Lecture 3: Matrix multiplication

different ways of matrix multiplication

Definition:

$$A_{m*n} \cdot B_{n*p} = C_{m*p}$$

where $c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$ is theelements of C_{m*p} .

Normarlly, we do matrix multiplication like ablove, but let's look at in whole columns and rows.

- Consider columns of matrix B:

  we have $A \cdot b_i = c_i$, where b_i is the column vector of B, and i is in range p, c_i is the combination of the columns of A.

- Consider rows of matrix A:

  we have $a_i \cdot B = c_i$, where a_i is the row vector of A, and i is in range m, c_i is the combination of the rows of B.

- Or suppose all of the columns and rows:

  $A \cdot B = \text{sum of (cols of A)} \cdot \text{rows of B}$

- partitioned multiplication

Inverses(square matrix)

For a square matrix A, if A is invertible, then exist a unique matrix A^{-1} satisfy that:

$$A \cdot A^{-1} = I = A^{-1} \cdot A$$

- Matrix A must be a square matrix, if not, then matrix A_{-1} on the left side and right side may have different size, it's aginst the unique casd.

Look at matrix $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$, according to the value of determinant $\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0$ we can know this matrix is inreversible. But let's see the columns of the matrix, vector $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ is the multiple of vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, which means one of them insignificance to its linear combination.

Thus,

$$\text{if exist a non-zero vector } x \text{, makes } Ax = 0 \text{ then } A \text{ is invertible.}$$

solution

solve the inverse of matrix $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.

Gauss-Jordan method

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \Longrightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \Longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

use augmented matrix and do matrix exchange.

After some operations, matrix $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ become matrix I, in *Lecture 2* we know matrix multipliction can present this process, so we can suppose exist a exchange matrix E makes $EA = I$ and E is equal A^{-1}.

And matrix I on the right side of the augmented matrix did the same operation then become $\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ can see as exchange matrix $EI = E = A^{-1}$, thus matrix $\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ is A^{-1}.