

Table 19-3. Thus,

$$\gamma = \frac{C_p}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = 1.40.$$

Solving Eq. 19-68 for  $T_f$  and inserting known data then yield

$$T_f = \frac{T_i V_i^{\gamma-1}}{V_f^{\gamma-1}} = \frac{(310 \text{ K})(12 \text{ L})^{1.40-1}}{(19 \text{ L})^{1.40-1}} = (310 \text{ K})\left(\frac{12}{19}\right)^{0.40} = 258 \text{ K.} \quad (\text{Answer})$$

(b) What would be the final temperature and pressure if, instead, the gas expands freely to the new volume, from an initial pressure of 2.0 Pa?

### KEY IDEA

The temperature does not change in a free expansion because there is nothing to change the kinetic energy of the molecules.

**Calculation:** Thus, the temperature is

$$T_f = T_i = 310 \text{ K.} \quad (\text{Answer})$$

We find the new pressure using Eq. 19-63, which gives us

$$p_f = p_i \frac{V_i}{V_f} = (2.0 \text{ Pa}) \frac{12 \text{ L}}{19 \text{ L}} = 1.3 \text{ Pa.} \quad (\text{Answer})$$

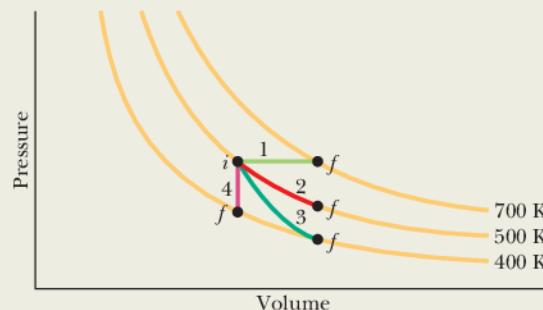
### Problem-Solving Tactics A Graphical Summary of Four Gas Processes

In this chapter we have discussed four special processes that an ideal gas can undergo. An example of each (for a monoatomic ideal gas) is shown in Fig. 19-16, and some associated characteristics are given in Table 19-4, including two process names (isobaric and isochoric) that we have not used but that you might see in other courses.



#### Checkpoint 5

Rank paths 1, 2, and 3 in Fig. 19-16 according to the energy transfer to the gas as heat, greatest first.



**Figure 19-16** A  $p$ - $V$  diagram representing four special processes for an ideal monatomic gas.

**Table 19-4** Four Special Processes

Path in Fig. 19-16	Constant Quantity	Process Type	Some Special Results	
			$(\Delta E_{\text{int}} = Q - W \text{ and } \Delta E_{\text{int}} = nC_V\Delta T \text{ for all paths})$	$Q = nC_p\Delta T; W = p\Delta V$ $Q = W = nRT \ln(V_f/V_i); \Delta E_{\text{int}} = 0$ $Q = 0; W = -\Delta E_{\text{int}}$ $Q = \Delta E_{\text{int}} = nC_V\Delta T; W = 0$
1	$p$	Isobaric		$Q = nC_p\Delta T; W = p\Delta V$
2	$T$	Isothermal		$Q = W = nRT \ln(V_f/V_i); \Delta E_{\text{int}} = 0$
3	$pV^\gamma, TV^{\gamma-1}$	Adiabatic		$Q = 0; W = -\Delta E_{\text{int}}$
4	$V$	Isochoric		$Q = \Delta E_{\text{int}} = nC_V\Delta T; W = 0$



Additional examples, video, and practice available at WileyPLUS

## Review & Summary

**Kinetic Theory of Gases** The *kinetic theory of gases* relates the *macroscopic* properties of gases (for example, pressure and temperature) to the *microscopic* properties of gas molecules (for example, speed and kinetic energy).

**Avogadro's Number** One mole of a substance contains  $N_A$  (*Avogadro's number*) elementary units (usually atoms or molecules), where  $N_A$  is found experimentally to be

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \quad (\text{Avogadro's number}). \quad (19-1)$$

One molar mass  $M$  of any substance is the mass of one mole of the substance. It is related to the mass  $m$  of the individual molecules of the substance by

$$M = mN_A. \quad (19-4)$$

The number of moles  $n$  contained in a sample of mass  $M_{\text{sam}}$ , consisting of  $N$  molecules, is given by

$$n = \frac{N}{N_A} = \frac{M_{\text{sam}}}{M} = \frac{M_{\text{sam}}}{mN_A}. \quad (19-2, 19-3)$$

**Ideal Gas** An *ideal gas* is one for which the pressure  $p$ , volume  $V$ , and temperature  $T$  are related by

$$pV = nRT \quad (\text{ideal gas law}). \quad (19-5)$$

Here  $n$  is the number of moles of the gas present and  $R$  is a constant ( $8.31 \text{ J/mol}\cdot\text{K}$ ) called the **gas constant**. The ideal gas law can also be written as

$$pV = NkT, \quad (19-9)$$

where the **Boltzmann constant**  $k$  is

$$k = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}. \quad (19-7)$$

**Work in an Isothermal Volume Change** The work done by an ideal gas during an **isothermal** (constant-temperature) change from volume  $V_i$  to volume  $V_f$  is

$$W = nRT \ln \frac{V_f}{V_i} \quad (\text{ideal gas, isothermal process}). \quad (19-14)$$

**Pressure, Temperature, and Molecular Speed** The pressure exerted by  $n$  moles of an ideal gas, in terms of the speed of its molecules, is

$$p = \frac{nMv_{\text{rms}}^2}{3V}, \quad (19-21)$$

where  $v_{\text{rms}} = \sqrt{\langle v^2 \rangle_{\text{avg}}}$  is the **root-mean-square speed** of the molecules of the gas. With Eq. 19-5 this gives

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}. \quad (19-22)$$

**Temperature and Kinetic Energy** The average translational kinetic energy  $K_{\text{avg}}$  per molecule of an ideal gas is

$$K_{\text{avg}} = \frac{3}{2}kT. \quad (19-24)$$

**Mean Free Path** The *mean free path*  $\lambda$  of a gas molecule is its average path length between collisions and is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}, \quad (19-25)$$

where  $N/V$  is the number of molecules per unit volume and  $d$  is the molecular diameter.

**Maxwell Speed Distribution** The *Maxwell speed distribution*  $P(v)$  is a function such that  $P(v) dv$  gives the *fraction* of molecules with speeds in the interval  $dv$  at speed  $v$ :

$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}. \quad (19-27)$$

Three measures of the distribution of speeds among the molecules of

## Questions

- 1 For four situations for an ideal gas, the table gives the energy transferred to or from the gas as heat  $Q$  and either the work  $W$  done by the gas or the work  $W_{\text{on}}$  done on the gas, all in joules. Rank the four situations in terms of the temperature change of the gas, most positive first.
- |                 | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
|-----------------|----------|----------|----------|----------|
| $Q$             | −50      | +35      | −15      | +20      |
| $W$             | −50      | +35      |          |          |
| $W_{\text{on}}$ |          |          | −40      | +40      |

a gas are

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} \quad (\text{average speed}), \quad (19-31)$$

$$v_P = \sqrt{\frac{2RT}{M}} \quad (\text{most probable speed}), \quad (19-35)$$

and the rms speed defined above in Eq. 19-22.

**Molar Specific Heats** The molar specific heat  $C_V$  of a gas at constant volume is defined as

$$C_V = \frac{Q}{n \Delta T} = \frac{\Delta E_{\text{int}}}{n \Delta T}, \quad (19-39, 19-41)$$

in which  $Q$  is the energy transferred as heat to or from a sample of  $n$  moles of the gas,  $\Delta T$  is the resulting temperature change of the gas, and  $\Delta E_{\text{int}}$  is the resulting change in the internal energy of the gas. For an ideal monatomic gas,

$$C_V = \frac{3}{2}R = 12.5 \text{ J/mol}\cdot\text{K}. \quad (19-43)$$

The molar specific heat  $C_p$  of a gas at constant pressure is defined to be

$$C_p = \frac{Q}{n \Delta T}, \quad (19-46)$$

in which  $Q$ ,  $n$ , and  $\Delta T$  are defined as above.  $C_p$  is also given by

$$C_p = C_V + R. \quad (19-49)$$

For  $n$  moles of an ideal gas,

$$E_{\text{int}} = nC_V T \quad (\text{ideal gas}). \quad (19-44)$$

If  $n$  moles of a confined ideal gas undergo a temperature change  $\Delta T$  due to *any* process, the change in the internal energy of the gas is

$$\Delta E_{\text{int}} = nC_V \Delta T \quad (\text{ideal gas, any process}). \quad (19-45)$$

**Degrees of Freedom and  $C_V$**  The *equipartition of energy* theorem states that every *degree of freedom* of a molecule has an energy  $\frac{1}{2}kT$  per molecule ( $= \frac{1}{2}RT$  per mole). If  $f$  is the number of degrees of freedom, then  $E_{\text{int}} = (f/2)nRT$  and

$$C_V = \left( \frac{f}{2} \right)R = 4.16f \text{ J/mol}\cdot\text{K}. \quad (19-51)$$

For monatomic gases  $f = 3$  (three translational degrees); for diatomic gases  $f = 5$  (three translational and two rotational degrees).

**Adiabatic Process** When an ideal gas undergoes an adiabatic volume change (a change for which  $Q = 0$ ),

$$pV^\gamma = \text{a constant} \quad (\text{adiabatic process}), \quad (19-53)$$

in which  $\gamma (= C_p/C_V)$  is the ratio of molar specific heats for the gas. For a free expansion, however,  $pV = \text{a constant}$ .

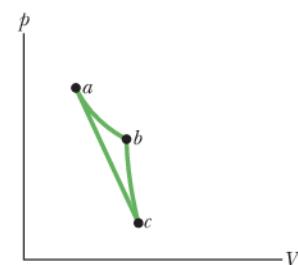


Figure 19-17 Question 2.

in the internal energy of the gas when it is taken along the straight path from *a* to *c*?

- 3** For a temperature increase of  $\Delta T_1$ , a certain amount of an ideal gas requires 30 J when heated at constant volume and 50 J when heated at constant pressure. How much work is done by the gas in the second situation?

- 4** The dot in Fig. 19-18*a* represents the initial state of a gas, and the vertical line through the dot divides the *p*-*V* diagram into regions 1 and 2. For the following processes, determine whether the work *W* done by the gas is positive, negative, or zero: (a) the gas moves up along the vertical line, (b) it moves down along the vertical line, (c) it moves to anywhere in region 1, and (d) it moves to anywhere in region 2.

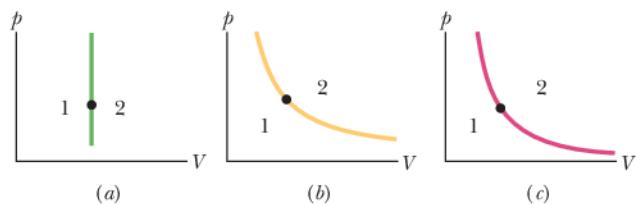


Figure 19-18 Questions 4, 6, and 8.

- 5** A certain amount of energy is to be transferred as heat to 1 mol of a monatomic gas (a) at constant pressure and (b) at constant volume, and to 1 mol of a diatomic gas (c) at constant pressure and (d) at constant volume. Figure 19-19 shows four paths from an initial point to four

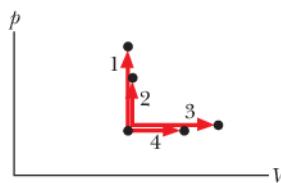


Figure 19-19 Question 5.

final points on a *p*-*V* diagram for the two gases. Which path goes with which process? (e) Are the molecules of the diatomic gas rotating?

- 6** The dot in Fig. 19-18*b* represents the initial state of a gas, and the isotherm through the dot divides the *p*-*V* diagram into regions 1 and 2. For the following processes, determine whether the change  $\Delta E_{\text{int}}$  in the internal energy of the gas is positive, negative, or zero: (a) the gas moves up along the isotherm, (b) it moves down along the isotherm, (c) it moves to anywhere in region 1, and (d) it moves to anywhere in region 2.

- 7** (a) Rank the four paths of Fig. 19-16 according to the work done by the gas, greatest first. (b) Rank paths 1, 2, and 3 according to the change in the internal energy of the gas, most positive first and most negative last.

- 8** The dot in Fig. 19-18*c* represents the initial state of a gas, and the adiabat through the dot divides the *p*-*V* diagram into regions 1 and 2. For the following processes, determine whether the corresponding heat *Q* is positive, negative, or zero: (a) the gas moves up along the adiabat, (b) it moves down along the adiabat, (c) it moves to anywhere in region 1, and (d) it moves to anywhere in region 2.

- 9** An ideal diatomic gas, with molecular rotation but without any molecular oscillation, loses a certain amount of energy as heat *Q*. Is the resulting decrease in the internal energy of the gas greater if the loss occurs in a constant-volume process or in a constant-pressure process?

- 10** Does the temperature of an ideal gas increase, decrease, or stay the same during (a) an isothermal expansion, (b) an expansion at constant pressure, (c) an adiabatic expansion, and (d) an increase in pressure at constant volume?

## Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

**WWW** Worked-out solution is at

**ILW** Interactive solution is at

<http://www.wiley.com/college/halliday>

### Module 19-1 Avogadro's Number

- 1** Find the mass in kilograms of  $7.50 \times 10^{24}$  atoms of arsenic, which has a molar mass of 74.9 g/mol.
- 2** Gold has a molar mass of 197 g/mol. (a) How many moles of gold are in a 2.50 g sample of pure gold? (b) How many atoms are in the sample?

### Module 19-2 Ideal Gases

- 3 SSM** Oxygen gas having a volume of 1000 cm<sup>3</sup> at 40.0°C and  $1.01 \times 10^5$  Pa expands until its volume is 1500 cm<sup>3</sup> and its pressure is  $1.06 \times 10^5$  Pa. Find (a) the number of moles of oxygen present and (b) the final temperature of the sample.

- 4** A quantity of ideal gas at 10.0°C and 100 kPa occupies a volume of 2.50 m<sup>3</sup>. (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature is raised to 30.0°C, how much volume does the gas occupy? Assume no leaks.

- 5** The best laboratory vacuum has a pressure of about  $1.00 \times 10^{-18}$  atm, or  $1.01 \times 10^{-13}$  Pa. How many gas molecules are there per cubic centimeter in such a vacuum at 293 K?

- 6 Flying Circus** Water bottle in a hot car. In the American Southwest, the temperature in a closed car parked in sunlight during the summer can be high enough to burn flesh. Suppose a bottle of water at a refrigerator temperature of 5.00°C is opened, then closed, and then left in a closed car with an internal temperature of 75.0°C. Neglecting the thermal expansion of the water and the bottle, find the pressure in the air pocket trapped in the bottle. (The pressure can be enough to push the bottle cap past the threads that are intended to keep the bottle closed.)

- 7** Suppose 1.80 mol of an ideal gas is taken from a volume of 3.00 m<sup>3</sup> to a volume of 1.50 m<sup>3</sup> via an isothermal compression at 30°C. (a) How much energy is transferred as heat during the compression, and (b) is the transfer *to* or *from* the gas?

- 8** Compute (a) the number of moles and (b) the number of molecules in 1.00 cm<sup>3</sup> of an ideal gas at a pressure of 100 Pa and a temperature of 220 K.

- 9** An automobile tire has a volume of  $1.64 \times 10^{-2}$  m<sup>3</sup> and contains air at a gauge pressure (pressure above atmospheric pressure) of 165 kPa when the temperature is 0.00°C. What is the gauge

pressure of the air in the tires when its temperature rises to 27.0°C and its volume increases to  $1.67 \times 10^{-2} \text{ m}^3$ ? Assume atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ .

**•10** A container encloses 2 mol of an ideal gas that has molar mass  $M_1$  and 0.5 mol of a second ideal gas that has molar mass  $M_2 = 3M_1$ . What fraction of the total pressure on the container wall is attributable to the second gas? (The kinetic theory explanation of pressure leads to the experimentally discovered law of partial pressures for a mixture of gases that do not react chemically: *The total pressure exerted by the mixture is equal to the sum of the pressures that the several gases would exert separately if each were to occupy the vessel alone.* The molecule–vessel collisions of one type would not be altered by the presence of another type.)

**•11 SSM ILW WWW** Air that initially occupies  $0.140 \text{ m}^3$  at a gauge pressure of 103.0 kPa is expanded isothermally to a pressure of 101.3 kPa and then cooled at constant pressure until it reaches its initial volume. Compute the work done by the air. (Gauge pressure is the difference between the actual pressure and atmospheric pressure.)

**•12 GO Submarine rescue.** When the U.S. submarine *Squalus* became disabled at a depth of 80 m, a cylindrical chamber was lowered from a ship to rescue the crew. The chamber had a radius of 1.00 m and a height of 4.00 m, was open at the bottom, and held two rescuers. It slid along a guide cable that a diver had attached to a hatch on the submarine. Once the chamber reached the hatch and clamped to the hull, the crew could escape into the chamber. During the descent, air was released from tanks to prevent water from flooding the chamber. Assume that the interior air pressure matched the water pressure at depth  $h$  as given by  $p_0 + \rho gh$ , where  $p_0 = 1.000 \text{ atm}$  is the surface pressure and  $\rho = 1024 \text{ kg/m}^3$  is the density of seawater. Assume a surface temperature of 20.0°C and a submerged water temperature of -30.0°C. (a) What is the air volume in the chamber at the surface? (b) If air had not been released from the tanks, what would have been the air volume in the chamber at depth  $h = 80.0 \text{ m}$ ? (c) How many moles of air were needed to be released to maintain the original air volume in the chamber?

**•13 GO** A sample of an ideal gas is taken through the cyclic process *abca* shown in Fig. 19-20. The scale of the vertical axis is set by  $p_b = 7.5 \text{ kPa}$  and  $p_{ac} = 2.5 \text{ kPa}$ . At point *a*,  $T = 200 \text{ K}$ . (a) How many moles of gas are in the sample? What are (b) the temperature of the gas at point *b*, (c) the temperature of the gas at point *c*, and (d) the net energy added to the gas as heat during the cycle?

**•14** In the temperature range 310 K to 330 K, the pressure  $p$  of a certain nonideal gas is related to volume  $V$  and temperature  $T$  by

$$p = (24.9 \text{ J/K}) \frac{T}{V} - (0.00662 \text{ J/K}^2) \frac{T^2}{V}.$$

How much work is done by the gas if its temperature is raised from 315 K to 325 K while the pressure is held constant?

**•15** Suppose 0.825 mol of an ideal gas undergoes an isothermal expansion as energy is added to it as heat  $Q$ . If Fig. 19-21 shows the final volume  $V_f$  versus  $Q$ , what is the gas temperature? The scale of

the vertical axis is set by  $V_{fs} = 0.30 \text{ m}^3$ , and the scale of the horizontal axis is set by  $Q_s = 1200 \text{ J}$ .

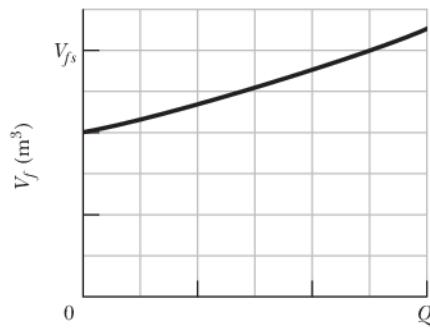


Figure 19-21 Problem 15.

**•16** An air bubble of volume  $20 \text{ cm}^3$  is at the bottom of a lake 40 m deep, where the temperature is 4.0°C. The bubble rises to the surface, which is at a temperature of 20°C. Take the temperature of the bubble's air to be the same as that of the surrounding water. Just as the bubble reaches the surface, what is its volume?

**•17 GO** Container A in Fig. 19-22 holds an ideal gas at a pressure of  $5.0 \times 10^5 \text{ Pa}$  and a temperature of 300 K. It is connected by a thin tube (and a closed valve) to container B, with four times the volume of A. Container B holds the same ideal gas at a pressure of  $1.0 \times 10^5 \text{ Pa}$  and a temperature of 400 K. The valve is opened to allow the pressures to equalize, but the temperature of each container is maintained. What then is the pressure?

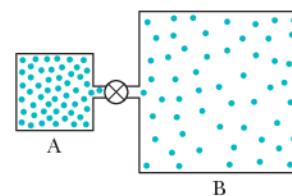


Figure 19-22 Problem 17.

### Module 19-3 Pressure, Temperature, and RMS Speed

**•18** The temperature and pressure in the Sun's atmosphere are  $2.00 \times 10^6 \text{ K}$  and  $0.0300 \text{ Pa}$ . Calculate the rms speed of free electrons (mass  $9.11 \times 10^{-31} \text{ kg}$ ) there, assuming they are an ideal gas.

**•19** (a) Compute the rms speed of a nitrogen molecule at 20.0°C. The molar mass of nitrogen molecules ( $N_2$ ) is given in Table 19-1. At what temperatures will the rms speed be (b) half that value and (c) twice that value?

**•20** Calculate the rms speed of helium atoms at 1000 K. See Appendix F for the molar mass of helium atoms.

**•21 SSM** The lowest possible temperature in outer space is 2.7 K. What is the rms speed of hydrogen molecules at this temperature? (The molar mass is given in Table 19-1.)

**•22** Find the rms speed of argon atoms at 313 K. See Appendix F for the molar mass of argon atoms.

**•23** A beam of hydrogen molecules ( $H_2$ ) is directed toward a wall, at an angle of 55° with the normal to the wall. Each molecule in the beam has a speed of 1.0 km/s and a mass of  $3.3 \times 10^{-24} \text{ g}$ . The beam strikes the wall over an area of  $2.0 \text{ cm}^2$ , at the rate of  $10^{23}$  molecules per second. What is the beam's pressure on the wall?

**•24** At 273 K and  $1.00 \times 10^{-2} \text{ atm}$ , the density of a gas is  $1.24 \times 10^{-5} \text{ g/cm}^3$ . (a) Find  $v_{rms}$  for the gas molecules. (b) Find the molar mass of the gas and (c) identify the gas. See Table 19-1.

### Module 19-4 Translational Kinetic Energy

**•25** Determine the average value of the translational kinetic energy of the molecules of an ideal gas at temperatures (a) 0.00°C

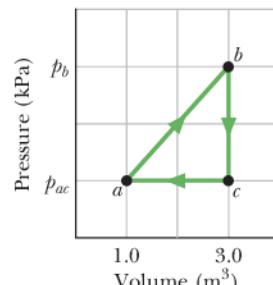


Figure 19-20 Problem 13.

and (b) 100°C. What is the translational kinetic energy per mole of an ideal gas at (c) 0.00°C and (d) 100°C?

**•26** What is the average translational kinetic energy of nitrogen molecules at 1600 K?

**•27** Water standing in the open at 32.0°C evaporates because of the escape of some of the surface molecules. The heat of vaporization (539 cal/g) is approximately equal to  $\varepsilon n$ , where  $\varepsilon$  is the average energy of the escaping molecules and  $n$  is the number of molecules per gram. (a) Find  $\varepsilon$ . (b) What is the ratio of  $\varepsilon$  to the average kinetic energy of H<sub>2</sub>O molecules, assuming the latter is related to temperature in the same way as it is for gases?

#### Module 19-5 Mean Free Path

**•28** At what frequency would the wavelength of sound in air be equal to the mean free path of oxygen molecules at 1.0 atm pressure and 0.00°C? The molecular diameter is  $3.0 \times 10^{-8}$  cm.

**•29 SSM** The atmospheric density at an altitude of 2500 km is about 1 molecule/cm<sup>3</sup>. (a) Assuming the molecular diameter of  $2.0 \times 10^{-8}$  cm, find the mean free path predicted by Eq. 19-25. (b) Explain whether the predicted value is meaningful.

**•30** The mean free path of nitrogen molecules at 0.0°C and 1.0 atm is  $0.80 \times 10^{-5}$  cm. At this temperature and pressure there are  $2.7 \times 10^{19}$  molecules/cm<sup>3</sup>. What is the molecular diameter?

**•31** In a certain particle accelerator, protons travel around a circular path of diameter 23.0 m in an evacuated chamber, whose residual gas is at 295 K and  $1.00 \times 10^{-6}$  torr pressure. (a) Calculate the number of gas molecules per cubic centimeter at this pressure. (b) What is the mean free path of the gas molecules if the molecular diameter is  $2.00 \times 10^{-8}$  cm?

**•32** At 20°C and 750 torr pressure, the mean free paths for argon gas (Ar) and nitrogen gas (N<sub>2</sub>) are  $\lambda_{\text{Ar}} = 9.9 \times 10^{-6}$  cm and  $\lambda_{\text{N}_2} = 27.5 \times 10^{-6}$  cm. (a) Find the ratio of the diameter of an Ar atom to that of an N<sub>2</sub> molecule. What is the mean free path of argon at (b) 20°C and 150 torr, and (c) -40°C and 750 torr?

#### Module 19-6 The Distribution of Molecular Speeds

**•33 SSM** The speeds of 10 molecules are 2.0, 3.0, 4.0, ..., 11 km/s. What are their (a) average speed and (b) rms speed?

**•34** The speeds of 22 particles are as follows ( $N_i$  represents the number of particles that have speed  $v_i$ ):

$N_i$	2	4	6	8	2
$v_i$ (cm/s)	1.0	2.0	3.0	4.0	5.0

What are (a)  $v_{\text{avg}}$ , (b)  $v_{\text{rms}}$ , and (c)  $v_p$ ?

**•35** Ten particles are moving with the following speeds: four at 200 m/s, two at 500 m/s, and four at 600 m/s. Calculate their (a) average and (b) rms speeds. (c) Is  $v_{\text{rms}} > v_{\text{avg}}$ ?

**•36** The most probable speed of the molecules in a gas at temperature  $T_2$  is equal to the rms speed of the molecules at temperature  $T_1$ . Find  $T_2/T_1$ .

**•37 SSM WWW** Figure 19-23 shows a hypothetical speed distribution for a sample of  $N$  gas particles (note that  $P(v) = 0$  for speed  $v > 2v_0$ ). What are the values of (a)  $av_0$ , (b)  $v_{\text{avg}}/v_0$ , and (c)  $v_{\text{rms}}/v_0$ ? (d) What fraction of the particles has a speed between  $1.5v_0$  and  $2.0v_0$ ?

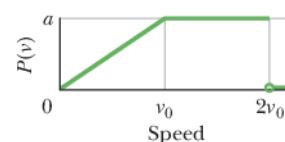


Figure 19-23 Problem 37.

**•38** Figure 19-24 gives the probability distribution for nitrogen gas. The scale of the horizontal axis is set by  $v_s = 1200$  m/s. What are the (a) gas temperature and (b) rms speed of the molecules?

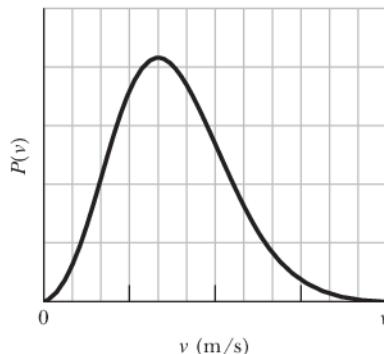


Figure 19-24 Problem 38.

**•39** At what temperature does the rms speed of (a) H<sub>2</sub> (molecular hydrogen) and (b) O<sub>2</sub> (molecular oxygen) equal the escape speed from Earth (Table 13-2)? At what temperature does the rms speed of (c) H<sub>2</sub> and (d) O<sub>2</sub> equal the escape speed from the Moon (where the gravitational acceleration at the surface has magnitude 0.16g)? Considering the answers to parts (a) and (b), should there be much (e) hydrogen and (f) oxygen high in Earth's upper atmosphere, where the temperature is about 1000 K?

**•40** Two containers are at the same temperature. The first contains gas with pressure  $p_1$ , molecular mass  $m_1$ , and rms speed  $v_{\text{rms}1}$ . The second contains gas with pressure  $2.0p_1$ , molecular mass  $m_2$ , and average speed  $v_{\text{avg}2} = 2.0v_{\text{rms}1}$ . Find the mass ratio  $m_1/m_2$ .

**•41** A hydrogen molecule (diameter  $1.0 \times 10^{-8}$  cm), traveling at the rms speed, escapes from a 4000 K furnace into a chamber containing cold argon atoms (diameter  $3.0 \times 10^{-8}$  cm) at a density of  $4.0 \times 10^{19}$  atoms/cm<sup>3</sup>. (a) What is the speed of the hydrogen molecule? (b) If it collides with an argon atom, what is the closest their centers can be, considering each as spherical? (c) What is the initial number of collisions per second experienced by the hydrogen molecule? (Hint: Assume that the argon atoms are stationary. Then the mean free path of the hydrogen molecule is given by Eq. 19-26 and not Eq. 19-25.)

#### Module 19-7 The Molar Specific Heats of an Ideal Gas

**•42** What is the internal energy of 1.0 mol of an ideal monatomic gas at 273 K?

**•43 GO** The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0°C without the pressure of the gas changing. The molecules in the gas rotate but do not oscillate. (a) How much energy is transferred to the gas as heat? (b) What is the change in the internal energy of the gas? (c) How much work is done by the gas? (d) By how much does the rotational kinetic energy of the gas increase?

**•44 GO** One mole of an ideal diatomic gas goes from *a* to *c* along the diagonal path in Fig. 19-25. The scale of the vertical axis is set by  $p_{ab} = 5.0$  kPa and  $p_c = 2.0$  kPa, and the scale of the horizontal axis is set by  $V_{bc} = 4.0$  m<sup>3</sup> and  $V_a = 2.0$  m<sup>3</sup>. During the transition, (a) what is the change in internal energy of the gas, and (b) how

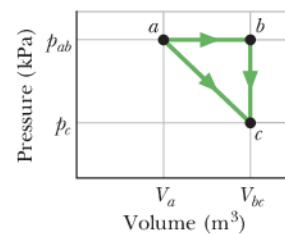


Figure 19-25 Problem 44.

much energy is added to the gas as heat? (c) How much heat is required if the gas goes from *a* to *c* along the indirect path *abc*?

**••45 ILW** The mass of a gas molecule can be computed from its specific heat at constant volume  $C_V$ . (Note that this is not  $C_p$ .) Take  $C_V = 0.075 \text{ cal/g} \cdot \text{C}^\circ$  for argon and calculate (a) the mass of an argon atom and (b) the molar mass of argon.

