gesis Leibniz Institute for the Social Sciences

39. Methodenseminar: Big Data Module II

Introduction to Social Network Science with Python

Meso-Scale Analysis

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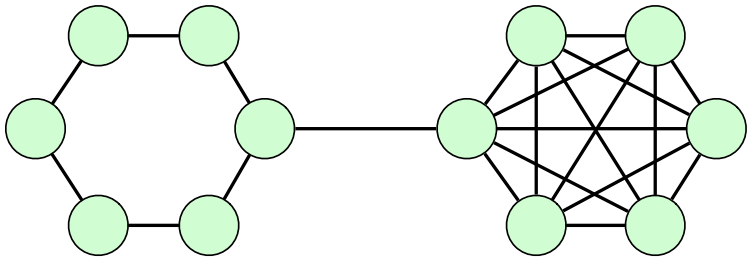
July 16, 2019

Leibniz Association

Meso-Scale Structures in Networks

- Real-world networks are heterogeneous.



$$C = 0.82$$

Meso-Scale Structures in Networks

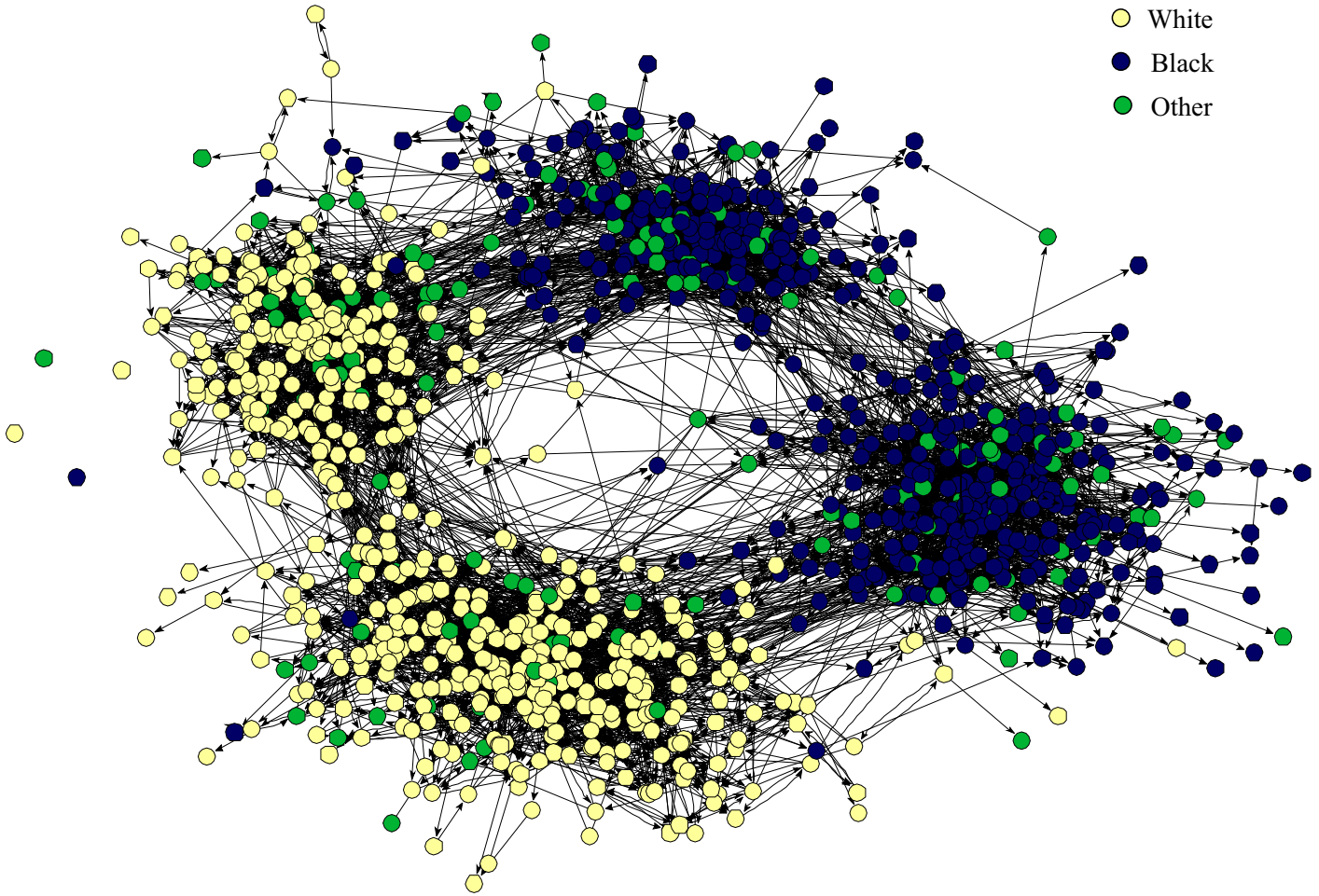
- We want a simplified model of something very complicated.
  - Networks are often too large and complex to be analyzed with some kind of simplification.

- Some uses
  - Extrapolation
  - Interpolation
  - Generalization
  - Mechanisms
  - Explanations
  - Useful division
  - Simplifications

Barabási, Albert-László. Network science. Cambridge University Press, 2016. From Clauset's and Larremore's class notes.

Meso-Scale Structures in Networks

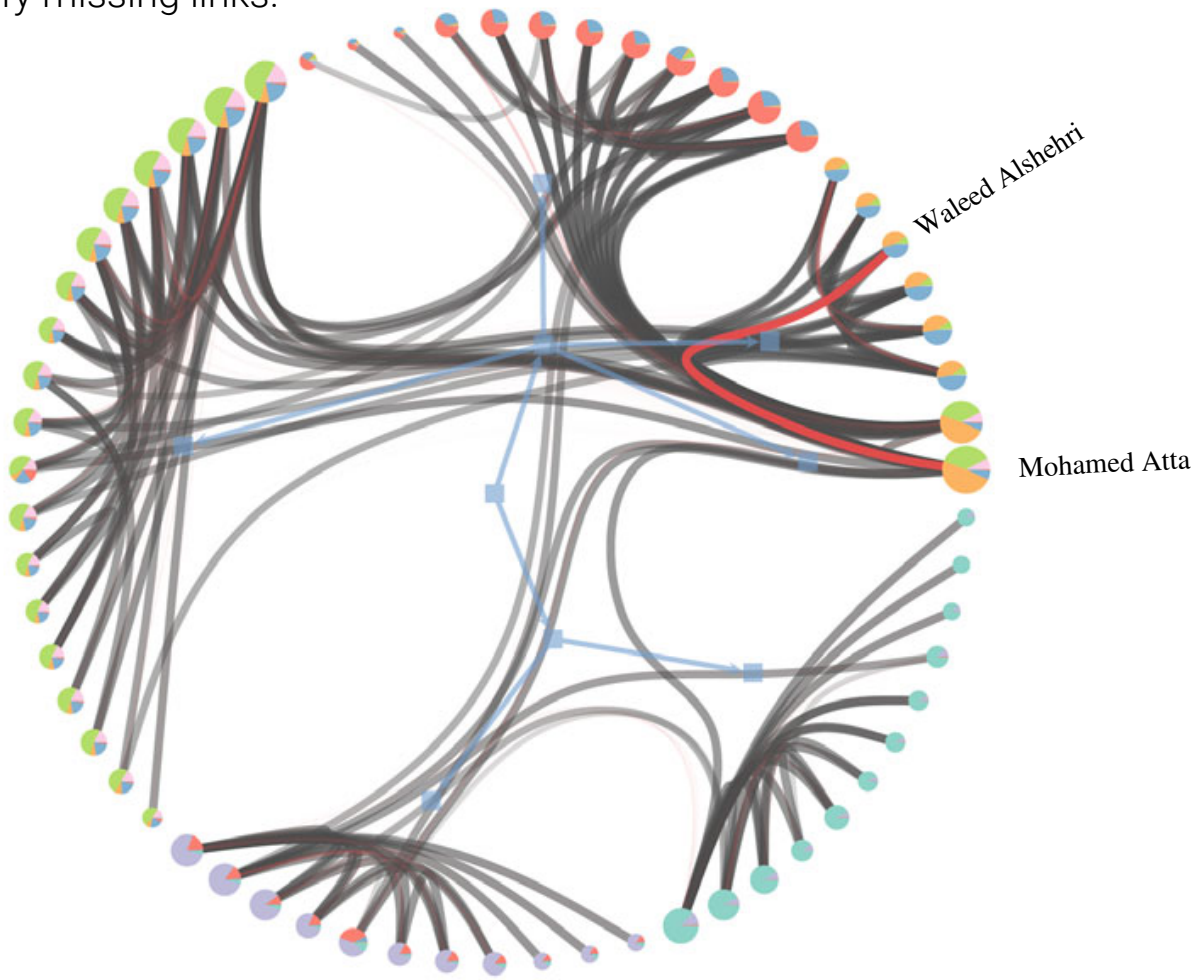
- Why?
  - Extrapolation
    - Make prediction for unseen nodes in the network.



