Main Idea: Current Minima at Energy Levels

One of the primary steps in our experiment is determining at what energies electrons are able to have collisions and how many collisions they can have at these energies. We observe this through minima in the measured current; the first minimum is where electrons have enough energy for one collision, the next is where they have energy for two, and so on. The reason we are able to measure this as current is because the number of electrons passing through an area is proportional to the current:

$$I = neAv$$

So, these minima occur where there is also a minimum of passing electrons. They occur because the electrons must overcome an electric potential from the cathode to the collection plate of 1.5[V]. This potential is associated with an electric field which exerts a force on the electrons in a direction opposite their travel. For the electrons travel through this region, their kinetic energy upon reaching the cathode must be greater than or equal to the potential energy of the field:

$$\frac{m_e}{2}v^2 = KE \geq U = q\Delta V.$$

If it is not, the electrons will stop moving toward the collection plate when their kinetic energy – and therefore velocity — reaches zero, and they will eventually start to turn around. As the energy of the electrons increases, we generally see more current because more electrons have sufficient energy to cross the potential, but we see minima at roughly 4.6[eV]*n (where n is the number of collisions) because at these energies, electrons are able to give up almost all of their energy by colliding n times. We see a smooth curve rather than discrete functions or sharp peaks both because the velocities have some variation and because not all electrons will collide n times, as the probability of collision is less than unity (since the cross-section for inelastic collisions is very small).

Equipment: Inside the Quartz Tube

One of the key elements of our setup is the tube inside the temperature-adjusting box and all of the components therein. In this tube are several key components of our circuit. We have the anode, which is the filament from which the electrons are boiled off; the cathode, which is the grid over which we have a potential swept by the Keithley 6487 from 0 to 40[V]; the collection plate, which is swept by Keithley as well to keep it at a bias voltage of 1.5[V] with respect to the cathode; and the mercury vapor through which the electrons travel, and with which they have a small chance to collide. The anode is a thermionic filament. This means that it "boils off" electrons. It is able to do this because it has a high resistance, so as electrons pass through it, it heats up significantly. Eventually, it has a high enough temperature that the electrons on the surface have sufficient energy to break free of the potential energy barrier binding them to the metal. The vacuum tube itself is made from quartz, which is convenient for many reasons. The most obvious of these is that it is transparent, so we can see into the tube; though this isn't entirely necessary, if we were able to increase the temperature much further than we did in our experiment, we would be able to see the mercury vapor glow as the electrons run through it, and we can look into easily it if we suspect anything has gone awry internally. Quartz is also useful because it is electromagnetically non-conductive, preventing it from affecting current, and because it has a relatively average thermal conductivity, so it doesn't significantly affect the rate at which temperature is increased or decreased. Additionally, quartz is one of the most common minerals on Earth, and it is not particularly difficult to shape compared to other minerals, making it a relatively cheap option, and it doesn't shatter in high temperatures like glass will. The tube is vacuum-sealed for three reasons: this keeps the mercury vapor and electrons from escaping; it keeps other gases from entering, interacting with the electrons, and interfering with our data; and it keeps the pressure low enough that the mercury is able to stay vaporized. At atmospheric pressure, mercury's boiling point is over 600[K], but the tube's pressure should be on the order of 10^{-3}[atm], making the vapor easy to maintain. Mercury is also useful because its excited state is at a reasonable level — near 5[eV], it is easy to detect but not difficult to achieve, allowing us to get several data points.

Data Analysis: Finding the Cross-Section for Collisions

One of the values we calculated in this experiment was the cross-section for collisions. This value is probabilistic, being associated with the likelihood that an inelastic collision will occur between a mercury atom and a passing electron. A very literal way of thinking of this is that, for the inelastic collision to occur, the electron must be passing through a very small cross-sectional area that increases with the temperature. We can determine the cross-section using

$$\sigma = \frac{k_B T}{P\lambda},$$

Where T is our temperature, P is the pressure, and λ is the mean free path. Pressure can be determined by temperature — within our temperature range, pressure can be approximated as

$$P = 8.7 * 10^{9-\frac{3110[K]}{T}}[Pa].$$

Temperature was controlled within ± 4[K]. The mean free path was determined from our voltage and current data. Voltage data directly correlated to energy data, as our carriers are electrons, so volts directly translate to electron volts. We then found the minima in the current and wrote down their *n*-values and corresponding energies. Next, we recorded $\Delta E_n = E_n - E_{n-1}$, then plotted these against n and took a curve fit. By extrapolating the curve fit to n=0.5, we were able to determine E_a. Using

$$\Delta E = \left[1 + \frac{\lambda}{L}(2n - 1)\right] E_a,$$

Where L is the distance from the anode to the cathode, we were able to determine the mean free path from the slope of our curve fit as

$$\frac{d\Delta E}{dn} = \frac{\lambda}{L} 2E_a.$$

Substituting all of our measured and calculated values into our original equation, we were able to calculate the cross-sections at four different temperatures. However, we don't see the patterns we would expect; the cross-section isn't monotonically increasing with temperature. Rather, it seems to be all over the place. Additionally, our uncertainties are proportionally very high; the smallest of our uncertainties is roughly 20% of the associated value, though it makes sense that measuring something so indirectly would give us a very high uncertainty. Additionally, we do see values on the expected order of magnitude for mercury in this temperature range.

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