**••46** Under constant pressure, the temperature of 2.00 mol of an ideal monatomic gas is raised 15.0 K. What are (a) the work  $W$  done by the gas, (b) the energy transferred as heat  $Q$ , (c) the change  $\Delta E_{\text{int}}$  in the internal energy of the gas, and (d) the change  $\Delta K$  in the average kinetic energy per atom?

**••47** The temperature of 2.00 mol of an ideal monatomic gas is raised 15.0 K at constant volume. What are (a) the work  $W$  done by the gas, (b) the energy transferred as heat  $Q$ , (c) the change  $\Delta E_{\text{int}}$  in the internal energy of the gas, and (d) the change  $\Delta K$  in the average kinetic energy per atom?

**••48 GO** When 20.9 J was added as heat to a particular ideal gas, the volume of the gas changed from 50.0 cm<sup>3</sup> to 100 cm<sup>3</sup> while the pressure remained at 1.00 atm. (a) By how much did the internal energy of the gas change? If the quantity of gas present was  $2.00 \times 10^{-3}$  mol, find (b)  $C_p$  and (c)  $C_V$ .

**••49 SSM** A container holds a mixture of three nonreacting gases: 2.40 mol of gas 1 with  $C_{V1} = 12.0 \text{ J/mol} \cdot \text{K}$ , 1.50 mol of gas 2 with  $C_{V2} = 12.8 \text{ J/mol} \cdot \text{K}$ , and 3.20 mol of gas 3 with  $C_{V3} = 20.0 \text{ J/mol} \cdot \text{K}$ . What is  $C_V$  of the mixture?

#### Module 19-8 Degrees of Freedom and Molar Specific Heats

**••50** We give 70 J as heat to a diatomic gas, which then expands at constant pressure. The gas molecules rotate but do not oscillate. By how much does the internal energy of the gas increase?

**••51 ILW** When 1.0 mol of oxygen (O<sub>2</sub>) gas is heated at constant pressure starting at 0°C, how much energy must be added to the gas as heat to double its volume? (The molecules rotate but do not oscillate.)

**••52 GO** Suppose 12.0 g of oxygen (O<sub>2</sub>) gas is heated at constant atmospheric pressure from 25.0°C to 125°C. (a) How many moles of oxygen are present? (See Table 19-1 for the molar mass.) (b) How much energy is transferred to the oxygen as heat? (The molecules rotate but do not oscillate.) (c) What fraction of the heat is used to raise the internal energy of the oxygen?

**••53 SSM WWW** Suppose 4.00 mol of an ideal diatomic gas, with molecular rotation but not oscillation, experienced a temperature increase of 60.0 K under constant-pressure conditions. What are (a) the energy transferred as heat  $Q$ , (b) the change  $\Delta E_{\text{int}}$  in internal energy of the gas, (c) the work  $W$  done by the gas, and (d) the change  $\Delta K$  in the total translational kinetic energy of the gas?

#### Module 19-9 The Adiabatic Expansion of an Ideal Gas

**••54** We know that for an adiabatic process  $pV^\gamma = \text{constant}$ . Evaluate “a constant” for an adiabatic process involving exactly 2.0 mol of an ideal gas passing through the state having exactly  $p = 1.0 \text{ atm}$  and  $T = 300 \text{ K}$ . Assume a diatomic gas whose molecules rotate but do not oscillate.

**••55** A certain gas occupies a volume of 4.3 L at a pressure of 1.2 atm and a temperature of 310 K. It is compressed adiabatically to a volume of 0.76 L. Determine (a) the final pressure and (b) the final temperature, assuming the gas to be an ideal gas for which  $\gamma = 1.4$ .

**••56** Suppose 1.00 L of a gas with  $\gamma = 1.30$ , initially at 273 K and 1.00 atm, is suddenly compressed adiabatically to half its initial volume. Find its final (a) pressure and (b) temperature. (c) If the gas is then cooled to 273 K at constant pressure, what is its final volume?

**••57** The volume of an ideal gas is adiabatically reduced from 200 L to 74.3 L. The initial pressure and temperature are 1.00 atm and 300 K. The final pressure is 4.00 atm. (a) Is the gas monatomic, diatomic, or polyatomic? (b) What is the final temperature? (c) How many moles are in the gas?

**••58 GO** **Opening champagne.** In a bottle of champagne, the pocket of gas (primarily carbon dioxide) between the liquid and the cork is at pressure of  $p_i = 5.00 \text{ atm}$ . When the cork is pulled from the bottle, the gas undergoes an adiabatic expansion until its pressure matches the ambient air pressure of 1.00 atm. Assume that the ratio of the molar specific heats is  $\gamma = \frac{4}{3}$ . If the gas has initial temperature  $T_i = 5.00^\circ\text{C}$ , what is its temperature at the end of the adiabatic expansion?

**••59 GO** Figure 19-26 shows two paths that may be taken by a gas from an initial point *i* to a final point *f*. Path 1 consists of an isothermal expansion (work is 50 J in magnitude), an adiabatic expansion (work is 40 J in magnitude), an isothermal compression (work is 30 J in magnitude), and then an adiabatic compression (work is 25 J in magnitude). What is the change in the internal energy of the gas if the gas goes from point *i* to point *f* along path 2?

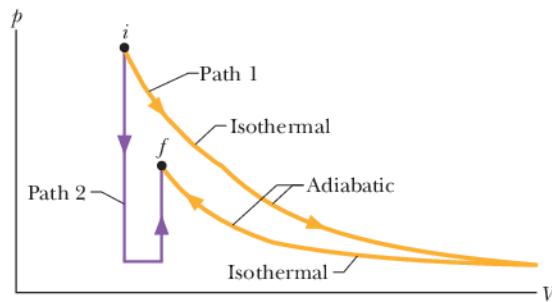


Figure 19-26 Problem 59.

**••60 GO** **Adiabatic wind.** The normal airflow over the Rocky Mountains is west to east. The air loses much of its moisture content and is chilled as it climbs the western side of the mountains. When it descends on the eastern side, the increase in pressure toward lower altitudes causes the temperature to increase. The flow, then called a chinook wind, can rapidly raise the air temperature at the base of the mountains. Assume that the air pressure  $p$  depends on altitude  $y$  according to  $p = p_0 \exp(-ay)$ , where  $p_0 = 1.00 \text{ atm}$  and  $a = 1.16 \times 10^{-4} \text{ m}^{-1}$ . Also assume that the ratio of the molar specific heats is  $\gamma = \frac{4}{3}$ . A parcel of air with an initial temperature of  $-5.00^\circ\text{C}$  descends adiabatically from  $y_1 = 4267 \text{ m}$  to  $y = 1567 \text{ m}$ . What is its temperature at the end of the descent?

**••61 GO** A gas is to be expanded from initial state *i* to final state *f* along either path 1 or path 2 on a *p*-*V* diagram. Path 1 consists of three steps: an isothermal expansion (work is 40 J in magnitude), an adiabatic expansion (work is 20 J in magnitude), and another isothermal expansion (work is 30 J in magnitude). Path 2 consists of two steps: a pressure reduction at constant volume and an expansion at constant pressure. What is the change in the internal energy of the gas along path 2?

**••62 GO** An ideal diatomic gas, with rotation but no oscillation, undergoes an adiabatic compression. Its initial pressure and volume are

1.20 atm and  $0.200\text{ m}^3$ . Its final pressure is 2.40 atm. How much work is done by the gas?

- 63** Figure 19-27 shows a cycle undergone by 1.00 mol of an ideal monatomic gas. The temperatures are  $T_1 = 300\text{ K}$ ,  $T_2 = 600\text{ K}$ , and  $T_3 = 455\text{ K}$ . For  $1 \rightarrow 2$ , what are (a) heat  $Q$ , (b) the change in internal energy  $\Delta E_{\text{int}}$ , and (c) the work done  $W$ ? For  $2 \rightarrow 3$ , what are (d)  $Q$ , (e)  $\Delta E_{\text{int}}$ , and (f)  $W$ ? For  $3 \rightarrow 1$ , what are (g)  $Q$ , (h)  $\Delta E_{\text{int}}$ , and (i)  $W$ ? For the full cycle, what are (j)  $Q$ , (k)  $\Delta E_{\text{int}}$ , and (l)  $W$ ? The initial pressure at point 1 is 1.00 atm ( $= 1.013 \times 10^5\text{ Pa}$ ). What are the (m) volume and (n) pressure at point 2 and the (o) volume and (p) pressure at point 3?

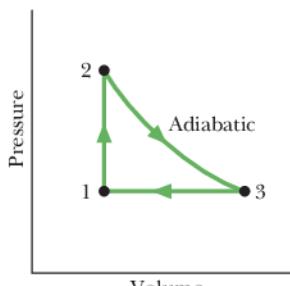


Figure 19-27 Problem 63.

### Additional Problems

- 64** Calculate the work done by an external agent during an isothermal compression of 1.00 mol of oxygen from a volume of  $22.4\text{ L}$  at  $0^\circ\text{C}$  and 1.00 atm to a volume of  $16.8\text{ L}$ .

- 65** An ideal gas undergoes an adiabatic compression from  $p = 1.0\text{ atm}$ ,  $V = 1.0 \times 10^6\text{ L}$ ,  $T = 0.0^\circ\text{C}$  to  $p = 1.0 \times 10^5\text{ atm}$ ,  $V = 1.0 \times 10^3\text{ L}$ . (a) Is the gas monatomic, diatomic, or polyatomic? (b) What is its final temperature? (c) How many moles of gas are present? What is the total translational kinetic energy per mole (d) before and (e) after the compression? (f) What is the ratio of the squares of the rms speeds before and after the compression?

- 66** An ideal gas consists of 1.50 mol of diatomic molecules that rotate but do not oscillate. The molecular diameter is 250 pm. The gas is expanded at a constant pressure of  $1.50 \times 10^5\text{ Pa}$ , with a transfer of 200 J as heat. What is the change in the mean free path of the molecules?

- 67** An ideal monatomic gas initially has a temperature of  $330\text{ K}$  and a pressure of 6.00 atm. It is to expand from volume  $500\text{ cm}^3$  to volume  $1500\text{ cm}^3$ . If the expansion is isothermal, what are (a) the final pressure and (b) the work done by the gas? If, instead, the expansion is adiabatic, what are (c) the final pressure and (d) the work done by the gas?

- 68** In an interstellar gas cloud at  $50.0\text{ K}$ , the pressure is  $1.00 \times 10^{-8}\text{ Pa}$ . Assuming that the molecular diameters of the gases in the cloud are all 20.0 nm, what is their mean free path?

- 69 SSM** The envelope and basket of a hot-air balloon have a combined weight of 2.45 kN, and the envelope has a capacity (volume) of  $2.18 \times 10^3\text{ m}^3$ . When it is fully inflated, what should be the temperature of the enclosed air to give the balloon a *lifting capacity* (force) of 2.67 kN (in addition to the balloon's weight)? Assume that the surrounding air, at  $20.0^\circ\text{C}$ , has a weight per unit volume of  $11.9\text{ N/m}^3$  and a molecular mass of 0.028 kg/mol, and is at a pressure of 1.0 atm.

- 70** An ideal gas, at initial temperature  $T_1$  and initial volume  $2.0\text{ m}^3$ , is expanded adiabatically to a volume of  $4.0\text{ m}^3$ , then expanded isothermally to a volume of  $10\text{ m}^3$ , and then compressed adiabatically back to  $T_1$ . What is its final volume?

- 71 SSM** The temperature of  $2.00\text{ mol}$  of an ideal monatomic gas is raised  $15.0\text{ K}$  in an adiabatic process. What are (a) the work  $W$  done by the gas, (b) the energy transferred as heat  $Q$ , (c) the change  $\Delta E_{\text{int}}$  in internal energy of the gas, and (d) the change  $\Delta K$  in the average kinetic energy per atom?

- 72** At what temperature do atoms of helium gas have the same rms speed as molecules of hydrogen gas at  $20.0^\circ\text{C}$ ? (The molar masses are given in Table 19-1.)

- 73 SSM** At what frequency do molecules (diameter 290 pm) collide in (an ideal) oxygen gas ( $\text{O}_2$ ) at temperature  $400\text{ K}$  and pressure  $2.00\text{ atm}$ ?

- 74** (a) What is the number of molecules per cubic meter in air at  $20^\circ\text{C}$  and at a pressure of  $1.0\text{ atm}$  ( $= 1.01 \times 10^5\text{ Pa}$ )? (b) What is the mass of  $1.0\text{ m}^3$  of this air? Assume that 75% of the molecules are nitrogen ( $\text{N}_2$ ) and 25% are oxygen ( $\text{O}_2$ ).

- 75** The temperature of  $3.00\text{ mol}$  of a gas with  $C_V = 6.00\text{ cal/mol}\cdot\text{K}$  is to be raised  $50.0\text{ K}$ . If the process is at *constant volume*, what are (a) the energy transferred as heat  $Q$ , (b) the work  $W$  done by the gas, (c) the change  $\Delta E_{\text{int}}$  in internal energy of the gas, and (d) the change  $\Delta K$  in the total translational kinetic energy? If the process is at *constant pressure*, what are (e)  $Q$ , (f)  $W$ , (g)  $\Delta E_{\text{int}}$ , and (h)  $\Delta K$ ? If the process is *adiabatic*, what are (i)  $Q$ , (j)  $W$ , (k)  $\Delta E_{\text{int}}$ , and (l)  $\Delta K$ ?

- 76** During a compression at a constant pressure of  $250\text{ Pa}$ , the volume of an ideal gas decreases from  $0.80\text{ m}^3$  to  $0.20\text{ m}^3$ . The initial temperature is  $360\text{ K}$ , and the gas loses  $210\text{ J}$  as heat. What are (a) the change in the internal energy of the gas and (b) the final temperature of the gas?

- 77 SSM** Figure 19-28 shows a hypothetical speed distribution for particles of a certain gas:  $P(v) = Cv^2$  for  $0 < v \leq v_0$  and  $P(v) = 0$  for  $v > v_0$ . Find (a) an expression for  $C$  in terms of  $v_0$ , (b) the average speed of the particles, and (c) their rms speed.

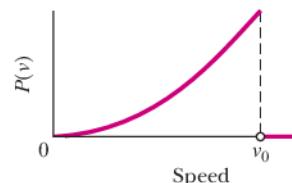


Figure 19-28 Problem 77.

- 78** (a) An ideal gas initially at pressure  $p_0$  undergoes a free expansion until its volume is  $3.00$  times its initial volume. What then is the ratio of its pressure to  $p_0$ ? (b) The gas is next slowly and adiabatically compressed back to its original volume. The pressure after compression is  $(3.00)^{1/3}p_0$ . Is the gas monatomic, diatomic, or polyatomic? (c) What is the ratio of the average kinetic energy per molecule in this final state to that in the initial state?

- 79 SSM** An ideal gas undergoes isothermal compression from an initial volume of  $4.00\text{ m}^3$  to a final volume of  $3.00\text{ m}^3$ . There is  $3.50\text{ mol}$  of the gas, and its temperature is  $10.0^\circ\text{C}$ . (a) How much work is done by the gas? (b) How much energy is transferred as heat between the gas and its environment?

- 80** Oxygen ( $\text{O}_2$ ) gas at  $273\text{ K}$  and  $1.0\text{ atm}$  is confined to a cubical container 10 cm on a side. Calculate  $\Delta U_g/K_{\text{avg}}$ , where  $\Delta U_g$  is the change in the gravitational potential energy of an oxygen molecule falling the height of the box and  $K_{\text{avg}}$  is the molecule's average translational kinetic energy.

- 81** An ideal gas is taken through a complete cycle in three steps: adiabatic expansion with work equal to 125 J, isothermal contraction at  $325\text{ K}$ , and increase in pressure at constant volume. (a) Draw a  $p$ - $V$  diagram for the three steps. (b) How much energy is transferred as heat in step 3, and (c) is it transferred *to* or *from* the gas?

- 82** (a) What is the volume occupied by  $1.00\text{ mol}$  of an ideal gas at standard conditions—that is,  $1.00\text{ atm}$  ( $= 1.01 \times 10^5\text{ Pa}$ ) and  $273\text{ K}$ ? (b) Show that the number of molecules per cubic centimeter (the *Loschmidt number*) at standard conditions is  $2.69 \times 10^{25}$ .

- 83 SSM** A sample of ideal gas expands from an initial pressure

and volume of 32 atm and 1.0 L to a final volume of 4.0 L. The initial temperature is 300 K. If the gas is monatomic and the expansion isothermal, what are the (a) final pressure  $p_f$ , (b) final temperature  $T_f$ , and (c) work  $W$  done by the gas? If the gas is monatomic and the expansion adiabatic, what are (d)  $p_f$ , (e)  $T_f$ , and (f)  $W$ ? If the gas is diatomic and the expansion adiabatic, what are (g)  $p_f$ , (h)  $T_f$ , and (i)  $W$ ?

**84** An ideal gas with 3.00 mol is initially in state 1 with pressure  $p_1 = 20.0$  atm and volume  $V_1 = 1500 \text{ cm}^3$ . First it is taken to state 2 with pressure  $p_2 = 1.50p_1$  and volume  $V_2 = 2.00V_1$ . Then it is taken to state 3 with pressure  $p_3 = 2.00p_1$  and volume  $V_3 = 0.500V_1$ . What is the temperature of the gas in (a) state 1 and (b) state 2? (c) What is the net change in internal energy from state 1 to state 3?

**85** A steel tank contains 300 g of ammonia gas ( $\text{NH}_3$ ) at a pressure of  $1.35 \times 10^6$  Pa and a temperature of 77°C. (a) What is the volume of the tank in liters? (b) Later the temperature is 22°C and the pressure is  $8.7 \times 10^5$  Pa. How many grams of gas have leaked out of the tank?

**86** In an industrial process the volume of 25.0 mol of a monatomic ideal gas is reduced at a uniform rate from  $0.616 \text{ m}^3$  to  $0.308 \text{ m}^3$  in 2.00 h while its temperature is increased at a uniform rate from 27.0°C to 450°C. Throughout the process, the gas passes through thermodynamic equilibrium states. What are (a) the cumulative work done on the gas, (b) the cumulative energy absorbed by the gas as heat, and (c) the molar specific heat for the process? (Hint: To evaluate the integral for the work, you might use

$$\int \frac{a + bx}{A + Bx} dx = \frac{bx}{B} + \frac{aB - bA}{B^2} \ln(A + Bx),$$

an indefinite integral.) Suppose the process is replaced with a two-step process that reaches the same final state. In step 1, the gas volume is reduced at constant temperature, and in step 2 the temperature is increased at constant volume. For this process, what are (d) the cumulative work done on the gas, (e) the cumulative energy absorbed by the gas as heat, and (f) the molar specific heat for the process?

**87** Figure 19-29 shows a cycle consisting of five paths:  $AB$  is isothermal at 300 K,  $BC$  is adiabatic with work = 5.0 J,  $CD$  is at a constant pressure of 5 atm,  $DE$  is isothermal, and  $EA$  is adiabatic with a change in internal energy of 8.0 J. What is the change in internal energy of the gas along path  $CD$ ?

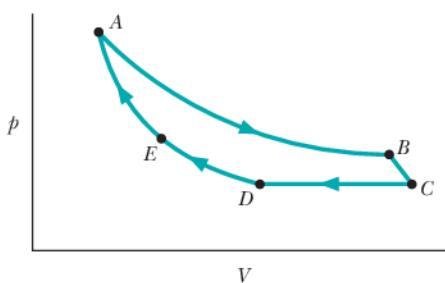


Figure 19-29 Problem 87.

**88** An ideal gas initially at 300 K is compressed at a constant pressure of  $25 \text{ N/m}^2$  from a volume of  $3.0 \text{ m}^3$  to a volume of  $1.8 \text{ m}^3$ . In the process, 75 J is lost by the gas as heat. What are (a) the change in internal energy of the gas and (b) the final temperature of the gas?

**89** A pipe of length  $L = 25.0$  m that is open at one end contains air at atmospheric pressure. It is thrust vertically into a freshwater lake

until the water rises halfway up in the pipe (Fig. 19-30). What is the depth  $h$  of the lower end of the pipe? Assume that the temperature is the same everywhere and does not change.

**90** In a motorcycle engine, a piston is forced down toward the crankshaft when the fuel in the top of the piston's cylinder undergoes combustion. The mixture of gaseous combustion products then expands adiabatically as the piston descends. Find the average power in (a) watts and (b) horsepower that is involved in this expansion when the engine is running at 4000 rpm, assuming that the gauge pressure immediately after combustion is 15 atm, the initial volume is  $50 \text{ cm}^3$ , and the volume of the mixture at the bottom of the stroke is  $250 \text{ cm}^3$ . Assume that the gases are diatomic and that the time involved in the expansion is one-half that of the total cycle.

**91** For adiabatic processes in an ideal gas, show that (a) the bulk modulus is given by

$$B = -V \frac{dp}{dV} = \gamma p,$$

where  $\gamma = C_p/C_V$ . (See Eq. 17-2.) (b) Then show that the speed of sound in the gas is

$$v_s = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}},$$

where  $\rho$  is the density,  $T$  is the temperature, and  $M$  is the molar mass. (See Eq. 17-3.)

**92** Air at 0.000°C and 1.00 atm pressure has a density of  $1.29 \times 10^{-3}$  g/cm<sup>3</sup>, and the speed of sound is 331 m/s at that temperature. Compute the ratio  $\gamma$  of the molar specific heats of air. (Hint: See Problem 91.)

**93** The speed of sound in different gases at a certain temperature  $T$  depends on the molar mass of the gases. Show that

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}},$$

where  $v_1$  is the speed of sound in a gas of molar mass  $M_1$  and  $v_2$  is the speed of sound in a gas of molar mass  $M_2$ . (Hint: See Problem 91.)

**94** From the knowledge that  $C_V$ , the molar specific heat at constant volume, for a gas in a container is  $5.0R$ , calculate the ratio of the speed of sound in that gas to the rms speed of the molecules, for gas temperature  $T$ . (Hint: See Problem 91.)

**95** The molar mass of iodine is 127 g/mol. When sound at frequency 1000 Hz is introduced to a tube of iodine gas at 400 K, an internal acoustic standing wave is set up with nodes separated by 9.57 cm. What is  $\gamma$  for the gas? (Hint: See Problem 91.)

**96** For air near 0°C, by how much does the speed of sound increase for each increase in air temperature by 1°C? (Hint: See Problem 91.)

**97** Two containers are at the same temperature. The gas in the first container is at pressure  $p_1$  and has molecules with mass  $m_1$  and root-mean-square speed  $v_{rms1}$ . The gas in the second is at pressure  $2p_1$  and has molecules with mass  $m_2$  and average speed  $v_{avg2} = 2v_{rms1}$ . Find the ratio  $m_1/m_2$  of the masses of their molecules.

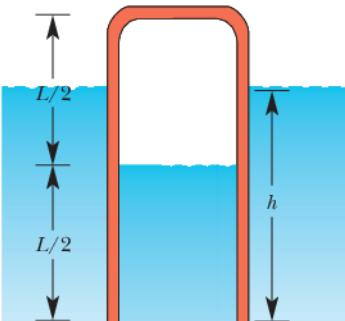


Figure 19-30 Problem 89.

# Entropy and the Second Law of Thermodynamics

## 20-1 ENTROPY

### Learning Objectives

After reading this module, you should be able to ...

- 20.01** Identify the second law of thermodynamics: If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes; it never decreases.
- 20.02** Identify that entropy is a state function (the value for a particular state of the system does not depend on how that state is reached).
- 20.03** Calculate the change in entropy for a process by integrating the inverse of the temperature (in kelvins) with respect to the heat  $Q$  transferred during the process.
- 20.04** For a phase change with a constant temperature process, apply the relationship between the entropy change  $\Delta S$ , the total transferred heat  $Q$ , and the temperature  $T$  (in kelvins).

### Key Ideas

- An irreversible process is one that cannot be reversed by means of small changes in the environment. The direction in which an irreversible process proceeds is set by the change in entropy  $\Delta S$  of the system undergoing the process. Entropy  $S$  is a state property (or state function) of the system; that is, it depends only on the state of the system and not on the way in which the system reached that state. The entropy postulate states (in part): If an irreversible process occurs in a closed system, the entropy of the system always increases.
- The entropy change  $\Delta S$  for an irreversible process that takes a system from an initial state  $i$  to a final state  $f$  is exactly equal to the entropy change  $\Delta S$  for any reversible process that takes the system between those same two states. We can compute the latter (but not the former) with

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}.$$

Here  $Q$  is the energy transferred as heat to or from the system during the process, and  $T$  is the temperature of the system in kelvins during the process.

- For a reversible isothermal process, the expression for an entropy change reduces to

- 20.05** For a temperature change  $\Delta T$  that is small relative to the temperature  $T$ , apply the relationship between the entropy change  $\Delta S$ , the transferred heat  $Q$ , and the average temperature  $T_{\text{avg}}$  (in kelvins).
- 20.06** For an ideal gas, apply the relationship between the entropy change  $\Delta S$  and the initial and final values of the pressure and volume.
- 20.07** Identify that if a process is an irreversible one, the integration for the entropy change must be done for a reversible process that takes the system between the same initial and final states as the irreversible process.
- 20.08** For stretched rubber, relate the elastic force to the rate at which the rubber's entropy changes with the change in the stretching distance.

$$\Delta S = S_f - S_i = \frac{Q}{T}.$$

- When the temperature change  $\Delta T$  of a system is small relative to the temperature (in kelvins) before and after the process, the entropy change can be approximated as

$$\Delta S = S_f - S_i \approx \frac{Q}{T_{\text{avg}}},$$

where  $T_{\text{avg}}$  is the system's average temperature during the process.

- When an ideal gas changes reversibly from an initial state with temperature  $T_i$  and volume  $V_i$  to a final state with temperature  $T_f$  and volume  $V_f$ , the change  $\Delta S$  in the entropy of the gas is

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}.$$

- The second law of thermodynamics, which is an extension of the entropy postulate, states: If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases. In equation form,

$$\Delta S \geq 0.$$

## What Is Physics?

Time has direction, the direction in which we age. We are accustomed to many one-way processes—that is, processes that can occur only in a certain sequence (the right way) and never in the reverse sequence (the wrong way). An egg is dropped onto a floor, a pizza is baked, a car is driven into a lamppost, large waves erode a sandy beach—these one-way processes are **irreversible**, meaning that they cannot be reversed by means of only small changes in their environment.

One goal of physics is to understand why time has direction and why one-way processes are irreversible. Although this physics might seem disconnected from the practical issues of everyday life, it is in fact at the heart of any engine, such as a car engine, because it determines how well an engine can run.

The key to understanding why one-way processes cannot be reversed involves a quantity known as *entropy*.

## Irreversible Processes and Entropy

The one-way character of irreversible processes is so pervasive that we take it for granted. If these processes were to occur *spontaneously* (on their own) in the wrong way, we would be astonished. Yet *none* of these wrong-way events would violate the law of conservation of energy.

For example, if you were to wrap your hands around a cup of hot coffee, you would be astonished if your hands got cooler and the cup got warmer. That is obviously the wrong way for the energy transfer, but the total energy of the closed system (*hands + cup of coffee*) would be the same as the total energy if the process had run in the right way. For another example, if you popped a helium balloon, you would be astonished if, later, all the helium molecules were to gather together in the original shape of the balloon. That is obviously the wrong way for molecules to spread, but the total energy of the closed system (*molecules + room*) would be the same as for the right way.

Thus, changes in energy within a closed system do not set the direction of irreversible processes. Rather, that direction is set by another property that we shall discuss in this chapter—the *change in entropy*  $\Delta S$  of the system. The change in entropy of a system is defined later in this module, but we can here state its central property, often called the *entropy postulate*:



If an irreversible process occurs in a *closed* system, the entropy  $S$  of the system always increases; it never decreases.

Entropy differs from energy in that entropy does *not* obey a conservation law. The *energy* of a closed system is conserved; it always remains constant. For irreversible processes, the *entropy* of a closed system always increases. Because of this property, the change in entropy is sometimes called “the arrow of time.” For example, we associate the explosion of a popcorn kernel with the forward direction of time and with an increase in entropy. The backward direction of time (a videotape run backwards) would correspond to the exploded popcorn reforming the original kernel. Because this backward process would result in an entropy decrease, it never happens.

There are two equivalent ways to define the change in entropy of a system: (1) in terms of the system’s temperature and the energy the system gains or loses as heat, and (2) by counting the ways in which the atoms or molecules that make up the system can be arranged. We use the first approach in this module and the second in Module 20-4.

## Change in Entropy

Let's approach this definition of *change in entropy* by looking again at a process that we described in Modules 18-5 and 19-9: the free expansion of an ideal gas. Figure 20-1a shows the gas in its initial equilibrium state *i*, confined by a closed stopcock to the left half of a thermally insulated container. If we open the stopcock, the gas rushes to fill the entire container, eventually reaching the final equilibrium state *f* shown in Fig. 20-1b. This is an irreversible process; all the molecules of the gas will never return to the left half of the container.

The *p-V* plot of the process, in Fig. 20-2, shows the pressure and volume of the gas in its initial state *i* and final state *f*. Pressure and volume are *state properties*, properties that depend only on the state of the gas and not on how it reached that state. Other state properties are temperature and energy. We now assume that the gas has still another state property—its entropy. Furthermore, we define the **change in entropy**  $S_f - S_i$  of a system during a process that takes the system from an initial state *i* to a final state *f* as

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T} \quad (\text{change in entropy defined}). \quad (20-1)$$

Here *Q* is the energy transferred as heat to or from the system during the process, and *T* is the temperature of the system in kelvins. Thus, an entropy change depends not only on the energy transferred as heat but also on the temperature at which the transfer takes place. Because *T* is always positive, the sign of  $\Delta S$  is the same as that of *Q*. We see from Eq. 20-1 that the SI unit for entropy and entropy change is the joule per kelvin.