Newman, M. E. J. (2003). The Structure and Function of Complex Networks. SIAM Review, 45(2), 167–256.

Meso-Scale Structures in Networks

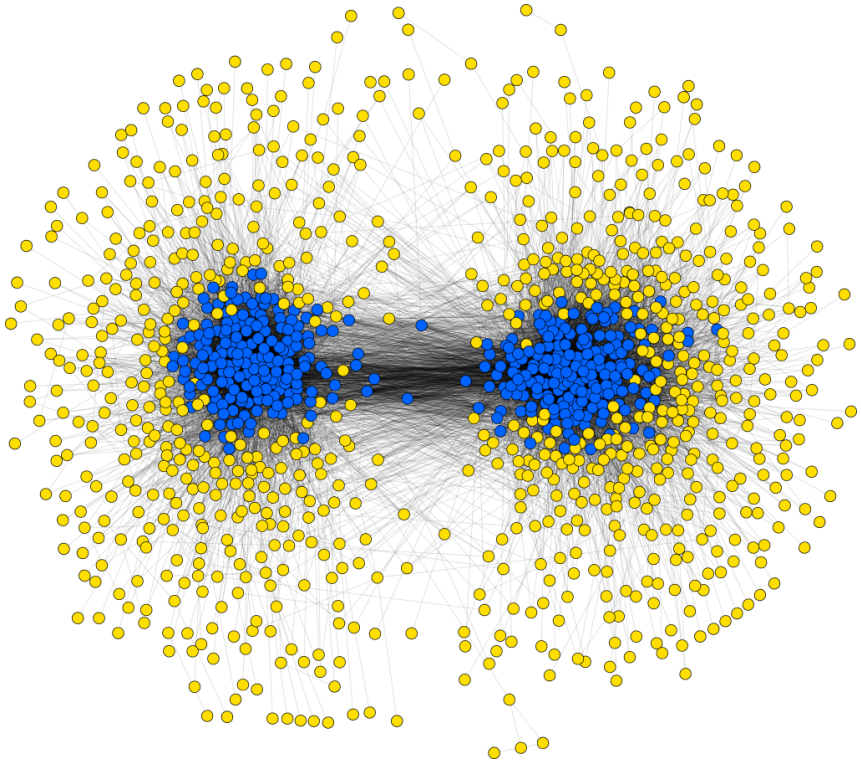
- Why?
  - Interpolation
    - Identify missing links.



Peixoto, Tiago P. "Reconstructing networks with unknown and heterogeneous errors." Physical Review X 8.4 (2018): 041011.

Meso-Scale Structures in Networks

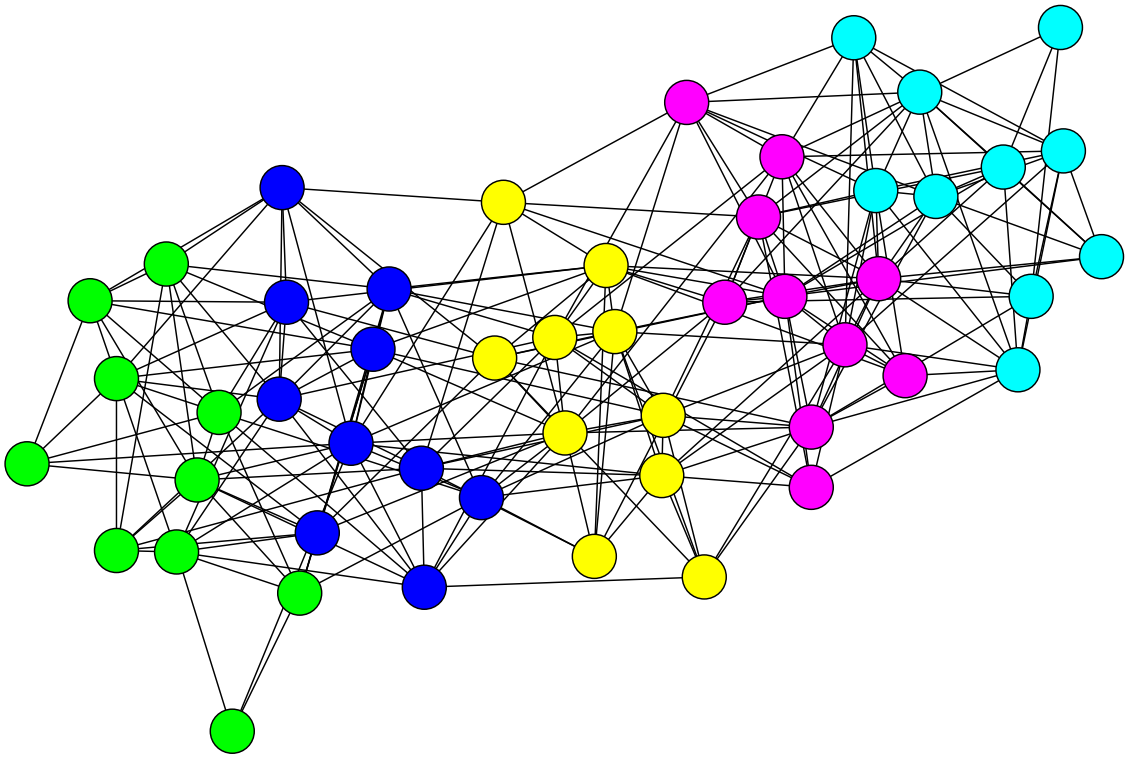
- Why?
  - Generalization
    - Nodes of this type are like the others of the same type.



Zhang, X., Martin, T., & Newman, M. E. J. (2015). Identification of core-periphery structure in networks. PRE 91(3), 1–10.

