Interferometry
Prelab

1) E_1, E_2 at time t travelled different distance (z) but have the same polarization direction $\hat{p}$,

z_1, z_2

$E_2\hat{p}\cos(\omega t - kz_2)$ Wavenumber $k = \frac{2\pi}{\lambda}$ and

$E_1\hat{p}\cos(\omega t - kz_1)$ angular frequency $\omega = ck = 2\pi\frac{c}{\nu}$

where λ = wavelength, c is ~~sp~~ and ν is frequency.

a) What's the time-averaged I_1, I_2?

$I = cn\varepsilon_0 (E)^2$ $\quad I_1 = cn\varepsilon_0 E_1^2 \cancel{\hat{p}^2}^{1} \cos^2(\omega t - kz_1)$

Air has $n \approx 1$ 1.00029

$$\boxed{I_{1avg} = c\varepsilon_0\frac{E_1^2}{2}}$$

$$\boxed{I_{2avg} = c\varepsilon_0\frac{E_2^2}{2}}$$

b) If the waves are coherent, $E = E_1 + E_2$. What is an expression for $I_{tot\,avg}$?

$\cancel{I_{tot\,avg} = c\varepsilon_0\frac{(E_1+E_2)^2}{2}}$ $\quad E_{tot} = \hat{p}(E_1\cos(\omega t - kz_1) + E_2\cos(\omega t - kz_2))$

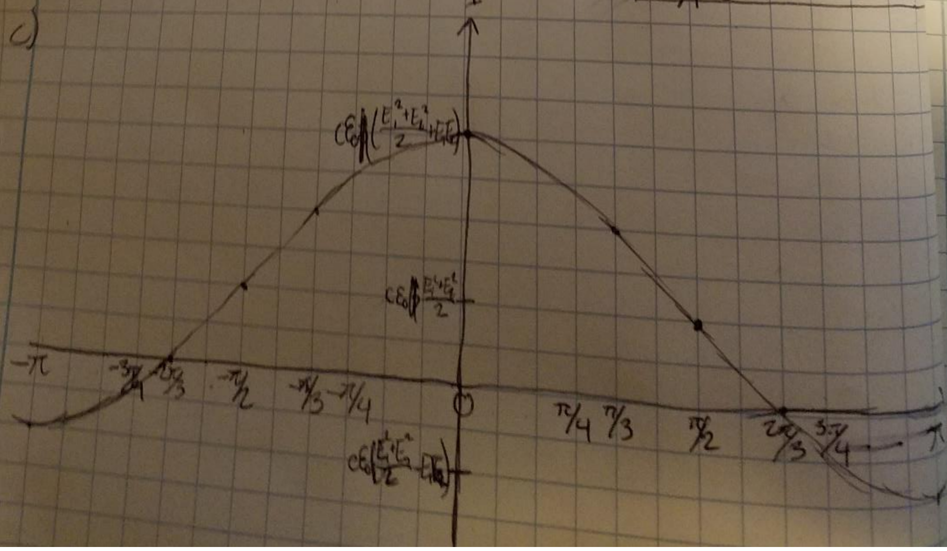
$\cancel{I_{tot} = c\varepsilon_0 \hat{p}^2 (E_1\sin(\omega t)\sin(kz_1) + E_1\cos(\omega t)\sin(kz_1) + E_2\sin(\omega t)\sin(kz_2)}$

$I_{tot} = c\varepsilon_0 \cancel{\hat{p}^2}(E_1^2\cos^2(\omega t - kz_1) + E_2^2\cos^2(\omega t - kz_2) + 2E_1E_2\cos(\omega t - kz_1)\cos(\omega t$

$= c\varepsilon_0 \cancel{\hat{p}^2}(E_1^2\cos^2(\omega t - kz_1) + E_2^2\cos^2(\omega t - kz_2) + \cancel{2}E_1E_2\frac{1}{\cancel{2}}(\cos(2\omega t - k(z_1 - z_2$

$I_{tot\,avg} = c\varepsilon_0\cancel{\hat{p}}\left(\frac{E_1^2+E_2^2}{2} + E_1E_2(0 + \cos(k(z_1 - z_2))\right) = \boxed{c\varepsilon_0\cancel{\hat{p}}\left(\frac{E_1^2+E_2^2}{2} + E_1E_2\cos\right.}$

c)



d) $I_{tot\,max} = c\varepsilon_0\left(\frac{E_1^2}{2}\right.$

$Vis = $

2) Now suppose on... while the othe... parallel lines, ... depend on ... the combinat... magnitude ... that each ...

Since th... at each loc...

