Plasma Physics: Langmuir Probe

Prelab

$pV = NkT, \quad k = 1.3806 \cdot 10^{-23}\,[J/K]$

[Q1] What is the gas number density in cm^{-3} at $0[°C]$ and $760[torr]$?

$\frac{N}{V} = \frac{P}{kT} \qquad T = 273.2\,[K] \qquad P_{stp} = 101325\,[Pa]$

$\frac{N}{V} = \frac{101325\,[Pa]}{1.3806 \cdot 10^{-23}\,[J/K]\,273.2\,[K]} = \frac{268.6}{\cancel{1.3...}}\,[m^{-3}] \cdot 10^{+23} = \cancel{\frac{268.6}{1.339 \cdot 10^{...}}\,[m^{-3}]}$

$m^{-3} \Rightarrow cm^{-3} = (m^{-1} \rightarrow cm^{-1})^3 \qquad (100\,cm \rightarrow 1\,m) \Rightarrow (1\,m^{-1} \rightarrow 10^{2}\,cm^{-1})$

$\cancel{\ldots \cdot 10^{+23}\,[cm^{-3}]}$ $\boxed{2.686 \cdot 10^{19}\,[cm^{-3}]}$

[Q2] What is N/V in $[cm^{-3}]$ for $T = 20[°C]$ and $P = 1\,[torr]$?

$\frac{N}{V} = \frac{133.3\,[Pa]}{293.2\,[K]} \frac{10^{23}\,[K]}{1.3806\,[J]} = \frac{0.4546\,[N/m^2]}{1.3806\,[J/m]} 10^{23} = 0.3293 \cdot 10^{23}\,[m^{-3}]$

$= 3.293\,[m^{-3}] \cdot 10^{22} = \boxed{3.293 \cdot 10^{16}\,[cm^{-3}]}$

[Q3] How many molecules (N) are in a plasma tube $l = 80cm$, $d = 5cm$, $P = 1$ torr, $T = 20[°C]$

$\cancel{V = l\pi r^2 = l\pi\left(\frac{d}{2}\right)^2 = l\pi\frac{d^2}{4} = 80[cm]\pi\frac{(5[cm])^2}{4} = 20\pi(25)[cm^3] = 500\pi\,[cm^3]}$

$\cancel{P = 1.333 \cdot 10^{2}\,[Pa]}$

$\cancel{T = 2.932 \cdot 10^{2}\,[K]}$

$\cancel{N = \frac{PV}{kT} = \frac{1.333 \cdot 10^{2}\,[Pa]\,5 \cdot 10^{2}\pi\,[cm^3]}{1.3806\,[J/K]\,2.932 \cdot 10^{2}\,[K]} 10^{23} = 20.4387\,[N/m^2]\,[cm^3]}$

$N = V \cdot Q2 = 3.293 \cdot 10^{16}\,[\cancel{cm^{-3}}] \cdot 500\pi\,[\cancel{cm^3}] = 51.726 \cdot 10^{18} = \boxed{5.173 \cdot 10^{19}}$

[Q4] What would happen to the gas temperature and pressure if a warmer electrode were introduced (systems are closed? diff)?

Temperature should rise. As the temperature rises, in the absence of other changes, the pressure should increase in closed (constant N) systems.

$(x, x+dx), (y, y+dy), (z, z+dz), (v_x, v_x+dv_x), (v_y, v_y+dv_y), (v_z, v_z+dv_z)$

$$f(x,y,z,v_x,v_y,v_z) = n_{(x,y,z)} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-(v_x^2+v_y^2+v_z^2)m/(2kT)}$$

$v = \sqrt{v_x^2+v_y^2+v_z^2}$ so

$$f_{speed}(x,y,z,v) = n_{(x,y,z)} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}$$

$\boxed{Q5}$ Find expressions for:

$\boxed{v_p}$ Most probable speed

Should occur at a maximum point, with respect to v.

$$\cancel{\frac{\partial f}{\partial v} = \underbrace{n_{(x,y,z)} \left(\frac{m}{2\pi kT}\right)^{3/2}}_{\text{"constant"}} \frac{-m}{2kT} 2 v e^{-mv^2/2kT}}$$

