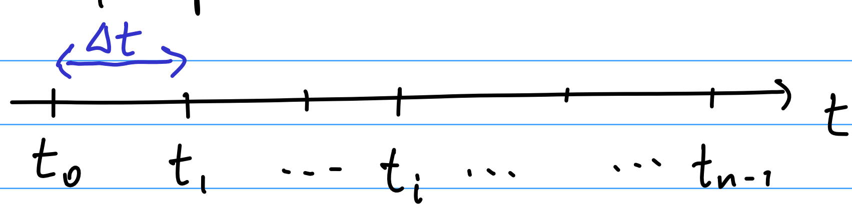
Equações diferenciais ordinárias

→ métodos fundamentais:

- Euler: $\dfrac{\partial}{\partial t}\vec{y}(t) = \vec{f}(\vec{y}, t)$
- Runge-Kutta: ↗
- Verlet: $\dfrac{\partial^2 \vec{y}(t)}{\partial t^2} = \vec{a}(\vec{y}, t)$

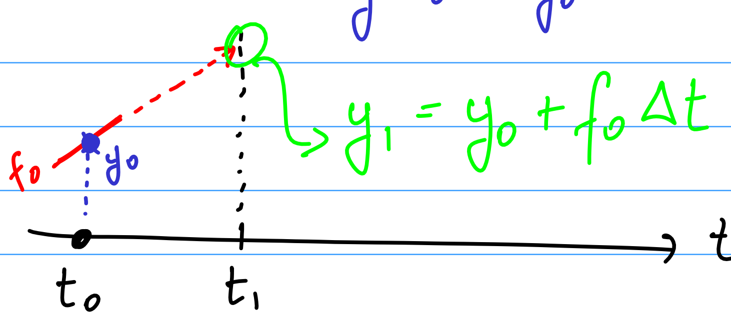
Euler

Tempo discreto



Quero resolver: $\dfrac{\partial y}{\partial t} = f(y, t)$

$y(t_0) = y_0$: condição inicial



$y_1 = y_0 + f_0 \Delta t$

Euler

Definição de derivada: $\dfrac{\partial y}{\partial t} = \lim\limits_{\Delta t \to 0} \boxed{\dfrac{y(t+\Delta t) - y(t)}{\Delta t} = f(y,t)}$

Assim, o método de Euler fica:

$$y_{i+1} = y_i + f(y_i, t_i)\,\Delta t = y_i + f_i\,\Delta t$$

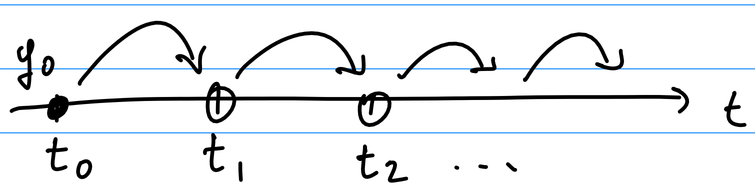
cond. inicial: y_0

$i=0$: $y_1 = y_0 + f_0\,\Delta t$

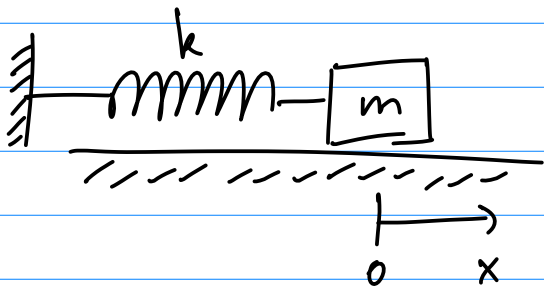
$i=1$: $y_2 = y_1 + f_1\,\Delta t$

$i=2$: $y_3 = y_2 + f_2\,\Delta t$

...



Ex.: Pendulo ou mola



$$m \frac{\partial^2 x}{\partial t^2} = -kx - bv$$

sendo $v = \frac{\partial x}{\partial t}$

$\omega^2 = \frac{k}{m}$ $\qquad \gamma = \frac{b}{m}$

$\Rightarrow$ **Redução de ordem**

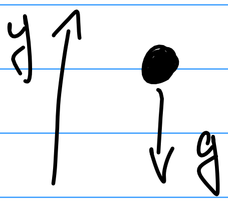
$$v = \frac{\partial x}{\partial t} \longrightarrow \frac{\partial v}{\partial t} = -\omega^2 x - \gamma v = a(x, v, t)$$

juntá-las numa única eq. vetorial

$$\frac{\partial}{\partial t} \underbrace{\begin{pmatrix} x \\ v \end{pmatrix}}_{=\vec{y}} = \underbrace{\begin{pmatrix} v \\ a(x, v, t) \end{pmatrix}}_{\vec{f}(y, t)}$$

$$\vec{y} = \begin{pmatrix} x \\ v \end{pmatrix} \qquad \vec{f}(\vec{y}, t) = \begin{pmatrix} y_1 \\ a(y_0, y_1, t) \end{pmatrix}$$

Ex: Queda livre



$$\frac{\partial^2 y}{\partial t^2} = -g - \gamma v$$

$$\boxed{\frac{\partial v}{\partial t} = -g - \gamma v}$$

cond. inicial

$v(0) = v_0 = 0$