

A.1

Correction of angles for vertically aligned masses:

$$c_{12} = (\theta_1 + \theta_2 - \frac{\pi}{2})$$

$$c_{123} = (\theta_1 + \theta_2 + \theta_3 - \frac{\pi}{2})$$

Handwritten derivation of the gravity torque vector $G(q_1, q_2, q_3)$ for a 3-link robot arm. The derivation shows the individual torque components τ_{m1} , τ_{m2} , and τ_{m3} and then combines them into the total gravity torque vector G .

$$A1. \quad G(q_1, q_2, q_3) = \tau_{m1} + \tau_{m2} + \tau_{m3}$$

$$\tau_{m1} = - \begin{pmatrix} r_1 c_1 g m_1 \\ 0 \\ 0 \end{pmatrix} \quad \tau_{m2} = - \begin{pmatrix} (L_1 c_1 + r_2 c_{12}) g m_2 \\ r_2 c_{12} g m_2 \\ 0 \end{pmatrix}$$

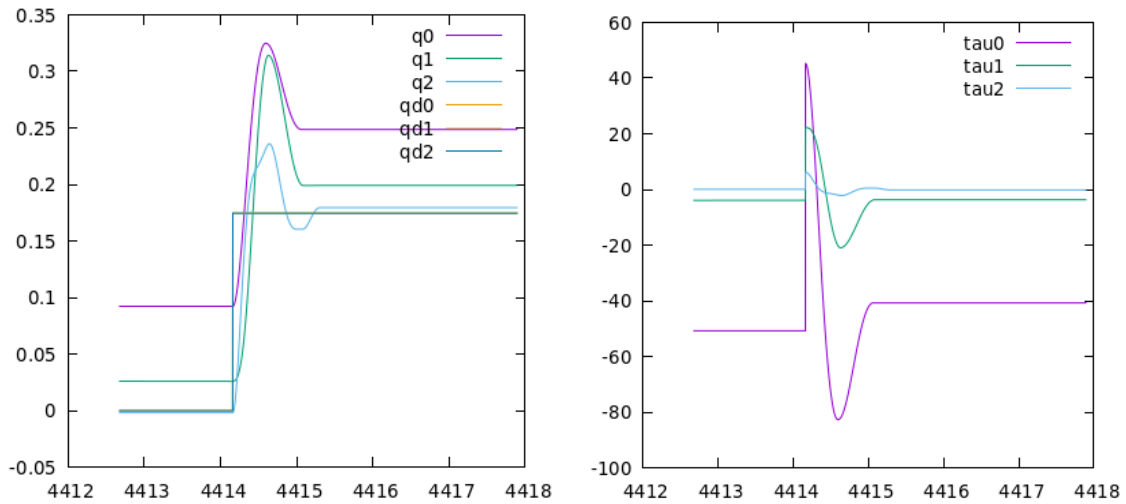
$$\tau_{m3} = - \begin{pmatrix} (L_1 c_1 + L_2 c_{12} + r_3 c_{123}) g m_3 \\ (L_2 c_{12} + r_3 c_{123}) g m_3 \\ r_3 c_{123} g m_3 \end{pmatrix}$$

$$G(q_1, q_2, q_3) = - \begin{pmatrix} (L_1 c_1 + L_2 c_{12} + r_3 c_{123}) m_3 + (L_1 c_1 + r_2 c_{12}) m_2 + r_1 c_1 m_1 \\ (L_2 c_{12} + r_3 c_{123}) m_3 + r_2 c_{12} m_2 \\ r_3 c_{123} m_3 \end{pmatrix} g$$

B.2

- One can observe that higher gain values lead to higher maximum torque values and higher precision to reach a certain position but they also lead to overshoot and oscillation. Lower gain values have the opposite effect. They lead to higher position error but less oscillation and less overshoot. Since the joints are connected, tuning one joint can improve or worsen the behavior of all other joints. One needs to find a balance between a gain value high enough to reach a good enough position precision with only limited oscillation/overshoot and without exceeding the torque limits.
- The gain values are different for each joint because the weight that is affected by a resulting torque is different. For example the joint at the shoulder of the robot has to account for the same weight as the elbow joint plus the weight of the connection between the shoulder and the elbow joint.
- best tested gains: $kp(550, 150, 35)$

