Relational Database Design

Chapter 8: Relational Database Design

- Features of Good Relational Design
- Atomic Domains and First Normal Form
- Decomposition Using Functional Dependencies
- Functional Dependency Theory
- Algorithms for Functional Dependencies
- Decomposition Using Multivalued Dependencies
- More Normal Form
- Database-Design Process
- Modeling Temporal Data

Combine Schemas?

- Suppose we combine *instructor* and *department* into *inst_dept*
  - *(No connection to relationship set inst_dept)*
- Result is possible repetition of information

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

What About Smaller Schemas?

- Suppose we had started with *inst_dept*. How would we know to split up (**decompose**) it into *instructor* and *department*?
- Write a rule "if there were a schema (*dept_name*, *building*, *budget*), then *dept_name* would be a candidate key"
- Denote as a **functional dependency**:

  $$dept\_name \rightarrow building, budget$$

- In *inst_dept*, because *dept_name* is not a candidate key, the building and budget of a department may have to be repeated.
  - This indicates the need to decompose *inst_dept*
- Not all decompositions are good. Suppose we decompose *employee(ID, name, street, city, salary)* into

  employee1 (*ID*, *name*)

  employee2 (*name*, *street*, *city*, *salary*)

- The next slide shows how we lose information -- we cannot reconstruct the original *employee* relation -- and so, this is a **lossy decomposition**.

A Lossy Decomposition

ID	*name*	*street*	*city*	*salary*
⋮				
57766	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

employee

ID	*name*
⋮	
57766	Kim
98776	Kim
⋮	

name	*street*	*city*	*salary*
⋮			
Kim	Main	Perryridge	75000
Kim	North	Hampton	67000
⋮			

natural join

ID	*name*	*street*	*city*	*salary*
⋮				
57766	Kim	Main	Perryridge	75000
57766	Kim	North	Hampton	67000
98776	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

