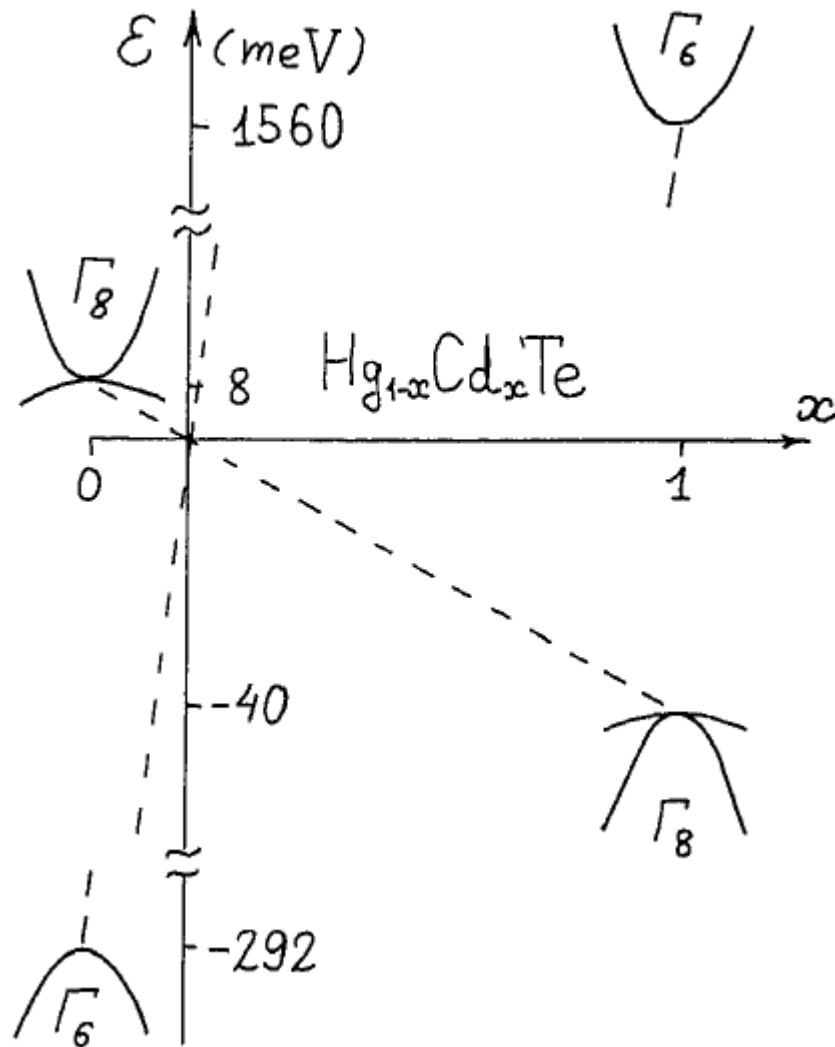


# Modelo BHZ



**Volkov, Pankratov: 1985**

[JETP Lett. 42, 178 ('85)]

[Solid State Comm. 61, 93 ('87)]

[Landau Level Spectroscopy, Chap. 14 ('91)]

**Bernevig, Hughes, Zhang: 2006**

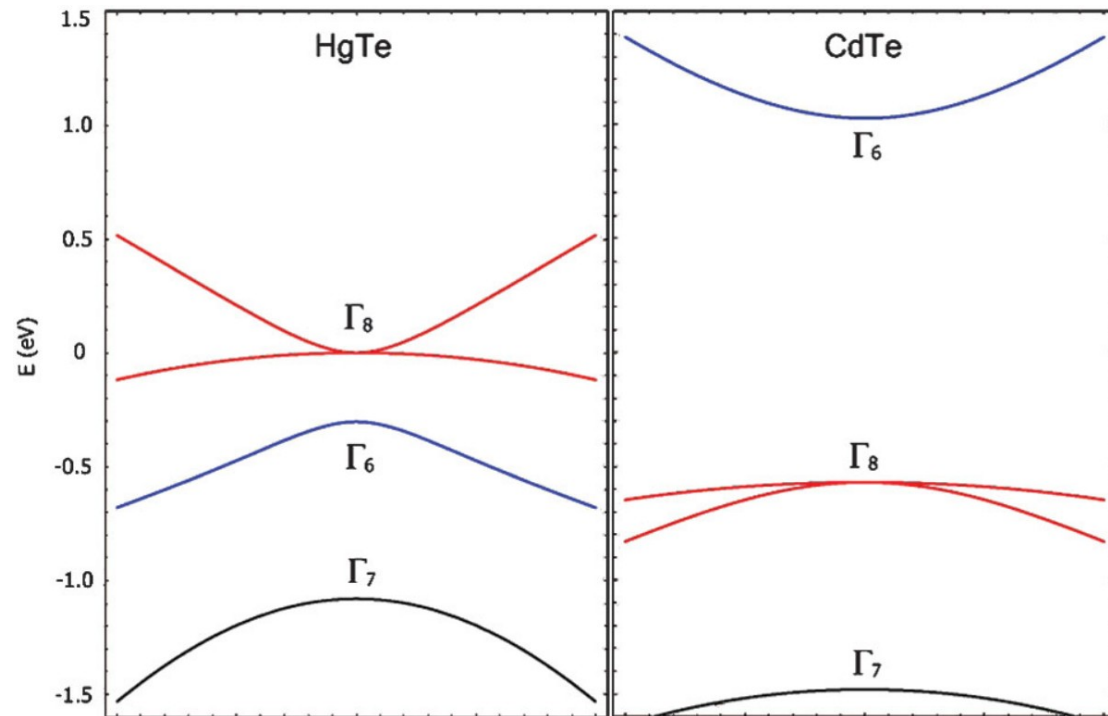
[Science 314, 1757 (2006)]

Bandas de condução/valência do **HgTe** ( $x=0$ ), não podem ser **continuamente deformadas** nas bandas do **CdTe** ( $x=1$ )