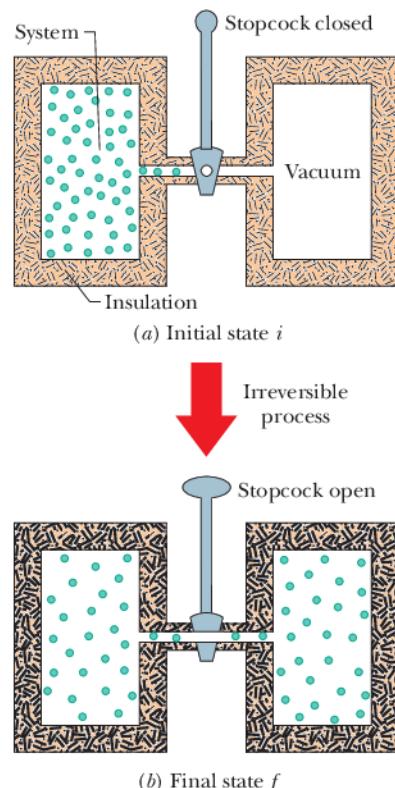
There is a problem, however, in applying Eq. 20-1 to the free expansion of Fig. 20-1. As the gas rushes to fill the entire container, the pressure, temperature, and volume of the gas fluctuate unpredictably. In other words, they do not have a sequence of well-defined equilibrium values during the intermediate stages of the change from initial state *i* to final state *f*. Thus, we cannot trace a pressure–volume path for the free expansion on the *p-V* plot of Fig. 20-2, and we cannot find a relation between *Q* and *T* that allows us to integrate as Eq. 20-1 requires.

However, if entropy is truly a state property, the difference in entropy between states *i* and *f* must depend *only on those states* and not at all on the way the system went from one state to the other. Suppose, then, that we replace the irreversible free expansion of Fig. 20-1 with a *reversible* process that connects states *i* and *f*. With a reversible process we can trace a pressure–volume path on a *p-V* plot, and we can find a relation between *Q* and *T* that allows us to use Eq. 20-1 to obtain the entropy change.

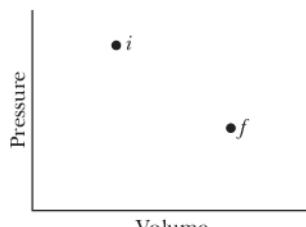
We saw in Module 19-9 that the temperature of an ideal gas does not change during a free expansion:  $T_i = T_f = T$ . Thus, points *i* and *f* in Fig. 20-2 must be on the same isotherm. A convenient replacement process is then a reversible isothermal expansion from state *i* to state *f*, which actually proceeds *along* that isotherm. Furthermore, because *T* is constant throughout a reversible isothermal expansion, the integral of Eq. 20-1 is greatly simplified.

Figure 20-3 shows how to produce such a reversible isothermal expansion. We confine the gas to an insulated cylinder that rests on a thermal reservoir maintained at the temperature *T*. We begin by placing just enough lead shot on the movable piston so that the pressure and volume of the gas are those of the initial state *i* of Fig. 20-1a. We then remove shot slowly (piece by piece) until the pressure and volume of the gas are those of the final state *f* of Fig. 20-1b. The temperature of the gas does not change because the gas remains in thermal contact with the reservoir throughout the process.

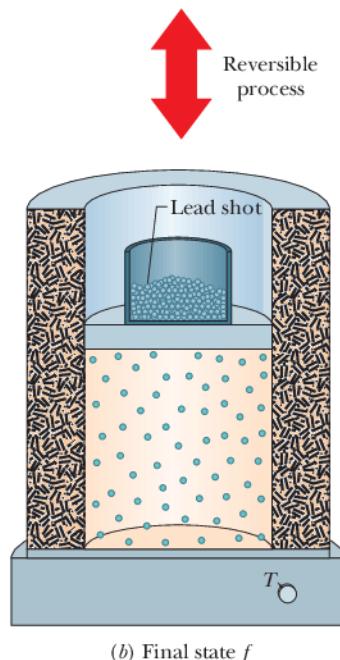
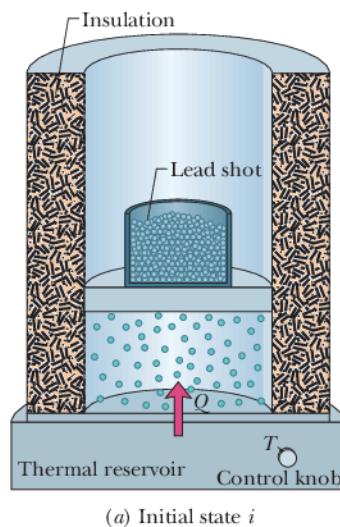
The reversible isothermal expansion of Fig. 20-3 is physically quite different from the irreversible free expansion of Fig. 20-1. However, *both processes have the same initial state and the same final state and thus must have the same change in*



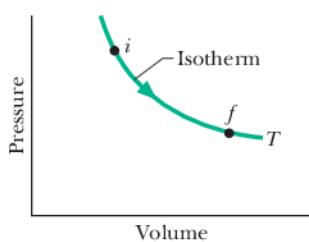
**Figure 20-1** The free expansion of an ideal gas. (a) The gas is confined to the left half of an insulated container by a closed stopcock. (b) When the stopcock is opened, the gas rushes to fill the entire container. This process is irreversible; that is, it does not occur in reverse, with the gas spontaneously collecting itself in the left half of the container.



**Figure 20-2** A *p-V* diagram showing the initial state *i* and the final state *f* of the free expansion of Fig. 20-1. The intermediate states of the gas cannot be shown because they are not equilibrium states.



**Figure 20-3** The isothermal expansion of an ideal gas, done in a reversible way. The gas has the same initial state *i* and same final state *f* as in the irreversible process of Figs. 20-1 and 20-2.



**Figure 20-4** A *p*-*V* diagram for the reversible isothermal expansion of Fig. 20-3. The intermediate states, which are now equilibrium states, are shown.

*entropy*. Because we removed the lead shot slowly, the intermediate states of the gas are equilibrium states, so we can plot them on a *p*-*V* diagram (Fig. 20-4).

To apply Eq. 20-1 to the isothermal expansion, we take the constant temperature *T* outside the integral, obtaining

$$\Delta S = S_f - S_i = \frac{1}{T} \int_i^f dQ.$$

Because  $\int dQ = Q$ , where *Q* is the total energy transferred as heat during the process, we have

$$\Delta S = S_f - S_i = \frac{Q}{T} \quad (\text{change in entropy, isothermal process}). \quad (20-2)$$

To keep the temperature *T* of the gas constant during the isothermal expansion of Fig. 20-3, heat *Q* must have been energy transferred *from* the reservoir *to* the gas. Thus, *Q* is positive and the entropy of the gas *increases* during the isothermal process and during the free expansion of Fig. 20-1.

To summarize:



To find the entropy change for an irreversible process, replace that process with any reversible process that connects the same initial and final states. Calculate the entropy change for this reversible process with Eq. 20-1.

When the temperature change  $\Delta T$  of a system is small relative to the temperature (in kelvins) before and after the process, the entropy change can be approximated as

$$\Delta S = S_f - S_i \approx \frac{Q}{T_{\text{avg}}}, \quad (20-3)$$

where  $T_{\text{avg}}$  is the average temperature of the system in kelvins during the process.



### Checkpoint 1

Water is heated on a stove. Rank the entropy changes of the water as its temperature rises (a) from 20°C to 30°C, (b) from 30°C to 35°C, and (c) from 80°C to 85°C, greatest first.

### Entropy as a State Function

We have assumed that entropy, like pressure, energy, and temperature, is a property of the state of a system and is independent of how that state is reached. That entropy is indeed a *state function* (as state properties are usually called) can be deduced only by experiment. However, we can prove it is a state function for the special and important case in which an ideal gas is taken through a reversible process.

To make the process reversible, it is done slowly in a series of small steps, with the gas in an equilibrium state at the end of each step. For each small step, the energy transferred as heat to or from the gas is *dQ*, the work done by the gas is *dW*, and the change in internal energy is *dE<sub>int</sub>*. These are related by the first law of thermodynamics in differential form (Eq. 18-27):

$$dE_{\text{int}} = dQ - dW.$$

Because the steps are reversible, with the gas in equilibrium states, we can use Eq. 18-24 to replace *dW* with *p dV* and Eq. 19-45 to replace *dE<sub>int</sub>* with *nC<sub>V</sub> dT*. Solving for *dQ* then leads to

$$dQ = p dV + nC_V dT.$$

Using the ideal gas law, we replace *p* in this equation with *nRT/V*. Then we divide each term in the resulting equation by *T*, obtaining

$$\frac{dQ}{T} = nR \frac{dV}{V} + nC_V \frac{dT}{T}.$$

Now let us integrate each term of this equation between an arbitrary initial state  $i$  and an arbitrary final state  $f$  to get

$$\int_i^f \frac{dQ}{T} = \int_i^f nR \frac{dV}{V} + \int_i^f nC_V \frac{dT}{T}.$$

The quantity on the left is the entropy change  $\Delta S$  ( $= S_f - S_i$ ) defined by Eq. 20-1. Substituting this and integrating the quantities on the right yield

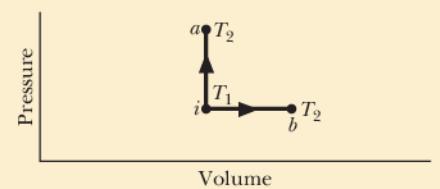
$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}. \quad (20-4)$$

Note that we did not have to specify a particular reversible process when we integrated. Therefore, the integration must hold for all reversible processes that take the gas from state  $i$  to state  $f$ . Thus, the change in entropy  $\Delta S$  between the initial and final states of an ideal gas depends only on properties of the initial state ( $V_i$  and  $T_i$ ) and properties of the final state ( $V_f$  and  $T_f$ );  $\Delta S$  does not depend on how the gas changes between the two states.



### Checkpoint 2

An ideal gas has temperature  $T_1$  at the initial state  $i$  shown in the  $p$ - $V$  diagram here. The gas has a higher temperature  $T_2$  at final states  $a$  and  $b$ , which it can reach along the paths shown. Is the entropy change along the path to state  $a$  larger than, smaller than, or the same as that along the path to state  $b$ ?



### Sample Problem 20.01 Entropy change of two blocks coming to thermal equilibrium

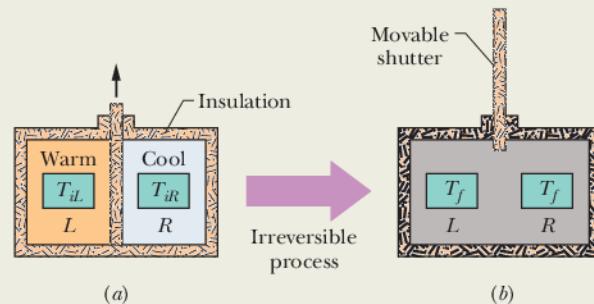
Figure 20-5a shows two identical copper blocks of mass  $m = 1.5 \text{ kg}$ ; block  $L$  at temperature  $T_{iL} = 60^\circ\text{C}$  and block  $R$  at temperature  $T_{iR} = 20^\circ\text{C}$ . The blocks are in a thermally insulated box and are separated by an insulating shutter. When we lift the shutter, the blocks eventually come to the equilibrium temperature  $T_f = 40^\circ\text{C}$  (Fig. 20-5b). What is the net entropy change of the two-block system during this irreversible process? The specific heat of copper is  $386 \text{ J/kg} \cdot \text{K}$ .

#### KEY IDEA

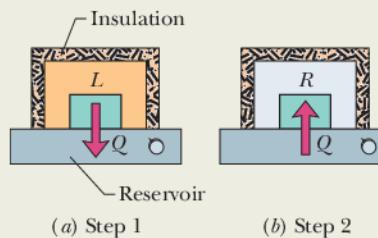
To calculate the entropy change, we must find a reversible process that takes the system from the initial state of Fig. 20-5a to the final state of Fig. 20-5b. We can calculate the net entropy change  $\Delta S_{\text{rev}}$  of the reversible process using Eq. 20-1, and then the entropy change for the irreversible process is equal to  $\Delta S_{\text{rev}}$ .

**Calculations:** For the reversible process, we need a thermal reservoir whose temperature can be changed slowly (say, by turning a knob). We then take the blocks through the following two steps, illustrated in Fig. 20-6.

**Step 1:** With the reservoir's temperature set at  $60^\circ\text{C}$ , put block  $L$  on the reservoir. (Since block and reservoir are at the same temperature, they are already in thermal equilib-



**Figure 20-5** (a) In the initial state, two copper blocks  $L$  and  $R$ , identical except for their temperatures, are in an insulating box and are separated by an insulating shutter. (b) When the shutter is removed, the blocks exchange energy as heat and come to a final state, both with the same temperature  $T_f$ .



**Figure 20-6** The blocks of Fig. 20-5 can proceed from their initial state to their final state in a reversible way if we use a reservoir with a controllable temperature (a) to extract heat reversibly from block  $L$  and (b) to add heat reversibly to block  $R$ .

rium.) Then slowly lower the temperature of the reservoir and the block to 40°C. As the block's temperature changes by each increment  $dT$  during this process, energy  $dQ$  is transferred as heat from the block to the reservoir. Using Eq. 18-14, we can write this transferred energy as  $dQ = mc dT$ , where  $c$  is the specific heat of copper. According to Eq. 20-1, the entropy change  $\Delta S_L$  of block  $L$  during the full temperature change from initial temperature  $T_{iL}$  ( $= 60^\circ\text{C} = 333\text{ K}$ ) to final temperature  $T_f$  ( $= 40^\circ\text{C} = 313\text{ K}$ ) is

$$\begin{aligned}\Delta S_L &= \int_i^f \frac{dQ}{T} = \int_{T_{iL}}^{T_f} \frac{mc}{T} dT = mc \int_{T_{iL}}^{T_f} \frac{dT}{T} \\ &= mc \ln \frac{T_f}{T_{iL}}.\end{aligned}$$

Inserting the given data yields

$$\begin{aligned}\Delta S_L &= (1.5\text{ kg})(386\text{ J/kg}\cdot\text{K}) \ln \frac{313\text{ K}}{333\text{ K}} \\ &= -35.86\text{ J/K}.\end{aligned}$$

**Step 2:** With the reservoir's temperature now set at 20°C,

put block  $R$  on the reservoir. Then slowly raise the temperature of the reservoir and the block to 40°C. With the same reasoning used to find  $\Delta S_L$ , you can show that the entropy change  $\Delta S_R$  of block  $R$  during this process is

$$\begin{aligned}\Delta S_R &= (1.5\text{ kg})(386\text{ J/kg}\cdot\text{K}) \ln \frac{313\text{ K}}{293\text{ K}} \\ &= +38.23\text{ J/K}.\end{aligned}$$

The net entropy change  $\Delta S_{\text{rev}}$  of the two-block system undergoing this two-step reversible process is then

$$\begin{aligned}\Delta S_{\text{rev}} &= \Delta S_L + \Delta S_R \\ &= -35.86\text{ J/K} + 38.23\text{ J/K} = 2.4\text{ J/K}.\end{aligned}$$

Thus, the net entropy change  $\Delta S_{\text{irrev}}$  for the two-block system undergoing the actual irreversible process is

$$\Delta S_{\text{irrev}} = \Delta S_{\text{rev}} = 2.4\text{ J/K}. \quad (\text{Answer})$$

This result is positive, in accordance with the entropy postulate.

### Sample Problem 20.02 Entropy change of a free expansion of a gas

Suppose 1.0 mol of nitrogen gas is confined to the left side of the container of Fig. 20-1a. You open the stopcock, and the volume of the gas doubles. What is the entropy change of the gas for this irreversible process? Treat the gas as ideal.

#### KEY IDEAS

- (1) We can determine the entropy change for the irreversible process by calculating it for a reversible process that provides the same change in volume. (2) The temperature of the gas does not change in the free expansion. Thus, the reversible process should be an isothermal expansion—namely, the one of Figs. 20-3 and 20-4.

**Calculations:** From Table 19-4, the energy  $Q$  added as heat to the gas as it expands isothermally at temperature  $T$  from an initial volume  $V_i$  to a final volume  $V_f$  is

$$Q = nRT \ln \frac{V_f}{V_i},$$

in which  $n$  is the number of moles of gas present. From Eq. 20-2 the entropy change for this reversible process in which the temperature is held constant is

$$\Delta S_{\text{rev}} = \frac{Q}{T} = \frac{nRT \ln(V_f/V_i)}{T} = nR \ln \frac{V_f}{V_i}.$$

Substituting  $n = 1.00\text{ mol}$  and  $V_f/V_i = 2$ , we find

$$\begin{aligned}\Delta S_{\text{rev}} &= nR \ln \frac{V_f}{V_i} = (1.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(\ln 2) \\ &= +5.76\text{ J/K}.\end{aligned}$$

Thus, the entropy change for the free expansion (and for all other processes that connect the initial and final states shown in Fig. 20-2) is

$$\Delta S_{\text{irrev}} = \Delta S_{\text{rev}} = +5.76\text{ J/K}. \quad (\text{Answer})$$

Because  $\Delta S$  is positive, the entropy increases, in accordance with the entropy postulate.



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## The Second Law of Thermodynamics

Here is a puzzle. In the process of going from (a) to (b) in Fig. 20-3, the entropy change of the gas (our system) is positive. However, because the process is reversible, we can also go from (b) to (a) by, say, gradually adding lead shot to the piston, to restore the initial gas volume. To maintain a constant temperature, we need to remove energy as heat, but that means  $Q$  is negative and thus the entropy change is also. Doesn't this entropy decrease violate the entropy postulate: en-

tropy always increases? No, because the postulate holds only for irreversible processes in closed systems. Here, the process is *not* irreversible and the system is *not* closed (because of the energy transferred to and from the reservoir as heat).

However, if we include the reservoir, along with the gas, as part of the system, then we do have a closed system. Let's check the change in entropy of the enlarged system *gas + reservoir* for the process that takes it from (b) to (a) in Fig. 20-3. During this reversible process, energy is transferred as heat from the gas to the reservoir—that is, from one part of the enlarged system to another. Let  $|Q|$  represent the absolute value (or magnitude) of this heat. With Eq. 20-2, we can then calculate separately the entropy changes for the gas (which loses  $|Q|$ ) and the reservoir (which gains  $|Q|$ ). We get

$$\Delta S_{\text{gas}} = -\frac{|Q|}{T}$$

and

$$\Delta S_{\text{res}} = +\frac{|Q|}{T}.$$

The entropy change of the closed system is the sum of these two quantities: 0.

With this result, we can modify the entropy postulate to include both reversible and irreversible processes:



If a process occurs in a *closed* system, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases.

Although entropy may decrease in part of a closed system, there will always be an equal or larger entropy increase in another part of the system, so that the entropy of the system as a whole never decreases. This fact is one form of the **second law of thermodynamics** and can be written as

$$\Delta S \geq 0 \quad (\text{second law of thermodynamics}), \quad (20-5)$$

where the greater-than sign applies to irreversible processes and the equals sign to reversible processes. Equation 20-5 applies only to closed systems.

In the real world almost all processes are irreversible to some extent because of friction, turbulence, and other factors, so the entropy of real closed systems undergoing real processes always increases. Processes in which the system's entropy remains constant are always idealizations.

### Force Due to Entropy

To understand why rubber resists being stretched, let's write the first law of thermodynamics

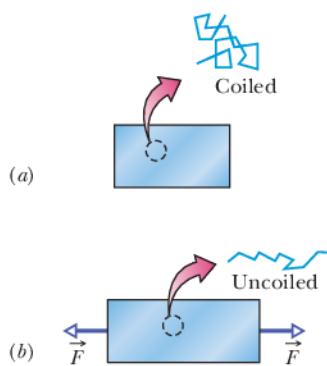
$$dE = dQ - dW$$

for a rubber band undergoing a small increase in length  $dx$  as we stretch it between our hands. The force from the rubber band has magnitude  $F$ , is directed inward, and does work  $dW = -F dx$  during length increase  $dx$ . From Eq. 20-2 ( $\Delta S = Q/T$ ), small changes in  $Q$  and  $S$  at constant temperature are related by  $dS = dQ/T$ , or  $dQ = T dS$ . So, now we can rewrite the first law as

$$dE = T dS + F dx. \quad (20-6)$$

To good approximation, the change  $dE$  in the internal energy of rubber is 0 if the total stretch of the rubber band is not very much. Substituting 0 for  $dE$  in Eq. 20-6 leads us to an expression for the force from the rubber band:

$$F = -T \frac{dS}{dx}. \quad (20-7)$$



**Figure 20-7** A section of a rubber band (a) unstretched and (b) stretched, and a polymer within it (a) coiled and (b) uncoiled.

This tells us that  $F$  is proportional to the rate  $dS/dx$  at which the rubber band's entropy changes during a small change  $dx$  in the rubber band's length. Thus, you can *feel* the effect of entropy on your hands as you stretch a rubber band.

To make sense of the relation between force and entropy, let's consider a simple model of the rubber material. Rubber consists of cross-linked polymer chains (long molecules with cross links) that resemble three-dimensional zig-zags (Fig. 20-7). When the rubber band is at its rest length, the polymers are coiled up in a spaghetti-like arrangement. Because of the large disorder of the molecules, this rest state has a high value of entropy. When we stretch a rubber band, we uncoil many of those polymers, aligning them in the direction of stretch. Because the alignment decreases the disorder, the entropy of the stretched rubber band is less. That is, the change  $dS/dx$  in Eq. 20-7 is a negative quantity because the entropy decreases with stretching. Thus, the force on our hands from the rubber band is due to the tendency of the polymers to return to their former disordered state and higher value of entropy.

## 20-2 ENTROPY IN THE REAL WORLD: ENGINES

### Learning Objectives

After reading this module, you should be able to . . .

- 20.09 Identify that a heat engine is a device that extracts energy from its environment in the form of heat and does useful work and that in an *ideal* heat engine, all processes are reversible, with no wasteful energy transfers.
- 20.10 Sketch a  $p$ - $V$  diagram for the cycle of a Carnot engine, indicating the direction of cycling, the nature of the processes involved, the work done during each process (including algebraic sign), the net work done in the cycle, and the heat transferred during each process (including algebraic sign).
- 20.11 Sketch a Carnot cycle on a temperature–entropy diagram, indicating the heat transfers.

### Key Ideas

- An engine is a device that, operating in a cycle, extracts energy as heat  $|Q_H|$  from a high-temperature reservoir and does a certain amount of work  $|W|$ . The efficiency  $\varepsilon$  of any engine is defined as

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}.$$

- In an ideal engine, all processes are reversible and no wasteful energy transfers occur due to, say, friction and turbulence.
- A Carnot engine is an ideal engine that follows the cycle of Fig. 20-9. Its efficiency is

$$\varepsilon_C = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H},$$

- 20.12 Determine the net entropy change around a Carnot cycle.

- 20.13 Calculate the efficiency  $\varepsilon_C$  of a Carnot engine in terms of the heat transfers and also in terms of the temperatures of the reservoirs.

- 20.14 Identify that there are no perfect engines in which the energy transferred as heat  $Q$  from a high temperature reservoir goes entirely into the work  $W$  done by the engine.
- 20.15 Sketch a  $p$ - $V$  diagram for the cycle of a Stirling engine, indicating the direction of cycling, the nature of the processes involved, the work done during each process (including algebraic sign), the net work done in the cycle, and the heat transfers during each process.

in which  $T_H$  and  $T_L$  are the temperatures of the high- and low-temperature reservoirs, respectively. Real engines always have an efficiency lower than that of a Carnot engine. Ideal engines that are not Carnot engines also have efficiencies lower than that of a Carnot engine.

- A perfect engine is an imaginary engine in which energy extracted as heat from the high-temperature reservoir is converted completely to work. Such an engine would violate the second law of thermodynamics, which can be restated as follows: No series of processes is possible whose sole result is the absorption of energy as heat from a thermal reservoir and the complete conversion of this energy to work.

### Entropy in the Real World: Engines

A **heat engine**, or more simply, an **engine**, is a device that extracts energy from its environment in the form of heat and does useful work. At the heart of every engine is a *working substance*. In a steam engine, the working substance is water,

in both its vapor and its liquid form. In an automobile engine the working substance is a gasoline–air mixture. If an engine is to do work on a sustained basis, the working substance must operate in a *cycle*; that is, the working substance must pass through a closed series of thermodynamic processes, called *strokes*, returning again and again to each state in its cycle. Let us see what the laws of thermodynamics can tell us about the operation of engines.

### A Carnot Engine

We have seen that we can learn much about real gases by analyzing an ideal gas, which obeys the simple law  $pV = nRT$ . Although an ideal gas does not exist, any real gas approaches ideal behavior if its density is low enough. Similarly, we can study real engines by analyzing the behavior of an **ideal engine**.



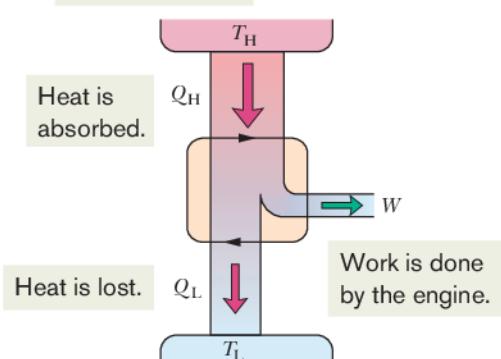
In an ideal engine, all processes are reversible and no wasteful energy transfers occur due to, say, friction and turbulence.

We shall focus on a particular ideal engine called a **Carnot engine** after the French scientist and engineer N. L. Sadi Carnot (pronounced “car-no”), who first proposed the engine’s concept in 1824. This ideal engine turns out to be the best (in principle) at using energy as heat to do useful work. Surprisingly, Carnot was able to analyze the performance of this engine before the first law of thermodynamics and the concept of entropy had been discovered.

Figure 20-8 shows schematically the operation of a Carnot engine. During each cycle of the engine, the working substance absorbs energy  $|Q_H|$  as heat from a thermal reservoir at constant temperature  $T_H$  and discharges energy  $|Q_L|$  as heat to a second thermal reservoir at a constant lower temperature  $T_L$ .

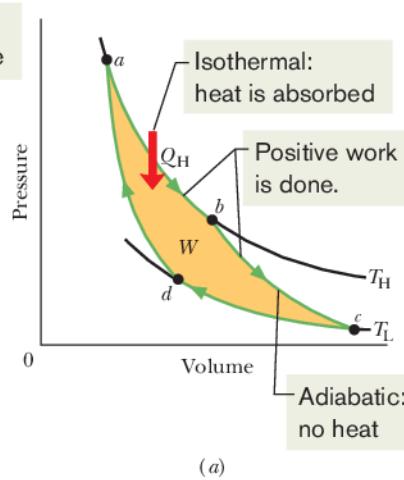
Figure 20-9 shows a *p*-*V* plot of the *Carnot cycle*—the cycle followed by the working substance. As indicated by the arrows, the cycle is traversed in the clockwise direction. Imagine the working substance to be a gas, confined to an insulating cylinder with a weighted, movable piston. The cylinder may be placed at will on either of the two thermal reservoirs, as in Fig. 20-6, or on an insulating slab. Figure 20-9a shows that, if we place the cylinder in contact with the high-temperature reservoir at temperature  $T_H$ , heat  $|Q_H|$  is transferred to the working substance *from* this reservoir as the gas undergoes an isothermal *expansion* from volume  $V_a$  to volume  $V_b$ . Similarly, with the working substance in contact with the low-temperature reservoir at temperature  $T_L$ , heat  $|Q_L|$  is transferred *from*

Schematic of a Carnot engine

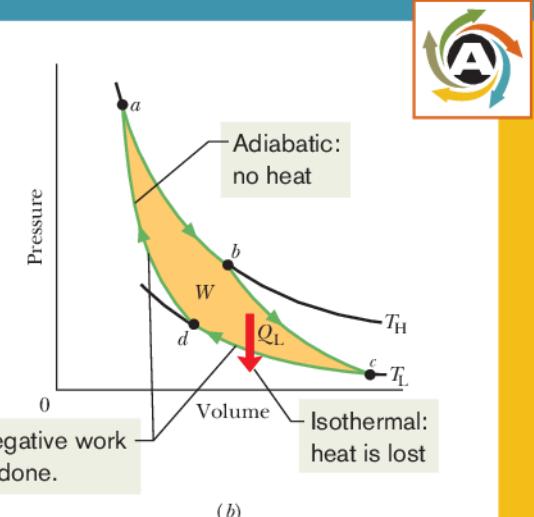


**Figure 20-8** The elements of a Carnot engine. The two black arrowheads on the central loop suggest the working substance operating in a cycle, as if on a *p*-*V* plot. Energy  $|Q_H|$  is transferred as heat from the high-temperature reservoir at temperature  $T_H$  to the working substance. Energy  $|Q_L|$  is transferred as heat from the working substance to the low-temperature reservoir at temperature  $T_L$ . Work  $W$  is done by the engine (actually by the working substance) on something in the environment.

Stages of a Carnot engine



**Figure 20-9** A pressure–volume plot of the cycle followed by the working substance of the Carnot engine in Fig. 20-8. The cycle consists of two isothermal (*ab* and *cd*) and two adiabatic processes (*bc* and *da*). The shaded area enclosed by the cycle is equal to the work  $W$  per cycle done by the Carnot engine.



the working substance *to* the low-temperature reservoir as the gas undergoes an isothermal compression from volume  $V_c$  to volume  $V_d$  (Fig. 20-9b).

In the engine of Fig. 20-8, we assume that heat transfers to or from the working substance can take place *only* during the isothermal processes *ab* and *cd* of Fig. 20-9. Therefore, processes *bc* and *da* in that figure, which connect the two isotherms at temperatures  $T_H$  and  $T_L$ , must be (reversible) adiabatic processes; that is, they must be processes in which no energy is transferred as heat. To ensure this, during processes *bc* and *da* the cylinder is placed on an insulating slab as the volume of the working substance is changed.

During the processes *ab* and *bc* of Fig. 20-9*a*, the working substance is expanding and thus doing positive work as it raises the weighted piston. This work is represented in Fig. 20-9*a* by the area under curve *abc*. During the processes *cd* and *da* (Fig. 20-9*b*), the working substance is being compressed, which means that it is doing negative work on its environment or, equivalently, that its environment is doing work on it as the loaded piston descends. This work is represented by the area under curve *cda*. The *net work per cycle*, which is represented by  $W$  in both Figs. 20-8 and 20-9, is the difference between these two areas and is a positive quantity equal to the area enclosed by cycle *abcda* in Fig. 20-9. This work  $W$  is performed on some outside object, such as a load to be lifted.