Example of Lossless-Join Decomposition

- **Lossless join decomposition**
- Decomposition of $R = (A, B, C)$

$$R_1 = (A, B) \qquad R_2 = (B, C)$$

A	B	C
α	1	A
β	2	B

r

A	B
α	1
β	2

$\Pi_{A,B}(r)$

B	C
1	A
2	B

$\Pi_{B,C}(r)$

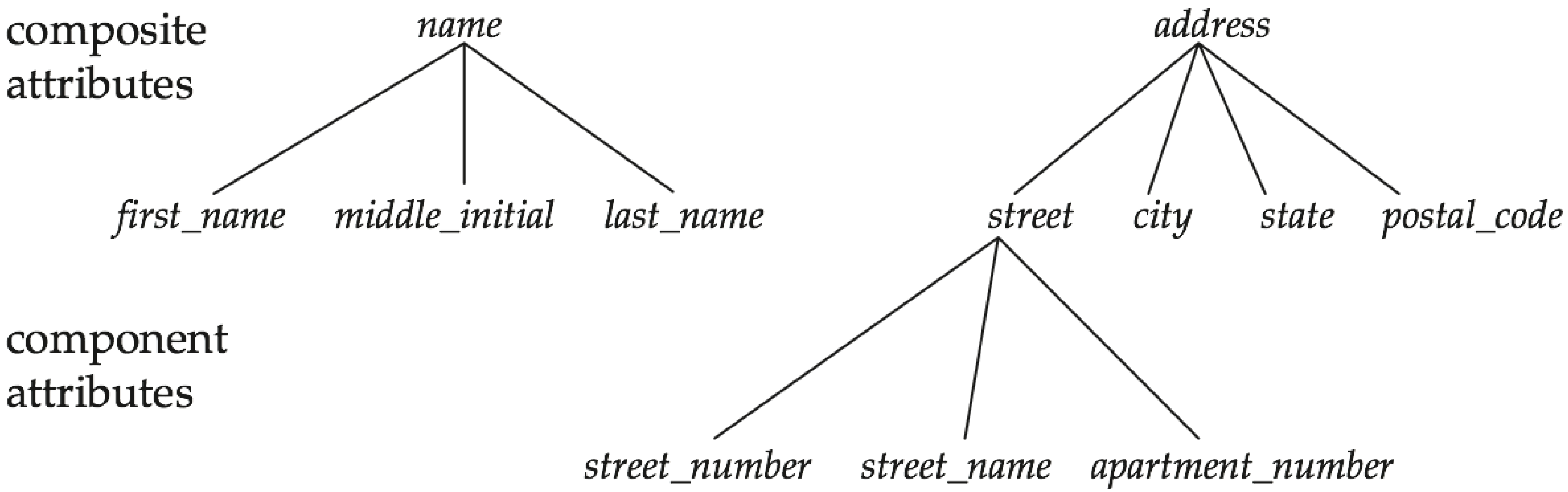
$$\Pi_{A}(r) \bowtie \Pi_{B}(r)$$

A	B	C
α	1	A
β	2	B

First Normal Form

- Domain is **atomic** if its elements are considered to be indivisible units
  - Examples of non-atomic domains:
    - Set of names, composite attributes
    - Identification numbers like CS101 that can be broken up into parts
- A relational schema R is in **first normal form** if the domains of all attributes of R are atomic
- Non-atomic values complicate storage and encourage redundant (repeated) storage of data
  - Example: Set of accounts stored with each customer, and set of owners stored with each account
  - We assume all relations are in first normal form

Composite Attributes



Notation to Express Entity with Complex Attributes

instructor

- *ID*
- *name*
  - *first_name*
  - *middle_initial*
  - *last_name*
- *address*
  - *street*
    - *street_number*
    - *street_name*
    - *apt_number*
  - *city*
  - *state*
  - *zip*
- *{ phone_number }*
- *date_of_birth*
- *age ( )*

First Normal Form (Cont'd)

- Atomicity is actually a property of how the elements of the domain are used.
  - Example: Strings would normally be considered indivisible
  - Suppose that students are given roll numbers which are strings of the form *CS0012* or *EE1127*
  - If the first two characters are extracted to find the department, the domain of roll numbers is not atomic.
  - Doing so is a bad idea: leads to encoding of information in application program rather than in the database.

Goal — Devise a Theory for the Following

- Decide whether a particular relation R is in "good" form.
- In the case that a relation R is not in "good" form, decompose it into a set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - each relation is in good form
  - the decomposition is a lossless-join decomposition
- Our theory is based on:
  - functional dependencies
  - multivalued dependencies

Functional Dependencies

- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key*.

Functional Dependencies (Cont.)

- Let R be a relation schema

$$\alpha \subseteq R \ \ and \ \ \beta \subseteq R$$

- The **functional dependency**

$$\alpha \rightarrow \beta$$

holds on R if and only if for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2[\alpha] \ \ \Rightarrow \ \ t_1[\beta] = t_2[\beta]$$

- Example: Consider r(A,B) with the following instance of r.

1	4
1	5
3	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.

Functional Dependencies (Cont.)

- K is a superkey for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R if and only if
  - $K \rightarrow R$, and
  - for no $\alpha \subset K$, $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

  inst_dept (<u>*ID,* </u>*name*, *salary*<u>, *dept_name*, </u>*building*, *budget*).

  We expect these functional dependencies to hold:

  $$dept\_name \rightarrow building \quad \text{AND} \quad ID \rightarrow building$$

  but would not expect the following to hold:

  $$dept\_name \rightarrow salary$$

Use of Functional Dependencies

- We use functional dependencies to:
  - test relations to see if they are legal under a given set of functional dependencies.
    - If a relation *r* is legal under a set *F* of functional dependencies, we say that *r* **satisfies** *F*.
  - specify constraints on the set of legal relations
    - We say that *F* **holds on** *R* if all legal relations on *R* satisfy the set of functional dependencies *F*.
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
  - For example, a specific instance of *instructor* may, by chance, satisfy
    $name \rightarrow ID$.

Functional Dependencies (Cont.)

- *A* functional dependency is **trivial** if it is satisfied by all instances of a relation
  - Example:
    - $ID, name \rightarrow ID$
    - $name \rightarrow name$
  - In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

Closure of a Set of Functional Dependencies

- Given a set F of functional dependencies, there are certain other functional dependencies that are logically implied by F.
  - For example: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of **all** functional dependencies logically implied by F is the **closure** of F.
- We denote the *closure* of F by $\mathbf{F^+}$.
- F^+ is a superset of F.

Boyce-Codd Normal Form

- *A relation schema R is in BCNF with respect to a set F of functional dependencies if for all functional dependencies in F+ of the form*

  $$\alpha \rightarrow \beta$$

  where $\alpha \subseteq R$ *and* $\beta \subseteq R$*, at least one of the following holds:*
  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
  - α is a superkey for *R*

- Example schema *not* in BCNF:

  instr_dept (<u>*ID,* </u>*name*, *salary*<u>*, dept_name,* </u>*building*, *budget*)

- because *dept_name* $\rightarrow$ *building*, *budget* holds on *instr_dept*, but *dept_name* is not a superkey

Decomposing a Schema into BCNF

- Suppose we have a schema R and a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF.

