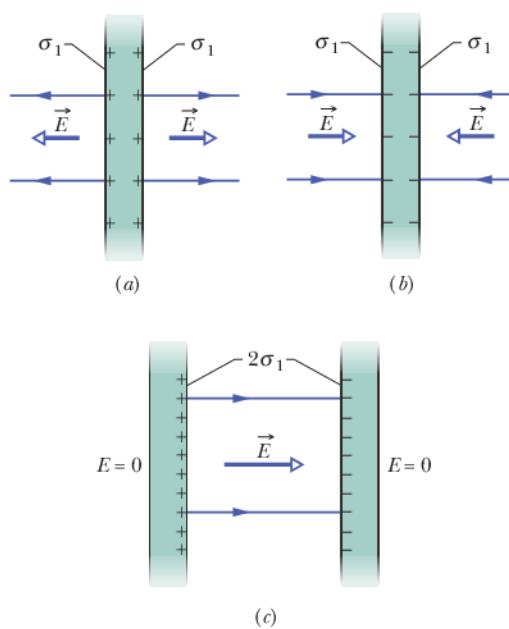


Figure 23-18 (a) A thin, very large conducting plate with excess positive charge. (b) An identical plate with excess negative charge. (c) The two plates arranged so they are parallel and close.

Two Conducting Plates

Figure 23-18a shows a cross section of a thin, infinite conducting plate with excess positive charge. From Module 23-3 we know that this excess charge lies on the surface of the plate. Since the plate is thin and very large, we can assume that essentially all the excess charge is on the two large faces of the plate.

If there is no external electric field to force the positive charge into some particular distribution, it will spread out on the two faces with a uniform surface charge density of magnitude σ_1 . From Eq. 23-11 we know that just outside the plate this charge sets up an electric field of magnitude $E = \sigma_1/\epsilon_0$. Because the excess charge is positive, the field is directed away from the plate.

Figure 23-18b shows an identical plate with excess negative charge having the same magnitude of surface charge density σ_1 . The only difference is that now the electric field is directed toward the plate.

Suppose we arrange for the plates of Figs. 23-18a and b to be close to each other and parallel (Fig. 23-18c). Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates as in Fig. 23-18c. With twice as much charge now on each inner face, the new surface charge density (call it σ) on each inner face is twice σ_1 . Thus, the electric field at any point between the plates has the magnitude

$$E = \frac{2\sigma_1}{\epsilon_0} = \frac{\sigma}{\epsilon_0}. \quad (23-14)$$

This field is directed away from the positively charged plate and toward the negatively charged plate. Since no excess charge is left on the outer faces, the electric field to the left and right of the plates is zero.

Because the charges moved when we brought the plates close to each other, the charge distribution of the two-plate system is not merely the sum of the charge distributions of the individual plates.

One reason why we discuss seemingly unrealistic situations, such as the field set up by an infinite sheet of charge, is that analyses for “infinite” situations yield good approximations to many real-world problems. Thus, Eq. 23-13 holds well for a finite nonconducting sheet as long as we are dealing with points close to the sheet and not too near its edges. Equation 23-14 holds well for a pair of finite conducting plates as long as we consider points that are not too close to their edges. The trouble with the edges is that near an edge we can no longer use planar symmetry to find expressions for the fields. In fact, the field lines there are curved (said to be an *edge effect* or *fringing*), and the fields can be very difficult to express algebraically.



Sample Problem 23.07 Electric field near two parallel nonconducting sheets with charge

Figure 23-19a shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8 \mu\text{C}/\text{m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3 \mu\text{C}/\text{m}^2$ for the negatively charged sheet.

Find the electric field \vec{E} (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-19a by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets

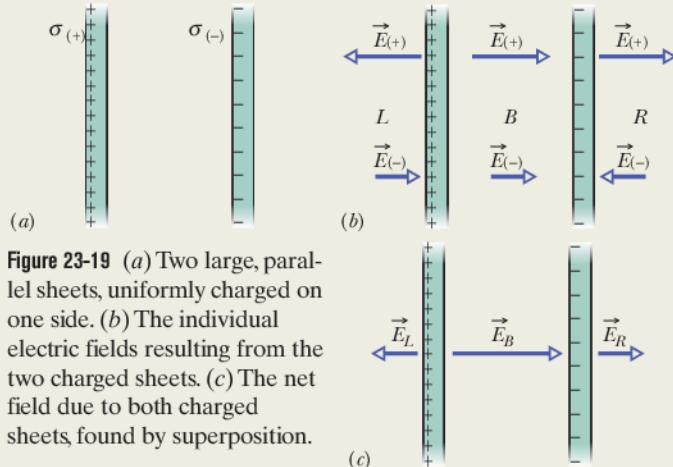


Figure 23-19 (a) Two large, parallel sheets, uniformly charged on one side. (b) The individual electric fields resulting from the two charged sheets. (c) The net field due to both charged sheets, found by superposition.