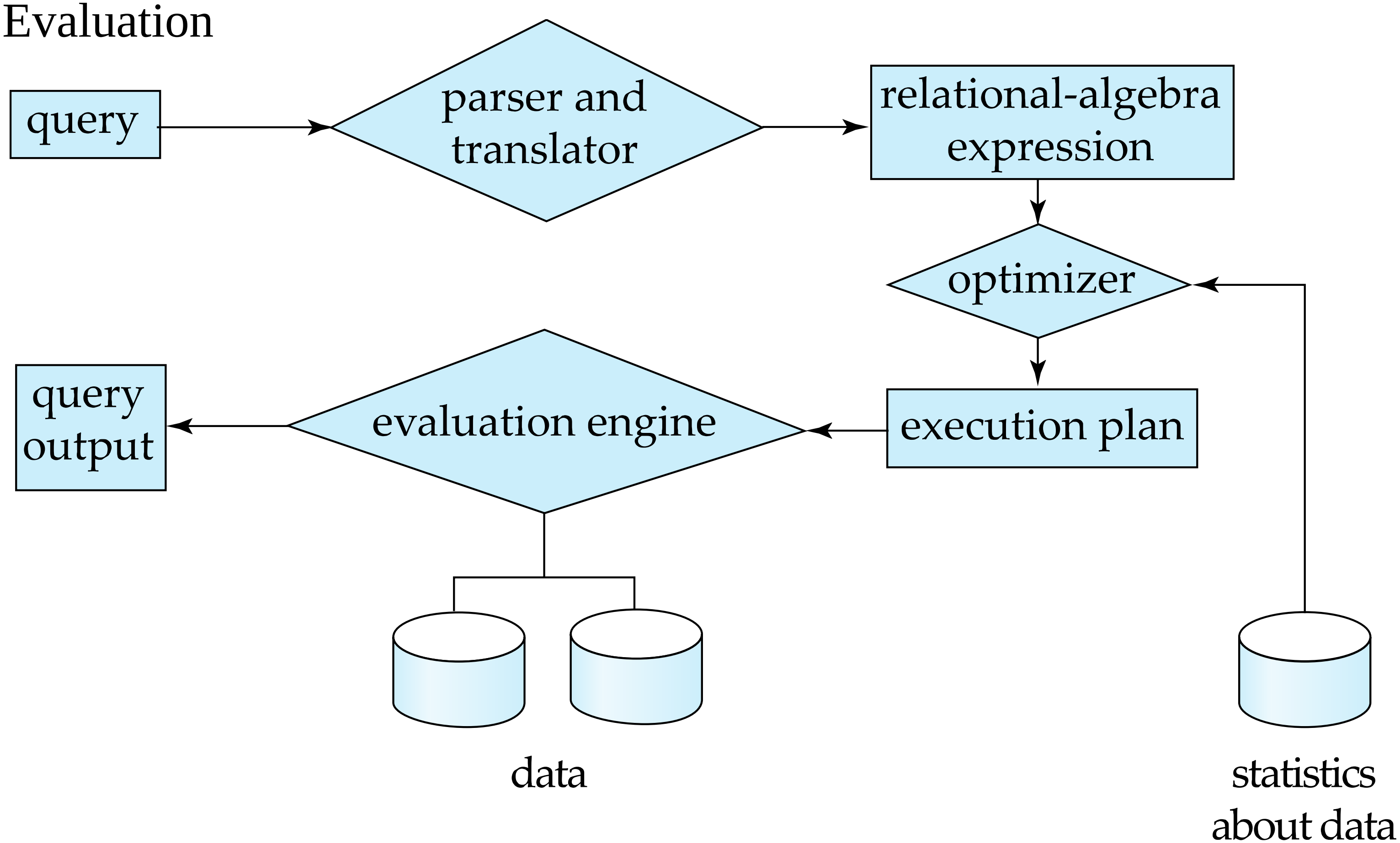
Query Processing & Optimization

Basic Steps in Query Processing

1. Parsing and translation
2. Optimization
3. Evaluation



Basic Steps in Query Processing (Cont.)

- Parsing and translation
  - translate the query into its internal form. This is then translated into relational algebra.
  - Parser checks syntax, verifies relations
- Evaluation
  - The query-execution engine takes a query-evaluation plan, executes that plan, and returns the answers to the query.

Basic Steps in Query Processing : Optimization

- A relational algebra expression may have many equivalent expressions
  - E.g., $\sigma_{salary<75000}(\Pi_{salary}(instructor))$ is equivalent to
    $\Pi_{salary}(\sigma_{salary<75000}(instructor))$
- Each relational algebra operation can be evaluated using one of several different algorithms
  - Correspondingly, a relational-algebra expression can be evaluated in many ways.
- Annotated expression specifying detailed evaluation strategy is called an **evaluation-plan**.
  - E.g., can use an index on *salary* to find instructors with salary < 75000,
  - or can perform complete relation scan and discard instructors with salary ≥ 75000

Basic Steps: Optimization (Cont.)

- **Query Optimization**: Amongst all equivalent evaluation plans choose the one with lowest cost.
  - Cost is estimated using statistical information from the database catalog
    - e.g. number of tuples in each relation, size of tuples, etc.

Measures of Query Cost

- Cost is generally measured as total elapsed time for answering query
  - Many factors contribute to time cost
    - *disk accesses*, *CPU*, or even network *communication*
- Typically disk access is the predominant cost, and is also relatively easy to estimate. Measured by taking into account
  - Number of seeks * average-seek-cost
  - Number of blocks read * average-block-read-cost
  - Number of blocks written * average-block-write-cost
    - Cost to write a block is greater than cost to read a block

      data is read back after being written to ensure that the write was successful

Measures of Query Cost (Cont.)