  We decompose R into:

  - $(\alpha \cup \beta)$
  - $(R - (\beta - \alpha))$

- In our example,

  - α = *dept_name*
  - β = *building*, *budget*

  and *inst_dept* is replaced by

  - $(\alpha \cup \beta)$ = (*dept_name*, *building*, *budget*)
  - $(R - (\beta - \alpha))$ = (*ID*, *name*, *salary*, *dept_name*)

BCNF and Dependency Preservation

- Constraints, including functional dependencies, are costly to check in practice unless they pertain to only one relation
- If it is sufficient to test only those dependencies on each individual relation of a decomposition in order to ensure that *all* functional dependencies hold, then that decomposition is *dependency preserving.*
- Because it is not always possible to achieve both BCNF and dependency preservation, we consider a weaker normal form, known as *third normal form.*

Third Normal Form

- A relation schema R is in **third normal form (3NF)** if for all:

  $$\alpha \rightarrow \beta \text{ in } F^+$$

  at least one of the following holds:

  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
  - α is a superkey for R
  - Each attribute A in $\beta - \alpha$ is contained in a candidate key for R.

    (**NOTE**: each attribute may be in a different candidate key)

- If a relation is in BCNF it is in 3NF (since in BCNF one of the first two conditions above must hold).
- Third condition is a minimal relaxation of BCNF to ensure dependency preservation.

Goals of Normalization

- Let R be a relation scheme with a set F of functional dependencies.
- Decide whether a relation scheme R is in “good” form.
- In the case that a relation scheme R is not in “good” form, decompose it into a set of relation scheme $\{R_1, R_2, ..., R_n\}$ such that
  - each relation scheme is in good form
  - the decomposition is a lossless-join decomposition
  - Preferably, the decomposition should be dependency preserving.

How good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a relation

inst_info (ID, child_name, phone)

  - where an instructor may have more than one phone and can have multiple children

ID	*child_name*	*phone*
99999	David	512-555-1234
99999	David	512-555-4321
99999	William	512-555-1234
99999	Willian	512-555-4321

inst_info

How good is BCNF? (Cont.)

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if we add a phone 981-992-3443 to 99999, we need to add two tuples

(99999, David, 981-992-3443)
(99999, William, 981-992-3443)

How good is BCNF? (Cont.)

- Therefore, it is better to decompose *inst_info* into:

inst_child

ID	*child_name*
99999	David
99999	David
99999	William
99999	Willian

inst_phone

ID	*phone*
99999	512-555-1234
99999	512-555-4321
99999	512-555-1234
99999	512-555-4321

- This suggests the need for higher normal forms, such as Fourth Normal Form (4NF).

Functional-Dependency Theory

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving

Closure of a Set of Functional Dependencies

- Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - For e.g.: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow$ C
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by $\boldsymbol{F^+}$.

Closure of a Set of Functional Dependencies

- We can find F^+, the closure of F, by repeatedly applying **Armstrong's Axioms:**
  - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ (**reflexivity**)
  - if $\alpha \rightarrow \beta$, then $\gamma\,\alpha \rightarrow \gamma\,\beta$ (**augmentation**)
  - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ (**transitivity**)
- These rules are
  - **sound** (generate only functional dependencies that actually hold), and
  - **complete** (generate all functional dependencies that hold).

Example

- $R = (A, B, C, G, H, I)$

  $F = \{ A \rightarrow B$

  $A \rightarrow C$

  $CG \rightarrow H$

  $CG \rightarrow I$

  $B \rightarrow H\}$

- some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity

Procedure for Computing F^+

- To compute the closure of a set of functional dependencies F:

$F^+ = F$
repeat
    for each functional dependency f in F^+
        apply reflexivity and augmentation rules on f
        add the resulting functional dependencies to F^+
    for each pair of functional dependencies f_1 and f_2 in F^+
        if f_1 and f_2 can be combined using transitivity
            then add the resulting functional dependency to F^+
until F^+ does not change any further

Closure of Functional Dependencies (Cont.)

- Additional rules:
  - If $\alpha \to \beta$ holds *a*nd $\alpha \to \gamma$ holds, then $\alpha \to \beta\gamma$ holds (**union**)
  - If $\alpha \to \beta\gamma$ holds, then $\alpha \to \beta$ holds and $\alpha \to \gamma$ holds (**decomposition**)
  - If $\alpha \to \beta$ holds *a*nd $\gamma\beta \to \delta$ holds, then $\alpha\gamma \to \delta$ holds (**pseudotransitivity**)

  The above rules can be inferred from Armstrong's axioms.