Meso-Scale Structures in Networks

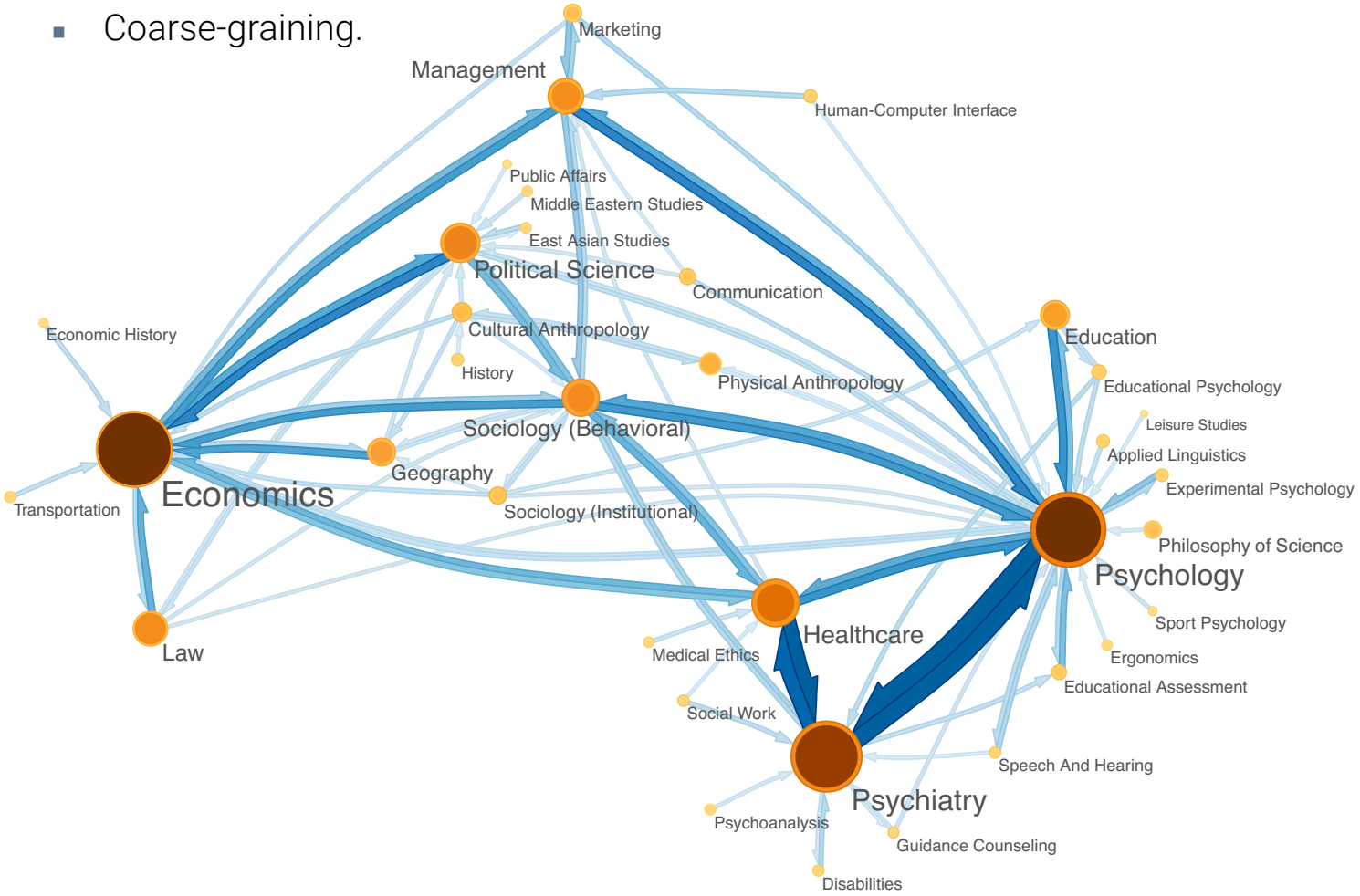
- Why?
  - Mechanisms
    - How did this network arise? What are the underlying mechanisms?



From Aaron Clauset's class notes.

Meso-Scale Structures in Networks

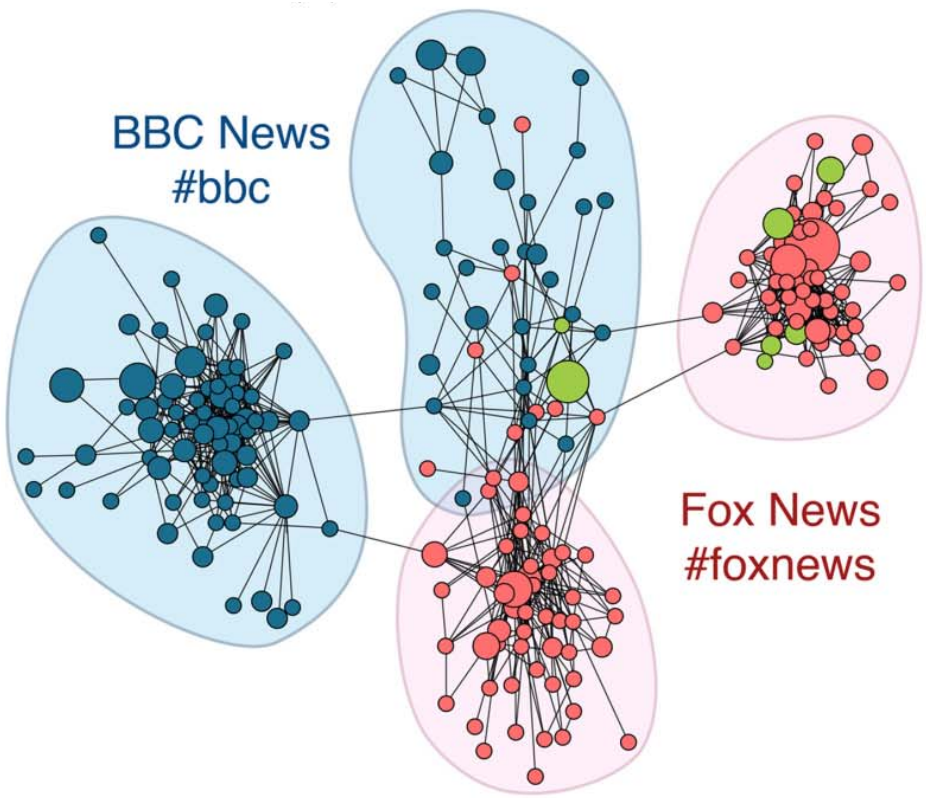
- Why?
  - Explanations
    - Coarse-graining.



Rosvall, M., & Bergstrom, C. T. (2008). Maps of random walks on complex networks reveal community structure. Proceedings of the National Academy of Sciences, 105(4), 1118–1123.

Meso-Scale Structures in Networks

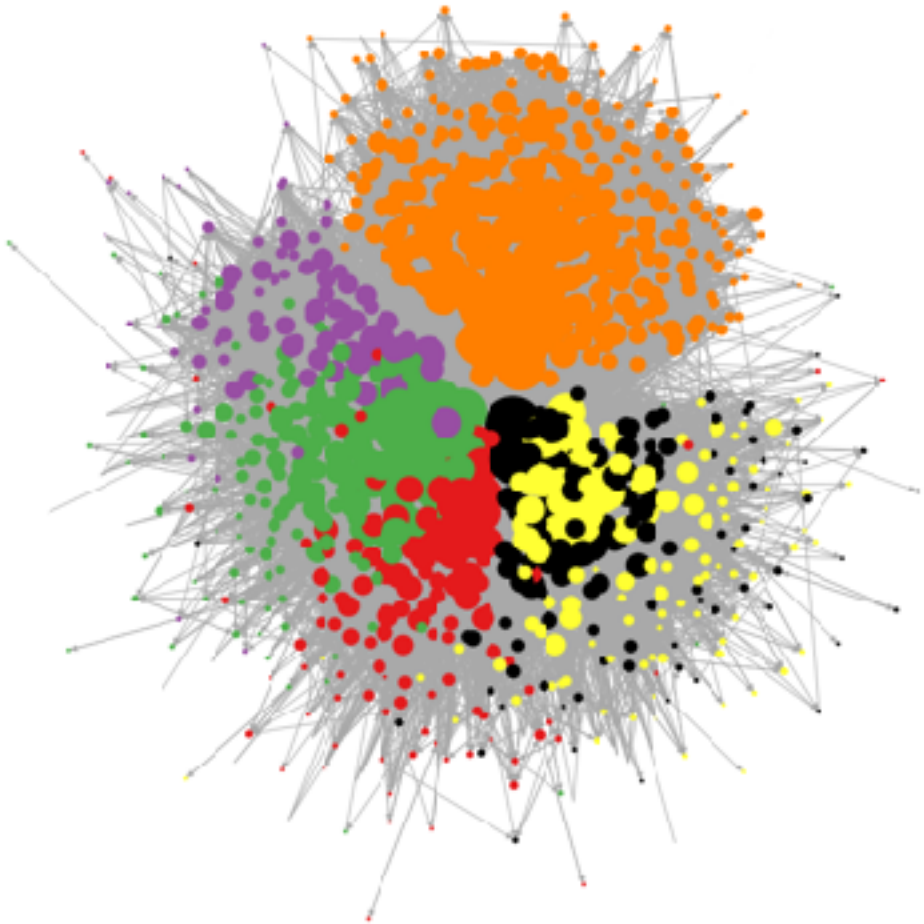
- Why?
  - Useful division
    - Finding groups to assign treatments.