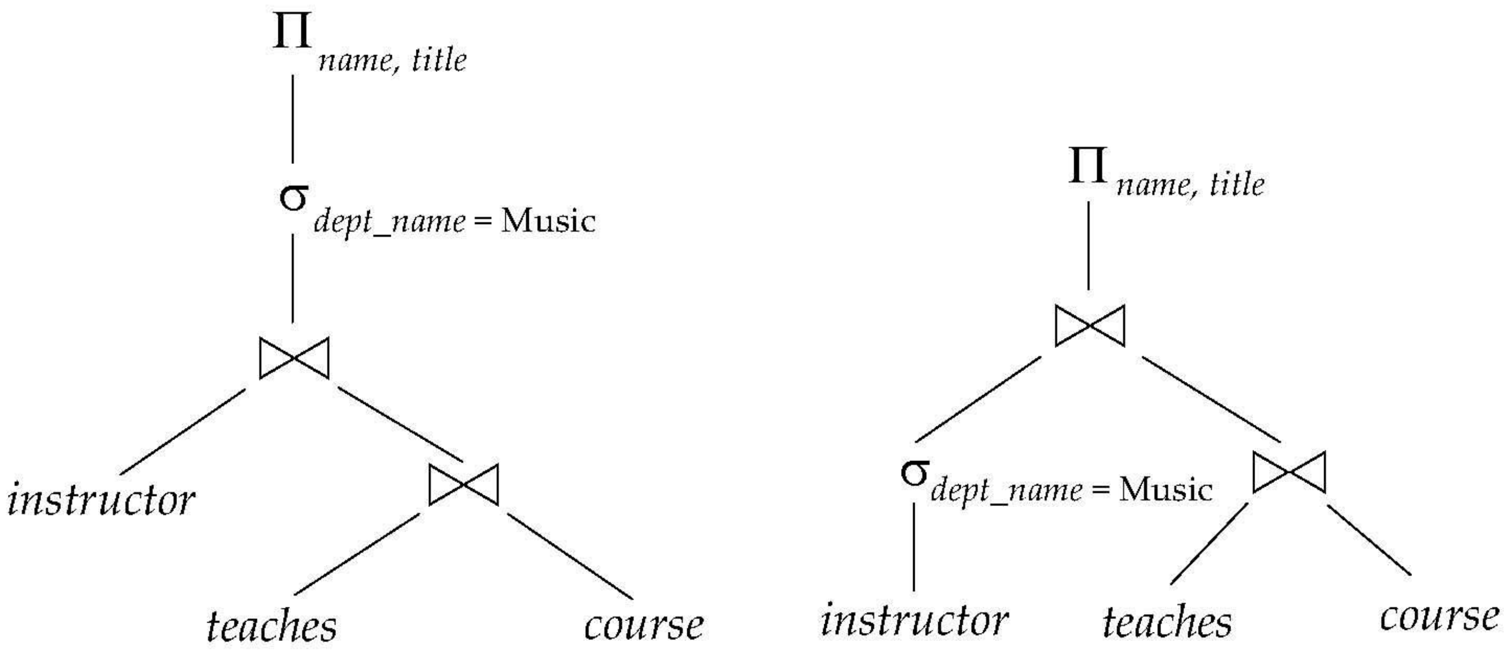
- For simplicity we just use the **number of block transfers** *from disk and the* **number of seeks** as the cost measures
  - t_T – time to transfer one block
  - t_S – time for one seek
  - Cost for b block transfers plus S seeks

$$b * t_T + S * t_S$$

- We ignore CPU costs for simplicity
  - Real systems do take CPU cost into account
- We do not include cost to writing output to disk in our cost formulae

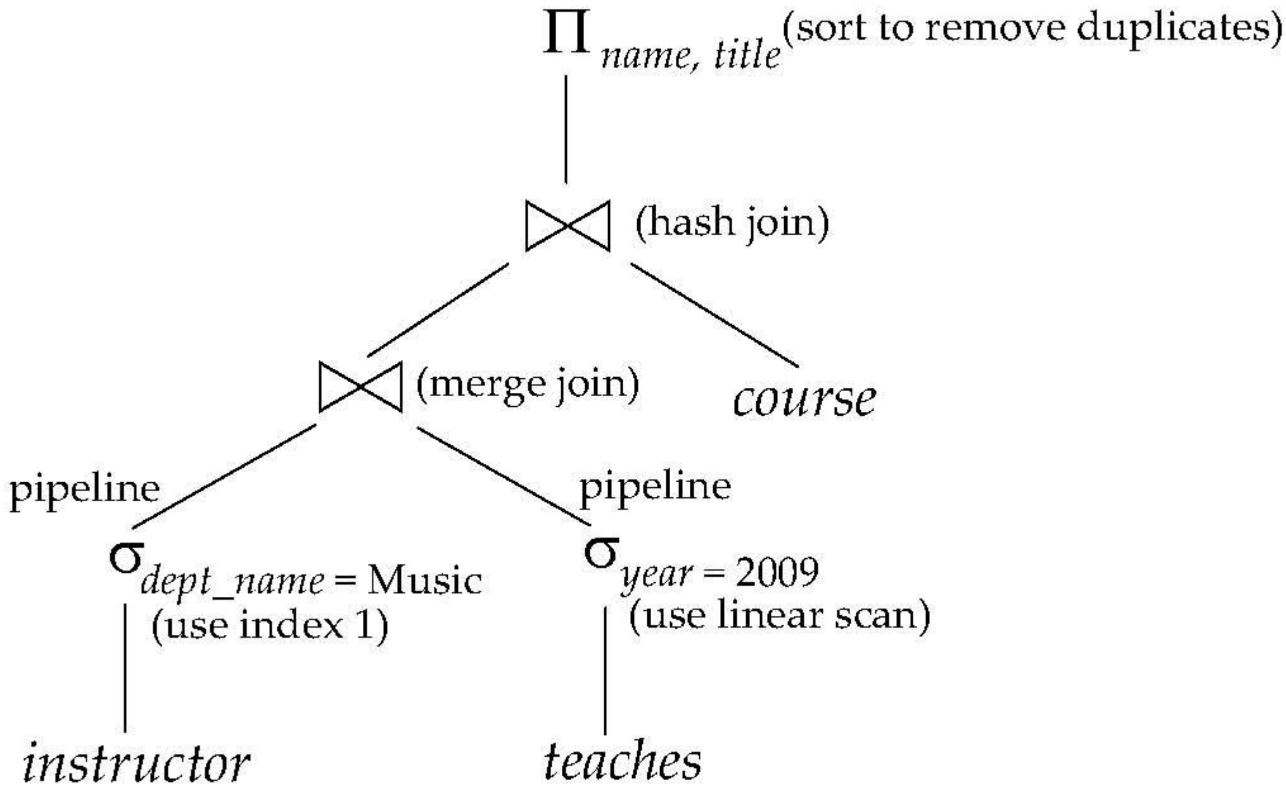
Optimization

- Alternative ways of evaluating a given query
  - Equivalent expressions
  - Different algorithms for each operation



Optimization (Cont.)

