Impedance Spectroscopy

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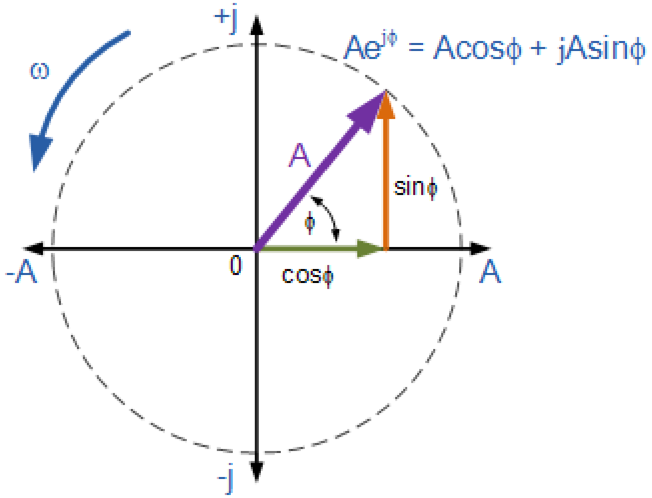
Phasor Notation

- Phasors are essentially a shorthand for oscillating functions expressed as functions of their phase and magnitude
- Name is portmanteau (“phase vector”)
- Consolidates signal information, for example:
  - $e(t)=\sqrt{(2)}A\cos(\omega t+\varphi)$
  - Becomes $A\angle\phi = A*e\wedge(j\omega+\varphi)$ where $j=\sqrt{(-1)}$
- Allows for use of complex impedance
  - $Z = R + jX$
    - Z = total complex impedance
    - R = resistor impedance, “resistance”
    - X = capacitor/inductor impedance, “reactance”
  - Complies with Kirchhoff's circuit laws and Ohm’s law of resistivity

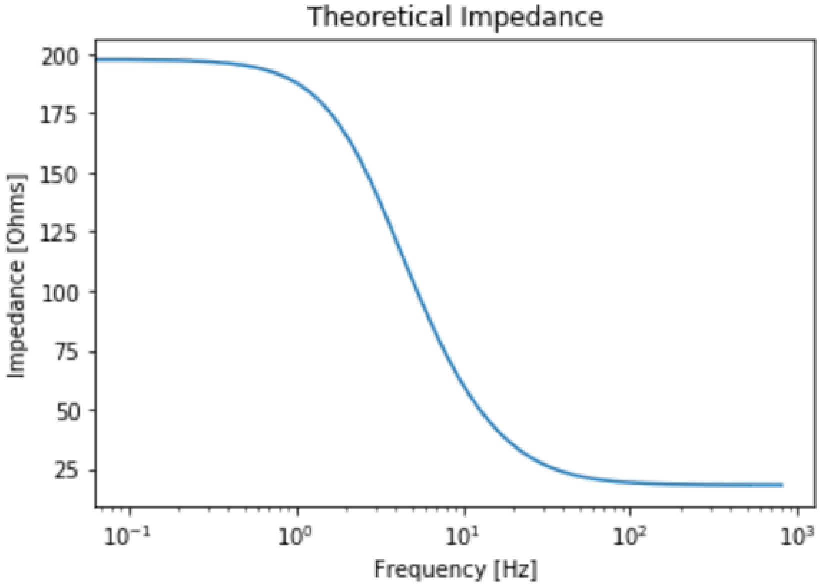


Frequency Dependence of Impedance

- **Impedance** is a measure of how current flow is hindered in a system.
- $Z_{series} = \sum_i Z$ and $\frac{1}{Z_{parallel}} = \sum_i \frac{1}{Z}$
- $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$
- So, for our system under test, we end up with

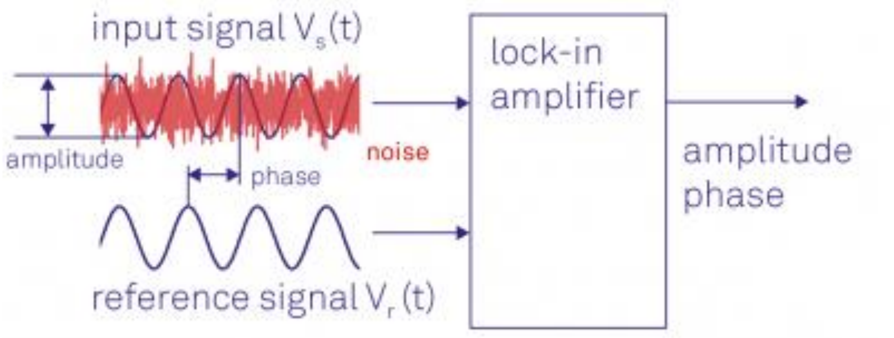
$$Z = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{\frac{1}{j\omega C} + R_2}\right)^{-1} = \frac{\mathrm{R}_1(1 + \omega^2 C^2 R_2(R_1 + R_2) - j\omega C R_1)}{1 + \omega^2 C^2 (R_1 + R_2)^2}$$