# Inversão de bandas em bulk

$\Gamma_6$   
Banda tipo S

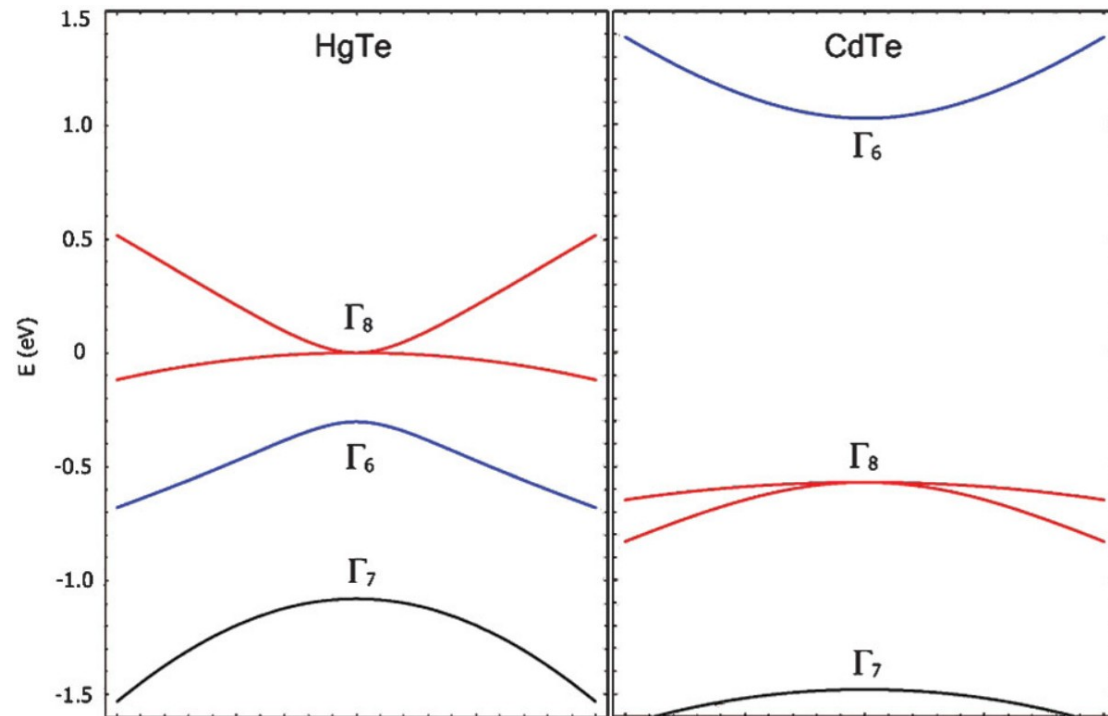
$\Gamma_8$   
Banda tipo P



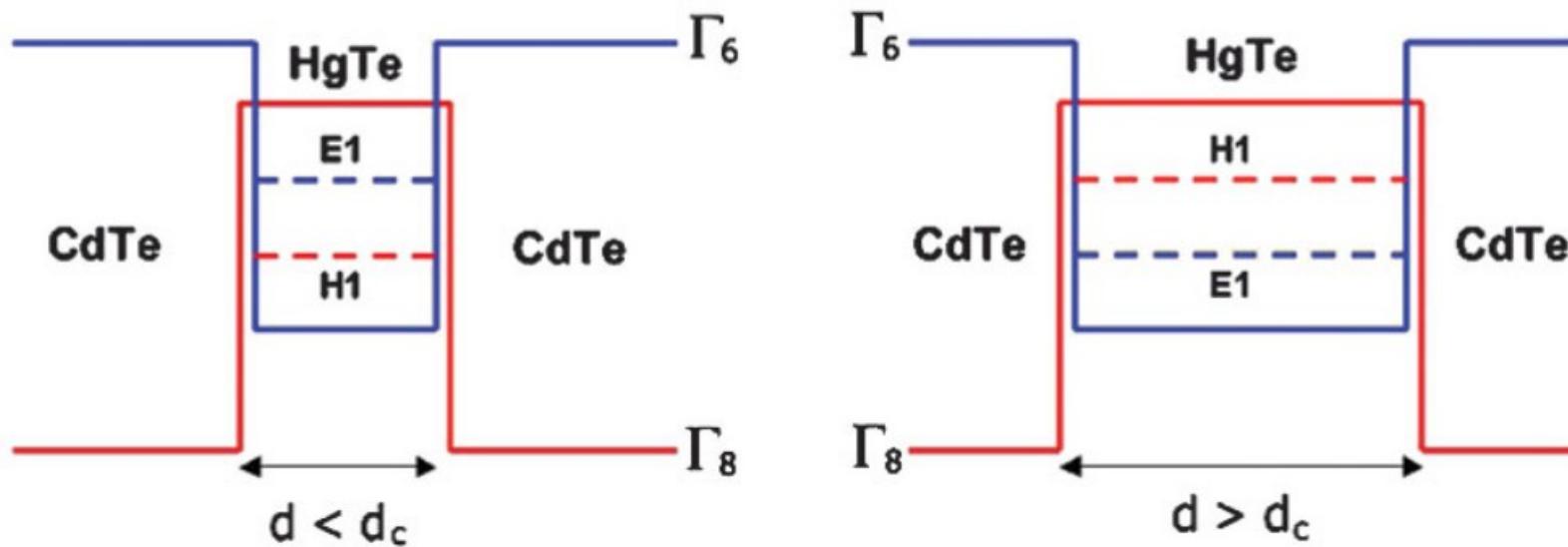
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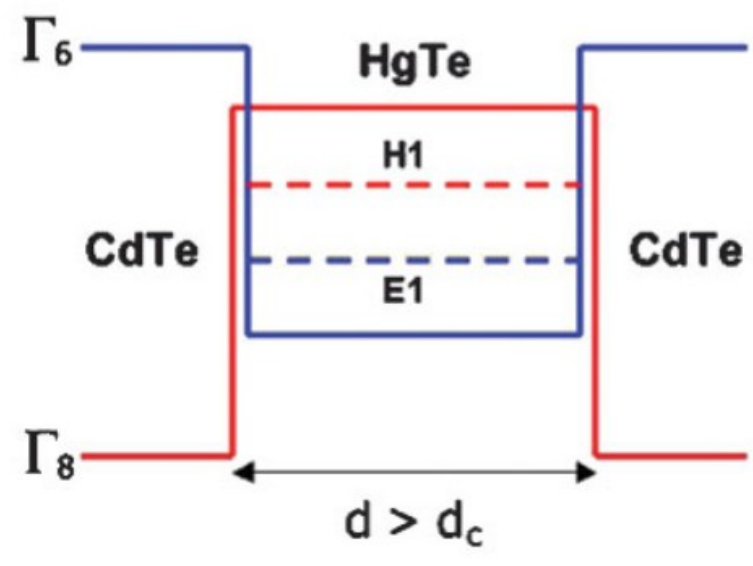
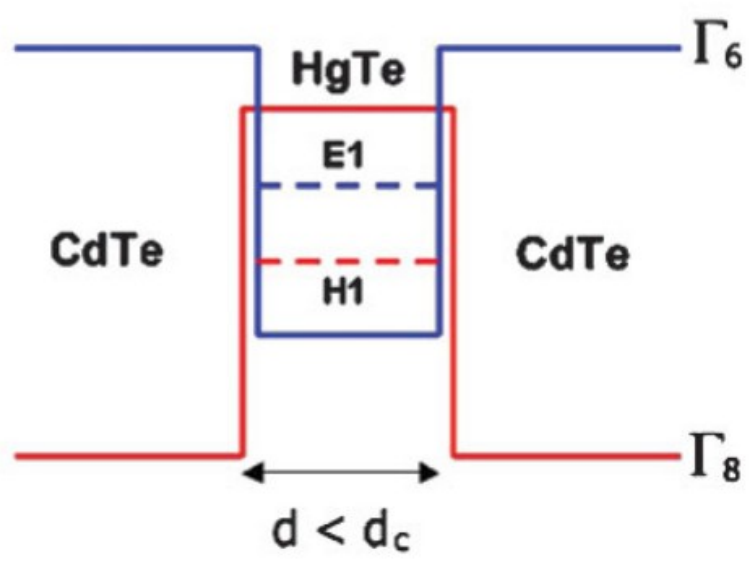
$\Gamma_6$   
Banda tipo S