Weng, Lilian, Filippo Menczer, and Yong-Yeol Ahn. "Virality prediction and community structure in social networks." Scientific reports 3 (2013): 2522.

Meso-Scale Structures in Networks

- Why?
  - Simplification
    - Regression models need ranks or groups.



Lietz, Haiko, et al. "When politicians talk: Assessing online conversational practices of political parties on twitter." Eighth International AAAI Conference on Weblogs and Social Media. 2014.

Meso-Scale Structures: Communities

Meso-Scale Structures: Communities

- Communities
  - In network science, we call a community **a group of nodes that have a higher likelihood of connecting to each other than to nodes from other communities**.

  - Homophily or assortative mixing: “like links with like.”

Barabási, Albert-László. Network science. Cambridge University Press, 2016. From Aaron Clauset’s class notes.

Meso-Scale Structures: Communities

- The so-called “hypotheses of community detection”:
  - H1: Fundamental hypothesis
    - A network’s community structure is uniquely encoded in its wiring diagram.
  - H2: Connectedness and density hypothesis
    - A community is a locally dense connected subgraph in a network.
  - H3: Random hypothesis
    - Randomly wired networks lack an inherent community structure.
  - H4: Maximal modularity hypothesis

Barabási, Albert-László. Network science. Cambridge University Press, 2016.

Meso-Scale Structures: Communities

- Modularity
  - How much more often do attributes match across edges than expected at random?

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - p_{ij} \right) \delta(x_i, x_j)$$

$\delta(x_i, x_j) = 1, \text{if } x_i = x_j$
$\delta(x_i, x_j) = 0, \text{otherwise}$

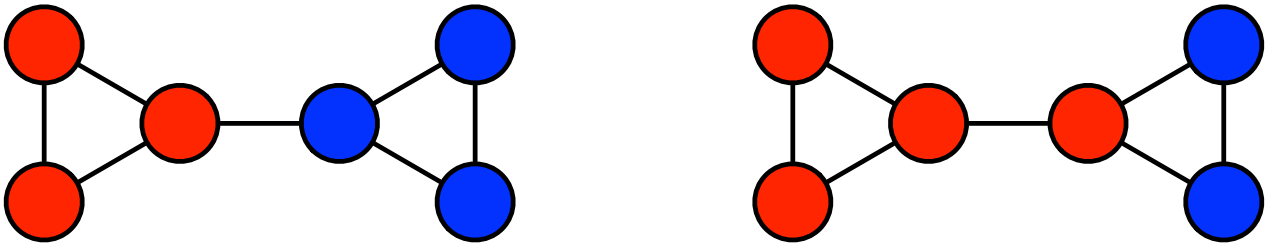
  - A good null model should account for the degree of the nodes:

$$p_{ij} = \frac{k_i k_j}{2m}$$

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(x_i, x_j)$$

Meso-Scale Structures: Communities

- Modularity
  - An example:



  - Resolution limit:
    - Modularity has an artifact in that small communities might be merged into a large one.
    - We will see this during the demo.

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(x_i, x_j)$$

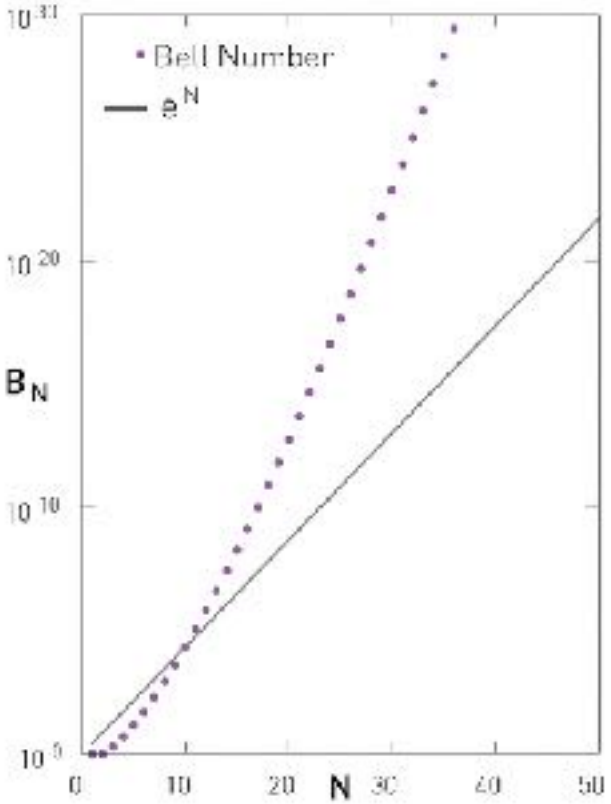
Meso-Scale Structures: Communities

- The hypotheses of community detection:
  - H1: Fundamental hypothesis
    - A network's community structure is uniquely encoded in its wiring diagram.
  - H2: Connectedness and density hypothesis
    - A community is a locally dense connected subgraph in a network.
  - H3: Random hypothesis
    - Randomly wired networks lack an inherent community structure.
  - H4: Maximal modularity hypothesis
    - For a given network the partition with maximum modularity corresponds to the optimal community structure.

Barabási, Albert-László. Network science. Cambridge University Press, 2016.

Meso-Scale Structures: Communities

- Community detection algorithms
  - Clauset-Newman-Moore: greedy modularity
  - The Louvain method
  - Girvan-Newman: a centrality-based method
  - Label propagation



Barabási, Albert-László. Network science. Cambridge University Press, 2016.

Meso-Scale Structures: Communities

- Community detection algorithms: greedy modularity.
  - General idea:
    - Start with every vertex being in its own group, then recursively and greedily choose a pair of groups to merge based on modularity.
  - Algorithm:

    1. Start with a every node being its own group.

    2. Choose the pair of groups that their merge would produce the greatest increase (or smallest decrease) in the modularity.

    3. Repeat step **2.** until there is only one group remaining.
  - Advantage: simple to implement and understand.
  - Disadvantage: not the fastest.

Meso-Scale Structures: Communities

- Community detection algorithms: the Louvain method.
  - Algorithm:

1. Start with a every node being its own group.

Phase 1:

2. For each node **i**

2.1. Evaluate the gain in modularity if node **i** joins the community of its neighbors **j**.

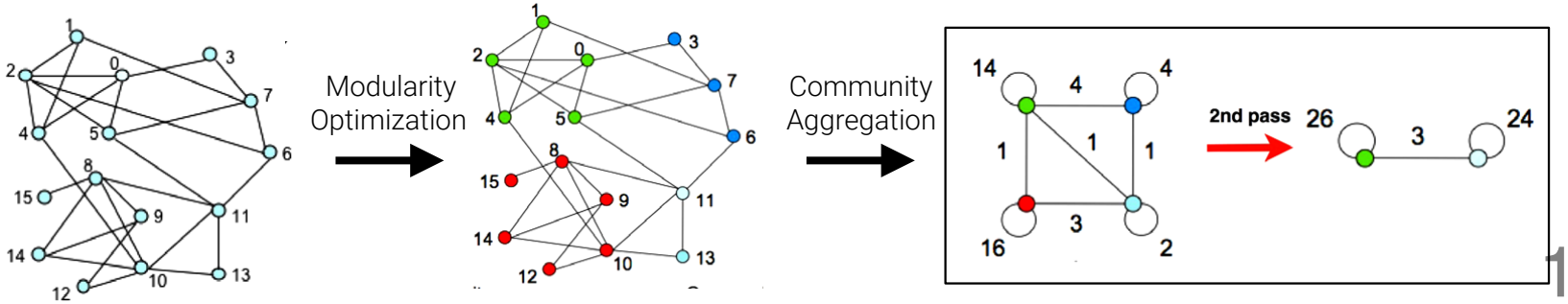
2.2. The node **i** is then placed in the community of maximum gain.