- As frequency approaches zero, the system behaves more like a DC circuit and charge cannot cross the branch containing the capacitor, so $Z \cong R_1$.
- As frequency becomes arbitrarily large, the current can arc across the capacitor, effectively ignoring its presence, and $Z \cong \frac{R_1 R_2}{R_1 + R_2}$.
- $V = I|Z|$



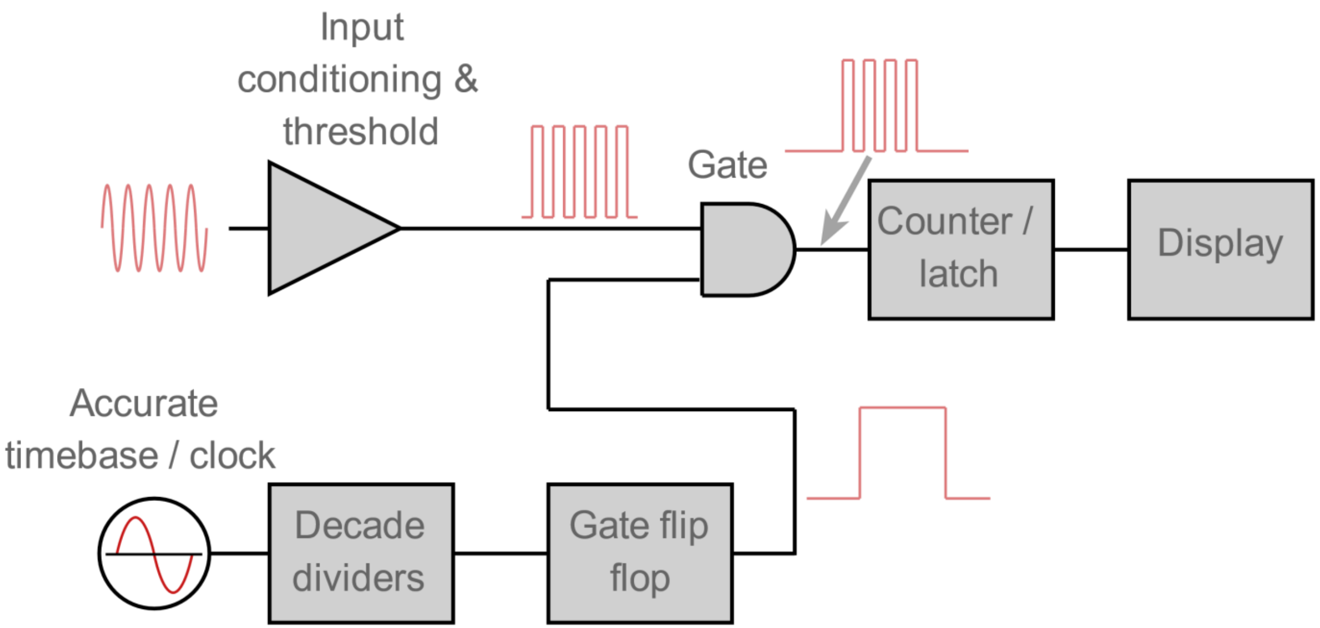
Lock-In Amplifier

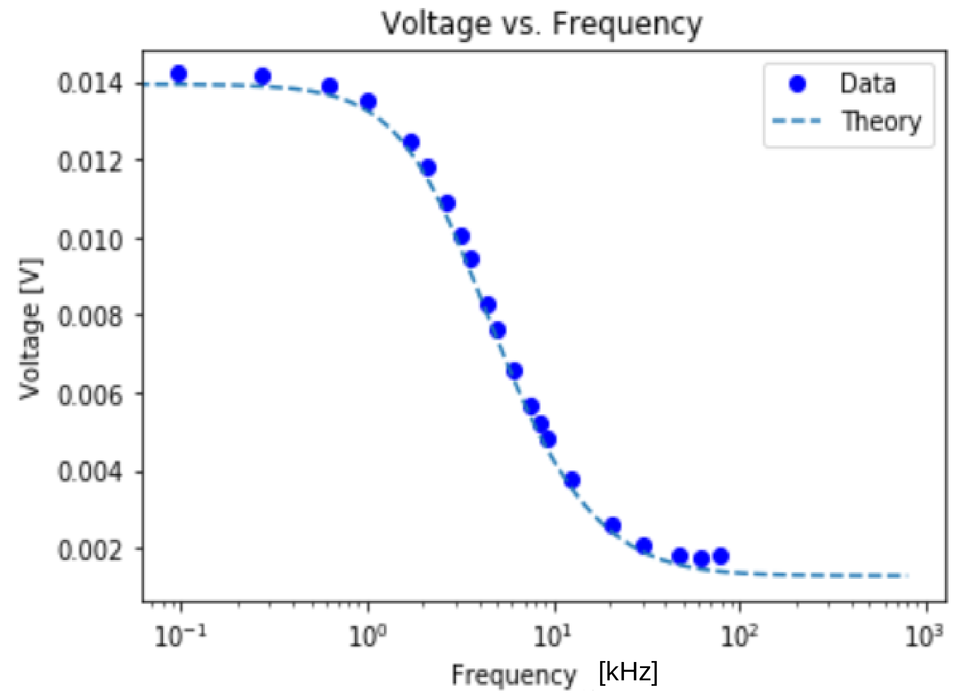
- Especially at low current, background noise conflicts with data
  - Need a way to remove unwanted signals
- One solution: Lock-in Amplifier
  - Phase sensitive detection / demodulation
  - Synchronizes internal oscillator to external reference signal
- Multiplies and integrates two signals (reference and input):
  - $A_{sig}\sin(\omega_{sig}t+\varphi_{sig}) \times A_{ref}\sin(\omega_{ref}t+\varphi_{ref})$
- Output becomes:
  - 0 for differing frequencies