Closure of Attribute Sets

- Given a set of attributes α, define the ***closure*** of α **under** F (denoted by α^+) as the set of attributes that are functionally determined by α under F

- Algorithm to compute α^+, the closure of α under F

> *result* := α;
> **while** (changes to *result*) **do**
>   **for each** $\beta \rightarrow \gamma$ **in** F **do**
>   **begin**
>   **if** $\beta \subseteq$ *result* **then** *result* := *result* $\cup\ \gamma$
>   **end**

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- $(AG)^+$
  1. $result = AG$
  2. $result = ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. $result = ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. $result = ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is *AG* a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R?$ == Is $(AG)^+ \supseteq R$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
     2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute α^{+}, and check if α^{+} contains all attributes of R.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^{+}), just check if $\beta \subseteq \alpha^{+}$.
  - That is, we compute α^{+} by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^{+}, and for each $S \subseteq \gamma^{+}$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, \quad B \rightarrow C, A \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g.: $\{A \rightarrow B, \quad B \rightarrow C, \quad A \rightarrow CD\}$ can be simplified to
      $\{A \rightarrow B, \quad B \rightarrow C, \quad A \rightarrow D\}$
    - E.g.: $\{\mathrm{A} \rightarrow B, \quad B \rightarrow C, \quad AC \rightarrow D\}$ can be simplified to
      $\{\mathrm{A} \rightarrow B, \quad B \rightarrow C, \quad A \rightarrow D\}$
- Intuitively, a canonical cover of F is a "minimal" set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
  - Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is **extraneous** in β if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (I.e. the result of dropping B from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C

Testing if an Attribute is Extraneous

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
- To test if attribute $A \in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
  2. check that $(\{\alpha\} - A)^+$ contains β; if it does, A is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in
     $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$,
  2. check that α^+ contains A; if it does, A is extraneous in β

Canonical Cover

- A **canonical cover** for *F* is a set of dependencies F_c such that
  - *F* logically implies all dependencies in F_c, and
  - F_c logically implies all dependencies in *F*, and
  - No functional dependency in F_c contains an extraneous attribute, and
  - Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for *F*:

  repeat

  Use the union rule to replace any dependencies in *F*

  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$

  Find a functional dependency $\alpha \rightarrow \beta$ with an

  extraneous attribute either in α or in β

  /* Note: test for extraneous attributes done using F_c, not F*/

  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$

  until *F* does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.

      Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$

Lossless-join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless join if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
  - Can be decomposed in two different ways
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$

  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$

  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \quad R_2$)

Dependency Preservation

- Let F_i be the set of dependencies F^+ that include only attributes in R_i.
  - A decomposition is **dependency preserving**, if
    $(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$
  - If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive.

Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into R_1, R_2, …, R_n we apply the following test (with attribute closure done with respect to F)
  - *result* = α
    while (changes to *result*) do
    for each R_i in the decomposition
    $t = (result \cap R_i)^+ \cap R_i$
    $result = result \cup t$
  - If *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B$
  $B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$
  - R_1 and R_2 in BCNF
  - Lossless-join decomposition
  - Dependency preserving

Testing for BCNF

- To check if a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF
  1. compute α^+ (the attribute closure of α), and
  2. verify that it includes all attributes of R, that is, it is a superkey of R.
- **Simplified test**: To check if a relation schema R is in BCNF, it suffices to check only the dependencies in the given set F for violation of BCNF, rather than checking all dependencies in F^+.
  - If none of the dependencies in F causes a violation of BCNF, then none of the dependencies in F^+ will cause a violation of BCNF either.
- However, **simplified test using only *F* is incorrect when testing a relation in a decomposition of R**
  - Consider $R = (A, B, C, D, E)$, with $F = \{ A \rightarrow B, BC \rightarrow D\}$
    - Decompose R into $R_1 = (A,B)$ and $R_2 = (A,C,D, E)$
    - Neither of the dependencies in F contain only attributes from (A,C,D,E) so we might be mislead into thinking R_2 satisfies BCNF.
    - In fact, dependency $AC \rightarrow D$ in F^+ shows R_2 is not in BCNF.

Testing Decomposition for BCNF

- To check if a relation R_i in a decomposition of R is in BCNF,
  - Either test R_i for BCNF with respect to the **restriction** of F to R_i (that is, all FDs in F^+ that contain only attributes from R_i)
  - or use the original set of dependencies F that hold on R, but with the following test:

    for every set of attributes $\alpha \subseteq R_i$, check that α^+ (the attribute closure of α) either includes no attribute of $R_i - \alpha$, or includes all attributes of R_i.