3. Repeat step **2.** until no individual move can improve the modularity.

Phase 2:

4. Create a network from the communities found in Phase 1.

5. Use this new network as input for the Phase 1 until there are no more changes.



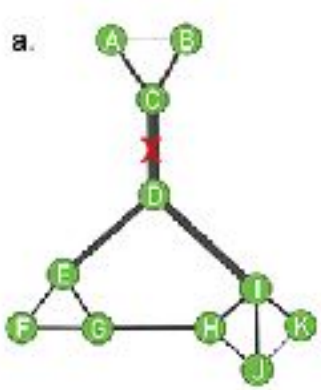
Meso-Scale Structures: Communities

- Community detection algorithms: the Louvain method.
  - Advantage: quite fast.

	Karate	Arxiv	Internet	Web nd.edu	Phone	Web uk-2005	Web WebBase 2001	
Nodes/ links	34/77	9k/24k	70k/351k	325k/1M	2.04M/5.4M	39M/783M	118M/1B	
	0.38/0 s	0.772/3.6 s	0.692/799 s	0.927/5034 s	—/—	—/—	—/—	Greedy
	0.42/0 s	0.813/0 s	0.781/1 s	0.935/3 s	0.76/44 s	0.979/738 s	0.984/152 mn	Louvain

Meso-Scale Structures: Communities

- Community detection algorithms: a centrality-based method.
  - General idea: remove links concerning nodes that belong to different communities, eventually breaking a network into communities.



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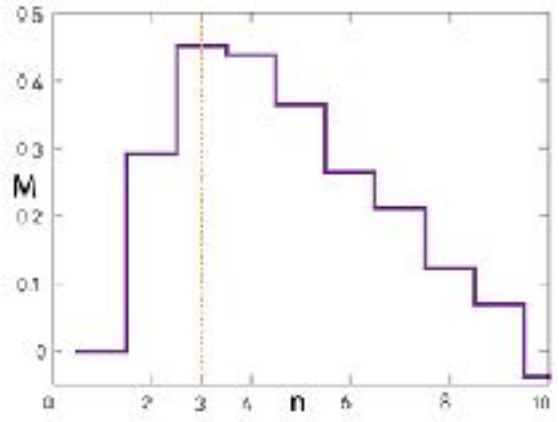

Meso-Scale Structures: Communities

- Community detection algorithms: label propagation.
  - General idea: labels are propagated across the network; the algorithm converges when each node has the majority label of its neighborhood.
  - Algorithm:

    1. Start with every node with a unique label.

    2. For each node **i**

    2.1. Set its label to the label occurring with the highest frequency among its neighbors.

    3. Repeat step **2.** until every node has a label that the maximum number of neighbors have.

Meso-Scale Structures: Communities

- Other measures of quality
  - Coverage
    - The ratio of *intra-community edges* by the total number of edges.
    - When all the communities are disconnected from each other, coverage is equal to 1.

$$\frac{\#\text{ intra-community edges}}{\text{total number of edges}}$$

  - Performance
    - The ratio between the number of connections inside the communities plus the non-edges between communities and the total number of potential edges
    - It counts the number of correctly “interpreted" pair of vertices.

$$\frac{\#\text{ intra-community edges} + \#\text{ inter-community non-edges}}{\text{total possible number of edges}}$$

Meso-Scale Structures: Communities

- Being critical about community detection
  - Do we really have communities?
    - Community detection algorithms will find communities, regardless they exist or not.
  - Hypotheses of community detection?
    - We cannot prove the correctness; how to turn them into theorems?
  - What does modularity really mean?
    - High modularity means the presence of significant deviations from the null model.
    - It is all about assortative mixing.
- Other meso-scale structures might explain the data better.
  - We will see this in the SBM part of the course.

Next: Community Detection (Demo)

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The Stochastic Block Model

If you don't have graph-tool installed, go to:
https://github.com/gesiscss/methods_seminar_2019

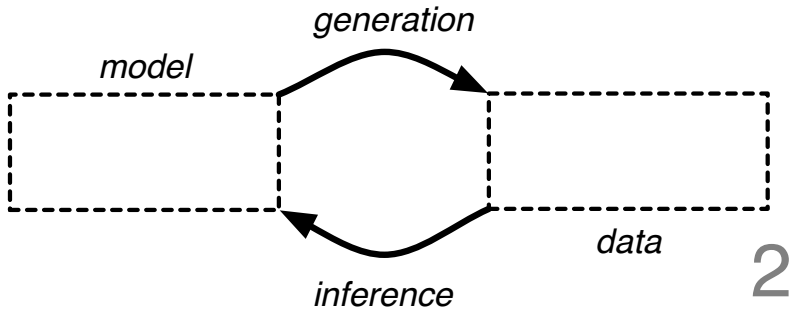
Then click on launch binder

Meso-Scale Structures in Networks: the SBM

- Block models
  - Divide the network into a set of nodes where all nodes in the same block had the same patterns of connection to nodes in other blocks.

- The Stochastic Block Model (SBM)
  - They are random graph ensembles in which:
    - **vertices** are separated into groups (or 'blocks'),
    - and the probability of an **edge** existing between two vertices is determined according to their group membership.
  - The *standard* stochastic block model assumes that all vertices belonging to the same block are statistically indistinguishable, they all have the same .

- A generative model approach:



Meso-Scale Structures in Networks: the SBM

- The Stochastic Block Model definition:

N Number of vertices

B Number of blocks

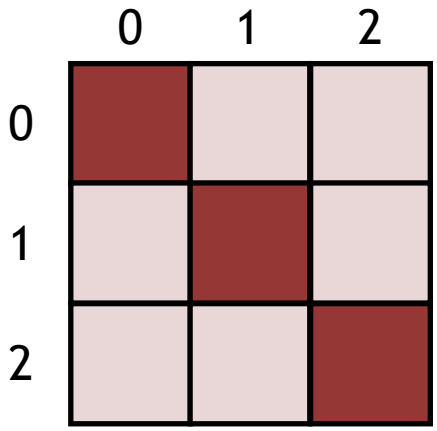
n_r Number of vertices in block $r \in [0, B-1]$

e_{rs} The number of edges between blocks r and s (micro-canonical)

Meso-Scale Structures in Networks: the SBM

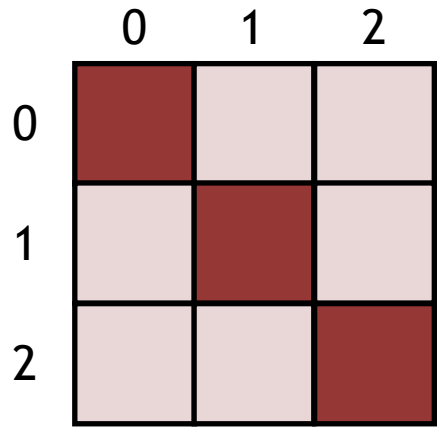
- SBM generation

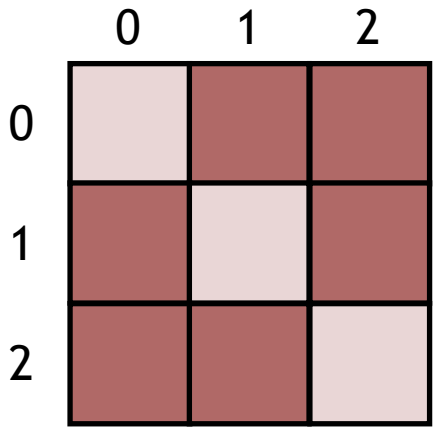
$$N = 99 \quad B = 3 \quad n_r = 33$$

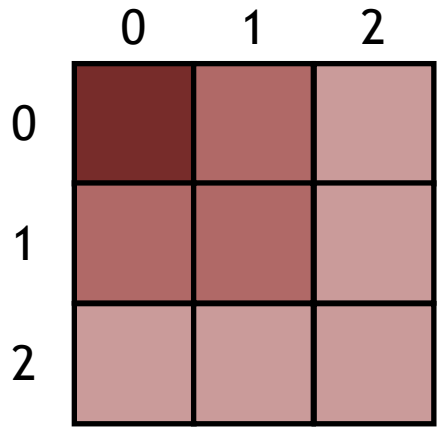


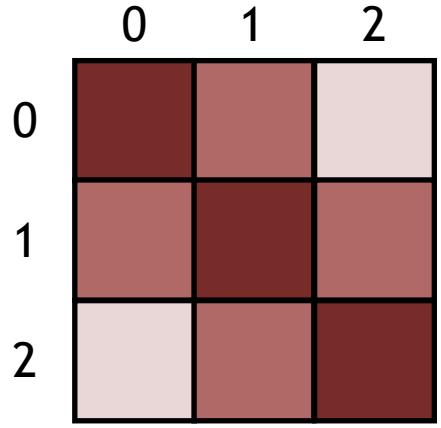
Meso-Scale Structures in Networks: the SBM