  - $V_{out}=(1/2)V_{sig}*V_{ref}*\cos(\varphi)$
- Result is isolation of signals with frequencies of interest, effectively reduces background noise

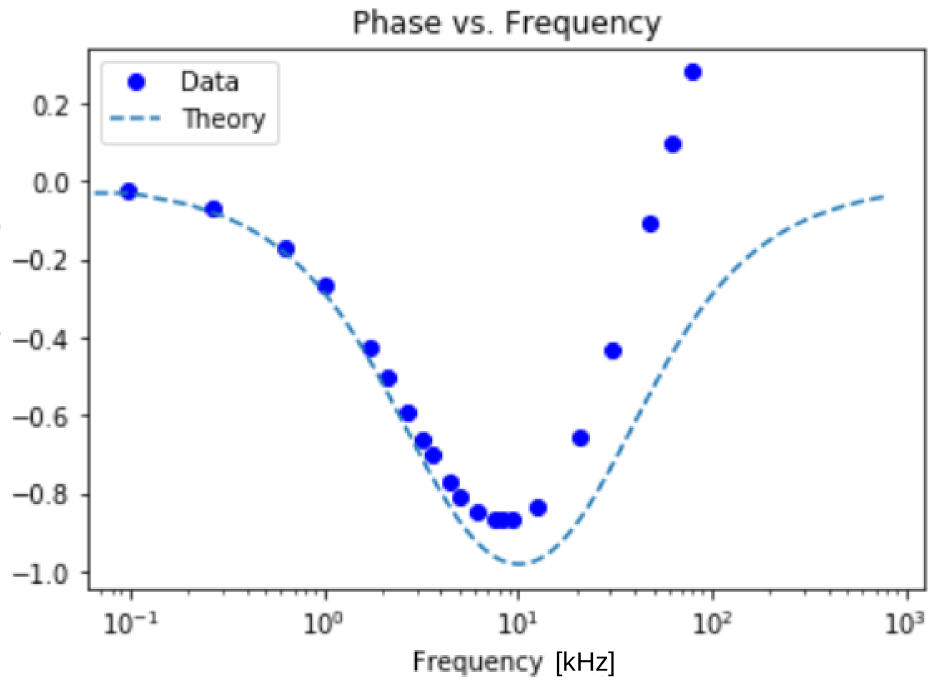


Frequency Meter

- A digital frequency counter measures frequency by taking counts of a periodic function over a **gate time.**
- **Direct counting** methods measure how many times the voltage (or other property being measured) passes a given threshold.
- **Reciprocal** frequency counters measure the period of a cycle and take the inverse.
- A clock produces a signal which is divided by the **decade dividers** into the appropriate gate time.
  - The "gate flip flop" receives this information and signals when the next set of counts should begin.
  - The **latch** holds the last value so that it can continue to be displayed while the counter is updating based on a new set of data.







