Main Idea: Frequency Dependence of Impedance

Impedance in an AC circuit is analogous to resistance in a DC circuit; it is a measure of how current flow is hindered in a system. Impedances in series are added linearly; impedances in parallel are added inversely – that is, $Z_{series} = \sum_i Z$ and $\frac{1}{Z_{parallel}} = \sum_i \frac{1}{Z}$. Our system under test uses a resistor in series with a capacitor both in parallel with a resistor. The impedances for each of these individually are $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$ (where *j* is indicative of imaginary numbers, which arise from our oscillating AC current). So, for our system under test, we end up with

$$Z = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{\frac{1}{j\omega C}+R_2}\right)^{-1} = \frac{\mathrm{R}_1\left(1+\omega^2 C^2 R_2 (R_1+R_2) - j\omega C R_1\right)}{1+\omega^2 C^2 (R_1+R_2)^2} = R_1 \frac{1 - R_1 j\omega C + R_2 \omega^2 C^2 (R_1+R_2)}{1+\omega^2 C^2 (R_1+R_2)^2},$$

Using $V = I|Z|$, and knowing that our current into the system under test is equal to an input voltage divided by a current-setting resistance, we get a voltage out:

$$V = \frac{V_0}{R_0} R_1 \frac{\sqrt{\left(1+(2\pi f)^2 C^2 R_2 (R_1+R_2)\right)^2 + (2\pi f C)^2}}{1+(2\pi f)^2 C^2 (R_1+R_2)^2}.$$

Using $\phi = \frac{Im\{Z\}}{Re\{Z\}}$, we are able to determine the phase:

$$\phi = -\frac{2\pi\mathrm{f}\mathrm{C}}{1+(2\pi f)^2 C^2 R_2 (R_1+R_2)^2}.$$

Therefore, as frequency approaches zero, the system behaves more like a DC circuit and charge cannot cross the branch containing the capacitor, meaning $Z \cong R_1$. Also, as frequency becomes arbitrarily large, the current can arc across the capacitor, effectively ignoring its presence, and $Z \cong \frac{R_1 R_2}{R_1+R_2}$.

Equipment: Frequency Meter

Our experiment required us to be able to measure frequencies of the current flowing through our AC circuit. In a system with more visible measurements, such as the swing of a pendulum, we might approach this by counting oscillations over a period time and finding an average frequency. A digital frequency counter using a direct counting method functions very similarly; it measures frequency by taking counts of a periodic function over what is known as a gate time. This gate time is based on a clock which produces a reliable time signal. This signal is then divided by what are known as decade dividers into the appropriate gate time. At intervals of the gate time, a "gate flip-flop" signals to the gate — which allows or blocks input — when the next set of counts should begin. Once the new count has started, a latch holds the last value so that it can continue to be displayed while the counter is updating based on a new set of data. At this point, the counter takes the number of times the signal passed a certain threshold (ideally, low enough that the system measures each oscillation but high enough that the system doesn't measure noise) and divides it by the gate time to give a frequency reading. There are some counters which, instead, measure the period of a cycle by counting how long it takes to get one count and taking the inverse of that time; these are known as reciprocal frequency counters.

Analysis: Theory vs. Data

We plugged in measured values (R_0 = 10020[Ohms], R_1 = 197.7[Ohms], R_2 = 20.0[Ohms], and C = 2.38[nF]) for the expressions for both voltage and phase to get curves representing the predicted outcomes of our experiment. We plotted these on the same graphs as our data and took the standard deviations for both as a way of measuring discrepancy.

In our voltage data, we saw an almost perfect correlation between the theory and the data we collected, especially in the center, far away from the limits. The limits also match fairly well; The voltage approaches 0.014[V] for both at low frequencies and approaches 0.001[V] for both at high frequencies. The standard deviation of voltage data compared with expected voltage was $8*10^{-8}$[V].

The phase data, however, doesn't entirely agree with the expected phase. The left half of the plot appears to be roughly correct, with agreement that the low-frequency limit is just below zero (with the voltage and current almost in phase with one another). The minimum occurs at the same frequency for both – around 10[Hz] – but the phases given for these minima are different; the minima in the theory is almost at -1 while the minima for the data only reaches down to about -0.9 before it starts climbing back up. This results in the phase being higher for the data than the theory for the remainder of the plot. The phase we measure then goes above zero, becoming positive; this should be impossible with an ideal circuit with our setup. This indicates to me that there must be something providing impedance opposite the rest of the system; that is, there must be something with negative impedance that is only having a major effect at high frequencies. Therefore, it is possible that something within the circuit is behaving as a very weak inductor; this could be something as simple as imperfections in the wire. The standard deviation between the data and theory for the phase was 0.05 [radians].

References

How Does a Frequency Counter Work. (n.d.). Retrieved from Electronics Notes: https://www.electronics-notes.com/articles/test-methods/frequency-counter-timer/how-does-a-frequency-counter-work-operation.php

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