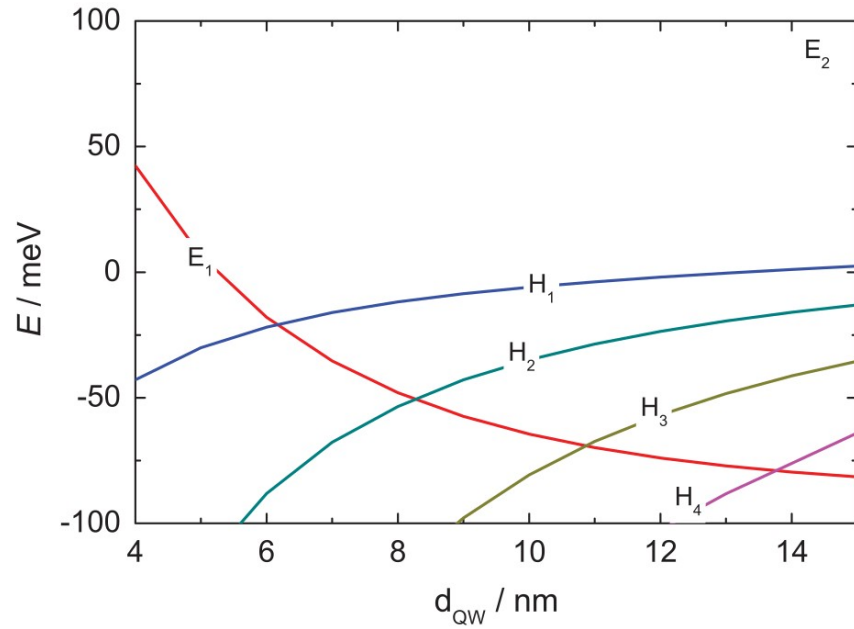
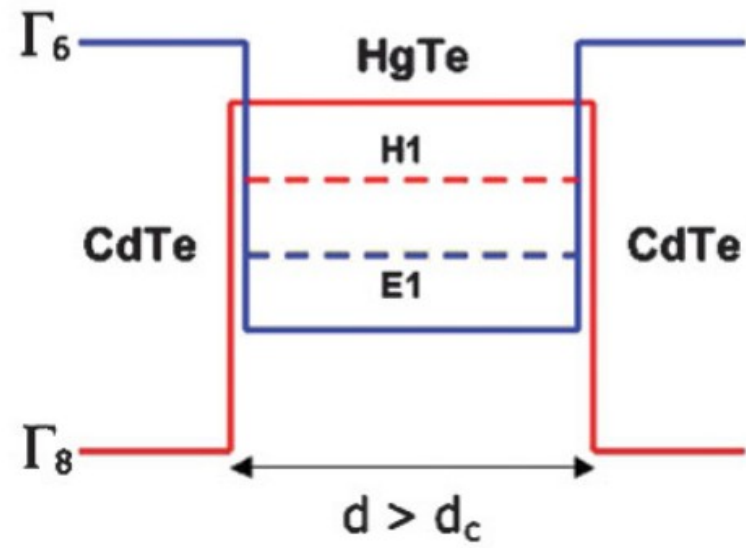
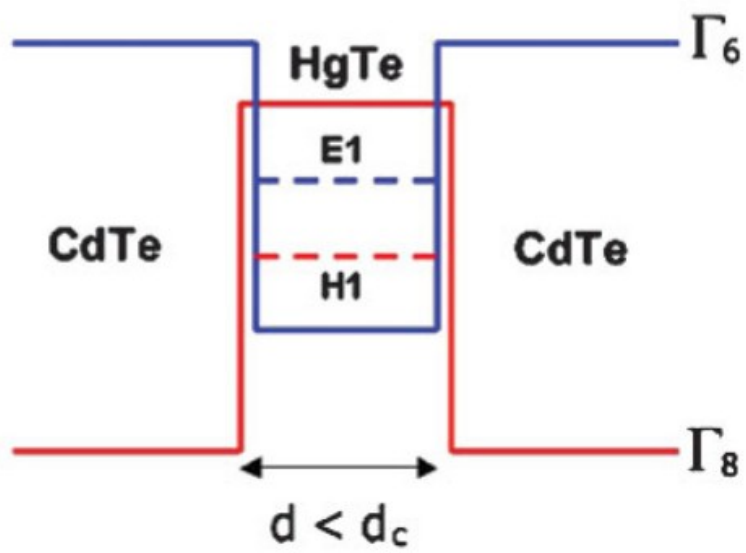
$\Gamma_8$   
Banda tipo P

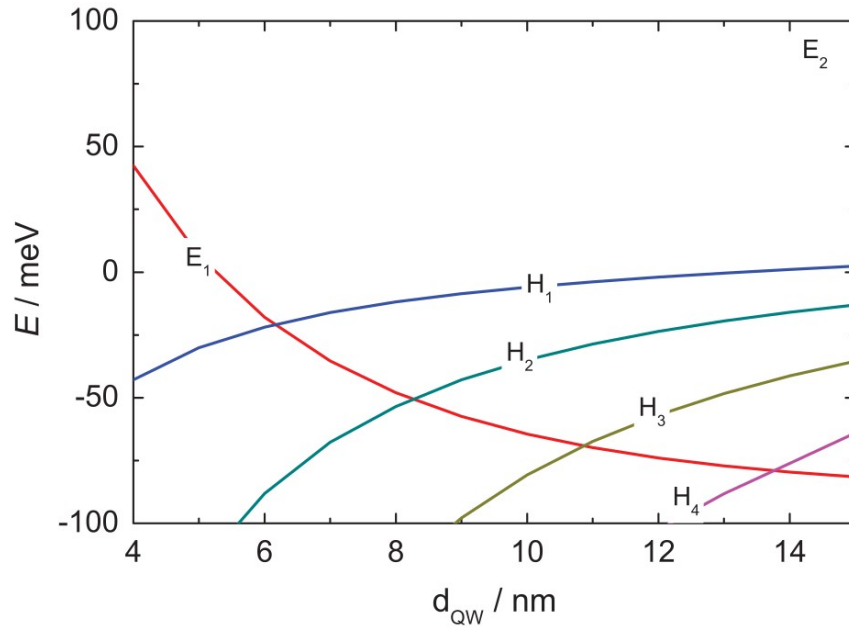
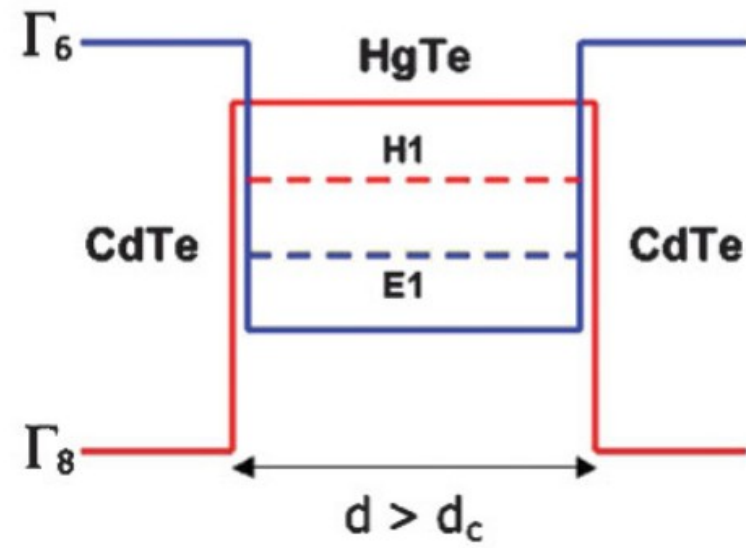
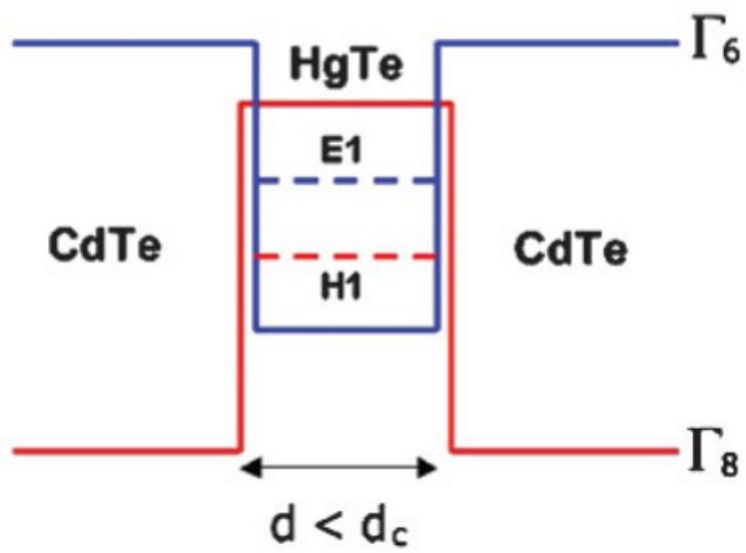