- Block matrix





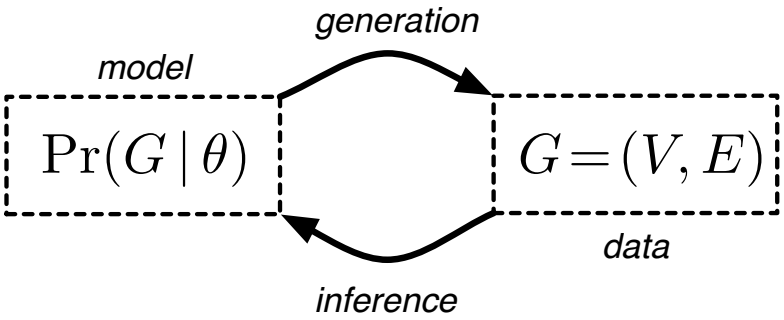




We will play with these during the demo.

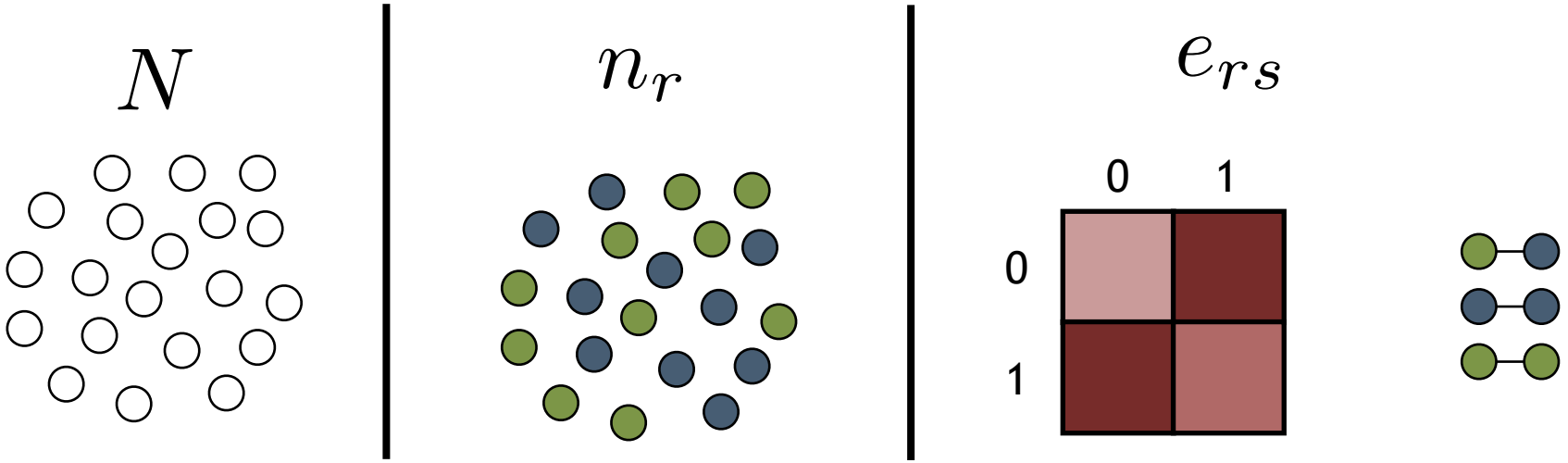
Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - Given that we know B



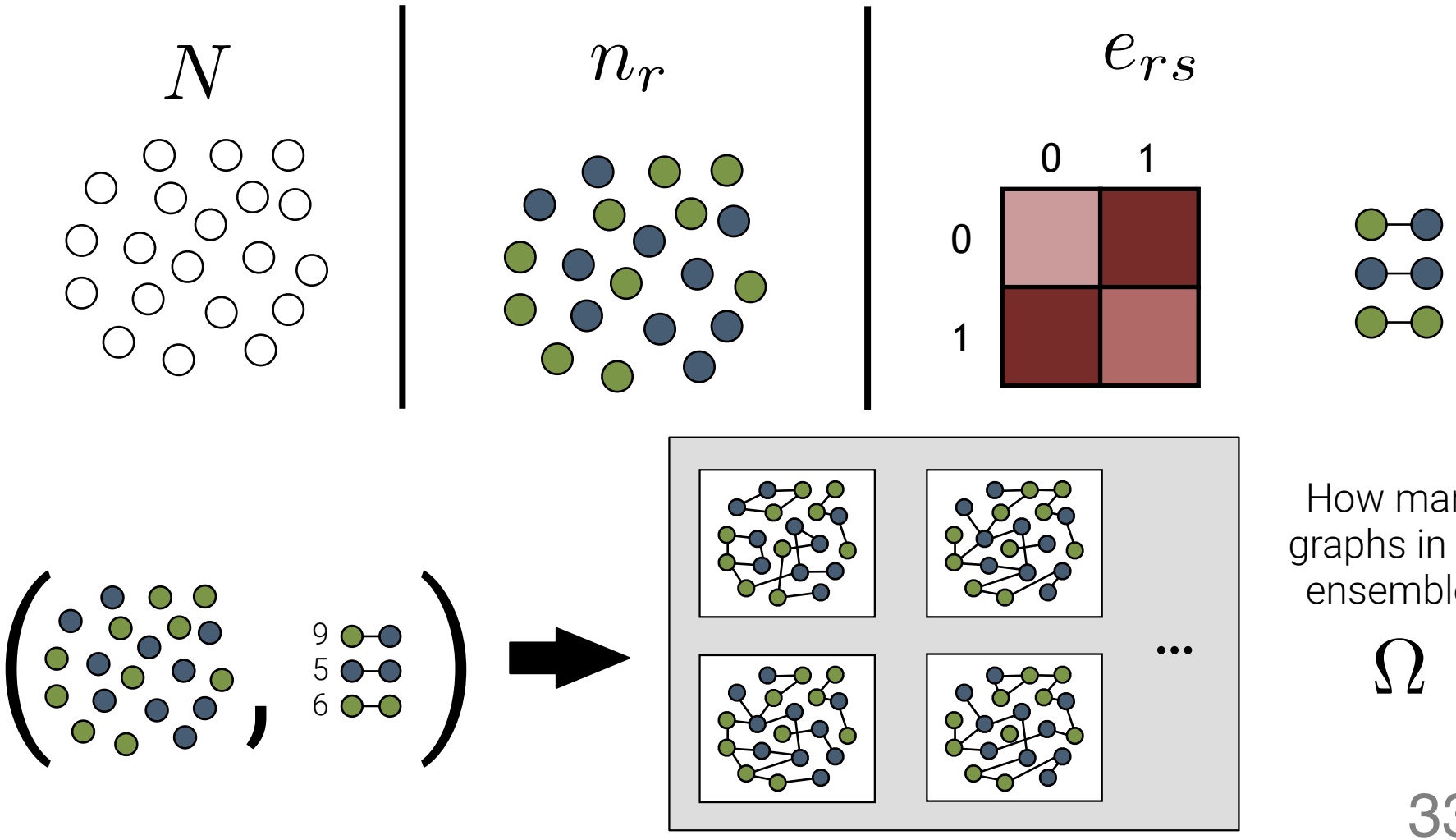
Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - Generating data for a specific value of B