Equation 20-1 ( $\Delta S = \int dQ/T$ ) tells us that any energy transfer as heat must involve a change in entropy. To see this for a Carnot engine, we can plot the Carnot cycle on a temperature–entropy (*T*-*S*) diagram as in Fig. 20-10. The lettered points *a*, *b*, *c*, and *d* there correspond to the lettered points in the *p*-*V* diagram in Fig. 20-9. The two horizontal lines in Fig. 20-10 correspond to the two isothermal processes of the cycle. Process *ab* is the isothermal expansion of the cycle. As the working substance (reversibly) absorbs energy  $|Q_H|$  as heat at constant temperature  $T_H$  during the expansion, its entropy increases. Similarly, during the isothermal compression *cd*, the working substance (reversibly) loses energy  $|Q_L|$  as heat at constant temperature  $T_L$ , and its entropy decreases.

The two vertical lines in Fig. 20-10 correspond to the two adiabatic processes of the Carnot cycle. Because no energy is transferred as heat during the two processes, the entropy of the working substance is constant during them.

**The Work** To calculate the net work done by a Carnot engine during a cycle, let us apply Eq. 18-26, the first law of thermodynamics ( $\Delta E_{\text{int}} = Q - W$ ), to the working substance. That substance must return again and again to any arbitrarily selected state in the cycle. Thus, if  $X$  represents any state property of the working substance, such as pressure, temperature, volume, internal energy, or entropy, we must have  $\Delta X = 0$  for every cycle. It follows that  $\Delta E_{\text{int}} = 0$  for a complete cycle of the working substance. Recalling that  $Q$  in Eq. 18-26 is the *net* heat transfer per cycle and  $W$  is the *net* work, we can write the first law of thermodynamics for the Carnot cycle as

$$W = |Q_H| - |Q_L|. \quad (20-8)$$

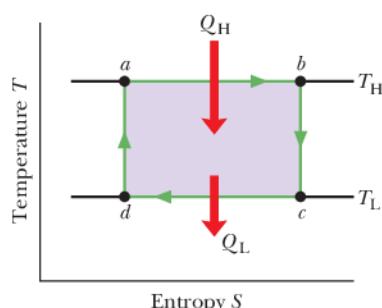
**Entropy Changes** In a Carnot engine, there are *two* (and only two) reversible energy transfers as heat, and thus two changes in the entropy of the working substance—one at temperature  $T_H$  and one at  $T_L$ . The net entropy change per cycle is then

$$\Delta S = \Delta S_H + \Delta S_L = \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L}. \quad (20-9)$$

Here  $\Delta S_H$  is positive because energy  $|Q_H|$  is *added to* the working substance as heat (an increase in entropy) and  $\Delta S_L$  is negative because energy  $|Q_L|$  is *removed from* the working substance as heat (a decrease in entropy). Because entropy is a state function, we must have  $\Delta S = 0$  for a complete cycle. Putting  $\Delta S = 0$  in Eq. 20-9 requires that

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}. \quad (20-10)$$

Note that, because  $T_H > T_L$ , we must have  $|Q_H| > |Q_L|$ ; that is, more energy is



**Figure 20-10** The Carnot cycle of Fig. 20-9 plotted on a temperature–entropy diagram. During processes *ab* and *cd* the temperature remains constant. During processes *bc* and *da* the entropy remains constant.

extracted as heat from the high-temperature reservoir than is delivered to the low-temperature reservoir.

We shall now derive an expression for the efficiency of a Carnot engine.

### Efficiency of a Carnot Engine

The purpose of any engine is to transform as much of the extracted energy  $Q_H$  into work as possible. We measure its success in doing so by its **thermal efficiency**  $\varepsilon$ , defined as the work the engine does per cycle (“energy we get”) divided by the energy it absorbs as heat per cycle (“energy we pay for”):

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|} \quad (\text{efficiency, any engine}). \quad (20-11)$$

For a Carnot engine we can substitute for  $W$  from Eq. 20-8 to write Eq. 20-11 as

$$\varepsilon_C = \frac{|Q_H| - |Q_L|}{Q_H} = 1 - \frac{|Q_L|}{|Q_H|}. \quad (20-12)$$

Using Eq. 20-10 we can write this as

$$\varepsilon_C = 1 - \frac{T_L}{T_H} \quad (\text{efficiency, Carnot engine}), \quad (20-13)$$

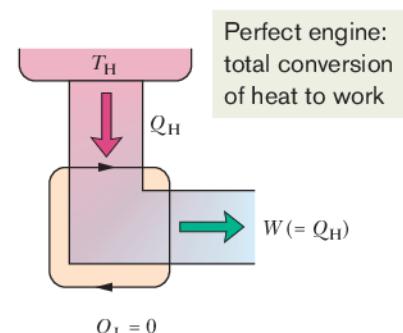
where the temperatures  $T_L$  and  $T_H$  are in kelvins. Because  $T_L < T_H$ , the Carnot engine necessarily has a thermal efficiency less than unity—that is, less than 100%. This is indicated in Fig. 20-8, which shows that only part of the energy extracted as heat from the high-temperature reservoir is available to do work, and the rest is delivered to the low-temperature reservoir. We shall show in Module 20-3 that no real engine can have a thermal efficiency greater than that calculated from Eq. 20-13.

Inventors continually try to improve engine efficiency by reducing the energy  $|Q_L|$  that is “thrown away” during each cycle. The inventor’s dream is to produce the *perfect engine*, diagrammed in Fig. 20-11, in which  $|Q_L|$  is reduced to zero and  $|Q_H|$  is converted completely into work. Such an engine on an ocean liner, for example, could extract energy as heat from the water and use it to drive the propellers, with no fuel cost. An automobile fitted with such an engine could extract energy as heat from the surrounding air and use it to drive the car, again with no fuel cost. Alas, a perfect engine is only a dream: Inspection of Eq. 20-13 shows that we can achieve 100% engine efficiency (that is,  $\varepsilon = 1$ ) only if  $T_L = 0$  or  $T_H \rightarrow \infty$ , impossible requirements. Instead, experience gives the following alternative version of the second law of thermodynamics, which says in short, *there are no perfect engines*:

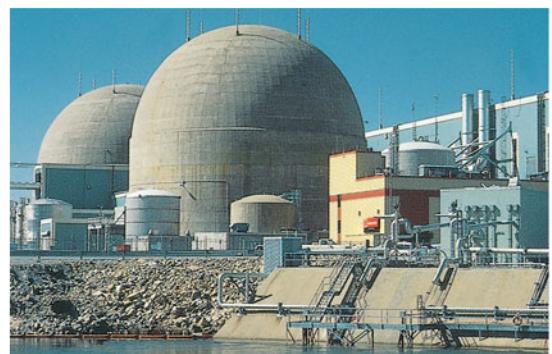


No series of processes is possible whose sole result is the transfer of energy as heat from a thermal reservoir and the complete conversion of this energy to work.

To summarize: The thermal efficiency given by Eq. 20-13 applies only to Carnot engines. Real engines, in which the processes that form the engine cycle are not reversible, have lower efficiencies. If your car were powered by a Carnot engine, it would have an efficiency of about 55% according to Eq. 20-13; its actual efficiency is probably about 25%. A nuclear power plant (Fig. 20-12), taken in its entirety, is an engine. It extracts energy as heat from a reactor core, does work by means of a turbine, and discharges energy as heat to a nearby river. If the power plant operated as a Carnot engine, its efficiency would be about 40%; its actual efficiency is about 30%. In designing engines of any type, there is simply no way to beat the efficiency limitation imposed by Eq. 20-13.

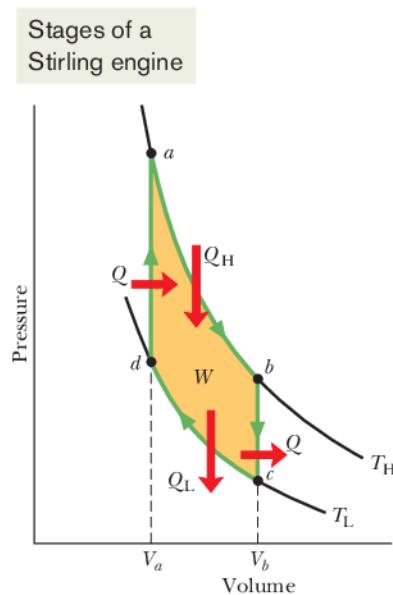


**Figure 20-11** The elements of a perfect engine—that is, one that converts heat  $Q_H$  from a high-temperature reservoir directly to work  $W$  with 100% efficiency.



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**Figure 20-12** The North Anna nuclear power plant near Charlottesville, Virginia, which generates electric energy at the rate of 900 MW. At the same time, by design, it discards energy into the nearby river at the rate of 2100 MW. This plant and all others like it throw away more energy than they deliver in useful form. They are real counterparts of the ideal engine of Fig. 20-8.



**Figure 20-13** A  $p$ - $V$  plot for the working substance of an ideal Stirling engine, with the working substance assumed for convenience to be an ideal gas.

### Stirling Engine

Equation 20-13 applies not to all ideal engines but only to those that can be represented as in Fig. 20-9—that is, to Carnot engines. For example, Fig. 20-13 shows the operating cycle of an ideal **Stirling engine**. Comparison with the Carnot cycle of Fig. 20-9 shows that each engine has isothermal heat transfers at temperatures  $T_H$  and  $T_L$ . However, the two isotherms of the Stirling engine cycle are connected, not by adiabatic processes as for the Carnot engine but by constant-volume processes. To increase the temperature of a gas at constant volume reversibly from  $T_L$  to  $T_H$  (process  $da$  of Fig. 20-13) requires a transfer of energy as heat to the working substance from a thermal reservoir whose temperature can be varied smoothly between those limits. Also, a reverse transfer is required in process  $bc$ . Thus, reversible heat transfers (and corresponding entropy changes) occur in all four of the processes that form the cycle of a Stirling engine, not just two processes as in a Carnot engine. Thus, the derivation that led to Eq. 20-13 does not apply to an ideal Stirling engine. More important, the efficiency of an ideal Stirling engine is lower than that of a Carnot engine operating between the same two temperatures. Real Stirling engines have even lower efficiencies.

The Stirling engine was developed in 1816 by Robert Stirling. This engine, long neglected, is now being developed for use in automobiles and spacecraft. A Stirling engine delivering 5000 hp (3.7 MW) has been built. Because they are quiet, Stirling engines are used on some military submarines.



### Checkpoint 3

Three Carnot engines operate between reservoir temperatures of (a) 400 and 500 K, (b) 600 and 800 K, and (c) 400 and 600 K. Rank the engines according to their thermal efficiencies, greatest first.



### Sample Problem 20.03 Carnot engine, efficiency, power, entropy changes

Imagine a Carnot engine that operates between the temperatures  $T_H = 850$  K and  $T_L = 300$  K. The engine performs 1200 J of work each cycle, which takes 0.25 s.

(a) What is the efficiency of this engine?

#### KEY IDEA

The efficiency  $\varepsilon$  of a Carnot engine depends only on the ratio  $T_L/T_H$  of the temperatures (in kelvins) of the thermal reservoirs to which it is connected.

**Calculation:** Thus, from Eq. 20-13, we have

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{850 \text{ K}} = 0.647 \approx 65\%. \quad (\text{Answer})$$

(b) What is the average power of this engine?

#### KEY IDEA

The average power  $P$  of an engine is the ratio of the work  $W$  it does per cycle to the time  $t$  that each cycle takes.

**Calculation:** For this Carnot engine, we find

$$P = \frac{W}{t} = \frac{1200 \text{ J}}{0.25 \text{ s}} = 4800 \text{ W} = 4.8 \text{ kW}. \quad (\text{Answer})$$

(c) How much energy  $|Q_H|$  is extracted as heat from the high-temperature reservoir every cycle?

#### KEY IDEA

The efficiency  $\varepsilon$  is the ratio of the work  $W$  that is done per cycle to the energy  $|Q_H|$  that is extracted as heat from the high-temperature reservoir per cycle ( $\varepsilon = W/|Q_H|$ ).

**Calculation:** Here we have

$$|Q_H| = \frac{W}{\varepsilon} = \frac{1200 \text{ J}}{0.647} = 1855 \text{ J}. \quad (\text{Answer})$$

(d) How much energy  $|Q_L|$  is delivered as heat to the low-temperature reservoir every cycle?

#### KEY IDEA

For a Carnot engine, the work  $W$  done per cycle is equal to the difference in the energy transfers as heat:  $|Q_H| - |Q_L|$ , as in Eq. 20-8.

**Calculation:** Thus, we have

$$|Q_L| = |Q_H| - W \\ = 1855 \text{ J} - 1200 \text{ J} = 655 \text{ J}. \quad (\text{Answer})$$

- (e) By how much does the entropy of the working substance change as a result of the energy transferred to it from the high-temperature reservoir? From it to the low-temperature reservoir?

### KEY IDEA

The entropy change  $\Delta S$  during a transfer of energy as heat  $Q$  at constant temperature  $T$  is given by Eq. 20-2 ( $\Delta S = Q/T$ ).

**Calculations:** Thus, for the *positive* transfer of energy  $Q_H$  from the high-temperature reservoir at  $T_H$ , the change in the

### Sample Problem 20.04 Impossibly efficient engine

An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and freezing points of water. Is this possible?

### KEY IDEA

The efficiency of a real engine must be less than the efficiency of a Carnot engine operating between the same two temperatures.



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entropy of the working substance is

$$\Delta S_H = \frac{Q_H}{T_H} = \frac{1855 \text{ J}}{850 \text{ K}} = +2.18 \text{ J/K.} \quad (\text{Answer})$$

Similarly, for the *negative* transfer of energy  $Q_L$  to the low-temperature reservoir at  $T_L$ , we have

$$\Delta S_L = \frac{Q_L}{T_L} = \frac{-655 \text{ J}}{300 \text{ K}} = -2.18 \text{ J/K.} \quad (\text{Answer})$$

Note that the net entropy change of the working substance for one cycle is zero, as we discussed in deriving Eq. 20-10.

**Calculation:** From Eq. 20-13, we find that the efficiency of a Carnot engine operating between the boiling and freezing points of water is

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{(0 + 273) \text{ K}}{(100 + 273) \text{ K}} = 0.268 \approx 27\%.$$

Thus, for the given temperatures, the claimed efficiency of 75% for a real engine (with its irreversible processes and wasteful energy transfers) is impossible.

## 20-3 REFRIGERATORS AND REAL ENGINES

### Learning Objectives

After reading this module, you should be able to . . .

**20.16** Identify that a refrigerator is a device that uses work to transfer energy from a low-temperature reservoir to a high-temperature reservoir, and that an ideal refrigerator is one that does this with reversible processes and no wasteful losses.

**20.17** Sketch a  $p$ - $V$  diagram for the cycle of a Carnot refrigerator, indicating the direction of cycling, the nature of the processes involved, the work done during each process (including algebraic sign), the net work done in the cycle,

### Key Ideas

● A refrigerator is a device that, operating in a cycle, has work  $W$  done on it as it extracts energy  $|Q_L|$  as heat from a low-temperature reservoir. The coefficient of performance  $K$  of a refrigerator is defined as

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}.$$

● A Carnot refrigerator is a Carnot engine operating in reverse. Its coefficient of performance is

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|} = \frac{T_L}{T_H - T_L}.$$

and the heat transferred during each process (including algebraic sign).

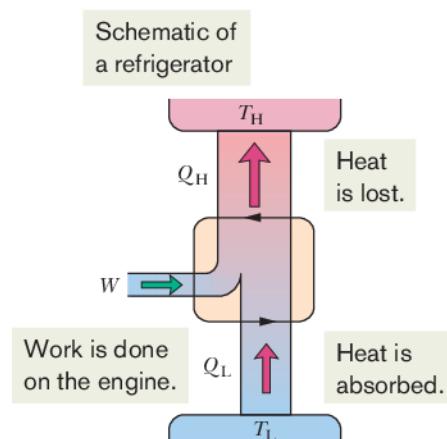
**20.18** Apply the relationship between the coefficient of performance  $K$  and the heat exchanges with the reservoirs and the temperatures of the reservoirs.

**20.19** Identify that there is no ideal refrigerator in which all of the energy extracted from the low-temperature reservoir is transferred to the high-temperature reservoir.

**20.20** Identify that the efficiency of a real engine is less than that of the ideal Carnot engine.

● A perfect refrigerator is an entirely imaginary refrigerator in which energy extracted as heat from the low-temperature reservoir is somehow converted completely to heat discharged to the high-temperature reservoir without any need for work.

● A perfect refrigerator would violate the second law of thermodynamics, which can be restated as follows: No series of processes is possible whose sole result is the transfer of energy as heat from a reservoir at a given temperature to a reservoir at a higher temperature (without work being involved).



**Figure 20-14** The elements of a refrigerator. The two black arrowheads on the central loop suggest the working substance operating in a cycle, as if on a  $p$ - $V$  plot. Energy is transferred as heat  $Q_L$  to the working substance from the low-temperature reservoir. Energy is transferred as heat  $Q_H$  to the high-temperature reservoir from the working substance. Work  $W$  is done on the refrigerator (on the working substance) by something in the environment.

## Entropy in the Real World: Refrigerators

A **refrigerator** is a device that uses work in order to transfer energy from a low-temperature reservoir to a high-temperature reservoir as the device continuously repeats a set series of thermodynamic processes. In a household refrigerator, for example, work is done by an electrical compressor to transfer energy from the food storage compartment (a low-temperature reservoir) to the room (a high-temperature reservoir).

Air conditioners and heat pumps are also refrigerators. For an air conditioner, the low-temperature reservoir is the room that is to be cooled and the high-temperature reservoir is the warmer outdoors. A heat pump is an air conditioner that can be operated in reverse to heat a room; the room is the high-temperature reservoir, and heat is transferred to it from the cooler outdoors.

Let us consider an *ideal refrigerator*:



In an ideal refrigerator, all processes are reversible and no wasteful energy transfers occur as a result of, say, friction and turbulence.

Figure 20-14 shows the basic elements of an ideal refrigerator. Note that its operation is the reverse of how the Carnot engine of Fig. 20-8 operates. In other words, all the energy transfers, as either heat or work, are reversed from those of a Carnot engine. We can call such an ideal refrigerator a **Carnot refrigerator**.

The designer of a refrigerator would like to extract as much energy  $|Q_L|$  as possible from the low-temperature reservoir (what we want) for the least amount of work  $|W|$  (what we pay for). A measure of the efficiency of a refrigerator, then, is

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|} \quad (\text{coefficient of performance, any refrigerator}), \quad (20-14)$$

where  $K$  is called the *coefficient of performance*. For a Carnot refrigerator, the first law of thermodynamics gives  $|W| = |Q_H| - |Q_L|$ , where  $|Q_H|$  is the magnitude of the energy transferred as heat to the high-temperature reservoir. Equation 20-14 then becomes

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|}. \quad (20-15)$$

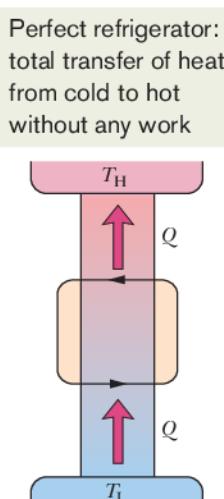
Because a Carnot refrigerator is a Carnot engine operating in reverse, we can combine Eq. 20-10 with Eq. 20-15; after some algebra we find

$$K_C = \frac{T_L}{T_H - T_L} \quad (\text{coefficient of performance, Carnot refrigerator}). \quad (20-16)$$

For typical room air conditioners,  $K \approx 2.5$ . For household refrigerators,  $K \approx 5$ . Perversely, the value of  $K$  is higher the closer the temperatures of the two reservoirs are to each other. That is why heat pumps are more effective in temperate climates than in very cold climates.

It would be nice to own a refrigerator that did not require some input of work—that is, one that would run without being plugged in. Figure 20-15 represents another “inventor’s dream,” a *perfect refrigerator* that transfers energy as heat  $Q$  from a cold reservoir to a warm reservoir without the need for work. Because the unit operates in cycles, the entropy of the working substance does not change during a complete cycle. The entropies of the two reservoirs, however, do change: The entropy change for the cold reservoir is  $-|Q|/T_L$ , and that for the warm reservoir is  $+|Q|/T_H$ . Thus, the net entropy change for the entire system is

$$\Delta S = -\frac{|Q|}{T_L} + \frac{|Q|}{T_H}.$$



**Figure 20-15** The elements of a perfect refrigerator—that is, one that transfers energy from a low-temperature reservoir to a high-temperature reservoir without any input of work.

Because  $T_H > T_L$ , the right side of this equation is negative and thus the net change in entropy per cycle for the closed system *refrigerator + reservoirs* is also negative. Because such a decrease in entropy violates the second law of thermodynamics (Eq. 20-5), a perfect refrigerator does not exist. (If you want your refrigerator to operate, you must plug it in.)

Here, then, is another way to state the second law of thermodynamics:



No series of processes is possible whose sole result is the transfer of energy as heat from a reservoir at a given temperature to a reservoir at a higher temperature.

In short, *there are no perfect refrigerators*.



#### Checkpoint 4

You wish to increase the coefficient of performance of an ideal refrigerator. You can do so by (a) running the cold chamber at a slightly higher temperature, (b) running the cold chamber at a slightly lower temperature, (c) moving the unit to a slightly warmer room, or (d) moving it to a slightly cooler room. The magnitudes of the temperature changes are to be the same in all four cases. List the changes according to the resulting coefficients of performance, greatest first.

## The Efficiencies of Real Engines

Let  $\varepsilon_C$  be the efficiency of a Carnot engine operating between two given temperatures. Here we prove that no real engine operating between those temperatures can have an efficiency greater than  $\varepsilon_C$ . If it could, the engine would violate the second law of thermodynamics.

Let us assume that an inventor, working in her garage, has constructed an engine *X*, which she claims has an efficiency  $\varepsilon_X$  that is greater than  $\varepsilon_C$ :

$$\varepsilon_X > \varepsilon_C \quad (\text{a claim}). \quad (20-17)$$

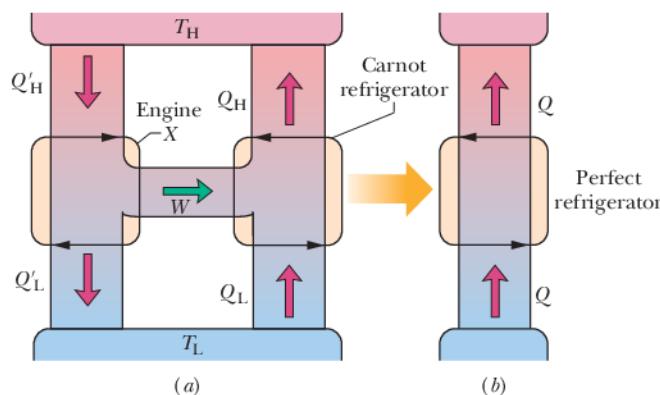
Let us couple engine *X* to a Carnot refrigerator, as in Fig. 20-16a. We adjust the strokes of the Carnot refrigerator so that the work it requires per cycle is just equal to that provided by engine *X*. Thus, no (external) work is performed on or by the combination *engine + refrigerator* of Fig. 20-16a, which we take as our system.

If Eq. 20-17 is true, from the definition of efficiency (Eq. 20-11), we must have

$$\frac{|W|}{|Q'_H|} > \frac{|W|}{|Q_H|},$$

where the prime refers to engine *X* and the right side of the inequality is the efficiency of the Carnot refrigerator when it operates as an engine. This inequality requires that

$$|Q_H| > |Q'_H|. \quad (20-18)$$



**Figure 20-16** (a) Engine *X* drives a Carnot refrigerator. (b) If, as claimed, engine *X* is more efficient than a Carnot engine, then the combination shown in (a) is equivalent to the perfect refrigerator shown here. This violates the second law of thermodynamics, so we conclude that engine *X* *cannot* be more efficient than a Carnot engine.

Because the work done by engine  $X$  is equal to the work done on the Carnot refrigerator, we have, from the first law of thermodynamics as given by Eq. 20-8,

$$|Q_H| - |Q_L| = |Q'_H| - |Q'_L|,$$

which we can write as

$$|Q_H| - |Q'_H| = |Q_L| - |Q'_L| = Q. \quad (20-19)$$

Because of Eq. 20-18, the quantity  $Q$  in Eq. 20-19 must be positive.

Comparison of Eq. 20-19 with Fig. 20-16 shows that the net effect of engine  $X$  and the Carnot refrigerator working in combination is to transfer energy  $Q$  as heat from a low-temperature reservoir to a high-temperature reservoir without the requirement of work. Thus, the combination acts like the perfect refrigerator of Fig. 20-15, whose existence is a violation of the second law of thermodynamics.

Something must be wrong with one or more of our assumptions, and it can only be Eq. 20-17. We conclude that *no real engine can have an efficiency greater than that of a Carnot engine when both engines work between the same two temperatures*. At most, the real engine can have an efficiency equal to that of a Carnot engine. In that case, the real engine *is* a Carnot engine.

## 20-4 A STATISTICAL VIEW OF ENTROPY

### Learning Objectives

After reading this module, you should be able to . . .

**20.21** Explain what is meant by the configurations of a system of molecules.

**20.22** Calculate the multiplicity of a given configuration.

**20.23** Identify that all microstates are equally probable but

the configurations with more microstates are more probable than the other configurations.

**20.24** Apply Boltzmann's entropy equation to calculate the entropy associated with a multiplicity.

### Key Ideas

- The entropy of a system can be defined in terms of the possible distributions of its molecules. For identical molecules, each possible distribution of molecules is called a microstate of the system. All equivalent microstates are grouped into a configuration of the system. The number of microstates in a configuration is the multiplicity  $W$  of the configuration.

- For a system of  $N$  molecules that may be distributed between the two halves of a box, the multiplicity is given by

$$W = \frac{N!}{n_1! n_2!},$$

in which  $n_1$  is the number of molecules in one half of the box and  $n_2$  is the number in the other half. A basic assumption of statistical mechanics is that all the microstates are equally probable.

Thus, configurations with a large multiplicity occur most often. When  $N$  is very large (say,  $N = 10^{22}$  molecules or more), the molecules are nearly always in the configuration in which  $n_1 = n_2$ .

- The multiplicity  $W$  of a configuration of a system and the entropy  $S$  of the system in that configuration are related by Boltzmann's entropy equation:

$$S = k \ln W,$$

where  $k = 1.38 \times 10^{-23}$  J/K is the Boltzmann constant.

- When  $N$  is very large (the usual case), we can approximate  $\ln N!$  with Stirling's approximation:

$$\ln N! \approx N(\ln N) - N.$$

### A Statistical View of Entropy

In Chapter 19 we saw that the macroscopic properties of gases can be explained in terms of their microscopic, or molecular, behavior. Such explanations are part of a study called **statistical mechanics**. Here we shall focus our attention on a single problem, one involving the distribution of gas molecules between the two halves of an insulated box. This problem is reasonably simple to analyze, and it allows us to use statistical mechanics to calculate the entropy change for the free expansion of an ideal gas. You will see that statistical mechanics leads to the same entropy change as we would find using thermodynamics.

Figure 20-17 shows a box that contains six identical (and thus indistinguishable) molecules of a gas. At any instant, a given molecule will be in either the left or the right half of the box; because the two halves have equal volumes, the molecule has the same likelihood, or probability, of being in either half.

Table 20-1 shows the seven possible *configurations* of the six molecules, each configuration labeled with a Roman numeral. For example, in configuration I, all six molecules are in the left half of the box ( $n_1 = 6$ ) and none are in the right half ( $n_2 = 0$ ). We see that, in general, a given configuration can be achieved in a number of different ways. We call these different arrangements of the molecules *microstates*. Let us see how to calculate the number of microstates that correspond to a given configuration.

Suppose we have  $N$  molecules, distributed with  $n_1$  molecules in one half of the box and  $n_2$  in the other. (Thus  $n_1 + n_2 = N$ .) Let us imagine that we distribute the molecules “by hand,” one at a time. If  $N = 6$ , we can select the first molecule in six independent ways; that is, we can pick any one of the six molecules. We can pick the second molecule in five ways, by picking any one of the remaining five molecules; and so on. The total number of ways in which we can select all six molecules is the product of these independent ways, or  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ . In mathematical shorthand we write this product as  $6! = 720$ , where  $6!$  is pronounced “six factorial.” Your hand calculator can probably calculate factorials. For later use you will need to know that  $0! = 1$ . (Check this on your calculator.)

However, because the molecules are indistinguishable, these 720 arrangements are not all different. In the case that  $n_1 = 4$  and  $n_2 = 2$  (which is configuration III in Table 20-1), for example, the order in which you put four molecules in one half of the box does not matter, because after you have put all four in, there is no way that you can tell the order in which you did so. The number of ways in which you can order the four molecules is  $4! = 24$ . Similarly, the number of ways in which you can order two molecules for the other half of the box is simply  $2! = 2$ . To get the number of *different* arrangements that lead to the (4, 2) split of configuration III, we must divide 720 by 24 and also by 2. We call the resulting quantity, which is the number of microstates that correspond to a given configuration, the *multiplicity W* of that configuration. Thus, for configuration III,

$$W_{\text{III}} = \frac{6!}{4! 2!} = \frac{720}{24 \times 2} = 15.$$

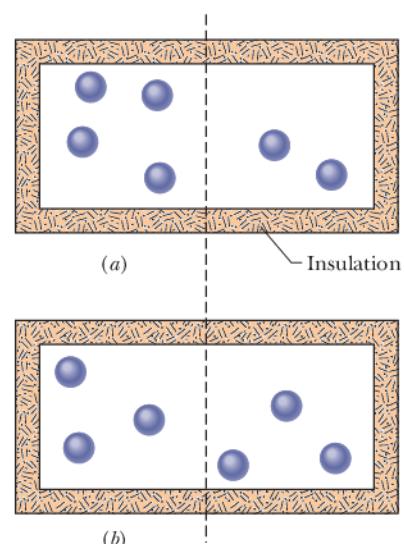
Thus, Table 20-1 tells us there are 15 independent microstates that correspond to configuration III. Note that, as the table also tells us, the total number of microstates for six molecules distributed over the seven configurations is 64.

