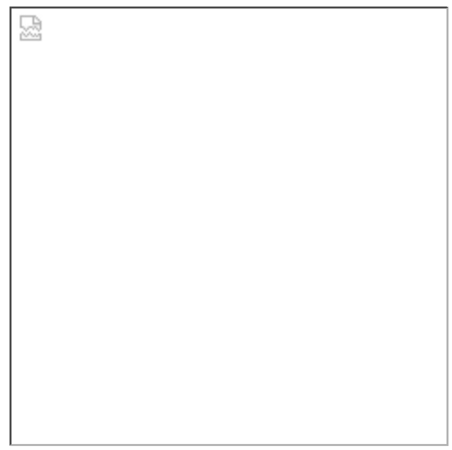
Impedance Spectroscopy

Phasor Notation

A **phasor** (a portmanteau of phase and vector) is a compact way of expressing a sinusoidal function with time-invariant amplitude A, frequency ω, and initial phase ϕ. It allows for easier mathematical manipulation and contains the same information that a sine-representation does. The idea of a phasor is derived from Euler's formula,
$e^{j\omega} + e^{-j\omega} = cos(\omega t) + jsin(\omega t)$.

Where the sinusoidal form of a signal may be written:
$e(t) = \sqrt{2}E_{max}cos(\omega t + \phi)$

The phasor in angle or complex notation becomes:
$\mathbf{E} = E_{max}\angle\phi = E_{max}e^{j\omega+\phi}$ where $j = \sqrt{-1}$



The image to the right is an example of a **phasor diagram**, which visualises a sinusoidal function in the Real-Imaginary plane. The horizontal axis represents the real portion of the complex function, and the vertical axis represents the imaginary. Because the amplitude, frequency, and phase are all time invariant, time is not explicitly represented in a phasor diagram. The phasor does, however rotate. A phasor diagram is distinguished from a vector diagram in that it rotates about the origin with frequency.

The amplitude of the any sinusoidal phasor is taken to be the RMS (root mean square) value of the oscillating function. With voltage and current as examples:
$V_{RMS} = \frac{1}{\sqrt{2}}V_{peak-to-peak}$
$I_{RMS} = \frac{1}{\sqrt{2}}I_{peak-to-peak}$

Impedance, which is a more complete form of resistance, can be expressed in complex form where resistance R of resistors is on the real axis, and reactance X of inductors and capacitors is on the imaginary axis:
$\vec{Z} = R + iX$

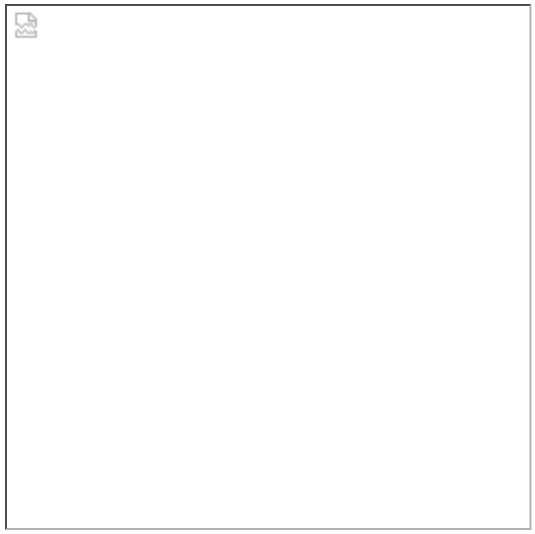
In phasor form, complex voltage, current, and impedance can all be used with Ohm's law and Kirchoff's laws of circuits.

Major Equipment - Lock-In Amplifier

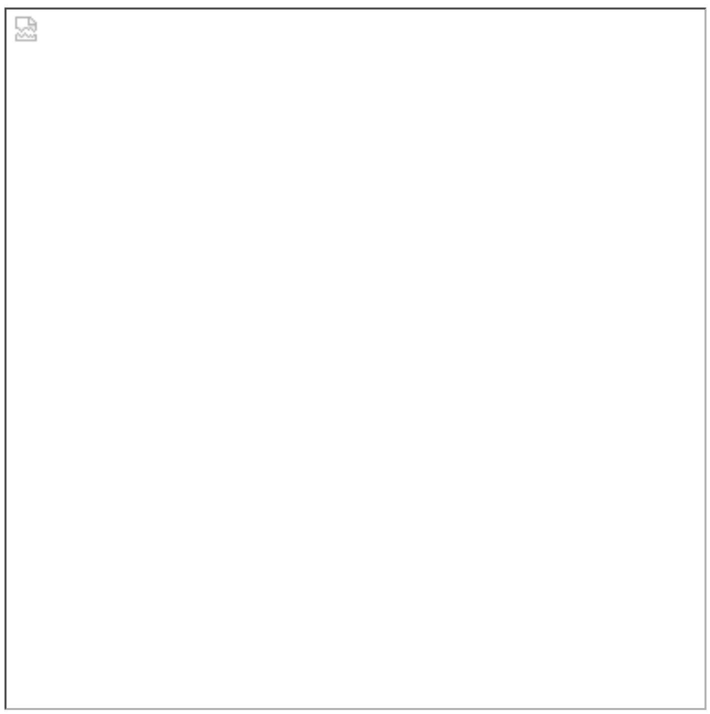
Background noise in data collection is a pervasive problem that affects a wide variety of applications, from dark matter detectors to recording studios. Especially at smaller voltages like the nanovolt scale, background noise can be several times stronger, making signal analysis particularly tricky by burying the true signal among a slurry of conflicting frequencies (exemplified in the image below).

Data Analysis - Estimating Unknown Circuit Values

Assuming the values of the the circuit components are unknown, but the circuit diagram is the same as in the experiment:



Then the values can be estimated using Python modules such as Numpy and SciPy. Expressions for voltage and phi as functions of frequency can be derived, with circuit components as parameters:



`scipy.curve_fit` is a callable function from SciPy that uses regression to fit a custom function to data, and return optimized values for given parameters.

First, import modules used for analysis:

```
import numpy as np
from scipy.stats import chisquare
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt

pi = np.pi
```