UNIVERSITAT POLITÈCNICA DE CATALUNYA
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COMPUTATIONAL ENGINEERING

Project – Wing modelling

Consider the wing prototype with a NACA0018 airfoil depicted in Fig. 1. The prototype wing is to be tested in a wind tunnel to analyze its structural response under aerodynamic loads. To do so, the root section is clamped.

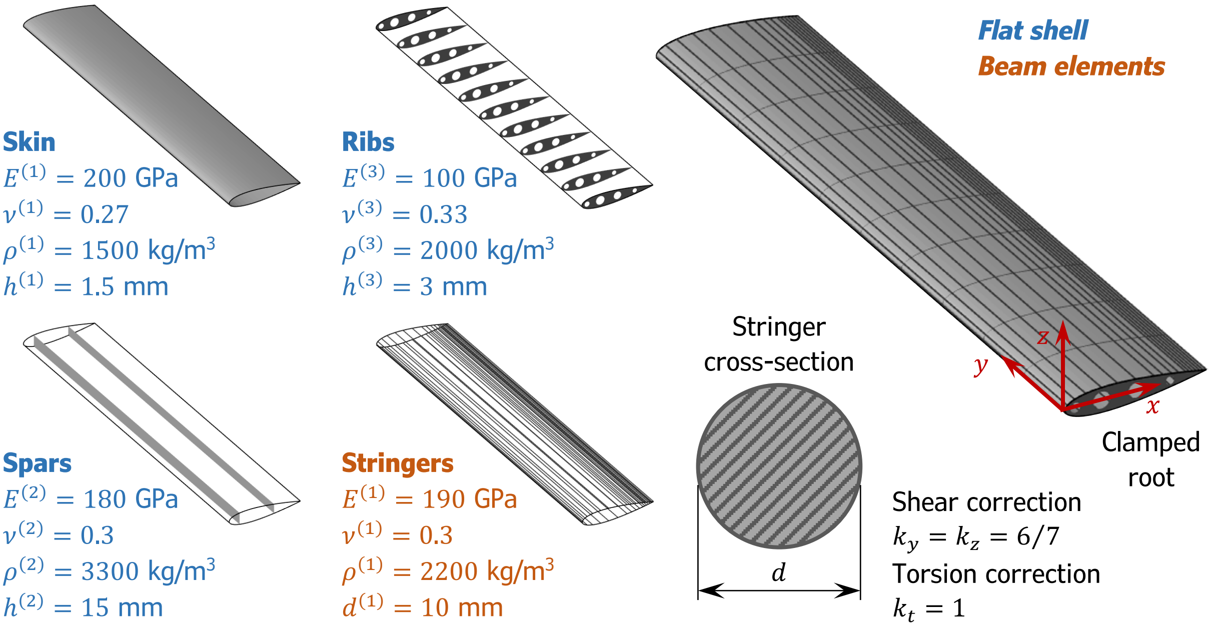
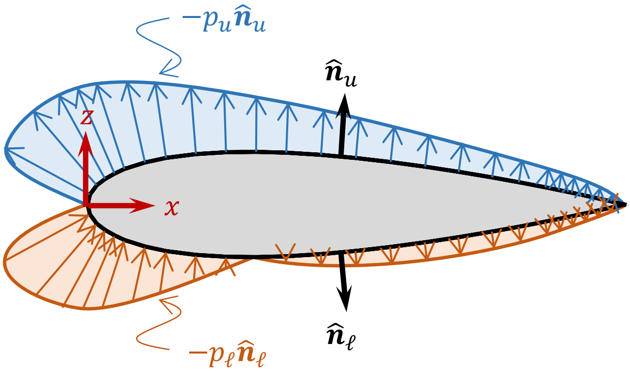


Fig. 1 Wing in a wind tunnel setting.