Extrapolating from six molecules to the general case of  $N$  molecules, we have

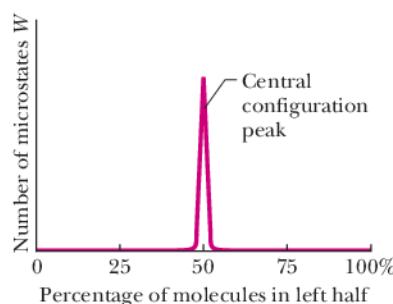
$$W = \frac{N!}{n_1! n_2!} \quad (\text{multiplicity of configuration}). \quad (20-20)$$

**Table 20-1 Six Molecules in a Box**

Configuration Label	$n_1$	$n_2$	Multiplicity $W$ (number of microstates)	Calculation of $W$ (Eq. 20-20)	Entropy $10^{-23} \text{ J/K}$ (Eq. 20-21)
I	6	0	1	$6!/(6! 0!) = 1$	0
II	5	1	6	$6!/(5! 1!) = 6$	2.47
III	4	2	15	$6!/(4! 2!) = 15$	3.74
IV	3	3	20	$6!/(3! 3!) = 20$	4.13
V	2	4	15	$6!/(2! 4!) = 15$	3.74
VI	1	5	6	$6!/(1! 5!) = 6$	2.47
VII	0	6	1	$6!/(0! 6!) = 1$	0
Total = 64					



**Figure 20-17** An insulated box contains six gas molecules. Each molecule has the same probability of being in the left half of the box as in the right half. The arrangement in (a) corresponds to configuration III in Table 20-1, and that in (b) corresponds to configuration IV.



**Figure 20-18** For a large number of molecules in a box, a plot of the number of microstates that require various percentages of the molecules to be in the left half of the box. Nearly all the microstates correspond to an approximately equal sharing of the molecules between the two halves of the box; those microstates form the *central configuration peak* on the plot. For  $N \approx 10^{22}$ , the central configuration peak is much too narrow to be drawn on this plot.

You should verify the multiplicities for all the configurations in Table 20-1. The basic assumption of statistical mechanics is that



All microstates are equally probable.

In other words, if we were to take a great many snapshots of the six molecules as they jostle around in the box of Fig. 20-17 and then count the number of times each microstate occurred, we would find that all 64 microstates would occur equally often. Thus the system will spend, on average, the same amount of time in each of the 64 microstates.

Because all microstates are equally probable but different configurations have different numbers of microstates, the configurations are *not* all equally probable. In Table 20-1 configuration IV, with 20 microstates, is the *most probable configuration*, with a probability of  $20/64 = 0.313$ . This result means that the system is in configuration IV 31.3% of the time. Configurations I and VII, in which all the molecules are in one half of the box, are the least probable, each with a probability of  $1/64 = 0.016$  or 1.6%. It is not surprising that the most probable configuration is the one in which the molecules are evenly divided between the two halves of the box, because that is what we expect at thermal equilibrium. However, it is surprising that there is *any* probability, however small, of finding all six molecules clustered in half of the box, with the other half empty.

For large values of  $N$  there are extremely large numbers of microstates, but nearly all the microstates belong to the configuration in which the molecules are divided equally between the two halves of the box, as Fig. 20-18 indicates. Even though the measured temperature and pressure of the gas remain constant, the gas is churning away endlessly as its molecules “visit” all probable microstates with equal probability. However, because so few microstates lie outside the very narrow central configuration peak of Fig. 20-18, we might as well assume that the gas molecules are always divided equally between the two halves of the box. As we shall see, this is the configuration with the greatest entropy.



### Sample Problem 20.05 Microstates and multiplicity

Suppose that there are 100 indistinguishable molecules in the box of Fig. 20-17. How many microstates are associated with the configuration  $n_1 = 50$  and  $n_2 = 50$ , and with the configuration  $n_1 = 100$  and  $n_2 = 0$ ? Interpret the results in terms of the relative probabilities of the two configurations.

#### KEY IDEA

The multiplicity  $W$  of a configuration of indistinguishable molecules in a closed box is the number of independent microstates with that configuration, as given by Eq. 20-20.

**Calculations:** Thus, for the  $(n_1, n_2)$  configuration  $(50, 50)$ ,

$$\begin{aligned} W &= \frac{N!}{n_1! n_2!} = \frac{100!}{50! 50!} \\ &= \frac{9.33 \times 10^{157}}{(3.04 \times 10^{64})(3.04 \times 10^{64})} \\ &= 1.01 \times 10^{29}. \end{aligned} \quad (\text{Answer})$$

Similarly, for the configuration  $(100, 0)$ , we have

$$W = \frac{N!}{n_1! n_2!} = \frac{100!}{100! 0!} = \frac{1}{0!} = \frac{1}{1} = 1. \quad (\text{Answer})$$

**The meaning:** Thus, a 50–50 distribution is more likely than a 100–0 distribution by the enormous factor of about  $1 \times 10^{29}$ . If you could count, at one per nanosecond, the number of microstates that correspond to the 50–50 distribution, it would take you about  $3 \times 10^{12}$  years, which is about 200 times longer than the age of the universe. Keep in mind that the 100 molecules used in this sample problem is a very small number. Imagine what these calculated probabilities would be like for a mole of molecules, say about  $N = 10^{24}$ . Thus, you need never worry about suddenly finding all the air molecules clustering in one corner of your room, with you gasping for air in another corner. So, you can breathe easy because of the physics of entropy.



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## Probability and Entropy

In 1877, Austrian physicist Ludwig Boltzmann (the Boltzmann of Boltzmann's constant  $k$ ) derived a relationship between the entropy  $S$  of a configuration of a gas and the multiplicity  $W$  of that configuration. That relationship is

$$S = k \ln W \quad (\text{Boltzmann's entropy equation}). \quad (20-21)$$

This famous formula is engraved on Boltzmann's tombstone.

It is natural that  $S$  and  $W$  should be related by a logarithmic function. The total entropy of two systems is the *sum* of their separate entropies. The probability of occurrence of two independent systems is the *product* of their separate probabilities. Because  $\ln ab = \ln a + \ln b$ , the logarithm seems the logical way to connect these quantities.

Table 20-1 displays the entropies of the configurations of the six-molecule system of Fig. 20-17, computed using Eq. 20-21. Configuration IV, which has the greatest multiplicity, also has the greatest entropy.

When you use Eq. 20-20 to calculate  $W$ , your calculator may signal "OVERFLOW" if you try to find the factorial of a number greater than a few hundred. Instead, you can use **Stirling's approximation** for  $\ln N!$ :

$$\ln N! \approx N(\ln N) - N \quad (\text{Stirling's approximation}). \quad (20-22)$$

The Stirling of this approximation was an English mathematician and not the Robert Stirling of engine fame.



### Checkpoint 5

A box contains 1 mol of a gas. Consider two configurations: (a) each half of the box contains half the molecules and (b) each third of the box contains one-third of the molecules. Which configuration has more microstates?

### Sample Problem 20.06 Entropy change of free expansion using microstates

In Sample Problem 20.01, we showed that when  $n$  moles of an ideal gas doubles its volume in a free expansion, the entropy increase from the initial state  $i$  to the final state  $f$  is  $S_f - S_i = nR \ln 2$ . Derive this increase in entropy by using statistical mechanics.

#### KEY IDEA

We can relate the entropy  $S$  of any given configuration of the molecules in the gas to the multiplicity  $W$  of microstates for that configuration, using Eq. 20-21 ( $S = k \ln W$ ).

**Calculations:** We are interested in two configurations: the final configuration  $f$  (with the molecules occupying the full volume of their container in Fig. 20-1b) and the initial configuration  $i$  (with the molecules occupying the left half of the container). Because the molecules are in a closed container, we can calculate the multiplicity  $W$  of their microstates with Eq. 20-20. Here we have  $N$  molecules in the  $n$  moles of the gas. Initially, with the molecules all in the left

half of the container, their  $(n_1, n_2)$  configuration is  $(N, 0)$ . Then, Eq. 20-20 gives their multiplicity as

$$W_i = \frac{N!}{N! 0!} = 1.$$

Finally, with the molecules spread through the full volume, their  $(n_1, n_2)$  configuration is  $(N/2, N/2)$ . Then, Eq. 20-20 gives their multiplicity as

$$W_f = \frac{N!}{(N/2)! (N/2)!}.$$

From Eq. 20-21, the initial and final entropies are

$$S_i = k \ln W_i = k \ln 1 = 0$$

and

$$S_f = k \ln W_f = k \ln(N!) - 2k \ln[(N/2)!]. \quad (20-23)$$

In writing Eq. 20-23, we have used the relation

$$\ln \frac{a}{b^2} = \ln a - 2 \ln b.$$

Now, applying Eq. 20-22 to evaluate Eq. 20-23, we find that

$$\begin{aligned} S_f &= k \ln(N!) - 2k \ln[(N/2)!] \\ &= k[N(\ln N) - N] - 2k[(N/2)\ln(N/2) - (N/2)] \\ &= k[N(\ln N) - N - N\ln(N/2) + N] \\ &= k[N(\ln N) - N(\ln N - \ln 2)] = Nk \ln 2. \end{aligned} \quad (20-24)$$

From Eq. 19-8 we can substitute  $nR$  for  $Nk$ , where  $R$  is the universal gas constant. Equation 20-24 then becomes

$$S_f = nR \ln 2.$$

The change in entropy from the initial state to the final is



Additional examples, video, and practice available at *WileyPLUS*

thus

$$\begin{aligned} S_f - S_i &= nR \ln 2 - 0 \\ &= nR \ln 2, \end{aligned} \quad (\text{Answer})$$

which is what we set out to show. In the first sample problem of this chapter we calculated this entropy increase for a free expansion with thermodynamics by finding an equivalent reversible process and calculating the entropy change for *that* process in terms of temperature and heat transfer. In this sample problem, we calculate the same increase in entropy with statistical mechanics using the fact that the system consists of molecules. In short, the two, very different approaches give the same answer.

## Review & Summary

**One-Way Processes** An **irreversible process** is one that cannot be reversed by means of small changes in the environment. The direction in which an irreversible process proceeds is set by the *change in entropy*  $\Delta S$  of the system undergoing the process. Entropy  $S$  is a *state property* (or *state function*) of the system; that is, it depends only on the state of the system and not on the way in which the system reached that state. The *entropy postulate* states (in part): *If an irreversible process occurs in a closed system, the entropy of the system always increases*.

**Calculating Entropy Change** The **entropy change**  $\Delta S$  for an irreversible process that takes a system from an initial state  $i$  to a final state  $f$  is exactly equal to the entropy change  $\Delta S$  for *any reversible process* that takes the system between those same two states. We can compute the latter (but not the former) with

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}. \quad (20-1)$$

Here  $Q$  is the energy transferred as heat to or from the system during the process, and  $T$  is the temperature of the system in kelvins during the process.

For a reversible isothermal process, Eq. 20-1 reduces to

$$\Delta S = S_f - S_i = \frac{Q}{T}. \quad (20-2)$$

When the temperature change  $\Delta T$  of a system is small relative to the temperature (in kelvins) before and after the process, the entropy change can be approximated as

$$\Delta S = S_f - S_i \approx \frac{Q}{T_{\text{avg}}}, \quad (20-3)$$

where  $T_{\text{avg}}$  is the system's average temperature during the process.

When an ideal gas changes reversibly from an initial state with temperature  $T_i$  and volume  $V_i$  to a final state with temperature  $T_f$  and volume  $V_f$ , the change  $\Delta S$  in the entropy of the gas is

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}. \quad (20-4)$$

**The Second Law of Thermodynamics** This law, which is an extension of the entropy postulate, states: *If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes. It never decreases.* In equation form,

$$\Delta S \geq 0. \quad (20-5)$$

**Engines** An **engine** is a device that, operating in a cycle, extracts energy as heat  $|Q_H|$  from a high-temperature reservoir and does a certain amount of work  $|W|$ . The *efficiency*  $\varepsilon$  of any engine is defined as

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}. \quad (20-11)$$

In an **ideal engine**, all processes are reversible and no wasteful energy transfers occur due to, say, friction and turbulence. A **Carnot engine** is an ideal engine that follows the cycle of Fig. 20-9. Its efficiency is

$$\varepsilon_C = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H}, \quad (20-12, 20-13)$$

in which  $T_H$  and  $T_L$  are the temperatures of the high- and low-temperature reservoirs, respectively. Real engines always have an efficiency lower than that given by Eq. 20-13. Ideal engines that are not Carnot engines also have lower efficiencies.

A **perfect engine** is an imaginary engine in which energy extracted as heat from the high-temperature reservoir is converted completely to work. Such an engine would violate the second law of thermodynamics, which can be restated as follows: No series of processes is possible whose sole result is the absorption of energy as heat from a thermal reservoir and the complete conversion of this energy to work.

**Refrigerators** A **refrigerator** is a device that, operating in a cycle, has work  $W$  done on it as it extracts energy  $|Q_L|$  as heat from a low-temperature reservoir. The coefficient of performance  $K$  of a refrigerator is defined as

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}. \quad (20-14)$$

A **Carnot refrigerator** is a Carnot engine operating in reverse.

For a Carnot refrigerator, Eq. 20-14 becomes

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|} = \frac{T_L}{T_H - T_L}. \quad (20-15, 20-16)$$

A *perfect refrigerator* is an imaginary refrigerator in which energy extracted as heat from the low-temperature reservoir is converted completely to heat discharged to the high-temperature reservoir, without any need for work. Such a refrigerator would violate the second law of thermodynamics, which can be restated as follows: No series of processes is possible whose sole result is the transfer of energy as heat from a reservoir at a given temperature to a reservoir at a higher temperature.

**Entropy from a Statistical View** The entropy of a system can be defined in terms of the possible distributions of its molecules. For identical molecules, each possible distribution of molecules is called a **microstate** of the system. All equivalent microstates are grouped into

## Questions

- 1 Point *i* in Fig. 20-19 represents the initial state of an ideal gas at temperature  $T$ . Taking algebraic signs into account, rank the entropy changes that the gas undergoes as it moves, successively and reversibly, from point *i* to points *a*, *b*, *c*, and *d*, greatest first.

- 2 In four experiments, blocks *A* and *B*, starting at different initial temperatures, were brought together in an insulating box and allowed to reach a common final temperature. The entropy changes for the blocks in the four experiments had the following values (in joules per kelvin), but not necessarily in the order given. Determine which values for *A* go with which values for *B*.

Block	Values			
<i>A</i>	8	5	3	9
<i>B</i>	-3	-8	-5	-2

- 3 A gas, confined to an insulated cylinder, is compressed adiabatically to half its volume. Does the entropy of the gas increase, decrease, or remain unchanged during this process?

- 4 An ideal monatomic gas at initial temperature  $T_0$  (in kelvins) expands from initial volume  $V_0$  to volume  $2V_0$  by each of the five processes indicated in the  $T$ - $V$  diagram of Fig. 20-20. In which process is the expansion

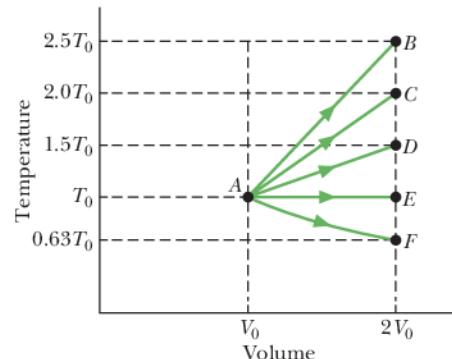


Figure 20-20  
Question 4.

a **configuration** of the system. The number of microstates in a configuration is the **multiplicity**  $W$  of the configuration.

For a system of  $N$  molecules that may be distributed between the two halves of a box, the multiplicity is given by

$$W = \frac{N!}{n_1! n_2!}, \quad (20-20)$$

in which  $n_1$  is the number of molecules in one half of the box and  $n_2$  is the number in the other half. A basic assumption of **statistical mechanics** is that all the microstates are equally probable. Thus, configurations with a large multiplicity occur most often.

The multiplicity  $W$  of a configuration of a system and the entropy  $S$  of the system in that configuration are related by Boltzmann's entropy equation:

$$S = k \ln W, \quad (20-21)$$

where  $k = 1.38 \times 10^{-23}$  J/K is the Boltzmann constant.

sion (a) isothermal, (b) isobaric (constant pressure), and (c) adiabatic? Explain your answers. (d) In which processes does the entropy of the gas decrease?

- 5 In four experiments, 2.5 mol of hydrogen gas undergoes reversible isothermal expansions, starting from the same volume but at different temperatures. The corresponding  $p$ - $V$  plots are shown in Fig. 20-21. Rank the situations according to the change in the entropy of the gas, greatest first.

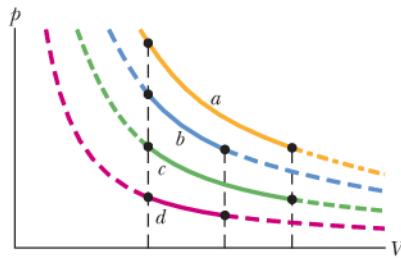


Figure 20-21 Question 5.

- 6 A box contains 100 atoms in a configuration that has 50 atoms in each half of the box. Suppose that you could count the different microstates associated with this configuration at the rate of 100 billion states per second, using a supercomputer. Without written calculation, guess how much computing time you would need: a day, a year, or much more than a year.

- 7 Does the entropy per cycle increase, decrease, or remain the same for (a) a Carnot engine, (b) a real engine, and (c) a perfect engine (which is, of course, impossible to build)?

- 8 Three Carnot engines operate between temperature limits of (a) 400 and 500 K, (b) 500 and 600 K, and (c) 400 and 600 K. Each engine extracts the same amount of energy per cycle from the high-temperature reservoir. Rank the magnitudes of the work done by the engines per cycle, greatest first.

- 9 An inventor claims to have invented four engines, each of which operates between constant-temperature reservoirs at 400 and 300 K. Data on each engine, per cycle of operation, are: engine A,  $Q_H = 200$  J,  $Q_L = -175$  J, and  $W = 40$  J; engine B,  $Q_H = 500$  J,  $Q_L = -200$  J, and  $W = 400$  J; engine C,  $Q_H = 600$  J,  $Q_L = -200$  J, and  $W = 400$  J; engine D,  $Q_H = 100$  J,  $Q_L = -90$  J, and  $W = 10$  J. Of the first and second laws of thermodynamics, which (if either) does each engine violate?

- 10 Does the entropy per cycle increase, decrease, or remain the same for (a) a Carnot refrigerator, (b) a real refrigerator, and (c) a perfect refrigerator (which is, of course, impossible to build)?



## Problems

Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign

Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty

Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://flyingcircusofphysics.com)

Worked-out solution is at



Interactive solution is at

<http://www.wiley.com/college/halliday>**Module 20-1 Entropy**

- 1 SSM** Suppose 4.00 mol of an ideal gas undergoes a reversible isothermal expansion from volume  $V_1$  to volume  $V_2 = 2.00V_1$  at temperature  $T = 400\text{ K}$ . Find (a) the work done by the gas and (b) the entropy change of the gas. (c) If the expansion is reversible and adiabatic instead of isothermal, what is the entropy change of the gas?

- 2** An ideal gas undergoes a reversible isothermal expansion at  $77.0^\circ\text{C}$ , increasing its volume from 1.30 L to 3.40 L. The entropy change of the gas is  $22.0\text{ J/K}$ . How many moles of gas are present?

- 3 ILW** A 2.50 mol sample of an ideal gas expands reversibly and isothermally at  $360\text{ K}$  until its volume is doubled. What is the increase in entropy of the gas?

- 4** How much energy must be transferred as heat for a reversible isothermal expansion of an ideal gas at  $132^\circ\text{C}$  if the entropy of the gas increases by  $46.0\text{ J/K}$ ?

- 5 ILW** Find (a) the energy absorbed as heat and (b) the change in entropy of a  $2.00\text{ kg}$  block of copper whose temperature is increased reversibly from  $25.0^\circ\text{C}$  to  $100^\circ\text{C}$ . The specific heat of copper is  $386\text{ J/kg}\cdot\text{K}$ .

- 6** (a) What is the entropy change of a  $12.0\text{ g}$  ice cube that melts completely in a bucket of water whose temperature is just above the freezing point of water? (b) What is the entropy change of a  $5.00\text{ g}$  spoonful of water that evaporates completely on a hot plate whose temperature is slightly above the boiling point of water?

- 7 ILW** A  $50.0\text{ g}$  block of copper whose temperature is  $400\text{ K}$  is placed in an insulating box with a  $100\text{ g}$  block of lead whose temperature is  $200\text{ K}$ . (a) What is the equilibrium temperature of the two-block system? (b) What is the change in the internal energy of the system between the initial state and the equilibrium state? (c) What is the change in the entropy of the system? (See Table 18-3.)

- 8** At very low temperatures, the molar specific heat  $C_V$  of many solids is approximately  $C_V = AT^3$ , where  $A$  depends on the particular substance. For aluminum,  $A = 3.15 \times 10^{-5}\text{ J/mol}\cdot\text{K}^4$ . Find the entropy change for 4.00 mol of aluminum when its temperature is raised from  $5.00\text{ K}$  to  $10.0\text{ K}$ .

- 9** A  $10\text{ g}$  ice cube at  $-10^\circ\text{C}$  is placed in a lake whose temperature is  $15^\circ\text{C}$ . Calculate the change in entropy of the cube–lake system as the ice cube comes to thermal equilibrium with the lake. The specific heat of ice is  $2220\text{ J/kg}\cdot\text{K}$ . (*Hint:* Will the ice cube affect the lake temperature?)

- 10** A  $364\text{ g}$  block is put in contact with a thermal reservoir. The block is initially at a lower temperature than the reservoir. Assume that the consequent transfer of energy as heat from the reservoir to the block is reversible. Figure 20-22

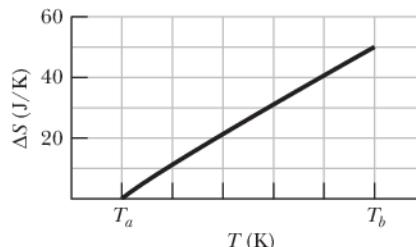


Figure 20-22 Problem 10.

gives the change in entropy  $\Delta S$  of the block until thermal equilibrium is reached. The scale of the horizontal axis is set by  $T_a = 280\text{ K}$  and  $T_b = 380\text{ K}$ . What is the specific heat of the block?

- 11 SSM WWW** In an experiment,  $200\text{ g}$  of aluminum (with a specific heat of  $900\text{ J/kg}\cdot\text{K}$ ) at  $100^\circ\text{C}$  is mixed with  $50.0\text{ g}$  of water at  $20.0^\circ\text{C}$ , with the mixture thermally isolated. (a) What is the equilibrium temperature? What are the entropy changes of (b) the aluminum, (c) the water, and (d) the aluminum–water system?

- 12** A gas sample undergoes a reversible isothermal expansion. Figure 20-23 gives the change  $\Delta S$  in entropy of the gas versus the final volume  $V_f$  of the gas. The scale of the vertical axis is set by  $\Delta S_s = 64\text{ J/K}$ . How many moles are in the sample?

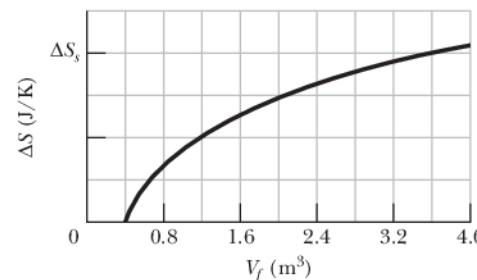


Figure 20-23 Problem 12.

- 13** In the irreversible process of Fig. 20-5, let the initial temperatures of the identical blocks  $L$  and  $R$  be  $305.5$  and  $294.5\text{ K}$ , respectively, and let  $215\text{ J}$  be the energy that must be transferred between the blocks in order to reach equilibrium. For the reversible processes of Fig. 20-6, what is  $\Delta S$  for (a) block  $L$ , (b) its reservoir, (c) block  $R$ , (d) its reservoir, (e) the two-block system, and (f) the system of the two blocks and the two reservoirs?

- 14** (a) For 1.0 mol of a monatomic ideal gas taken through the cycle in Fig. 20-24, where  $V_1 = 4.00V_0$ , what is  $W/p_0V_0$  as the gas goes from state  $a$  to state  $c$  along path  $abc$ ? What is  $\Delta E_{\text{int}}/p_0V_0$  in going (b) from  $b$  to  $c$  and (c) through one full cycle? What is  $\Delta S$  in going (d) from  $b$  to  $c$  and (e) through one full cycle?

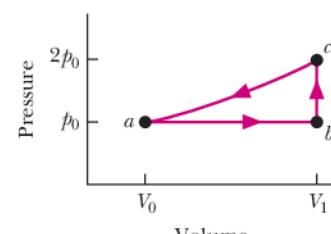


Figure 20-24 Problem 14.

- 15** A mixture of  $1773\text{ g}$  of water and  $227\text{ g}$  of ice is in an initial equilibrium state at  $0.00^\circ\text{C}$ . The mixture is then, in a reversible process, brought to a second equilibrium state where the water–ice ratio, by mass, is  $1.00:1.00$  at  $0.00^\circ\text{C}$ . (a) Calculate the entropy change of the system during this process. (The heat of fusion for water is  $333\text{ kJ/kg}$ .) (b) The system is then returned to the initial equilibrium state in an irreversible process (say, by using a Bunsen burner). Calculate the entropy change of the system during this process. (c) Are your answers consistent with the second law of thermodynamics?

**••16 GO** An 8.0 g ice cube at  $-10^{\circ}\text{C}$  is put into a Thermos flask containing  $100\text{ cm}^3$  of water at  $20^{\circ}\text{C}$ . By how much has the entropy of the cube–water system changed when equilibrium is reached? The specific heat of ice is  $2220\text{ J/kg}\cdot\text{K}$ .

**••17** In Fig. 20-25, where  $V_{23} = 3.00V_1$ ,  $n$  moles of a diatomic ideal gas are taken through the cycle with the molecules rotating but not oscillating. What are (a)  $p_2/p_1$ , (b)  $p_3/p_1$ , and (c)  $T_3/T_1$ ? For path  $1 \rightarrow 2$ , what are (d)  $W/nRT_1$ , (e)  $Q/nRT_1$ , (f)  $\Delta E_{\text{int}}/nRT_1$ , and (g)  $\Delta S/nR$ ? For path  $2 \rightarrow 3$ , what are (h)  $W/nRT_1$ , (i)  $Q/nRT_1$ , (j)  $\Delta E_{\text{int}}/nRT_1$ , (k)  $\Delta S/nR$ ? For path  $3 \rightarrow 1$ , what are (l)  $W/nRT_1$ , (m)  $Q/nRT_1$ , (n)  $\Delta E_{\text{int}}/nRT_1$ , and (o)  $\Delta S/nR$ ?

**••18 GO** A 2.0 mol sample of an ideal monatomic gas undergoes the reversible process shown in Fig. 20-26. The scale of the vertical axis is set by  $T_s = 400.0\text{ K}$  and the scale of the horizontal axis is set by  $S_s = 20.0\text{ J/K}$ . (a) How much energy is absorbed as heat by the gas? (b) What is the change in the internal energy of the gas? (c) How much work is done by the gas?

**••19** Suppose 1.00 mol of a monatomic ideal gas is taken from initial pressure  $p_1$  and volume  $V_1$  through two steps: (1) an isothermal expansion to volume  $2.00V_1$  and (2) a pressure increase to  $2.00p_1$  at constant volume. What is  $Q/p_1V_1$  for (a) step 1 and (b) step 2? What is  $W/p_1V_1$  for (c) step 1 and (d) step 2? For the full process, what are (e)  $\Delta E_{\text{int}}/p_1V_1$  and (f)  $\Delta S$ ? The gas is returned to its initial state and again taken to the same final state but now through these two steps: (1) an isothermal compression to pressure  $2.00p_1$  and (2) a volume increase to  $2.00V_1$  at constant pressure. What is  $Q/p_1V_1$  for (g) step 1 and (h) step 2? What is  $W/p_1V_1$  for (i) step 1 and (j) step 2? For the full process, what are (k)  $\Delta E_{\text{int}}/p_1V_1$  and (l)  $\Delta S$ ?

**••20** Expand 1.00 mol of an monatomic gas initially at 5.00 kPa and 600 K from initial volume  $V_i = 1.00\text{ m}^3$  to final volume  $V_f = 2.00\text{ m}^3$ . At any instant during the expansion, the pressure  $p$  and volume  $V$  of the gas are related by  $p = 5.00 \exp[(V_i - V)/a]$ , with  $p$  in kilopascals,  $V_i$  and  $V$  in cubic meters, and  $a = 1.00\text{ m}^3$ . What are the final (a) pressure and (b) temperature of the gas? (c) How much work is done by the gas during the expansion? (d) What is  $\Delta S$  for the expansion? (Hint: Use two simple reversible processes to find  $\Delta S$ .)

**••21 GO** Energy can be removed from water as heat at and even below the normal freezing point ( $0.0^{\circ}\text{C}$  at atmospheric pressure) without causing the water to freeze; the water is then said to be *supercooled*. Suppose a 1.00 g water drop is supercooled until its temperature is that of the surrounding air, which is at  $-5.00^{\circ}\text{C}$ . The drop then suddenly and irreversibly freezes, transferring energy to the air as heat. What is the entropy change for the drop? (Hint: Use a three-step reversible process as if the water were taken through the normal freezing point.) The specific heat of ice is  $2220\text{ J/kg}\cdot\text{K}$ .

**••22 GO** An insulated Thermos contains 130 g of water at  $80.0^{\circ}\text{C}$ . You put in a 12.0 g ice cube at  $0^{\circ}\text{C}$  to form a system of

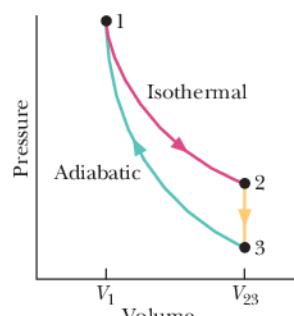


Figure 20-25 Problem 17.