- An **evaluation plan** defines exactly what algorithm is used for each operation, and how the execution of the operations is coordinated.



Optimization (Cont.)

- Cost difference between evaluation plans for a query can be enormous
  - E.g. seconds vs. days in some cases
- Steps in **cost-based query optimization**
  1. Generate logically equivalent expressions using **equivalence rules**
  2. Annotate resultant expressions to get alternative query plans
  3. Choose the cheapest plan based on **estimated cost**
- Estimation of plan cost based on:
  - Statistical information about relations. Examples:
    - number of tuples, number of distinct values for an attribute
  - Statistics estimation for intermediate results
    - to compute cost of complex expressions
  - Cost formulae for algorithms, computed using statistics

Equivalence Rules

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections.

$$\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

2. Selection operations are commutative.

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

3. Only the last in a sequence of projection operations is needed, the others can be omitted.

$$\Pi_{L_1}(\Pi_{L_2}(\ldots(\Pi_{Ln}(E))\ldots)) = \Pi_{L_1}(E)$$

4. Selections can be combined with Cartesian products and theta joins.

   a. $\sigma_{\theta}(E_1 \text{ X } E_2) = E_1 \bowtie_{\theta} E_2$

   b. $\sigma_{\theta 1}(E_1 \bowtie_{\theta 2} E_2) = E_1 \bowtie_{\theta 1 \wedge \theta 2} E_2$

Equivalence Rules (Cont.)

5. Theta-join operations (and natural joins) are commutative.

$$E_1 \bowtie_\theta E_2 = E_2 \bowtie_\theta E_1$$

6. (a) Natural join operations are associative:

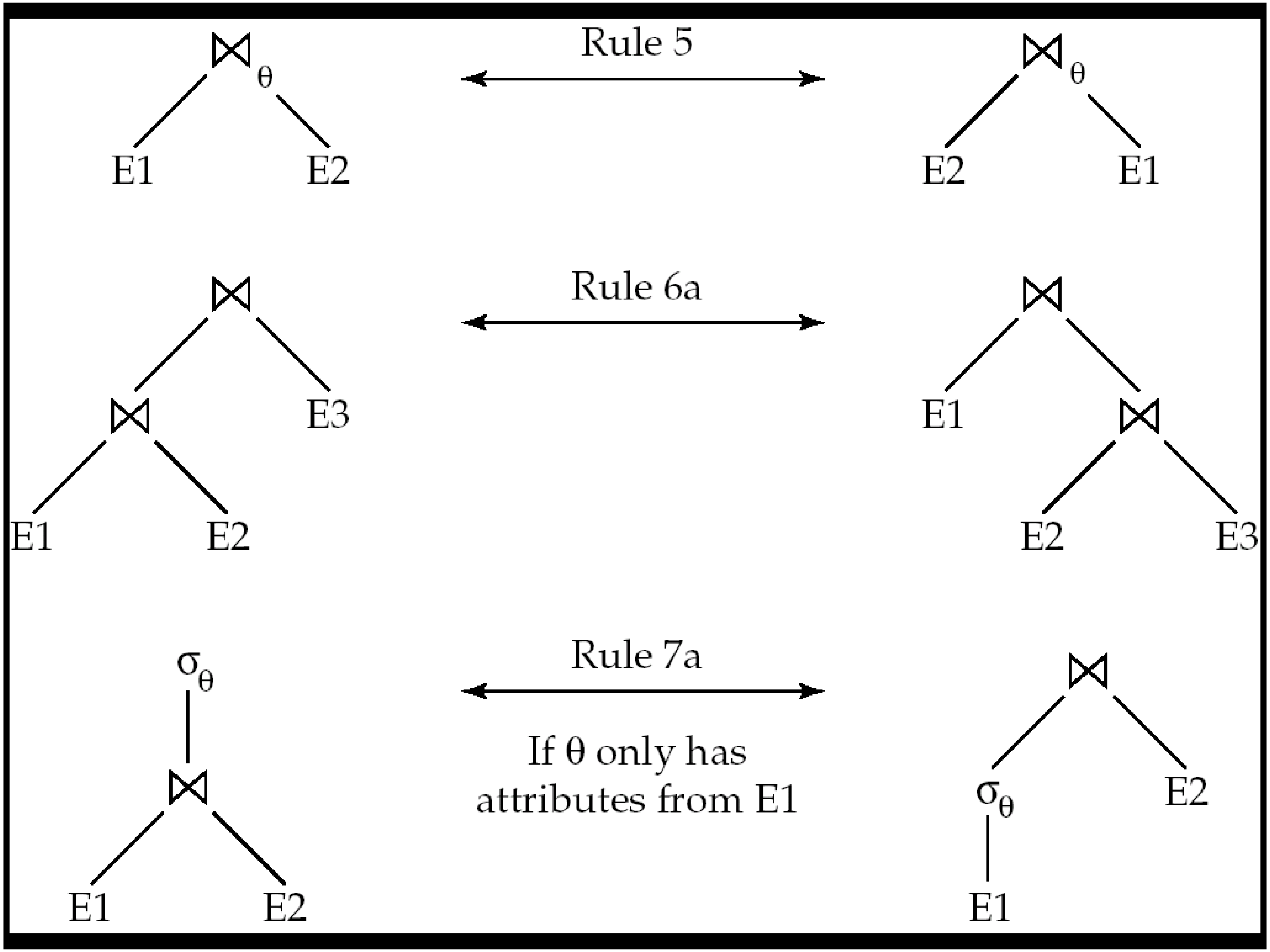
$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$$

(b) Theta joins are associative in the following manner:

$$(E_1 \bowtie_{\theta 1} E_2) \bowtie_{\theta 2 \wedge \theta 3} E_3 = E_1 \bowtie_{\theta 1 \wedge \theta 3} (E_2 \bowtie_{\theta 2} E_3)$$

where θ_2 involves attributes from only E_2 and E_3.