Heteroestrutura

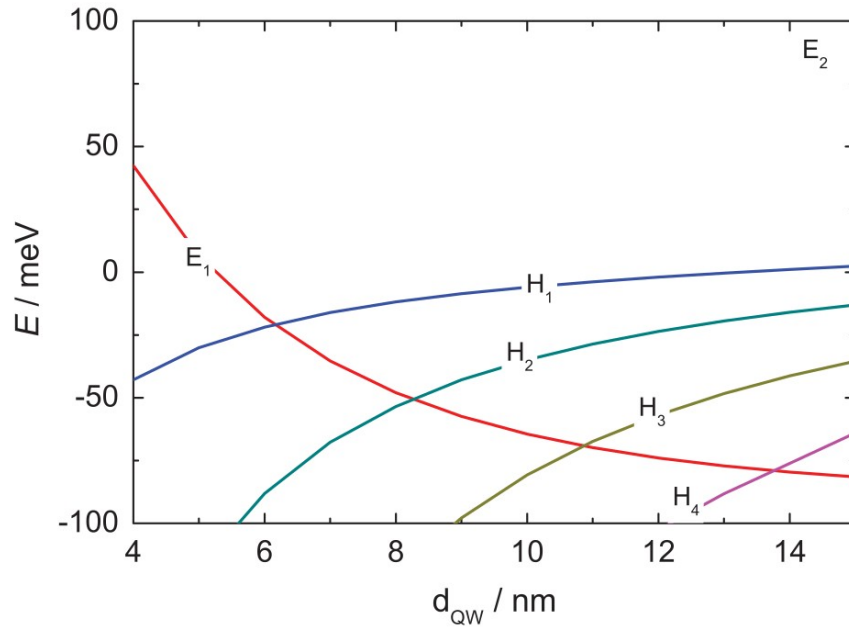
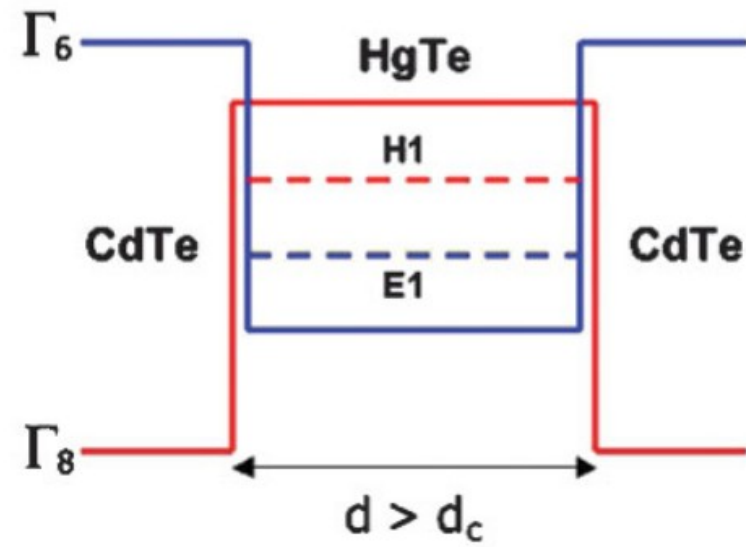
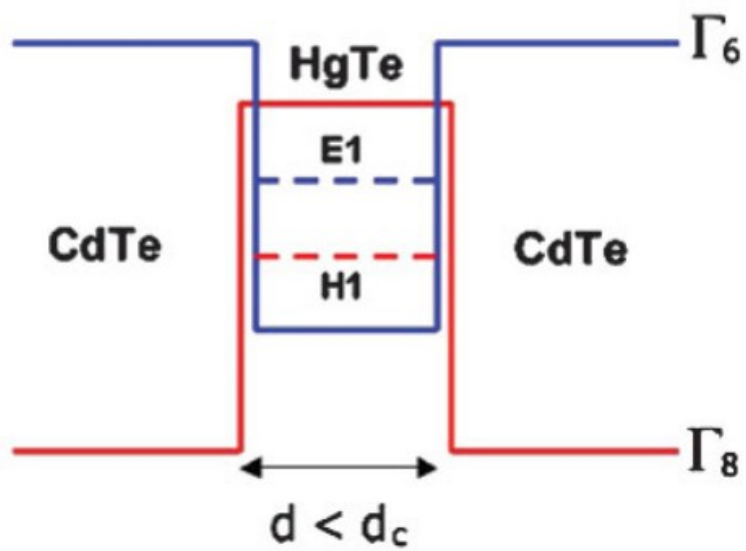






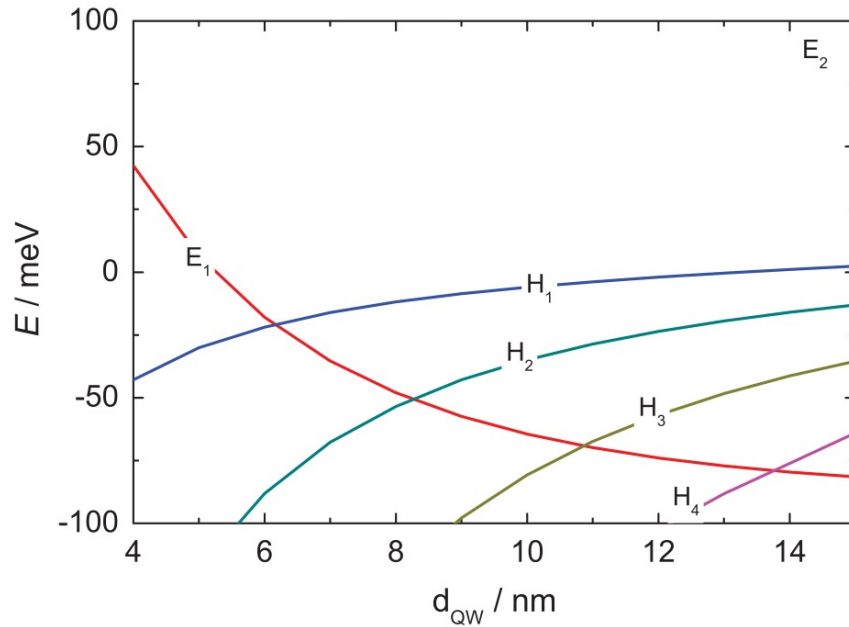
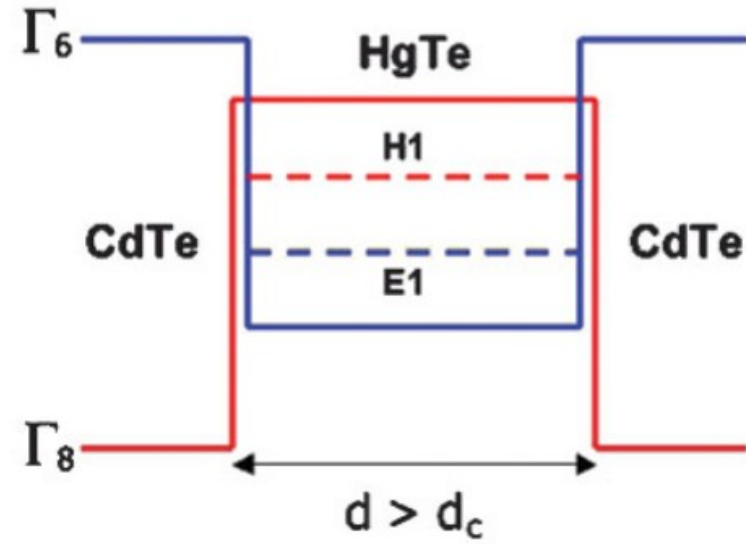
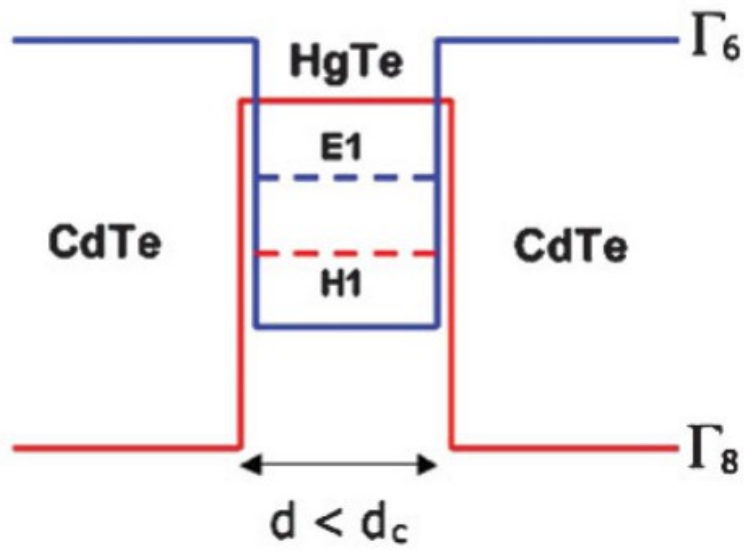