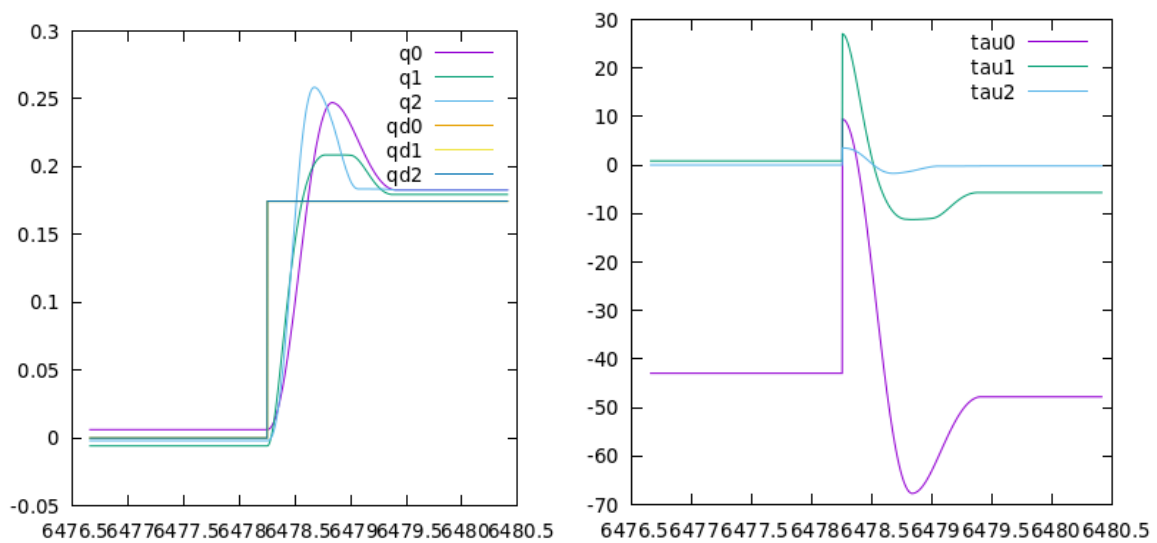
B.3



The left plot shows position q values over time and the right plot shows torque values τ over time.

B.6

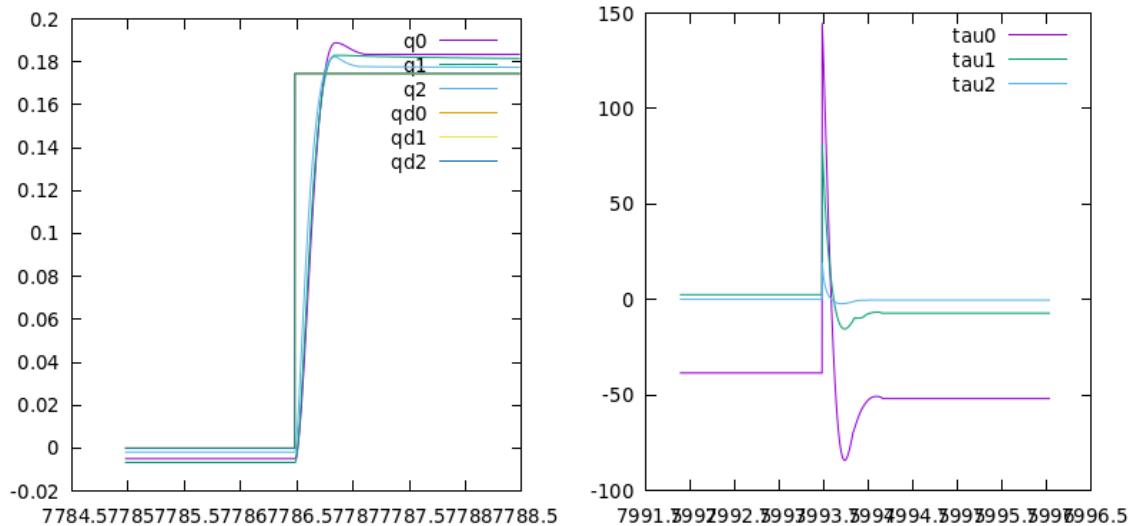
best tested gains: $k_p(300, 200, 10)$



The left plot shows position q values over time and the right plot shows torque values τ over time

B.7

best tested gains: $k_p(1050, 450, 110)$



The left plot shows position q values over time and the right plot shows torque values τ over time. The max torques are approx. $\tau_{max} \approx [149, 81, 19.4]$

The gain values used for the PD-Controller can be higher compared to the gain values used for the P-Controller, because the PD-Controller accounts for the velocity of the robot arm while computing the torques for the joints of the robot. By doing this overshoot and oscillation can be avoided while benefiting from higher position precision and faster movements.

Contribution

	B1	B4	B5	B6	B7
Jarne Jüchser	x	x	x	x	x
Lukas Vormwald	x	x	x	x	x
Florian Hoffmann	x	x	x	x	x