~~0 at v=0 ???~~

Hyperphysics: $\boxed{v_p = \sqrt{\frac{2kT}{m}}}$

Relative ratio:

$v_p : v_{av} : v_{rms} = \sqrt{2} : 2\frac{\sqrt{2}}{\pi} : \sqrt{3}$

$\boxed{v_{av}}$ Average v

Hyperphysics: $v_{av} = \bar{v} = \sqrt{\frac{8kT}{\pi m}} = \boxed{2\sqrt{\frac{2kT}{\pi m}}}$

$\boxed{v_{rms}}$ Root mean square v

Hyperphysics: $\boxed{v_{rms} = \sqrt{\frac{3kT}{m}}}$

$\boxed{Q6}$ What is v_{rms} (m/s) for N_2 at $T = 20 [°C] = 293.2 [K]$?
How does this compare to the speed of sound?

$$\sqrt{\frac{3k\, 293.2 [K]}{\cancel{2 (m_N)}\, m_{N_2}}} = \boxed{510.94 [m/s]}$$

The speed of sound is 295.1 [m/s] so $\boxed{v_{rms} \approx 1.3\, v_{sound}}$

Q7 What is the average KE of a molecule ~~(N_2)~~ at $T = 20[°C]$? Give your answer in [eV]. Does m matter?

v_{avg}: $KE = \frac{m}{2}v^2 = \frac{m}{2}v_{avg}^2 = \frac{m}{2}\left(2\sqrt{\frac{2kT}{\pi m}}\right)^2 = \frac{\cancel{m}}{\cancel{2}}4\left(\frac{\cancel{2}kT}{\pi \cancel{m}}\right) = \frac{4}{\pi}kT = \frac{4}{\pi}k\,293.2[K]$

$\boxed{KE = 0.032[eV]}$

The molecule's mass is $\boxed{\text{irrelevant}}$ because $v^2 \propto \frac{T}{m}$ and $KE \propto mv^2$, so $KE \propto T$, and not to m or any term containing it in a meaningful way.

At room temperature, $kT = 0.0253[eV]$ or $\sim \frac{1}{40}[eV]$

If a tube has cross-sectional area σ and length equal to the mean-free-path l_m, that volume should have 1 molecule, on average. Let n be density

$$n\sigma l_m = 1 \rightarrow l_m = \frac{1}{n\sigma}$$

Identical molecules will collide if distance between their centers is d.

$$\sigma = \pi d^2$$
$$l_m = \frac{1}{n\pi d^2}$$

But we have a distribution of relative velocities so $l_m = (\sqrt{2}n\pi d^2)^{-1}$, but we'll ignore thi

Applying gas law, $l_m = \frac{kT}{p\pi d^2}$

Q8 If the gas is 100% N_2, $T = 20[°C]$, and $P = 1[torr]$ (as in Q2), what is l_m?

Q2: $\frac{N}{V} = \frac{P}{kT} = 3.293 \cdot 10^{16}[cm^{-3}] = Q$

$$l_m = \frac{1}{Q\pi d^2}$$

According to wikipedia, the kinetic diameter of N_2 is 364[pm]

$\boxed{l_m = 7.296 \cdot 10^7[pm]}$

[Q9] Suppose we want to make a plasma inside a capillary with $l_m = 0.1[mm]$. If $T = 20[°C]$, what is the necessary P? (N_2)

$$l_m = 0.1[mm] = \frac{kT}{P\pi d^2}$$

$$l_m P\pi d^2 = kT \rightarrow P = \frac{kT}{l_m \pi d^2} = \boxed{97.2[Pa]}$$

If electrons collide with molecules our cross-section uses r instead of d.

$$\sigma = \pi r^2 = \pi \frac{d^2}{4}$$

[Q10] What is σ for N_2 and e^-?

$$\sigma = \pi \frac{(364[pm])^2}{4} = \boxed{0.1041[nm]^2}$$

If an object enters a random location in the cross section A, it has a $\frac{\sigma}{A}$ chance of colliding with a given target.