Estados confinados



Estados confinados

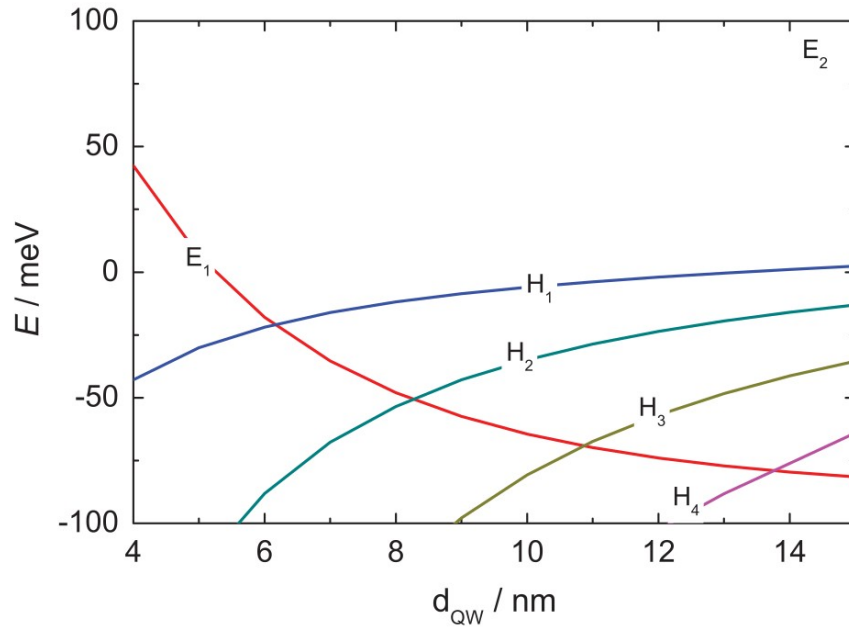
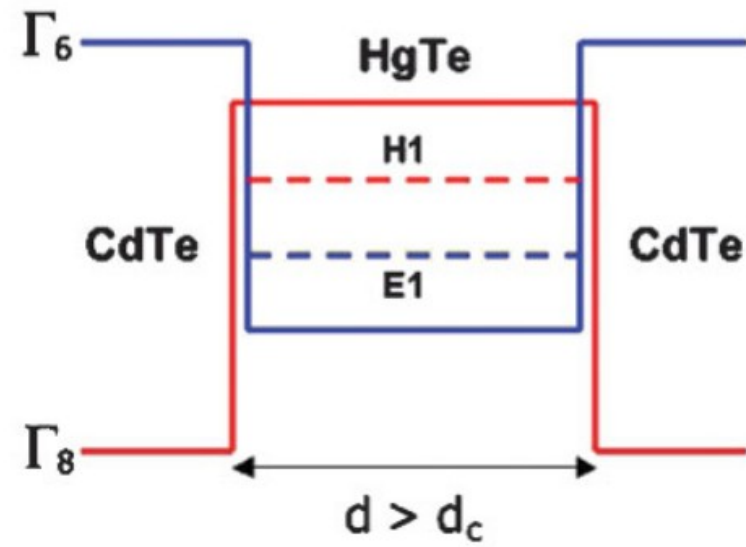
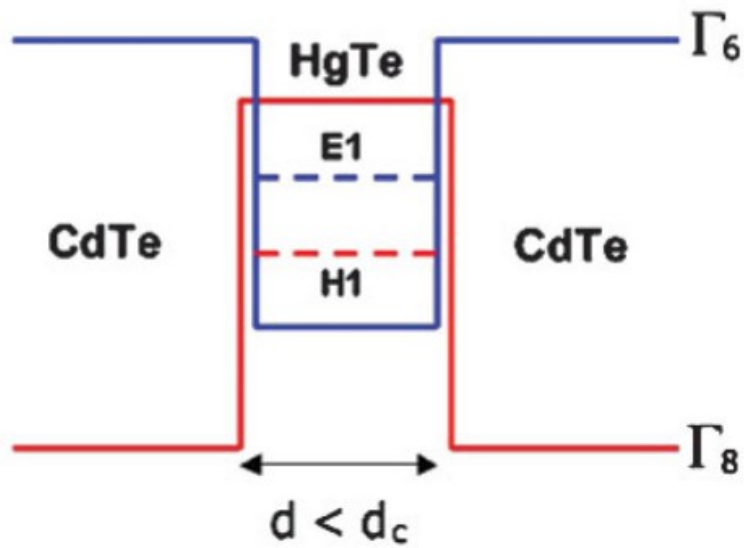
Trocam de ordem como função



Estados confinados

Trocam de ordem como função  
da largura do poço quântico

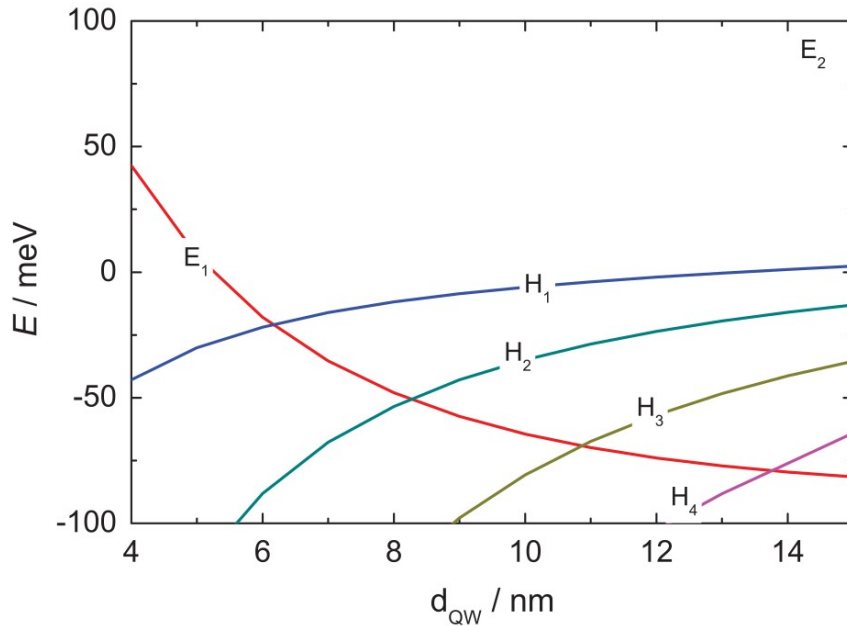
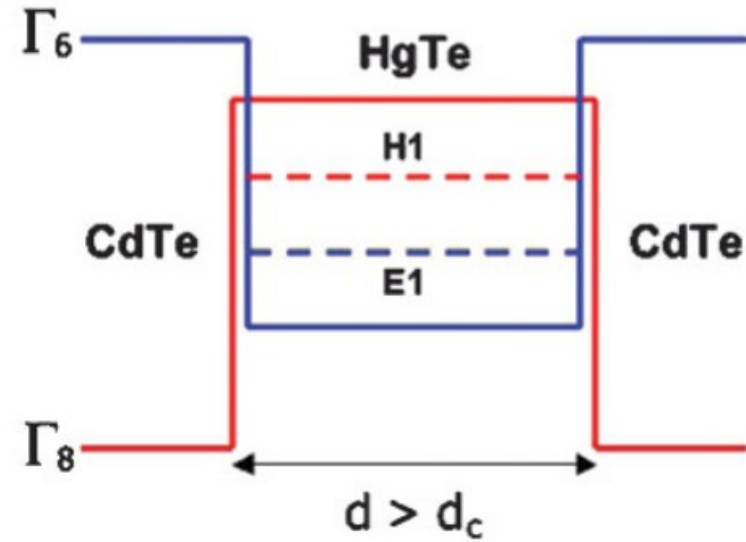
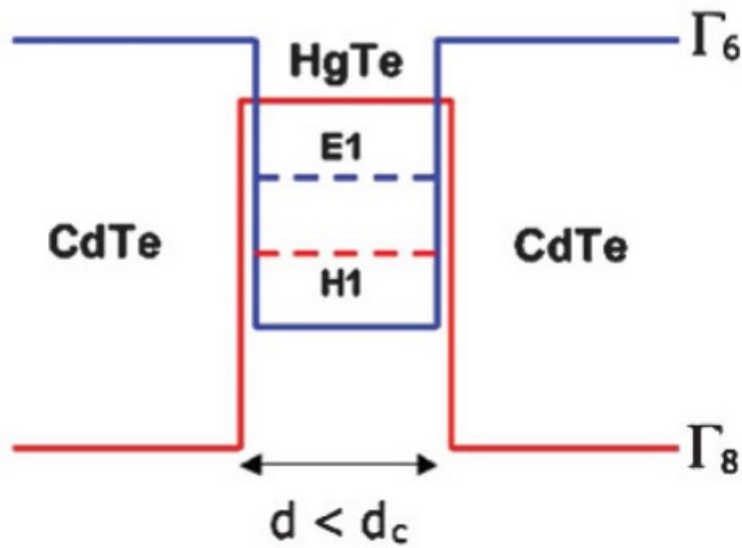