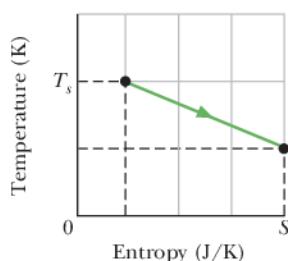


Figure 20-26 Problem 18.

ice + original water. (a) What is the equilibrium temperature of the system? What are the entropy changes of the water that was originally the ice cube (b) as it melts and (c) as it warms to the equilibrium temperature? (d) What is the entropy change of the original water as it cools to the equilibrium temperature? (e) What is the net entropy change of the ice + original water system as it reaches the equilibrium temperature?

### Module 20-2 Entropy in the Real World: Engines

**•23** A Carnot engine whose low-temperature reservoir is at  $17^{\circ}\text{C}$  has an efficiency of 40%. By how much should the temperature of the high-temperature reservoir be increased to increase the efficiency to 50%?

**•24** A Carnot engine absorbs 52 kJ as heat and exhausts 36 kJ as heat in each cycle. Calculate (a) the engine's efficiency and (b) the work done per cycle in kilojoules.

**•25** A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by  $75.0\text{ }^{\circ}\text{C}$ . What is the temperature of the (a) lower-temperature and (b) higher-temperature reservoir?

**•26** In a hypothetical nuclear fusion reactor, the fuel is deuterium gas at a temperature of  $7 \times 10^8\text{ K}$ . If this gas could be used to operate a Carnot engine with  $T_L = 100^{\circ}\text{C}$ , what would be the engine's efficiency? Take both temperatures to be exact and report your answer to seven significant figures.

**•27 SSM WWW** A Carnot engine operates between  $235^{\circ}\text{C}$  and  $115^{\circ}\text{C}$ , absorbing  $6.30 \times 10^4\text{ J}$  per cycle at the higher temperature. (a) What is the efficiency of the engine? (b) How much work per cycle is this engine capable of performing?

**•28** In the first stage of a two-stage Carnot engine, energy is absorbed as heat  $Q_1$  at temperature  $T_1$ , work  $W_1$  is done, and energy is expelled as heat  $Q_2$  at a lower temperature  $T_2$ . The second stage absorbs that energy as heat  $Q_2$ , does work  $W_2$ , and expels energy as heat  $Q_3$  at a still lower temperature  $T_3$ . Prove that the efficiency of the engine is  $(T_1 - T_3)/T_1$ .

**•29 GO** Figure 20-27 shows a reversible cycle through which 1.00 mol of a monatomic ideal gas is taken. Assume that  $p = 2p_0$ ,  $V = 2V_0$ ,  $p_0 = 1.01 \times 10^5\text{ Pa}$ , and  $V_0 = 0.0225\text{ m}^3$ . Calculate (a) the work done during the cycle, (b) the energy added as heat during stroke abc, and (c) the efficiency of the cycle. (d) What is the efficiency of a Carnot engine operating between the highest and lowest temperatures that occur in the cycle? (e) Is this greater than or less than the efficiency calculated in (c)?

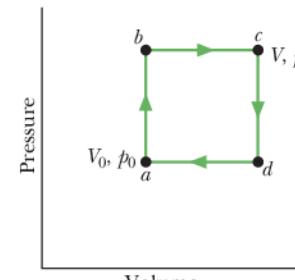


Figure 20-27 Problem 29.

**•30** A 500 W Carnot engine operates between constant-temperature reservoirs at  $100^{\circ}\text{C}$  and  $60.0^{\circ}\text{C}$ . What is the rate at which energy is (a) taken in by the engine as heat and (b) exhausted by the engine as heat?

**•31** The efficiency of a particular car engine is 25% when the engine does 8.2 kJ of work per cycle. Assume the process is reversible. What are (a) the energy the engine gains per cycle as heat  $Q_{\text{gain}}$  from the fuel combustion and (b) the energy the engine loses per cycle as heat  $Q_{\text{lost}}$ ? If a tune-up increases the efficiency to 31%, what are (c)  $Q_{\text{gain}}$  and (d)  $Q_{\text{lost}}$  at the same work value?

- 32 GO** A Carnot engine is set up to produce a certain work  $W$  per cycle. In each cycle, energy in the form of heat  $Q_H$  is transferred to the working substance of the engine from the higher-temperature thermal reservoir, which is at an adjustable temperature  $T_H$ . The lower-temperature thermal reservoir is maintained at temperature  $T_L = 250$  K. Figure 20-28 gives  $Q_H$  for a range of  $T_H$ . The scale of the vertical axis is set by  $Q_{Hs} = 6.0$  kJ. If  $T_H$  is set at 550 K, what is  $Q_H$ ?

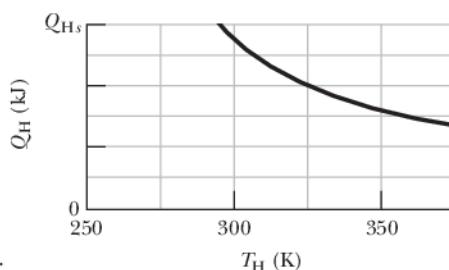


Figure 20-28 Problem 32.

- 33 SSM ILW** Figure 20-29 shows a reversible cycle through which 1.00 mol of a monatomic ideal gas is taken. Volume  $V_c = 8.00V_b$ . Process  $bc$  is an adiabatic expansion, with  $p_b = 10.0$  atm and  $V_b = 1.00 \times 10^{-3}$  m<sup>3</sup>. For the cycle, find (a) the energy added to the gas as heat, (b) the energy leaving the gas as heat, (c) the net work done by the gas, and (d) the efficiency of the cycle.

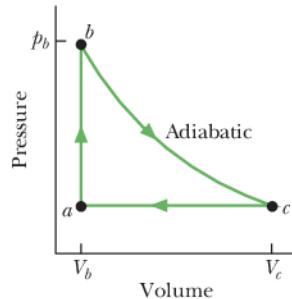


Figure 20-29 Problem 33.

- 34 GO** An ideal gas (1.0 mol) is the working substance in an engine that operates on the cycle shown in Fig. 20-30. Processes  $BC$  and  $DA$  are reversible and adiabatic. (a) Is the gas monatomic, diatomic, or polyatomic? (b) What is the engine efficiency?

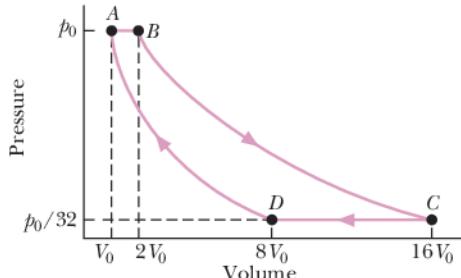


Figure 20-30 Problem 34.

- 35** The cycle in Fig. 20-31 represents the operation of a gasoline internal combustion engine. Volume  $V_3 = 4.00V_1$ . Assume the gasoline-air intake mixture is an ideal gas with  $\gamma = 1.30$ . What are the ratios (a)  $T_2/T_1$ , (b)  $T_3/T_1$ , (c)  $T_4/T_1$ , (d)  $p_3/p_1$ , and (e)  $p_4/p_1$ ? (f) What is the engine efficiency?

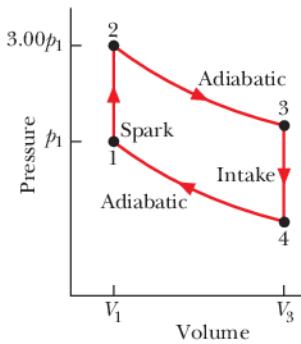


Figure 20-31 Problem 35.

### Module 20-3 Refrigerators and Real Engines

- 36** How much work must be done by a Carnot refrigerator to transfer 1.0

J as heat (a) from a reservoir at 7.0°C to one at 27°C, (b) from a reservoir at -73°C to one at 27°C, (c) from a reservoir at -173°C to one at 27°C, and (d) from a reservoir at -223°C to one at 27°C?

- 37 SSM** A heat pump is used to heat a building. The external temperature is less than the internal temperature. The pump's coefficient of performance is 3.8, and the heat pump delivers 7.54 MJ as heat to the building each hour. If the heat pump is a Carnot engine working in reverse, at what rate must work be done to run it?

- 38** The electric motor of a heat pump transfers energy as heat from the outdoors, which is at -5.0°C, to a room that is at 17°C. If the heat pump were a Carnot heat pump (a Carnot engine working in reverse), how much energy would be transferred as heat to the room for each joule of electric energy consumed?

- 39 SSM** A Carnot air conditioner takes energy from the thermal energy of a room at 70°F and transfers it as heat to the outdoors, which is at 96°F. For each joule of electric energy required to operate the air conditioner, how many joules are removed from the room?

- 40** To make ice, a freezer that is a reverse Carnot engine extracts 42 kJ as heat at -15°C during each cycle, with coefficient of performance 5.7. The room temperature is 30.3°C. How much (a) energy per cycle is delivered as heat to the room and (b) work per cycle is required to run the freezer?

- 41 ILW** An air conditioner operating between 93°F and 70°F is rated at 4000 Btu/h cooling capacity. Its coefficient of performance is 27% of that of a Carnot refrigerator operating between the same two temperatures. What horsepower is required of the air conditioner motor?

- 42** The motor in a refrigerator has a power of 200 W. If the freezing compartment is at 270 K and the outside air is at 300 K, and assuming the efficiency of a Carnot refrigerator, what is the maximum amount of energy that can be extracted as heat from the freezing compartment in 10.0 min?

- 43 GO** Figure 20-32 represents a Carnot engine that works between temperatures  $T_1 = 400$  K and  $T_2 = 150$  K and drives a Carnot refrigerator that works between temperatures  $T_3 = 325$  K and  $T_4 = 225$  K. What is the ratio  $Q_3/Q_1$ ?

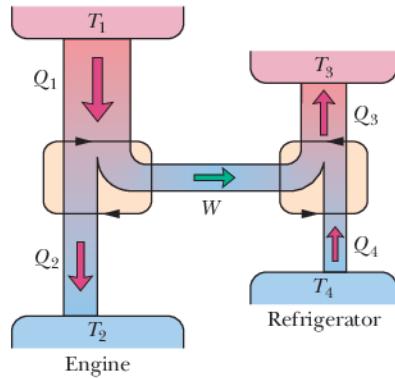


Figure 20-32 Problem 43.

- 44** (a) During each cycle, a Carnot engine absorbs 750 J as heat from a high-temperature reservoir at 360 K, with the low-temperature reservoir at 280 K. How much work is done per cycle? (b) The engine is then made to work in reverse to function as a Carnot refrigerator between those same two reservoirs. During each cycle, how much work is required to remove 1200 J as heat from the low-temperature reservoir?

### Module 20-4 A Statistical View of Entropy

- 45** Construct a table like Table 20-1 for eight molecules.

- 46** A box contains  $N$  identical gas molecules equally divided between its two halves. For  $N = 50$ , what are (a) the multiplicity  $W$  of the central configuration, (b) the total number of microstates, and (c) the percentage of the time the system spends in the central configuration? For  $N = 100$ , what are (d)  $W$  of the central configura-

tion, (e) the total number of microstates, and (f) the percentage of the time the system spends in the central configuration? For  $N = 200$ , what are (g)  $W$  of the central configuration, (h) the total number of microstates, and (i) the percentage of the time the system spends in the central configuration? (j) Does the time spent in the central configuration increase or decrease with an increase in  $N$ ?

**•••47 SSM WWW** A box contains  $N$  gas molecules. Consider the box to be divided into three equal parts. (a) By extension of Eq. 20-20, write a formula for the multiplicity of any given configuration. (b) Consider two configurations: configuration *A* with equal numbers of molecules in all three thirds of the box, and configuration *B* with equal numbers of molecules in each half of the box divided into two equal parts rather than three. What is the ratio  $W_A/W_B$  of the multiplicity of configuration *A* to that of configuration *B*? (c) Evaluate  $W_A/W_B$  for  $N = 100$ . (Because 100 is not evenly divisible by 3, put 34 molecules into one of the three box parts of configuration *A* and 33 in each of the other two parts.)

#### Additional Problems

**48** Four particles are in the insulated box of Fig. 20-17. What are (a) the least multiplicity, (b) the greatest multiplicity, (c) the least entropy, and (d) the greatest entropy of the four-particle system?

**49** A cylindrical copper rod of length 1.50 m and radius 2.00 cm is insulated to prevent heat loss through its curved surface. One end is attached to a thermal reservoir fixed at 300°C; the other is attached to a thermal reservoir fixed at 30.0°C. What is the rate at which entropy increases for the rod–reservoirs system?

**50** Suppose 0.550 mol of an ideal gas is isothermally and reversibly expanded in the four situations given below. What is the change in the entropy of the gas for each situation?

Situation	(a)	(b)	(c)	(d)
Temperature (K)	250	350	400	450
Initial volume ( $\text{cm}^3$ )	0.200	0.200	0.300	0.300
Final volume ( $\text{cm}^3$ )	0.800	0.800	1.20	1.20

**51 SSM** As a sample of nitrogen gas ( $\text{N}_2$ ) undergoes a temperature increase at constant volume, the distribution of molecular speeds increases. That is, the probability distribution function  $P(v)$  for the molecules spreads to higher speed values, as suggested in Fig. 19-8b. One way to report the spread in  $P(v)$  is to measure the difference  $\Delta v$  between the most probable speed  $v_p$  and the rms speed  $v_{\text{rms}}$ . When  $P(v)$  spreads to higher speeds,  $\Delta v$  increases. Assume that the gas is ideal and the  $\text{N}_2$  molecules rotate but do not oscillate. For 1.5 mol, an initial temperature of 250 K, and a final temperature of 500 K, what are (a) the initial difference  $\Delta v_i$ , (b) the final difference  $\Delta v_f$ , and (c) the entropy change  $\Delta S$  for the gas?

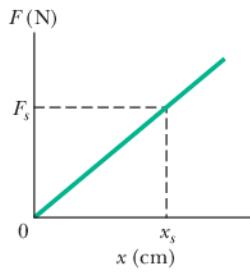
**52** Suppose 1.0 mol of a monatomic ideal gas initially at 10 L and 300 K is heated at constant volume to 600 K, allowed to expand isothermally to its initial pressure, and finally compressed at constant pressure to its original volume, pressure, and temperature. During the cycle, what are (a) the net energy entering the system (the gas) as heat and (b) the net work done by the gas? (c) What is the efficiency of the cycle?

**53 AO** Suppose that a deep shaft were drilled in Earth's crust near one of the poles, where the surface temperature is  $-40^\circ\text{C}$ , to a depth where the temperature is  $800^\circ\text{C}$ . (a) What is the theoretical limit to the efficiency of an engine operating between these

temperatures? (b) If all the energy released as heat into the low-temperature reservoir were used to melt ice that was initially at  $-40^\circ\text{C}$ , at what rate could liquid water at  $0^\circ\text{C}$  be produced by a 100 MW power plant (treat it as an engine)? The specific heat of ice is  $2220 \text{ J/kg}\cdot\text{K}$ ; water's heat of fusion is  $333 \text{ kJ/kg}$ . (Note that the engine can operate only between  $0^\circ\text{C}$  and  $800^\circ\text{C}$  in this case. Energy exhausted at  $-40^\circ\text{C}$  cannot warm anything above  $-40^\circ\text{C}$ .)

**54** What is the entropy change for 3.20 mol of an ideal monatomic gas undergoing a reversible increase in temperature from 380 K to 425 K at constant volume?

**55** A 600 g lump of copper at  $80.0^\circ\text{C}$  is placed in 70.0 g of water at  $10.0^\circ\text{C}$  in an insulated container. (See Table 18-3 for specific heats.) (a) What is the equilibrium temperature of the copper–water system? What entropy changes do (b) the copper, (c) the water, and (d) the copper–water system undergo in reaching the equilibrium temperature?

**56**  Figure 20-33 gives the force magnitude  $F$  versus stretch distance  $x$  for a rubber band, with the scale of the  $F$  axis set by  $F_s = 1.50 \text{ N}$  and the scale of the  $x$  axis set by  $x_s = 3.50 \text{ cm}$ . The temperature is  $2.00^\circ\text{C}$ . When the rubber band is stretched by  $x = 1.70 \text{ cm}$ , at what rate does the entropy of the rubber band change during a small additional stretch?

**57** The temperature of 1.00 mol of a monatomic ideal gas is raised reversibly from 300 K to 400 K, with its volume kept constant. What is the entropy change of the gas?

**58** Repeat Problem 57, with the pressure now kept constant.

**59 SSM** A 0.600 kg sample of water is initially ice at temperature  $-20^\circ\text{C}$ . What is the sample's entropy change if its temperature is increased to  $40^\circ\text{C}$ ?

**60** A three-step cycle is undergone by 3.4 mol of an ideal diatomic gas: (1) the temperature of the gas is increased from 200 K to 500 K at constant volume; (2) the gas is then isothermally expanded to its original pressure; (3) the gas is then contracted at constant pressure back to its original volume. Throughout the cycle, the molecules rotate but do not oscillate. What is the efficiency of the cycle?

**61** An inventor has built an engine X and claims that its efficiency  $\epsilon_X$  is greater than the efficiency  $\epsilon$  of an ideal engine operating between the same two temperatures. Suppose you couple engine X to an ideal refrigerator (Fig. 20-34a) and adjust the cycle

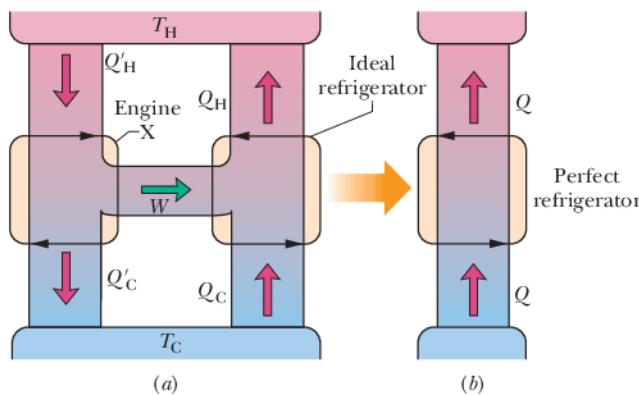


Figure 20-34 Problem 61.

of engine X so that the work per cycle it provides equals the work per cycle required by the ideal refrigerator. Treat this combination as a single unit and show that if the inventor's claim were true (if  $\varepsilon_X > \varepsilon$ ), the combined unit would act as a perfect refrigerator (Fig. 20-34b), transferring energy as heat from the low-temperature reservoir to the high-temperature reservoir without the need for work.

**62** Suppose 2.00 mol of a diatomic gas is taken reversibly around the cycle shown in the  $T$ - $S$  diagram of Fig. 20-35, where  $S_1 = 6.00 \text{ J/K}$  and  $S_2 = 8.00 \text{ J/K}$ . The molecules do not rotate or oscillate. What is the energy transferred as heat  $Q$  for (a) path  $1 \rightarrow 2$ , (b) path  $2 \rightarrow 3$ , and (c) the full cycle? (d) What is the work  $W$  for the isothermal process? The volume  $V_1$  in state 1 is  $0.200 \text{ m}^3$ . What is the volume in (e) state 2 and (f) state 3?

What is the change  $\Delta E_{\text{int}}$  for (g) path  $1 \rightarrow 2$ , (h) path  $2 \rightarrow 3$ , and (i) the full cycle? (*Hint:* (h) can be done with one or two lines of calculation using Module 19-7 or with a page of calculation using Module 19-9.) (j) What is the work  $W$  for the adiabatic process?

**63** A three-step cycle is undergone reversibly by 4.00 mol of an ideal gas: (1) an adiabatic expansion that gives the gas 2.00 times its initial volume, (2) a constant-volume process, (3) an isothermal compression back to the initial state of the gas. We do not know whether the gas is monatomic or diatomic; if it is diatomic, we do not know whether the molecules are rotating or oscillating. What are the entropy changes for (a) the cycle, (b) process 1, (c) process 3, and (d) process 2?

**64** (a) A Carnot engine operates between a hot reservoir at 320 K and a cold one at 260 K. If the engine absorbs 500 J as heat per cycle at the hot reservoir, how much work per cycle does it deliver? (b) If the engine working in reverse functions as a refrigerator between the same two reservoirs, how much work per cycle must be supplied to remove 1000 J as heat from the cold reservoir?

**65** A 2.00 mol diatomic gas initially at 300 K undergoes this cycle: It is (1) heated at constant volume to 800 K, (2) then allowed to expand isothermally to its initial pressure, (3) then compressed at constant pressure to its initial state. Assuming the gas molecules neither rotate nor oscillate, find (a) the net energy transferred as heat to the gas, (b) the net work done by the gas, and (c) the efficiency of the cycle.

**66** An ideal refrigerator does 150 J of work to remove 560 J as heat from its cold compartment. (a) What is the refrigerator's coefficient of performance? (b) How much heat per cycle is exhausted to the kitchen?

**67** Suppose that 260 J is conducted from a constant-temperature reservoir at 400 K to one at (a) 100 K, (b) 200 K, (c) 300 K, and (d) 360 K. What is the net change in entropy  $\Delta S_{\text{net}}$  of the reservoirs in each case? (e) As the temperature difference of the two reservoirs decreases, does  $\Delta S_{\text{net}}$  increase, decrease, or remain the same?

**68** An apparatus that liquefies helium is in a room maintained at 300 K. If the helium in the apparatus is at 4.0 K, what is the minimum ratio  $Q_{\text{to}}/Q_{\text{from}}$ , where  $Q_{\text{to}}$  is the energy delivered as heat to the room and  $Q_{\text{from}}$  is the energy removed as heat from the helium?

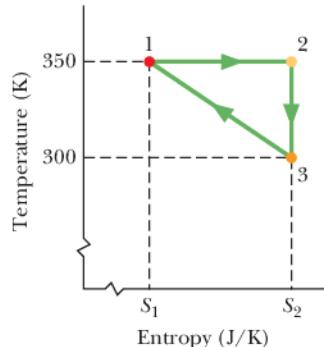


Figure 20-35 Problem 62.

**69 GO** A brass rod is in thermal contact with a constant-temperature reservoir at 130°C at one end and a constant-temperature reservoir at 24.0°C at the other end. (a) Compute the total change in entropy of the rod–reservoirs system when 5030 J of energy is conducted through the rod, from one reservoir to the other. (b) Does the entropy of the rod change?

**70** A 45.0 g block of tungsten at 30.0°C and a 25.0 g block of silver at -120°C are placed together in an insulated container. (See Table 18-3 for specific heats.) (a) What is the equilibrium temperature? What entropy changes do (b) the tungsten, (c) the silver, and (d) the tungsten–silver system undergo in reaching the equilibrium temperature?

**71** A box contains  $N$  molecules. Consider two configurations: configuration *A* with an equal division of the molecules between the two halves of the box, and configuration *B* with 60.0% of the molecules in the left half of the box and 40.0% in the right half. For  $N = 50$ , what are (a) the multiplicity  $W_A$  of configuration *A*, (b) the multiplicity  $W_B$  of configuration *B*, and (c) the ratio  $f_{B/A}$  of the time the system spends in configuration *B* to the time it spends in configuration *A*? For  $N = 100$ , what are (d)  $W_A$ , (e)  $W_B$ , and (f)  $f_{B/A}$ ? For  $N = 200$ , what are (g)  $W_A$ , (h)  $W_B$ , and (i)  $f_{B/A}$ ? (j) With increasing  $N$ , does  $f$  increase, decrease, or remain the same?

**72** Calculate the efficiency of a fossil-fuel power plant that consumes 380 metric tons of coal each hour to produce useful work at the rate of 750 MW. The heat of combustion of coal (the heat due to burning it) is 28 MJ/kg.

**73 SSM** A Carnot refrigerator extracts 35.0 kJ as heat during each cycle, operating with a coefficient of performance of 4.60. What are (a) the energy per cycle transferred as heat to the room and (b) the work done per cycle?

**74** A Carnot engine whose high-temperature reservoir is at 400 K has an efficiency of 30.0%. By how much should the temperature of the low-temperature reservoir be changed to increase the efficiency to 40.0%?

**75 SSM** System *A* of three particles and system *B* of five particles are in insulated boxes like that in Fig. 20-17. What is the least multiplicity  $W$  of (a) system *A* and (b) system *B*? What is the greatest multiplicity  $W$  of (c) *A* and (d) *B*? What is the greatest entropy of (e) *A* and (f) *B*?

**76** Figure 20-36 shows a Carnot cycle on a  $T$ - $S$  diagram, with a scale set by  $S_s = 0.60 \text{ J/K}$ . For a full cycle, find (a) the net heat transfer and (b) the net work done by the system.

**77** Find the relation between the efficiency of a reversible ideal heat engine and the coefficient of performance of the reversible refrigerator obtained by running the engine backwards.

**78** A Carnot engine has a power of 500 W. It operates between heat reservoirs at 100°C and 60.0°C. Calculate (a) the rate of heat input and (b) the rate of exhaust heat output.

**79** In a real refrigerator, the low-temperature coils are at -13°C, and the compressed gas in the condenser is at 26°C. What is the theoretical coefficient of performance?

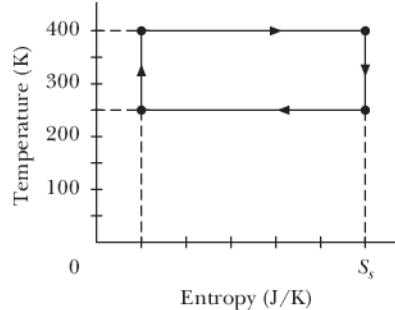


Figure 20-36 Problem 76.

# Coulomb's Law

## 21-1 COULOMB'S LAW

### Learning Objectives

After reading this module, you should be able to . . .

- 21.01** Distinguish between being electrically neutral, negatively charged, and positively charged and identify excess charge.
- 21.02** Distinguish between conductors, nonconductors (insulators), semiconductors, and superconductors.
- 21.03** Describe the electrical properties of the particles inside an atom.
- 21.04** Identify conduction electrons and explain their role in making a conducting object negatively or positively charged.
- 21.05** Identify what is meant by "electrically isolated" and by "grounding."
- 21.06** Explain how a charged object can set up induced charge in a second object.
- 21.07** Identify that charges with the same electrical sign repel each other and those with opposite electrical signs attract each other.
- 21.08** For either of the particles in a pair of charged particles, draw a free-body diagram, showing the electrostatic force (Coulomb force) on it and anchoring the tail of the force vector on that particle.
- 21.09** For either of the particles in a pair of charged particles, apply Coulomb's law to relate the magnitude of the electrostatic force, the charge magnitudes of the particles, and the separation between the particles.

### Key Ideas

- The strength of a particle's electrical interaction with objects around it depends on its electric charge (usually represented as  $q$ ), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other.
  - An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.
  - Conductors are materials in which a significant number of electrons are free to move. The charged particles in nonconductors (insulators) are not free to move.
  - Electric current  $i$  is the rate  $dq/dt$  at which charge passes a point:
- $$i = \frac{dq}{dt}$$
- Coulomb's law describes the electrostatic force (or electric

- 21.10** Identify that Coulomb's law applies only to (point-like) particles and objects that can be treated as particles.
- 21.11** If more than one force acts on a particle, find the net force by adding all the forces as vectors, not scalars.
- 21.12** Identify that a shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated as a particle at the shell's center.
- 21.13** Identify that if a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.
- 21.14** Identify that if excess charge is put on a spherical conductor, it spreads out uniformly over the external surface area.
- 21.15** Identify that if two identical spherical conductors touch or are connected by conducting wire, any excess charge will be shared equally.
- 21.16** Identify that a nonconducting object can have any given distribution of charge, including charge at interior points.
- 21.17** Identify current as the rate at which charge moves through a point.
- 21.18** For current through a point, apply the relationship between the current, a time interval, and the amount of charge that moves through the point in that time interval.

force) between two charged particles. If the particles have charges  $q_1$  and  $q_2$ , are separated by distance  $r$ , and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1| |q_2|}{r^2} \quad (\text{Coulomb's law}),$$

where  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$  is the permittivity constant. The ratio  $1/4\pi\epsilon_0$  is often replaced with the electrostatic constant (or Coulomb constant)  $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ .

- The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge).
- If multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

- Shell theorem 1: A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.
- Shell theorem 2: A charged particle inside a shell with

charge uniformly distributed on its surface has no net force acting on it due to the shell.

- Charge on a conducting spherical shell spreads uniformly over the (external) surface.

## What Is Physics?

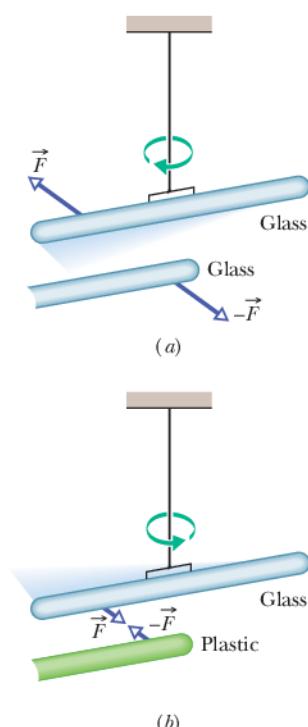
You are surrounded by devices that depend on the physics of electromagnetism, which is the combination of electric and magnetic phenomena. This physics is at the root of computers, television, radio, telecommunications, household lighting, and even the ability of food wrap to cling to a container. This physics is also the basis of the natural world. Not only does it hold together all the atoms and molecules in the world, it also produces lightning, auroras, and rainbows.

The physics of electromagnetism was first studied by the early Greek philosophers, who discovered that if a piece of amber is rubbed and then brought near bits of straw, the straw will jump to the amber. We now know that the attraction between amber and straw is due to an electric force. The Greek philosophers also discovered that if a certain type of stone (a naturally occurring magnet) is brought near bits of iron, the iron will jump to the stone. We now know that the attraction between magnet and iron is due to a magnetic force.

From these modest origins with the Greek philosophers, the sciences of electricity and magnetism developed separately for centuries—until 1820, in fact, when Hans Christian Oersted found a connection between them: an electric current in a wire can deflect a magnetic compass needle. Interestingly enough, Oersted made this discovery, a big surprise, while preparing a lecture demonstration for his physics students.