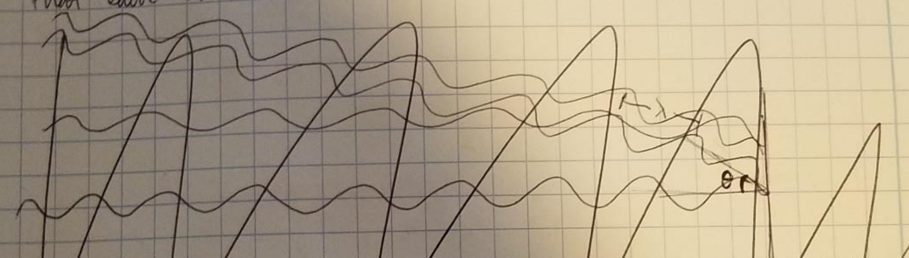
$E_{2x} = E_2\cos($

$I_{tot} = c\varepsilon_0($

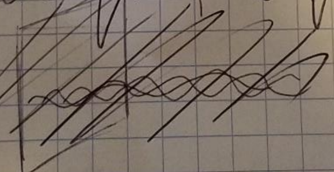
d) $I_{tot\,max} = c\varepsilon_0\left(\frac{E_1^2+E_2^2}{2} + E_1E_2\right)$, $I_{tot\,min} = c\varepsilon_0\left(\frac{E_1^2+E_2^2}{2} - E_1E_2\right)$

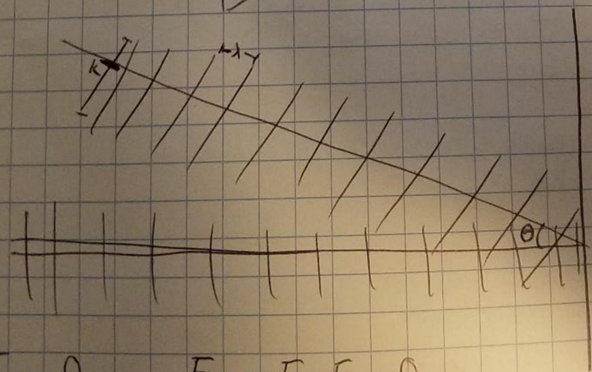
$$Vis = \frac{\cancel{c\varepsilon_0}\left(\cancel{\frac{E_1^2+E_2^2}{2}} + E_1E_2 + \cancel{\frac{E_1^2+E_2^2}{2}} + E_1E_2\right)}{\cancel{c\varepsilon_0}\left(\cancel{\frac{E_1^2+E_2^2}{2}} + \cancel{E_1E_2} + \cancel{\frac{E_1^2+E_2^2}{2}} - \cancel{E_1E_2}\right)} = \boxed{\frac{2E_1E_2}{E_1^2+E_2^2} = \text{Visibility}}$$

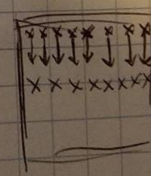
2) Now suppose one of the plane waves runs parallel to the detector while the other strikes at an angle θ. Are the resulting fringes parallel lines, concentric circles, or something else? How does the spacing depend on λ and θ? (You may do this geometrically or by taking the combination of field amplitudes. Assume both have the same magnitude k but the components must be determined by that angle that each wave vector makes to the normal to the surface.)



Since the plane waves will intersect at the same angle each time at each location, we should see parallel lines







$E_{2x} = E_2 \cos\theta$ $\qquad$ $E_{tot\,x} = E_1 + E_2\cos\theta$

$$I_{tot} = c\varepsilon_0\left(E_1^2\cos^2(\omega t - kz_1) + E_2^2\cos^2\theta\cos^2(\omega t - kz_2) + E_1E_2\cos\theta\left(\cos(\omega t - k(z_1-z_2)) + \cos(k(z_1-z_2))\right)\right)$$

For simplicity, $z_1 = z_2 = 0 \longrightarrow I_{tot} = c\varepsilon_0\left(E_1^2\cos^2(\omega t) + E_2^2\cos^2\theta\cos^2(\omega t) + E_1E_2\cos\theta(\cos(\omega t) + 1)\right)$

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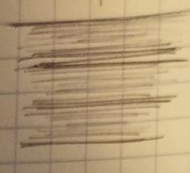
Since we have plane waves, we should see interference pattern in terms of parallel lines:



which dim and brighten as we travel vertically/across the detector.

The distance between peaks in intensity will increase with the wavelength, as the interference is going to be constructive where the ~~distan~~ difference in distance travelled is a multiple of the wavelength.

The distance between peaks should decrease with θ since $\theta < 90°$ if we see interference and $\Delta x = \frac{2\pi}{k\sin(\frac{\theta}{2})}$

3) $E_0 \hat{x} \frac{w_0}{w(z)} e^{-(r/w(z))^2} e^{-i(kz+kr^2/2R(z)-\psi(z))}$

For a plane wave, R = radius of curvature $\rightarrow \infty$, so

$$E \rightarrow E_0 \hat{x} \frac{w_0}{w(z)} e^{-(r/w(z))^2} \underbrace{e^{-i(kz-\psi(z))}}$$

real part is $\cos(kz-\psi(z))$, which parallels the plane wave.

direction as in the plane wave

amplitude parallels plane wave

So this is a good approximation if $\frac{w_0}{w(z)} e^{-(r/w(z))^2} \simeq 1$.
For a plane wave, $\frac{w_0}{w(z)} = 1$, so we need $e^{-(r/w(z))^2} \simeq 1$, or $\left(\frac{r}{w(z)}\right)^2 \simeq 0$, where r is distance from source and $w(z)$ is a property of the beam.

So, the Gaussian beam is a good approximation for a plane wave when $w(z) \gg r$.