If it has speed v, then volume produced in a time interval is $dV = Avdt$ and the average number of targets in that volume is $dN = nAvdt$

$$dN_{hit} = dN\frac{\sigma}{A} = nAvdt\frac{\sigma}{A} = n\sigma v dt$$

On average, the rate of hits is $\nu \equiv \frac{dN_{hit}}{dt} = nv\sigma$

Time interval between hits is $\tau = 1/\nu = \frac{1}{nv\sigma}$

[Q11] Let $V = 1[m^3]$, $N = 100$, $d = 10[cm]$, $v = 60[m/s]$. What are ν and τ?

$$\nu = nv\sigma = nv\pi\frac{d^2}{4} = \frac{N}{V}v\frac{\pi}{4}d^2 = \frac{100}{[m^3]}60[m/s]\frac{\pi}{4}(10[cm])^2 = \boxed{47.12[s^{-1}]}$$

$$\boxed{\tau = 0.02122[s]}$$

$$l_m = v\tau = \frac{1}{n\sigma}$$

Let there be n_{BB} BB's entering the box. Then collision rate is

$$r = n_{BB}\nu = n_{BB}nv\sigma = Kn_{BB}n \quad (K \equiv v\sigma)$$

Ionization rate due to collision is found by performing a weighted average of $n_g v \sigma_i$ over e^- speeds using

$f(v) = \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} 4\pi v^2 e^{-m_e v^2/2kT_e}$. Then $\overline{KE} = \langle \frac{m}{2} v^2 \rangle = \frac{3}{2} k T_e$

$\mathcal{E}_i$ = ionization energy

$$\sigma_i \simeq C_i \left(\tfrac{m}{2} v^2 - \mathcal{E}_i\right) \text{ for } \tfrac{m}{2} v^2 \geq \mathcal{E}_i \text{ and } 0 \text{ for } \tfrac{m}{2} v^2 \leq \mathcal{E}_i$$

C_i: coefficient of the slope of linear growth in ionization cross-section vs $KE_e > \mathcal{E}_i$ and is found either empirically from σ_i or from more detailed theoretical models.

If we define $\sigma \equiv C_i \mathcal{E}_i$, $\sigma_i = \sigma_i\left(\frac{m}{2}\frac{v^2}{\mathcal{E}_i} - 1\right)$ for $\frac{m_e}{2} v^2 > \mathcal{E}_i$ and 0 for $\frac{m_e}{2} v^2 \leq \mathcal{E}_i$

Then $\nu_i = \int_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty} n_g v \sigma_i \left(\frac{m_e}{\mathcal{E}_i} v^2 - 1\right)\left(\frac{m_e}{2\pi k T_e}\right)^{3/2} 4\pi v^2 e^{-m_e v^2/2kT_e} dv$, which yields

$$\nu_i = n_g \sigma_i \sqrt{\frac{8\mathcal{E}_i}{\pi m_e}}\, 2\sqrt{\frac{kT_e}{\mathcal{E}_i}} \left(1 - 2\frac{kT_e}{\mathcal{E}_i}\right) e^{-\mathcal{E}_i/kT_e} \simeq n_g \sigma_i \sqrt{\frac{8kT_e}{\pi m_e}}\, e^{-\mathcal{E}_i/kT_e} \left(1 + 2\frac{kT_e}{\mathcal{E}_i}\right)$$

(we could get higher-order terms if we expanded the whole thing)

$\boxed{2.12}$ Derive $\nu_i = n_g \sigma_i \sqrt{\frac{8kT_e}{\pi m_e}} \left(1 + 2\frac{kT_e}{\mathcal{E}_i}\right) e^{-\mathcal{E}_i/kT_e}$

$$\nu_i = n_g \sigma_i \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} 4\pi \int_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty} v \left(\frac{m_e v^2}{2\mathcal{E}_i} - 1\right) v^2 e^{-m_e v^2/2kT_e} dv$$

$\frac{m_e}{2kT_e} = u$ $\quad \nu_i \simeq n_g \sigma_i \left(\frac{u}{\pi}\right)^{3/2} 4\pi \int_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty} v^3 e^{-uv^2} \left(\frac{m_e v^2}{2\mathcal{E}_i} - 1\right) dv$