Estados confinados

Trocam de ordem como função  
da largura do poço quântico

“massa”:  $M = E1 - H1$



Estados confinados

Trocam de ordem como função da largura do poço quântico

“massa”:  $M = E1 - H1$

$$h(\mathbf{k}_{\parallel}) = C - D\mathbf{k}_{\parallel}^2 + A(\sigma_x k_x - \sigma_y k_y) + (M - B\mathbf{k}_{\parallel}^2)\sigma_z$$

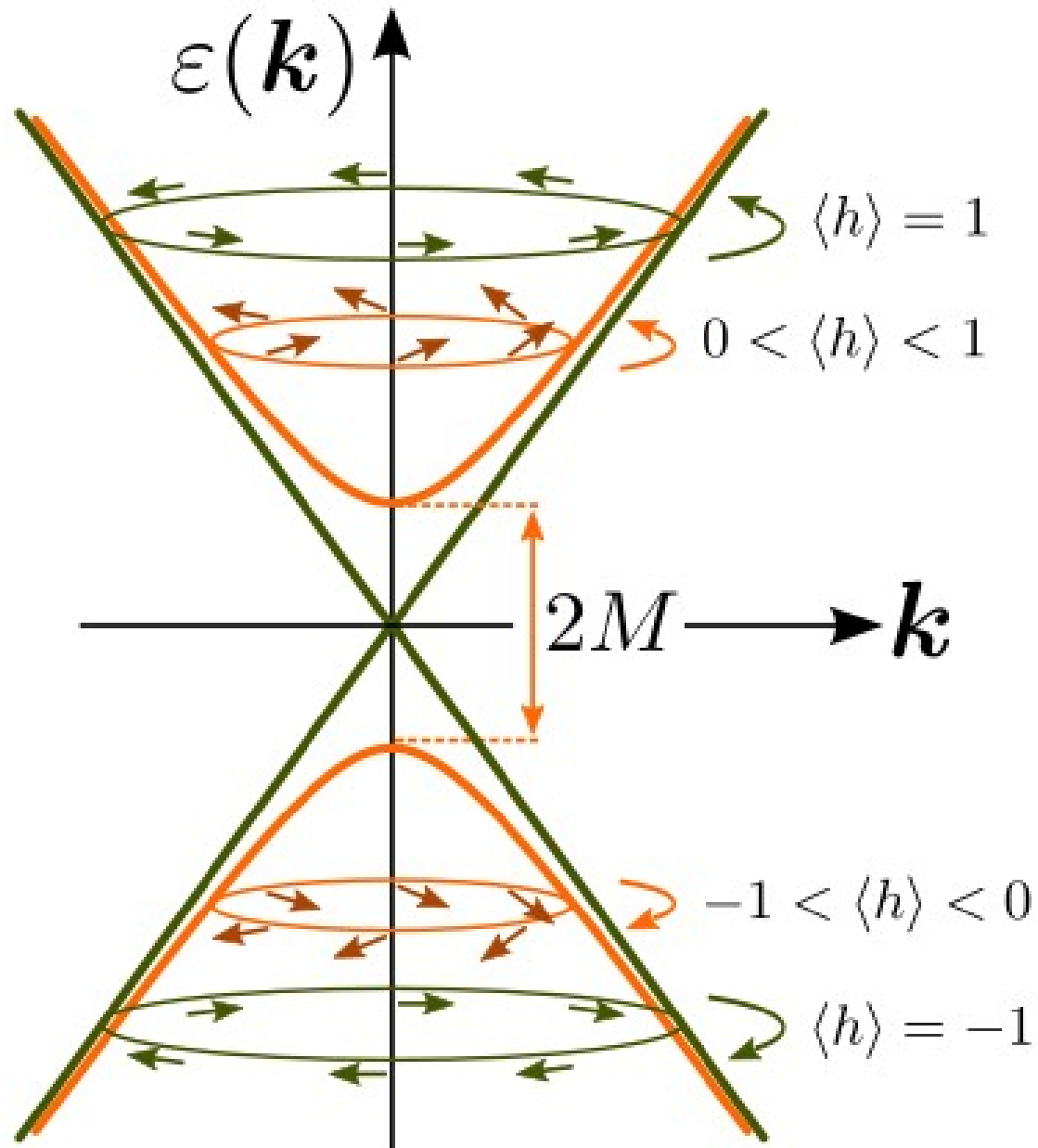
Massa  $M??$

*Massa  $M$ ??*

$$H = A(\sigma_x k_x - \sigma_y k_y) + M\sigma_z$$

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# *Estados de borda: bulk-boundary correspondence*