Pictorial Depiction of Equivalence Rules



Equivalence Rules (Cont.)

7. The selection operation distributes over the theta join operation under the following two conditions:
   (a) When all the attributes in θ_0 involve only the attributes of one of the expressions (E_1) being joined.

$$\sigma_{\theta 0}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta 0}(E_1)) \bowtie_{\theta} E_2$$

   (b) When θ_1 involves only the attributes of E_1 and θ_2 involves only the attributes of E_2.

$$\sigma_{\theta 1 \wedge \theta 2}(E_1 \bowtie_{\theta} E_2) = (\sigma_{\theta 1}(E_1)) \bowtie_{\theta} (\sigma_{\theta 2}(E_2))$$

Equivalence Rules (Cont.)

8. The projection operation distributes over the theta join operation as follows:

   (a) if θ involves only attributes from $L_1 \cup L_2$:

$$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = (\Pi_{L_1}(E_1)) \bowtie_\theta (\Pi_{L_2}(E_2))$$

   (b) Consider a join $E_1 \bowtie_\theta E_2$.

   - Let L_1 and L_2 be sets of attributes from E_1 and E_2, respectively.
   - Let L_3 be attributes of E_1 that are involved in join condition θ, but are not in $L_1 \cup L_2$, and
   - let L_4 be attributes of E_2 that are involved in join condition θ, but are not in $L_1 \cup L_2$.

$$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1 \cup L_2}((\Pi_{L_1 \cup L_3}(E_1)) \bowtie_\theta (\Pi_{L_2 \cup L_4}(E_2)))$$

Equivalence Rules (Cont.)

9. The set operations union and intersection are commutative

$$E_1 \cup E_2 = E_2 \cup E_1$$
$$E_1 \cap E_2 = E_2 \cap E_1$$

(set difference is not commutative).

10. Set union and intersection are associative.

$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3)$$
$$(E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

11. The selection operation distributes over $\cup$, $\cap$ and $-$.

$$\sigma_\theta (E_1 - E_2) = \sigma_\theta (E_1) - \sigma_\theta(E_2)$$

and similarly for $\cup$ and $\cap$ in place of $-$

Also: $\sigma_\theta (E_1 - E_2) = \sigma_\theta(E_1) - E_2$

and similarly for $\cap$ in place of $-$, but not for $\cup$

12. The projection operation distributes over union

$$\Pi_L(E_1 \cup E_2) = (\Pi_L(E_1)) \cup (\Pi_L(E_2))$$

Transformation Example: Pushing Selections

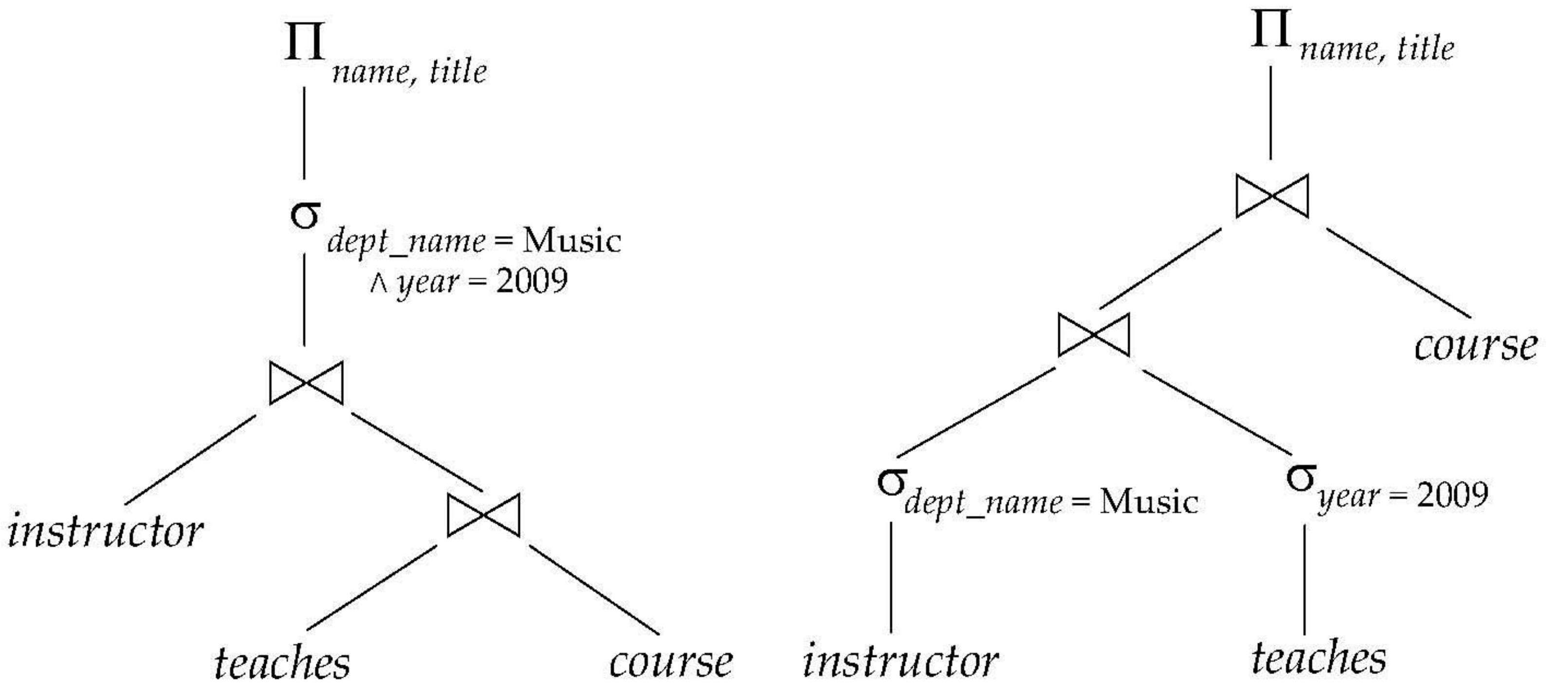
- Query: Find the names of all instructors in the Music department, along with the titles of the courses that they teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Music”}}(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course))))$
- Transformation using rule 7a.
  - $\Pi_{name,\ title}((\sigma_{dept\_name=\text{“Music”}}(instructor)) \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course)))$
- Performing the selection as early as possible reduces the size of the relation to be joined.