Meso-Scale Structures in Networks: the SBM

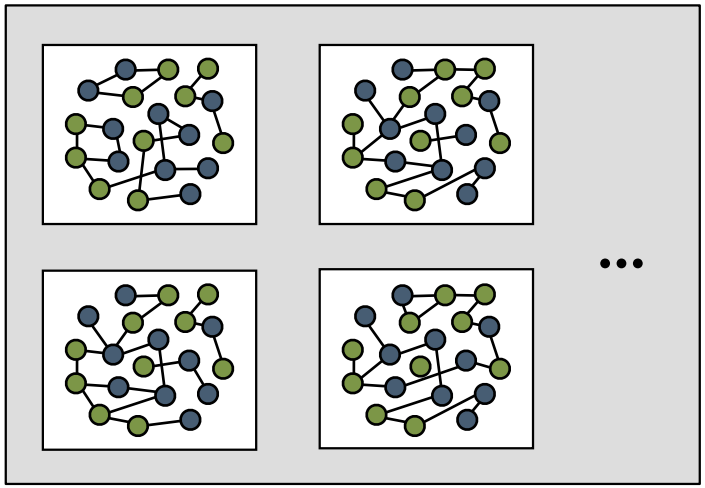
- SBM Inference
  - Generating data for a specific value of B



Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - The entropy of an SBM

$$N, n_r, e_{rs}$$



Ω Number of graphs in the ensemble

The entropy is given as: $\mathcal{S} = \ln \Omega$

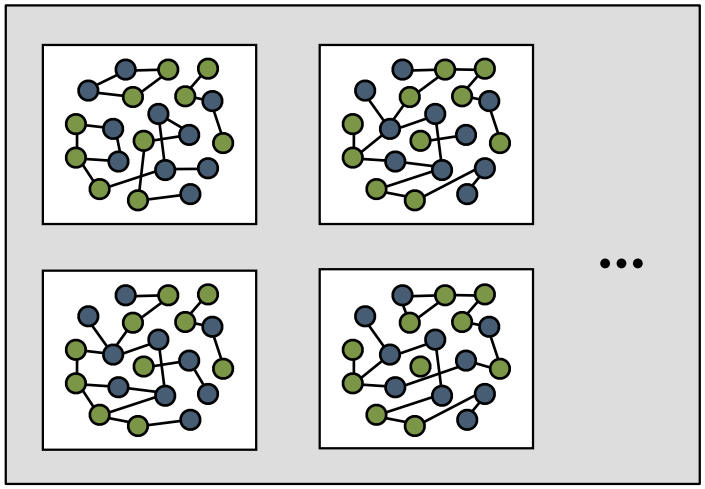
Entropy measures the degree of "order" of an ensemble. Higher entropy = more disorder.

Peixoto, Tiago P. "Entropy of stochastic blockmodel ensembles." Physical Review E 85.5 (2012): 056122.

Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - The entropy of an SBM

$$N, n_r, e_{rs}$$



Ω Number of graphs in the ensemble

The entropy is given as: $\mathcal{S} = \ln \Omega$

Entropy measures the degree of "order" of an ensemble. Higher entropy = more disorder.

  - Deriving a log-likelihood function:

If we assume that each graph has the same probability: $\mathcal{P} = 1/\Omega$

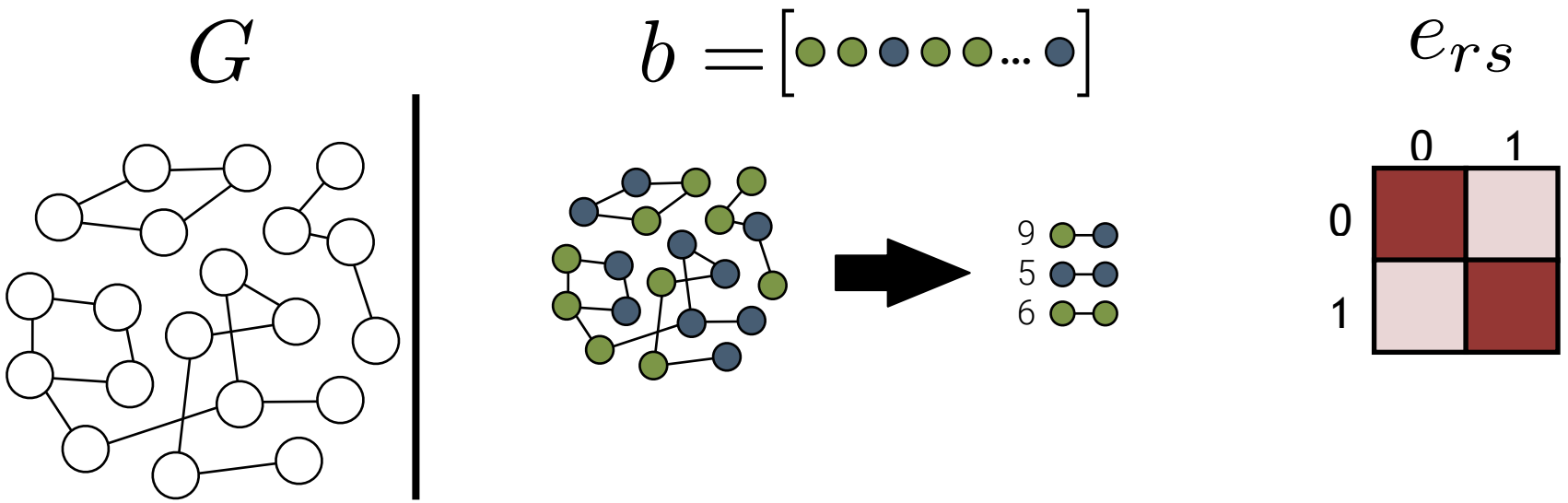
We can define the log-likelihood function as: $L = \ln \mathcal{P}$

$$L = -\mathcal{S}$$

Peixoto, Tiago P. "Entropy of stochastic blockmodel ensembles." Physical Review E 85.5 (2012): 056122.

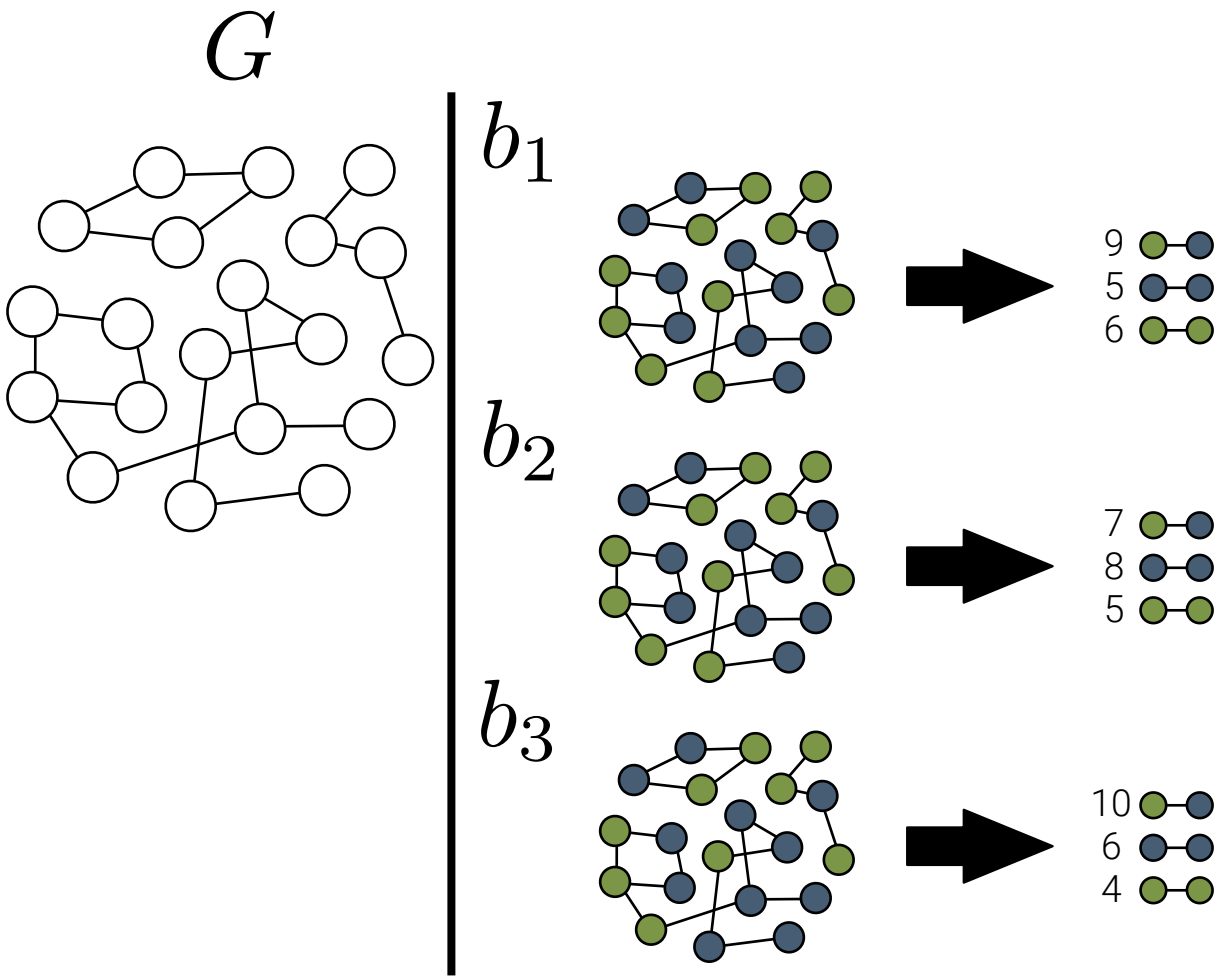
Meso-Scale Structures in Networks: the SBM

