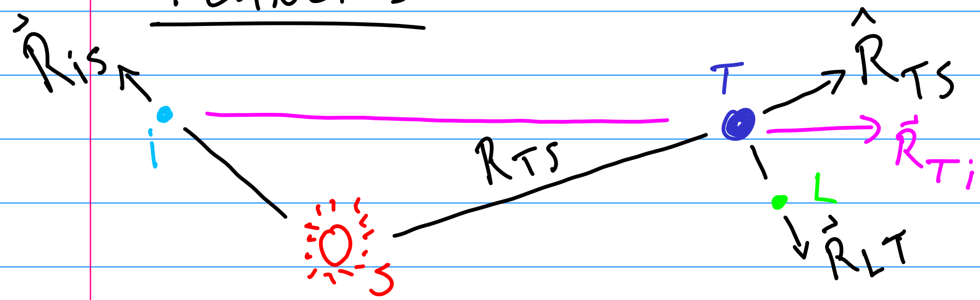
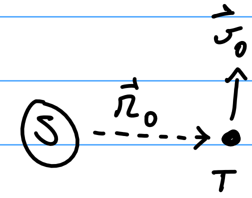
Planetas

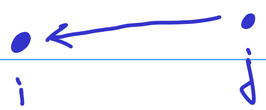




$$\vec{R}_{TS} = \vec{R}_T - \vec{R}_S \,, \qquad \vec{R}_S = 0$$

$$\vec{R}_{ij} = \vec{R}_i - \vec{R}_j$$

Força: $$\vec{F}_{ij} = G M_i M_j \cdot \frac{\vec{R}_{ij}}{R_{ij}^3}$$



$$\vec{F}_{ij} = M_j \, \vec{a}_{ij}$$

$$\vec{a}_{ij} = G M_i \frac{\vec{R}_{ij}}{R_{ij}^3}$$

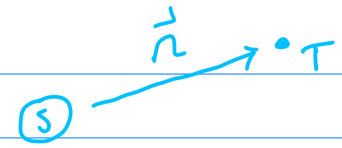
Aceleração Total:

$$\vec{a}_j = \sum_{i \neq j} \vec{a}_{ij}$$

$$\vec{z}(t) = \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}$$

Runge-Kutta: $$\frac{\partial \vec{z}}{\partial t} = \begin{pmatrix} \vec{v} \\ \vec{a} \end{pmatrix} = f(\vec{z}, t) = \begin{pmatrix} v_x \\ v_y \\ a_x \\ a_y \end{pmatrix}$$

Terra - Sol



$\vec{r} = (x, y)$

$$\cancel{m_T} \frac{\partial^2 \vec{r}}{\partial t^2} = - G \cancel{m_T} \cdot M_s \frac{\vec{r}}{r^3}$$

Linear: Redução de ordem

$$\vec{z}(t) = \begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \end{pmatrix}$$

$$\begin{cases} \dfrac{\partial}{\partial t} \vec{r}(t) = \vec{v}(t) \qquad \dfrac{\partial}{\partial t} \vec{v}(t) = \vec{a}(\vec{r}(t)) \\ \vec{a}(\vec{r}) = - G M_s \dfrac{\vec{r}}{r^3} \end{cases}$$

$$\frac{\partial}{\partial t} \vec{z}(t) = \vec{f}(\vec{z}(t), t)$$

RK

Euler

...

odeint ⇝

+ cond. inicial

$$\vec{f}(\vec{z}, \cancel{t}) = \begin{pmatrix} \vec{v} \\ \vec{a} \end{pmatrix}$$

$\vec{z} = (x, y, v_x, v_y)$

$\vec{a} = \vec{a}(\vec{r})$

$t=0 \quad \Delta t \quad 2\Delta t \quad \ldots \quad t$

$\vec{z}_0 \begin{cases} \vec{r}_0 \\ \vec{v}_0 \end{cases} \qquad \vec{z}_1 = \begin{pmatrix} \vec{r}_1 \\ \vec{v}_1 \end{pmatrix}$

Euler: $\vec{z}_1 = \vec{z}_0 + \Delta t \, \vec{f}(\vec{z}_0, t_0)$

$$\vec{r}_1 = \vec{r}_0 + \Delta t \, \vec{v}_0$$
$$\vec{v}_1 = \vec{v}_0 + \Delta t \, \vec{a}(\vec{r}_0)$$