Example with Multiple Transformations

- Query: Find the names of all instructors in the Music department who have taught a course in 2009, along with the titles of the courses that they taught
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\ \text{"Music"} \land year\ =\ 2009}$
    $(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}\ (course))))$
- Transformation using join associatively (Rule 6a):
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\ \text{"Music"} \land gear\ =\ 2009}$
    $((instructor \bowtie teaches) \bowtie \Pi_{course\_id,\ title}\ (course)))$
- Second form provides an opportunity to apply the "perform selections e arly" rule, resulting in the subexpression

$$\sigma_{dept\_name\ =\ \text{"Music"}}\ (instructor) \bowtie \sigma_{\ year\ =\ 2009}\ (teaches)$$

Multiple Transformations (Cont.)



(a) Initial expression tree

(b) Tree after multiple transformations

Transformation Example: Pushing Projections

- Consider: $\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches)$
  $\bowtie \Pi_{course\_id,\ title}(course))))$
- When we compute

  $(\sigma_{dept\_name = \text{"Music"}}(instructor \bowtie teaches)$

  we obtain a relation whose schema is:
  (*ID*, *name*, *dept_name*, *salary*, *course_id*, *sec_id*, *semester*, *year*)
- Push projections using equivalence rules 8a and 8b; eliminate unneeded attributes from intermediate results to get:

  $$\Pi_{name,\ title}(\Pi_{name,\ course\_id}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches)) \bowtie \Pi_{course\_id,\ title}(course))))$$
- Performing the projection as early as possible reduces the size of the relation to be joined.

Join Ordering Example

- For all relations $r_1, r_2,$ and r_3,

$$(r_1 \bowtie r_2) \bowtie r_3 = r_1 \bowtie (r_2 \bowtie r_3)$$

  (Join Associativity)

- If $r_2 \bowtie r_3$ is quite large and $r_1 \bowtie r_2$ is small, we choose

$$(r_1 \bowtie r_2) \bowtie r_3$$

  so that we compute and store a smaller temporary relation.

Join Ordering Example (Cont.)

- Consider the expression

$$\Pi_{name,\ title}(\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches) \bowtie \Pi_{course\_id,\ title}(course))))$$

- Could compute $teaches \bowtie \Pi_{course\_id,\ title}(course)$ first, and join result with

$$\sigma_{dept\_name=\text{"Music"}}(instructor)$$

but the result of the first join is likely to be a large relation.

- Only a small fraction of the university's instructors are likely to be from the Music department
  - it is better to compute

$$\sigma_{dept\_name=\text{"Music"}}(instructor) \bowtie teaches$$

first.

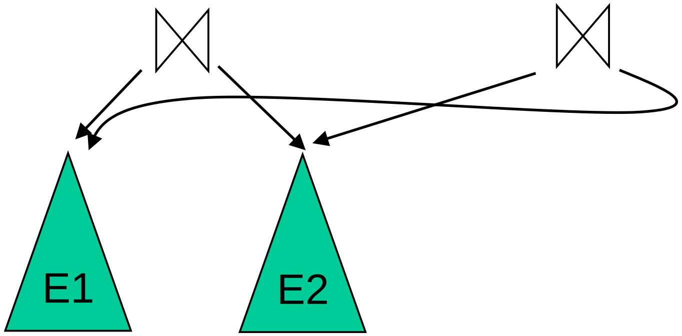
Enumeration of Equivalent Expressions

- Query optimizers use equivalence rules to **systematically** generate expressions equivalent to the given expression
- Can generate all equivalent expressions as follows:
  - Repeat
    - apply all applicable equivalence rules on every subexpression of every equivalent expression found so far
    - add newly generated expressions to the set of equivalent expressions

    Until no new equivalent expressions are generated above
- The above approach is very expensive in space and time
  - Two approaches
    - Optimized plan generation based on transformation rules
    - Special case approach for queries with only selections, projections and joins

Implementing Transformation Based Optimization

- Space requirements reduced by sharing common sub-expressions:
  - when E1 is generated from E2 by an equivalence rule, usually only the top level of the two are different, subtrees below are the same and can be shared using pointers
    - E.g. when applying join commutativity



  - Same sub-expression may get generated multiple times
    - Detect duplicate sub-expressions and share one copy
- Time requirements are reduced by not generating all expressions
  - Dynamic programming
    - We will study only the special case of dynamic programming for join order optimization

Cost Estimation

- Cost of each operator computed as described in Chapter 12
  - Need statistics of input relations
    - E.g. number of tuples, sizes of tuples
- Inputs can be results of sub-expressions
  - Need to estimate statistics of expression results
  - To do so, we require additional statistics
    - E.g. number of distinct values for an attribute
- More on cost estimation later

Choice of Evaluation Plans

- Must consider the interaction of evaluation techniques when choosing evaluation plans
  - choosing the cheapest algorithm for each operation independently may not yield best overall algorithm. E.g.
    - merge-join may be costlier than hash-join, but may provide a sorted output which reduces the cost for an outer level aggregation.
    - nested-loop join may provide opportunity for pipelining
- Practical query optimizers incorporate elements of the following two broad approaches:
  1. Search all the plans and choose the best plan in a cost-based fashion.
  2. Uses heuristics to choose a plan.