    - If the condition is violated by some $\alpha \rightarrow \beta$ in F, the dependency
      $\alpha \rightarrow (\alpha^+ - \alpha) \cap R_i$
      can be shown to hold on R_i, and R_i violates BCNF.
    - We use above dependency to decompose R_i

BCNF Decomposition Algorithm

result := $\{R\}$;
done := false;
compute F^+;
while (not *done)* **do**
  if (there is a schema R_i in *result* that is not in BCNF)
    then begin
      let $\alpha \rightarrow \beta$ be a nontrivial functional dependency that
      holds on R_i such that $\alpha \rightarrow R_i$ is not in F^+,
      and $\alpha \cap \beta = \emptyset$;
      result := $(result - R_i) \cup (R_i - \beta) \cup (\alpha, \beta)$;
    end
    else *done* := **true;**

Note: each R_i is in BCNF, and decomposition is lossless-join.

Example of BCNF Decomposition

- $R = (A, B, C)$
  $F = \{A \rightarrow B$
  $\quad B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF ($B \rightarrow C$ but B is not superkey)
- Decomposition
  - $R_1 = (B, C)$
  - $R_2 = (A,B)$

Example of BCNF Decomposition

- *class* (*course_id*, *title*, *dept_name*, *credits*, *sec_id*, *semester*, *year*, *building*, *room_number*, *capacity*, *time_slot_id*)
- Functional dependencies:
  - *course_id*→ *title*, *dept_name*, *credits*
  - *building*, *room_number*→*capacity*
  - *course_id*, *sec_id*, *semester*, *year*→*building*, *room_number*, *time_slot_id*
- A candidate key {*course_id*, *sec_id*, *semester*, *year*}.
- BCNF Decomposition:
  - *course_id*→ *title*, *dept_name*, *credits* holds
    - but *course_id* is not a superkey.
  - We replace *class* by:
    - *course*(*course_id*, *title*, *dept_name*, *credits*)
    - *class-1* (*course_id*, *sec_id*, *semester*, *year*, *building*, *room_number*, *capacity*, *time_slot_id*)

BCNF Decomposition (Cont.)

- *course* is in BCNF
  - How do we know this?
- *building*, *room_number*→*capacity* holds on *class-1*
  - but {*building*, *room_number*} is not a superkey for *class-1*.
  - We replace *class-1* by:
    - *classroom* (*building*, *room_number*, *capacity*)
    - *section* (*course_id*, *sec_id*, *semester*, *year*, *building*, *room_number*, *time_slot_id*)
- *classroom* and *section* are in BCNF.

BCNF and Dependency Preservation

- It is not always possible to get a BCNF decomposition that is
  - dependency preserving

- $R = (J, K, L)$
  $F = \{JK \rightarrow L$
  $\quad L \rightarrow K\}$
  Two candidate keys = JK and JL
- R is not in BCNF
- Any decomposition of R will fail to preserve

$$JK \rightarrow L$$

This implies that testing for $JK \rightarrow L$ requires a join

Third Normal Form: Motivation

- There are some situations where
  - BCNF is not dependency preserving, and
  - efficient checking for FD violation on updates is important
- Solution: define a weaker normal form, called Third Normal Form (3NF)
  - Allows some redundancy (with resultant problems; we will see examples later)
  - But functional dependencies can be checked on individual relations without computing a join.
  - There is always a lossless-join, dependency-preserving decomposition into 3NF.

3NF Example

- Relation *dept_advisor*:
  - *dept_advisor* (*s_ID*, *i_ID*, *dept_name)*
    $F = \{s\_ID, dept\_name \rightarrow i\_ID,\ i\_ID \rightarrow dept\_name\}$
  - Two candidate keys: *s_ID*, *dept_name*, and *i_ID*, *s_ID*
  - *R* is in 3NF
    - $s\_ID, dept\_name \rightarrow i\_ID$ *s_ID*

      dept_name is a superkey
    - $i\_ID \rightarrow dept\_name$

      dept_name is contained in a candidate key

Redundancy in 3NF

- There is some redundancy in this schema
- Example of problems due to redundancy in 3NF
  - $R = (J, K, L)$
    $F = \{JK \rightarrow L, L \rightarrow K\}$

J	L	K
j_1	l_1	k_1
j_2	l_1	k_1
j_3	l_1	k_1
null	l_2	k_2

- repetition of information (e.g., the relationship l_1, k_1)
  - (i_ID, dept_name)
- need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J).
  - (i_ID, dept_name_I) if there is no separate relation mapping instructors to departments