The new science of electromagnetism was developed further by workers in many countries. One of the best was Michael Faraday, a truly gifted experimenter with a talent for physical intuition and visualization. That talent is attested to by the fact that his collected laboratory notebooks do not contain a single equation. In the mid-nineteenth century, James Clerk Maxwell put Faraday's ideas into mathematical form, introduced many new ideas of his own, and put electromagnetism on a sound theoretical basis.

Our discussion of electromagnetism is spread through the next 16 chapters. We begin with electrical phenomena, and our first step is to discuss the nature of electric charge and electric force.



**Figure 21-1** (a) The two glass rods were each rubbed with a silk cloth and one was suspended by thread. When they are close to each other, they repel each other. (b) The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

## Electric Charge

Here are two demonstrations that seem to be magic, but our job here is to make sense of them. After rubbing a glass rod with a silk cloth (on a day when the humidity is low), we hang the rod by means of a thread tied around its center (Fig. 21-1a). Then we rub a second glass rod with the silk cloth and bring it near the hanging rod. The hanging rod magically moves away. We can see that a force repels it from the second rod, but how? There is no contact with that rod, no breeze to push on it, and no sound wave to disturb it.

In the second demonstration we replace the second rod with a plastic rod that has been rubbed with fur. This time, the hanging rod moves toward the nearby rod (Fig. 21-1b). Like the repulsion, this attraction occurs without any contact or obvious communication between the rods.

In the next chapter we shall discuss how the hanging rod knows of the presence of the other rods, but in this chapter let's focus on just the forces that are involved. In the first demonstration, the force on the hanging rod was *repulsive*, and

in the second, *attractive*. After a great many investigations, scientists figured out that the forces in these types of demonstrations are due to the *electric charge* that we set up on the rods when they are in contact with silk or fur. Electric charge is an intrinsic property of the fundamental particles that make up objects such as the rods, silk, and fur. That is, charge is a property that comes automatically with those particles wherever they exist.

**Two Types.** There are two types of electric charge, named by the American scientist and statesman Benjamin Franklin as positive charge and negative charge. He could have called them anything (such as cherry and walnut), but using algebraic signs as names comes in handy when we add up charges to find the net charge. In most everyday objects, such as a mug, there are about equal numbers of negatively charged particles and positively charged particles, and so the net charge is zero, the charge is said to be *balanced*, and the object is said to be *electrically neutral* (or just *neutral* for short).

**Excess Charge.** Normally you are approximately neutral. However, if you live in regions where the humidity is low, you know that the charge on your body can become slightly unbalanced when you walk across certain carpets. Either you gain negative charge from the carpet (at the points of contact between your shoes with the carpet) and become negatively charged, or you lose negative charge and become positively charged. Either way, the extra charge is said to be an *excess charge*. You probably don't notice it until you reach for a door handle or another person. Then, if your excess charge is enough, a spark leaps between you and the other object, eliminating your excess charge. Such sparking can be annoying and even somewhat painful. Such *charging* and *discharging* does not happen in humid conditions because the water in the air *neutralizes* your excess charge about as fast as you acquire it.

Two of the grand mysteries in physics are (1) *why* does the universe have particles with electric charge (what is it, really?) and (2) *why* does electric charge come in two types (and not, say, one type or three types). We just do not know. Nevertheless, with lots of experiments similar to our two demonstrations scientists discovered that

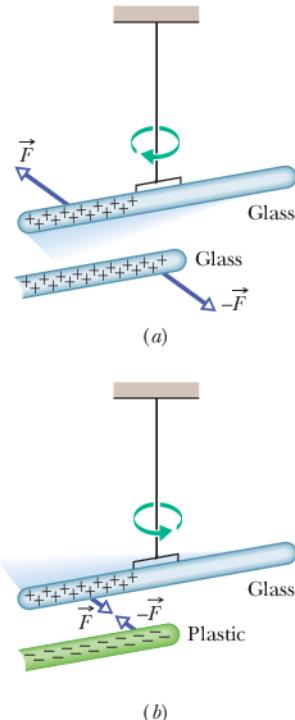


Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.

In a moment we shall put this rule into quantitative form as Coulomb's law of *electrostatic force* (or *electric force*) between charged particles. The term *electrostatic* is used to emphasize that, relative to each other, the charges are either stationary or moving only very slowly.

**Demos.** Now let's get back to the demonstrations to understand the motions of the rod as being something other than just magic. When we rub the glass rod with a silk cloth, a small amount of negative charge moves from the rod to the silk (a transfer like that between you and a carpet), leaving the rod with a small amount of excess positive charge. (Which way the negative charge moves is not obvious and requires a lot of experimentation.) We *rub* the silk over the rod to increase the number of contact points and thus the amount, still tiny, of transferred charge. We hang the rod from the thread so as to *electrically isolate* it from its surroundings (so that the surroundings cannot neutralize the rod by giving it enough negative charge to rebalance its charge). When we rub the second rod with the silk cloth, it too becomes positively charged. So when we bring it near the first rod, the two rods repel each other (Fig. 21-2a).

Next, when we rub the plastic rod with fur, it gains excess negative charge from the fur. (Again, the transfer direction is learned through many experiments.) When we bring the plastic rod (with negative charge) near the hanging glass rod (with positive charge), the rods are attracted to each other (Fig. 21-2b). All this is subtle. You cannot see the charge or its transfer, only the results.



**Figure 21-2** (a) Two charged rods of the same sign repel each other. (b) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

## Conductors and Insulators

We can classify materials generally according to the ability of charge to move through them. **Conductors** are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water. **Nonconductors**—also called **insulators**—are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water. **Semiconductors** are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips. **Superconductors** are materials that are *perfect* conductors, allowing charge to move without *any* hindrance. In these chapters we discuss only conductors and insulators.

**Conducting Path.** Here is an example of how conduction can eliminate excess charge on an object. If you rub a copper rod with wool, charge is transferred from the wool to the rod. However, if you are holding the rod while also touching a faucet, you cannot charge the rod in spite of the transfer. The reason is that you, the rod, and the faucet are all conductors connected, via the plumbing, to Earth's surface, which is a huge conductor. Because the excess charges put on the rod by the wool repel one another, they move away from one another by moving first through the rod, then through you, and then through the faucet and plumbing to reach Earth's surface, where they can spread out. The process leaves the rod electrically neutral.

In thus setting up a pathway of conductors between an object and Earth's surface, we are said to *ground* the object, and in neutralizing the object (by eliminating an unbalanced positive or negative charge), we are said to *discharge* the object. If instead of holding the copper rod in your hand, you hold it by an insulating handle, you eliminate the conducting path to Earth, and the rod can then be charged by rubbing (the charge remains on the rod), as long as you do not touch it directly with your hand.

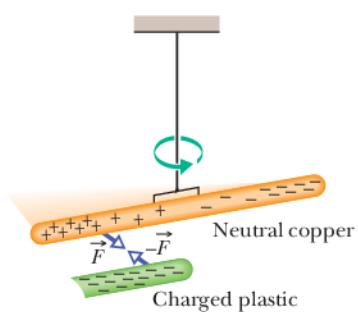
**Charged Particles.** The properties of conductors and insulators are due to the structure and electrical nature of atoms. Atoms consist of positively charged *protons*, negatively charged *electrons*, and electrically neutral *neutrons*. The protons and neutrons are packed tightly together in a central *nucleus*.

The charge of a single electron and that of a single proton have the same magnitude but are opposite in sign. Hence, an electrically neutral atom contains equal numbers of electrons and protons. Electrons are held near the nucleus because they have the electrical sign opposite that of the protons in the nucleus and thus are attracted to the nucleus. Were this not true, there would be no atoms and thus no you.

When atoms of a conductor like copper come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (*positive ions*). We call the mobile electrons *conduction electrons*. There are few (if any) free electrons in a nonconductor.

**Induced Charge.** The experiment of Fig. 21-3 demonstrates the mobility of charge in a conductor. A negatively charged plastic rod will attract either end of an isolated neutral copper rod. What happens is that many of the conduction electrons in the closer end of the copper rod are repelled by the negative charge on the plastic rod. Some of the conduction electrons move to the far end of the copper rod, leaving the near end depleted in electrons and thus with an unbalanced positive charge. This positive charge is attracted to the negative charge in the plastic rod. Although the copper rod is still neutral, it is said to have an *induced charge*, which means that some of its positive and negative charges have been separated due to the presence of a nearby charge.

Similarly, if a positively charged glass rod is brought near one end of a neutral copper rod, induced charge is again set up in the neutral copper rod but now the near end gains conduction electrons, becomes negatively charged, and is attracted to the glass rod, while the far end is positively charged.



**Figure 21-3** A neutral copper rod is electrically isolated from its surroundings by being suspended on a nonconducting thread. Either end of the copper rod will be attracted by a charged rod. Here, conduction electrons in the copper rod are repelled to the far end of that rod by the negative charge on the plastic rod. Then that negative charge attracts the remaining positive charge on the near end of the copper rod, rotating the copper rod to bring that near end closer to the plastic rod.

Note that only conduction electrons, with their negative charges, can move; positive ions are fixed in place. Thus, an object becomes positively charged only through the *removal of negative charges*.

### Blue Flashes from a Wintergreen LifeSaver

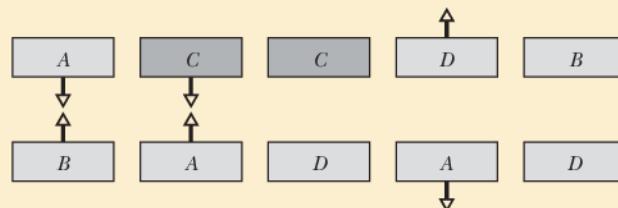
Indirect evidence for the attraction of charges with opposite signs can be seen with a wintergreen LifeSaver (the candy shaped in the form of a marine lifesaver). If you adapt your eyes to darkness for about 15 minutes and then have a friend chomp on a piece of the candy in the darkness, you will see a faint blue flash from your friend's mouth with each chomp. Whenever a chomp breaks a sugar crystal into pieces, each piece will probably end up with a different number of electrons. Suppose a crystal breaks into pieces *A* and *B*, with *A* ending up with more electrons on its surface than *B* (Fig. 21-4). This means that *B* has positive ions (atoms that lost electrons to *A*) on its surface. Because the electrons on *A* are strongly attracted to the positive ions on *B*, some of those electrons jump across the gap between the pieces.

As *A* and *B* move away from each other, air (primarily nitrogen,  $N_2$ ) flows into the gap, and many of the jumping electrons collide with nitrogen molecules in the air, causing the molecules to emit ultraviolet light. You cannot see this type of light. However, the wintergreen molecules on the surfaces of the candy pieces absorb the ultraviolet light and then emit blue light, which you *can* see—it is the blue light coming from your friend's mouth.



### Checkpoint 1

The figure shows five pairs of plates: *A*, *B*, and *D* are charged plastic plates and *C* is an electrically neutral copper plate. The electrostatic forces between the pairs of plates are shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other?

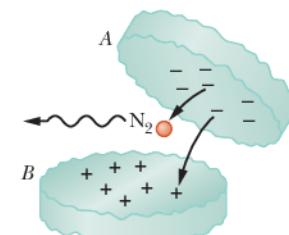


### Coulomb's Law

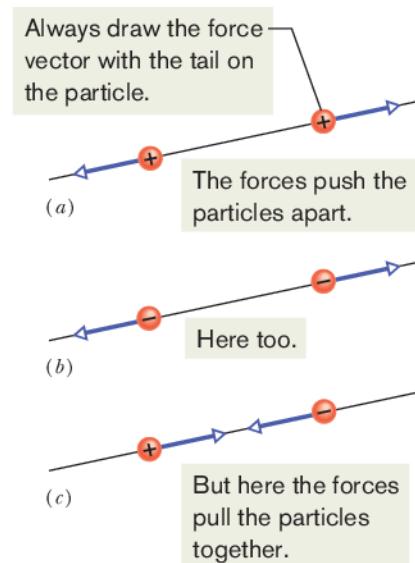
Now we come to the equation for Coulomb's law, but first a caution. This equation works for only charged particles (and a few other things that can be treated as particles). For extended objects, with charge located in many different places, we need more powerful techniques. So, here we consider just charged particles and not, say, two charged cats.

If two charged particles are brought near each other, they each exert an **electrostatic force** on the other. The direction of the force vectors depends on the signs of the charges. If the particles have the same sign of charge, they repel each other. That means that the force vector on each is directly away from the other particle (Figs. 21-5*a* and *b*). If we release the particles, they accelerate away from each other. If, instead, the particles have opposite signs of charge, they attract each other. That means that the force vector on each is directly toward the other particle (Fig. 21-5*c*). If we release the particles, they accelerate toward each other.

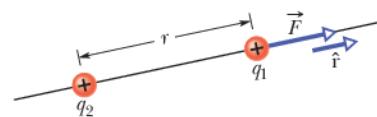
The equation for the electrostatic forces acting on the particles is called **Coulomb's law** after Charles-Augustin de Coulomb, whose experiments in 1785 led him to it. Let's write the equation in vector form and in terms of the particles shown in Fig. 21-6, where particle 1 has charge  $q_1$  and particle 2 has charge  $q_2$ . (These symbols can represent either positive or negative charge.) Let's also focus on particle 1 and write the force acting on it in terms of a unit vector  $\hat{r}$  that points along a radial



**Figure 21-4** Two pieces of a wintergreen LifeSaver candy as they fall away from each other. Electrons jumping from the negative surface of piece *A* to the positive surface of piece *B* collide with nitrogen ( $N_2$ ) molecules in the air.



**Figure 21-5** Two charged particles repel each other if they have the same sign of charge, either (a) both positive or (b) both negative. (c) They attract each other if they have opposite signs of charge.



**Figure 21-6** The electrostatic force on particle 1 can be described in terms of a unit vector  $\hat{r}$  along an axis through the two particles, radially away from particle 2.

axis extending through the two particles, radially away from particle 2. (As with other unit vectors,  $\hat{r}$  has a magnitude of exactly 1 and no unit; its purpose is to point, like a direction arrow on a street sign.) With these decisions, we write the electrostatic force as

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} \quad (\text{Coulomb's law}), \quad (21-1)$$

where  $r$  is the separation between the particles and  $k$  is a positive constant called the *electrostatic constant* or the *Coulomb constant*. (We'll discuss  $k$  below.)

Let's first check the direction of the force on particle 1 as given by Eq. 21-1. If  $q_1$  and  $q_2$  have the same sign, then the product  $q_1 q_2$  gives us a positive result. So, Eq. 21-1 tells us that the force on particle 1 is in the direction of  $\hat{r}$ . That checks, because particle 1 is being repelled from particle 2. Next, if  $q_1$  and  $q_2$  have opposite signs, the product  $q_1 q_2$  gives us a negative result. So, now Eq. 21-1 tells us that the force on particle 1 is in the direction opposite  $\hat{r}$ . That checks because particle 1 is being attracted toward particle 2.

**An Aside.** Here is something that is very curious. The form of Eq. 21-1 is the same as that of Newton's equation (Eq. 13-3) for the gravitational force between two particles with masses  $m_1$  and  $m_2$  and separation  $r$ :

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r} \quad (\text{Newton's law}), \quad (21-2)$$

where  $G$  is the gravitational constant. Although the two types of forces are wildly different, both equations describe inverse square laws (the  $1/r^2$  dependences) that involve a product of a property of the interacting particles—the charge in one case and the mass in the other. However, the laws differ in that gravitational forces are always attractive but electrostatic forces may be either attractive or repulsive, depending on the signs of the charges. This difference arises from the fact that there is only one type of mass but two types of charge.

**Unit.** The SI unit of charge is the **coulomb**. For practical reasons having to do with the accuracy of measurements, the coulomb unit is derived from the SI unit *ampere* for electric current  $i$ . We shall discuss current in detail in Chapter 26, but here let's just note that current  $i$  is the rate  $dq/dt$  at which charge moves past a point or through a region:

$$i = \frac{dq}{dt} \quad (\text{electric current}). \quad (21-3)$$

Rearranging Eq. 21-3 and replacing the symbols with their units (coulombs C, amperes A, and seconds s) we see that

$$1 \text{ C} = (1 \text{ A})(1 \text{ s}).$$

**Force Magnitude.** For historical reasons (and because doing so simplifies many other formulas), the electrostatic constant  $k$  in Eq. 21-1 is often written as  $1/4\pi\epsilon_0$ . Then the magnitude of the electrostatic force in Coulomb's law becomes

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2} \quad (\text{Coulomb's law}). \quad (21-4)$$

The constants in Eqs. 21-1 and 21-4 have the value

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2. \quad (21-5)$$

The quantity  $\epsilon_0$ , called the **permittivity constant**, sometimes appears separately in equations and is

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2. \quad (21-6)$$

**Working a Problem.** Note that the charge magnitudes appear in Eq. 21-4, which gives us the force magnitude. So, in working problems in this chapter, we use Eq. 21-4 to find the magnitude of a force on a chosen particle due to a second

particle and we separately determine the direction of the force by considering the charge signs of the two particles.

**Multiple Forces.** As with all forces in this book, the electrostatic force obeys the principle of superposition. Suppose we have  $n$  charged particles near a chosen particle called particle 1; then the net force on particle 1 is given by the vector sum

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}, \quad (21-7)$$

in which, for example,  $\vec{F}_{14}$  is the force on particle 1 due to the presence of particle 4.

This equation is the key to many of the homework problems, so let's state it in words. If you want to know the net force acting on a chosen charged particle that is surrounded by other charged particles, first clearly identify that chosen particle and then find the force on it due to each of the other particles. Draw those force vectors in a free-body diagram of the chosen particle, with the tails anchored on the particle. (That may sound trivial, but failing to do so easily leads to errors.) Then add all those forces *as vectors* according to the rules of Chapter 3, not as scalars. (You cannot just willy-nilly add up their magnitudes.) The result is the net force (or resultant force) acting on the particle.

Although the vector nature of the forces makes the homework problems harder than if we simply had scalars, be thankful that Eq. 21-7 works. If two force vectors did not simply add but for some reason amplified each other, the world would be very difficult to understand and manage.

**Shell Theories.** Analogous to the shell theories for the gravitational force (Module 13-1), we have two shell theories for the electrostatic force:



Shell theory 1. A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.



Shell theory 2. A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

(In the first theory, we assume that the charge on the shell is much greater than the particle's charge. Thus the presence of the particle has negligible effect on the distribution of charge on the shell.)

### Spherical Conductors

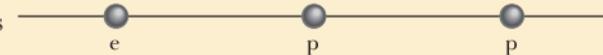
If excess charge is placed on a spherical shell that is made of conducting material, the excess charge spreads uniformly over the (external) surface. For example, if we place excess electrons on a spherical metal shell, those electrons repel one another and tend to move apart, spreading over the available surface until they are uniformly distributed. That arrangement maximizes the distances between all pairs of the excess electrons. According to the first shell theorem, the shell then will attract or repel an external charge as if all the excess charge on the shell were concentrated at its center.

If we remove negative charge from a spherical metal shell, the resulting positive charge of the shell is also spread uniformly over the surface of the shell. For example, if we remove  $n$  electrons, there are then  $n$  sites of positive charge (sites missing an electron) that are spread uniformly over the shell. According to the first shell theorem, the shell will again attract or repel an external charge as if all the shell's excess charge were concentrated at its center.



### Checkpoint 2

The figure shows two protons (symbol p) and one electron (symbol e) on an axis. On the central proton, what is the direction of (a) the force due to the electron, (b) the force due to the other proton, and (c) the net force?



e

p

p



### Sample Problem 21.01 Finding the net force due to two other particles

This sample problem actually contains three examples, to build from basic stuff to harder stuff. In each we have the same charged particle 1. First there is a single force acting on it (easy stuff). Then there are two forces, but they are just in opposite directions (not too bad). Then there are again two forces but they are in very different directions (ah, now we have to get serious about the fact that they are vectors). The key to all three examples is to draw the forces correctly *before* you reach for a calculator, otherwise you may be calculating nonsense on the calculator. (Figure 21-7 is available in WileyPLUS as an animation with voiceover.)

(a) Figure 21-7a shows two positively charged particles fixed in place on an  $x$  axis. The charges are  $q_1 = 1.60 \times 10^{-19} \text{ C}$  and  $q_2 = 3.20 \times 10^{-19} \text{ C}$ , and the particle separation is  $R = 0.0200 \text{ m}$ . What are the magnitude and direction of the electrostatic force  $\vec{F}_{12}$  on particle 1 from particle 2?

#### KEY IDEAS

Because both particles are positively charged, particle 1 is repelled by particle 2, with a force magnitude given by Eq. 21-4. Thus, the direction of force  $\vec{F}_{12}$  on particle 1 is *away from* particle 2, in the negative direction of the  $x$  axis, as indicated in the free-body diagram of Fig. 21-7b.

**Two particles:** Using Eq. 21-4 with separation  $R$  substituted for  $r$ , we can write the magnitude  $F_{12}$  of this force as

$$\begin{aligned} F_{12} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{R^2} \\ &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(0.0200 \text{ m})^2} \\ &= 1.15 \times 10^{-24} \text{ N}. \end{aligned}$$

Thus, force  $\vec{F}_{12}$  has the following magnitude and direction (relative to the positive direction of the  $x$  axis):

$$1.15 \times 10^{-24} \text{ N} \text{ and } 180^\circ. \quad (\text{Answer})$$

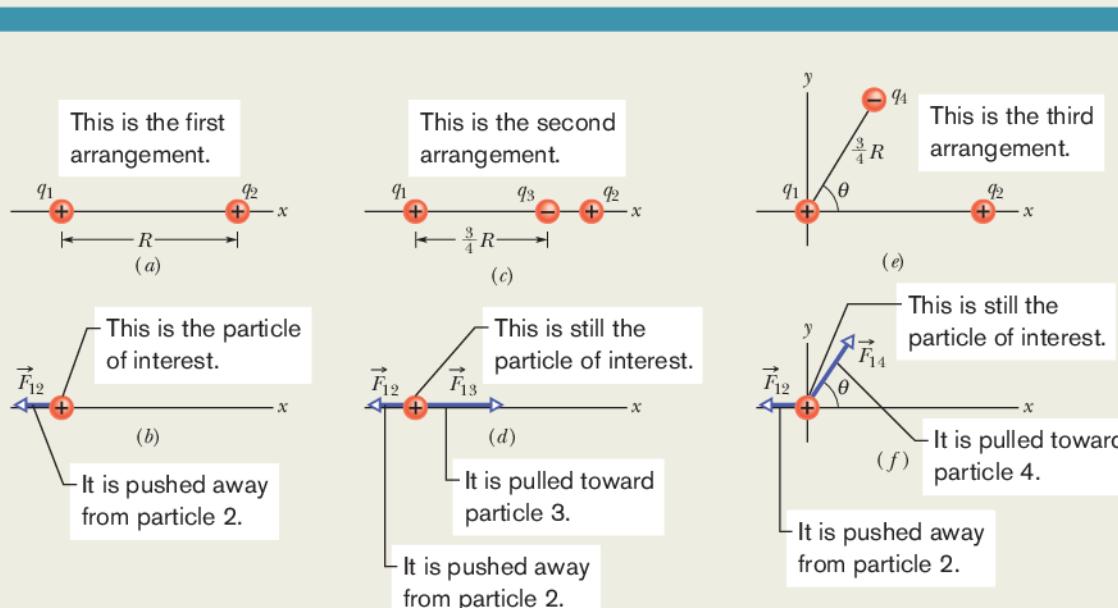
We can also write  $\vec{F}_{12}$  in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24} \text{ N})\hat{i}. \quad (\text{Answer})$$

(b) Figure 21-7c is identical to Fig. 21-7a except that particle 3 now lies on the  $x$  axis between particles 1 and 2. Particle 3 has charge  $q_3 = -3.20 \times 10^{-19} \text{ C}$  and is at a distance  $\frac{3}{4}R$  from particle 1. What is the net electrostatic force  $\vec{F}_{1,\text{net}}$  on particle 1 due to particles 2 and 3?

#### KEY IDEA

The presence of particle 3 does not alter the electrostatic force on particle 1 from particle 2. Thus, force  $\vec{F}_{12}$  still acts on particle 1. Similarly, the force  $\vec{F}_{13}$  that acts on particle 1 due to particle 3 is not affected by the presence of particle 2. Because particles 1



**Figure 21-7** (a) Two charged particles of charges  $q_1$  and  $q_2$  are fixed in place on an  $x$  axis. (b) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (c) Particle 3 included. (d) Free-body diagram for particle 1. (e) Particle 4 included. (f) Free-body diagram for particle 1.

and 3 have charge of opposite signs, particle 1 is attracted to particle 3. Thus, force  $\vec{F}_{13}$  is directed *toward* particle 3, as indicated in the free-body diagram of Fig. 21-7d.

**Three particles:** To find the magnitude of  $\vec{F}_{13}$ , we can rewrite Eq. 21-4 as

$$\begin{aligned} F_{13} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_3|}{(\frac{3}{4}R)^2} \\ &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(\frac{3}{4})^2(0.0200 \text{ m})^2} \\ &= 2.05 \times 10^{-24} \text{ N}. \end{aligned}$$

We can also write  $\vec{F}_{13}$  in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24} \text{ N})\hat{i}.$$

The net force  $\vec{F}_{1,\text{net}}$  on particle 1 is the vector sum of  $\vec{F}_{12}$  and  $\vec{F}_{13}$ ; that is, from Eq. 21-7, we can write the net force  $\vec{F}_{1,\text{net}}$  on particle 1 in unit-vector notation as

$$\begin{aligned} \vec{F}_{1,\text{net}} &= \vec{F}_{12} + \vec{F}_{13} \\ &= -(1.15 \times 10^{-24} \text{ N})\hat{i} + (2.05 \times 10^{-24} \text{ N})\hat{i} \\ &= (9.00 \times 10^{-25} \text{ N})\hat{i}. \end{aligned} \quad (\text{Answer})$$

Thus,  $\vec{F}_{1,\text{net}}$  has the following magnitude and direction (relative to the positive direction of the  $x$  axis):

$$9.00 \times 10^{-25} \text{ N} \quad \text{and} \quad 0^\circ. \quad (\text{Answer})$$

(c) Figure 21-7e is identical to Fig. 21-7a except that particle 4 is now included. It has charge  $q_4 = -3.20 \times 10^{-19} \text{ C}$ , is at a distance  $\frac{3}{4}R$  from particle 1, and lies on a line that makes an angle  $\theta = 60^\circ$  with the  $x$  axis. What is the net electrostatic force  $\vec{F}_{1,\text{net}}$  on particle 1 due to particles 2 and 4?

### KEY IDEA

The net force  $\vec{F}_{1,\text{net}}$  is the vector sum of  $\vec{F}_{12}$  and a new force  $\vec{F}_{14}$  acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. Thus, force  $\vec{F}_{14}$  on particle 1 is directed *toward* particle 4, at angle  $\theta = 60^\circ$ , as indicated in the free-body diagram of Fig. 21-7f.

**Four particles:** We can rewrite Eq. 21-4 as

$$\begin{aligned} F_{14} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_4|}{(\frac{3}{4}R)^2} \\ &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(\frac{3}{4})^2(0.0200 \text{ m})^2} \\ &= 2.05 \times 10^{-24} \text{ N}. \end{aligned}$$

Then from Eq. 21-7, we can write the net force  $\vec{F}_{1,\text{net}}$  on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces  $\vec{F}_{12}$  and  $\vec{F}_{14}$  are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

**Method 1. Summing directly on a vector-capable calculator.** For  $\vec{F}_{12}$ , we enter the magnitude  $1.15 \times 10^{-24}$  and the angle  $180^\circ$ . For  $\vec{F}_{14}$ , we enter the magnitude  $2.05 \times 10^{-24}$  and the angle  $60^\circ$ . Then we add the vectors.

**Method 2. Summing in unit-vector notation.** First we rewrite  $\vec{F}_{14}$  as

$$\vec{F}_{14} = (F_{14} \cos \theta)\hat{i} + (F_{14} \sin \theta)\hat{j}.$$

Substituting  $2.05 \times 10^{-24} \text{ N}$  for  $F_{14}$  and  $60^\circ$  for  $\theta$ , this becomes

$$\vec{F}_{14} = (1.025 \times 10^{-24} \text{ N})\hat{i} + (1.775 \times 10^{-24} \text{ N})\hat{j}.$$

Then we sum:

$$\begin{aligned} \vec{F}_{1,\text{net}} &= \vec{F}_{12} + \vec{F}_{14} \\ &= -(1.15 \times 10^{-24} \text{ N})\hat{i} \\ &\quad + (1.025 \times 10^{-24} \text{ N})\hat{i} + (1.775 \times 10^{-24} \text{ N})\hat{j} \\ &\approx (-1.25 \times 10^{-25} \text{ N})\hat{i} + (1.78 \times 10^{-24} \text{ N})\hat{j}. \end{aligned} \quad (\text{Answer})$$

**Method 3. Summing components axis by axis.** The sum of the  $x$  components gives us

$$\begin{aligned} F_{1,\text{net},x} &= F_{12,x} + F_{14,x} = F_{12} + F_{14} \cos 60^\circ \\ &= -1.15 \times 10^{-24} \text{ N} + (2.05 \times 10^{-24} \text{ N})(\cos 60^\circ) \\ &= -1.25 \times 10^{-25} \text{ N}. \end{aligned}$$

The sum of the  $y$  components gives us

$$\begin{aligned} F_{1,\text{net},y} &= F_{12,y} + F_{14,y} = 0 + F_{14} \sin 60^\circ \\ &= (2.05 \times 10^{-24} \text{ N})(\sin 60^\circ) \\ &= 1.78 \times 10^{-24} \text{ N}. \end{aligned}$$

The net force  $\vec{F}_{1,\text{net}}$  has the magnitude

$$F_{1,\text{net}} = \sqrt{F_{1,\text{net},x}^2 + F_{1,\text{net},y}^2} = 1.78 \times 10^{-24} \text{ N}. \quad (\text{Answer})$$

To find the direction of  $\vec{F}_{1,\text{net}}$ , we take

$$\theta = \tan^{-1} \frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0^\circ.$$

However, this is an unreasonable result because  $\vec{F}_{1,\text{net}}$  must have a direction between the directions of  $\vec{F}_{12}$  and  $\vec{F}_{14}$ . To correct  $\theta$ , we add  $180^\circ$ , obtaining

$$-86.0^\circ + 180^\circ = 94.0^\circ. \quad (\text{Answer})$$



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**Checkpoint 3**

The figure here shows three arrangements of an electron  $e$  and two protons  $p$ . (a) Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first. (b) In situation  $c$ , is the angle between the net force on the electron and the line labeled  $d$  less than or more than  $45^\circ$ ?