$A = n_g \sigma_i \frac{4}{\sqrt{\pi}}$ $\quad$ ~~$\nu_i = A u^{3/2} \left(v^3 \left(-\frac{1}{u}\right) e^{-uv^2} - \int 3v^2 -\right.$~~

w substitution $e^{-uv^2} = e^{w}$ $\quad dw = -2uv\,dv$

$\nu_i = A u^{3/2} \int_{v=\sqrt{2\mathcal{E}_i/m_e}}^{v=\infty} -\frac{1}{2} v^2 e^{-w} dw = -\frac{1}{2} A \sqrt{u} \left(v^2 e^{-w} \frac{1}{2uv} - \int 2v\,dv\, e^{-w}\right)\Big|_{v=\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$

$\nu_i = -\frac{A}{2}\sqrt{u}\left(v e^{-uv^2}\right.$

$$\nu_i = n_g \sigma_i \left(\frac{u}{\pi}\right)^{3/2} 4\pi \left(\int_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty} \frac{m_e}{2\mathcal{E}_i} v^5 e^{-uv^2} dv - \int_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty} v^3 e^{-uv^2} dv\right) = 4Au^{3/2} \left(\int \frac{m_e}{2\mathcal{E}_i} v^5 e^{-uv^2} dv - \int v^3 e^{-uv^2} dv\right)\Bigg|_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$$

$$v_i = 4Au^{3/2}\left(\frac{m_e}{2\mathcal{E}_i}\int v^5 e^{-uv^2}dv - \int v^3 e^{-uv^2}dv\right)\Big|_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$$

If $-uv^2 = w$, $dw = -2uv\,dv \Rightarrow dv = \frac{-dw}{2uv}$

$$\int v^5 e^{-uv^2}dv = \int v^5 e^{-uv^2}\left(\frac{-dw}{2uv}\right) = \frac{-1}{2u}\int v^4 e^w dw$$

$$= -\frac{1}{2u}\left(v^4e^w - \int 4v^3 e^w dv\right) = \frac{1}{2u}\left(\cancel{\frac{-1}{2u}\int 4v^2 e^w}\ 4\int v^3 e^w dv - v^4 e^w\right)$$

$$\int v^3 e^w dv = \int v^3\left(\frac{-1}{2uv}\right)e^w dw = -\frac{1}{2u}\int v^2 e^w dw = -\frac{1}{2u}\left(v^2e^w - \int ve^w dw\right)$$

$$= \frac{1}{2u}\left(\int \cancel{\frac{2v}{-2uv}} e^w dw - v^2e^w\right) = \frac{-1}{2u}\left(\frac{1}{u}e^w + v^2e^w\right)$$

$$V_i = 4Au^{3/2}\left(\frac{m_e}{2\mathcal{E}_i}\left(\frac{1}{2u}4\int v^3 e^w dv - v^4e^w\right) - \int v^3 e^w dw\right)\Big|_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$$

At $v = \sqrt{2\mathcal{E}_i/m_e}$, $w = -2u\mathcal{E}_i/m_e$

$$= 4Au^{3/2}\left(\frac{m_e}{2\mathcal{E}_i}\left(\frac{2}{u}\int v^3e^w dv - v^4e^w\right) - \int v^3 e^w dv\right)\Big|_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$$

$$= 4Au^{3/2}\left(\left(\frac{m_e}{\mathcal{E}_i u} - 1\right)\int v^3 e^w dv - \frac{m_e}{2\mathcal{E}_i}v^4e^w\right)\Big|_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$$

$$= 4Au^{3/2}\left(\frac{m_e - \mathcal{E}_i u}{\mathcal{E}_i u}\left(\frac{-1}{2u}\right)\left(\frac{1}{u}e^w + v^2e^w\right) - \frac{m_e}{2\mathcal{E}_i}v^4e^w\right)\Big|_{\sqrt{2\mathcal{E}_i/m_e}}^{\infty}$$

$$= 4Au^{3/2}\left(\cancel{\frac{m_e - \mathcal{E}_i u}{}}\ \frac{\mathcal{E}_i u - m_e}{\mathcal{E}_i u}\left(\frac{1}{2u}\right)\left(\frac{1}{u}\left(0 - e^{-2u\mathcal{E}_i/m_e}\right) + \left(0 - 2\frac{\mathcal{E}_i}{m_e}e^{-2u\mathcal{E}_i/m_e}\right)\right) - \frac{m_e}{2\mathcal{E}_i}\frac{4\mathcal{E}_i^2}{m_e^2}\ldots\right)$$