Testing for 3NF

- Optimization: Need to check only FDs in *F*, need not check all FDs in F^+.
- Use attribute closure to check for each dependency $\alpha \rightarrow \beta$, if α is a superkey.
- If α is not a superkey, we have to verify if each attribute in β is contained in a candidate key of *R*
  - this test is rather more expensive, since it involve finding candidate keys
  - testing for 3NF has been shown to be NP-hard
  - Interestingly, decomposition into third normal form (described shortly) can be done in polynomial time

3NF Decomposition Algorithm

Let F_c be a canonical cover for F;
$i := 0$;
for each functional dependency $\alpha \rightarrow \beta$ in F_c **do**
 if none of the schemas R_j, $1 \leq j \leq i$ contains $\alpha\ \beta$
  then begin
   $i := i + 1$;
   $R_i := \alpha\ \beta$
  end
if none of the schemas R_j, $1 \leq j \leq i$ contains a candidate key for R
 then begin
  $i := i + 1$;
  $R_i :=$ any candidate key for R;
 end
/* Optionally, remove redundant relations */

repeat
if any schema R_j is contained in another schema R_k
 then /* delete R_j ***/**
  $R_j = R;$;
  $i=i-1;$
return $(R_1, R_2, ..., R_i)$

3NF Decomposition Algorithm (Cont.)

- Above algorithm ensures:
  - each relation schema R_i is in 3NF
  - decomposition is dependency preserving and lossless-join
  - Proof of correctness is at end of this presentation (click here)

3NF Decomposition: An Example

- Relation schema:

  cust_banker_branch = (*<u>customer_id, employee_id</u>*, *branch_name*, *type*)

- The functional dependencies for this relation schema are:

  1. *customer_id*, *employee_id* $\rightarrow$ *branch_name*, *type*
  2. *employee_id* $\rightarrow$ *branch_name*
  3. *customer_id*, *branch_name* $\rightarrow$ *employee_id*

- We first compute a canonical cover

  - *branch_name* is extraneous in the r.h.s. of the 1^{st} dependency
  - No other attribute is extraneous, so we get F_C =

    customer_id, *employee_id* $\rightarrow$ *type*
    employee_id $\rightarrow$ *branch_name*
    customer_id, *branch_name* $\rightarrow$ *employee_id*

3NF Decompsition Example (Cont.)

- The **for** loop generates following 3NF schema:

  (*customer_id*, *employee_id*, *type*)

  (<u>*employee_id*</u>, *branch_name*)

  (*customer_id*, *branch_name*, *employee_id)*

  - Observe that (*customer_id*, *employee_id*, *type*) contains a candidate key of the original schema, so no further relation schema needs be added

- At end of for loop, detect and delete schemas, such as (<u>*employee_id*</u>, *branch_name*), which are subsets of other schemas

  - result will not depend on the order in which FDs are considered

- The resultant simplified 3NF schema is:

  (*customer_id*, *employee_id*, *type*)

  (*customer_id*, *branch_name*, *employee_id)*

Comparison of BCNF and 3NF

- It is always possible to decompose a relation into a set of relations that are in 3NF such that:
  - the decomposition is lossless
  - the dependencies are preserved
- It is always possible to decompose a relation into a set of relations that are in BCNF such that:
  - the decomposition is lossless
  - it may not be possible to preserve dependencies.

Design Goals

- Goal for a relational database design is:
  - BCNF.
  - Lossless join.
  - Dependency preservation.
- If we cannot achieve this, we accept one of
  - Lack of dependency preservation
  - Redundancy due to use of 3NF
- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys.

  Can specify FDs using assertions, but they are expensive to test, (and currently not supported by any of the widely used databases!)
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left hand side is not a key.

Multivalued Dependencies

- Suppose we record names of children, and phone numbers for instructors:
  - *inst_child*(*ID*, *child_name*)
  - *inst_phone*(*ID*, *phone_number*)
- If we were to combine these schemas to get
  - *inst_info*(*ID*, *child_name*, *phone_number*)
  - Example data:
    (99999, David, 512-555-1234)
    (99999, David, 512-555-4321)
    (99999, William, 512-555-1234)
    (99999, William, 512-555-4321)
- This relation is in BCNF
  - Why?