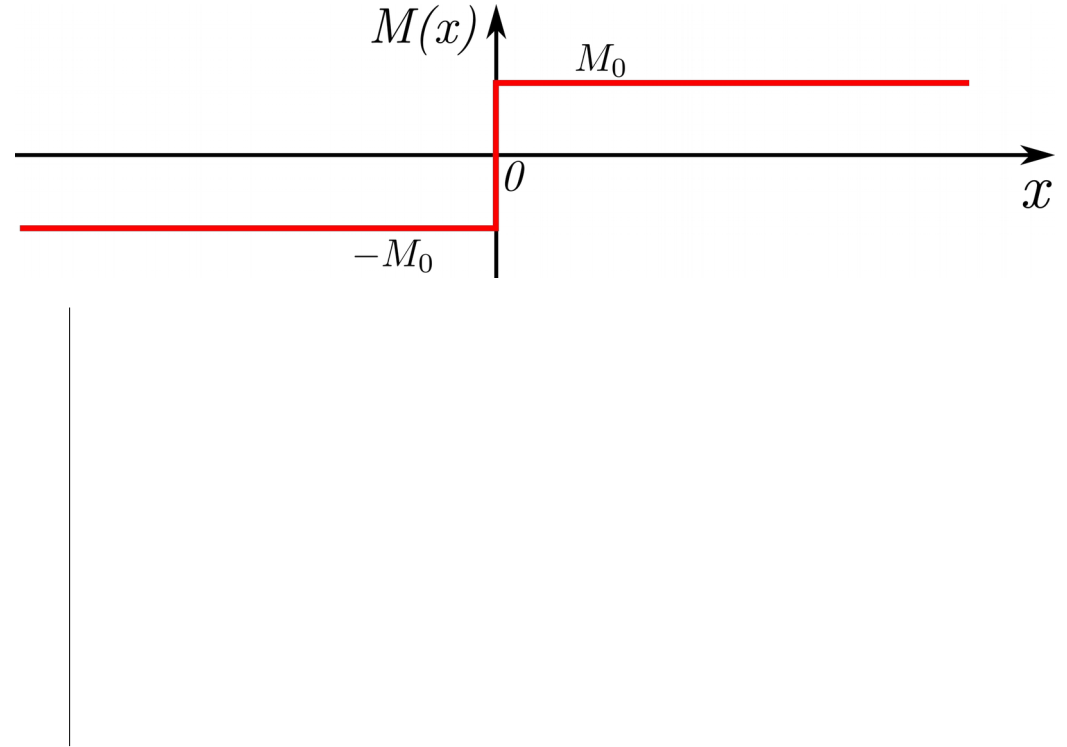
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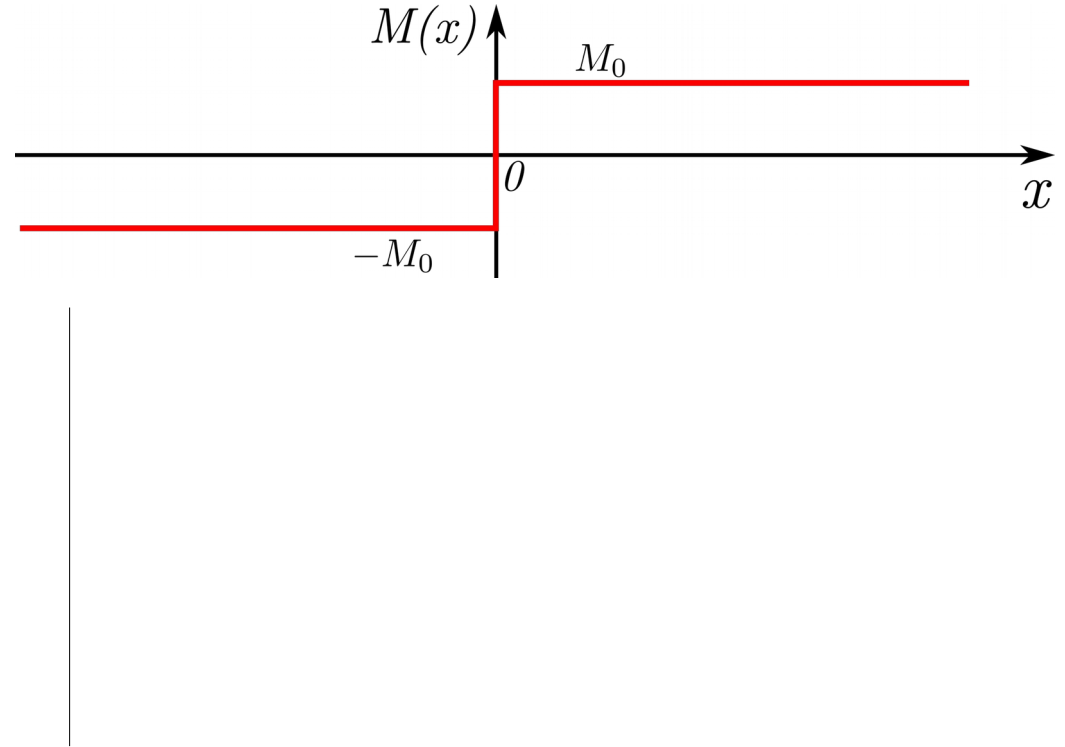




# Estados de borda: bulk-boundary correspondence

$$H = A(\sigma_x k_x - \sigma_y k_y) + M(x)\sigma_z$$

$$M(x) = M_0 \text{sgn}(x)$$

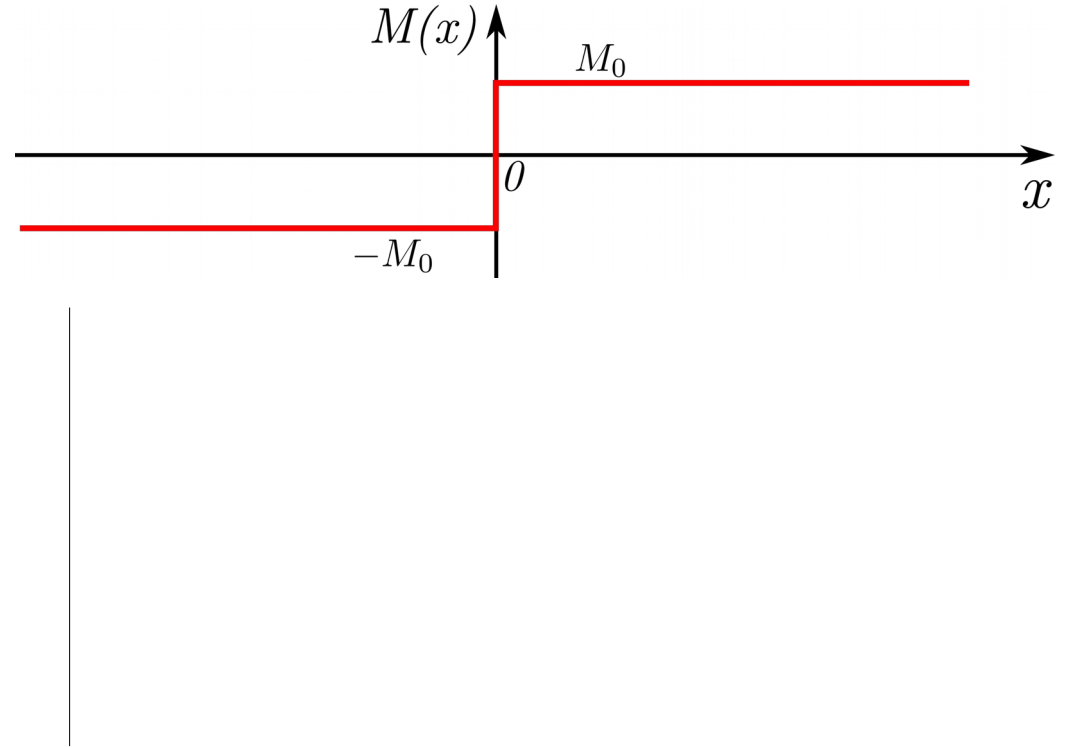


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Considerar  $k_y=0$  por simplicidade



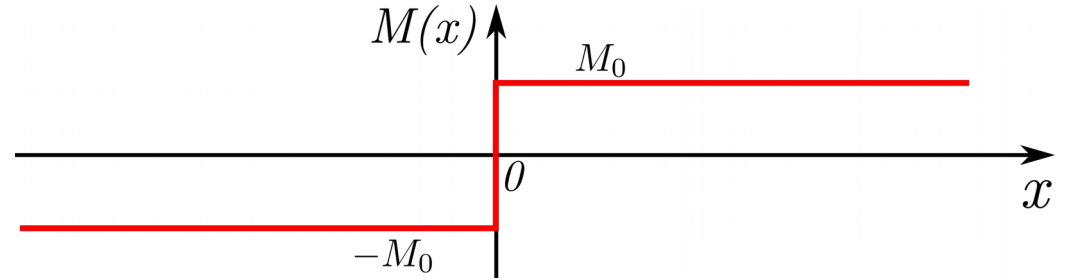
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Para  $x < 0$