- SBM Inference



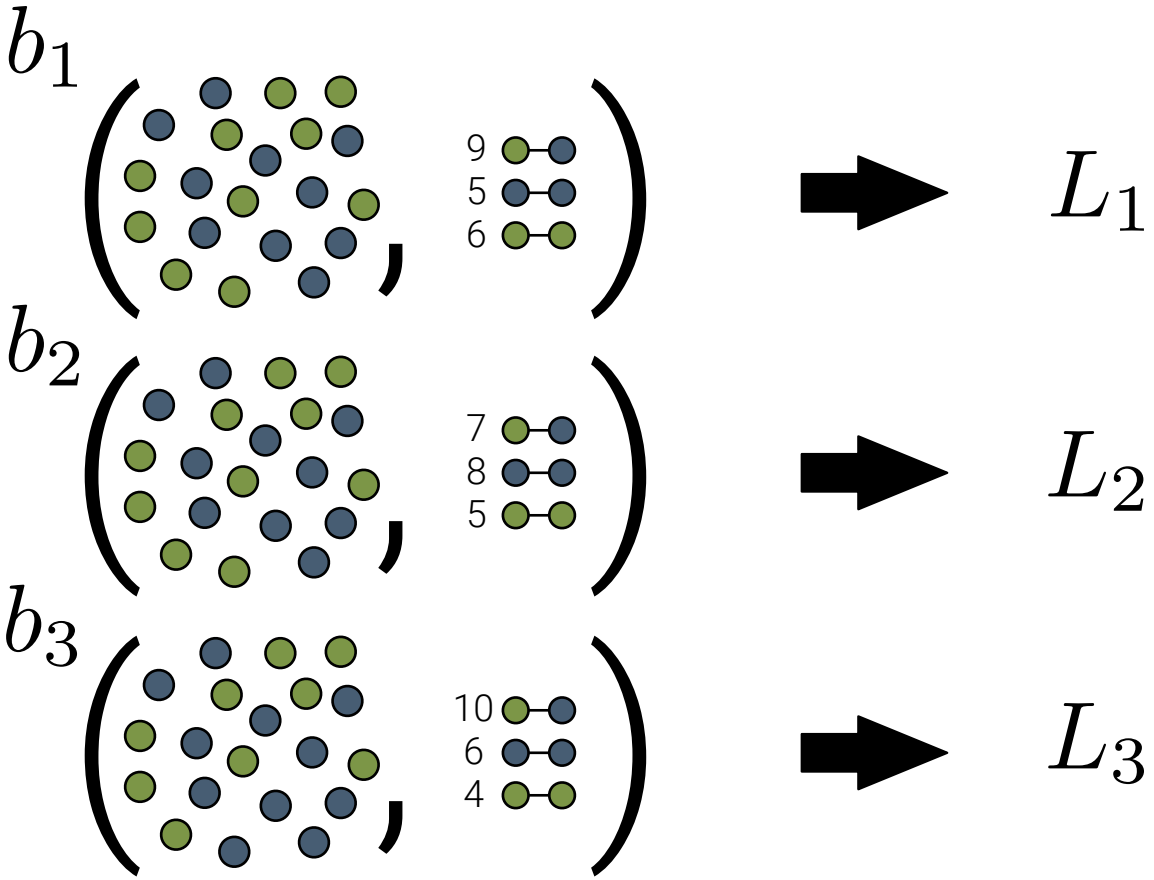
Meso-Scale Structures in Networks: the SBM

- SBM Inference



Meso-Scale Structures in Networks: the SBM

- SBM Inference



Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - Maximize the likelihood L (or equivalently, minimize $\mathcal{S}$)
  - A simple "greedy" algorithm:
    1. Start with a random partition of b
    2. For each vertex in the graph:
      2.1. Change its block membership to the value that maximizes the likelihood.
    3. Repeat step **2.** until no further improvement is possible.
  - We need many runs to find the optimal solution.

Peixoto, Tiago P. "Entropy of stochastic blockmodel ensembles." Physical Review E 85.5 (2012): 056122.

Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - Most of the time we are unaware of B
  - As B increases, we start to memorize data.
  - The “best model” is when $B = N$ (a.k.a. the worst model).

Peixoto, Tiago P. “Parsimonious module inference in large networks.” Physical Review Letters 110.14 (2013): 148701.

Meso-Scale Structures in Networks: the SBM

- SBM Inference
  - The Description Length:
    - The information-theoretic definition: “the amount of bits required to send the compressed message plus the number of bits in the encoding scheme”
  - The Description Length of the Stochastic Block Model:
    - The entropy of data given the model plus the entropy of the model.

$$\Sigma = \mathcal{S} + \mathcal{L}$$

    - It is basically a way to penalize the complexity of the model.
  - To find blocks, we follow the Minimum Description Length principle:
    - The best choice of model which fits given data is the one which most compresses it; the one that minimizes the total amount of information required to describe it.

Peixoto, Tiago P. “Parsimonious module inference in large networks.” Physical Review Letters 110.14 (2013): 148701.

Meso-Scale Structures in Networks: the SBM

- SBM Inference when we are unaware of B
  - Minimizing the description length $\sum$
  - A straightforward algorithm:
    1. Start with B = 0.
    2. Increase the number of blocks B.
    3. Fit the SBM with B blocks and check the description length of it.
    4. Repeat step **2.** until the description length stop decreasing.
  - The fitting part (i.e., step **3.**) is based on Metropolis-Hasting:
    - Move across the "partition space" by proposing to move a node from a group into another group.
    - For this, we use local-level information about the neighbors of the node.

Peixoto, Tiago P. "Parsimonious module inference in large networks." Physical Review Letters 110.14 (2013): 148701.

Meso-Scale Structures in Networks: the SBM

- Degree-corrected Stochastic Block Model (DC-SBM)
  - The standard stochastic block model assumes that all vertices belonging to the same block are statistically indistinguishable, they all have the same.
  - Many observed networks, however, show a broad range of degrees.

Karrer, B., & Newman, M. E. J. (2011). “Stochastic blockmodels and community structure in networks.” PRE, 83(1), 016107.

Next: Stochastic Block Model (Demo)

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