Cost-Based Optimization

- Consider finding the best join-order for $r_1 \bowtie r_2 \bowtie \ldots r_n$.
- There are $(2(n-1))!/(n-1)!$ different join orders for above expression. With $n = 7$, the number is 665280, with $n = 10$, the number is greater than 176 billion!
- No need to generate all the join orders. Using dynamic programming, the least-cost join order for any subset of
  $\{r_1, r_2, \ldots r_n\}$ is computed only once and stored for future use.

Dynamic Programming in Optimization

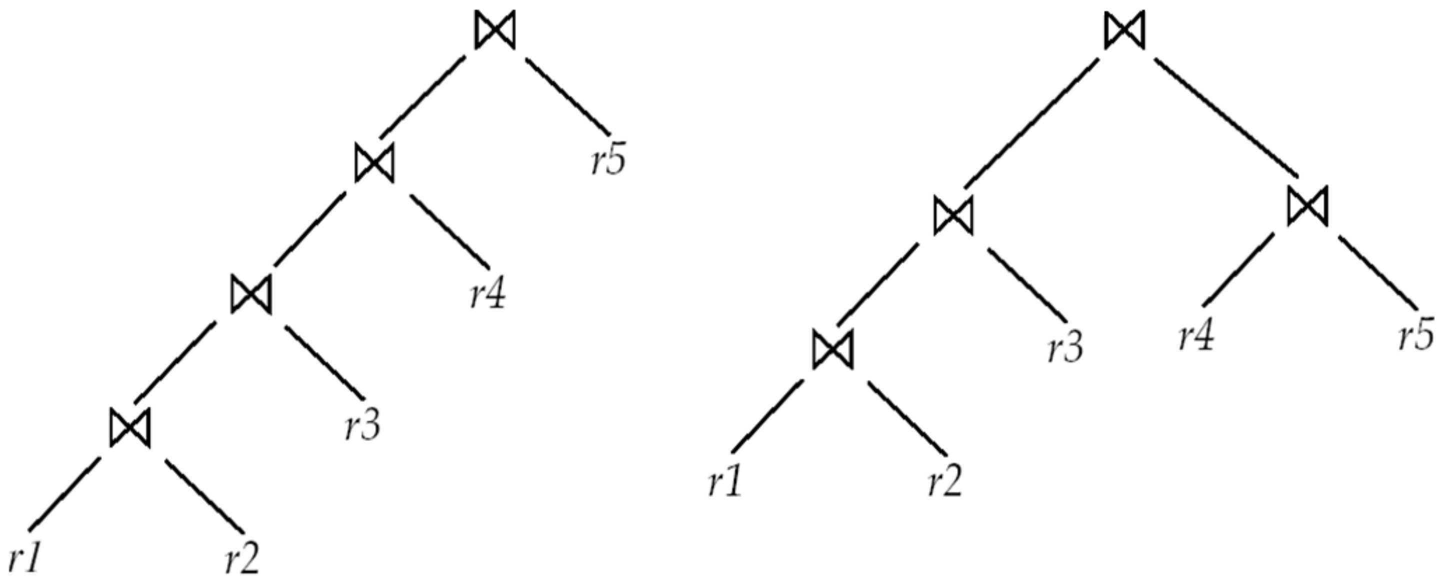
- To find best join tree for a set of n relations:
  - To find best plan for a set S of n relations, consider all possible plans of the form: S_1 $(S - S_1)$ where S_1 is any non-empty subset of S.
  - Recursively compute costs for joining subsets of S to find the cost of each plan. Choose the cheapest of the $2^n - 2$ alternatives.
  - Base case for recursion: single relation access plan
    - Apply all selections on R_i using best choice of indices on R_i
  - When plan for any subset is computed, store it and reuse it when it is required again, instead of recomputing it
    - Dynamic programming

Join Order Optimization Algorithm

```
procedure findbestplan(S)
    if (bestplan[S].cost ≠ ∞)
        return bestplan[S]
    // else bestplan[S] has not been computed earlier, compute it now
    if (S contains only 1 relation)
        set bestplan[S].plan and bestplan[S].cost based on the best way
        of accessing S  /* Using selections on S and indices on S */
    else for each non-empty subset S1 of S such that S1 ≠ S
        P1= findbestplan(S1)
        P2= findbestplan(S - S1)
        A = best algorithm for joining results of P1 and P2
        cost = P1.cost + P2.cost + cost of A
        if cost < bestplan[S].cost
            bestplan[S].cost = cost
            bestplan[S].plan = "execute P1.plan; execute P2.plan;
                                join results of P1 and P2 using A"
    return bestplan[S]
```

Left Deep Join Trees

- In **left-deep join trees,** the right-hand-side input for each join is a relation, not the result of an intermediate join.



(a) Left-deep join tree

(b) Non-left-deep join tree

Cost of Optimization

- With dynamic programming time complexity of optimization with bushy trees is $O(3^n)$.
  - With n = 10, this number is 59000 instead of 176 billion!
- Space complexity is $O(2^n)$
- To find best left-deep join tree for a set of n relations:
  - Consider n alternatives with one relation as right-hand side input and the other relations as left-hand side input.
  - Modify optimization algorithm:
    - Replace "**for each** non-empty subset $S1$ of S such that $S1 \neq S$"
    - By: **for each** relation r in S
let S1 = S – r .
- If only left-deep trees are considered, time complexity of finding best join order is $O(n\ 2^n)$
  - Space complexity remains at $O(2^n)$
- Cost-based optimization is expensive, but worthwhile for queries on large datasets (typical queries have small n, generally < 10)

Interesting Sort Orders

- Consider the expression $(r_1 \bowtie r_2) \bowtie r_3 \bowtie$ (with A as common attribute)
- An **interesting sort order** is a particular sort order of tuples that could be useful for a later operation
  - Using merge-join to compute $r_1 \bowtie r_2$ may be costlier than hash join but generates result sorted on A
  - Which in turn may make merge-join with r_3 cheaper, which may reduce cost of join with r_3 and minimizing overall cost
  - Sort order may also be useful for order by and for grouping
- Not sufficient to find the best join order for each subset of the set of n given relations
  - must find the best join order for each subset, **for each interesting sort order**
  - Simple extension of earlier dynamic programming algorithms
  - Usually, number of interesting orders is quite small and doesn't affect time/space co mplexity significantly

