
MA2202S Homework 4

Qi Ji (A0167793L)

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1

- i. We already have $|S| = m$, just need to show $|gS| = m$. From definition of gS , we see that it is the image of $(g*)(S)$ the left-multiply by g map under S . Since the left-multiply map $(g*)$ is injective we have $|gS| = |S| = m$. \square
- ii. Let $S \in X$, we can verify that $\pi'(e, S) = eS = S$. Now let $g, h \in G$, $\pi'(g, \pi'(h, S)) = \pi'(g, hS) = ghS$. On the other hand $\pi'(gh, S) = ghS$. \square

2

- i. Since x_i and x_j are in the same orbit, we have a $g \in G$ such that

$$x_i = \pi(g)x_j.$$

Suppose $z \in G_{x_i}$ such that $\pi(z)x_i = x_i$, then we see that

$$\pi(g^{-1}zg)x_j = \pi(g^{-1}z)x_i = \pi(g^{-1})x_i = x_j$$

so $g^{-1}zg \in G_{x_j}$. We see that $z \mapsto g^{-1}zg$ defines a map $G_{x_i} \rightarrow G_{x_j}$. This map of conjugation is bijective, as a symmetric argument shows that $z' \mapsto gz'g^{-1}$ defines a map $G_{x_j} \rightarrow G_{x_i}$, which is its inverse. \square

- ii. By part (i) and proposition 79,

$$\sum_{i=1}^r |G_{x_i}| = r |G_{x_1}| = |Gx_1| |G_{x_1}| = |G|. \quad \square$$

3

- i. From definition of the matrix A , we have

$$\sum_{i=1}^n a_{ij} = |\{g_i \in G : \pi(g_i)x_j = x_j\}| = |G_{x_j}|. \quad \square$$

- ii. Also from definition of matrix A , we have

$$\sum_{j=1}^m a_{ij} = |\{x_j \in X : \pi(g_i)x_j = x_j\}| = |F(g_i)|. \quad \square$$

iii. By parts (i) and (ii),

$$\begin{aligned}
 \sum_{j=1}^m |G_{x_j}| &= \sum_{j=1}^m \sum_{i=1}^n a_{ij} \\
 &= \sum_{i=1}^n \sum_{j=1}^m a_{ij} \\
 &= \sum_{i=1}^n |F(g_i)|. \quad \square
 \end{aligned}$$

iv. By part (ii) of previous question,

$$\sum_{j=1}^m |G_{x_j}| = |G| \cdot |\{Gx : x \in X\}|.$$

By part (iii) we have the number of G -orbits being

$$\frac{1}{|G|} \sum_{j=1}^m |G_{x_j}| = \frac{1}{|G|} \sum_{i=1}^n |F(g_i)|. \quad \square$$