$$= 4Au^{3/2}\left(\frac{m_e - \mathcal{E}_i u}{2\mathcal{E}_i u^2}\frac{1}{u}e^{-2u\mathcal{E}_i/m_e} \cancel{- 2\frac{\mathcal{E}_i}{m_e}e^{-2u\mathcal{E}_i/m_e}} \cancel{+ 2\frac{\mathcal{E}_i}{m_e}e^{-2u\mathcal{E}_i/m_e}}\right)$$

$$= 2Au^{3/2}\left(\frac{m_e - \mathcal{E}_i u}{\mathcal{E}_i u^3}e^{-2u\mathcal{E}_i/m_e}\right) = \text{[illegible]}\quad n_g\sigma_i\sqrt{\frac{8k}{\pi m_e}}$$

$$\boxed{= n_g\sigma_i\sqrt{\frac{8kT_e}{\pi m_e}}\left(1 + 2\frac{kT_e}{\mathcal{E}_i}\right)e^{-\mathcal{E}_i/kT_e}}$$

Finding Electron Temperature

We can use the above requirement on the ionization rate to find the electron temperature (T_e) by ~~replacing v_i~~ expressing v_i in terms of gas density (n_g), proportionality constant (σ_i) for ionization's x-section dependence on energy above ionization energy ($\mathcal{E}_i$), and electron temperature (T_e)

$$n_g \sigma_i \left(\frac{8\mathcal{E}_i}{\pi m_e}\right)^{1/2} \left(\frac{kT_e}{\mathcal{E}_i}\right)^{1/2} e^{-\mathcal{E}_i/kT_e} = \mu_i \frac{kT_e}{e} \left(\frac{2.4048}{R}\right)^2$$

$$\sqrt{\frac{\mathcal{E}_i}{kT_e}}\, e^{\mathcal{E}_i/kT_e} = \frac{e n_g C_i}{\mu_i} \left(\frac{8\mathcal{E}_i}{\pi m_e}\right)^{1/2} \left(\frac{R}{2.4048}\right)^2 \quad \text{using } \sigma_i \equiv C_i \mathcal{E}_i$$

Define $a \equiv \frac{e n_g C_i}{p_0}$ and $eV_i \equiv \mathcal{E}_i$. V_i is the ionization potential of gas molecules in volts, while a is ionization per length per gas pressure per volt of electron potential above V_i (electron kinetic energy given in volts: $\frac{m}{2}v^2 \frac{1}{q}$)

n_g is number density (not molar density)

At $T_0 = 273.15\,[K]$, $P_0 = n_g k T_0$, so $a = \frac{eC_i}{kT_0}$ is an inherent property of the gas.

Now

$$\left(\frac{eV_i}{kT_e}\right)^{-1/2} e^{eV_i/kT_e} = \frac{a p_0}{\mu_i} \left(\frac{8eV_i}{\pi m_e}\right)^{1/2} \left(\frac{R}{2.4048}\right)^2$$

which rearranges to

$$\left(\frac{kT_e}{eV_i}\right)^{1/2} e^{eV_i/kT_e} = \sqrt{\frac{8e}{\pi m_e}}\, \frac{1}{2.4048^2}\, \frac{a\sqrt{V_i}}{\mu_i p_0} (p_0 R)^2$$

$\boxed{Q3}$. Verify that the units work out in SI

$$\frac{k\ \text{Temperature}}{\text{charge}\cdot\text{potential}}\, e^{\text{charge}\cdot\text{potential}/k\ \text{Temperature}} = \sqrt{\frac{\text{charge}}{\text{mass}}}\, \frac{a\sqrt{\text{potential}}}{\mu_i\ \text{pressure}} (\text{pressure}\cdot\text{length})^2$$

$$\cancel{k\ \text{Temp}}\ \cancel{\frac{\text{Energy}}{\text{Energy}}}\ \text{unitless} = \sqrt{\frac{\text{charge}}{\text{mass}}}\, \frac{\text{charge}\ C_i}{\text{volume}\ \cancel{\text{pressure}}}\, \frac{\sqrt{\text{potential}}}{\mu_i\ \cancel{\text{pressure}}} (\text{pressure}\cdot\text{length})^2$$

unitless = $\frac{\text{charge} \cdot \text{energy}}{\text{mass}}$ $\frac{\text{[illegible]}}{\text{volume}}$ $\frac{\text{length}^{2}}{\mu_i}$ = charge $\cdot \sqrt{\frac{\text{energy}}{\text{mass}}}$ $\frac{C_1}{\text{length}}$ $\frac{1}{\mu_i}$