Cost Based Optimization with Equivalence Rules

- **Physical equivalence rules** allow logical query plan to be converted to physical query plan specifying what algorithms are used for each operation.
- Efficient optimizer based on equivalent rules depends on
  - A space efficient representation of expressions which avoids making multiple copies of subexpressions
  - Efficient techniques for detecting duplicate derivations of expressions
  - A form of dynamic programming based on **memoization**, which stores the best plan for a subexpression the first time it is optimized, and reuses in on repeated optimization calls on same subexpression
  - Cost-based pruning techniques that avoid generating all plans
- Pioneered by the Volcano project and implemented in the SQL Server optimizer

Heuristic Optimization

- Cost-based optimization is expensive, even with dynamic programming.
- Systems may use *heuristics* to reduce the number of choices that must be made in a cost-based fashion.
- Heuristic optimization transforms the query-tree by using a set of rules that typically (but not in all cases) improve execution performance:
  - Perform selection early (reduces the number of tuples)
  - Perform projection early (reduces the number of attributes)
  - Perform most restrictive selection and join operations (i.e. with smallest result size) before other similar operations.
  - Some systems use only heuristics, others combine heuristics with partial cost-based optimization.

Structure of Query Optimizers

- Many optimizers considers only left-deep join orders.
  - Plus heuristics to push selections and projections down the query tree
  - Reduces optimization complexity and generates plans amenable to pipelined evaluation.
- Heuristic optimization used in some versions of Oracle:
  - Repeatedly pick "best" relation to join next
    - Starting from each of n starting points. Pick best among these
- Intricacies of SQL complicate query optimization
  - E.g. nested subqueries

Structure of Query Optimizers (Cont.)

- Some query optimizers integrate heuristic selection and the generation of alternative access plans.
  - Frequently used approach
    - heuristic rewriting of nested block structure and aggregation
    - followed by cost-based join-order optimization for each block
  - Some optimizers (e.g. SQL Server) apply transformations to entire query and do not depend on block structure
  - **Optimization cost budget** to stop optimization early (if cost of plan is less than cost of optimization)
  - **Plan caching** to reuse previously computed plan if query is resubmitted
    - Even with different constants in query
- Even with the use of heuristics, cost-based query optimization imposes a substantial overhead.
  - But is worth it for expensive queries
  - Optimizers often use simple heuristics for very cheap queries, and perform exhaustive enumeration for more expensive queries

Statistics for Cost Estimation

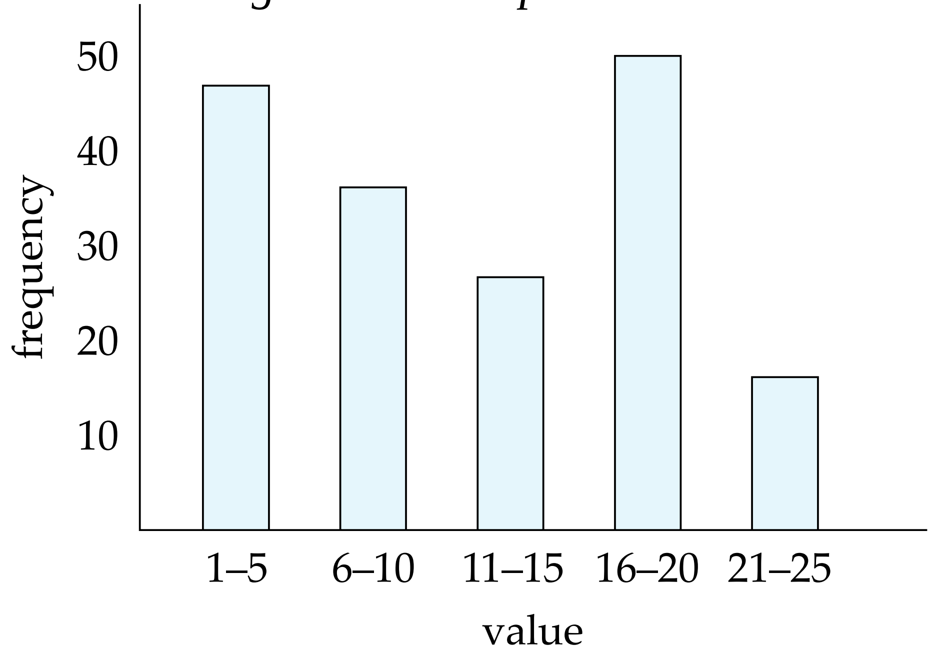
Statistical Information for Cost Estimation

- n_r: number of tuples in a relation r.
- b_r: number of blocks containing tuples of r.
- l_r: size of a tuple of r.
- f_r: blocking factor of r — i.e., the number of tuples of r that fit into one block.
- $V(A, r)$: number of distinct values that appear in r for attribute A; same as the size of $\prod_A(r)$.
- If tuples of r are stored together physically in a file, then:

$$b_r = \left\lceil \frac{n_r}{f_r} \right\rceil$$

Histograms

- Histogram on attribute *age* of relation *person*



- **Equi-width** histograms
- **Equi-depth** histograms

Selection Size Estimation

- $\sigma_{A=v}(r)$
  - $n_r / V(A,r)$: number of records that will satisfy the selection
  - Equality condition on a key attribute: *size estimate* = 1
- $\sigma_{A \leq V}(r)$ (case of $\sigma_{A \geq V}(r)$ is symmetric)
  - Let c denote the estimated number of tuples satisfying the condition.
  - If min(A,r) and max(A,r) are available in catalog
    - c = 0 if v < min(A,r)
    - c = $n_r \cdot \frac{v - \min(A,r)}{\max(A,r) - \min(A,r)}$
  - If histograms available, can refine above estimate
  - In absence of statistical information *c* is assumed to be $n_r / 2$.