Multivalued Dependencies (MVDs)

- Let R be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The **multivalued dependency**

$$\alpha \rightarrow\rightarrow \beta$$

holds on R if in any legal relation $r(R)$, for all pairs for tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that:

$$\begin{aligned}
t_1[\alpha] = t_2[\alpha] &= t_3[\alpha] = t_4[\alpha] \\
t_3[\beta] &= t_1[\beta] \\
t_3[R - \beta] &= t_2[R - \beta] \\
t_4[\beta] &= t_2[\beta] \\
t_4[R - \beta] &= t_1[R - \beta]
\end{aligned}$$

MVD (Cont.)

- Tabular representation of $\alpha \rightarrow\rightarrow \beta$

	α	β	$R - \alpha - \beta$
t_1	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	$a_{j+1} \ldots a_n$
t_2	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	$b_{j+1} \ldots b_n$
t_3	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	$b_{j+1} \ldots b_n$
t_4	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	$a_{j+1} \ldots a_n$

Example

- Let R be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

$$Y, Z, W$$

- We say that $Y \to\to Z$ (Y **multidetermines** Z)
  if and only if for all possible relations $r(R)$

$$< y_1, z_1, w_1 > \in r \text{ and } < y_1, z_2, w_2 > \in r$$

  then

$$< y_1, z_1, w_2 > \in r \text{ and } < y_1, z_2, w_1 > \in r$$

- Note that since the behavior of Z and W are identical it follows that

  $Y \to\to Z$ if $Y \to\to W$

Example (Cont.)

- In our example:

$$ID \rightarrow\rightarrow child\_name$$
$$ID \rightarrow\rightarrow phone\_number$$

- The above formal definition is supposed to formalize the notion that given a particular value of Y (ID) it has associated with it a set of values of Z *(child_name)* and a set of values of W *(phone_number)*, and these two sets are in some sense independent of each other.
- Note:
  - If $Y \rightarrow Z$ then $Y \rightarrow\rightarrow Z$
  - Indeed we have (in above notation) $Z_1 = Z_2$
    The claim follows.

Use of Multivalued Dependencies

- We use multivalued dependencies in two ways:
  1. To test relations to **determine** whether they are legal under a given set of functional and multivalued dependencies
  2. To specify **constraints** on the set of legal relations. We shall thus concern ourselves *only* with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation *r* fails to satisfy a given multivalued dependency, we can construct a relations *r′* that does satisfy the multivalued dependency by adding tuples to *r*.

Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
  - If $\alpha \rightarrow \beta$, then $\alpha \rightarrow\rightarrow \beta$

  That is, every functional dependency is also a multivalued dependency
- The **closure** D^+ of *D* is the set of all functional and multivalued dependencies logically implied by *D*.
  - We can compute D^+ from *D*, using the formal definitions of functional dependencies and multivalued dependencies.
  - We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice
  - For complex dependencies, it is better to reason about sets of dependencies using a system of inference rules (see Appendix C).

$\rightarrow\rightarrow$

Fourth Normal Form

- A relation schema R is in **4NF** with respect to a set D of functional and multivalued dependencies if for all multivalued dependencies in D^+ of the form $\alpha \rightarrow\rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
  - $\alpha \rightarrow\rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$ or $\alpha \cup \beta = R$)
  - α is a superkey for schema R
- If a relation is in 4NF it is in BCNF

Restriction of Multivalued Dependencies

- The restriction of D to R_i is the set D_i consisting of
  - All functional dependencies in D^+ that include only attributes of R_i
  - All multivalued dependencies of the form

    $$\alpha \rightarrow\rightarrow (\beta \cap R_i)$$

    where $\alpha \subseteq R_i$ and $\alpha \rightarrow\rightarrow \beta$ is in D^+

4NF Decomposition Algorithm

result: = {*R*};
done := false;
compute D^+;
Let D_i denote the restriction of D^+ to R_i

while (**not** *done*)
    if (there is a schema $\mathbf{R}_i$ in *result* that is not in 4NF) **then**
        begin
            let $\alpha \rightarrow\rightarrow \beta$ be a nontrivial multivalued dependency that holds on R_i such that $\alpha \rightarrow R_i$ is not in D_i, and $\alpha \cap \beta = \phi$;
            result := $(result - R_i) \cup (R_i - \beta) \cup (\alpha, \beta)$;
        end
    else *done*:= true;

Note: each R_i is in 4NF, and decomposition is lossless-join

Example

- $R = (A, B, C, G, H, I)$

  $F = \{ A \rightarrow\rightarrow B$

  $B \rightarrow\rightarrow HI$

  $CG \rightarrow\rightarrow H \}$
- R is not in 4NF since $A \rightarrow\rightarrow B$ and A is not a superkey for R
- Decomposition

  a) $R_1 = (A, B)$ (R_1 is in 4NF)

  b) $R_2 = (A, C, G, H, I)$ (R_2 is not in 4NF, decompose into R_3 and R_4)

  c) $R_3 = (C, G, H)$ (R_3 is in 4NF)

  d) $R_4 = (A, C, G, I)$ (R_4 is not in 4NF, decompose into R_5 and R_6)