Prediction vs. Data

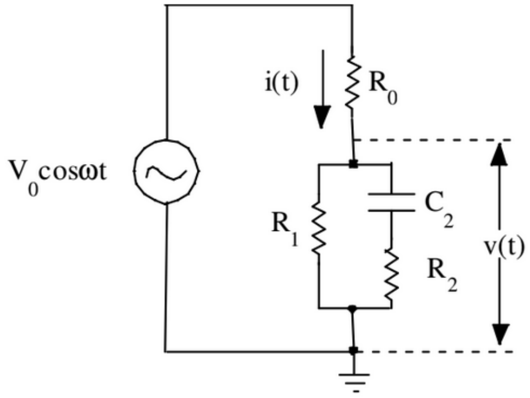
Voltage Data

- We see an almost perfect correlation here, especially in the center.
- Asymptotes match fairly well
- Standard deviation: $8*10^{-8}$[V]

Phase Data

- The left half of the plot appears to be roughly correct.
- The dip occurs at the right point.
- We have positive phase; this should be impossible with an ideal circuit of our type
- Possible explanation: Something within the circuit is behaving as an inductor, producing a field opposing the current, causing the positive phase.
- Standard deviation: 0.05 [radians]

Curve Fitting

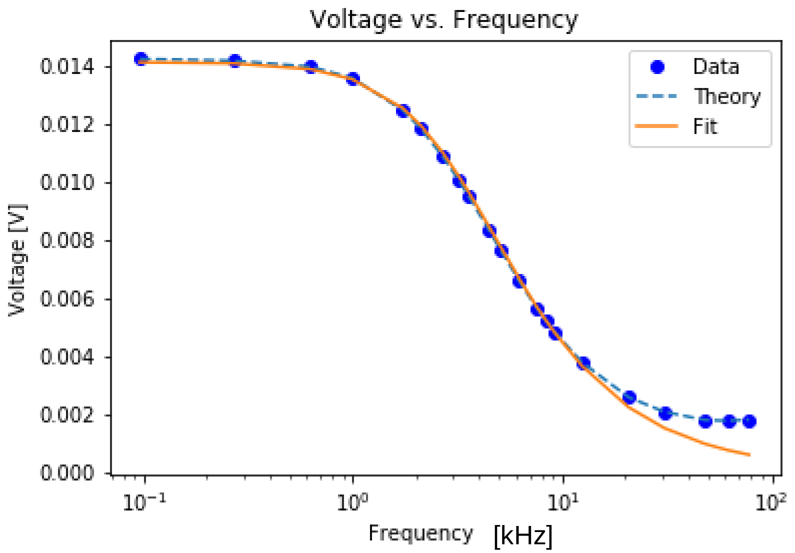


- Assuming a circuit with the same arrangement but different component values, Python can be used to fit a curve and approximate unknown circuit values

$$V = \frac{V_0}{R_0} R_1 \frac{\sqrt{\left(1+(2\pi f)^2 C^2 R_2 (R_1+R_2)\right)^2 + (2\pi f C)^2}}{1+(2\pi f)^2 C^2 (R_1+R_2)^2}$$

$$\phi = -\frac{2\pi f C}{1+(2\pi f)^2 C^2 R_2 (R_1+R_2)^2}$$

- SciPy.curve_fit() allows users to define a custom function with parameters to be optimized
- Values from curve_fit
  - R0 = 6.82329841e+04
    R1 = 1.36179032e+03
    R2 = 1.00003488e+00
    C = -2.20289657e-04
  - Covariance Matrix:
    [7.29720286e+21 1.45637188e+20 2.08120685e+14 2.35187976e+13]
    [1.45637188e+20 2.90661927e+18 4.15366154e+12 4.69386915e+11]
    [2.07711078e+14 4.14548663e+12 2.78138962e+07 6.67042843e+05]
    [2.35188426e+13 4.69387815e+11 6.68364286e+05 7.58012135e+04]
- Fit is close until higher frequency limit
  - Possible background inductance affecting derivation for V(f) due to affect on total system impedance



References

- https://www.instructables.com/id/Digital-Frequency-Counter/
- https://www.electronics-notes.com/articles/test-methods/frequency-counter-timer/how-does-a-frequency-counter-work-operation.php