1. (3 points) In class, it was suggested that if one had an ensemble of admissible heuristics h_1, h_2, ..., h_N, one can define a new heuristic h_{max} that gives for any node n:

$$h_{\max}(n) = \max_i h_i(n)$$

a) (1 point) Show that h_{max} is an admissible heuristic.

b) (2 point) Show that h_{max} dominates all of the other h_i's.

Problem Set 2

- $h_1, h_2, h_3, ..., h_N$ are admissible.
- For any node n, $h_{\text{max}}(n)$ is equal to $h_i(n)$ for some $i \in 1...N$.
- So we know it has to underestimate the cost from n to goal!

Problem Set 2

- $h_{\max}(n)$ is the greatest $h_i(n)$
- So all other $h_j(n)$ must be less than or equal to that.

Problem Set 2

1. In class, it was suggested that if one had an ensemble of heuristics $h_1, h_2, ...h_N$, one can define a new heuristic $h_{\max}$ that gives for any node n:

$$h_{\max}(n) = \max_i h_i(n)$$

   a) Show that $h_{\max}$ is an admissible heuristic.
   b) Show that $h_{\max}$ dominates all of the other h_i's.

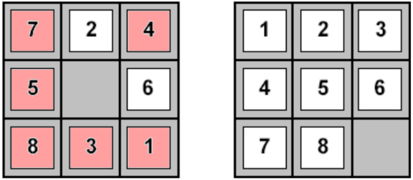
2. (4 points) Recall the 8-puzzle problem discussed in class.

a) (1 point) Show that the “misplaced tiles” heuristic is admissible.

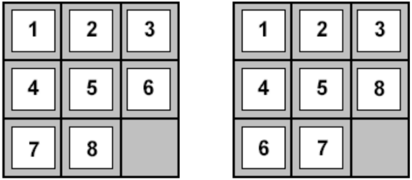
b) (1 point) Show that the “sum of Manhattan distances” heuristic is admissible.

c) (2 points) Explain why the “misplaced tiles” heuristic is not as good as the “sum of Manhattan distances” heuristic for the 8-puzzle.

- The heuristic comes from relaxation of the actual problem.
- The actual case will always take more work.



Problem Set 2



- Misplaced tiles: 3
- Manhattan distance: 6
- Actual moves required to solve: 12

Admissible Heuristic

“The total number of misplaced tiles”: It is clear that this heuristic is admissible since the total number of moves to order the tiles correctly is at least the number of misplaced tiles (each tile not in place must be moved at least once).

“Manhattan distance”: The Manhattan distance is an admissible heuristic in this case because every tile will have to be moved at least the number of spots in between itself and its correct position.

“Manhattan distance” h1 dominates “The number of misplaced tiles” h2: for any misplaced tile, the h2 is 1, but the h1>=1, h1>=h2. Since we can only move one tile to the empty position which means only one tile could change it’s position, the sum(h1) >= sum(h2) holds.

Problem Set 2

- Consider some configuration n
- n' is one move away from n
- A move can either increment or decrement that tile's Manhattan distance by 1
- Suppose the move decreases it
- $h(n) \leq h(n') + c(n, n')$
- If the move increases it, this still holds
- So h is consistent, and admissible!

3. (8 points) Consider the relaxed 8-puzzle problem where any tile can be swapped with the blank space, not just tiles directly adjacent to the blank space. The distance to the goal state in this relaxed problem is an admissible heuristic to the original 8-puzzle problem.

   a) (2 points) Explain why this heuristic is at least as good as the "misplaced tiles" heuristic.

   b) (3 points) Give an example state where this heuristic gives a better value than the "sum of Manhattan distances" heuristic. (draw figure and compute the heuristic)

   c) (3 points) Describe an algorithm for how to compute this heuristic. (pseudo code)

Gaschnig’s Heuristic

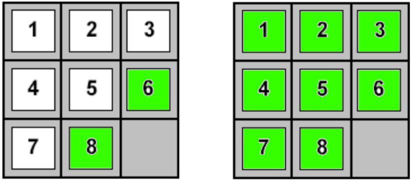
“The distance to the goal state” means “the number of moves”.

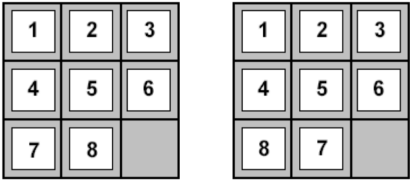
“This heuristic” refers to “Gaschnig’s Heuristic”

if the location of the blank space should be occupied by a tile, move that tile to the blank space;

else,

move any misplaced tile to the blank space.


1 2 3
4 5 6
7 8
1 2 3
4 5 6
7 8



- Misplaced: 2
- Manhattan: 2
- New heuristic: 3

```
while not at the goal state:
    if the blank is in the right spot:
        move a misplaced tile there
    else:
        move the tile that should be there, there
```