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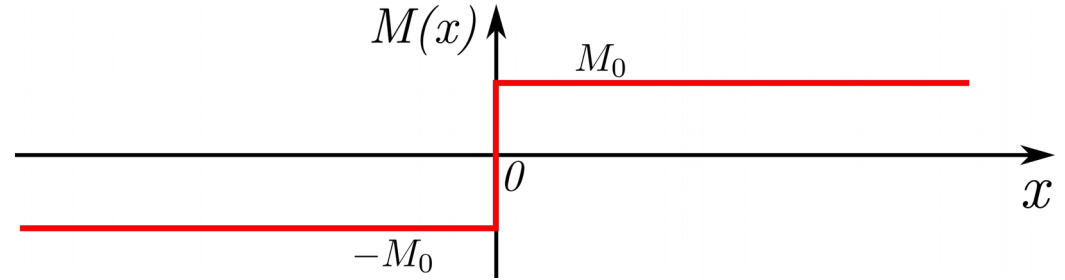
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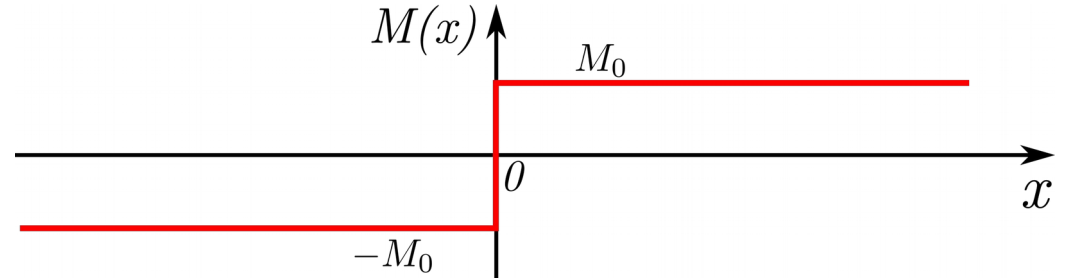
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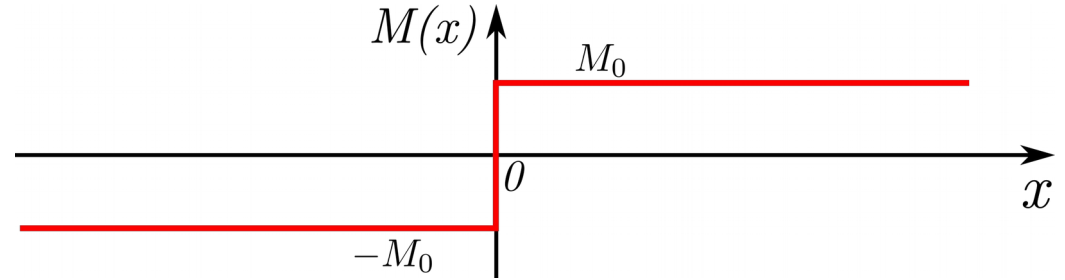


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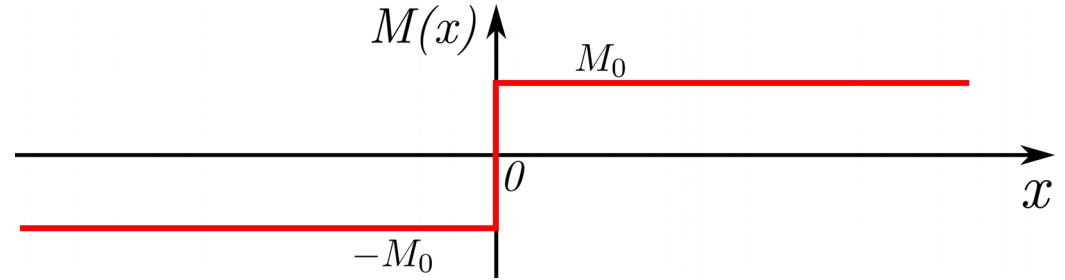
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Para  $x > 0$

$$\varphi_R(x) = N e^{-\lambda x} \phi_R$$

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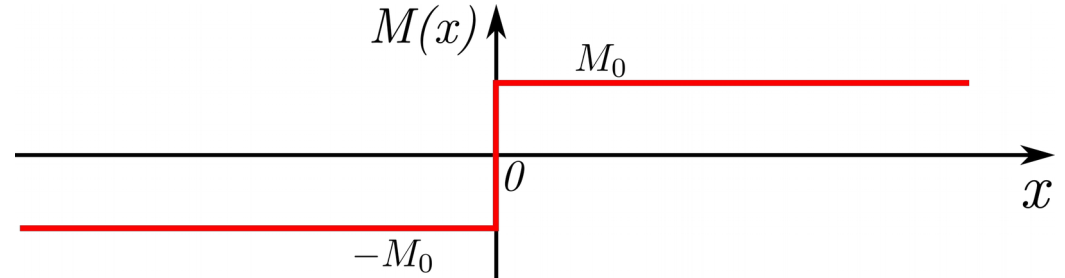
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Continuidade na interface  $\rightarrow \lambda = M_0/A$

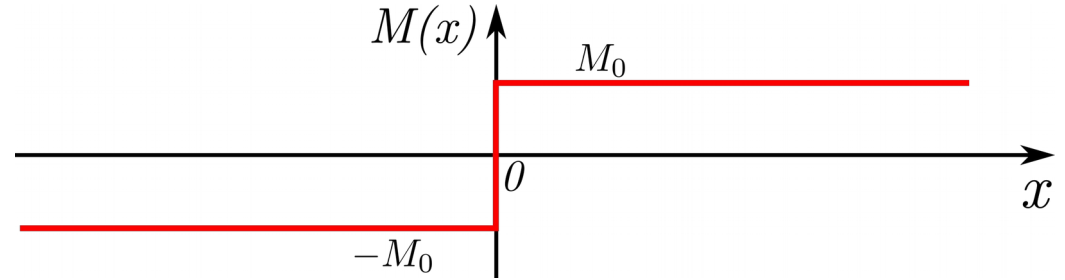


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Para  $k_y \neq 0$

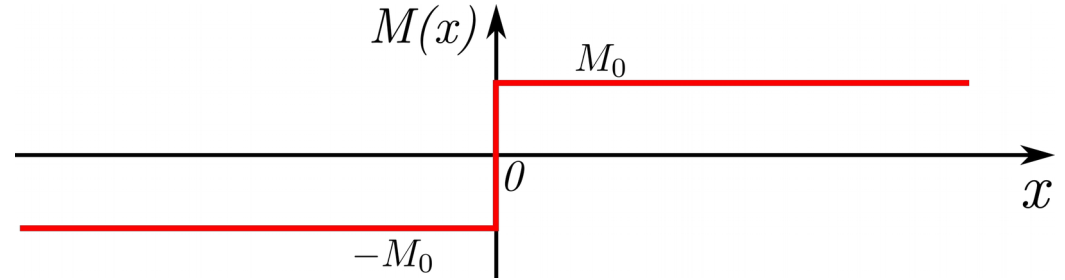
$$\varphi(x, y) = N e^{ik_y y} e^{-\lambda|x|} \phi$$

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Considerar  $k_y=0$  por simplicidade



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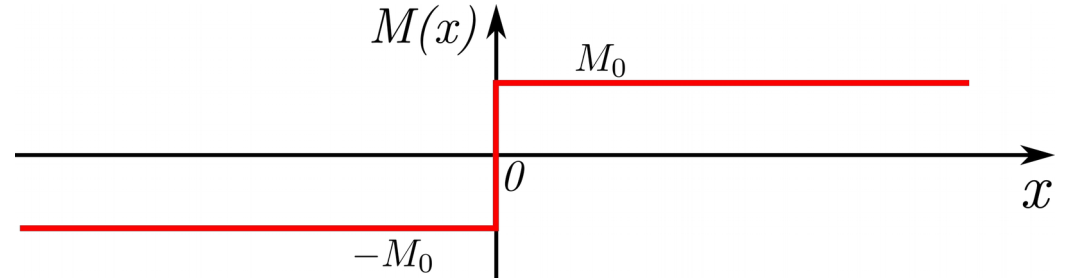
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