If velocity $= \mu_i E$ then μ_i = velocity / energy

unitless = charge $\cdot \sqrt{\frac{\text{energy}}{\text{mass}}}$ $\frac{C_1}{\text{length}}$ $\frac{\text{energy}}{\text{velocity}}$

If $\sigma_i = C_1 E$ proportionality $= C_1$ energy then $C_1 = \cancel{\frac{\text{energy}}{\sigma_i}}$ $\frac{\sigma_i}{\text{energy}}$

unitless = charge $\cdot \sqrt{\frac{\text{energy}}{\text{mass}}}$ $\frac{\text{energy}}{\text{length}}^{2}$ $\frac{V}{\cancel{\rho_i}}$ $\frac{1}{\text{velocity}}$ $\frac{\sigma_i}{\cancel{\text{energy}}}$

E = mass velocity^2

unitless = charge $\cdot \cancel{\sqrt{\frac{\text{mass} \cdot \text{velocity}^2}{\text{mass}}}}$ $\frac{\text{energy}}{\text{length}}$ $\frac{V}{\rho_i}$ $\cancel{\frac{1}{\text{velocity}}}$ $\sigma_i = \sigma_i \frac{\text{charge}}{\text{length}}$ = unitless

$\boxed{\text{Both sides are unitless}}$

Now we define $c^2 = \frac{e n \sqrt{V_i}}{\mu_i \rho_0}$, or $c^2 = \frac{e n_0 C_1 \sqrt{V_i}}{\mu_i \rho_0} = \frac{e n_0 \sigma_i \sqrt{V_i}}{E \mu_i \rho_0}$

Note that $\mu_i \rho_0$ is a gas property independent of most imposed conditions, since $\mu_i \propto \frac{1}{\text{[illegible]}}$ of pressure

$$\frac{\sqrt{\frac{eV_i}{kT_e}}}{} e^{eV_i / kT_e} = 1.16 \cdot 10^{7} (c p_0 R)^2$$

The numerical term has units [cm/s $\sqrt{V}$], the "Von Engel-Steenbeck" "constant" (c) has units [[illegible] / torr cm], and $p_0 R$ has units [torr cm], making both sides unitless

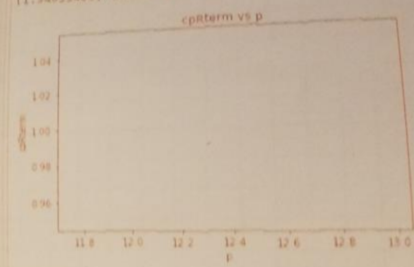
Computationally, it's easy to take the natural log: $\frac{eV_i}{kT_e} - \frac{1}{2} \ln\left(\frac{eV_i}{kT_e}\right) = 16.27 + 2 \ln(cp$

We can estimate $\frac{eV_i}{kT_e} \approx 16.27 + 2 \ln(c p_0 R)$

Different gases have different gas constants

```
def etempfunc(etempRatio,cpRterm):

    fvalue = etempRatio-0.5*math.log(etempRatio)-cpRterm

    return fvalue
from scipy.optimize import fsolve


etempRatioopt.fsolve(etempfunc,etempRatioGuess,args=(cpRterm))
etemp=Vi/etempRatio
print(                                                    .format(etemp[0],11605*et





result = fsolve (etempfunc,1.325,1.104)
print(result,11605*result)

import matplotlib.pyplot as plt
plt.plot(cpRterm, [p in range(5,15,1)])
plt.ylabel(        )
plt.xlabel(   )
plt.title(          )
plt.show
```

```
In [49]: runfile(                                   )
          , wdir=
1.253 eV 14542 K - first guess of electron
temperature
1.133 eV 13151 K - refined estimate of electron
temperature
[1.34053403] [15556.89745907]
```




```
In [50]:
```