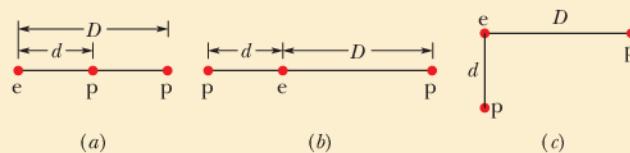
**Sample Problem 21.02 Equilibrium of two forces on a particle**

Figure 21-8a shows two particles fixed in place: a particle of charge  $q_1 = +8q$  at the origin and a particle of charge  $q_2 = -2q$  at  $x = L$ . At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)

**KEY IDEA**

If  $\vec{F}_1$  is the force on the proton due to charge  $q_1$  and  $\vec{F}_2$  is the force on the proton due to charge  $q_2$ , then the point we seek is where  $\vec{F}_1 + \vec{F}_2 = 0$ . Thus,

$$\vec{F}_1 = -\vec{F}_2. \quad (21-8)$$

This tells us that at the point we seek, the forces acting on the proton due to the other two particles must be of equal magnitudes,

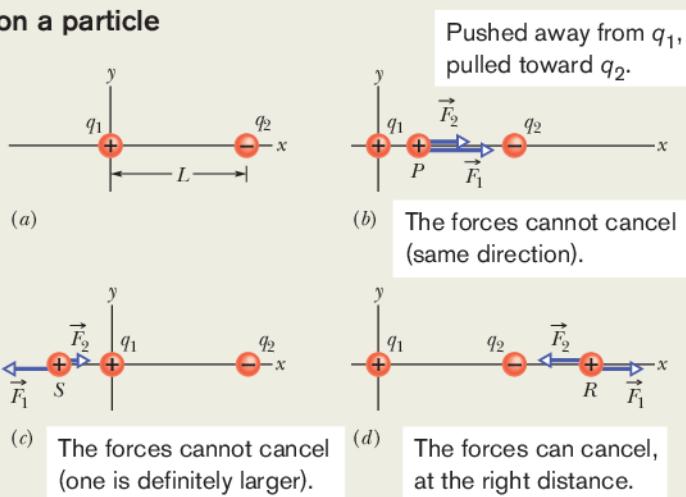
$$F_1 = F_2, \quad (21-9)$$

and that the forces must have opposite directions.

**Reasoning:** Because a proton has a positive charge, the proton and the particle of charge  $q_1$  are of the same sign, and force  $\vec{F}_1$  on the proton must point away from  $q_1$ . Also, the proton and the particle of charge  $q_2$  are of opposite signs, so force  $\vec{F}_2$  on the proton must point toward  $q_2$ . “Away from  $q_1$ ” and “toward  $q_2$ ” can be in opposite directions only if the proton is located on the  $x$  axis.

If the proton is on the  $x$  axis at any point between  $q_1$  and  $q_2$ , such as point  $P$  in Fig. 21-8b, then  $\vec{F}_1$  and  $\vec{F}_2$  are in the same direction and not in opposite directions as required. If the proton is at any point on the  $x$  axis to the left of  $q_1$ , such as point  $S$  in Fig. 21-8c, then  $\vec{F}_1$  and  $\vec{F}_2$  are in opposite directions. However, Eq. 21-4 tells us that  $\vec{F}_1$  and  $\vec{F}_2$  cannot have equal magnitudes there:  $F_1$  must be greater than  $F_2$ , because  $F_1$  is produced by a closer charge (with lesser  $r$ ) of greater magnitude ( $8q$  versus  $2q$ ).

Finally, if the proton is at any point on the  $x$  axis to the right of  $q_2$ , such as point  $R$  in Fig. 21-8d, then  $\vec{F}_1$  and  $\vec{F}_2$  are again in opposite directions. However, because now the charge of greater magnitude ( $q_1$ ) is *further* away from the proton than the charge of lesser magnitude, there is a point at which  $F_1$  is equal to  $F_2$ . Let  $x$  be the coordinate of this point, and let  $q_p$  be the charge of the proton.



**Figure 21-8** (a) Two particles of charges  $q_1$  and  $q_2$  are fixed in place on an  $x$  axis, with separation  $L$ . (b)–(d) Three possible locations  $P$ ,  $S$ , and  $R$  for a proton. At each location,  $\vec{F}_1$  is the force on the proton from particle 1 and  $\vec{F}_2$  is the force on the proton from particle 2.

**Calculations:** With Eq. 21-4, we can now rewrite Eq. 21-9:

$$\frac{1}{4\pi\epsilon_0} \frac{8qq_p}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{2qq_p}{(x-L)^2}. \quad (21-10)$$

(Note that only the charge magnitudes appear in Eq. 21-10. We already decided about the directions of the forces in drawing Fig. 21-8d and do not want to include any positive or negative signs here.) Rearranging Eq. 21-10 gives us

$$\left(\frac{x-L}{x}\right)^2 = \frac{1}{4}.$$

After taking the square roots of both sides, we find

$$\frac{x-L}{x} = \frac{1}{2}$$

and

$$x = 2L. \quad (\text{Answer})$$

The equilibrium at  $x = 2L$  is unstable; that is, if the proton is displaced leftward from point  $R$ , then  $F_1$  and  $F_2$  both increase but  $F_2$  increases more (because  $q_2$  is closer than  $q_1$ ), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both  $F_1$  and  $F_2$  decrease but  $F_2$  decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.



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### Sample Problem 21.03 Charge sharing by two identical conducting spheres

In Fig. 21-9a, two identical, electrically isolated conducting spheres *A* and *B* are separated by a (center-to-center) distance *a* that is large compared to the spheres. Sphere *A* has a positive charge of  $+Q$ , and sphere *B* is electrically neutral. Initially, there is no electrostatic force between the spheres. (The large separation means there is no induced charge.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

#### KEY IDEAS

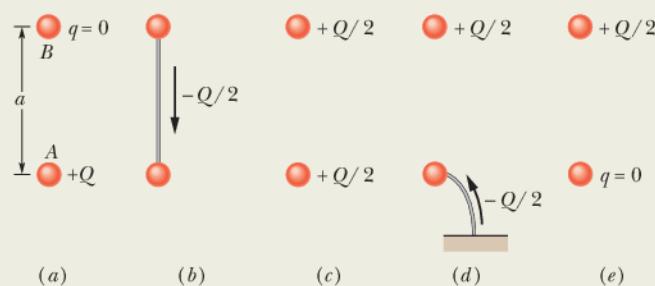
- (1) Because the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount).
- (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

**Reasoning:** When the spheres are wired together, the (negative) conduction electrons on *B*, which repel one another, have a way to move away from one another (along the wire to positively charged *A*, which attracts them—Fig. 21-9b). As *B* loses negative charge, it becomes positively charged, and as *A* gains negative charge, it becomes *less* positively charged. The transfer of charge stops when the charge on *B* has increased to  $+Q/2$  and the charge on *A* has decreased to  $+Q/2$ , which occurs when  $-Q/2$  has shifted from *B* to *A*.

After the wire has been removed (Fig. 21-9c), we can assume that the charge on either sphere does not disturb the uniformity of the charge distribution on the other sphere, because the spheres are small relative to their separation. Thus, we can apply the first shell theorem to each sphere. By Eq. 21-4 with  $q_1 = q_2 = Q/2$  and  $r = a$ ,



Additional examples, video, and practice available at WileyPLUS



**Figure 21-9** Two small conducting spheres *A* and *B*. (a) To start, sphere *A* is charged positively. (b) Negative charge is transferred from *B* to *A* through a connecting wire. (c) Both spheres are then charged positively. (d) Negative charge is transferred through a grounding wire to sphere *A*. (e) Sphere *A* is then neutral.

$$F = \frac{1}{4\pi\epsilon_0} \frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\epsilon_0} \left(\frac{Q}{a}\right)^2. \quad (\text{Answer})$$

The spheres, now positively charged, repel each other.

(b) Next, suppose sphere *A* is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?

**Reasoning:** When we provide a conducting path between a charged object and the ground (which is a huge conductor), we neutralize the object. Were sphere *A* negatively charged, the mutual repulsion between the excess electrons would cause them to move from the sphere to the ground. However, because sphere *A* is positively charged, electrons with a total charge of  $-Q/2$  move *from* the ground up onto the sphere (Fig. 21-9d), leaving the sphere with a charge of 0 (Fig. 21-9e). Thus, the electrostatic force is again zero.

## 21-2 CHARGE IS QUANTIZED

### Learning Objectives

After reading this module, you should be able to . . .

**21.19** Identify the elementary charge.

### Key Ideas

- Electric charge is quantized (restricted to certain values).
- The charge of a particle can be written as  $ne$ , where  $n$  is a positive or negative integer and  $e$  is the elementary charge,

**21.20** Identify that the charge of a particle or object must be a positive or negative integer times the elementary charge.

which is the magnitude of the charge of the electron and proton ( $\approx 1.602 \times 10^{-19}$  C).

## Charge Is Quantized

In Benjamin Franklin's day, electric charge was thought to be a continuous fluid—an idea that was useful for many purposes. However, we now know that

fluids themselves, such as air and water, are not continuous but are made up of atoms and molecules; matter is discrete. Experiment shows that “electrical fluid” is also not continuous but is made up of multiples of a certain elementary charge. Any positive or negative charge  $q$  that can be detected can be written as

$$q = ne, \quad n = \pm 1, \pm 2, \pm 3, \dots, \quad (21-11)$$

in which  $e$ , the **elementary charge**, has the approximate value

$$e = 1.602 \times 10^{-19} \text{ C.} \quad (21-12)$$

The elementary charge  $e$  is one of the important constants of nature. The electron and proton both have a charge of magnitude  $e$  (Table 21-1). (Quarks, the constituent particles of protons and neutrons, have charges of  $\pm e/3$  or  $\pm 2e/3$ , but they apparently cannot be detected individually. For this and for historical reasons, we do not take their charges to be the elementary charge.)

You often see phrases—such as “the charge on a sphere,” “the amount of charge transferred,” and “the charge carried by the electron”—that suggest that charge is a substance. (Indeed, such statements have already appeared in this chapter.) You should, however, keep in mind what is intended: *Particles* are the substance and charge happens to be one of their properties, just as mass is.

When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is **quantized**. It is possible, for example, to find a particle that has no charge at all or a charge of  $+10e$  or  $-6e$ , but not a particle with a charge of, say,  $3.57e$ .

The quantum of charge is small. In an ordinary 100 W lightbulb, for example, about  $10^{19}$  elementary charges enter the bulb every second and just as many leave. However, the graininess of electricity does not show up in such large-scale phenomena (the bulb does not flicker with each electron).



### Checkpoint 4

Initially, sphere  $A$  has a charge of  $-50e$  and sphere  $B$  has a charge of  $+20e$ . The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere  $A$ ?



### Sample Problem 21.04 Mutual electric repulsion in a nucleus

The nucleus in an iron atom has a radius of about  $4.0 \times 10^{-15} \text{ m}$  and contains 26 protons.

- (a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by  $4.0 \times 10^{-15} \text{ m}$ ?

#### KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

**Calculation:** Table 21-1 tells us that the charge of a proton is  $+e$ . Thus, Eq. 21-4 gives us

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \\ &= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{(4.0 \times 10^{-15} \text{ m})^2} \\ &= 14 \text{ N.} \end{aligned}$$

(Answer)

**No explosion:** This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force to be acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force.

- (b) What is the magnitude of the gravitational force between those same two protons?

#### KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-2).

**Calculation:** With  $m_p (= 1.67 \times 10^{-27} \text{ kg})$  representing the

mass of a proton, Eq. 21-2 gives us

$$\begin{aligned} F &= G \frac{m_p^2}{r^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.67 \times 10^{-27} \text{ kg})^2}{(4.0 \times 10^{-15} \text{ m})^2} \\ &= 1.2 \times 10^{-35} \text{ N.} \quad (\text{Answer}) \end{aligned}$$

**Weak versus strong:** This result tells us that the (attractive) gravitational force is far too weak to counter the repulsive electrostatic forces between protons in a nucleus. Instead, the protons are bound together by an enormous force called

(aptly) the *strong nuclear force*—a force that acts between protons (and neutrons) when they are close together, as in a nucleus.

Although the gravitational force is many times weaker than the electrostatic force, it is more important in large-scale situations because it is always attractive. This means that it can collect many small bodies into huge bodies with huge masses, such as planets and stars, that then exert large gravitational forces. The electrostatic force, on the other hand, is repulsive for charges of the same sign, so it is unable to collect either positive charge or negative charge into large concentrations that would then exert large electrostatic forces.



Additional examples, video, and practice available at WileyPLUS



## 21-3 CHARGE IS CONSERVED

### Learning Objectives

After reading this module, you should be able to . . .

**21.21** Identify that in any isolated physical process, the net charge cannot change (the net charge is always conserved).

### Key Ideas

- The net electric charge of any isolated system is always conserved.
- If two charged particles undergo an annihilation process,

**21.22** Identify an annihilation process of particles and a pair production of particles.

**21.23** Identify mass number and atomic number in terms of the number of protons, neutrons, and electrons.

they have opposite signs of charge.

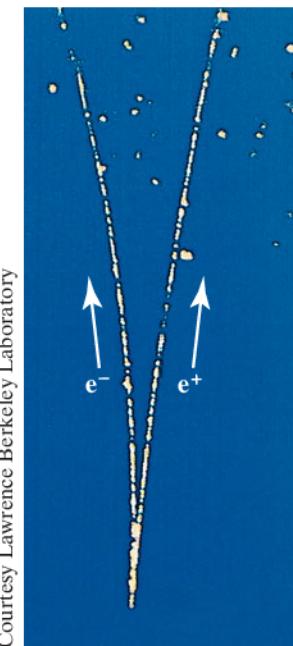
- If two charged particles appear as a result of a pair production process, they have opposite signs of charge.

### Charge Is Conserved

If you rub a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body during the process. This hypothesis of **conservation of charge**, first put forward by Benjamin Franklin, has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles. No exceptions have ever been found. Thus, we add electric charge to our list of quantities—including energy and both linear momentum and angular momentum—that obey a conservation law.

Important examples of the conservation of charge occur in the *radioactive decay* of nuclei, in which a nucleus transforms into (becomes) a different type of nucleus. For example, a uranium-238 nucleus ( $^{238}\text{U}$ ) transforms into a thorium-234 nucleus ( $^{234}\text{Th}$ ) by emitting an *alpha particle*. Because that particle has the same makeup as a helium-4 nucleus, it has the symbol  $^4\text{He}$ . The number used in the name of a nucleus and as a superscript in the symbol for the nucleus is called the *mass number* and is the total number of the protons and neutrons in the nucleus. For example, the total number in  $^{238}\text{U}$  is 238. The number of protons in a nucleus is the *atomic number Z*, which is listed for all the elements in Appendix F. From that list we find that in the decay





Courtesy Lawrence Berkeley Laboratory

**Figure 21-10** A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom. Being electrically neutral, the gamma ray did not generate a telltale trail of bubbles along its path, as the electron and positron did.

the *parent* nucleus  $^{238}\text{U}$  contains 92 protons (a charge of  $+92e$ ), the *daughter* nucleus  $^{234}\text{Th}$  contains 90 protons (a charge of  $+90e$ ), and the emitted alpha particle  $^4\text{He}$  contains 2 protons (a charge of  $+2e$ ). We see that the total charge is  $+92e$  before and after the decay; thus, charge is conserved. (The total number of protons and neutrons is also conserved: 238 before the decay and  $234 + 4 = 238$  after the decay.)

Another example of charge conservation occurs when an electron  $e^-$  (charge  $-e$ ) and its antiparticle, the *positron*  $e^+$  (charge  $+e$ ), undergo an *annihilation process*, transforming into two *gamma rays* (high-energy light):

$$e^- + e^+ \rightarrow \gamma + \gamma \quad (\text{annihilation}). \quad (21-14)$$

In applying the conservation-of-charge principle, we must add the charges algebraically, with due regard for their signs. In the annihilation process of Eq. 21-14 then, the net charge of the system is zero both before and after the event. Charge is conserved.

In *pair production*, the converse of annihilation, charge is also conserved. In this process a gamma ray transforms into an electron and a positron:

$$\gamma \rightarrow e^- + e^+ \quad (\text{pair production}). \quad (21-15)$$

Figure 21-10 shows such a pair-production event that occurred in a bubble chamber. (This is a device in which a liquid is suddenly made hotter than its boiling point. If a charged particle passes through it, tiny vapor bubbles form along the particle's trail.) A gamma ray entered the chamber from the bottom and at one point transformed into an electron and a positron. Because those new particles were charged and moving, each left a trail of bubbles. (The trails were curved because a magnetic field had been set up in the chamber.) The gamma ray, being electrically neutral, left no trail. Still, you can tell exactly where it underwent pair production—at the tip of the curved V, which is where the trails of the electron and positron begin.

## Review & Summary

**Electric Charge** The strength of a particle's electrical interaction with objects around it depends on its **electric charge** (usually represented as  $q$ ), which can be either positive or negative. Particles with the same sign of charge repel each other, and particles with opposite signs of charge attract each other. An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged and has an excess charge.

**Conductors** are materials in which a significant number of electrons are free to move. The charged particles in **nonconductors (insulators)** are not free to move.

Electric current  $i$  is the rate  $dq/dt$  at which charge passes a point:

$$i = \frac{dq}{dt} \quad (\text{electric current}). \quad (21-3)$$

**Coulomb's Law** Coulomb's law describes the electrostatic force (or electric force) between two charged particles. If the particles have charges  $q_1$  and  $q_2$ , are separated by distance  $r$ , and are at rest (or moving only slowly) relative to each other, then the magnitude of the force acting on each due to the other is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1| |q_2|}{r^2} \quad (\text{Coulomb's law}), \quad (21-4)$$

where  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$  is the **permittivity constant**. The ratio  $1/4\pi\epsilon_0$  is often replaced with the **electrostatic constant** (or **Coulomb constant**)  $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ .

The electrostatic force vector acting on a charged particle due to a second charged particle is either directly toward the second particle (opposite signs of charge) or directly away from it (same sign of charge). As with other types of forces, if multiple electrostatic forces act on a particle, the net force is the vector sum (not scalar sum) of the individual forces.

The two shell theories for electrostatics are

*Shell theorem 1:* A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

*Shell theorem 2:* A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

Charge on a conducting spherical shell spreads uniformly over the (external) surface.

**The Elementary Charge** Electric charge is quantized (restricted to certain values). The charge of a particle can be written as  $ne$ , where  $n$  is a positive or negative integer and  $e$  is the elementary charge, which is the magnitude of the charge of the electron and proton ( $\approx 1.602 \times 10^{-19} \text{ C}$ ).

**Conservation of Charge** The net electric charge of any isolated system is always conserved.

## Questions

- 1** Figure 21-11 shows four situations in which five charged particles are evenly spaced along an axis. The charge values are indicated except for the central particle, which has the same charge in all four situations. Rank the situations according to the magnitude of the net electrostatic force on the central particle, greatest first.

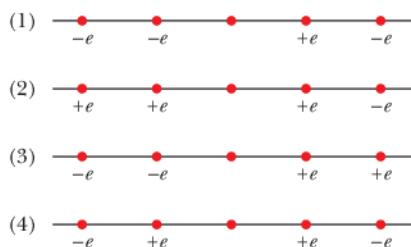


Figure 21-11 Question 1.

- 2** Figure 21-12 shows three pairs of identical spheres that are to be touched together and then separated. The initial charges on them are indicated. Rank the pairs according to (a) the magnitude of the charge transferred during touching and (b) the charge left on the positively charged sphere, greatest first.

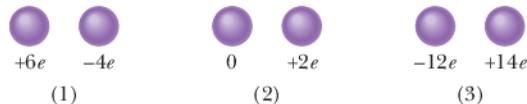


Figure 21-12 Question 2.

- 3** Figure 21-13 shows four situations in which charged particles are fixed in place on an axis. In which situations is there a point to the left of the particles where an electron will be in equilibrium?

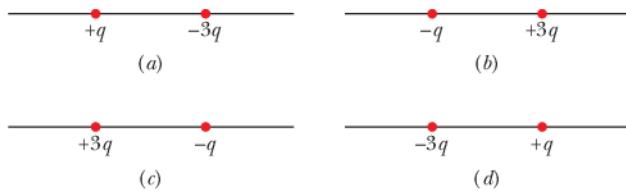


Figure 21-13 Question 3.

- 4** Figure 21-14 shows two charged particles on an axis. The charges are free to move. However, a third charged particle can be placed at a certain point such that all three particles are then in equilibrium. (a) Is that point to the left of the first two particles, to their right, or between them? (b) Should the third particle be positively or negatively charged? (c) Is the equilibrium stable or unstable?



Figure 21-14 Question 4.

- 5** In Fig. 21-15, a central particle of charge  $-q$  is surrounded by two circular rings of charged particles. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (Hint: Consider symmetry.)

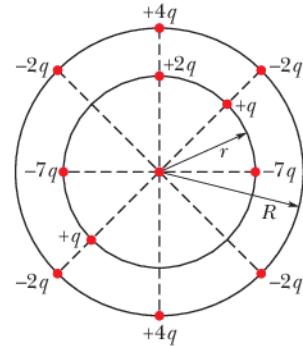


Figure 21-15 Question 5.

- 6** A positively charged ball is brought close to an electrically neutral isolated conductor. The conductor is then grounded while the ball is kept close. Is the conductor charged positively, charged negatively, or neutral if (a) the ball is first taken away and then the

ground connection is removed and (b) the ground connection is first removed and then the ball is taken away?

- 7** Figure 21-16 shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. Rank the situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.

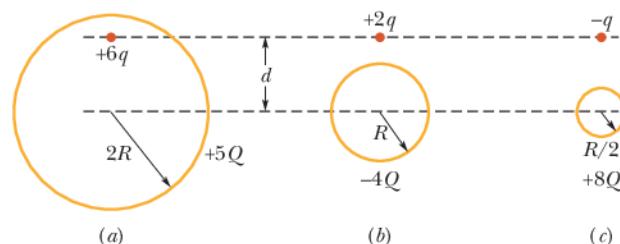


Figure 21-16 Question 7.

- 8** Figure 21-17 shows four arrangements of charged particles. Rank the arrangements according to the magnitude of the net electrostatic force on the particle with charge  $+Q$ , greatest first.

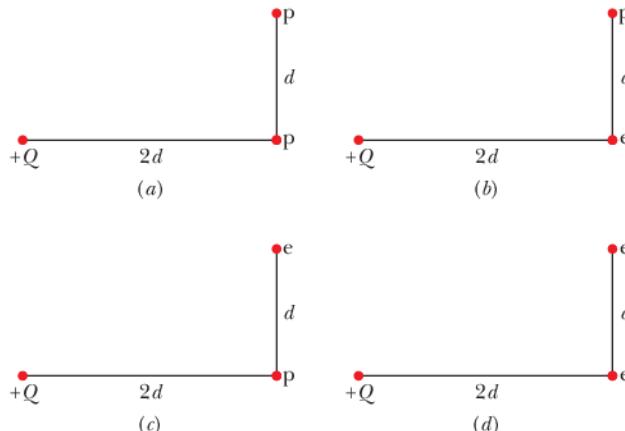


Figure 21-17 Question 8.

- 9** Figure 21-18 shows four situations in which particles of charge  $+q$  or  $-q$  are fixed in place. In each situation, the parti-

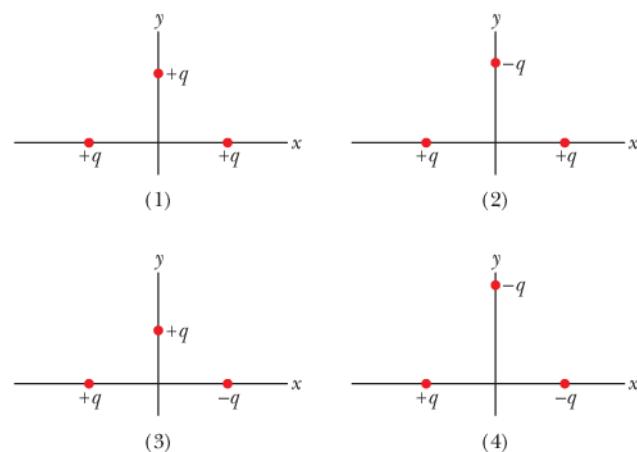


Figure 21-18 Question 9.

cles on the  $x$  axis are equidistant from the  $y$  axis. First, consider the middle particle in situation 1; the middle particle experiences an electrostatic force from each of the other two particles. (a) Are the magnitudes  $F$  of those forces the same or different? (b) Is the magnitude of the net force on the middle particle equal to, greater than, or less than  $2F$ ? (c) Do the  $x$  components of the two forces add or cancel? (d) Do their  $y$  components add or cancel? (e) Is the direction of the net force on the middle particle that of the canceling components or the adding components? (f) What is the direction of that net force? Now consider the remaining situations: What is the direction of the net force on the middle particle in (g) situation 2, (h) situation 3, and (i) situation 4? (In each situation, consider the symmetry of the charge distribution and determine the canceling components and the adding components.)

**10** In Fig. 21-19, a central particle of charge  $-2q$  is surrounded by a square array of charged particles, separated by either distance  $d$  or  $d/2$  along the perimeter of the square. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consideration of symmetry can greatly reduce the amount of work required here.)

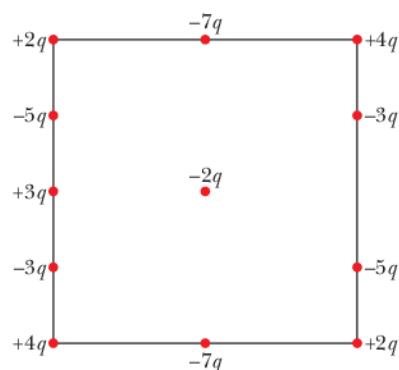


Figure 21-19 Question 10.

**11** Figure 21-20 shows three identical conducting bubbles  $A$ ,  $B$ , and  $C$  floating in a con-

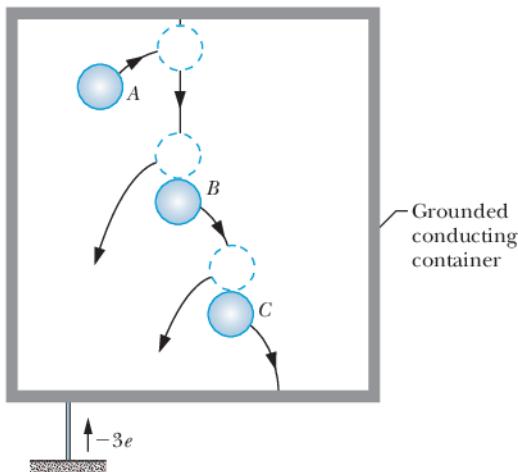


Figure 21-20 Question 11.

ducting container that is grounded by a wire. The bubbles initially have the same charge. Bubble  $A$  bumps into the container's ceiling and then into bubble  $B$ . Then bubble  $B$  bumps into bubble  $C$ , which then drifts to the container's floor. When bubble  $C$  reaches the floor, a charge of  $-3e$  is transferred upward through the wire, from the ground to the container, as indicated. (a) What was the initial charge of each bubble? When (b) bubble  $A$  and (c) bubble  $B$  reach the floor, what is the charge transfer through the wire? (d) During this whole process, what is the total charge transfer through the wire?

**12** Figure 21-21 shows four situations in which a central proton is partially surrounded by protons or electrons fixed in place along a half-circle. The angles  $\theta$  are identical; the angles  $\phi$  are also. (a) In each situation, what is the direction of the net force on the central proton due to the other particles? (b) Rank the four situations according to the magnitude of that net force on the central proton, greatest first.

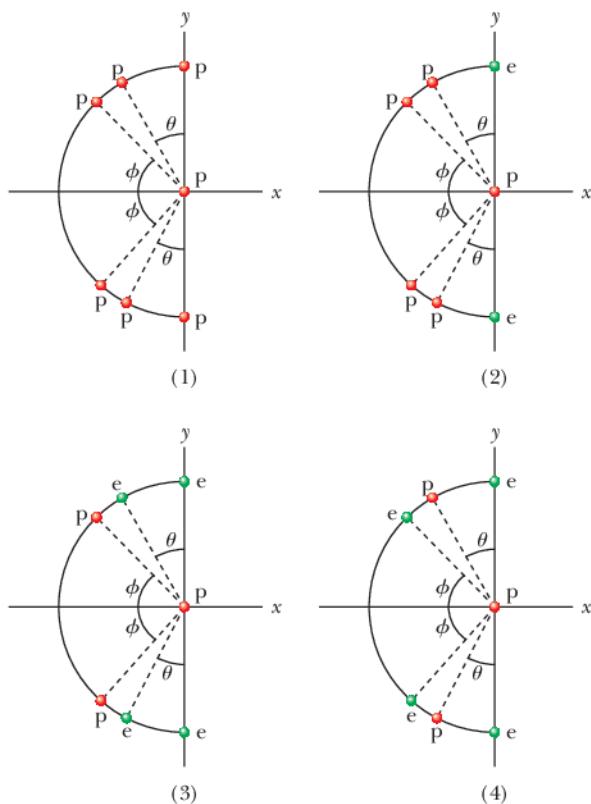


Figure 21-21 Question 12.

## Problems



Tutoring problem available (at instructor's discretion) in *WileyPLUS* and *WebAssign*



Worked-out solution available in *Student Solutions Manual*

**WWW** Worked-out solution is at

<http://www.wiley.com/college/halliday>



Number of dots indicates level of problem difficulty

**ILW** Interactive solution is at



Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://flyingcircusofphysics.com)

### Module 21-1 Coulomb's Law

- **1 SSM ILW** Of the charge  $Q$  initially on a tiny sphere, a portion  $q$  is to be transferred to a second, nearby sphere. Both spheres

can be treated as particles and are fixed with a certain separation. For what value of  $q/Q$  will the electrostatic force between the two spheres be maximized?