  - $A \rightarrow\rightarrow B$ and $B \rightarrow\rightarrow HI$ ➔ $A \rightarrow\rightarrow HI$, (MVD transitivity), and
  - and hence $A \rightarrow\rightarrow I$ *(MVD restriction to R_4)*

  e) $R_5 = (A, I)$ (R_5 is in 4NF)

  f) $R_6 = (A, C, G)$ (R_6 is in 4NF)

Figure 8.02

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

Figure 8.03

ID	*name*	*street*	*city*	*salary*
⋮				
57766	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

employee

ID	*name*
⋮	
57766	Kim
98776	Kim
⋮	

name	*street*	*city*	*salary*
⋮			
Kim	Main	Perryridge	75000
Kim	North	Hampton	67000
⋮			

natural join

ID	*name*	*street*	*city*	*salary*
⋮				
57766	Kim	Main	Perryridge	75000
57766	Kim	North	Hampton	67000
98776	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

Figure 8.04

A	B	C	D
a_1	b_1	c_1	d_1
a_1	b_2	c_1	d_2
a_2	b_2	c_2	d_2
a_2	b_3	c_2	d_3
a_3	b_3	c_2	d_4

Figure 8.05

building	*room_number*	*capacity*
Packard	101	500
Painter	514	10
Taylor	3128	70
Watson	100	30
Watson	120	50

Figure 8.06

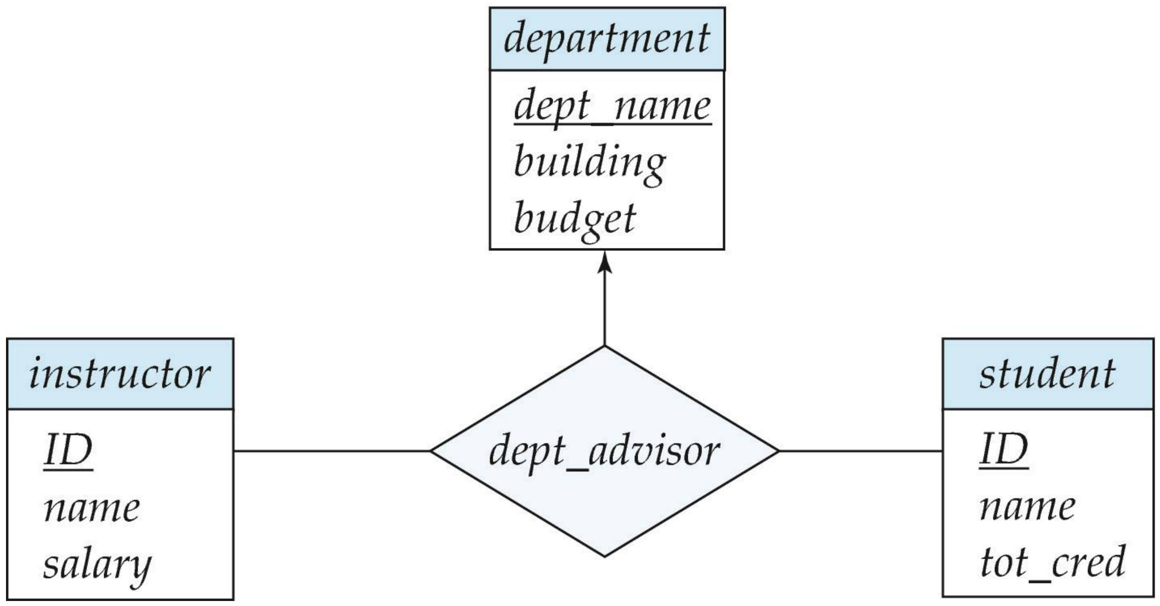

department
dept_name
building
budget
instructor
ID
name
salary
dept_advisor
student
ID
name
tot_cred

Figure 8.14

dept_name	*ID*	*street*	*city*
Physics	22222	North	Rye
Physics	22222	Main	Manchester
Finance	12121	Lake	Horseneck

Figure 8.15

dept_name	*ID*	*street*	*city*
Physics	22222	North	Rye
Math	22222	Main	Manchester

Figure 8.17

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_2	b_1	c_1
a_2	b_1	c_3