Size Estimation of Complex Selections

- The **selectivity** of a condition θ_i is the probability that a tuple in the relation r satisfies θ_i .
  - If s_i is the number of satisfying tuples in r, the selectivity of θ_i is given by s_i / n_r.
- **Conjunction:** $\sigma_{\theta 1 \wedge \theta 2 \wedge \ldots \wedge \theta n}(r)$. *Assuming indepdence*, estimate of tuples in the result is:

$$n_r * \frac{s_1 * s_2 * \ldots * s_n}{n_r^n}$$

- **Disjunction:** $\sigma_{\theta 1 \vee \theta 2 \vee \ldots \vee \theta n}(r)$. Estimated number of tuples:

$$n_r * \left(1 - (1 - \frac{s_1}{n_r}) * (1 - \frac{s_2}{n_r}) * \ldots * (1 - \frac{s_n}{n_r})\right)$$

- **Negation:** $\sigma_{\neg\theta}(r)$. Estimated number of tuples:

  $n_r - size(\sigma_\theta(r))$

Join Operation: Running Example

Running example:

$student \bowtie takes$

Catalog information for join examples:

- $n_{student}$ = 5,000.
- $f_{student}$ = 50, which implies that
  $b_{student}$ =5000/50 = 100.
- n_{takes} = 10000.
- f_{takes} = 25, which implies that
  b_{takes} = 10000/25 = 400.
- *V(ID, takes)* = 2500, which implies that on average, each student who has taken a course has taken 4 courses.
  - Attribute *ID* in *takes* is a foreign key referencing *student.*
  - *V*(*ID, student*) = 5000 (*primary key!*)

Estimation of the Size of Joins

- The Cartesian product $r \times s$ contains $n_r . n_s$ tuples; each tuple occupies $s_r + s_s$ bytes.
- If $R \cap S = \varnothing$, then $r \bowtie s$ is the same as $r \times s$.
- If $R \cap S$ is a key for R, then a tuple of s will join with at most one tuple from r
  - therefore, the number of tuples in $r \bowtie s$ is no greater than the number of tuples in s.
- If $R \cap S$ *in* S is a foreign key in S referencing R, then the number of tuples in $r \bowtie s$ is exactly the same as the number of tuples in s.
  - The case for $R \cap S$ being a foreign key referencing S is symmetric.
- In the example query *student* $\bowtie$ *takes, ID* in *takes* is a foreign key referencing *student*
  - hence, the result has exactly n_{takes} tuples, which is 10000

Estimation of the Size of Joins (Cont.)

- If $R \cap S = \{A\}$ is not a key for R or S.
  If we assume that every tuple t in R produces tuples in $R \bowtie S$, the number of tuples in $R \bowtie S$ is estimated to be:

$$\frac{n_r * n_s}{V(A,s)}$$

  If the reverse is true, the estimate obtained will be:

$$\frac{n_r * n_s}{V(A,r)}$$

  The lower of these two estimates is probably the more accurate one.
- Can improve on above if histograms are available
  - Use formula similar to above, for each cell of histograms on the two relations

Estimation of the Size of Joins (Cont.)

- Compute the size estimates for *depositor* $\bowtie$ *customer* without using information about foreign keys:
  - *V(ID, takes)* = 2500, and
    V(ID, student) = 5000
  - The two estimates are 5000 * 10000/2500 = 20,000 and 5000 * 10000/5000 = 10000
  - We choose the lower estimate, which in this case, is the same as our earlier computation using foreign keys.

Size Estimation for Other Operations

- Projection: estimated size of $\prod_A(r) = V(A,r)$
- Aggregation : estimated size of $_A\mathcal{g}_F(r) = V(A,r)$
- Set operations
  - For unions/intersections of selections on the same relation: rewrite and use size estimate for selections
    - E.g. $\sigma_{\theta1}(r) \cup \sigma_{\theta2}(r)$ can be rewritten as $\sigma_{\theta1 \vee \theta2}(r)$
  - For operations on different relations:
    - estimated size of $r \cup s$ = size of r + size of s.
    - estimated size of $r \cap s$ = minimum size of r and size of s.
    - estimated size of $r - s = r$.
    - <u>All the three estimates may be quite inaccurate, but provide upper bounds on the sizes</u>.

Size Estimation (Cont.)

- Outer join:
  - Estimated size of $r ⟕ s$ = *size of* $r \bowtie s$ + *size of* r
    - Case of right outer join is symmetric
  - Estimated size of $r ⟗ s$ = *size of* $r \bowtie s$ + size of r + size of s

Estimation of Number of Distinct Values

Selections: $\sigma_\theta(r)$

- If θ forces A to take a specified value: $V(A,\sigma_\theta(r)) = 1$.
  - e.g., $A = 3$
- If θ forces A to take on one of a specified set of values:
  $V(A,\sigma_\theta(r))$ = number of specified values.
  - (e.g., ($A = 1 \vee A = 3 \vee A = 4$)),
- If the selection condition θ is of the form A *op* r
  estimated $V(A,\sigma_\theta(r)) = V(A.r) * s$
  - where s is the selectivity of the selection.
- In all the other cases: use approximate estimate of
  $\min(V(A,r), n_{\sigma\theta(r)})$
  - More accurate estimate can be got using probability theory, but this one works fine generally

Estimation of Distinct Values (Cont.)

Joins: $r \bowtie s$

- If all attributes in A are from r
  estimated $V(A, r \bowtie s) = \min(V(A,r), n_{r \bowtie s})$
- If A contains attributes $A1$ from r and $A2$ from s, then estimated $V(A,r \bowtie s) =$

  $$\min(V(A1,r)*V(A2 - A1,s), V(A1 - A2,r)*V(A2,s), n_{r \bowtie s})$$

  - More accurate estimate can be got using probability theory, but this one works fine generally

Estimation of Distinct Values (Cont.)

- Estimation of distinct values are straightforward for projections.
  - They are the same in $\prod_{A}(r)$ as in *r*.
- The same holds for grouping attributes of aggregation.
- For aggregated values
  - For min(*A*) and max(*A*), the number of distinct values can be estimated as min(V(*A*,*r*), *V*(*G*,*r*)) where G denotes grouping attributes
  - For other aggregates, assume all values are distinct, and use *V*(*G*,*r*)