Finding the minimal cover:

S = {A→BD , AB→C , C→D , BC→D}

Step 1: Transform the FDs, so that each RHS contains only one attribute

Result of Step 1 = {A→B , A→D , AB→C , C→D , BC→D}

Step 2: Remove redundant attributes on LHS of each FD (check only those with > 1 LHS)

- For each FD (with > 1 LHS), cover each attribute and check if it can derive from S

AB→C:

1) Cover A, so we will have [redacted]B→C:
   a. Check if B→C can be implied by S
   b. B→C is not covered by S, so we will need A
2) Cover B, so we will have A[redacted]→C:
   a. Check if A→C can be implied by S
   b. Since A→BCD, therefore A→C can be implied by S, so B is **redundant** in AB→C

BC→D:

1) Cover B, so we will have [redacted]C→D:
   a. Check if C→D can be implied by S
   b. C→D is implied by S, so B is **redundant** in BC→D
2) Cover C, so we will have B[redacted]→D:
   a. Check if B→D can be implied by S
   b. B→D is not covered by S, so we will need C.

Result of Step 2 = {A→B , A→D , A→C , C→D}

Step 3: Remove redundant FDs by covering each FD, then check if it can be implied by the remaining

- For each FD, cover the FD and see whether it can be implied from the remaining FDs

1) Cover A→B
   - {[redacted] , A→D , A→C , C→D }
   - We are left with A→CD, which does not imply A→B. So we keep A→B.
2) Cover A→D
   - {A→B , [redacted] , A→C , C→D }
   - We are left with A→BCD, which implies A→D. So we **remove** A→D.
3) Cover A→C
   - {A→B , [redacted] , [redacted] , C→D }
   - We are left with A→B, which does not imply A→C. So we keep A→C.
4) Cover C→D
   - {A→B , [redacted] , A→C , [redacted] }
   - We are left with C→nothing, which does not imply C→D. So we keep C→D.

Final Results = {A→B , A→C , C→D} = {A→BC , C→D}