Lecture 5: transpose, permutation, vector space

Permutation matrix

Permutation matrix P: to execute row exchange.

For a good matrix A: inversible and no-zero on the pivot column, always satisfy the equation $A = LU$.

But not all the matrix are good so we need permutation matrix to finish row exchange and for any inversible matrix always have $PA = LU$.

Transpose

$$(A^T)_{ij} = (A)_{ji}$$

If $A^T = A$, the A is a symmetric matrix.

- $R^T R$ always a symmetric matrix: $(R^T R)^T = R$.

Vector space

vector space

So-called vector space satisfied that closed with linear operation:

- addition:$\forall v, w \in \mathbb{R}^n,\ (v + w) \in \mathbb{R}^n$.
- multiply by a real number: $\forall v \in \mathbb{R}^n, n \in R,\ n \cdot v \in \mathbb{R}^n$.
- any vector space has to include $\mathbf{0}$ vector.

Example: From $\mathbb{R}^2$ extend to $\mathbb{R}^n$

In lower dimension, we can describe the vector space with diagram, such as $\mathbb{R}^2$ is a plane and $\mathbb{R}^3$ is a 3D space people lived and we can draw column picture of them.

When it extend to n-dimension, and $n \to \infty$, it's hard to describe the vector space and the intuitional understanding is $\mathbb{R}^n$ include all n-dimension vector and every vector have n real components.

subspace

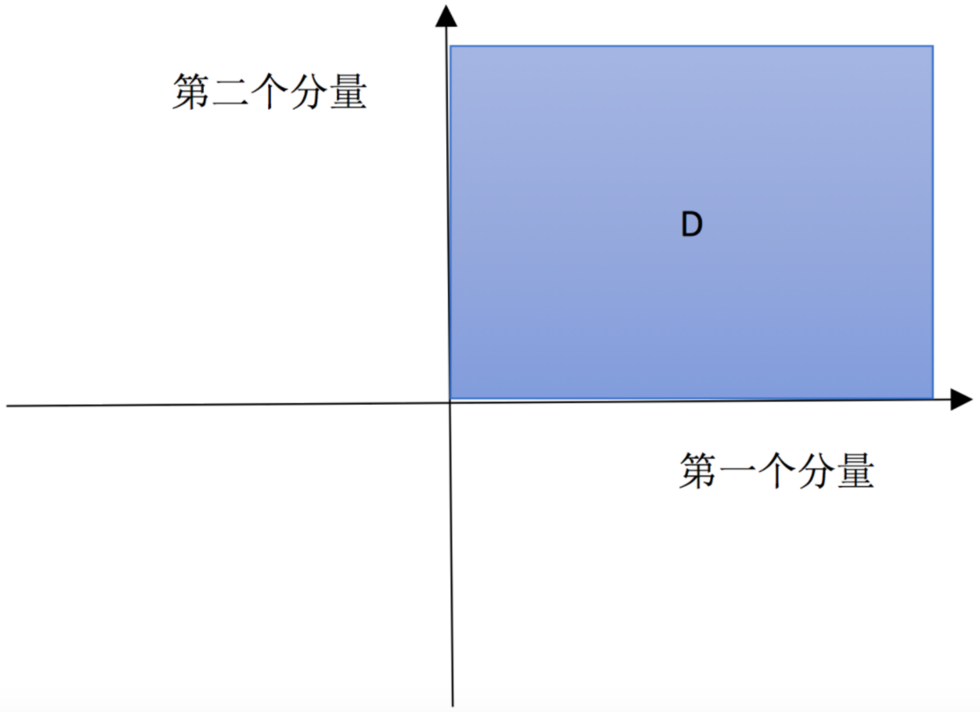
Consider: if we select a part of a vector space, will it construct a vector space?

Answer: it depends on whether it still satisfy the condition, some will be and some may not.

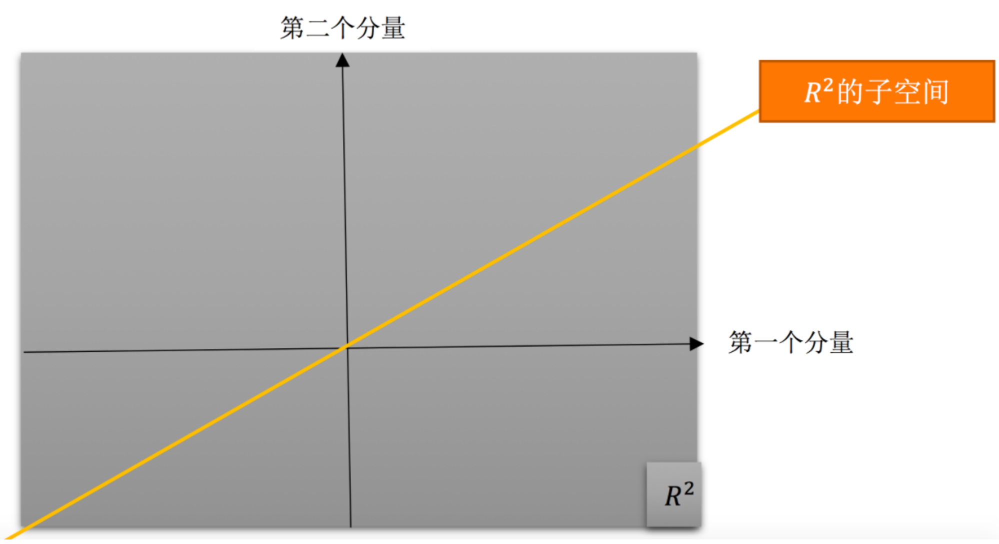
Example: $\mathbb{R}^2$

1. select the first quadrant as a sub-space $\mathbb{D}$

   It's obviously that if we multiply the vector in $\mathbb{D}$ with a negative, the result will in third quadrant rather than first quadrant, so sub-space $\mathbb{D}$ is not a subspace.



2. a line cross the original point: is a subspace of $\mathbb{R}^2$



Conclusion:

subspace for $\mathbb{R}^2$: $\mathbb{R}^2$, line across origin, $\vec{0}$.

subspace for $\mathbb{R}^3$: $\mathbb{R}^3$, plane across origin, line across origin, $\vec{0}$

Column space

Given a matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}$, how to build a subspace with it?

Look at the column vector, they all in $\mathbb{R}^3$, to satisfy the linear closed operation, a intuitional understanding is all its linear combinations form a subspace we called **column space** and denote as $C(A)$.