The wing supports its own weight and a pressure load distributed over the upper and lower skin surfaces. This pressure load is modelled as:



$$p_u(x,y) = -4\alpha P(y)\left[\left(1-\frac{x}{c}\right)^4 + \sqrt{1-\frac{x}{c}}\right]$$

$$p_\ell(x,y) = 4\alpha P(y)\left[\left(1-\frac{x}{c}\right)^4 - \frac{1}{4}\sqrt{1-\frac{x}{c}}\right]$$

$$P(y) = \begin{cases} p_\infty & \text{for } y \leq 0.8b \\ p_\infty \cos\left(\dfrac{\pi}{2}\dfrac{y-0.8b}{0.2b}\right) & \text{for } y > 0.8b \end{cases}$$

with α being the angle of attack (in rad), $c = 0.6$ m is the chord length, $b = 3$ m is the wingspan, and $p_\infty = 1.5625 \times 10^5$ Pa.

The following is asked:

(a) For $g = 9.81$ m/s^2 and an angle of attack of $\alpha = 10°$, plot the deformed state along with the Von Mises stress distribution over the upper and lower skins of the wing.

(b) Using the global stiffness and mass matrices, perform a modal analysis and plot the first 12 vibration modes.

(c) Solve the same static problem in (a) using a model-order reduction scheme based on a system projection onto the most relevant modes. Do it considering 2 and 6-modes approximations, respectively, and obtain:

   i. Deformed state and stress distribution for each case.
   ii. Plot the leading and trailing edges vertical displacement, u_z, along the span of the wing (y) for both cases (2 and 6 modes approximations) and compare the results with the same solution from (a).

A report with the requested results must be submitted in the corresponding task on Atenea. The assignment can be done in groups of maximum 2 people.

Important notes

Reference guidelines for the project resolution:

- The **`InputData.mat`** file contains the following variables:
  - **`X`** Nodal coordinates matrix
  - **`Tn_b`** Nodal connectivities matrix for all beam elements
  - **`Tm_b`** Material connectivities matrix for all beam elements
    - (1) – Stringers.
  - **`Tn_s`** Nodal connectivities matrix for all shell elements
  - **`Tm_s`** Material connectivities matrix for all shell elements
    - (1) – Skin elements.
    - (2) – Spar elements.
    - (3) – Rib elements.
  - **`I_root`** Root section nodes IDs (where DOFs need to be prescribed)
  - **`n_u`** Unit vector for upper skin nodes (to apply pressure force)
    - `n_u(:,1)` x-component
    - `n_u(:,2)` y-component
    - `n_u(:,3)` z-component
    - `n_u(:,4)` Corresponding node IDs
  - **`n_l`** Unit vector for lower skin nodes (to apply pressure force)
    - `n_l(:,1)` x-component
    - `n_l(:,2)` y-component
    - `n_l(:,3)` z-component
    - `n_l(:,4)` Corresponding node IDs
  - **`I_le`** Leading edge nodes IDs (for requested plots in (c))
  - **`I_te`** Trailing edge nodes IDs (for requested plots in (c))

- The **main** script file contains a reference structure for the general algorithm to solve the problem.

- The **plotWing** function can be used to plot the deformed state and requested Von Mises stress distributions (for questions (a) and (c)), for a given global displacements vector **u**, and the Von Mises stress at each Gauss point **SigVM** of all the flat shell elements (computed according to the *algorithm for problems involving flat shells*).

- The **plotModes** function can be used to plot the modes given the obtained modes matrix **Phi** and the squared natural frequencies **w2** (both computed according to the *algorithm for frequency analysis*).

- Given the relatively big size of the resulting system matrices, the assembly process and problem resolution can take some time (up to several minutes). Try to initialize the needed submatrices (using the **zeros** function) before entering long **for** loops whenever possible to speed up the process.

- <u>Once you are confident</u> about your global mass and stiffness matrices, you can save and load them in subsequent code runs avoiding having to recompute them and speed up the execution time. The functions:
  - `save('filename')` Stores the current workspace variables into a file called "filename.mat".
  - `load('filename')` Loads the workspace variables stored into a file called "filename.mat".

- <u>**Important**</u>: Initialize the global mass and stiffness matrices using the **sparse** function instead of the **zeros** function. This will allow you to use the **eigs** function for the modal analysis obtaining just the needed number of modes (trying to use the **eig** function with non-sparse matrices, i.e., initialized with the **zeros** function, can be dramatically slow).

- <u>**Important**</u>: Force the symmetry of the global mass and stiffness matrices as explained in the *algorithm for frequency analysis* to improve the performance of the modal analysis and avoid getting possible spurious modes.